

# EPSILON-CONSTRAINT METHOD FOR SOLUTION TO THE COMBINED ECONOMIC AND EMISSION DISPATCH PROBLEM

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**Abstract:** System operators are concerned with the economics of power system operation as well as the amount of harmful gases released during power generation. The combined economic and emission dispatch (CEED) determines the power output of each online thermal unit for which the operational cost and emissions are at their minimum. It is a multi-objective optimization problem (MOOP) with equality and inequality constraints. In this paper, the epsilon-constraint method is used to tackle the CEED problem. The General Algebraic Modelling Systems (GAMS), known for its speed and ability to handle large and complex power system optimization problems is used to solve the problem formulation. A six-generator test system is taken as the case study in this paper, to verify the CEED mathematical formulation. In comparison with results in literature, the proposed solution method yields lower operational costs.

**Key words:** GAMS, economic dispatch, emission constraint, epsilon-constraint method.

## 1. INTRODUCTION

Economic dispatch of thermal generating units is important in optimizing power system operation [1]. It minimizes operational cost by optimally allocating demand to generating units, whilst regarding the system constraints [2]. These constraints include transmission loss constraints, generator capability limits and power balance constraints among others [3]. Following the Clean Air Act Amendments of 1990, harmful emissions produced by utilities have become a subject of great concern. In addition to fuel cost minimization, combined economic and emission dispatch CEED also aims to minimize the amount of emissions. The problem therefore becomes a multi-objective optimization problem with competing objectives – fuel cost and emissions [4] [5]; minimization of one does not guarantee a reduction of the other.

Various approaches have been utilized by researchers to solve this problem. A nonlinear approach was proposed in [6], but as is common with any nonlinear optimization technique, problems may be encountered while trying to obtain the global optimum solution. The First Order Gradient method was reported in [7]. It is somewhat reliable, but may require re-initialization in some cases. Also, a large number of iterations may be required to obtain a solution. A comparative analysis of the performance of Genetic algorithm (GA), Ant Colony Search algorithm and the conventional lambda iteration method was carried out in [8]. Genetic algorithm yields a more optimal solution than lambda iteration method; however, it suffers from immature convergence and limited searching ability [8]. Although, Ant Colony Search algorithm gave a better solution than genetic algorithm, its computation time is higher [8]. The performance of certain Particle Swarm algorithm (PSO) variants with that of GAMS, in solving the economic

dispatch problem, was reviewed in [9]. For discontinuous cost functions, General Algebraic Modelling System (GAMS) is better than any variant of PSO [9]. The time taken for PSO technique is relatively larger than that taken by GAMS. Moreover, unlike GAMS, it increases largely with an increase in the complexity and scale of the problem. GAMS is a modelling language which was originally developed through a World Bank funded study in 1988 [10]. It is a mathematical specification language specially dedicated for the solution of optimization problems. Large problems can be represented in GAMS in a concise manner, and this can easily be altered [9] for testing and research purposes. GAMS uses mathematical operations to achieve optimal solutions; hence its solutions are always consistent. It is particularly useful for handling complex and large scale power systems optimization problems [11]. The use of GAMS for solution to power system optimization problems is therefore practicable.

In this paper, the epsilon-constraint approach to solving multi-objective optimization problems is employed in formulating the CEED problem which is then solved using the GAMS Modular In-core Nonlinear Optimization System (GAMS-MINOS) solver. The Epsilon-constraint method is suitable for the CEED problem as it allows specification of the maximum allowable amount of emissions (which the appropriate regulatory body, like the environmental protection agency in the United States, may have specified). The rest of the paper is organized as follows. The network considered in this study is shown in Section 2. Section 3 details the mathematical model of the CEED problem. The test data used in the study is shown in the tables under Section 4. Sections 5 and 6 present results obtained and a brief discussion of the results respectively. The paper is concluded in Section 7.

## 2. SYSTEM MODEL

Figure 1 is a diagrammatic representation of the power network considered in this study. It depicts the thermal units of a plant, situated close to one another and connected to the same bus from which power is exported to the grid. Since these are units located within the same premises, it can be assumed that they are all equidistant from the common bus, hence power loss can be neglected in the CEED formulation. The load shown in the figure is the demand to be met by the power plant.

