The Effects of Mathematical Game Play on the Cognitive and Affective
Development of Pre-Secondary Students

Patrick D. Galarza

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ABSTRACT

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Society has consistently sought means of improving extant effective tools and designing new effective tools for educational purposes. With the consistent progression of technology, mathematical games—especially mathematical educational video games—stand out as potentially powerful mediums for helping new mathematics learners make sense of formal mathematical ideas. The aim of this study was to understand the effects that the introduction and use of a specific mathematical video game had for the cognitive, affective, and content-retentive learning outcomes of eighth graders studying elementary algebra for the first time. The three research questions guiding the study were the following: 1) How does integrating mathematical game play into a traditional eighth grade algebra curriculum impact students' cognitive learning outcomes in elementary algebra?; 2) How does integrating mathematical game play into a traditional eighth grade algebra curriculum impact students' affective outcomes about both mathematics in general and algebra specifically?; 3) How does integrating mathematical game play into a traditional eighth grade algebra curriculum impact students' content retention in elementary algebra? In order to realistically implement mathematical educational games in typical mathematics classrooms, a holistic understanding of such games' effects must be understood through research addressing several aspects of students' learning experiences.

This study utilized a mixed methodology, drawing both quantitative and qualitative data from instruments administered to a class of eighth graders split into control and treatment groups. Quantitative data primarily entailed a series of three short examinations that tested

students on their algebraic equation-solving content knowledge. Some additional metrics from game play data were recorded and discussed as quantitative data by the principal researcher. Qualitative data primarily entailed two series of interviews—one in two parts and one in three parts—and one questionnaire. Some additional observations of student interactions were also recorded and discussed as qualitative data by the principal researcher. Data on student cognition and student affect were collected at the beginning, middle, and end of the treatment. Data on student content retention were collected following a one-month recess after the treatment.

This research suggests nine attributes that typified the mathematical game play experience found in this study: three attributes regarded student cognition, four attributes regarded student affect, and two attributes regarded student content retention. Additionally, the principal researcher designed and discussed a framework for assessing the cognitive mappings formed by student game players between content featured in mathematical game play and content of formal mathematical ideas. In analyzing these mappings, the principal researcher highlighted types of interspatial cognitive connections that proved to be either fruitless or, in fact, detrimental to student game players, damaging proper development and/or understanding of formal mathematical ideas. The study's results have implications for informing future considerations of educational game design and the practical implementation of educational games as pedagogical tools within classrooms.

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Dedication

To my Catalyst, Dr. Sabrina Goldberg: Thank you for convincing me that I was capable of embarking on and completing this journey. The brief time that I spent working with you was eye-opening for me. I cannot and will not undersell the influence you have had in my transformation as an academic.

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Chapter 1: Study Introduction

To capitalize on the consistent progression of technology, mathematics educators in the past several decades have sought to take advantage of available classroom tools, particularly as ease-of-access has increased. Only in the mid-to-late 1980s did educational computer programs gain notoriety and acceptance from the academic community, although nowadays, computer usage in most schools and universities is commonplace (Lepper & Gurtner, 1989; Sheingold & Hadly, 1990). With computer usage having become so widespread, educators have had opportunities to consider new ways of representing old pedagogical ideas by galvanizing them with novel technology and information in the hopes that more powerful mathematical learning tools will emerge for future use. Among these potentially powerful learning tools is the mathematical game (Bright, Harvey & Wheeler, 1985; Civil, 2002; Devlin, 2011; Plass, Homer, & Kinzer 2015).

Salen and Zimmerman's (2004) general definition of a game, which will be used in this study, is "[a game is a] dynamic, interactive system in which players engage in an artificial conflict with a quantifiable outcome" (p. 80). We then define a *mathematical* game as a game such that all aspects of the game space—including but not limited to things such as the game's rules, the game players' strategies, and the game's hypothetical outcomes—are explicitly connected to some kind of formal mathematics. In general, games can involve any number of human players and exist across any number of mediums (up to design and representation), and do not necessarily require the use of technology. Familiar examples of mathematical games include Chess, which relies on a lattice board configuration and point-to-point transitions, or Nim, which can be solved using binary digital sums under the "exclusive or" operation.

The argument for utilizing mathematical games for pedagogical purposes is multifaceted. One consideration is that games inherently evoke the concept of childhood and adolescent "play,"

which can be extremely useful for entertaining abstract ideas (Vygotsky, 1980). Appealing to situated learning theory, the environment and context in which this play exists allow learners meaningful interactions with mathematical concepts (Brown, Collins, & Duguid, 1988). Several studies have already been conducted showing that the use of games for the teaching and learning of mathematical ideas is viable (Ke, 2006; Kebritchi, 2008; McCue, 2011; van den Heuvel-Panhuizen, Kolovou & Robitzsch. 2013; Wijers, Jonker & Drijvers, 2010).

However, many individuals still express concern and caution regarding the misappropriation of games as tools in education. Some researchers warn that games, viewed as an efficient educational resource, run the risk of being overly relied upon, thus limiting peer-to-peer and peer-to-instructor interaction, while others argue that games' true utility in helping students achieve conceptual mastery may be sidelined by developers and educators who are satisfied with building only procedural accuracy (Devlin, 2011; Kitchen & Berk, 2016).

In response to these concerns, it is important that educators investigate the effects that meaningful mathematical game play has on mathematics learners; rather than merely instilling procedural accuracy, mathematics games should help students strive further, in the hopes of achieving conceptual understanding and strategic competence, among other attributes (National Research Council, 2001). Achieving this level of meaningful mathematical game play can be challenging, as multiple social and psychological factors may impact the experience of the learner(s). Researchers have reported differences in cognitive and affective changes for students in both traditional mathematics courses and mathematics courses that include mathematical games, as students of varying gender, prior knowledge, and socioeconomic status can have extremely different experiences even when studying in the same course with the same curriculum (Bryce & Rutter, 2003; de Jean, Upitis, Koch & Young, 1999; Feng, Spence, & Pratt, 2007; Fennema, 1978;

Jordan & Levine, 2009; Reyes & Stanic, 1988).

The limited extant literature is inconclusive on the matter of content retention (e.g. the ability to reproduce, explain, and comprehend mathematical ideas experienced earlier), sometimes arguing that games induce enhanced student retention because of their repetitive nature and other times stating that it is the dynamism of game play that provides students with memorable experiences that contribute to enhanced retention; competing perspectives must consider the variety of game attributes and qualities that create unique game spaces, making generalizations challenging. Still other studies say that it is not universally true that retention is enhanced by the playing of mathematical games, but rather, that there is no significant difference in students' retention (Arici, 2008; Chow, Woodford, & Maes, 2011; Pivec, Dziabenko & Schinnerl, 2003; Ricci, Salas, & Cannon-Bowers, 1996).

Based on the aforementioned literature, it can be argued that the implementation of curricula including mathematical games requires further research to create a more conclusive picture of different learners' changing cognition and affects with respect to mathematics, as well as students' abilities to retain content encountered in both a traditional classroom format and a non-traditional game-based format.

Purpose for Study

This research explored the cognitive and affective developments of eighth grade algebra students as they utilized, alongside their traditional curriculum, a mathematical game—in particular, one played on a technological device—to augment their learning experiences within the classroom. Additionally, students' abilities to retain content encountered in their main course of study and through the mathematical game were examined one month following the study's treatment phase. Students' gender and prior mathematical knowledge were considered and examined in order to draw conclusions about various benefits different learners might bring to or

receive from a game-enhanced curriculum.

The following research questions guided the study:

1. How does integrating mathematical game play into a traditional eighth grade algebra curriculum impact students' cognitive learning outcomes in elementary algebra?

2. How does integrating mathematical game play into a traditional eighth grade algebra curriculum impact students' affective outcomes about both mathematics in general and algebra specifically?

3. How does integrating mathematical game play into a traditional eighth grade algebra curriculum impact students' content retention in elementary algebra?

Procedure for Study

The study took place over a four-month period separated into three phases: the intervention-phase, the break-phase, and the retention-phase. The intervention-phase was the two-month period in which the bulk of the study was be conducted. The break-phase was the one-month winter recess period following the intervention-phase in which students did not have mathematics courses due to time off from school; this was important because the pause in mathematics learning allowed the study to collect meaningful data relating to content retention. The retention-phase was a week long period following the break-phase in which the study collected data for content retention.

The mathematical game used in this study, *Dragonbox Algebra 12+*, is a single-player game played on a personal computing device. It saves player progress through the game and allows opportunities for revisiting and reassessing completed problems.

The study focused on an eighth-grade algebra class with 30 students taught by a single instructor. The class was divided into a control and treatment group. The control group participated in its usual learning of algebra content, while the treatment group spent some of its class time

participating in a game-based algebra-learning experience. During game play sessions, the researcher acted as an aide to the primary instructor to help guide game integration; this effectively meant that he maintained the game play equipment and supervised students during their game play sessions with minimal interaction otherwise. Further, students who participated in the study were selected for interviews, and completed a selection of examinations and questionnaires, as described later.

To address research question 1, both quantitative and qualitative data were collected. First, quantitative data were collected to measure student cognition via the "Algebra Game's Ability Tests (AGATE 1, AGATE 2)," a pair of similar tests that were designed to measure the cognitive mathematics abilities and skills that the study's game imparted to or reinforced for students. The AGATE aligned with the standard algebra course curriculum and were verified as a set of content-appropriate examinations by the algebra course instructor. To ensure content alignment and facilitate the verification process, the examination's construction drew on questions from examinations used in previous iterations of the standard course of study. The AGATE 1 was utilized as a pretest administered at the start of the intervention-phase while the AGATE 2 was utilized as a posttest administered at the end of the intervention-phase. The two examinations were administered to both student populations. The primary difference between the two examinations was that, while each exam's questions covered identical content, numbers and variables were changed between the pretest and posttest examinations. This was a superficial change and did not meaningfully impact students' abilities to utilize algebraic knowledge. Results of the AGATE 1 were used to establish a baseline for individual and classroom knowledge. Additionally, the AGATE 1 was used to establish and verify the comparability of the treatment and control groups. Results of the AGATE2 were analyzed using the statistical techniques of analysis of co-variance

(ANCOVA) and emphasized the correlation between aspects such as a student's learning balance (control group vs. game-enhanced group) or gender, and AGATE 2 performance. The data of the AGATE 1 and AGATE 2 were reviewed on both an individual-level (thus granting utility for some qualitative data collection) and a class-level.

Second, qualitative data were collected via on-site researcher-student interviews to help make sense of changes and developments in student cognition. Extended dialogue between the researcher and students of the game-based condition was required to make sense of and track the aforementioned changes and developments, so several interviews were conducted at the half-way point and end of the intervention-phase. Students were pseudorandomly selected from the game-enhanced course to participate in both rounds of the cognition-focused interviews. The interview protocol sought to answer thematic questions such as "What's the connection between students' game play and students' corresponding mathematics output?," "How are students using the game?," and "How does game play influence students' approaches to mathematical (algebraic) tasks?" A sample question was "do you think that playing *Dragonbox* has changed the way you understand your regular Algebra course content, for better or worse? Why or why not?" Additionally, some interview questions were student-specific when drawing on data collected from the AGATE examinations, as mentioned earlier. Interviews were recorded and video data were replaced by transcriptions. Interview responses were axially coded, and emerging themes were paired with (or against) results from the quantitative data when applicable. Together, the qualitative and quantitative data were used to answer how student cognition was affected by game play.

To address research question 2, qualitative data were collected via on-site researcher-student interviews to help interpret students' changes and developments in affects. This protocol differed from the one used for collecting data on students' cognitive changes and did not have the

exact same set of participants. These interviews were conducted at the start, middle, and end of the intervention-phase of the study to show how student affects changed in the game-based condition. Students were chosen pseudorandomly from the game-enhanced course to participate in all rounds of the affect-focused interviews. The protocol for these interviews drew on ideas found in Tapia and Marsh's ATMI (2004), "Attitudes Towards Mathematics Inventory," among other sources. Like the ATMI, this interview protocol encouraged students to describe the intensity of affects and relationships, but unlike the typical selections on a Likert scale, these intensities were justified and examined through researcher-student dialogue to gather evidence for questions such as "What's the connection between students' game play and affect with respect to mathematics?" Sample prompts provided to students included "I think it's useful that I study mathematics in school?" or "I am often confused when doing mathematics." Interviews were recorded and video data were replaced by transcriptions. Interview data were axially coded to find emerging themes.

To address research question 3, quantitative data to measure content retention were collected via the Algebra Game's Ability Tests: Retention Module (AGATE 3), an examination structured and designed identically to the aforementioned AGATE 1 and AGATE 2. The AGATE3 was administered during the retention-phase of the study. It was administered to all students in both groups. Results of the AGATE3 were analyzed using ANCOVA, emphasizing the correlation between aspects such as a student's learning group (control classroom vs. game-enhanced classroom) or gender, and AGATE 3 performance.

To further address research question 3, qualitative data were collected via an open-ended questionnaire to help interpret what impacted students' content retention. The questionnaire was designed independently by the principal researcher and was administered only to students studying a game-enhanced curriculum. It was utilized following completion of the AGATE3 and sought to

answer questions such as "What aspects of mathematics learning do students find most memorable?" and "What aspects of game-based learning—if any—do students attribute to retention gains/detriments?" A sample question was "What content in your algebra course have you found most memorable? Why?" Questionnaire data were axially coded, and emerging themes were paired with (or against) results from the quantitative data when applicable.

Additional information about all instruments utilized is provided in Chapter 3.

Chapter 2: Literature Review

Introduction

Game-based learning, especially for mathematics study, has a history extending back to the times of ancient civilizations and a contemporary life in our modern era, finding special promise and excitement with the birth of new technologies, particularly thanks to digital media. In this literature review, I will examine the extant literature on the nature of mathematical game play and the general overview of mathematical game play's effects on student learning, cognition, affect, and retention, and several controversies and critiques about the use of games for education.

What is Play, What is Game Play, and Why Should We Care?

There is evidence of some form of game-playing in most societies across human history, and with good reason: play is essential to the development and maintenance of the human psyche, whether the player recognizes it or not. Although we might trace our records on the nature of play back to the ancient Greeks and Romans (Fagan, 2017; Goldhill, 2017), the watershed treatises describing the benefits of play—particularly for adolescent development— emerge in the mid-to-late 20th century by way of Lev Vygotsky and Jean Piaget.

In 1933, Vygotsky wrote on *Play and its Role in the Mental Development of the Child*, in which he attempted to characterize play before emphasizing its importance for the developing mind. Vygotsky wrote on play after discussing it in his 1930 text *Mind in Society* as a means by which a child's zone of proximal development (sometimes referred to as ZPD) may expand or shift. For an individual, the zone of proximal development is described by Vygotsky as "[the difference between a child's] actual developmental level as determined by independent problem solving and the level of [that child's] potential development as determined through problem

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solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, pp. 85-86). Notably, Vygotsky takes issue with characterizations of play as always yielding pleasure and being loosely structured. As counterexample to the former, Vygotsky might say that a player can reap no pleasure from play if some particularly important outcome was not achieved; as counterexample to the latter, he might suggest that a girl who plays as the mother of her doll subconsciously imposes upon herself the rule that "I will only do as I feel a mother would, and nothing else." He argues that there is a redefining of characteristics of some real-world situation—during play, a new world is imagined. However, Vygotsky also comments on how play evolves as the player's mind matures: "the development [from] an overt imaginary situation and covert rules to [a covert imaginary situation with overt rules] outlines the evolution of children's play" (p. 94). It is here that Vygotsky reveals his central theory of play: the individual's concept and enactment of play serves as an evolving psychological device that transitions the player from preferences for ambiguous purpose bereft of structure towards preferences for meaningful purpose reliant on structure. Vygotsky comments that "creating an imaginary situation can be regarded as a means of developing abstract thought. The corresponding development of rules leads to actions on the basis of which the division between work and play becomes possible" (p.100). Vygotsky concludes that it is this new understanding of abstract thought that allows developing minds to attribute meaning and purpose to the objects in their surrounding worlds, marking powerful developmental growth.

In *Play, Dreams, and Imitation in Childhood* (1952), Piaget describes play as being "a modification, varying in degree, of the conditions of equilibrium between reality and the ego" (p. 4). He draws primarily on the work of Groos, Hall, and Buytendijk while constructing a list of

criteria describing the characteristics of play. Piaget refers to his earlier theory of assimilation[1] and accommodation[2], put forth in *The Psychology of Intelligence* (Piaget, 2005) to describe play, first, as having an "opposition between assimilation of objects to the child's activity and accommodation of the child's activity to objects" (p. 2). In this sense, play acts as a real-world parallel to the psychological balancing between assimilation and accommodation, since players choose how to play corresponding to their understanding of the objects with which they play, but must also abide by some hidden mandates of those objects which informs the way(s) play is conducted; the player is afforded opportunities for both real and imagined reconceptualizing. Piaget's sense of play, much like Vygotsky's, imagines a new world. Although Piaget goes on to discuss play as being spontaneous, pleasurable (contrary to Vygotsky), disorganized (also contrary to Vygotsky), and free of conflict, Piaget's most salient point is his conclusion that play indicates a predominance of assimilation over accommodation in a developing mind. Corroborating Vygotsky's findings, Piaget writes that it is by considering and reconsidering the real-world meanings of the things a player encounters that he or she achieves a heightened understanding of the role or roles those things play. A comparison of Vygotsky's views and Piaget's views on play is included in Figure 2.1.

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[1] "…'Assimilation' may be used to describe the action of the organism on surrounding objects, in so far as this action depends on previous [behavior] involving the same or similar objects" (Piaget, 2005, p.7).
[2] "Conversely, the environment acts on the organism and…we can describe this converse action by the term 'accommodation'" (Piaget, 2005, p.7).

Theories of Play:
Play...

...is capable of expanding a learner's ZPD

...evolves from an overt imaginary situation with covert rules to a covert imaginary situation with overt rules

...transitions a learner towards finding purpose through structure

...is an opportunity to redefine real-world attributes

...is an opportunity to apply imagined attributes to reality

...is free from real-world conflicts

...helps develop abstract thought

...is always pleasurable

...is disorganized

...is spontaneous

...has a predominance of Assimilative actions over Accommodative actions

Vygotsky

Piaget

Figure 2.1: A comparison of Vygotsky and Piaget's theories on play.

Regarding play as being psychologically beneficial established a baseline for further inquiry, and academics refined the general notion of play into specific types of play. For our purposes, we look at some definitions specifically surrounding "game play."

As stated in Chapter 1, although there are many competing definitions for what a game is, this literature utilizes the one put forth by Salen and Zimmerman (2004) stating that a game is "a dynamic, interactive system in which [a player or several] players engage in an artificial conflict with a quantifiable outcome" (p. 80). This requires some definitional unpacking. Note first that

the game is portrayed as a system that a player may enter. By entering, a player is granted agency which manifests as interactivity with the system. Because of this interactivity, the system is put into a state of dynamic flux which allows it to change based on player actions. Presumably, these actions are set towards achieving an imagined goal: the resolution of some artificial conflict. Finally, there is some sort of quantifiable outcome recognizing the player's impact on the system and clarifying whether and potentially how the artificial conflict was resolved.

