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Geopolitical interactions from reduced Google matrix analysis of Wikipedia

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Abstract—**Interactions between countries originate from diverse aspects such as geographic proximity, trade, socio-cultural habits, language, religions, etc. Geopolitics studies the influence of a country's geographic space on its political power and its relationships with other countries. This work reveals the potential of Wikipedia mining for geopolitical study. Actually, Wikipedia offers solid knowledge and strong correlations among countries by linking web pages together for different types of information (e.g. economical, historical, political, and many others). The major finding of this paper is to show that meaningful results on the influence of country ties can be extracted from the hyperlinked structure of Wikipedia. We leverage a novel stochastic matrix representation of Markov chains of complex directed networks called the reduced Google matrix theory. For a selected small size set of nodes, the reduced Google matrix concentrates direct and indirect links of the million-node sized Wikipedia network into a small Perron-Frobenius matrix that preserves the PageRank probabilities of the global Wikipedia network. We perform a novel sensitivity analysis that leverages this reduced Google matrix to characterize the influence of relationships between countries from the global network. We apply this analysis to the set of 27 European Union countries. We show that with our sensitivity analysis we can exhibit easily very meaningful information on geopolitics from five different Wikipedia editions (English, Arabic, Russian, French and German).**

I. Introduction

Relationships between countries have always been of utmost interest to study for countries themselves as they have to be accounted for into any country's strategic and diplomatic plan. Studies are driven by observing the influence of a relationship between two countries on other countries from different perspectives listing economic exchanges, social changes, history, politics, religious, martial, regional as seen in [1]. The major finding of this paper is to show that meaningful results on geopolitics interactions could be extracted from Wikipedia for a given selection of countries. Therefore, it can be leveraged to provide a picture of countries relationships offering a new framework for long-term geopolitical studies. In [2], S. Javanmardi et al. show that even though anyone can edit a Wikipedia entry at any time, the average article quality increases as it goes through various edits. Wikipedia's accuracy for its scientific entries has been proved by comparing it to Encyclopedia Britannica and to PDQ - NCI's Comprehensive Database in [3], [4]. To sum up, Wikipedia has become the largest accurate reliable free online open source of knowledge.

TABLE I
List of EU countries.

Wikipedia edition			English	French	German
Countries	CC	Color	K	K	K
France*	FR	BL	1	1	2
United Kingdom*	GB	GN	2	4	24
Germany	DE	BL	3	2	1
Italy	IT	BL	4	3	4
Spain*	ES	OR	5	5	5
Poland*	PL	RD	6	8	6
Netherlands	NL	BL	7	7	7
Sweden*	SE	PK	8	11	8
Romania	RO	RD	9	18	17
Belgium	BE	BL	10	6	9
Austria	AT	PK	11	9	3
Greece	GR	OR	12	13	14
Portugal	PT	OR	13	12	11
Ireland	IE	GN	14	19	16
Denmark	DK	GN	15	14	10
Finland	FI	PK	16	17	15
Hungary	HU	RD	17	10	13
Czech Republic	CZ	RD	18	15	12
Bulgaria	BG	RD	19	20	20
Estonia	EE	RD	20	24	22
Slovenia	SI	RD	21	23	23
Slovakia	SK	RD	22	16	18
Lithuania	LT	RD	23	22	21
Cyprus	CY	RD	24	27	27
Latvia	LV	RD	25	25	25
Luxembourg	LU	BL	26	21	19
Malta	MT	RD	27	26	26

PageRank K for EnWiki, FrWiki and DeWiki. Color code groups EU countries into 5 subsets: Blue (BL) for Founders, Green (GN) for 1973 new member states, Orange (OR) for 1981 to 1986 new member states, Pink (PK) for 1995 new member states and Red (RD) for 2004 to 2007 new member states. Standard country codes (CC) are given as well. Countries in bold are the selected ones for each group.

Unique to Wikipedia is that articles make citations to each other, providing a direct relationship between webpages. As such, Wikipedia generates a larger directed network of article titles with a rather clear meaning. For these reasons, it is interesting to apply algorithms developed for search engines of World Wide Web, those like the PageRank algorithm [5], to analyze the ranking properties and relations between Wikipedia articles. For various language editions of Wikipedia it was shown that the PageRank vector produces a reliable ranking of historical figures over 35 centuries of human history [6]–[9] and a solid Wikipedia ranking of world universities (WRWU) [6], [11]. It has been shown that the Wikipedia ranking of historical figures is in a good agreement with the well-known Hart ranking [12], while the WRWU is in a good agreement with the Shanghai Academic ranking of world universities [13].

