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MAGNETICALLY INDUCED FLOW AND SURFACE STRUCTURES

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The flow fields as well as the surface structures of magnetic fluids can strongly be influenced via magnetic forces. Starting with the Hagen–Poiseuille flow in an alternating field, continuing with drop formation and the Faraday instability finally the normal field instability of a resting fluid is addressed. In this way four different levels of interaction of magnetically induced flow and surface structures are presented.

Introduction. Magnetic fluids offer the unique advantage to influence or even control their behavior strongly via an external magnetic field. In this way they have found widespread technical applications, ranging from rotary feed throughs to loud speakers (see Ref. [1] e.g.). Apart from their technical importance, structure formation in magnetic fluids is entering more and more the focus of basic research [2]. Depending on the boundary conditions magnetic forces may alter either flow or surface structures of the fluid or both together. In the following we present four representative examples, illuminating different levels of interaction between magnetically induced flow and surface structures.

We start with the Hagen–Poiseuille flow in an alternating field. Here the surface is completely determined by the surrounding pipe and the magnetic field may solely change the flow pattern. In section 2 measurements on the rupture of a liquid bridge of magnetic fluid are reported. Now flow- and surface structures are of equal importance. This is true as well for the twin peak pattern observed at the Faraday instability of magnetic fluids (section 3). Eventually we look at the normal field instability of a resting fluid. Here no flow takes place and the magnetic field solely determines the static surface structure.

1. Hagen–Poiseuille flow. In 1969 McTague investigated the viscosity of a Hagen–Poiseuille flow under influence of a *static* magnetic field [3]. The external field hinders the free rotation of the magnetic nano particles and thus increases the viscosity of the flow. A theoretical treatment for the magnetically induced relative viscosity change $\eta_{\mathrm{r}} = \Delta\eta/\eta$ was given by Shliomis [4, 5]. Later, in 1994, Shliomis and Morozov [6] investigated the additional viscosity generated in a flow with vorticity due to an *alternating*, linearly polarized field. They postulated a negative viscosity contribution ($\Delta\eta < 0$) for a certain range of the field strength and frequency of the applied magnetic field. This 'negative viscosity effect' can be understood as a transfer of energy from the magnetic field into rotational motion of the particles.

A first observation of the effect by means of a concentrated suspension of Co-ferrite particles was published together with model equations, to be solved numerically [7]. For a quantitative comparison of experiment and model equations independent measurements have been conducted with a dilute suspension of magnetite [8]. Fig. 1 presents the measured values of the reduced viscosity versus

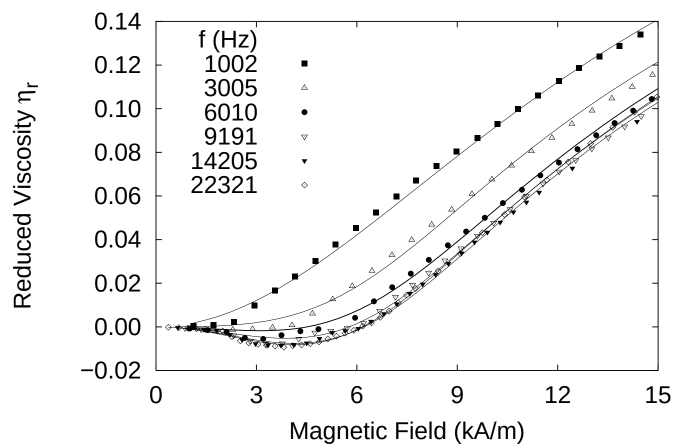

Fig. 1. Reduced viscosity as a function of the magnetic field for six different driving frequencies; from [8]. The symbols denote the experimental data, the solid lines the fit by a model first presented in [7].

the magnetic field strength together with a fit by the data obtained via integration of the model equations. A quantitative agreement of both data sets could only be achieved, when taking into account a *frequency dependence* of the fitting parameters, namely the Brownian relaxation time, the effective volume fraction, and the magnetic dipole moment. This difficulty was not expected and is still unexplained. Especially for the fitted Brownian relaxation time we have found a phenomenological scaling which is cum grano salis inversely proportional to the driving frequency [8].

For weak excitation amplitudes Shliomis and Morozov presented an analytical description of η_{r} [6]. We have tested this prediction experimentally. A reasonable fit between experiment and the presented function was only possible when again taking into account the above mentioned phenomenological scaling of the Brownian relaxation time with the driving frequency [9]. A thorough understanding of this scaling is still lacking. A possible ansatz would stress the reversible formation of aggregates of magnetic particles under the influence of the magnetic field. Due to their relatively large size these aggregates could dominate the relaxation time of the fluid. Assuming the time for the formation of aggregates to be proportional to the driving period, the observed frequency scaling is reproduced.