In preparing this definition, Salen and Zimmerman rigorously reviewed definitions of the term "game" put forth by other authors, as well as closely associated definitions, such as ones for "playing a game." From this definition, we can make further refinements, such as talking about a "mathematical game," which is defined by the author of this text as a game for which the entire framework of the game space is explicitly connected to some kind of formal mathematics; or perhaps more generally, an "educational game," which Hogle (1996) defines as "a game …designed to be used as a cognitive tool" (p. 7). Note that there are distinctions inherent among playing a mathematical game to learn new content, playing a mathematical game to better understand content one is in the process of learning, and playing a mathematical game to practice already known content; mathematical games can be played for any of these reasons, and literature has shown that playing a mathematical game has varying effects depending on if the game is played in a pre-instructional, co-instructional, or post-instructional phase of learning (Bright, Harvey & Wheeler, 1985). We may also preliminarily describe a "game space" as the physical, digital, and/or imagined locations in which the game's player(s) interact with the artificial conflict for the duration of the game. For example, in a game of basketball, the game space houses both the players' physical interactions on the basketball court and the imagined thoughts generated by each player to navigate the game. Figure 2.2 shows a hierarchy of game

definitions. With all our working definitions in place, we can turn our attention to the greater body of literature on the pedagogical uses of educational games.
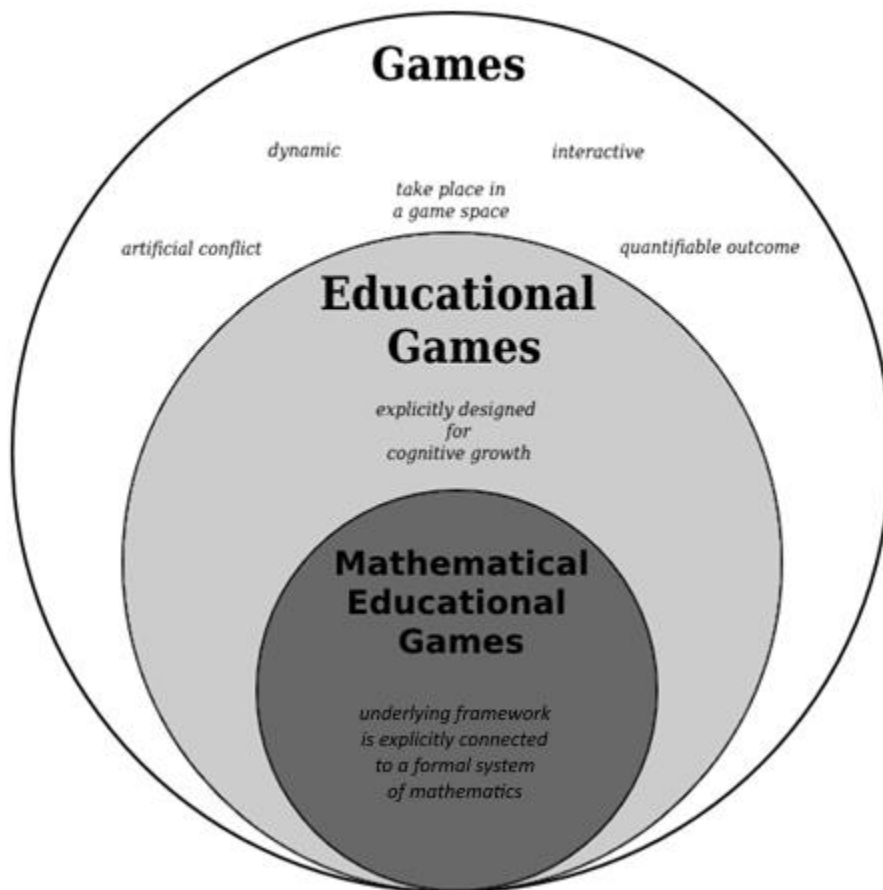
Figure 2.2: Our hierarchy of game definitions

The First Wave of Teaching and Learning with Mathematical Games: Research through the Late 1960s

Modern mathematical games research finds its catalysts—technological advancements and new theories of cognitive psychology—in the late 20th century. However, prior to this point, mathematical games research was still carried out, but with slightly variant research goals and a weaker foundation. This section and the following two provide a chronological analysis of the three epochs of research related to mathematical games as identified by the author.

Since the integration of digital learning technologies into most school and university

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classrooms didn't begin until the 1980s (Lepper & Gurtner, 1989; Sheingold & Hadley, 1989), the era here identified as the First Wave of game-based research in classrooms saw games represented or constructed via immobile physical utilities, thus complicating their implementation processes. Accordingly, among the little mathematical game-based research that exists up to 1970, the studies that were rigorously implemented were primarily concerned with student achievement and cognition, and took one of two approaches: they either masked extant drilling scenarios with a superficial conflict (e.g. timed equation solving, recognition games, etc.), or generated completely novel educational games whose full constructive processes and rules had to be included in the literature. For instance, early work done by both Hoover (1921) and Wheeler and Wheeler (1940) described flash card use for, respectively, playing an arithmetical drill game with third graders, and playing a bingo-esque numeral-recognition game with first graders. In contrast, Bastier's (1969) report on several arithmetic and geometric game play experiences with students ages 10 and 11 is accompanied by several pages of diagrams, lists of materials, and game play instructions. This allows readers to construct the games so that the study's results might be replicable and so that the games could be shared on a wider scale. Steiner and Kaufman (1969) write on a selection of their "operational systems games," meant for teaching algebra at the elementary and secondary levels; they describe only a few basic games, stating that a compendium more fully delineating the games will be published separately by McGraw-Hill. It is important to note a prevalent trend that will be challenged later: in the majority of early mathematics game-based research, students only encounter formal mathematical ideas at what Steiner and Kaufman call a "pre-mathematical level" (p. 445); students usually did not directly engage formal mathematics content in these games, but they found related ideas that could facilitate the learning of select concepts during students' later

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formal coursework. It wasn't necessarily the case that games directly engaging with formal

mathematics content were impossible to construct, but it was challenging to construct such

games while still making the game play experiences meaningful and distinct from the usual kind

of formal mathematics study; for example, in the case of Hoover's game, while students

technically engaged with formal mathematical ideas, the game play was virtually

indistinguishable from traditional drilling exercises. One notable counterexample showing a

game that does meaningfully integrate mathematical thinking into a distinct game play

experience arises in Layman Allen's game series *WFF 'N PROOF*, which allowed players to toss

sets of customized dice and compete to make mathematics statements based on the rolled

characters from their selected game version: classic *WFF 'N PROOF* for symbolic logic, *ON-

SETS* for set theory, or *EQUATIONS* for elementary arithmetic (Allen, Allen & Miller, 1966;

Allen, Allen & Ross, 1970; Allen, Jackson, Ross & White, 1978).

The Second Wave of Teaching and Learning with Mathematical Games: Research from the Early 1970s through the Late 1980s
The mid-20th century's emergence of theories validating the importance of play

galvanized what is here described as a Second Wave of game-based teaching and learning

research that emphasized finding generalizable properties of useful games for mathematics

learning and new methods for designing and sharing new games. This stands in contrast to the

First Wave, which primarily aimed to adapt extant commercial games for more limited classroom

utility.

Keith Edwards and David DeVries were prominent researchers in this new wave and

produced several early texts examining the questions "how should games be played in the

classroom, and can they affect more than just achievement?" In *Games and Teams: A Winning

Combination* (1972), Edwards, DeVries, and colleague John Snyder took Allen's *EQUATIONS*

and implemented their "Teams-Games-Tournament" (TGT) system for four classes of seventh graders over nine weeks, treating two classes as non-game-playing control groups, and two classes as game-playing experimental groups. They concluded that "combining...*EQUATIONS* with team competition significantly increased students' mathematics achievement over that of a traditionally taught class. The effect was observed for [game-specific skills] as well as more general arithmetic skills" (p. 20). Following this significant success for using mathematics games in the classroom to improve student cognition and achievement, Edwards and DeVries reimplemented their *EQUATIONS*/TGT system in further studies, this time not only revisiting student achievement in populations comparable to those of their initial study, but also analyzing multiple facets of student affect; they generally found that the game play improved student affect, specifically by encouraging peer-to-peer communication, lowering students' perceived course difficulty, and increasing overall student satisfaction (DeVries & Edwards, 1972; Edwards & DeVries, 1974; DeVries, Edwards & Slavin, 1978). Interestingly, Edwards and DeVries found that their TGT implementation did *not* produce significant changes to student achievement or affect in seventh grade social studies classes utilizing the commercially produced game *Ameri-card* (1974); the success of the *EQUATIONS* implementation, therefore, is mainly attributed to the notion that the game's concepts are ever-relevant in a mathematics course (whereas *Ameri-card* tested several bits of factual trivia, like the names of US states and geographical regions). Contemporary studies found that combining TGT with other mathematics games could help student achievement at the seventh-grade level (Hulten, 1974), but not necessarily at the fourth- and fifth-grade levels (Slavin & Karweit, 1979).

The resounding success of *EQUATIONS* allowed Allen to modify the game so that it could be one of the first mathematical games to be ported from a physically-playable format to a

digitally-playable format (Allen & Ross, 1975). Interestingly, the digital version of *EQUATIONS*, which was effectively single-player, did not improve student achievement and affect in the ways that the physical, multiplayer variant had (Moore, 1980). Nonetheless, several other mathematics games began getting ported to computer systems, or completely designed for computer systems from the ground up, including the following games: *POE* (Moore, 1980), *Fish Chase* (Kraus, 1981), *Nim* (Kraus, 1982), *Speedway* and *Tug-of-War* (McCann, 1977). This marked an important turning point for the actual means of game delivery and implementation.

With mathematical games quickly gaining popularity and seizing upon the benefits of digital programing, the literary corpus turned back towards the question "how should a mathematical game be designed?" This query is deeply treated throughout the prolific work of George Bright, John Harvey, and Margariete Wheeler. Their writings typically addressed questions about in-game mathematical representations and associated constraints (e.g. Should fractions be represented numerically? pictorially?) (1980a; 1981). Their paper, *Incorporating Instructional Objectives into the Rules for Playing a Game* (1979a), is an essential work of our Second Wave. In it, the authors affirmatively respond to the question "when instructional objectives of a game are incorporated directly into the rules, is learning interfered with or enhanced?" During play sessions of their various arithmetic games aimed at improving elementary schoolers' conceptions and procedural accuracy when working with multiplication and division questions, directly incorporating formal mathematical concepts and language into the game's rules did not make the game any less effective; this showed a feasible alternative to the "pre-mathematical" approaches described by Steiner and Kaufman and bolstered the viewpoint that more games like *EQUATIONS* could be produced. These results are elaborated upon in a pair of studies done by Bright, Harvey, and Wheeler (1979b; 1980b), culminating in

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their NCTM-commissioned report, *Learning and Mathematics Games* (1985), which discusses the best instructional and taxonomic levels to utilize mathematical games for learning. This report ultimately concluded that, based on a review of sixty-seven texts on learning with mathematical games, introducing students to games that incorporate formal mathematics content following some in-class instruction would be best for improving students' knowledge, comprehension, application skills, and mathematical analyses, stating that game play before or during the learning of formal content more frequently led to mixed results.

Additional support for mathematical game-based learning came from the emergence of situated cognition theory. In *Situated Cognition and the Culture of Learning* (Brown, Collins & Duguid, 1988), Brown, Collins, and Duguid write that "many teaching methods implicitly assume that conceptual knowledge is independent of the situations in which it is learned and used;" they instead propose a theory of teaching and learning called "cognitive apprenticeship," which emphasizes the environment wherein acquired information is "situated" (p.3). If knowledge is indeed situated, then it is not necessarily true that abstraction of information is the key to content acquisition and transfer. The authors argue that knowledge should be viewed instead as tools that require practical, situated uses (referred to as authentic [mathematical] activity) if they are to provide learners with deeper understanding. Situated cognition directly adds credibility to teaching and learning via mathematical games, as the games situate the learner's play within a game space that provides the learner with a context for engaging with and doing mathematics, forming the cognitive apprenticeship. Within this space, the learner acquires a practical, exercised, and consistent understanding of mathematical content that can potentially be used to scaffold an understanding of related abstract concepts.

The Third Wave of Teaching and Learning with Mathematical Games: Research from the Early 1990s to the Present

With all the necessary pieces in place by the end of the 1980s, mathematics game-based research seemed more viable than ever, but researchers were still reviewing past information and seeking new information about what such research should look like, and how it could benefit aspiring learners. With some ideas of generalizable good-design principles and practices discovered from the Second Wave, the following era—here identified as the Third Wave—emphasized instead game utility that could be specialized.

Early in the Third Wave, several overviews on the state of game-based learning (Amory, Naicker, Vincent & Adams, 1999; Hogle, 1996; Randel, Morris, Wetzel & Whitehill, 1992) were published. These texts typically reviewed extant literature from the First and Second Waves on whether game play in any subject could improve learner affect, content retention, reasoning skills, or higher order thinking. Reviews were mixed. The literature found that game-based learning was not generally more effective than classroom instruction for the purposes of improving cognition and learning outcomes, but that there was a higher chance of success in two cases: first, when specific content goals were targeted, and second, when computer games were utilized (Hogle, 1996; Randel et al., 1992). Both characteristics were emergent trends specifically in mathematics game-based learning. Discussion about games' potential for improving learner affect and retention also abounded and were accompanied by questions about what genres of games might be best for learning, as well as questions about whether gender differences impacted the game play experience (Amory et al., 1999; Hogle, 1996). Notably, during this time period, diverse game-based studies (many of which featured mathematics learning games, specifically) were conducted that began directly engaging with all of these questions (Blum & Yocom, 1996; Inkpen et al., 1994; Koran & McLaughlin, 1990; Lawry et al.,

1995; Murray, Mokros & Rubin, 1998; Ricci, Salas & Cannon-Bowers, 1996).

At the turn of the millennium, mathematics games research found itself as a budding field with a solid theoretical foundation, eager to capitalize on the advancements of digital technologies in the forms of more powerful personal computers, enhanced handheld devices, and improved video-game-rendering hardware from the commercial game industry. Games could now be shared more easily on a global scale and implemented en masse within the classroom in either single-player or multiple-player formats. Accordingly, mathematics games educators and researchers set out to begin deeply exploring the variety of queries that had been posed, but only topically analyzed in the pre-millennium era.

The Strands of Mathematical Proficiency

In 2001, the National Research Council published its landmark report on the state of mathematics education in the United States, *Adding It Up*; early in the report, the authors present five components (referred to as "strands") of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (p.5). They are defined in the text as follows:

- Conceptual Understanding—Comprehension of mathematical concepts, operations, and relations

- Procedural Fluency—Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

- Strategic Competence—Ability to formulate, represent, and solve mathematical problems

- Adaptive Reasoning—Capacity for logical thought, reflection, explanation, and justification

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- Productive Disposition—Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (p.116).

The first four strands deal specifically with learners' cognitive growth potentials, while the fifth strand deals with learners' affective growth potentials. Several authors have illustrated how mathematical games can directly empower learners to master most, if not all, of these strands and have pointed out the ways in which mathematical games provide learners with additional learning benefits. In the second half of this literature review, I will do the following: examine the five strands as they overlap and interweave with mathematical game-based learning for cognitive, affective, and retentive growth; introduce and briefly discuss the game chosen for the purposes of the study this review accompanies; and provide some popular critiques of mathematical game-based learning.

On Cognitive Change

"Cognition" and "cognitive change" are challenging terms to define, particularly because of the broad array of topics related to thinking that they may encapsulate. Typically, when these terms are encountered in a game-based setting designed to facilitate mathematics learning, they are somehow used to measure the fluidity of students' reasoning, the depth of students' understanding, how students justify and reason within the mathematical space, the mastery of learning outcomes, or other closely related ideas.

In the Second Wave of research described earlier, Brown, Collins, and Duguid constructed the theory of situated cognition, which advocates for the learning of new (in our case) mathematical content in environments that allow for authentic and meaningful exercise of mathematical ideas supporting the emergence of mathematical understanding. Explorations into

situating meaning and personalization for mathematics learning has proven effective in traditional mathematics classrooms (Bernacki & Walkington, 2014; Stephens & Konvalina, 1999; Toh, 2009; Walkington, Cooper & Howell, 2013), and the same strategy is at least as potent in game-based learning. Related to the theory of situated cognition is James Paul Gee's theory of "Meaning as Action Image," which argues that because humans usually think more experientially than logically, the most effective types of learning must emerge from routines in which learners are slowly able to acclimate to new concepts by having continued experiences with those concepts (Gee, 2004). However, it should be noted that the implementation of a teaching and learning approach based on the theories of situated cognition and/or meaning as action image may be experientially and practically different depending on the content being targeted for teaching. The case of situating tasks to develop students' cognition pertaining to formal mathematics content must intrinsically differ from the case of situating tasks to develop students' cognition pertaining to, for example, musical composition; not all strategies that effectively contribute to an apprenticeship of one field will be applicable during an individual's apprenticeship in another field because of, minimally, social differences, neural differences, and representational differences that exist between any two fields (Clancey, 1994). Collins, Brown, and Holum (1991) investigated differences between effective cognitive apprenticeships for teaching dialogue (by Palincsar) and mathematical problem solving (by Schoenfeld). In their observations, it's clear that although similar methodologies may be employed across disciplines to develop cognitive apprenticeships (e.g. modelling, coaching, scaffolding, etc.), the corresponding implementations differ mechanically across subjects—whereas Palincsar's students are heavily reliant on consistent verbal discussion and spoken or written dialogue to develop content mastery throughout their apprenticeships, Schoenfeld's students pause and

translate their plain-speech dialogue into written representations of polynomial equations, consider changes that might be made to those representations either with peers or in individual reflection, then reinitiate or rejoin the greater discussion. This example of dialogic teaching demonstrates a lesser reliance on visualized content than does this example of mathematics teaching, but both qualify as creating effective cognitive apprenticeships. Up to content, it's entirely possible that a different lesson of dialogic teaching and a different lesson of mathematical problem solving could have cognitive apprenticeships near-identical in structure; each apprenticeship is heavily influenced by the contexts and resources available to and for instruction, and many characteristics of games and game spaces afford instructors unique characteristics and opportunities that can be drawn upon to appropriately situate mathematics content for this type of learning.

Because games can provide new learners with "sandboxes" (places in which learners can explore concepts within the system's constraints, but usually without repercussions) and/or recurrent content structured via scaled difficulty gradients, they situate learning in a synthesized world that inherently makes the learner's trials and explorations meaningful and authentic (Gee, 2004). Because of difficulty gradients, game play can also be structured to ensure that new challenges are always within students' reach (Devlin, 2011); additionally, game designers and mathematics instructors can aid students gradually attempting these new challenges by scaffolding the learning of new content, defined by Wood, Bruner, and Ross (1976) as "[a] process that enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts" (p. 90). Together, these aspects effectively create an artificial version of Vygotsky's zone of proximal development.

Games that feature reusable, flexible resources allow learners opportunities to experiment

with new concepts to gain deeper understanding of them. This makes mathematical games (especially mathematical *video* games) strong candidates for helping students achieve procedural fluency, particularly because they provide sustained periods of time for learners to review and connect a myriad of examples related to any specific topic. However, even in cases in which players/learners may be forced to replay a game because of failure, learning emerges (Devlin, 2011; Squire & Barab, 2004). Oftentimes, failure in game play provides the player with an impetus to revisit his or her prior knowledge and seek out a new strategy, encouraging adaptive reasoning. Whereas many mathematics curricula fail to interweave topics, mathematical games can reintroduce concepts throughout the time spent playing; this can be done by creating special objectives that might challenge the player to deviate from the forward path and revisit or reexplore previously cleared content, or content that interweaves concepts from previous game experiences (Devlin, 2011).

Further, in a game designed chiefly for the learning of mathematics, the game world is inextricably linked to the doing of mathematics; together, they form an endogenous fantasy[3] that stands apart from the traditional classroom context (Ke, 2008). In cases for which the formal and informal learning environments are so tightly wound together, there are often opportunities for a type of adaptive reasoning that Holbert and Wilensky term "epistemological integration:" a database of knowledge seamlessly fused from both game experiences and formal content knowledge (2012). The benefits afforded by situating mathematics in an endogenous space are elaborated upon by many authors (Gee, 2005a; Gee, 2013; Huizenga, Admiraal, Akkerman & ten Dam, 2011; Pivec, Dziabenko & Schinnerl, 2003; Rosas et al., 2003; Wijers, Jonker & Drijvers, 2010; Van Eck, 2006a), but are particularly well characterized by Keith Devlin in his 2011 text

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[3] E.g. The game space and the mathematics space are seamlessly intertwined and overlapped.