This paper analyses the networks of articles extracted from 5 language editions of Wikipedia to study the influence of countries on each other. Previous work [16] has identified the strongest ties between countries, but this one focuses on capturing the impact of a change in the strength of a relationship between two countries on the overall network interactions of selected countries via the global network. The impact on the overall network structure is measured by calculating the variation of importance of the nodes in the network. We show that this sensitivity analysis renders a reasonable and meaningful idea of the influence of a given bilateral tie on the whole network.

We have conducted our geopolitics study for the target set of 27 European Union member states. As such, from the global network of articles of Wikipedia we have derived the reduced Google matrix $G_{\rm R}$ for these 27 EU states. Thus, $G_{\rm R}$ reflects in a 27-by-27 matrix the complete (direct and indirect) relationships between countries. To quantify the relative influence of one relationship between two nations on all other nations, we propose in this paper to compute a logarithmic derivative of the PageRank probabilities calculated from $G_{\rm R}$ and $\tilde{G}_{\rm R}$. PageRank probabilities are derived from $G_{\rm R}$ as explained later. They represent the importance of a node in the global network of articles. $\tilde{G}_{\rm R}$ is almost equal to $G_{\rm R}$. It only differs by the values of one column. If the relationship going from nation j to nation i, only the values of column j are changed to relatively inflate the probability $\tilde{G}_{\rm R}(i,j)$ of nation j ending in nation i compared to the other ones. This is done in practice by modifying $\tilde{G}_{\rm R}(i,j)$ and then normalizing the column again to unity to enforce the column normalization property of Google matrices. Results are derived for 5 different Wikipedia editions (*Data collected February 2013*) from the set of 24 analyzed in [9]: EnWiki, ArWiki, RuWiki, DeWiki and FrWiki that contain 4.212, 0.203 , 0.966, 1.533 and 1.353 millions of articles each. The selected countries are the 27 EU countries as of February 2013 (*Croatia joined in July 2013*) as mentioned in Table I.

The paper is organized as follows. At first we introduce the reduced Google matrix theory, together with a primer on Google Matrix and PageRank calculations. The reduced Google matrix is illustrated for the 27 EU set of states. Next, the methodology for our link sensitivity analysis is presented. A detailed analysis of EU countries is given in the Results section, with a special focus on the sensitivity analysis of important relationships among member states. Finally, conclusions are drawn in the last section.

II. Reduced Google Matrix theory

It is convenient to describe the network of N Wikipedia articles by the Google matrix G constructed from the adjacency matrix A_{ij} with elements 1 if article (node) j points to article (node) i and zero otherwise. Elements of the Google matrix take the standard form $G_{ij} = \alpha S_{ij} + (1-\alpha)/N$ [5], [10], where S is the matrix of Markov transitions with elements $S_{ij} = A_{ij}/k_{out}(j)$, $k_{out}(j) = \sum_{i=1}^{N} A_{ij} \neq 0$ being the node j out-degree (number of outgoing links) and with $S_{ij} = 1/N$ if j has no outgoing links. The damping factor $0 < \alpha < 1$ is which for a random surfer determines the probability $(1-\alpha)$ to jump to any node; below we use the standard value $\alpha = 0.85$. The right eigenvector of G with the unit eigenvalue gives the PageRank probabilities $P(j)$ to find a random surfer on a node j. We order nodes by decreasing P getting them ordered by the PageRank index $K = 1, 2, ...N$ with a maximal probability at $K = 1$. From this global ranking we capture the top local PageRank mentioned in Tab. I.

