

The Time of Arrival Statistics for Cellular Communication Using Multiple Concentric Annular Rings of Uniformly Distributed Scatterers

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Abstract. Mobile communication has become popular over the last couple of decades. The channel between the base station and the mobile device depends of the environment in which the mobile device is placed. The time of arrival statistics of the signal arriving from the base station to the mobile device depends on the channel. The time of arrival statistics is an important parameter as it assists in determining the maximum width of the pulse that should be transmitted over the channel. Geometrical channel model is one of the widely used channel modeling techniques for determining the time of arrival statistics of the signals. In this paper we approximate the gaussian distribution of scatterers around the mobile device by concentric annular rings of uniformly distributed scatterers to evaluate the time of arrival statistics. We subsequently compare the statistics obtained by both the methods for validation of the results

Keywords: Time of arrival, Geometrical Channel Model, One ring channel model.

1 Introduction

Mobile communication has gained popularity and brought in a revolution to the communication industry. The end to end link in cellular communication is achieved by routing the signal through a base station. The base station is generally located on top of a high rise building or at a highest point in the locality it serves. Thus the base station is devoid of scatterers around it. Whereas, the mobile device is located at the ground level and surrounded by objects like buildings, furnitures etc. The objects around the mobile device can be considered as scatterers. The signal from the base station reaches the mobile device after being reflected, refracted and dispersed by the scatterers. The time required for the different signals to reach the mobile device depends on the path it traverses from the base station to the mobile device via the scatterers. The signal may reach the mobile device from the base station after being scattered by a single scatterer or multiple scatterers. If the signal undergoes multiple scattering before reaching the mobile device, then the power of the received signal degrades drastically and contributes negligible power to the detectable signal. Hence in this paper we consider signal that reaches the mobile device after being scattered by a single scatterer.

2 Model Description

The geometrical based models are a widely used technique for modelling channels between the transmitter and receiver of wireless communication [1,2]. In this modeling technique, as shown in figure 1, the transmitter and the receiver are separated by a distance D . The receiver is surrounded by scatterers around it inside a circle of radius R . The distance between the transmitter and the receiver (D) is considered to be much greater than the radius (R) of the scattering circle, thus justifying the use of geometric optics and representation of the waves by rays. The scatterers around the mobile device can be considered to be gaussian or uniformly distributed depending on the environment it models. The number of scatterers do not affect the time of arrival statistics, as the time of arrival statistics is dependent on the distribution of the scatterers in the circle. But the number of scatterers should be adequately large to represent the distribution in the circle.

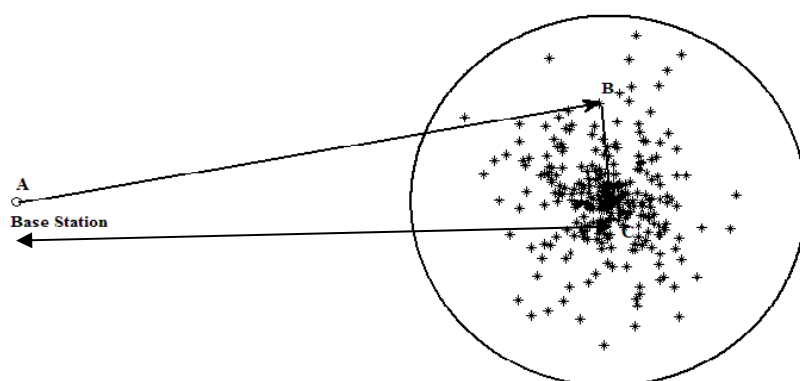


Figure 1. Geometrical based model of cellular channel

In the present work we consider the scatterers to have normal distribution around the mobile device. Similar work has been done before by R Janaswamy in [3,4]. The analytical approach in [3] is involved and cumbersome. In this paper we propose a much easier approach to find the time of arrival statistics of the signal at the mobile device. This is a more generalized approach and may be extended to any kind of scatterer distribution around the mobile device.

As mentioned earlier we consider the scatterers to have a normal distribution around

the mobile device confined in a circle of radius R. The probability density function and the cumulative density function of gaussian distribution is given by equation 1 and equation 2 respectively,

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (1)$$

$$F_x(x) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{x-\mu}{\sigma\sqrt{2}} \right) \right] \quad (2)$$

where,

x = random variable

μ = mean value of the normal distribution

σ^2 = variance

$$\operatorname{erf}(x) = \text{error function} = \frac{1}{\sqrt{\pi}} \int_{-x}^x e^{-t^2} dt$$