Mathematics Education for a New Era. Devlin notes that because funding and generating real-world problem-solving scenarios on a massive scale is often neither practical nor possible, capitalizing on the situated worlds of games (which can be computationally modified and generated) may help instructors assess student understanding and learning. He compares student learning experiences in the synthesized, virtual worlds of games to solving "real world" word problems, like those appearing on the NAEP surveys for the past several decades. These word problems often confound students with artificial, unrelatable scenarios and can inhibit learning; Devlin maintains that doing mathematics in game-based environments likely has the potential for developing more powerful strategic competencies. Additional support for developing strategic competencies via educational game play is bolstered by Gee's characterization of games as providing players/learners with information both "on demand" (e.g. easy-access to rules, controls, or hints) and "just in time" (e.g. scaffolding scenarios in time-aware, compact chunks that ensure information is not diffusely distributed), allowing learners to take full stock of their information base before approaching a problem (Gee, 2003; Gee 2004).

As reported by the National Research Council for science learning, game-based learning has also proven itself as a powerful means of improving concept concretization—that is, helping learners understand and make sense of concepts that initially seem abstract (2011). This is evident based on a wealth of studies describing game-based treatments producing improvements in learners' spatial cognition and visualization (Barlett, Vowels, Shanteau, Crow & Miller, 2008; Feng, Spence & Pratt, 2007; Granic, Lobel & Engles, 2014; Shute, Ventura & Ke, 2015; Terlecki & Newcombe, 2005) which are potentially helpful for teaching about geometric concepts. Being able to provide learners with a means of concretization makes game-based learning good, in general, for conceptual understanding—arguably the most challenging strand of mathematics

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proficiency. Devlin demonstrates that improper conceptual understanding can go undetected for years when he discusses the work of Uri Leron, an Israeli mathematician, who demonstrated that university students in mathematics and computer science maintain false conceptions of mathematical functions they acquired in grade school, such as that a function applied to an argument changes the argument (p. 114). Such students might be able to operate with procedural accuracy and fluidity, and yet will never be able to recognize their own conceptual misunderstandings. Because the challenges of conceptual understanding extend beyond the realm of mathematical games, there are naturally occurring difficulties, from a design perspective, with inducing conceptual understanding in an artificial game space. However, some studies have shown that intentionally concretizing abstract concepts can help students achieve baseline or improved conceptual understanding (Galarza, 2017; Kebritchi, Hirumi & Bai, 2010). In other cases, conceptual understanding can emerge as an unintended byproduct of playing a game, as was the case with a digital clothing and furniture designer who achieved a deep understanding of geometric properties just by working through her designs in the game *Sims* (Gee, 2013). A model summarizing the ways in which aspects of game-based learning induce cognitive change is presented in Figure 2.3.
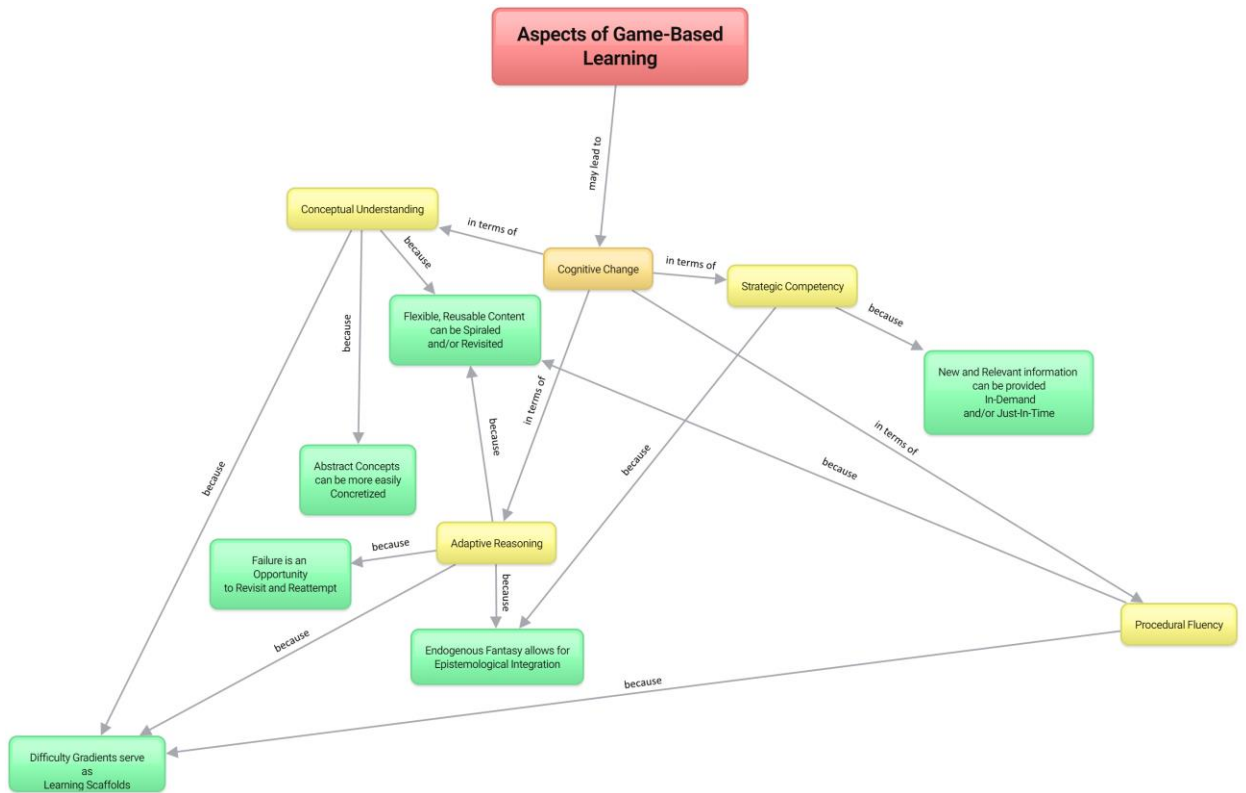
Figure 2.3: A mapping of how some aspects of game-based learning may contribute to cognitive change.

On Affective Change

"Affect" and "affective change" typically relate to an array of emotional aspects that may characterize a learner or group of learners. In studies concerning mathematics learning, these aspects may include the way(s) that a learner feels about the mathematics he or she is learning, the value that the learner perceives as being derived from his or her experiences, the opinion(s) that the learner has about himself or herself as a learner of mathematics, or the opinion(s) that the learner has about the ways in which the mathematics is being presented, just to name a few examples. Arguably, the positive variants of all these elements fall under the umbrella term "productive disposition" suggested by the NCTM, as they relate to learners being more interested in doing and learning mathematics.

One of the most important aspects contributing to a student's productive disposition in the mathematics classroom is a strong motivation. A great deal of research has been done on the effects that game-based learning has on students' motivations towards formal content acquisition, and while many authors have demonstrated cases in which motivation for formal learning has improved (Bragg, 2012; Ke, 2008; O' Rourke, Main & Ellis, 2012; Rosas et al., 2003; Squire & Barab, 2004), there have been roughly just as many cases demonstrating that motivations either did not change following a game-based learning intervention, or in fact declined (Bragg, 2007; Huizenga et al., 2011; Kebritchi, Hirumi & Bai, 2010; Tüzün, Yılmaz-Soylu, Karakus, Înal & Kızılkaya, 2008). Again, as reported by Ke (2008), a large part of motivation in game-based learning interventions is directly related to the student's connection to the endogenous fantasy that exists; in cases when games are designed for specific student populations—for example, in the handheld video games designed specifically for 1st and 2nd graders in Rosas et. al's study (2003)—this can manifest itself very clearly with overall improvements to student affect and specifically motivation. To the contrary, in cases in which games are built in a one-size-fits-all fashion, there is a seeming lesser chance of success. For example, in the Kebritchi et. al study (2010), although a treatment group of pre-algebra and algebra students were reported as having significant cognitive growth over their non-game-playing peers, those same students did not report any affinity for the games being played (a selection from Pearson's *DimensionM* series) and showed no changes in motivation as compared to the non-game-playing students. Bragg (2012) also showed that improved motivation can lead to improved focus when it comes to classroom activities: in a study assessing the effects of using games to motivate on-task behaviors in 5th and 6th grade mathematics classrooms, Bragg reported that during game-playing sessions, students were focused on their learning task 93% of the time, while during non-game-

playing sessions, they were only focused on their learning task 72% of the time. While improving motivation is always a plus, there are cases when game-playing can detract from motivation and focus. For example, Ke (2008) cautions that in cases when the game goals and learning goals are not entirely intertwined—that is, when there are aspects of game play that do not feed directly into mathematical learning—there sometimes arise opportunities for students to lose focus on the content goals. Bragg (2007) notes that game play sessions can be crippling to student motivation if the game played is too challenging; in a study playing mathematical games with 5th and 6th graders, Bragg noted that some students became disinterested in the content because, among other reasons, the concepts in the game were too advanced mathematically. Additional considerations for why student work motivations may dip during game play sessions could be that students are not as interested in games when the games are "prescribed" to them by instructors, or that students, in anticipation of playing a game in the classroom, get overly excited by the prospect of playing a commercial game, and become disappointed if the selected educational game does not meet their expectations (Wouters, van Nimwegen, van Oostendorp & van der Spek, 2013).

Another aspect of mathematics game-based learning that contributes to affective change is games' intrinsic potential for creating new social dynamics or fostering existing dynamics (Bryce & Rutter, 2003; Ito et al., 2009). Among the 97% of teens aged 12-17 that play electronic games recreationally, 76% noted that they do not strictly play games alone, indicating the ubiquity of social connections during game play (Lenhart et al., 2008). Typically, games can be categorized as being "single player," meaning that only one player engages in a conflict, or "multiplayer," meaning that multiple players work, either with or against each other, in the game space. However, regardless of whether players are working with or against each other in the

game space, they share experiences inside and outside of the game which help them form what Gee calls an "affinity space" (2005b). An affinity space is a space that arises from the shared experiences relating to some sort of content (in this case, game play) that takes on 11 specific properties; example properties include the space encouraging individual and distributed knowledge among space inhabitants, encouraging the dispersal of knowledge among space inhabitants, and not segregating inhabitants according to any personal characteristics or qualifiers. Gee discusses the websites (both official and player-generated) around a historical game, *Age of Mythology*, as being good examples of how educational game play can be used to create a pervasive learning experience that appeals to players even once game play has finished; players are encouraged to explore the affinity spaces in which they can discuss and reflect on their experiences with others, and actively seek greater learning opportunities.

Mathematical game-based learning may also help learners' affects by instilling them with a newfound sense of agency or control when doing mathematics. When playing a game, learners make choices that directly effect their in-game outcomes, adding weight and meaning to each decision (Pivec et al., 2003). Elements of identity, interaction, organic creation, risk-taking, and customization all contribute to players' sense of agency and ownership over in-game activities; these may not be readily available in a typical classroom environment (Gee, 2005a). Oftentimes, the highlight of these game-contextualized choices is that learners feel as though they are making independent decisions that help them fully understand and grasp their learning experiences (O'Rourke, Main & Ellis, 2012).

Finally, learner affect when playing mathematical games can also be influenced by changes to learners' outlooks, perceived values, and enjoyment of mathematics. Several studies have already shown that mathematical game players at the elementary- (Plass et al. 2013),

secondary- (Wijers et al., 2010), and university-levels (Amory et al., 1999) have all experienced improvements in their outlook on mathematics in general, and enjoyment of their specific mathematics courses' content. Devlin (2011) attempts to explain this by stating that because a game can be designed to purposefully embed mathematics into its player experience, mathematics in such a game space is inherently useful; this perceived usefulness of mathematics may then be taken back to the formal learning environment by the player. Granic et al. (2014) also points out that, "because [game play provides] players concrete, immediate feedback regarding specific *efforts* players have made" (p. 6), situating learning in game play is an ideal strategy for encouraging what Dweck calls an incremental[4] theory of intelligence, as opposed to an entity[5] theory of intelligence; learners who have or adopt the former theory are more likely to be motivated for success in formal learning environments (Dweck, 2000). A model summarizing the ways in which aspects of game-based learning induce affective change is presented in Figure 2.4.

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[4] "…intelligence is not a fixed trait…, but something [that is] cultivated through learning" (Dweck, 2013, p.3).
[5] "…intelligence is portrayed as an entity that dwells within us and that we can't change" (Dweck, 2013, p. 2).
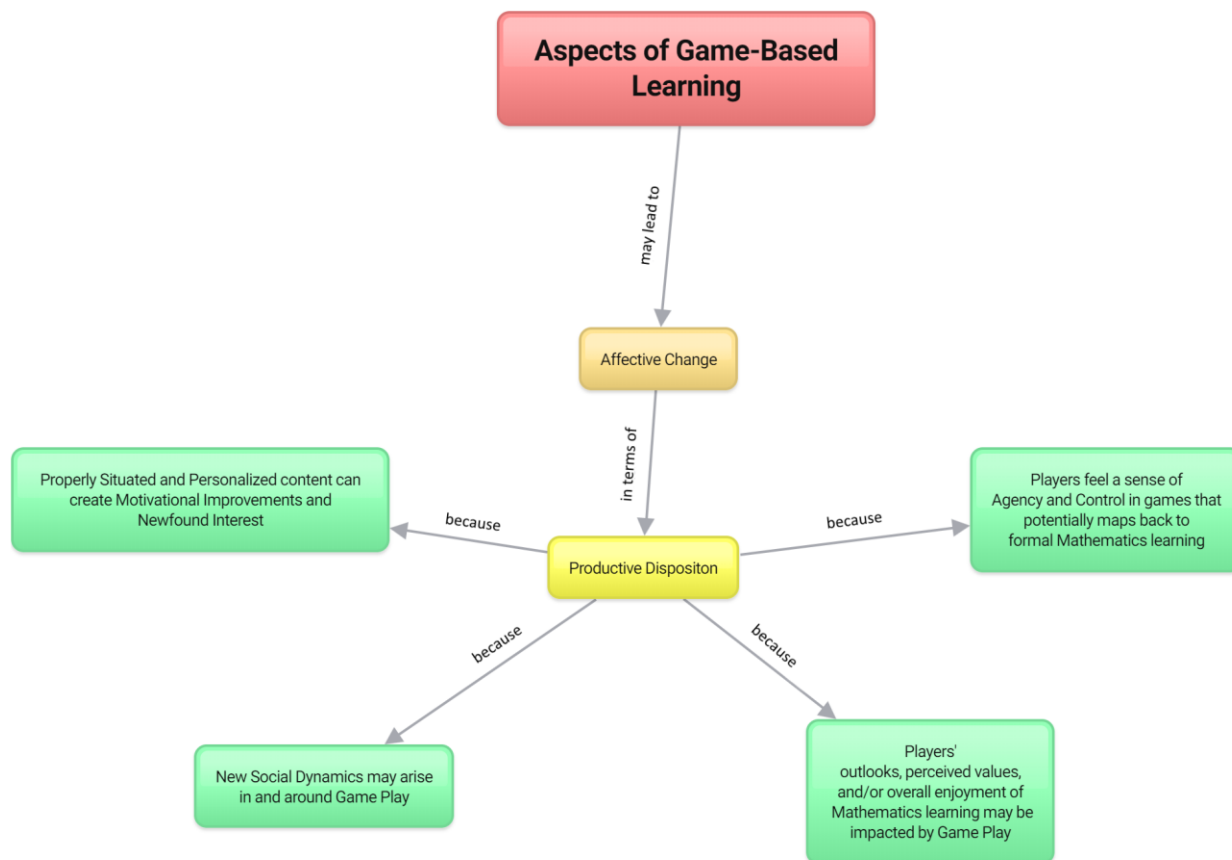
Figure 2.4: A mapping of how some aspects of game-based learning may contribute to affective change.

On Content-Retentive Change

When Bright, Harvey, and Wheeler published their report on the state of mathematics game-based research in 1985, they were careful to note that little-to-no significant research had been done about mathematics games for the sake of content retention (p. 131). Using a study to check for mathematics content retention can be challenging primarily because following the phase in which new content is learned during the study, there must be a gap in students' formal learning. Few institutions would be willing to pause students' formal mathematics learning for an extended period of time or treat the learned concepts as forbidden topics in the time between

concept learning and a potential retention check. Accordingly, retention studies conducted in formal learning environments frequently check for an imperfect sense of content retention on constrained windows of time, often varying between only a week and a month.

Hogle (1996) stated that extant literature comparing the retention rates of traditional learning methods and game-based learning methods seemed to favor the latter. This was supported by Pivec et al. (2003) who reported that, at the time of writing, of 11 studies carried out examining the retentive abilities of game-based learning as compared to traditional learning methods (e.g. Ricci et al., 1996), 8 studies favored game-based learning, while the other 3 showed no significant difference. While some studies done since these reviews were published have supported the use of game-based learning for the sake of content retention (Arici, 2008; Chow, Woodford & Maes, 2011; Wouters et al., 2013), others have rejected the notion (Hicks, 2007; Jain, 2012). Although the literature demonstrating the retentive benefits of mathematics game-based learning is positively oriented, it remains unconvincing. However, the literature has identified certain game attributes that could potentially improve leaners' retention of new content acquired via game play.

As stated in earlier sections, mathematics games may be constructed so that the game space is entirely enveloped by an endogenous fantasy binding game play to some targeted content knowledge. The retentive benefits of endogenous fantasy were investigated by Parker and Lepper (1992), who conducted two studies using *Logo*, a programming language designed to facilitate young learners' acquisition of problem-solving and formal mathematics skills. Across the two studies ($n = 47, n = 31$), third and fourth grade students were tasked with constructing geometric graphics and solving geometric problems in *Logo*; however, some students' lessons were situated in a fantasy context, while other students' lessons were not. Going further, of the

students whose learning was situated in a fantasy context, only half of those students were able to choose their context, while the other half of the students were randomly assigned a fantasy context. Across both studies, it was found that just situating work within a fantasy setting was sufficient for improving content retention—whether the student had chosen the fantasy context or not made no difference. However, Ke (2008) notes that if a fantasy is not truly endogenous— for example, if the content to be learned is only superficially applied over the fantasy setting— then the fantasy does run the risk of completely subverting important aspects of content acquisition, and later, retention.

Core to the notion of game-based learning are the concepts of spiraled, recurring and reusable game content as encouraging and enabling improved content retention. Van Eck (2007) notes that "things learned early in games are brought back in different, often more complex forms later. Players know that what they learn will be relevant in the short and long term" (p.15). Devlin (2011) mentioned that players were often encouraged by game objectives to revisit content that had been previously engaged, or sometimes forced to do so to overcome prior failings—and that this was not something for players to shirk from, but to embrace. Because failure is not usually viewed as an irredeemable turning point from either the player or designer's perspective in many educational games, and because key content goals can be consistently revisited, complexified, and improved upon, exploring content via mathematical game play can be a useful means of creating varied, interrelated, and memorable experiences with important information.