Reduced Google matrix is constructed for a selected subset of nodes (articles) following the method described in [14]–[16] and based on concepts of scattering theory used in different fields of mesoscopic and nuclear physics or quantum chaos. It captures in a N_r-by-N_r Perron-Frobenius matrix the full contribution of direct and indirect interactions happening in the full Google matrix between the N_r nodes of interest. In addition the PageRank probabilities of selected N_r nodes are the same as for the global network with N nodes, up to a constant multiplicative factor taking into account that the sum of PageRank probabilities over N_r nodes is unity. Elements of reduced matrix $G_{\rm R}(i,j)$ can be interpreted as the probability for a random surfer starting at web-page j to arrive in web-page i using direct and indirect interactions. Indirect interactions refer to paths composed in part of web-pages different from the N_r ones of interest. Even more interesting and unique to reduced Google matrix theory, we show here that intermediate computation steps of $G_{\rm R}$ offer a decomposition of $G_{\rm R}$ into matrices that clearly distinguish direct from indirect interactions: $G_{\rm R} = G_{rr} + G_{\rm pr} + G_{\rm qr}$ [15]. Here, G_{rr} is the submatrix of G representing the original direct links between the selected N_r nodes. Fig. 1 shows that $G_{\rm pr}$ is rather close to the matrix in which each column is given by the PageRank vector P_r, ensuring that PageRank probabilities of $G_{\rm R}$ are the same as for G (up to a constant multiplier). As such, $G_{\rm pr}$ doesn't provide relevant information to characterize the importance of links between the selected nodes. The one playing an interesting role is $G_{\rm qr}$, which captures the effect of all indirect paths connecting the selected N_r nodes in the full network of N nodes (see [14]–[16]). The matrix $G_{\rm qr} = G_{\rm qrd} + G_{\rm qrnd}$ has diagonal ($G_{\rm qrd}$) and non-diagonal ($G_{\rm qrnd}$) parts. $G_{\rm qrnd}$ is leveraged for the studies of Section III-B. Results of sections IV and V are based on $G_{\rm R}$ only. The complete theoretical background is to be found in [14]–[16].

III. Results: $G_{\rm R}$ properties

A. Reduced Google matrix of 27 EU set.

As an example, we have picked the EnWiki edition to plot the matrices $G_{\rm R}$, $G_{\rm pr}$, G_{rr}, and $G_{\rm qrnd}$ in Fig. 1. As $G_{\rm R}$ is per-column normalized and dominated by the projector $G_{\rm pr}$ contribution, which is proportional to the global PageRank

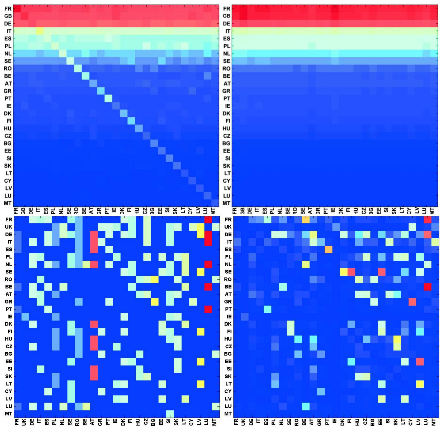

Fig. 1. **Density plots of G_{R} and its decomposition for 27 EU extracted from EnWiki.** G_{R} (top left), G_{pr} (top right), G_{rr} (bottom left) and G_{qrnd} (bottom right). Max values in red (0.14 in top panels; 0.003 in bottom left; 0.008 in bottom right), intermediate in green and min (≈ 0) in blue.

probabilities (more details in [14], [15]), this prevents a meaningful per-line analysis. G_{rr} provides information only on direct links between countries as it lists the genuine Google matrix probability for a random surfer to jump from node j to i. On the contrary, G_{qrnd} offers a much more unified view of countries interactions as it highlights more general indirect (or hidden) interactions views via the rest of nodes. It captures the contribution of all indirect paths connecting two nodes i and j in the full network of Wikipedia articles. For the three selected languages editions, we have identified very strong hidden links connecting Finland to Sweden. Other interesting hidden links are between Ireland and United Kingdom in DeWiki or in EnWiki, the hidden links connecting Luxembourg to France.

B. Networks of friends

As proposed in [16], it is possible to extract from G_{qrnd} a network of friendships to easily illustrate hidden links in the network. For the sake of simplicity, we refer next to G_{qrnd} using G_{qr} notation. To create these networks of friends, we divide the set of N_r nodes into representative groups as shown in Tab I. EU countries are grouped upon their accession date to the union. One leading country per EU member state group has been selected as well.