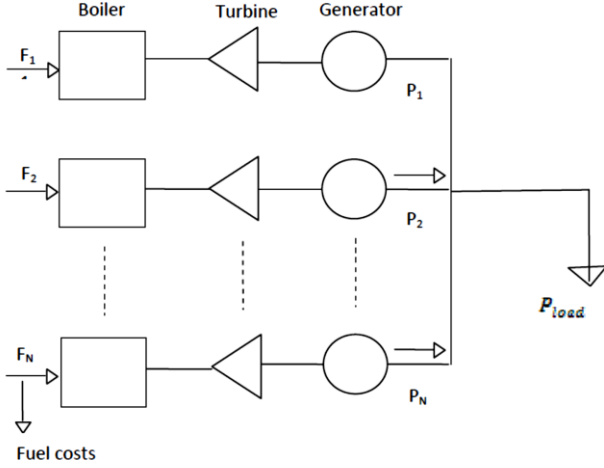


Figure 1: Common bus connection of thermal units [12]

## 3. PROBLEM FORMULATION

The CEED problem is, traditionally, a multi-objective optimization problem with generator limits and power balance constraints as follows:

$$\text{Min} (F(P_G), E(P_G)) \quad (1)$$

Subject to:

$$P_{Gimin} \leq P_{Gi} \leq P_{Gimax} \quad \forall i \in [1, n] \quad [13] \quad (2)$$

$$\sum_{i=1}^n P_{Gi} = P_D \quad (3)$$

$$f_i(P_{Gi}) = a_i(P_{Gi})^2 + b_i P_{Gi} + c_i \quad \forall i \in [1, n] \quad (4)$$

$$F(P_G) = \sum_{i=1}^n f_i(P_{Gi}) \quad (5)$$

$$e_i(P_{Gi}) = \alpha_i(P_{Gi})^2 + \beta_i P_{Gi} + \gamma_i \quad \forall i \in [1, n] \quad (6)$$

$$E(P_G) = \sum_{i=1}^n e_i(P_{Gi}) \quad (7)$$

The basic idea behind the epsilon-constraint approach to solving multi-objective optimization problems is the optimization of one of the objectives while limiting the other objectives to user-specified values. In other words, one objective is optimized while others become constraints. The resulting formulation after applying the Epsilon-constraint method is as follows:

$$\text{Min} (F(P_G)) \quad (8)$$

Subject to:

$$E(P_G) \leq \epsilon \quad (9)$$

$$P_{Gimin} \leq P_{Gi} \leq P_{Gimax} \quad \forall i \in [1, n] \quad (10)$$

$$\sum_{i=1}^n P_{Gi} = P_D \quad (11)$$

Where:

$i$  = Index of thermal unit

$n$  = Number of thermal units

$\epsilon$  = Permissible amount of emissions [kg/h]

$P_{Gi}$  = Power output by unit  $i$  [MW]

$P_{Gimin}$  = Minimum power output of unit  $i$  [MW]

$P_{Gimax}$  = Maximum power output of unit  $i$  [MW]

$a_i, b_i$  &  $c_i$  = Fuel cost coefficients of thermal units

$\alpha_i, \beta_i$  &  $\gamma_i$  = Emission coefficients of thermal units

$f_i(P_{Gi})$  = Cost of fuel consumed by unit  $i$  [\$/h]

$F(P_G)$  = Total cost of fuel consumed by all units [\$/h]

$e_i(P_{Gi})$  = Emissions produced by unit  $i$  [kg/h]

$E(P_G)$  = Total emissions produced by all plants [kg/h]

$P_D$  = Power demand from plant [MW]

## 4. SIMULATION

To compare the performance of the proposed approach with other solution approaches reported in literature, a six-generator test system used in [14] is taken as case study. The proposed approach with data for the test system is specified and solved in GAMS. Typically, a GAMS formulation follows the basic format detailed in [15].

The generator fuel cost coefficients, emission coefficients and generation limits of the system are shown in Tables 1, 2 and 3 respectively, and a demand of 500 MW is assumed.