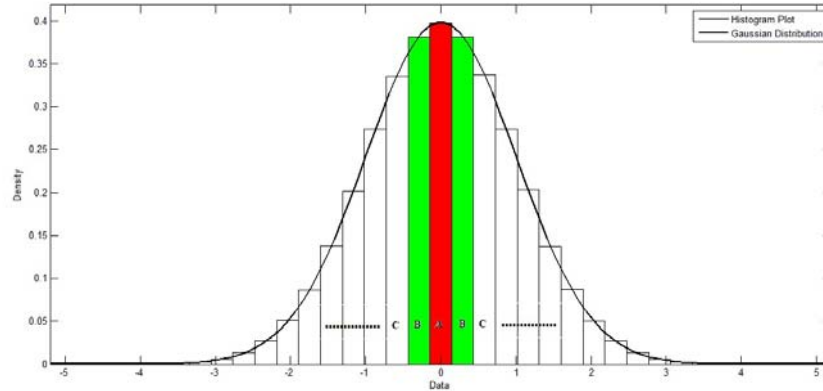


Figure 2. A plot of the Gaussian density function with zero mean and unity variance

Figure 2 shows a plot of the gaussian density function. It may be observed both from the plot and equation 2 that 99.7% of the data are within the -3σ to 3σ limits. The histograms in figure 2, gives the relative number of items in the bins. The items in each bin can be assumed to be containing uniformly distributed data selected over the bin width.

In this paper, we approximate the gaussian distributed scatterer in the scattering circle

by concentric rings of uniformly distributed scatterers as shown in figure 3. The number of scatterers in a particular ring is determined by the mean, standard deviation of the gaussian distribution and the distance of the ring from the center of the circle.

Let us assume that the radius of the scattering circle is R and the standard deviation of the gaussian distribution of the scatterers is σ . If we equate the radius of the circle to 3σ , then we can make sure that 99.7% of the scatterers are within the circle of radius R . Let us assume that the circle of radius R is divided into N concentric circles of equal width, then the width of each annular ring would be R/N . Larger the value of N , better is the approximation. Now we are required to put the scatterers in the annular rings in such a way that in each annular ring the scatterers are uniformly distributed whereas at the same time the scatterers should be gaussianly distributed over the circle [5]. Figure 3 shows the scattering circle being divided into annular rings. Correlating figure 2 and figure 3, the histogram A, B,C,... of figure 2 corresponds to the annular rings A, B, C,...of figure 3. The number of scatterers in an annular ring of figure 3 should be proportional to the area under corresponding histogram in figure 2. The area under the histogram can be obtained by putting the lower and upper limits to the integral of equation 1. It may be noted here that the integral of equation 1 is given by the cumulative density function represented in equation 2.

The time of arrival is evaluated by calculating the time taken by the signal from the base station to reach a scatterer and then to the mobile device. The statistics of the time of arrival can be obtained after calculating the time of arrival from each scatterer. It is evident that the pattern of time of arrival statistics depends on the way the scatterers are distributed. Whereas, the actual time of arrival also depends on the separation distance between the base station and the mobile device [6].

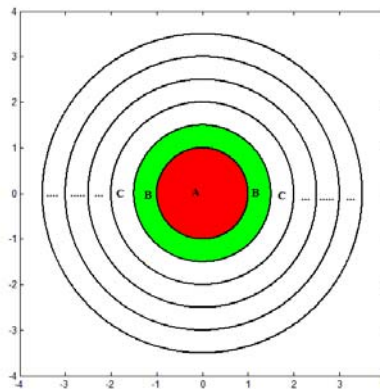


Figure 3. Concentric annular rings of scatterers

3 Results and Discussion

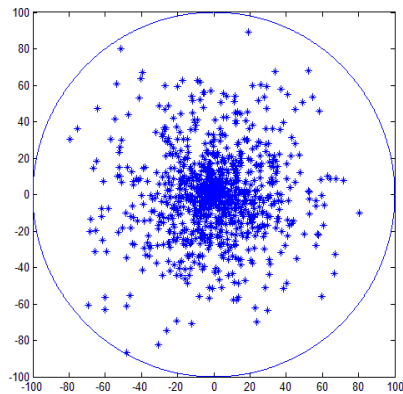


Figure 4 Scatterers generated using gaussian distribution directly.

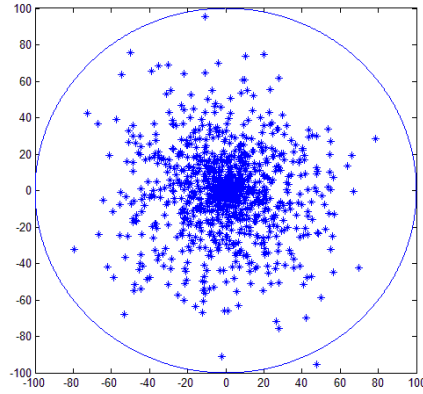


Figure 5. Gaussian distribution of Scatterers generated using annular rings with uniform distribution approach

Figure 4 above shows one thousand gaussian distributed scatterers generated in a circle of radius hundred meters and at whose center the mobile device is located. The position of the scatterers are generated by using the randn function of Matlab. This generates a set of points gaussianly distributed over the x-axis. Then to each point an angle is associated which is generated by using a uniform random generator which generates angles between 0 and 2π radians.