For each leading country j, we extract from G_{qr} the top 4 *Friends* given by the 4 best values of the elements of column j. In other words, it corresponds to destinations of the 4 strongest outgoing links of j. These networks of top 4 friends have been calculated for the five editions of Wikipedia. Top 4 friends of EU leading countries are plot on the graphs of Fig. 2. Results for EnWiki, FrWiki and DeWiki are presented here. The black thick arrows identify the top 4 friends interactions. Red arrows represent the friends of friends interactions that are computed recursively until no new edge is added to the graph. All graphs are visualized with Yifan Hu layout [17] using [18].

It can be noticed in Fig. 2 that the order of arrival of member states is meaningful. Indeed, nodes of the same color are closely interconnected. It is worth noting as well that Germany,

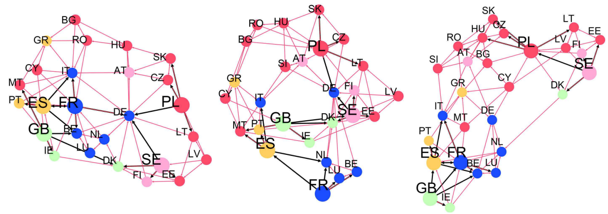

Fig. 2. **Relationship structure extracted from G_{qrnd} for the network of EU countries.** friends induced by the leading countries (FR, GB, ES, SE, PL) of each group. Results are plotted for EnWiki (left), FrWiki (middle) and DeWiki (right). Node colors represent geographic appartenance to a group of countries (cf. Table I for details). Selected countries points with a bold black arrow to its top 4 friends. Red arrows show friends of friends interactions computed until no new edges are added to the graph.

TABLE II
Cross-edition friends extracted from G_{qr} of EU countries.

Top country	G_{qr} Wiki friends present in		
	all 5 editions	4 out of 5 editions	3 out of 5 editions
FR	BE -ES	IT	
GB	IE		DK - FR
ES	IT - PT	FR	BE
SE	DK - FI		EE
PL	CZ		DE - HU - LT - SK

as one of the Founders, bridges the group of Founders to Sweden (the leader of the countries that have joined EU in 1995) and Poland (the leader of the countries that have joined EU between 2004 and 2007) in FrWiki and EnWiki. From EnWiki and DeWiki, strong ties are seen between Italy and France, while it is not the case for FrWiki authors. This is an example of cultural bias. However, lots of links are to seen in all three editions: GB-IE, SE-FI, ES-PT, PL-LT, IT-GR and many others. In all editions, Benelux and Nordic countries create a cluster densely interconnected. To underline this constant presence of links, we give in Table II the list of friends that are among the top 4 ones in all 5 editions, in 4 out of 5 and in 3 out of 5. For each leading country, around 2 to 3 top friends are present across all editions.

IV. Link sensitivity analysis

A. Influence analysis of geopolitical ties using G_{R}

Previous developments in this article show that G_{R} captures essential interactions between countries. These interactions are extracted from Wikipedia and thus stem from all links covering this very rich network of webpages.

The point is now to see how some ties between countries influence the whole network structure. More specifically, we focus here on capturing the impact of a change in the strength of a relationship between two countries on the importance of the nodes in the network. Therefore we have designed a sensitivity analysis that measures a logarithmic derivative of the PageRank probability when the transition probability of only one selected link is increased for a specific couple of nodes in G_{R}, relatively to the other nodes.

Our sensitivity analysis is performed for a directed link where the relationship going from country i to j is increased.

We investigate in the last part of this Section the imbalance between the influence of two opposite direction interactions. In other words, we conduct the aforementioned sensitivity analysis for the link going from country i to j, and for the link going in the opposite direction from j to i. For each pair of countries, we derive from this two-way sensitivity the relationship imbalance to identify the most important player in the relationship.

B. Sensitivity analysis

We define δ as the relative fraction to be added to the relationship from nation j to nation i in G_{R}. Knowing δ, a new modified matrix $\tilde{G}_{\mathrm{R}}$ is calculated in two steps. First, element $\tilde{G}_{\mathrm{R}}(i,j)$ is set to $(1+\delta)\cdot G_{\mathrm{R}}(i,j)$. Second, all elements of column j of $\tilde{G}_{\mathrm{R}}$ are normalized to 1 (including element i) to preserve the unity column-normalization property of the Google matrix. Now $\tilde{G}_{\mathrm{R}}$ reflects an increased probability for going from nation j to nation i.