Table 1 Fuel cost coefficients of test system

Units	Fuel Cost Coefficients		
	$a_i$	$b_i$	$c_i$
1	0.15247	38.53973	756.79886
2	0.10587	46.15916	451.32513
3	0.02803	40.39655	1049.9977
4	0.03546	38.30553	1243.5311
5	0.02111	36.32782	1658.5696
6	0.01799	38.27041	1356.6592

Table 2 Emission coefficients of test system

Units	Emission Coefficients		
	$\alpha_i$	$\beta_i$	$\gamma_i$
1	0.00419	0.32767	13.85932
2	0.00419	0.32767	13.85932
3	0.00683	-0.54551	40.2669
4	0.00683	-0.54551	40.2669

5	0.00461	-0.51116	42.89553
6	0.00461	-0.51116	42.89553

Table 3 Generator capacity limits

Units	Operating limits	
	Lower limit(MW)	Upper limit(MW)
1	10	125
2	10	150
3	35	225
4	35	210
5	130	325
6	125	315

## 5. RESULTS

For ease of comparison, the results (in bold) obtained in this study (using the epsilon-constraint approach) together with results (reported in literature) gotten by using the Lambda iteration method, recursive method and a genetic algorithm, are displayed in Table 4. The optimal power output of each thermal unit is also shown in the Table.

Table 4 Results obtained by various approaches

Method	Unit 1 power (MW)	Unit 2 power (MW)	Unit 3 power (MW)	Unit 4 power (MW)	Unit 5 power (MW)	Unit 6 Power (MW)	Fuel cost (\$/h)	Emission (kg/h)
Conv. $\lambda$ -iteration	21.119	22.047	79.214	99.611	149.418	128.591	27092.50	261.635
Recursive method	26.124	28.246	68.421	97.125	147.115	132.969	27092.50	261.634
Genetic algorithm variant	25.731	22.149	89.154	92.152	141.124	129.690	27089.79	261.419
<b>Epsilon-constraint method (proposed)</b>	<b>27.391</b>	<b>13.277</b>	<b>87.381</b>	<b>89.930</b>	<b>143.652</b>	<b>138.368</b>	<b>27088.01</b>	<b>261.150</b>