Where as, figure 5 above shows one thousand scatterers generated in a circle of radius hundred meters and at whose center the mobile device is located. Here the circle is first divided into thirty annular rings of equal width. Then the scatterers are uniformly distributed in these annular rings. The total number of scatterers were considered to be thousand. The number of scatterers in each ring decreases with its distance from the center and depends on the area of the histogram it corresponds to in the gaussian distribution.

As the coordinates of the scatterers generated by above two methods are known, the time of arrival of the signal to the mobile from the base station placed at 500 meters away from the mobile device can be calculated. The time of arrival was calculated for signal arriving the mobile device from each scatterer in both the cases. The probability density of the time of arrival from both the cases were then plotted, as shown in figure 6 below. It can be observed that the probability density function obtained by both the methods match each other.

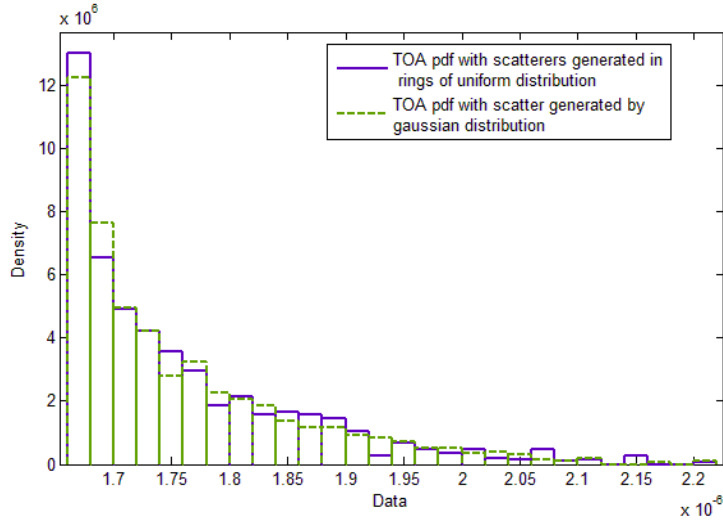


Figure 6 Probability density function of Time of Arrival obtained by scatterers generated by by gaussian distribution and annular rings with uniform distribution

4 Conclusion

The time of arrival of signal and its statistics is an important parameter in wireless communication system. The time of arrival is one of the parameters which determine the pulse width of the communication system. In this paper, we propose an alternative way to generate the gaussian distributed scatterers to evaluate the time of arrival statistics between the base station and the mobile device. This method of generation divides the scattering circle into equal width annular rings. Then the scatterers are uniformly distributed in the annular rings. The number of uniformly distributed scatterers in an annular rings depends on its distance from the center of the ring, the standard deviation of the gaussian distribution it needs to replicate. This method of generation of scatterers makes the mathematical formulation for the time of arrival more tractable. The proposed method was then validated with the standard method by evaluating the time of arrival statistics by both the approaches.

References

1. P. Almers, E. Bonek, A. Burr, N. Czink, M. Debbah, V. Degli-Esposti, H. Hofstetter, P. Kyosti, D. Laurenson, G. Matz, A. F. Molisch, C. Oestges, and H. Ozcelik. Survey of channel and radio propagation models for wireless MIMO systems. *EURASIP Journal on Wireless Communications and Networking*, 2007(1):56–75, January 2007.

2. M Patzold, and B O Hogstad, A space-time channel simulator for MIMO channels based on the geometrical one-ringscattering model, In Proceedings of the Vehicular Technology Conference (VTC 2004), Vol. 1, pages 144-9, Sep. 2004.
3. R. Janaswamy. Angle and time of arrival statistics for the gaussian scatter density model. IEEE Transactions on Wireless Communications, 1(3):488–497, July 2002.
4. R. Janaswamy. Radiowave Propagation And Smart Antennas For Wireless Communications. Kluwer Academic Publishers, 2001.
5. B. S. Paul and R. Bhattacharjee, Time and angle of arrival statistics of mobile-to-mobile communication channel employing dual annular strip model IETE Journal of Research, vol. 56, issue 6, pp 327 - 332 Nov.–Dec. 2010.
6. R. B. Ertel and J. H. Reed. Angle and time of arrival statistics for circular and elliptical scattering models. IEEE Journal on Selected Areas in Communication, 17(11):1829–1840, November 1999.