Additionally, greater engagement and on-task focus during the learning of new content has also been linked with improved content retention (Hannafin & Hooper, 1993). As discussed in the review of affective growth, mathematical games do have the potential to elucidate on-task

behavior, when implemented correctly (Bragg, 2012; Ke, 2008; O' Rourke, Main & Ellis, 2012; Rosas et al., 2003; Squire & Barab, 2004), so there is further evidence that such implementations may lead to improved content retention. Although some literature has claimed that seamless integration of content into a game play experience directly improves student engagement, and by extension, content retention (Titus & Ng'ambi, 2014), Hannafin and Hooper (1993) have argued that this is a non-obvious, non-generalizable conclusion, as student engagement, motivation, and effort may improve even in cases when a course's instruction via game play does not sufficiently address formal learning objectives. A model summarizing the ways in which aspects of game-based learning induce (content) retentive change is presented in Figure 2.5.
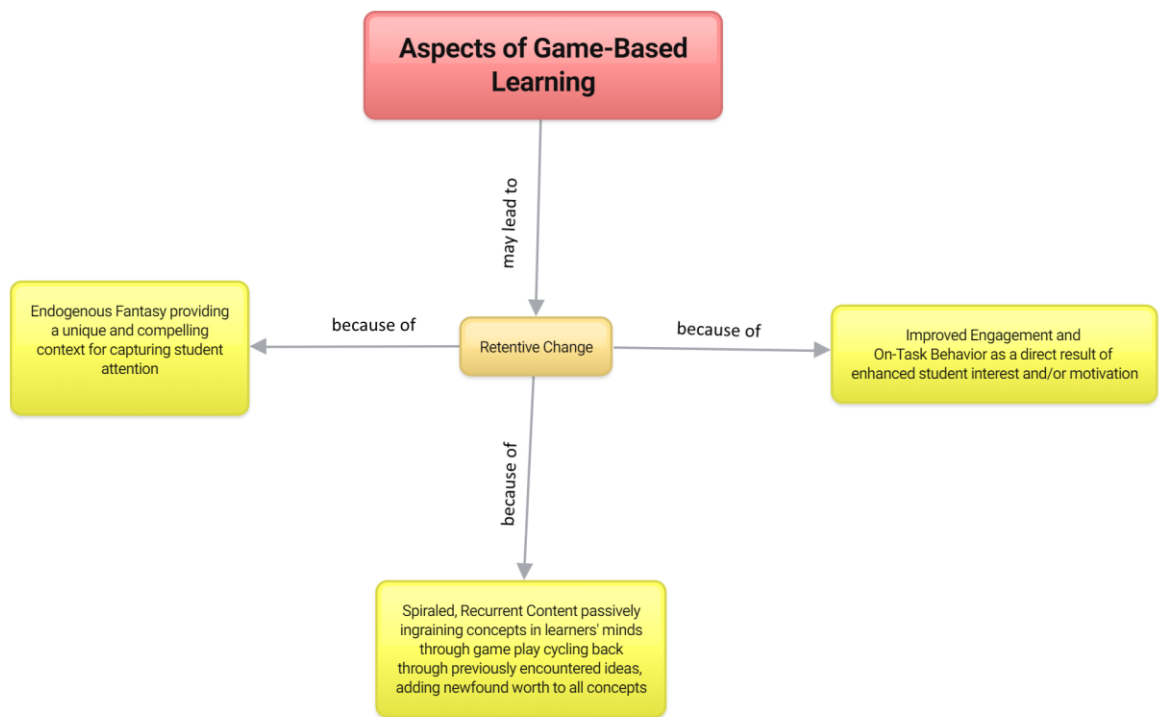
Figure 2.5: A mapping of how some aspects of game-based learning may contribute to (content) retentive change.

Critiques

For all the benefits that the literature suggests mathematical games, in general, can potentially provide, their implementation as formal learning tools has been subject to some controversy and criticism.

Since games are often viewed as a transplant to the educational industry from the commercial entertainment industry, educational games often are branded "edutainment," a term which evokes a half-hearted sense of both education and entertainment coming together for a product that achieves neither aspect quite perfectly (Hogle, 1996; Rosas et al., 2003). For example, one study showed that the *Lumosity* series of games—specifically constructed by neuroscientists to improve cognitive skills—failed to significantly improve players' cognitive skills for exercises such as association and matching tasks after 8 hours of play time by players ages 18 to 22; players from the same demographic who instead played the commercial game *Portal* for the same amount of time and under the same conditions were reported as greatly improving spatial skills, persistence, and problem-solving (Shute et al., 2015). Rebelling against the edutainment archetype, many educational game designers have branded themselves as "serious game" designers. Serious games remold the fused aspects that critics ascribed to the industry: they are games that don't have entertainment as their primary purpose, but may include it as a means of adding comfort and accessibility to an educational gaming experience (Michael & Chen, 2005; Djaouti, Alvarez & Jessel, 2011).

Joseph (2009) writes that "for years, video games have been blamed for turning children into mesmerized robots, agents of sexism and racism, and violent gun-toting psychopaths…" (p.253). On the notion of mesmerization, Ke (2008) cautions that educational game players can sometimes be distracted by goals of a game that are unrelated to content learning. This may

inhibit student achievement of content mastery due to distraction. Joseph hints specifically at portrayals of in-game violence as being one potential type of distraction that may influence learners not only when playing a game, but also once the game session has completed. However, while this claim has hounded the game design industry for years, it is chiefly unfounded; several studies have shown that violence in video games is likely not responsible for encouraging negative behavior outside of the game (Granic et al., 2014; Tear & Nielsen, 2013) and, in fact, some studies have even found that violent video games can potentially strongly improve players' cognitive skills for a variety of aspects, but most notably for spatial thinking (Barlett et al., 2009).

One final critique often lobbed at games of all kinds is that they unilaterally favor male players. However, this is a stereotype that has been rejected in the industry thanks to a variety of studies showing the contrary (Bryce & Rutter, 2003; Kahne, Middaugh & Evans, 2009). Although some literature has shown that among teens ages 12-17, more males (99%) than females (94%) regularly play some form of video game in their leisure time (Lenhart et al., 2008), these percentages are comparable. Additional studies have found that educational games can be designed in such a way that they can cater to both halves of a population, appealing equally to both males and females (Amory, 1999), or designed to specifically engage with tropes of importance to one gender specifically (de Jean, Upitis, Koch & Young, 2010).

Choosing a Mathematical Discipline and Mathematical Game for Research Based on this Literature Review

As discussed earlier, when used as an instructional tool, mathematical games have the potential to impact learners' cognitive outcomes, affective outcomes, and content-retentive outcomes in meaningful ways. Because of the literature's clear indication that the quality and learning outcomes of elementary algebra courses can be improved by both technological

interventions (Gilbert et al., 2008; Glickman & Dixon, 2002; Neurath & Stephens, 2006; Stephens & Konvalina, 1999) and content personalization (Bernacki & Walkington, 2014; Stephens & Konvalina, 1999; Toh, 2009; Walkington, Cooper & Howell, 2013), an elementary algebra course is a particularly strong candidate for game-based learning.

Usiskin (1988) describes the instruction of algebra as shifting students from numerically-dependent mindsets to variable-manipulating mindsets. He discusses how algebra may be viewed as a generalized form of arithmetic, a study of procedures for solving specific problem types, a study of relationships among quantities, and (especially in university-level algebra) the study of algebraic structures; his discussion makes it clear that the learning of algebra is a progression from a point of very concrete information (e.g. a basic understanding of well-defined arithmetical operations) towards increasingly more abstract ideas. Other authors (Devlin, 2000; Witzel, Mercer & Miller, 2003) have argued that good algebra instruction should facilitate and augment learners' progression through these stages. In particular, when it comes to equation solving, authors have indicated that exposure to varied representations of equations and different types of equation-solving strategies can give students greater conceptual flexibility and understanding of the nature of algebra as a whole, and problem-solving in general (Star & Rittle-Johnson, 2007). Therefore, a game that addresses algebra in a way that scales up from concrete to abstract ideas and provides a thorough treatment of equation-solving—with variation from the way the content is taught in class—would be an excellent candidate to assess game-based learning. Here, I will describe the game used co-instructionally in the study accompanying this literature review—*Dragonbox Algebra 12+*—which was selected because it is a strong representative of some of the most desirable qualities of games for mathematics education identified earlier.

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Dragonbox Algebra 12+ is a single-player video game divided up into a series of chapters, each containing twenty to thirty stages. It was intentionally designed from the ground up so that players could access algebraic equation solving experiences from a novel game-based perspective; accordingly, the goals within the game are directly aligned with learning goals in formal algebra content. Dolonen and Kluge (2015) do an excellent job of describing the game play experience within each stage:

> "[Each stage] consists of two large fields corresponding to the two sides of an equation, along with a storage located underneath consisting of objects that can be pulled out and placed within the two fields. The game is organized into chapters with increasing difficulty. A level ends when the main symbol—the dragon box (and later an "x")—stands alone in one field. Other evaluation criteria are whether the player has used the correct number of steps and whether there is an excessive number of objects in the other field that could have been eliminated. The player gets feedback on whether the criteria for successfully solving a level are met by getting one, two, or three stars. An object can be moved into a field and inside an equation in accordance with the four basic arithmetic operations [of addition, subtraction, multiplication, or division]. It may add to or subtract from a side in the equation depending on how the student has assigned a plus or a minus sign to the object in the store, act as a multiplier when placed beside another object in a field, and act as a divisor when it is placed beneath an object and thus creates a fraction bar or a multiplication of an existing divisor. When an object is drawn into a field, algorithmic rules are activated. When the player adds or subtracts an object on a field (one side of the equation), a dent appears in the other field (the other side of the equation), which shows that a corresponding object should be placed there. The student cannot progress further in the game until the dent has been filled with a corresponding object" (pp. 3-4).

In a typical *Dragonbox* stage, the solver is tasked with performing the in-game equivalent of isolating a single variable on one side of an equation; this experience is a concretized, manipulatable parallel to formal algebraic equation solving, affording solvers great agency and control over their actions and decisions. Throughout game play, strategies for the game's version of equation solving will change and evolve; players will begin the game by (covertly) learning about additive and multiplicative inverses in expressions, then dive into equations in which they'll be required to correctly utilize the various properties of equality in order to aid in variable

isolation. As players proceed through the stages and chapters, the fundamental elements of equation-solving are continually revisited and expanded upon for the sake of internalizing the information for the learner, thus potentially promoting cognitive growth for the learning of new content and instilling relevance into previously encountered content for the sake of retention. Later, the in-game equivalents of additional properties (e.g. the distributive property of multiplication over addition) and new equation-solving techniques (e.g. factoring of like-terms) are introduced via short pictorially-guided tutorials designated as "New Power" stages, introducing information "just in time." In many levels, these abilities act to facilitate and augment earlier encounted solving processes, but in some levels, utilization of new properties or techniques is mandatory, up to the resources the game has provided the solver.

It is worthwhile to consider the utility of the equation-solving processes that *Dragonbox* aims to teach to new learners. At some point in the possibly near future, technologies may arise that will invalidate the need to have a holistic understanding of the equation-solving process. However, even an individual utilizing an equation-solving utility would need to have an intimate understanding of, minimally, the fundamentals of how an equation works. That person would first need to recognize the origin of the equation and the relationship it is modeling. He or she might need to differentiate between constant values, variable values, known and unknown quantities, and the differences and meanings in represented operations or other symbols or ideas. These fundamental components are the things that *Dragonbox* is looking to instill in new learners' minds. The game provides a thorough treatment in understanding the properties of equality, which helps learners make sense of the game's representations of regular addition, subtraction, multiplication, and division. It introduces equations featuring many unknown quantities but emphasizes the differences between variable quantities being sought and those

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quantities that might be considered constants. Lastly, it also provides learners with a variety of tools and solving strategies that they will more than likely need when attempting to solve equations at a moment's notice in practical situations, such as financial budgeting, test score averages, or otherwise.

For many students, one of the largest challenges to overcome in transitioning from formal computational thinking towards structured algebraic thinking is fully understanding the distinction between variables and constants; Usiskin (1995) describes algebra as a language of generalization that, when properly utilized, allows solvers to move away from computations in which all numerical values are known towards computations in which not all values are explicitly known, but in which relationships between or among values may be deduced. *Dragonbox* attempts to circumvent this issue of representation by staging the game's earliest levels so that no reference is made to any formal mathematical symbology; additionally, the game never introduces formal terminology to describe any of the operations carried out within a stage. For instance, it refers to "isolating a variable" as "getting a dragon alone," thus helping build upon the game's endogenous fantasy (previously identified as being an important trait affecting the extent of epistemological integration, individual interest, and motivational improvements) of befriending and growing dragons from eggs to maturity. Similarly, for much of the game, "standard" representations of algebra content are not utilized. For example, the "variable being isolated" is not initially shown as, say, an x, but is the eponymous "dragonbox"—a literal box containing a shy dragon which will only reveal itself when it is alone (i.e. isolated). Although these representations may be viewed as asking mathematics learners to adopt a new perspective on mathematical/algebraic vernacular, this design choice is very intentional and aligns with the literature; building upon the endogenous fantasy of the

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Dragonbox world and introducing a vernacular with novel pictorial representations for mathematical ideas prevents students from shying away from the game as a possible "drill-generator." Instead, they identify the unique art style and representations as aspects of a potentially inviting puzzle-like game.

Nonetheless, the game does have a unique set of allowable actions (albeit most of which map to analogous mathematical processes for solving equations) and so, in this sense, students must learn two sets of processes (one in the algebra classroom and one in the game space), not just one. The premise is that since the two sets of processes are related, the learning in both spaces should be mutually beneficial; however, this could (unproductively) just be creating "additional" things to learn. In either case, it should be noted that not all actions that could be carried out in formal mathematics doing are allowed in game play—in particular, *Dragonbox* is designed with two prominent scaffolds aimed at facilitating the learning of the equation-solving process.

The first scaffold is termed here as the "pre-emptively corrective" mechanic. This mechanic prohibits players from making movements that would disrupt certain mathematical processes from being completed correctly (e.g. adding some non-zero value to *only* one side of an algebraic equation, then beginning a new mathematical process before balancing the equation). Notably, because of its pre-emptive nature, this scaffold precludes students from investigating in-game processes that would be incorrect when considered by their formal mathematical equivalents. Although an error-lite or error-free approach to the teaching and learning of new content has been advocated for by several studies in cognitive psychology (Ausubel, Novak & Hanesian, 1968; Bandura, 1986; Barnes & Underwood, 1959; Skinner, 1953), more recent work has proposed that learning is actually enhanced by allowing and then

correcting error instances (Kornell, Hays & Bjork, 2009; Metcalfe, 2017; Slamecka & Fevreiski, 1983). As discussed earlier, game play redefines and reshapes game players' psychological responses to failure in a typically less-negative (or even positive) fashion; although *Dragonbox* chooses to ignore some error opportunities with this scaffold, other game-based research has found that, when properly implemented, many error-corrective processes in game play have been beneficial to the learning of new content (Ivancic IV & Hesketh, 2000; Nowak, Plotkin & Krakauer, 1999; Ziv, Ben-David & Ziv, 2005).

The second scaffold is termed here as the "finite tile bank." This mechanic prohibits players from introducing terms or components of terms (i.e. in-game equivalents of variables or numbers) from outside those given to the player; this will be investigated more deeply with illustrations in Chapter 3. It should be recognized that this scaffolding decision reduces student agency over many decisions, but provides, in exchange, a targeted set of numbers and variables which are all guaranteed to be relevant, which is, intuitively, useful for first-time algebra learners and doers independently exploring content.

Both the chapters and the stages follow separate scaling difficulty gradients; for example, a later stage in Chapter 1 may be objectively harder to solve than an early stage in Chapter 2, but Chapter 2 will peak with more complicated content than anything presented in Chapter 1. This system ensures that players always have a challenge at hand.

By default, hints are not provided, but players can request hints during each stage, giving them access to information on-demand. Further, feedback on player work is provided at the end of each stage via a ranking from 1 to 3 stars; players may revisit levels to produce "more elegant" (e.g. fewer steps, fully simplified, etc.) mathematical solutions that may earn more stars than previous attempts, potentially providing players with a chance to revisit content for a more

whole understanding or overall fulfilling experience.

Lastly, because the game is single-player, each player can progress through the game at his or her own pace. However, because the game can be played on a variety of devices, it has the potential to create new social dynamics extending beyond the scope of a single individual, depending on how it is incorporated into formal learning activities, which may also be useful depending on one's learning goals. (I discuss more specific details about the game *Dragonbox* in Chapter 3.)

A wealth of literature has discussed the design decisions in the game's construction, as well as various studies that have used the game to examine cognitive and affective growth (Clark, Sengupta, Brady, Martinez-Garza & Killingsworth, 2015; Dolonen & Kluge, 2015; Gutiérrez‐Soto, Arnau & González‐Calero, 2015; Katirci, 2017; Nordahl, 2017; Siew, Geofrey & Lee, 2017). In a lecture entitled "The Role of the Teacher of the Future,"[6] (2015) given at the Universidad ORT Uruguay, *Dragonbox* director Jean-Baptiste Huynh discussed the University of Washington's Center for Game Science's June 2013 Algebra Challenge. The challenge aimed to improve algebra mastery in Washington K-12 schools by having students aged 7-17 play an adaptive version of the usual *Dragonbox* game. In the lecture, Huynh reported that "93% of children that played at least 1.5 hours learned basic equation-solving concepts,"[7] and that "children of all ages were able to learn the basic concepts for solving linear equations."[8] These early findings made *Dragonbox* an attractive learning tool for algebra-learning at all stages of academia. However, while affective growth (specifically increased confidence in

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[6] Original: "El Rol del Maestro del Futuro"

[7] Original: "93% de los niños que jugaron 1,5 horas aprendieron los conceptos básicos de resolución de ecuaciones."

[8] Original: "Niños de todas las edades pueden aprender los conceptos básicos de resolver ecuaciones lineales."

mathematics and comfort-level with algebra problems) has been consistently high for *Dragonbox*

players in these studies (Dolonen & Kluge, 2015; Katirci, 2017; Nordahl, 2017; Siew, Geofrey &

Lee, 2017), improvements to content mastery have varied, and no study has checked for any

form of content retention.

Gutiérrez‑Soto et al. (2015) reported that after using *Dragonbox* to supplement remedial

algebra learning for 9 early-college-level students during two 75 minute sessions across two

consecutive days, "…it seems that the students were able to recover or remember some solving

techniques [for algebraic equation solving] that relate to the actions used when problem-solving

with [*Dragonbox*],"[9] (p. 43). Supporting this is the Siew et al. study, in which 60 Malaysian

students aged 14 were split into two groups—a treatment group that would learn algebra by

playing *Dragonbox*, and a control group that would study algebra using a traditional classroom

setting—for a 16-hour algebra-learning session. Pretests and posttests were administered based

primarily on items from the TIMSS 2011 and the Malaysian curriculum for 7th and 8th grade

students; results showed that the control group mean rose from 13.5% to 49.6% correct answers,

while the treatment group mean rose from 13.2% to 71.1% correct answers.

However, concluding a study that pre-instructionally presented algebra game play and

then followed up with formal equation-solving, Katirci (2017) writes that, in the case of one

class of 7th grade American public-school students playing the game 10 minutes a day each

school day for five weeks, it seems that *Dragonbox* would be best used as a co-instructional

supplement to formal pre-algebra or algebra coursework; in Katirci's study, mastery of game

content did not immediately map to mastery of corresponding algebra concepts, and instructor

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[9] Original: "…parece que los alumnos han recuperado o recordado unos modos de resolución que se pueden
relacionar con alguna de las acciones que se usan cuando resuelven con el DragonBox Algebra©."

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guidance was required. In Dolonen and Kluge's study (2015), 75 Norwegian students aged 13-14 split into two groups for specialty mathematics learning using either *Dragonbox* or a non-game mathematics learning utility, *Kikora* (effectively used as a drill-generator for algebraic concepts), for 8 hours over the course of 4 weeks; while both groups did have significant gains in learning as measured by a pretest and posttest built around the course curriculum and TIMSS items released as of 2012, the *Kikora* group statistically improved twice as much as the *Dragonbox* group.

In the present study, *Dragonbox* game play was implemented co-instructionally while students were learning new algebra content; however, this learning experience was unique compared to the studies listed here incorporating *Dragonbox* game play. The cited texts typically either chose to pre-instructionally utilize the game (Katirci, 2017; Nordahl, 2017) or co-instructionally use the game, but with instructors drawing explicit connections between game play and formal content (e.g. Dolonen & Kluge, 2015; Siew, Geofrey & Lee, 2017). In one case the game was used post-instructionally as a tool for remediation (Gutiérrez‑Soto, Arnau & González‑Calero, 2015).

Closing and Intended Contributions to the Literature

In closing, educational games have the potential to profoundly transform formal mathematics learning for cognitive, affective, and retentive growth. Although much of the literature is still undecided on the best ways of doing this, there are strong indications that games which intentionally embed mathematics concepts into their designs have a heightened chance of success when they are properly introduced and implemented in the mathematics classroom.