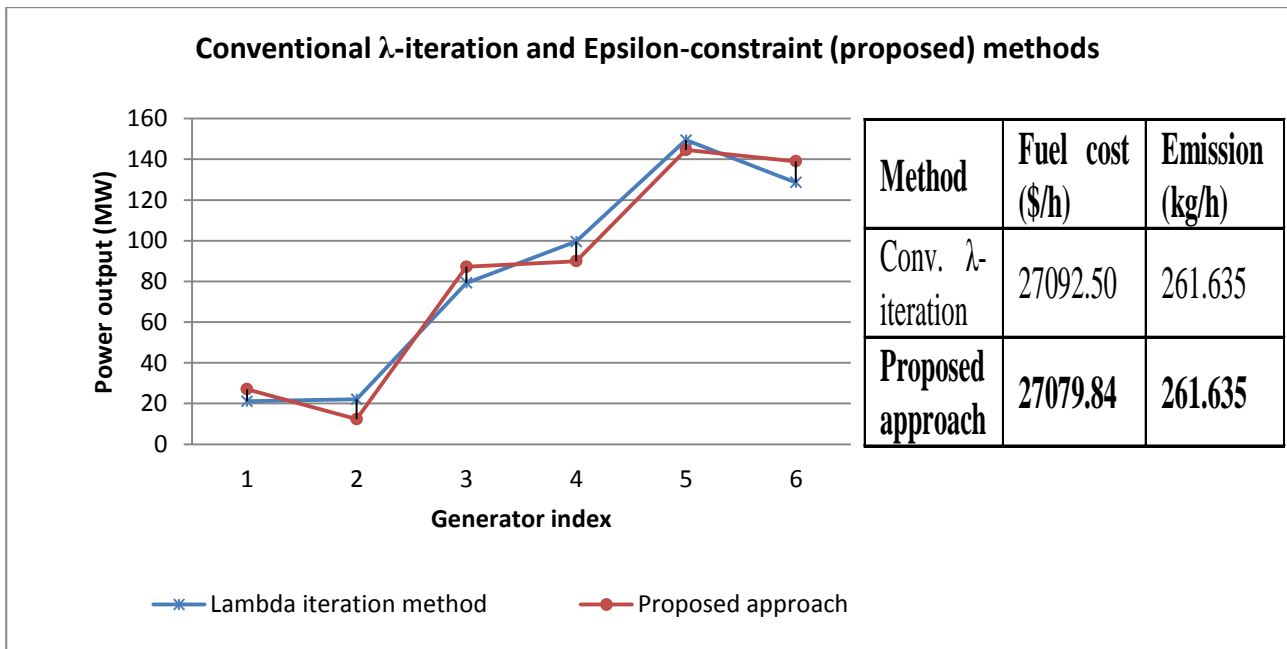


Figure 2: Optimal operating points and fuel cost by both  $\lambda$ -iteration and Epsilon-constraint methods

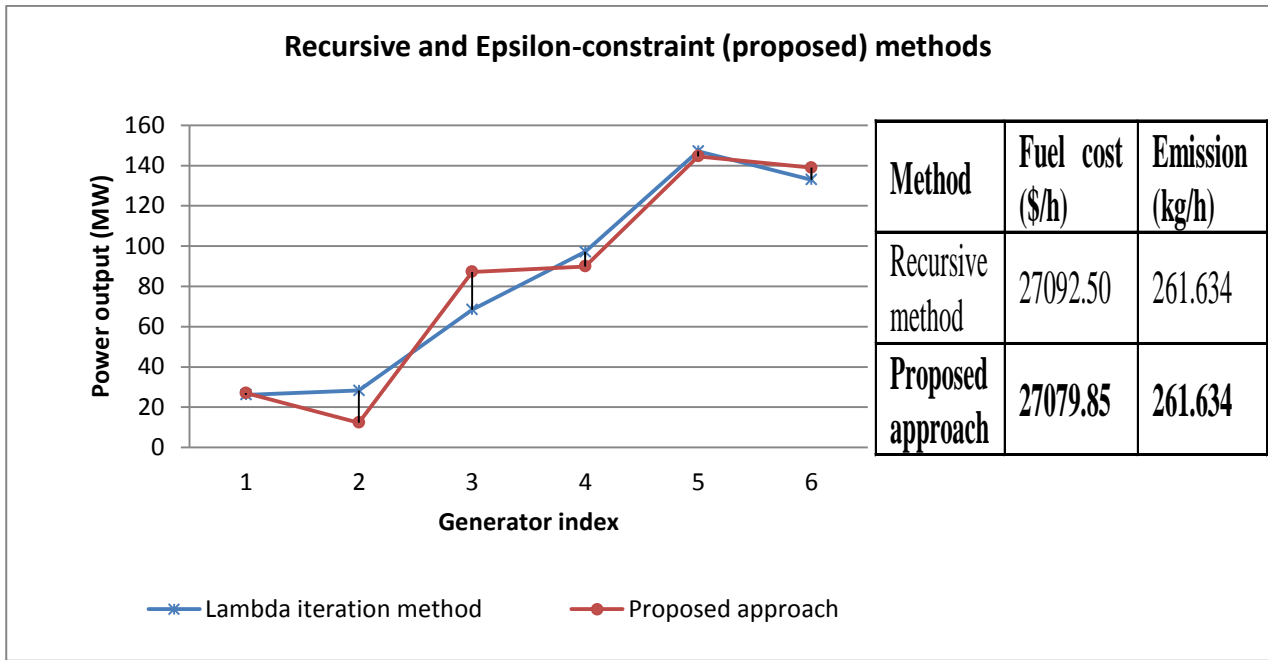


Figure 3: Optimal operating points and fuel cost by both recursive and Epsilon-constraint methods