2. Rupture of a magnetic liquid bridge Without the lateral support by a pipe wall the surface of a liquid column has additional degrees of freedom. The early stage of the developing instability is described by classical linear stability analysis first conducted by Rayleigh [10]. In the last stage of the surface tension driven instability drop formation occurs. The surface and flow structures immediately before drop pinch–off are described by universal scaling functions [11]. We have investigated whether this scaling laws for standard Newtonian liquids survive for the case of magnetic liquids subject to an axial magnetic field [12]. A magnetic liquid bridge is suspended in between the pole shoes of two electro magnets. Upon increase of the static magnetic field the bridge disintegrates. Fig. 2 displays a sequence of frames before the rupture of the bridge. During the last 3 ms before the rupture the measured neck diameter is found to follow the equation

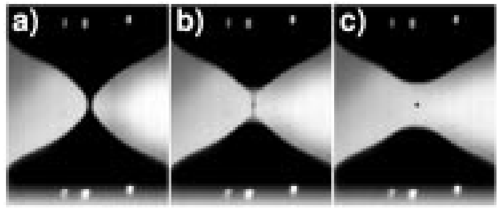

Fig. 2. Decay of a liquid bridge of magnetic fluid (APG J12 from Ferrofluidics) recorded by means of a high speed CCD–camera. The frames are taken at $t = 0$ ms (a), 2 ms (b), and 3 ms (c). From [12].

$d_{min} = -0.136\, V_\nu\, (t_0 - t)$. Here t_0 denotes the time of pinch–off, $V_\nu = \sigma/(\nu\rho)$ the intrinsic velocity, σ the surface tension, ν the kinematic viscosity, and ρ the density of the fluid. The factor 0.136 is close to the value 0.142 predicted by Papageorgiou for the case of a viscosity dominated flow [13] in agreement with the relatively large viscosity of the investigated magnetic fluid. It seems, that immediately before drop pinch–off the magnetic forces are of rather low importance.

3. Twin peaks at the Faraday instability. The Faraday instability belongs to the most popular experimental configurations for the investigation of parametrically excited instabilities, structure formation and spatio-temporal chaos [14]. Operating the experiment with magnetic fluid instead of the commonly used water or silicon oil is adding several interesting flavors. Firstly, instead of shaking the container, the instability can also be driven by periodic modulation of the applied magnetic field (see Ref. [15, 16, 17, 18], e.g.). Secondly, different orientations of the magnetic field with respect to the surface layer permit the realization of various symmetries [19]. Finally, the dispersion relation of magnetic fluids can be tuned by the external magnetic field. Especially, the advent of the normal field instability is accompanied by a non–monotonous dispersion relation [20, 15]

$$\omega_{\mathrm{D}}^2 = gk - \mu\frac{(\mu_{\mathrm{r}}-1)^2}{\mu_{\mathrm{r}}+1}\frac{1}{\rho}H^2k^2 + \frac{\sigma}{\rho}k^3. \tag{1}$$

Here ω_{D} denotes the driving frequency, k the wave number, H the strength of the external magnetic field, μ_{r} the relative magnetic permeability, $\mu = \mu_{\mathrm{r}}\mu_0$ the magnetic permeability, and μ_0 the magnetic field constant.

Experimentally, the non-monotonous dispersion relation was investigated by means of locally excited travelling waves in an annular channel [21], and in a circular container [22]. Due to the non-monotonousness up to three different wavenumbers can be excited with one single driving frequency. Which of the wavenumbers can actually be realized depends on the viscous dissipation in the bulk and in the bottom layer of the fluid [23]. For surface waves excited in a spatially homogeneous manner, the competition of the different wavenumbers was predicted to result in the spontaneous formation of domain structures [24]. This symmetry–breaking process could be experimentally demonstrated in an annular channel excited by vertical vibration [25]. In the annulus a domain of standing subharmonic waves with the wavenumber $k_1 = 34$ and another domain with $k_2 = 46$ evolved. In addition to the predicted domain formation in space, for different parameters a domain formation in time could also be detected [25]: A standing wave pattern of wavenumber k_1 collapses spontaneously in the whole annulus, and gives way to a pattern with wavenumber k_2. The latter however is not stable and forms a slowly

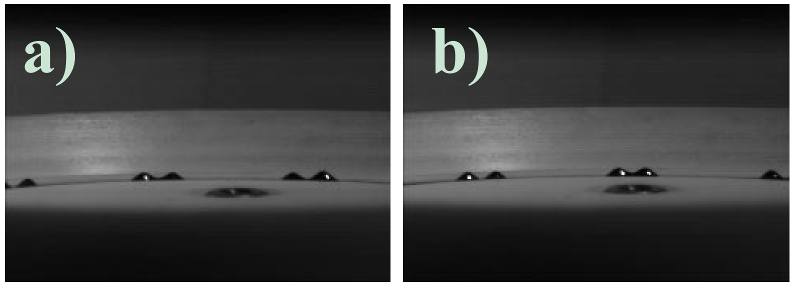

Fig. 3. Twin peak pattern in the non–monotonous regime of the dispersion relation. The time elapsed between picture a) and b) is one driving period.

shrinking domain, which finally vanishes in favour of k_1. This cycle is repeated in an irregular manner.