The study that this literature review accompanies will aim to contribute to the game-based research on cognition, affect, and content-retention by meeting the following goals: 1)

identify the cognitive connections generated by students linking their game play and formal mathematics experiences; 2) articulate the specific aspects of students' uses of formal algebraic equation solving techniques that are impacted by game play; 3) portray an image of students' essential affective changes in terms of factors including but not limited to engagement with mathematics, outlook on mathematics, and self-image as mathematics doers; 4) describe the aspects of game play experiences that prove memorable over an extended period of time when engaging content involving techniques for solving algebraic equations.

Chapter 3: Methodology

Introduction

This study was designed to explore the cognitive, affective, and content-retentive developments of pre-secondary students playing a mathematical game as a supplement to the students' formal mathematics coursework. As demonstrated by the literature review in the preceding chapter, the utilities of mathematical games for improving learners' cognition, affect, and content retention are still unclear, though generally positively oriented. This study was designed to contribute information to each of these three fields of consideration to clarify the uses of educational games for pedagogical purposes. Because elementary algebra courses were previously shown to be good candidates for studying the impacts that mathematical games can have on students' learning outcomes, and because there are a wealth of well-documented technological resources (including mathematical games) available specifically for use with this topic, this study examined an eighth grade elementary algebra course in which some students played the mathematical video game *Dragonbox Algebra 12+* as a co-instructional supplement to their formal algebra instruction. Although this was not a completely experimental study, for utility, this text uses the term "treatment group" to refer to the students who participated in the algebra game play supplement (which will be referred to as the "treatment"); similarly, the term "control group" is used to refer to those students who did not participate in the treatment and received their usual formal in-class algebra instruction.

Research Questions

In accordance with the stated goals, the following research questions, as discussed earlier, guide the study:

1. How does integrating mathematical game play into a traditional eighth grade algebra curriculum impact students' cognitive learning outcomes in elementary algebra?

2. How does integrating mathematical game play into a traditional eighth grade algebra curriculum impact students' affective outcomes about both mathematics in general and algebra specifically?

3. How does integrating mathematical game play into a traditional eighth grade algebra curriculum impact students' content retention in elementary algebra?

Setting and Participants

The study was conducted at a K-12 independent school in a large city on the eastern coast of the United States. The intervention phase of the study (in which students participated directly in the treatment, and cognitive and affective data were collected) took place over eight weeks in 2017 from late October to mid-to-late December. The second phase of the study (in which data on content-retention were collected) took place on two consecutive days in early January 2018. This timing was essential for adding validity to the retention data, as the period between the end of the intervention phase and the start of the retention phase constituted a winter recess in which students would not be expected to have formal algebra instruction due to classes not being in session.

The school's elementary algebra course, taught by one instructor who was not the principal researcher, had 30 matriculated students ages 13-14. The students were mixed in terms of gender, ethnicity, prior mathematics knowledge, and socioeconomic status. Students were recruited for the study earlier in the academic year via exposure to three on-site talks on using mathematical games as pedagogical tools given by the principal researcher to the whole 8th grade. Students were offered a digital copy of the *Dragonbox Algebra 12+* game as compensation for participation in the study. Because they were minors, students were only able to participate in the study pending receipt of both a parental permission form and a student

consent form.

Of the 12 affirmative respondents, 11 students were randomly selected to form the treatment group. This decision was necessary due to the soft-limitation of 11 students imposed by the school's available hardware in its computer lab; in this study, *Dragonbox Algebra 12+* was played on 11 personal computers using the Windows 10 operating system and the Microsoft Store software platform. Like the full course of 30 students, the 11 students in the treatment group were mixed in terms of gender, ethnicity, prior knowledge, and socioeconomic status. However, the treatment group ended up having slightly more males than females with a female-to-male ratio of 3:8.

Of the 19 students who were not chosen for the treatment group, 11 students' course profiles were randomly blindly sourced to form a control group for the study. The sourced data provided by the course instructor contained information only on students' genders and cognitive examination performances; the female-to-male ratio was 6:5. Because no data were sourced on the control group students' ethnicities or socioeconomic statuses, it is not possible to say with certainty whether these aspects were mixed; however, as mentioned earlier, the general population of 30 was mixed for all these aspects.

Students in the control group received strictly formal mathematics instruction throughout the entire intervention phase of the study. Typically, this meant that control group students would attend a class that would involve a combination of lecture and problem-working—usually, though not necessarily, in that same order each day. They would be joined by treatment students for the latter half of each session. Students in the treatment group received mixed mathematics instruction; for two of five class sessions each week, treatment students would spend the first half of the period utilizing the mathematical game as a learning supplement, then rejoin control

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students in the second half of class. Typically, this meant that treatment group students would usually miss some lecture component on game play days; however, for the other three days each week (i.e. when game play did not occur), students in both groups received identical instruction.

The Game Play Experience and Instructional Approach

As mentioned, the treatment group received a mixture of formal mathematics instruction and an algebra game play alternative. Game play sessions were substituted for traditional class time for twenty-minute sessions twice a week for 8 weeks; notably, class periods were 42 minutes long. This means that in a typical week during the intervention phase of the study, a student in the treatment group had roughly 160 minutes of formal algebra instruction and 40 minutes of *Dragonbox Algebra 12+* game play; note that a few minutes could be lost on either account due to students walking from the computer lab to the algebra class from game play sessions mid-period. Game play sessions with the treatment group students were always conducted during the first half of algebra periods.

In each session, students would meet the principal researcher in the computer lab. Usually, the principal researcher would have prepared each of the computers to have the *Dragonbox Algebra 12+* software on screen by the time students entered the room. Upon entering, students would immediately sit at their assigned computers and begin playing while the principal researcher observed the students for the roughly twenty-minute half-period. Students sat adjacent to and across from one another and were free to discuss in-game content.

Students who participated in the algebra game play experience played *Dragonbox* co-instructionally, in the sense that they were learning new algebra equation-solving content while playing through a game that was designed to help enhance their algebra equation-solving abilities. The principal researcher supervised game play but did not attempt to directly influence

students' exploration within game play or attempt to directly influence students' connection-making between the content encountered in game play and formal algebra equation-solving ideas. This approach was taken primarily for two reasons. First, few studies in the literature utilized *Dragonbox* co-instructionally, and no study utilizes this variant of co-instructional learning when working with *Dragonbox*. It is interesting to see the connections and themes that arise naturally for students from this experience – without any expert interference. Games, more generally, are often picked up and played by students outside of the classroom context, and so this study provides a perspective on what students might acquire from playing *Dragonbox* even outside of school settings. Second, because the principal researcher would bring biases to the teaching and learning process while actively facilitating connections between game play and formal mathematics content, he did not provide instruction so as not to exert any strong influences on the treatment group that would present a challenge of accountability during data analysis or conflict with the primary algebra teacher's algebra instruction.

Dragonbox Game Play Connections to Formally Expressed Algebraic Concepts

The game used in the study, *Dragonbox Algebra 12+*, contains many representations of concepts central to the algebraic equation-solving process. This section expounds upon what the principal researcher has decided are the game's most important representations and mechanics for ease of discussion in later sections. Table 3.1 is included at the end of this section for quick reference on in-game representations of equation-solving concepts. It is accompanied by Table 3.2, which displays some in-game mechanics associated with different in-game movements. Note that although game play does not initially utilize notations and representations found in equation solving typically, it steadily moves towards them, so that by the later chapters in the game, game play stages may appear precisely as if they were equations from, for example, a textbook.

Recalling part of Dolonen and Kluge's (2015) description of game play provided in the literature review, the game players' goals are here reiterated: "[each stage] consists of two large fields corresponding to the two sides of an equation, along with a storage located underneath consisting of objects that can be pulled out and placed within the two fields.... A level ends when the main symbol—the dragon box (and later an "x")—stands alone in one field" (pp. 3-4).

Chapter 1-9 is one of the earliest levels to feature an "equation" screen. In Figure 3.1, most of the screen is divided into two halves; the left half has two tiles on it, and the right half has one tile on it. Each of these tiles can be thought of as a variable, but the glowing box represents the variable that needs to be "solved for" or "isolated." This level may be thought of as a model of an equation such as $x + a = b$. In this first screen shot, we see that the bottom of the screen is a tile bank that allows the player to introduce new tiles into the board/equation; because any new tiles that the player wishes to introduce to the equation must come from this tile bank, players are bound to a finite (though potentially large) number of possible moves. To "solve for the variable/box" on this level, the player is supposed to drag the dark-creature tile from the tile bank onto the left hand side of the equation in order to have it "cancel out" the bright-creature tile; the game will then prompt the player to move another copy of the tile onto the right hand side of the model equation, demonstrating the addition/subtraction properties of equality and establishing a notion of additive inverses.

Figure 3.1: An early problem that emphasizes the addition/subtraction property of equality.

In game play, equivalents to a variety of arithmetic conventions can also be utilized.

Figure 3.2 demonstrates a fraction— "fish over fish"—that the player is prompted to reduce into

a white one-dot tile (representing the number one); in a much later level, the reverse is also

described (e.g. change the number 1 into "fish over fish"). Figure 3.3 then establishes that, if a

player has "one times some variable" (e.g. $1x$), represented here by a variable-box joined to a

white one-dot tile by a silver circle, the player can click on the white-dot tile to make it vanish

(e.g. treating $1x$ as just x).
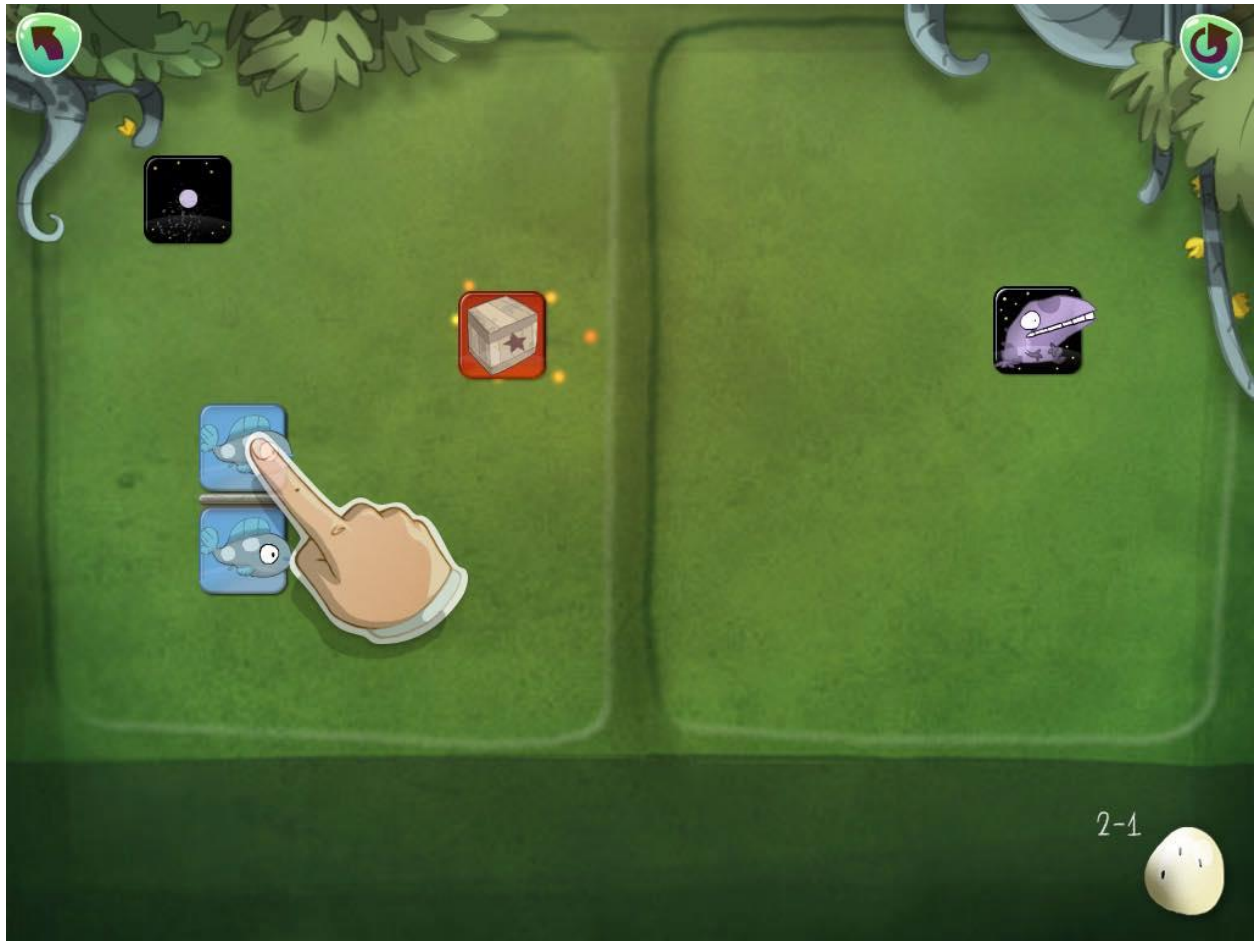
Figure 3.2: The concept of a "whole fraction" equaling 1 is suggested to the player.
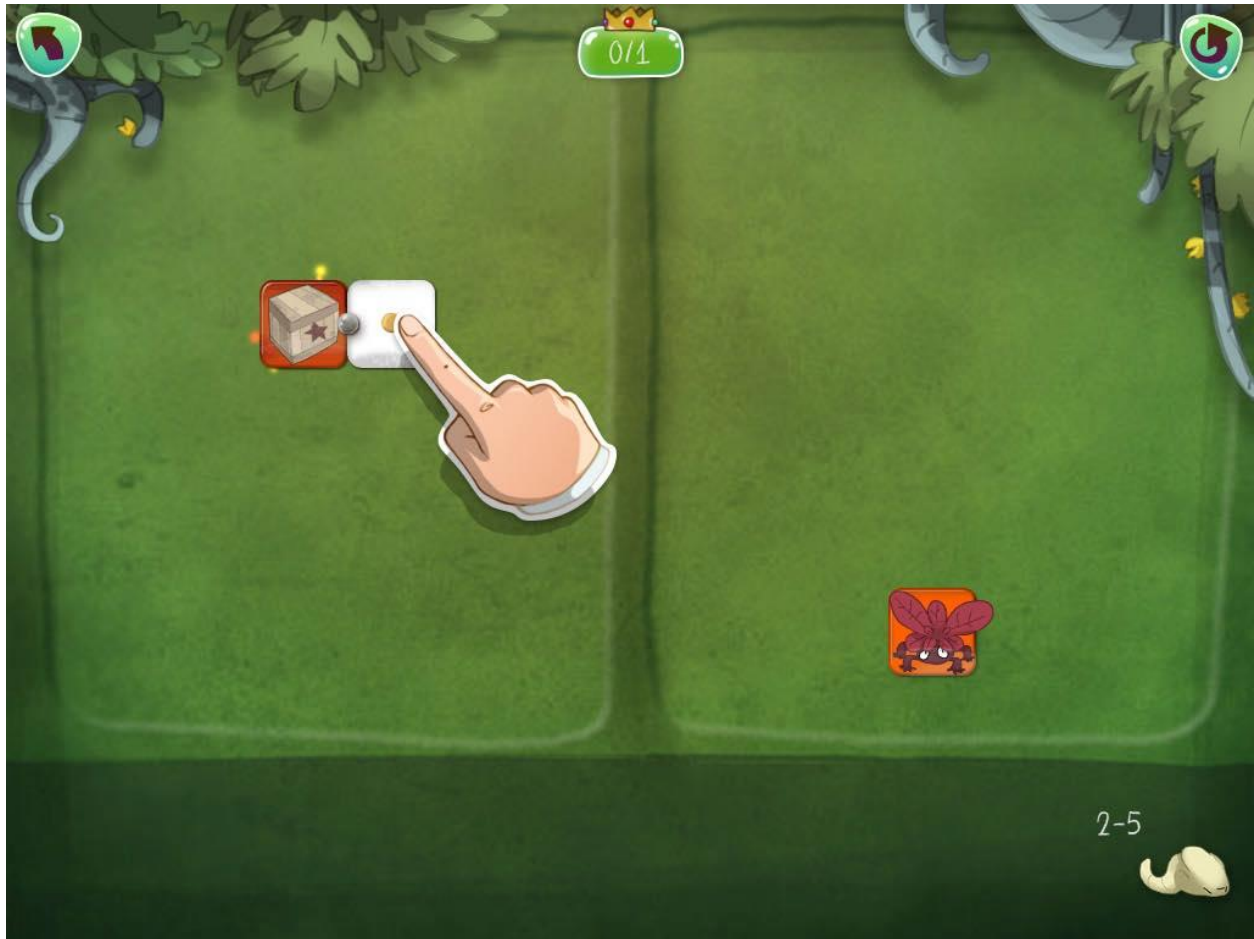
Figure 3.3: The convention of writing, for example, $1x$ as x is taught to the player.

Game play also features separate treatments for the multiplication and division properties of equality. In Figure 3.4, the player is instructed to "divide" both sides of the equation by inserting the only tile in his/her tile bank into an indent on the left-hand side of the equation; upon doing this, he/she is prompted to also place one copy of that tile under each term on the right-hand side. Until the player satisfies this prompt, no other progress can be made; the game is designed to force certain moves during play to guarantee players' adherence to certain mathematical ideas—in this case, the division property of equality is enforced once the player had indicated an interest in dividing somewhere in the equation. This board is effectively representing an equation such as $ax = b + c$, and showing players that the solution when solving

for x can be found by computing $\frac{ax}{a} = \frac{b}{a} + \frac{c}{a}$. Figure 3.5 demonstrates the game play equivalent

of the multiplication property of equality, using a similar set up.

Figure 3.4: A level analogous to the equation $ax = b + c$ representing

the division property of equality.

Figure 3.5: A level analogous to the equation $\frac{x}{a} = b + c$ representing

the multiplication property of equality.

Further levels demonstrate the notion of combining like terms and factoring within

fractions. In Figure 3.6, the player is instructed to drag a white two-dot tile onto a white three-dot

tile in order to create a white five-dot tile; the level can then be solved by "flipping" the white

five-dot tile in the tile bank into a black five-dot tile, and using the black five-dot tile to balance

the equation (effectively, one solves $x + 2 + 3 = a$ by first getting $x + 5 = a$, advancing to $x +$

$5 - 5 = a - 5$, and concluding that $x = a - 5$). Figure 3.7 shows an equation with a fraction for

which the player is instructed to tap the white six-dot tile, which will "factor" it into a white

three-dot tile and white two-dot tile.