It is now possible to calculate the modified PageRank eigenvector $\tilde{P}$ from $\tilde{G}_{\mathrm{R}}$ using the standard $\tilde{G}_{\mathrm{R}}\tilde{P}=\tilde{P}$ relation and compare it to the original PageRank probabilities P calculated with G_{R} using $G_{\mathrm{R}}P=P$. Due to the relative change of the transition probability between nodes i and j, steady state PageRank and CheiRank probabilities are modified. This reflects a structural modification of the network and entails a change of importance of nodes in the network. These changes are measured by a logarithmic derivative of the PageRank probability of node a given by:

$$D_{(j\to i)}(a) = (\mathrm{d}P_a/\mathrm{d}\delta_{ij})/P_a = (\tilde{P}_a - P_a)/(\delta_{ij}P_a) \quad (1)$$

Notation $(j\to i)$ indicates that the link from node j to node i has been modified. Element $D_{(j\to i)}(a)$ gives the logarithmic variation of PageRank probability for country a if the link from j to i has been modified. We will refer to this variation as the *sensitivity* of nation a to the relationship from nation i to nation j. If this sensitivity is negative, country i has lost importance in the network. On the opposite, a positive sensitivity expresses a gain in importance. The computation has been tested for values of $\delta=\pm0.01,\pm0.03,\pm0.05$. The result is not sensitive to δ and following results are given for $\delta=0.03$.

C. Relationship imbalance analysis

As introduced earlier, sensitivity $D_{(j\to i)}(a)$ of Eq (1) measures the change of importance of country a if the link from nation j to i has been changed. The sensitivity of node a to a change in one direction is not necessarily the same as its sensitivity to the change in the opposite direction. We define as such the *2-way sensitivity* of node a which is simply the sum of the sensitivities calculated for both directions:

$$D_{(i\leftrightarrow j)}(a) = D_{(i\to j)}(a) + D_{(j\to i)}(a) \quad (2)$$

The two-way sensitivity can be leveraged to find out, for a pair of countries a and b, which one has the most influence on the other one. Therefore, we define the following metric :

$$F(a,b) = D_{(a\leftrightarrow b)}(a) - D_{(a\leftrightarrow b)}(b) \quad (3)$$

Here, we measure the 2-way sensitivity for nodes a and b when the link between them is modified both ways in G_{R}. If $F(a,b)$ is positive, it means that the 2-way sensitivity of a is larger than the 2-way sensitivity of b. In this case, a is more influenced by b than b by a. We can say that b is the *strongest* country. If $F(a,b)$ is negative, we can say that a is the strongest country.

V. Sensitivity analysis results

The sensitivity analysis previously presented has been performed for the 27 EU reduced network with 3 Wikipedia editions: EnWiki, FrWiki and DeWiki. This analysis calculates for each directed link $j\to i$ of the reduced 27 EU network the sensitivity $D_{j\to i}(a)$ of each country a. From this, the relationship imbalance analysis has been calculated as well for each pair of nations. Note that if the modified link is clearly identified in the following, we will drop the index $i\to j$ in our sensitivity measure notation for clarity.

In order to better capture the countries sensitivities from a multicultural perspective, all sensitivity results are averaged over the three editions using $\bar{D}=\frac{1}{3}\sum_{i=1}^{3}D_i$, with i the index of a Wikipedia edition.

A. Sensitivity analysis

We start this analysis by introducing a first simple example where Italy increases its relationship with France. Then, we analyze the impact on the EU countries of Great Britain's exit (i.e. Brexit) from European Union. Next, we highlight the sensitivity of Luxembourg to the increase of Germany and France's cooperation with other member states. Finally, we present the results that underline the strong ties that exist between groups of countries that function together in Europe.

For each sensitivity analysis, we show an axial representation of the sensitivity $\bar{D}$ (cf. Fig. 5, Fig. 6, Fig. 3, Fig. 4). Each axis represents the sensitivity values obtained for a given link variation.

1) Great Britain ties to France and Germany: The United Kingdom has triggered article 50 on March 27, 2017 to leave the European Union as a consequence of the referendum of June 23rd, 2016 [23]. To understand its impact on EU countries with our dataset, we have reduced (and not increased as done in other studies) the G_{R} transition probability UK towards France or Germany. We remind that our network is dated by 2013 but it captures the strong UK influence. Results are shown in Fig. 3 and indicate that Ireland and Cyprus are by far the most negatively affected countries in both cases. Moreover, the sensitivity of UK is negative as it

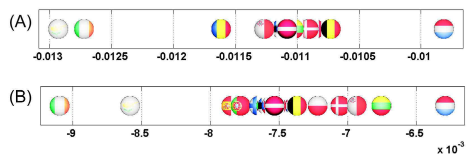

Fig. 3. **Axial representation of $\bar{D}$ for link modifications from {GB} to {FR or DE}.** (A): GB to FR. (B): GB to DE.