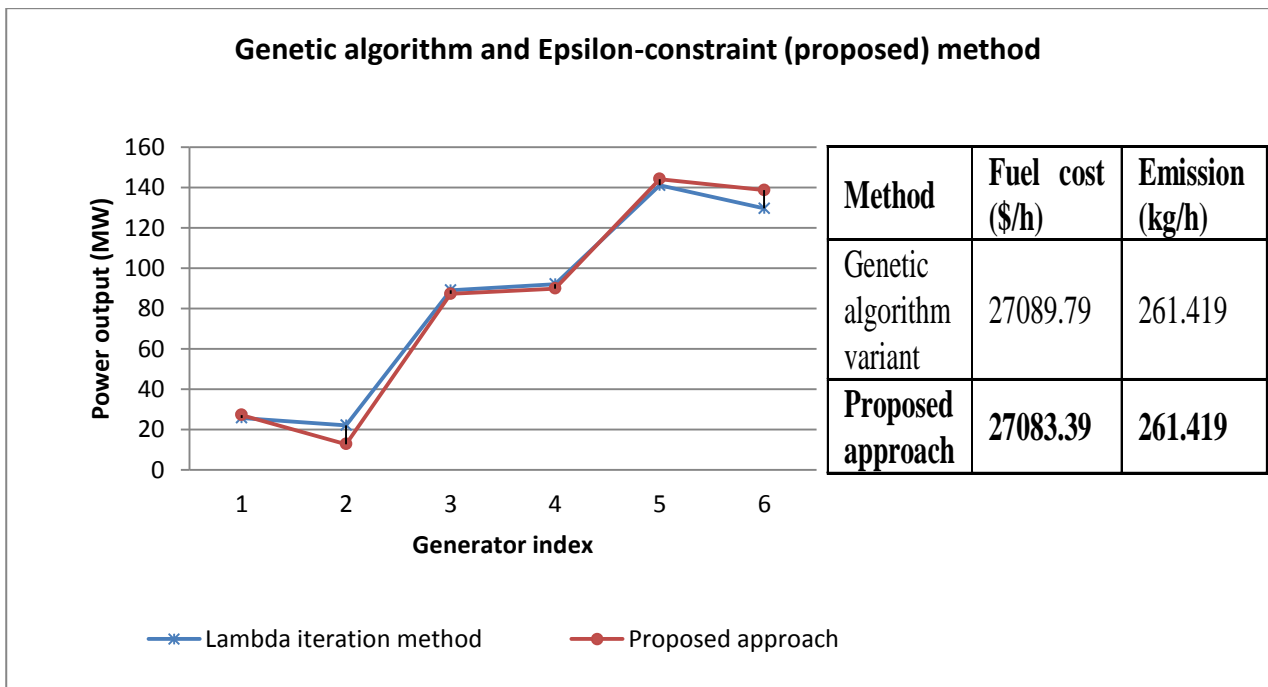


Figure 4: Optimal operating points and fuel cost by both Genetic Algorithm and Epsilon-constraint method

## 6. DISCUSSION OF RESULTS

The epsilon-constraint method allows the specification of a value for the maximum amount of emissions permissible; hence it reduces the traditionally bi-objective CEED problem to a single objective problem. This facilitates quicker and better solutions to the CEED problem.

Regarding the results shown in Table 4, the maximum quantity of emissions permitted was set to 261.150 kg/h, and a fuel cost of 27088.01\$/h was obtained. As seen in the table, the proposed approach realizes the lowest fuel cost at the lowest amount of emissions, in comparison with the other results shown.

In Figures 2 through 4, the proposed approach is also compared with the respective methods used by various researchers. In each case, the permissible amount of

emissions is set to that obtained using each respective method (as reported in literature), that is, 261.635 kg/h in Figure 2, 261.634 kg/h in Figure 3 and 261.419 kg/h in Figure 4. The optimal operating point (in MW) of each thermal unit and resulting fuel cost are obtained by applying the proposed method. The optimal operating points realized by each pair of methods are somewhat alike however, in the recursive-proposed method pair, a considerable margin is noticed with generators 2 and 3. Of the three pairs considered, the GA-proposed method pair gives the most similar optimal power outputs. From the figures, the proposed approach is seen to outperform the other methods. This demonstrates the advantage of the proposed epsilon-constraint method, coupled with the use of GAMS, in solving the CEED problem.

## 7. CONCLUSION

CEED, which is normally a multi-objective optimization problem, was formulated using the epsilon-constraint method and solved with GAMS-MINOS solver. Results from tests carried out on six-generator power plant show that the proposed approach outperforms the other methods considered. Moreover, the approach offers the flexibility of allowing the operator to set a maximum value for the permissible amount of emissions during power system operation. This flexibility is appropriate for the CEED problem as some power utilities have a limit on the amount of emissions they are allowed to produce.

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