Figure 3.6: A level representing the equation $x + 2 + 3 = a$ that teaches players about

combining like terms.
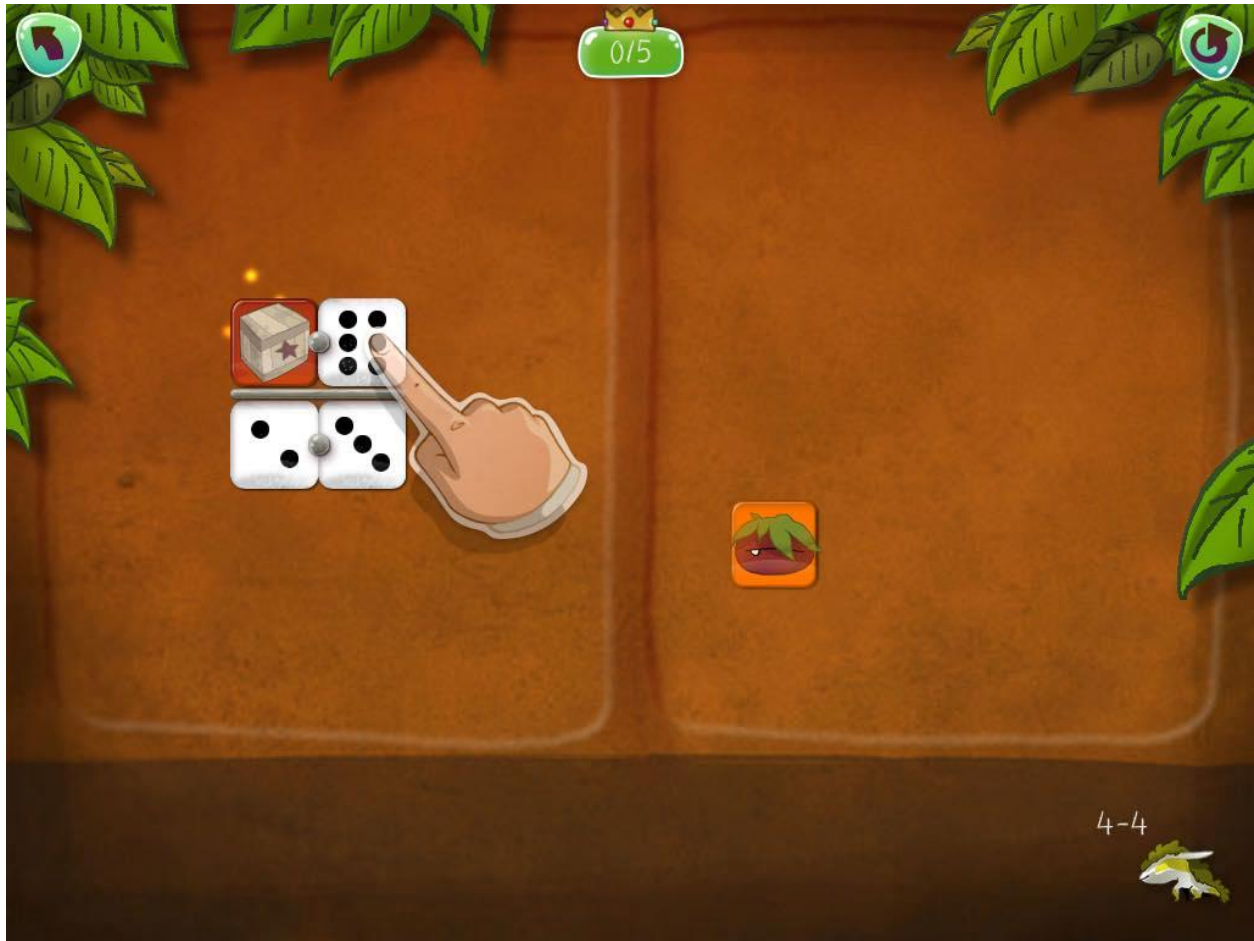
Figure 3.7: A level that instructs players on factoring within a fraction.

Levels in the second half of the game introduce concepts related to parentheses, which typically appear as bubbles (Figure 3.8). However, for cases in which the terms within the parentheses would be joined by a plus or minus sign, and there is a term multiplying or dividing the parenthetical terms, the bubble is instead a block of ice (which cannot instantly be "popped") to indicate that operations such as multiplication and division must interact with all of the terms in the parentheses. Figure 3.9 demonstrates the equivalent of the distributive property of multiplication over addition, while Figure 3.10 demonstrates the factoring of like terms.

Figure 3.8: A representation of the equation $(x) = a$, encouraging players to "pop the bubble."
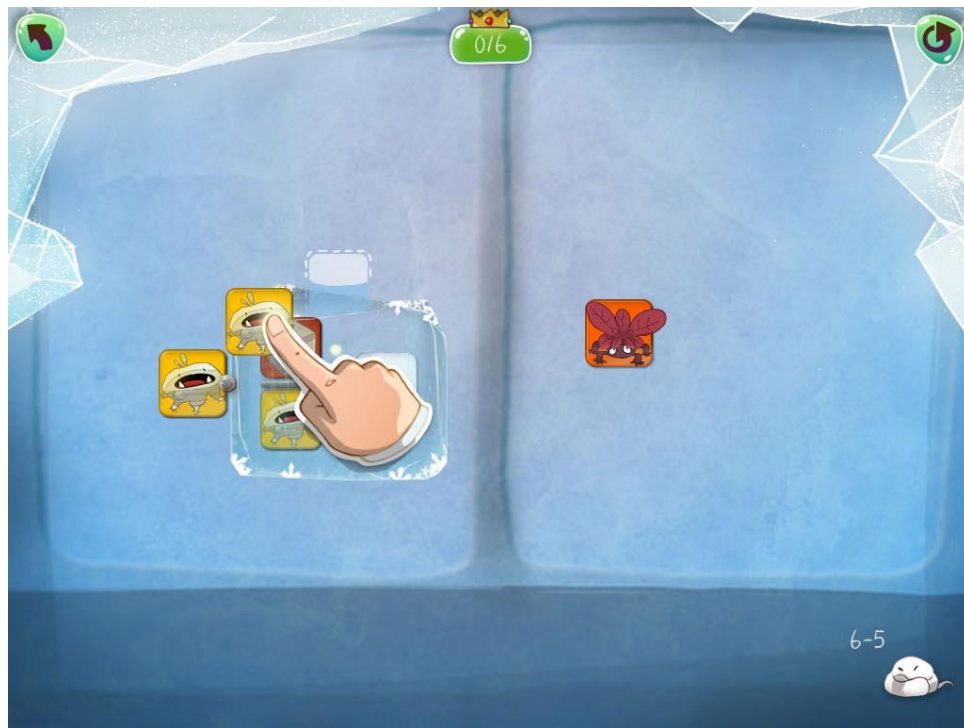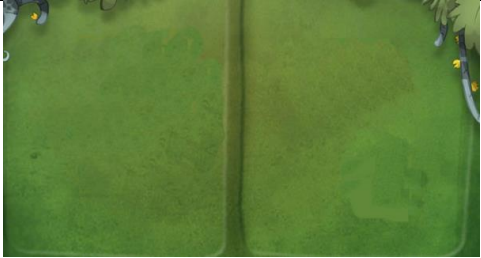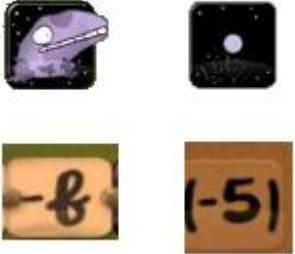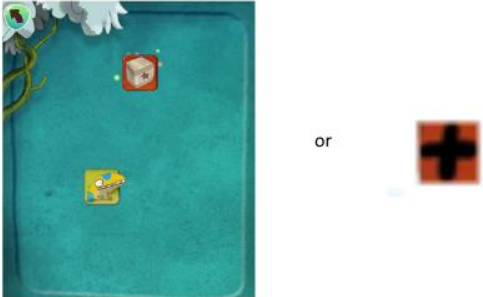
Figure 3.9: A level demonstrating the distributive property of multiplication over addition.

Figure 3.10: The equivalent of the equation $\left(\frac{x}{3} + \frac{x}{3}\right) = a + b$ is presented to the player; one

suggested method is to "factor out" the $\frac{1}{3}$ coefficient, first.

Table 3.1: Comparison of Essential Equation-Solving Concepts in Elementary Algebra and Representations of those Concepts in *Dragonbox Algebra 12+*	
Algebra Equation-Solving Concept	*Dragonbox* Representation Examples
Left-Hand Side, Equals Sign, Right-Hand Side of the Equation	Or (Above, a clear game board shows left and right halves split by a bar to indicate the two sides of an equation and equals sign; below, a typical equation that appears in late-game *Dragonbox*.)
Variable Being Solved For	(At left, the eponymous "Dragonbox" tile; at right, the standard tile for x.)
Other Variables	Any of the following, as examples: Above, random creature tiles; below, a standard tile for the random variable c.
Whole Numbers	At left, a tile with five dots to symbolize the number 5; at right, the tile for 6.

Negatives	Any of the following, as examples: At top left, the "negative" of a creature tile shown earlier; at top right, a tile with one dot on a dark palette to symbolize the number -1; at bottom left, a standard $-b$ tile; at bottom right, a standard -5 tile.
Addition/Subtraction	 At left, two free-floating variable tiles are treated as being added together; at right, the standard plus-sign may also be used. Notably, subtraction is dealt with by adding the negative tiles shown earlier.
Multiplication	 In both cases, a circular bullet is shown indicating a product between two terms.
Division/Fractions	 At left, the representation of a fraction like, say, $\frac{x}{a}$; at right, the standard fraction $\frac{c}{d}$.
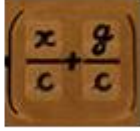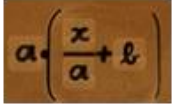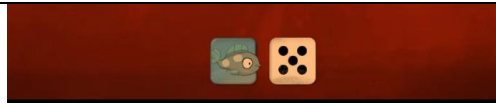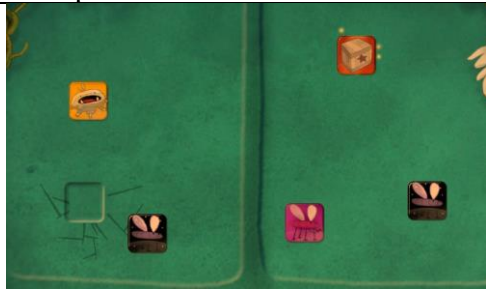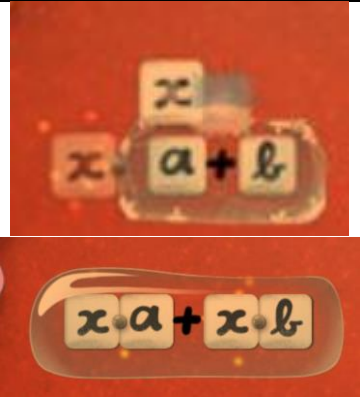
Parentheses (First Variant)	or At left, a bubble surrounds the sum of two fractions; at right, standard parentheses surround the sum of two fractions.
Parentheses (Second Variant)	or At left, a bullet binds a term to parentheses containing a sum—*Dragonbox* uses an ice-block instead of a bubble in this case in order to indicate that the Distributive Property of Multiplication over Addition may be used; at right, a bullet binds a to parentheses containing a sum, but standard parentheses are used.

Table 3.2: Examination of In-Game Mechanics Related to Essential Equation-Solving Actions	
Player's Intended Actions in terms of Formal Equation-Solving Language	In-Game Mechanics' Response(s)
If the player chooses to introduce any new term to the equation…	…he/she must choose a term from the tile-bank provided at the start of the level.
If the player chooses to "add a term…"	…he/she will see an indent on the screen indicating where else a term must be "added" to obey the Addition Property of Equality. This indent must be filled before continuing.
If the player chooses to "multiply by a term…"	…he/she will see several indents on the screen indicating where else a term must be "multiplied" to obey the Multiplication Property of Equality. These indents must be filled before continuing.
If the player chooses to "divide by a term…"	…he/she will see several indents on the

	screen indicating where else a term must be "divided" to obey the Division Property of Equality. These indents must be filled before continuing.
If the player chooses to "change 1 into an equivalent fraction…"	…he/she will swipe the 1 until it becomes a fraction of question-marks, then tap a different tile to populate the question-marks with that tile's content. These question-marks must be converted before continuing.
If the player chooses to create "parentheses…"	…he/she may drag a bubble icon onto the terms he/she wishes to put into the "parentheses." The "parentheses" can be removed by tapping to "pop" the bubble. The ice-block variant of parentheses discussed in the previous table can be created by combining the bubble variant of parentheses with the multiplication procedure.
If the player chooses to "factor…"	

	…he/she may remove one term from "parentheses;" he/she will then be prompted (by a slight glow) to remove other terms to complete the factoring. The factoring must be completed to continue.
If the player chooses to utilize the "Distributive Property of Multiplication over Addition…"	…he/she may drag the multiplying term onto the attached "parentheses"; the distribution will complete automatically.

Instruments and Data Collection

This study utilizes a convergent mixed-methods research design, defined by Merriam and Tisdell (2016) to be a "design in which the qualitative and quantitative data are collected more or less simultaneously; both data sets are analyzed and the results are compared" (p. 46). Chatterji (2010) argues that mixed-method designs considerably improve the flexibility of a study's data collection resources and allows for the quantitative and qualitative data to scaffold each other for increased support. In this study, the primary use of the collected qualitative data is to aid in the identification and description of trends found in the quantitative data, although throughout the study, both the quantitative and qualitative data were collected on ongoing overlapping intervals. The collection of qualitative data was deemed particularly important due to much extant

literature collecting primarily superficial qualitative data.

Instruments for collecting data on cognition.

Two quantitative instruments and two qualitative instruments were developed for collecting data on student cognition throughout the study.

The quantitative instruments were the first and second variants (of three) of the Algebra Games Abilities Tests (AGATE 1 and AGATE 2, respectively) designed by the principal researcher. To formulate these designs, the principal researcher first identified that, among the four strands of cognitive growth potentials scaffolding cognition research in mathematics education as defined in the literature review, procedural fluency, conceptual understanding, and strategic competence with respect to elementary algebra could be well-addressed via a quantitative instrument that could clarify each student's equation-solving prowess. He next reviewed the formal algebraic content that both paralleled the game play of *Dragonbox Algebra 12+* and also appeared in students' planned formal mathematics instruction. Finally, he presented a series of drafts of each AGATE to the algebra course instructor to confirm that by the end of the intervention phase of the study, all students would have had, minimally, some formal treatment of all concepts appearing on the exams. No references to game play were included in the AGATE designs; although this starkly separates the endogenous fantasy of game play and the formal mathematics, this decision was made because the AGATEs would be utilized by both the treatment and control groups, the latter of which would have no game-related knowledge. Each of the AGATEs contained 17 questions split across 8 parts—each part asks for the solver to isolate a different variable in various one-step and multiple-step equations. However, the 17 questions are organized conceptually into three subsections: questions 1-5 examine basic uses of the addition, subtraction, multiplication, and division properties of equality; questions 6-10 test

the former, but also introduce fractional multiplication and division; questions 11-17 test both of the former, but test also the distributive property of multiplication over addition, heightened mastery of inverse operations, and, to a limited extent, factoring skills. Changes to content between the two exams were superficial; for example, a question on the AGATE 1 may only have differed with a corresponding question on the AGATE 2 by way of the numbers and variables utilized in the problem. Because this was a superficial change, it didn't impact students' abilities to use their algebraic knowledge. The AGATE 1 was administered as a pretest to all students at the start of the intervention phase during their algebra period. Similarly, the AGATE 2 was administered as a posttest to all students at the end of the intervention phase during students' algebra period; however, two students were absent during the administration of the AGATE 2 at the end of the treatment phase. These students were unable to complete the AGATE 2 because they left the US several days early—and prior to the final two game play sessions—for travel during winter recess; they would only return following the one-month recess, by which point, potential AGATE 2 data would not be useful.

The qualitative instruments, Cognition-Focused Interview Protocols 1 and 2, were utilized only with students from the treatment group and were designed to help students vocalize the impact, if any, that their algebra game play experiences may have had on the way that they think about and solve algebraic equations. Students were chosen pseudorandomly to participate in the interviews; of the first four students asked to participate, two declined and two agreed. Declining students were replaced by two new pseudorandomly chosen students who agreed. The final four agreeing students participated in both protocols 1 and 2.

The first protocol was conducted half-way through the eight-week treatment phase and the second protocol was conducted at the end of the eight-week treatment. The protocol guided

the principal researcher through an open-ended, free-response interview with each of the students in roughly 20-minute blocks occurring outside of the algebra class (meaning on students' free periods or before or after class sessions). Most of the questions in the second protocol make some reference to student responses to the first protocol; for questions that do not make such reference, it is noted that the time of interest is "since the last interview," as opposed to the full-course experience.

The Cognition-Focused Interview Protocols were designed around a selection of themes that the literature review exposed as being important; although some of the language on the two protocols differs, each protocol contains 7 question-types which are presented in the same order in each protocol. Question-type 1 probes for how students have utilized the game, asking specifically if they've ever played it outside of the study in the past, or during the treatment duration outside of regular class periods or even outside of school; this may provide an explanation for the utility or interest a student may have in the game. Question-type 2 asks students to provide a description of game play, checking to see if they can articulate the game design's intentional parallels between game play and algebraic equation solving. This was potentially useful because it could provide insight regarding whether students were aware they may have been refining their algebra skills via game play. Question-type 3 directly asks students if they feel their understanding of content from elementary algebra was impacted by game play, and if so, whether it was impacted positively or negatively. This question-type naturally flows from the previous one, as both relate to the parallels between the mathematical game and the formal classroom mathematics as perceived by the learner/player. Question-type 4 asks students if any experience(s) from game play impacted their actions while doing mathematics during their formal algebra course; here, the potential of passive information transfer, from an informal

environment to a formal environment, is assessed. Question-type 5 asks whether anything from game play complicated or contradicted information found in the formal algebra course, probing for potential dissonance that could cause the student mathematical confusion. Question-type 6 asks students whether content encountered during game play appears to parallel content from their formal algebra course, and whether game content ever appears to parallel potentially confusing concepts from students' formal mathematics learning; this question particularly seeks to determine if there is any algebra content which was learned better by students through the informal game play experience. Finally, question-type 7 asks students to walk the principal researcher through a level(s) of *Dragonbox Algebra 12+* while justifying the actions made either via informal language or with algebra-specific vocabulary. The student is also asked whether he or she would be able to construct an algebraic model of the game level using numbers and variables. This last question-type elucidates how students express their algebraic thinking and potentially draw connections between game play content and formal algebraic ideas.

Instruments for collecting data on affect.

Three qualitative instruments were developed for collecting data on student affect throughout the study: Affect-Focused Interview Protocols 1, 2, and 3. These instruments were utilized only with students from the treatment group and were designed to help students vocalize the impact, if any, that their algebra game play experiences may have had on the ways they view the field of mathematics (and more specifically, algebra), themselves as mathematics doers, and related perspectives and ideas. Of the first four students pseudorandomly asked to participate in these interviews, three agreed and one declined; the declining student was replaced with a pseudorandomly chosen student who agreed to participate. The final four agreeing students participated in each of the three protocols. Notably, although there was some overlap in the

student volunteers from the Cognitive-Focused Interview Protocols and the Affective-Focused Interview protocols, they are not identical sets of students: exactly two students, Ivan and Harold, went through all Cognitive-Focused and Affective-Focused Interview Protocols.

The first protocol was conducted at the beginning of the eight-week treatment, the second protocol was conducted at the middle of the eight-week treatment phase, and the third protocol was conducted at the end of the eight-week treatment phase. Each protocol guided the principal researcher through an interview that asked a student to respond to each item in a series of prompts by first selecting a Likert scale position and then explaining why he or she chose that position. Each interview for each protocol occurred in roughly 20-minute blocks outside of the algebra class (meaning on students' free periods or before or after class sessions). The same selection of 16 questions appears across all three protocols.

The Affective-Focused Interview protocols draw on ideas from Tapia and Marsh's "Attitudes Towards Mathematics Inventory," sometimes referred to as ATMI (2004). The original ATMI utilizes a 5-point Likert scale that asks students to identify the intensity of their views across 40 prompts from "strongly disagree" to "strongly agree." A sample item from the original ATMI is the prompt "Mathematics is a worthwhile and necessary subject." However, the principal researcher believed that richer data could be collected if students talked out their views and explained how and why those views had been established. Therefore, in the Affective-Focused Interview protocols, a student first selects his or her position on the Likert scale relative to a prompt, and then elaborates as much as he or she chooses. However, this design choice required a shortening of the prompt list from 40 items to 16 items for time limitations. The principal researcher utilized the ATMI as a basis for the Affective-Focused Interview protocols, directly quoting some items from the original ATMI, adapting other items from the original

ATMI to be better suited for the new protocols/setting/population, and generating some novel prompts that look specifically at elements core to this research. An example of an adapted prompt is "I am often confused when doing mathematics;" the original prompt had been "I am always confused in my mathematics class." In this case, the prompt was changed to encourage a broader perspective on when, where, and how an individual might be utilizing mathematical thinking. An example of a novel prompt in the same spirit of the original ATMI follows: "There's no mathematics involved in playing games." This prompt is clearly aimed at determining students' views on whether game play can be mathematical in nature, or if games are something incompatible with mathematical learning or doing.

Instruments for collecting data on retention.