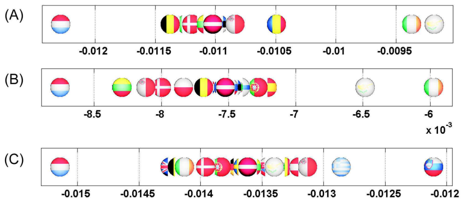

Fig. 4. **Axial representation of $\bar{D}$ for link modifications from {FR or DE} to {GB or IT}.** (A): FR to GB. (B): DE to GB. (C) FR to IT.

benefits less from France's or Germany's influence. These facts have been recently backed up by specialists. In [24], a study delivered by the London School of Economics discussing the consequences of Brexit forecasts that UK will loose 2.8% of its Gross domestic product (GDP)[1]. Similarly, [24] shows that Ireland will loose as well 2.3% of its GDP, which is the largest proportional loss caused by Brexit. Cyprus-UK Relations are strong as claimed by the official website of the Ministry of Foreign Affairs of Cyprus [26]. Referring to [20], UK is the 4^{th} top export destination for Cyprus with \$242M and the 2^{nd} import origin with \$508M. As such, this clear bond of UK with Cyprus explains that if GB suffers from Brexit, Cyprus will do as well. Our data strikingly exhibits the same conclusion as shown in Fig. 3.

2) Luxembourg's sensitivity to Germany and France: Luxembourg shares its borders with Belgium, Germany and France with whom it has strong and diverse relationships. Luxembourg has an open economy. Together with Belgium, they position themselves as the 12^{th} largest economy in the world. Two of the top three export and import countries of Belgium-Luxembourg are Germany (\$44.6B, \$50.4B) and France (\$43.8B, \$36.8B) [20]. Official languages in Luxembourg are Luxembourgish, French and German. Luxembourg has robust relationships with France [27], [29] and Germany [30] in various areas such as finance, culture, science, security or nuclear power. It is clear that Luxembourg will suffer if one of these EU countries reduces its exchanges with it. In Fig. 4, we clearly show that Luxembourg is strongly influenced by France and Germany. If France or Germany increases its relationships with Italy or Great Britain, Luxembourg is by far the most impacted country.

3) Clusters of countries: By analyzing the sensitivity of countries to various 2-nation relationships, we have noticed that several groups of nations function together. These groups are strongly interconnected, and if anyone of these group members increases its relationship strength with a country outside of the group, all group members loose importance in the network. We highlight two meaningful examples next: the cluster of Nordic countries and the cluster Austro-Hungarian cluster. Other clusters we have identified in our network are for instance the cluster of Benelux countries (e.g. Belgium, the Netherlands and Luxembourg) or the cluster of the Iberian peninsula (e.g. Portugal and Spain).

[1]GDP: monetary value of all the finished goods and services produced within a country's borders in a specific time period [25].

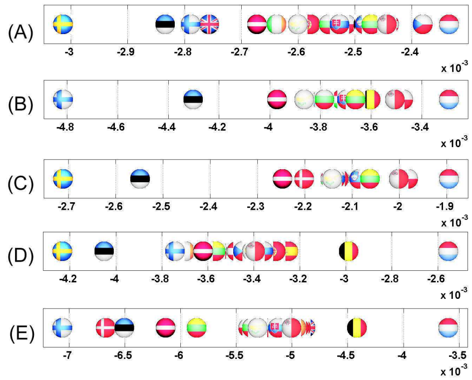

Fig. 5. **Axial representation of $\bar{D}$ for link modifications from Nordic countries to {FR or DE}.** (A): DK to DE. (B): SE to DE. (C): FI to DE. (D): DK to FR. (E): SE to FR.

For both investigated groups, we test the influence of an increase in collaboration from one member of the group to France or to Germany. France and Germany have been chosen as they are central members of European Union.