Recently, the competition between two different wavenumbers was found to be solved in a third way [26]. In the non–monotonous regime, for a magnetic field of $H = 0.98H_c$ and a driving frequency $\omega_{\mathrm{D}} = 9.615\,\mathrm{Hz}$ a novel pattern of twin peaks has been detected. Fig. 3 displays two snapshots taken one driving period apart. One clearly unveils a subharmonic standing wave. Apparently, instead of two separated domains, a bi–periodic structure in space has been established. Both dominant wavenumbers of the twin–peak pattern are found to be situated on the non–monotonous dispersion curve [26].

4. Measuring the Rosensweig instability Even without parametric excitation a flat layer of magnetic fluid, subject to a vertical magnetic field, is a fascinating object of structure formation. According to Eq. (1) the non–monotonous dispersion curve touches the $\omega_{\mathrm{D}}^2 = 0$ – line for a critical magnetic field H_c at the critical wavenumber $k_c = \sqrt{\rho g/\sigma}$. For $H > H_c$ the growth rate of a band of wavenumbers around k_c is getting positive and the liquid forms a stationary pattern of peaks which is called the normal field or Rosensweig instability.

The wavenumber of the emerging pattern, accessible with conventional CCD-cameras, has been investigated in several experiments. Especially the dependence of the wavenumber on the magnetic field and on the viscosity of the fluid was subject of recent theoretical and experimental efforts [27]–[31]. For further information please see the contribution by A. Lange *et al.* in this volume.

In contrast, only a few experiments are investigating the order parameter of the transition, i. e. the height of the liquid crests. This is mainly due to the difficulties in measuring a three dimensional profile of the instability. The fully developed crests are much to steep to be measured with the standard optical shadowgraphy, utilizing the slightly deformed surface as a focusing or defocusing mirror for a parallel beam of light [32, 22]. Another recently proposed method, which analyzes the reflections of a narrow laser beam in a Faraday experiment [33], yields only the local surface slope but not the local surface height.

The straight–forward technique, namely the lateral observation of the instability, is only possible for zero- or one dimensional systems, i.e. a single Rosensweig peak [34] or a chain of peaks [35, 25]. This requests a proper matching of the dimensions of the container to the wave length of the instability. In addition, a careful design of the container edges is necessary in order to minimize the influence of the meniscus on the profile of the instability. Despite both efforts the magnetic field gradient induced by the edges will still distort the true profile. Thus, for mea-

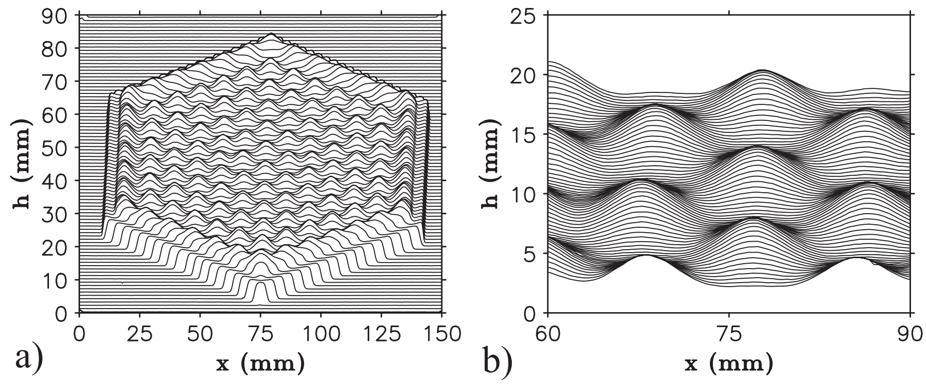

Fig. 4. Profile of the Rosensweig instability at a magnetic induction of $B = 22.95\,\mathrm{mT}$ (a) in a hexagonal shaped Teflon container of depth of 4 mm. Figure (b) displays a zoom of the center of the structure.

suring quantitatively the profile of the normal field instability, an extended layer of ferrofluid is most appropriate. There the region of interest can be selected far away from the container edges. However, for this extended layers a new detection technique has to be applied.

We measure the attenuation of x-rays passing the magnetic fluid layer in vertical direction. The transmitting radiation is recorded by means of an x–ray sensitive photo diode array based on amorphous silicon technology. For details of the experimental setup see [36]. The intensity of the transmitted radiation of a narrow, well collimated beam of monochromatic x-rays is found to decrease exponentially with the layer thickness $h(x, y)$

$$I(x, y) = I_0 \, \exp(-\mu \, h(x, y)), \tag{2}$$

where μ denotes the linear attenuation coefficient. For measured values of $I(x, y), I_0$ and μ, the local height $h(x, y)$ of the layer can then be calculated simply via the inverse of Eq. (2). For a good resolution one has to match the half-value thickness $h_{1/2} = 0.69/\mu$ to the height of the liquid structures by tuning the wavelength of the radiation. In preliminary studies we obtained a height resolution better than $50\,\mu$m. Fig. 4 provides first reliefs recorded by this radioscopic method.

To conclude, we have presented recent experimental efforts for a quantitative characterization and understanding of flow and surface structures of ferrofluids under influence of the magnetic field.

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