One quantitative instrument and one qualitative instrument were developed for collecting data on student content retention assessed one month after the intervention phase of the study. During the one-month interim, students were on winter recess in mid-to-late December and early January, and as such, were not expected to receive any instruction related to schoolwork from their formal courses or from the study.

The quantitative instrument was the third variant of the Algebra Games Abilities Tests (AGATE 3) designed by the principal researcher. The AGATE 3 was designed using the same process used for the AGATE 1 and AGATE 2. The AGATE 3 was also structurally identical to the AGATE 1 and 2, aside from some superficial changes; for example, a question on the AGATE 3 may only have differed with a corresponding question on the AGATE 1 or 2 by way of the numbers and variables utilized in the problem. As mentioned earlier, because this was a superficial change, it didn't impact students' abilities to use their algebraic knowledge. The AGATE 3 was administered to all students the first day they returned from winter recess in early

January; however, one student was absent during the administration of the AGATE 3 during the retention phase. This student indicated that he would be out of the country for an extended period following the winter recess, so it was unclear at what point in the future he would be able to complete the AGATE 3; he returned roughly a week after the study had concluded, so his data point for the AGATE 3 was invalidated since he had been doing his formal mathematics coursework from home during this time.

To complement the quantitative data from the AGATE 3, the qualitative instrument used for gathering data on content retention was the Effects of Mathematical Game Play Study Questionnaire. The questionnaire included five open-ended questions asking participants to discuss what aspects, in their current and previous mathematics course experiences, have been most memorable. It was administered the day after the administration of the AGATE 3. Students were asked to provide examples and justify their responses in the questionnaire instructions. A sample item from the questionnaire is "What learning experiences or activities in your study of algebra this academic year have you found most memorable? Why?" Although the questionnaire was administered to all members of the treatment group (and no members of the control group), no questions directly discuss or reference the algebra game play experience so as not to influence students to discuss game play. These data were collected specifically to determine the degree to which students correlate their content retention in their algebra course with the algebra game play experience and was examined by the principal researcher to identify themes of the treatment experience that could directly relate to changes in content retention.

Additional game play data.

At the end of the treatment, the principal researcher did record some in-game statistics that were available for each student's *Dragonbox Algebra 12+* profile. In particular, this included

the number of in-game levels each student completed (of 10) and the "quality" of a student's

solution to a problem (measured from 1 to 3) based on the number of moves taken to complete

the level and whether the final solution was in simplest form. These were combined to form a

metric referred to as "levels attempted" reintroduced later in analysis; to combine these data, the

principal researcher tripled the initial number of levels possible (thus representing Level X with

1, 2, or 3 as the response "quality") and computed the percentage students completed of the new

total of levels, for which a quality of 2 would include completion of a quality of 1, and a quality

of 3 would include completion of both qualities 1 and 2.

Timeline overview.

On the first day of the study, students in both the control and treatment groups completed

the AGATE 1 pretest exam, and four students from the treatment group agreed to participate in

the first affective-focused interview. The AGATE 1 exams from the treatment group were

collected by the principal researcher, and the algebra course instructor collected and held the

AGATE 1 exams from the control group; the principal researcher did not have access to exam

results at that time. The affective-focused interviews were all conducted according to the

protocol described earlier and were video recorded by the principal investigator. Following

transcription by the principal investigator, the recordings were deleted; this was the standard

procedure for all interviews conducted during the study.

Four weeks into the study, at the half-way point of the treatment phase, the second

affective-focused interview and first cognitive-focused interview were conducted, recorded, and

transcribed. In the final week of the treatment phase, eight weeks into the study, the AGATE 2

posttest exam was administered to students in both the treatment and control groups, and the

final rounds of both the cognitive-focused and the affective-focused interviews were conducted

with students from the treatment group. Again, the algebra course instructor maintained the AGATE 2 data, although all interview data were immediately available to the principal investigator.

Following the treatment phase, no data were collected during the 1-month winter recess. Following the recess, on the first day that students returned to school in mid-January, the principal researcher administered the AGATE 3 to the students in both the treatment and control groups and administered a retention-focused questionnaire to the students in the treatment group. Upon collecting the questionnaires and the AGATE 3 from the treatment group students, the principal researcher had collected all necessary data from the treatment group students. Once this was confirmed, the algebra course instructor randomly selected 11 student profiles from the control group, and the principal researcher was given access to each of those 11 students' AGATE 1, 2, and 3 results.

As two final clarifying remarks, note that all interviews were conducted outside of the usual algebra-period time whenever students had free periods, and the AGATEs were all administered during the formal algebra class time. The questionnaire for the treatment group was also administered outside of students' usual algebra-period time. Figure 3.11 provides a visual of the data-collection timeline.

Whole Study Timeline
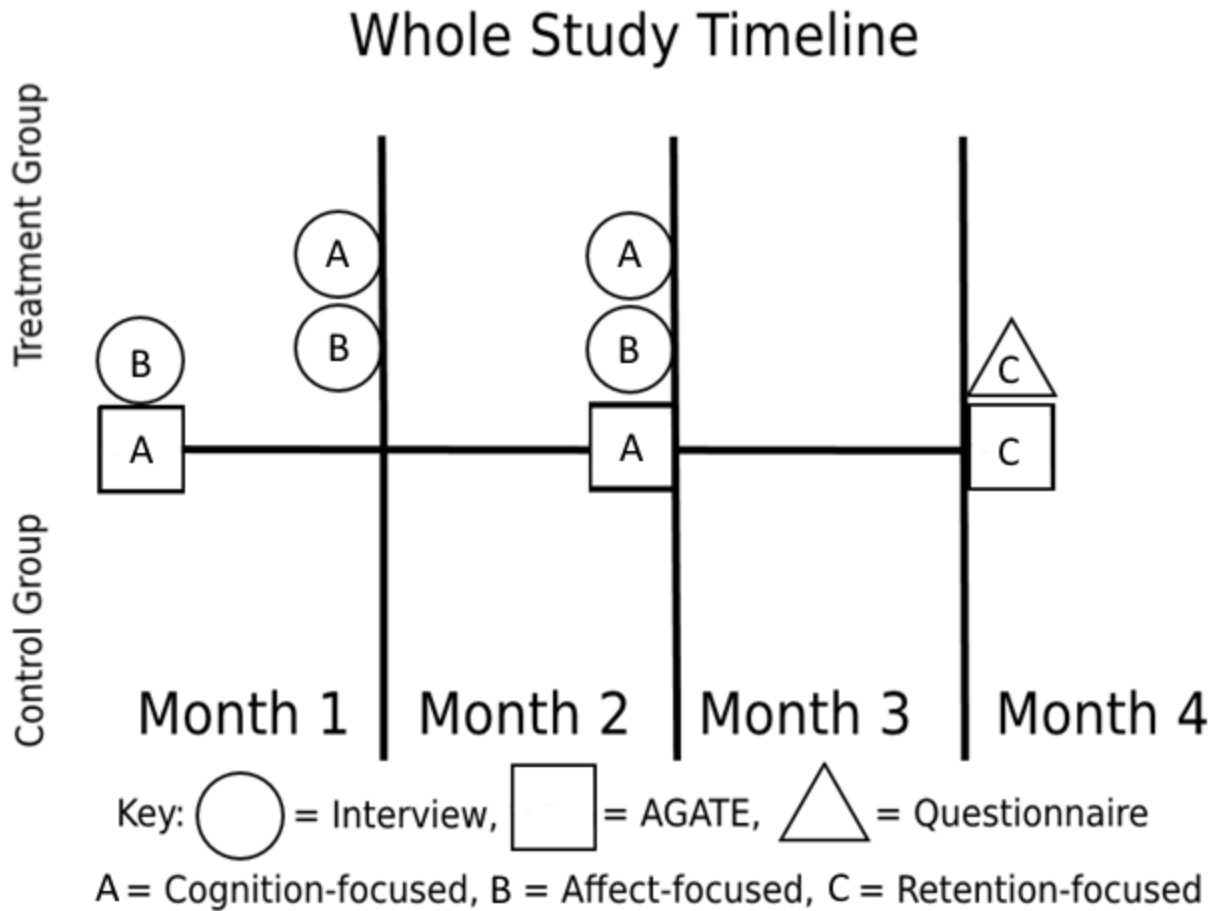
Figure 3.11: A timeline of the study identifying when, for what, and with whom each instrument was utilized.

Data Analysis

Addressing research question 1.

To address the first research question about cognitive learning outcomes, data on student cognition from the AGATE 1, AGATE 2, and cognition-focused interview protocols were analyzed in several ways. The themes resulting from those analyses were interwoven to provide a holistic answer to the first research question.

To begin analysis, the principal researcher first graded the AGATE 1 and AGATE 2;

notably, he was the sole grader in this process. Results of the AGATE 1 were used to establish a baseline of prior knowledge on both an individual and class-wide scale. All AGATEs were graded by the principal researcher in the following two ways: i) Each question was graded as being either correct (1) or incorrect (0), and no partial credit was awarded. Note that correct responses contained a question's solution without any notational errors, but did not have to be in simplest form (e.g. a response writing "$\frac{4}{2}x$" instead of "$2x$" would be considered correct if all other work leading to that point was correct). The decision to not penalize work for not being in simplest form was made because *Dragonbox* game play does not require solutions to be in the game-equivalent of simplest form—levels only require the in-game variable to be isolated. ii) In addition to grading responses as either correct or incorrect, the principal researcher coded incorrect responses as falling into one of the following error categories: 1) Computationally Erroneous; 2) Consistently Applying an Incorrect Conceptual Framework; 3) Omitted; 4) Attempted, but either Incomplete or Unjustified.

Here, these categories are briefly described. Work that was deemed "Computationally Erroneous" typically consisted of only one error that could be the result of carelessness, such as missing a negative sign in a final answer, or incorrectly summing two numbers together; "Computationally Erroneous" work did not contain compelling evidence that the student did not understand the processes necessary to solve the algebraic equation. Work in which the student was "Consistently Applying an Incorrect Conceptual Framework," however, did contain compelling evidence that the student did not have a correct understanding of how to solve the question at hand; this evidence was usually demonstrated across several recurrent errors either within the same question's work, or sometimes across multiple questions' work, for questions that involved identical or closely related understanding. Questions in which no work whatsoever

was provided were marked as being "Omitted." Questions that seemed to have some work, but presented no final answer, or which came up with nonsensical responses with or without work were marked as "Attempted, but either Incomplete or Unjustified."

Analysis based on grading (i) utilized the statistical techniques of Analysis of Co-Variance (ANCOVA) and the statistical software *R*, assessing each student's AGATE 2 results in conjunction with covariate data to identify cognitive development on individual and communal bases within and across the two groups over the course of the eight-week intervention phase. Two ANCOVAs were conducted; for both, the primary covariate was students' AGATE 1 results. This was done to verify that AGATE 1 scores were significant predictors of AGATE 2 scores. In the first ANCOVA, gender and group assignment were additional covariates. In the second ANCOVA, gender was maintained as a covariate, but a quantile for "in-game chapters attempted" was included as an explanatory variable, rather than the binary group assignment, while the group covariate was excluded. The number of in-game chapters attempted served as a rough estimate for the progress that students made through the game's 10 chapters; students with different quantiles for this measure might be considered as having completed "more of" or "less of" the treatment. Similarly, students in the control group could be considered to have completed 0 chapters.

Analysis based on grading (ii) examined and unpacked students' errors on both individual and communal scales, and compared responses from the AGATE 1 and AGATE 2. This analysis was done primarily by addressing patterns in students' responses that developed within each conceptual bucket of questions (i.e. within questions 1-5, questions 6-10, or questions 11-17). Themes of students' cognitive changes—especially students' misconceptions—that were identified from student work on the AGATEs were investigated and elaborated upon with respect

to potential connections to game play.

To support many of the claims made in reviewing the quantitative data, the principal researcher reviewed data harvested from the interview transcripts and, in analysis, described patterns of behavior, outlooks, and other cognitive changes that arose among interviewees. Each of the 8 interviews (2 for each of 4 students) were analyzed on both an individual scale (e.g. Student A Interview 1, Student A Interview 2) and a communal scale (e.g. all students' Interview 1 responses). Emergent themes were paired up with students' AGATE 1 and AGATE 2 responses. Themes were sought and viewed primarily through a lens of connecting game play to mathematics-doing, or mathematics-doing to game play.

The stated quantitative analyses helped the principal researcher recognize changes in cognition during the treatment that impacted students' cognitive outcomes, and the qualitative analyses helped assess the connections held by these changes and outcomes to the algebra game play experience. Together, these helped the principal researcher describe how integrating mathematical game play into a traditional eighth grade algebra curriculum impacted students' cognitive learning outcomes in elementary algebra.

Addressing research question 2.

To address the second research question about affective outcomes, data on student affect from the affect-focused interview protocols were analyzed to identify a variety of emergent themes related to the algebra game play experience.

The affect-focused interview protocols differed from the cognition-focused interview protocols in a notable way: although students still responded to prompts provided by the principal researcher, their responses were preceded by their given Likert-scale rating, which could present any of the following perspectives: strongly disagree, disagree, (feel) neutral, agree,

strongly agree. Students' Likert-scale responses from the 12 interviews (3 for each of 4 students) were first analyzed to chart instances of converging and diverging viewpoints from interviewees that spanned the course of the treatment. For example, initially similar Likert-scale responses during the first series of interviews changing to rather unlike responses during the second or third series of interviews indicated a divergence of opinion; the reverse indicated a convergence of opinion. Checking for convergences and divergences of opinions was done to identify interview prompts for which students' responses might suggest something about the game play experience's impact on students' affective outcomes. In particular, convergences suggest ways that game play might influence all students' affective outcomes universally, and divergences suggest ways that game play might influence each student's affective outcomes differentially.

Once these prompts were identified, students' responses were closely analyzed on both an individual scale (e.g. Student A Interview 1, Student A Interview 2) and a communal scale (e.g. all students' Interview 1 responses). Qualitative analysis of students' responses to such prompts identified recurring themes which the principal researcher coded and looked at relative to the algebra game play experience.

The stated qualitative analyses helped the principal researcher recognize changes in affect during the treatment that impacted students' affective outcomes. These analyses helped assess the connections held by these changes and outcomes to the algebra game play experience. Using these, the principal researcher described how integrating mathematical game play into a traditional eighth grade algebra curriculum impacted students' affective outcomes in relation to both elementary algebra and mathematics in general.

Addressing research question 3.

To address the third research question, data on content retention were attained by

collecting quantitative data from the AGATE 3 and qualitative data from a retention-focused questionnaire.

To begin analysis, the principal researcher first graded the AGATE 3 using precisely the same two methods as had been used for the AGATE 1 and AGATE 2. Then, using the statistical techniques of Analysis of Co-Variance (ANCOVA) and the statistical software *R*, each student's AGATE 3 results were assessed in conjunction with covariate data to identify the extent of students' content retention on individual and communal bases within and across the two groups following the one-month recess. Two ANCOVAs were conducted; for both, the primary covariate was students' AGATE 2 results. In the first ANCOVA, gender and group assignment were additional covariates. In the second ANCOVA, gender was maintained as a covariate, but a quantile for "in-game chapters attempted" was included as an explanatory variable instead of the binary group assignment covariate. As before, the number of in-game chapters attempted served as a rough estimate for the progress that students made through the game's 10 chapters; students with different quantiles for this measure might be considered as having completed "more of" or "less of" the treatment.

As had been done for the AGATE 1 and AGATE 2, in addition to grading responses as either correct or incorrect, the principal researcher coded incorrect responses as falling into one of the following error categories: 1) Computationally Erroneous; 2) Consistently Applying an Incorrect Conceptual Framework; 3) Omitted; 4) Attempted, but either Incomplete or Unjustified. After this, the principal researcher conducted an analysis of students' error types that examined and unpacked students' errors on both individual and communal scales, comparing responses across the AGATEs. This analysis was done primarily by addressing patterns in students' responses that developed within each conceptual bucket of questions (i.e. within

questions 1-5, questions 6-10, or questions 11-17). Themes related to students' content retention were investigated and elaborated upon with respect to potential connections to game play.

To support the claims made in reviewing the quantitative data, the principal researcher reviewed qualitative data harvested from the accompanying questionnaire. Emergent themes found analyzing the qualitative data were paired up with students' AGATE 2 and AGATE 3 responses. Themes were sought and viewed through a lens of connecting the algebra game play experience to memorable content learning or vice versa.

The stated quantitative analyses helped the principal researcher recognize the extent to which students retained content knowledge following the treatment phase, and the qualitative analysis helped assess the connections that students' content retentive reflections held to the algebra game play experience. Together, these helped the principal researcher describe how integrating mathematical game play into a traditional eighth grade algebra curriculum impacted students' content retention in elementary algebra.

Chapter 4: Results

Introduction

This chapter further elaborates on the analytical techniques used for reviewing each of the data sets mentioned in Chapter 3 and demonstrates results from the study. Quantitative data were collected from the AGATE 1 and AGATE 2 to compare data on cognition between the treatment and control group on both an individual and population-wide level. Quantitative data were also collected from the AGATE 3 to compare data on retention between the treatment and control groups on both an individual and population-wide level. Qualitative data were collected only from the treatment group from a set of two cognition-focused interview protocols, a set of three affect-focused interview protocols, and one retention-focused questionnaire.

A pseudorandom selection of 11 eighth-grade students (of a potential 30) formed the treatment group for this study, and a random selection of 11 of the remaining 19 students formed the control group. The principal researcher recognizes and comments that, because $n < 30$, the central limit theorem does not guarantee generalization of the following statistical results, and some quantitative data may be overly influenced by outliers.

To address concerns about normality of the data set, three Shapiro-Wilk tests were conducted using the results of the AGATE 1, AGATE 2, and AGATE 3. Considering the Shapiro-Wilk test's null hypothesis that each of the AGATE 1, AGATE 2, and AGATE 3 data sets used in this chapter are normally distributed, p-values of 0.0577, 0.2535, and 0.7133, respectively, were calculated, indicating that the null hypothesis should not be rejected in any of these cases; therefore, the following statistical considerations work with the assumption of the data being normally distributed. In addition to running statistical tests, several data plots are also included to

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make better sense of the results.

On Cognition

  Preliminary observations.

   The AGATE 1 was administered at the beginning of the study to determine the baseline

of both individual and classroom knowledge related to algebraic equation solving. A selection of

the measures of central tendency and dispersion (e.g. simple range, median, and interquartile

range) were calculated within and across each of the treatment and control groups' results to

assess students' content masteries at the very beginning of the study. Use of these measures was

selected over the use of alternatives (e.g. mean and standard deviation) to make the data more

robust, as the small population sizes in question may otherwise be more susceptible to outlier

data points. Note that the following calculations use 9 of the original 11 treatment students, as

two were unable to participate in the AGATE 2, so their corresponding AGATE 1 data points

were invalidated.

   Ranges were comparable; in both groups there was a floor of 0%. The treatment group

ceiling was 70.59%, while the control group ceiling was 64.71%. Treatment and control group

medians were identically 17.65%. However, the IQRs varied slightly with 23.53% for the

treatment group and 35.30% for the control group. This distribution of scores is displayed in

Chart 4.1.