The Nordic countries Denmark, Finland, and Sweden have much in common: their way of life, history, language and social structure [19]. After World War II, the first concrete step into unity was the introduction of a Nordic Passport Union in 1952. Nordic countries co-operate in the Nordic Council, a geopolitical forum. In the Nordic Statistical Yearbook [19], Klaus Munch illustrates that "The Nordic economies are among the countries in the Western World with the best macroeconomic performance in the recent ten years". Nordic countries should keep cooperating to stay strong. Thus, if any Nordic country attempts to abandon these relationships in favor of other countries, it will negatively impact the remaining Nordic countries. Our sensitivity analysis illustrates this impact in Fig. 5. In these figs, we show how the relationship increase between any Nordic country towards France or Germany induces a drop in sensitivity for Nordic countries.

Referring to [21], relations between Slovenia, Hungary and Austria are tight. Hungary has supported Slovenia for its NATO membership applications and Austria has assisted Slovenia in entering European Union. Relationships between Austria and Hungary are important for both countries in the economic, political and cultural fields [22]. Concerning economy [20], Austria is one of the top import origins for Hungary and Slovenia with \$5.54B and \$2.37B respectively. Similarly to the Nordic group of countries, if Austria, Slovenia or Hungary increases its relationships with another European country, the other two will be affected. Sensitivity analysis backs up this statement as seen in Fig. 6.

B. Relationship imbalance analysis

Relationship imbalance analysis has been derived for all pairs of European countries following Eq (3). Fig. 7 shows a density plot of $F(a, b)$. We recall that if $F(a, b)$ is negative, nation a has more influence on nation b than b on a. If $F(a, b)$

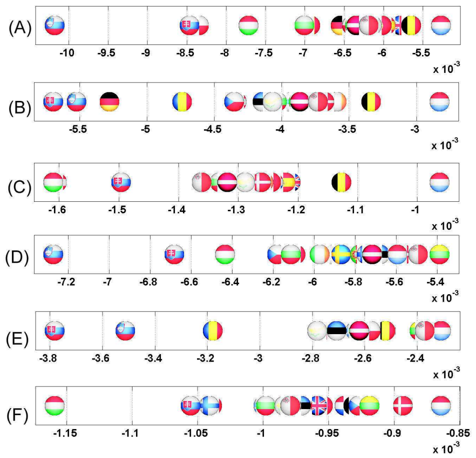

Fig. 6. **Axial representation of $\bar{D}$ for link modifications from {AT, HU and SI} to {FR or DE}.** (A): AT to FR. (B): HU to FR. (C): SI to FR. (D): AT to DE. (E): HU to DE. (F): SI to DE.

is positive, nation b dominates nation a. According to *The Globe of Economic Complexity* [28] and identical to our results in Fig. 7, Germany and France are the two largest economies in Europe. From $G_{\rm R}$ we can clearly see the dominance of France and Germany on other EU countries. Another interesting result of Fig. 7 is the equal influence between all pairs of countries created by one member of {GR, PT, IE, DK, FI, HU} and another of {BG, EE, SI, SK, LT, CY, LV, LU, MT}. These pairs have $F(a, b)$ close to zero and are plotted with orange color in Fig. 7.

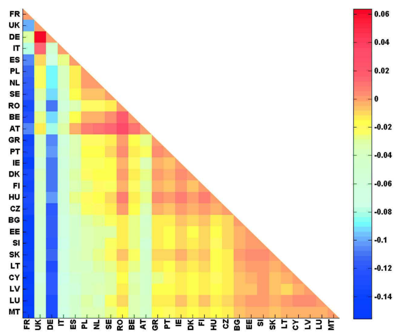

Fig. 7. **Relationship imbalance analysis: F-representation for 27 EU network.** X-axis and Y-axis represent a and b respectively.

VI. Conclusion

This work offers a new perspective for future geopolitics studies. It is possible to extract from multi-cultural Wikipedia networks a global understanding of the interactions between countries at a regional scale. Reduced Google matrix theory has been shown to exhibit hidden interactions among countries, resulting in new knowledge on geopolitics. Results show that our sensitivity analysis captures the importance of relationships on network structure. This analysis relies on the reduced Google matrix and leverages its capability of concentrating all Wikipedia knowledge in a small stochastic matrix. We stress that the obtained sensitivity of geopolitical relations between two countries and its influence on other world countries is obtained on a pure mathematical statistical analysis without any direct appeal to political, economical and social sciences.

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