Chart 4.1: Comparison of Number of Questions Answered Correctly by Control and Treatment Students on AGATE 1 (+/- 1 Standard Deviation)

Number of AGATE 1 Questions Answered Correctly (of 17)
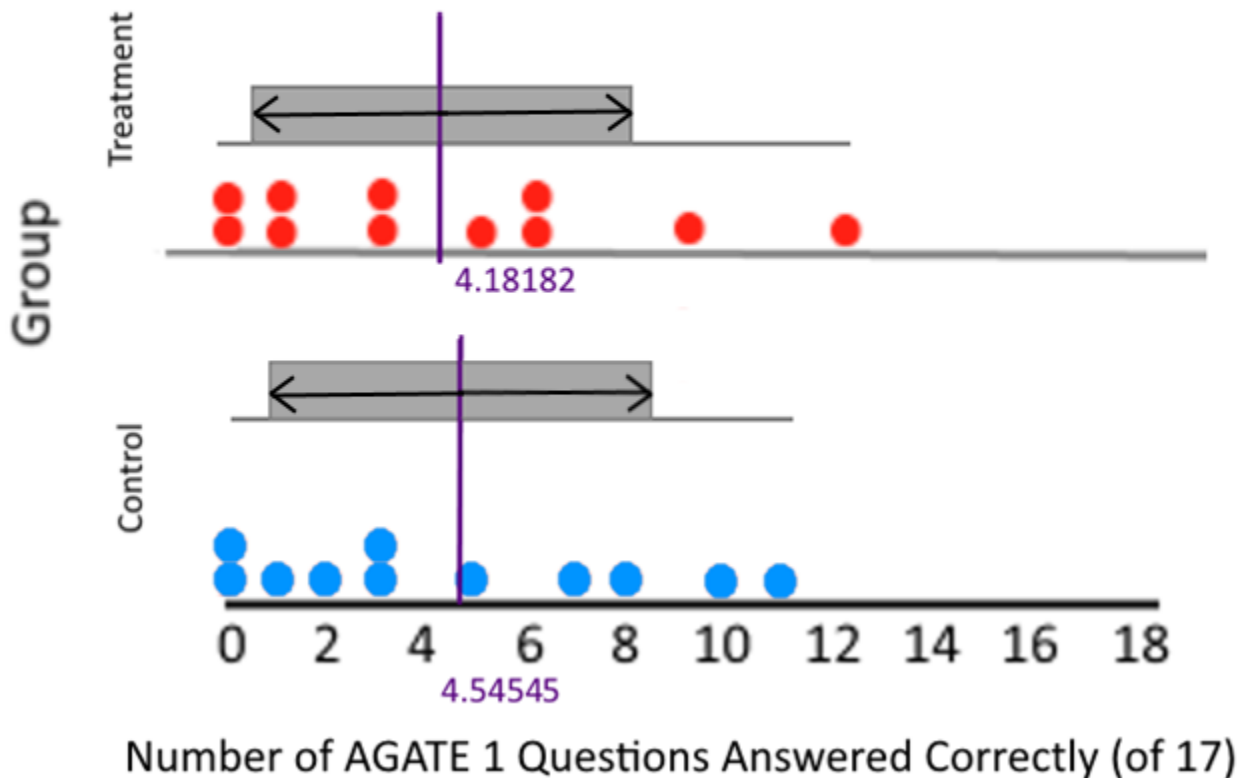
The AGATE 2 was administered at the end of the treatment phase of the study to determine changes to students' cognition regarding equation solving processes. For the AGATE 2, ranges were again comparable; in both groups, there was a floor of 0%. The treatment group ceiling was 88.24%, while the control group ceiling was 82.35%. The treatment group's class median remained at 17.65% across 9 students (recall that 2 students were absent on the day of examination administration, and their data could not be collected in the future); however, the control group's median of 11 students was 47.06%. The IQRs of both groups were more comparable, with 41.18% for the treatment group and 32.36% for the control group; notably, the IQR of the treatment group had grown considerably, while the IQR of the control group had

stayed roughly the same. Recognizing this, one also notes that the results of both groups were surprisingly low, given that the course instructor agreed that all students would have had some familiarity with each concept appearing on the AGATEs by the end of the treatment phase. The distribution of scores is displayed in Chart 4.2.

Chart 4.2: Comparison of Number of Questions Answered Correctly by Control and Treatment Students on AGATE 2 (+/- 1 Standard Deviation)

Number of AGATE 2 Questions Answered Correctly (of 17)
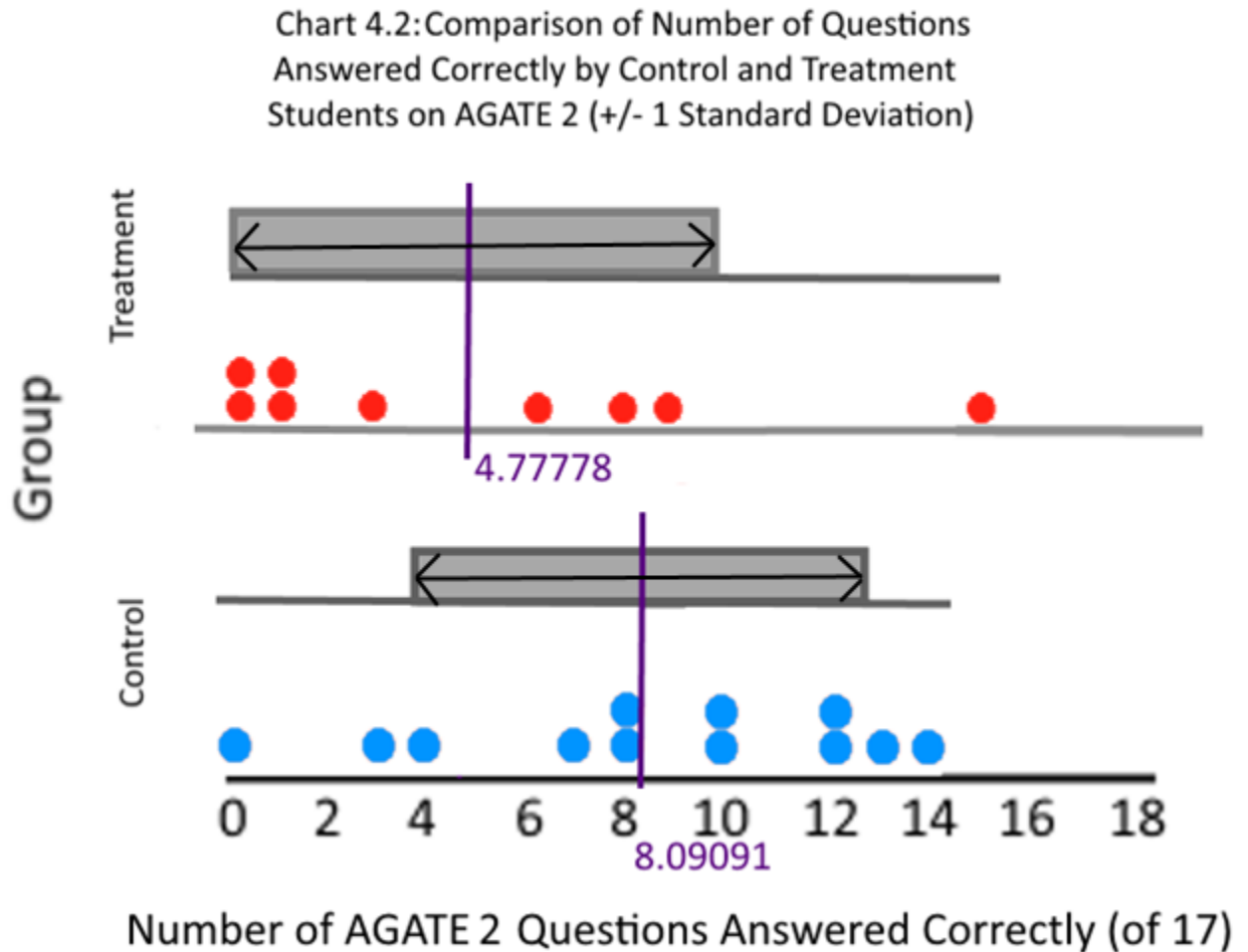
Additionally, the following Chart 4.3 summarizes each student's results on the AGATE 1 and AGATE 2 and Table 4.1 offers a summary of the measures of central tendency and dispersion from both groups' results across both examinations.

Chart 4.3: Comparison of Students' Results on the AGATE 1 and AGATE 2 Across Exams and Groups
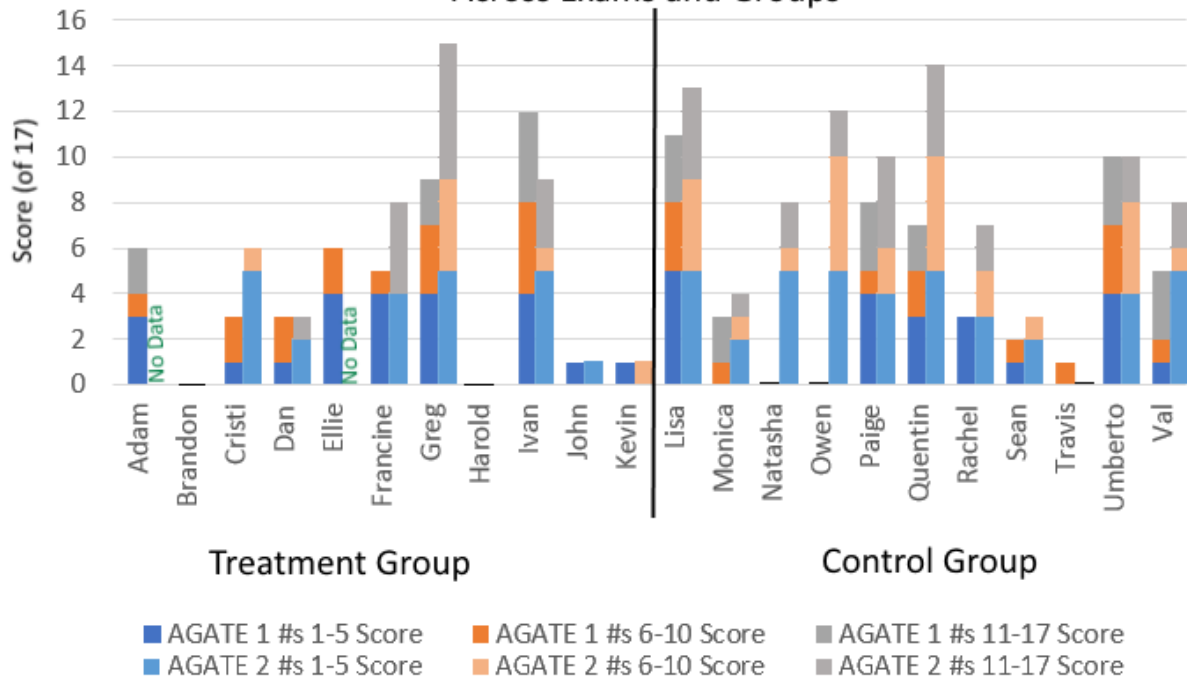
Table 4.1: A Comparison of the Measures of Central Tendency and Dispersion of Students' Results on the AGATE 1 and AGATE 2 Across Exams and Groups				
Data Type	AGATE 1 Treatment ($n = 9$)	AGATE 1 Control ($n = 11$)	AGATE 2 Treatment ($n = 9$)	AGATE 2 Control ($n = 11$)
Score Floor/Ceiling	0%/70.59%	0%/64.71%	0%/88.24%	0%/82.35%
Median	17.65%	17.65%	17.65%	47.06%
IQR	23.53%	35.30%	41.18%	32.36%

As discussed in Chapter 3, the AGATE 2 results of all control and treatment group students were also analyzed twice via analysis of covariates (ANCOVA) in which the primary covariate was students' AGATE 1 results. The following formula was used for the first analysis:

$$AGATE\ 2\ Results = (B_0 + B_1 * AGATE\ 1\ Results + B_2 * Gender + B_3 * Group).$$

This ANCOVA detected that AGATE 1 scores served as statistically significant predictors of

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AGATE 2 results ($f = 3.997, p = .001$) when controlling for gender and group assignment; no

other variables were found to be significant. A second ANCOVA was computed using the

following formula:

$$AGATE\ 2\ Results$$
$$= (B_0 + B_1 * AGATE\ 1\ Results + B_2 * Gender + B_3$$
$$* Game\ Chapters\ Attempted).$$

This ANCOVA detected that AGATE 1 scores served as statistically significant predictors of

AGATE 2 results ($f = 3.117, p = .0076$) when controlling for gender and considering the

explanatory variable for the number of game chapters attempted by students (i.e. a rough

measure of how much game content students encountered); no other variables were found to be

significant above and beyond AGATE 1 scores. Based on these two ANCOVA results, the only

claim that can be made is that students' prior knowledge at the start of treatment was the best

predictor of students' results obtained at the end of treatment. Further, the following Chart 4.3

demonstrates the relationship between the percentage of game chapters attempted by treatment

group students and each student's AGATE 2 performance for the 9 treatment group students who

completed the AGATE 1 and AGATE 2.
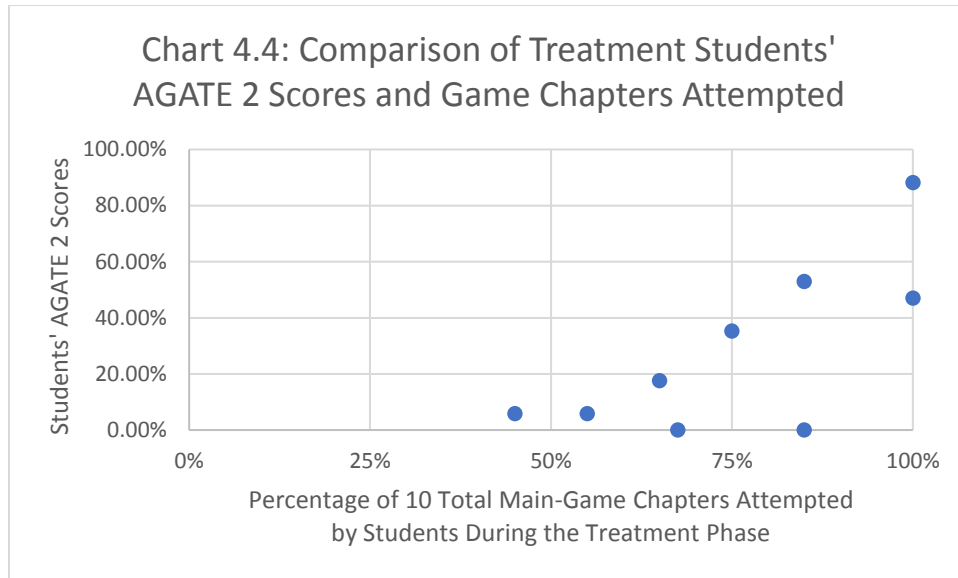
Chart 4.4: Comparison of Treatment Students' AGATE 2 Scores and Game Chapters Attempted

Additional quantitative data were collected on the number of error types students across the groups made on the AGATE 1 and AGATE 2. The principal researcher coded responses as either being correct or falling into one of the following error categories: 1) Computationally Erroneous; 2) Consistently Applying an Incorrect Conceptual Framework; 3) Omitted; 4) Attempted, but either Incomplete or Unjustified.

For the AGATE 1, the treatment group gave 187 responses in total[10]. Forty-six responses were correct, 11 were computationally erroneous, 31 showed consistent applications of an incorrect conceptual framework, 77 were omitted, and 22 were attempted, but either incomplete or unjustified. For the AGATE 2, the treatment group gave 153 responses in total[11]. Forty-three responses were correct, 3 were computationally erroneous, 24 showed consistent applications of an incorrect conceptual framework, 50 were omitted, and 33 were attempted, but either incomplete or unjustified. For the AGATE 1, the control group gave 187 responses in total. Fifty responses were correct, 7 were computationally erroneous, 42 showed consistent applications of

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[10] Each of 11 students answered the same 17 questions. The same is true of the control group for the AGATE 1 and the control group for the AGATE 2.

[11] As mentioned before, two students were absent, so each of 9 students answered the same 17 questions.

an incorrect conceptual framework, 57 were omitted, and 31 were attempted, but either incomplete or unjustified. For the AGATE 2, the control group gave 187 responses in total. Eighty-nine responses were correct, 11 were computationally erroneous, 38 showed consistent applications of an incorrect conceptual framework, 40 were omitted, and 9 were attempted, but either incomplete or unjustified. These data are visualized in Figure 4.1.

Figure 4.1: Student Error Profiles from the AGATE 1 and AGATE 2
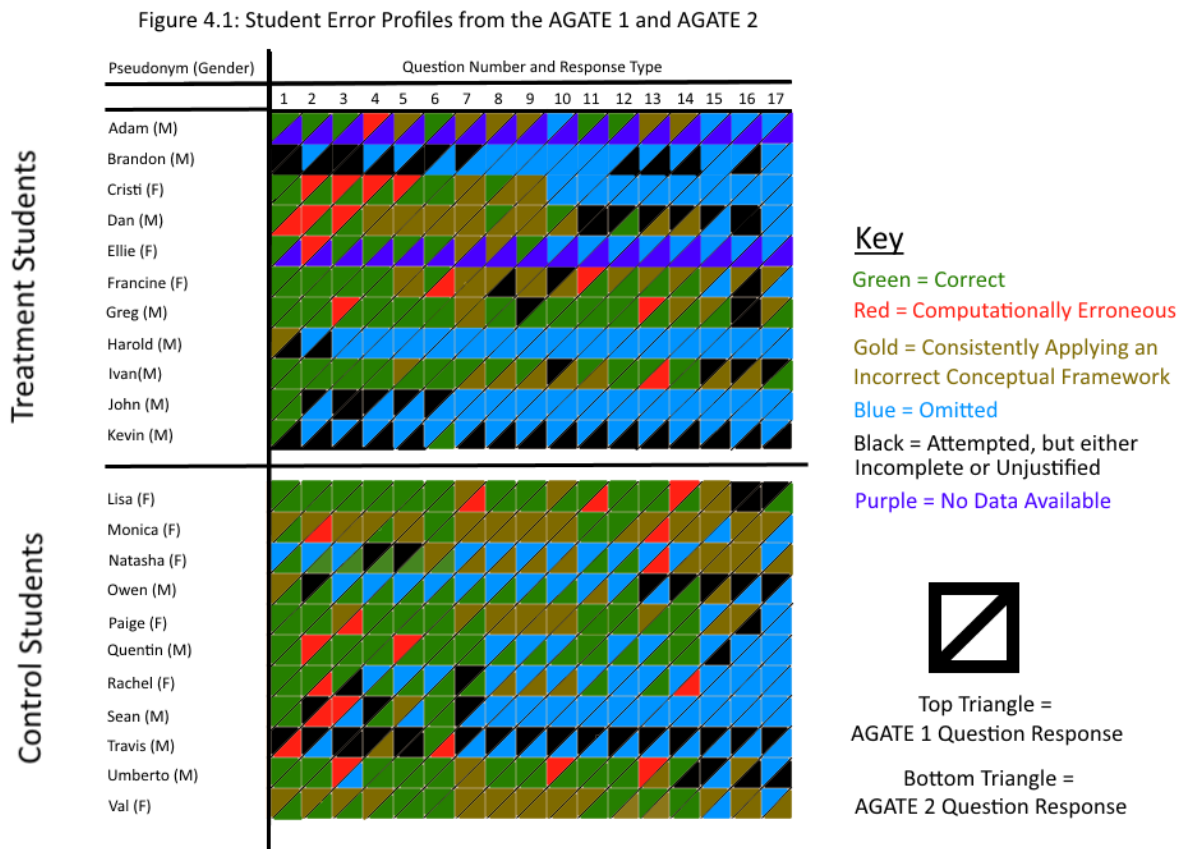
Figure 4.1 is rich with information but can be challenging to navigate. The most important error-related aspects for examination are boxes which contain at least one gold triangle. If a box's top triangle is gold, but its bottom triangle is green (e.g. Francine, Question 12), this is an indication that a student initially had some sort of conceptual misunderstanding but was able to correct it during the treatment. If a box's bottom triangle is gold, but its top triangle

is green (e.g. Ivan, Question 11), this is an indication that a student originally had the correct means of understanding a question but became confused about the process during the treatment. If both of a box's triangles are gold (e.g. Dan, Question 4), then a student held some misconception at the start of the treatment and, most likely, maintained that misconception throughout the treatment; however, there is also the possibility that the misconceptions seen at the beginning and end of the treatment are distinct.

Figure 4.1 may additionally be used to detect patterns of cognitive change within and between the groups. As mentioned in earlier chapters, the AGATEs' 17 questions may be thought of as testing concepts in three parts: questions 1-5 examine basic uses of the addition, subtraction, multiplication, and division properties of equality; questions 6-10 test the former, but introduce fractional multiplication and division; questions 11-17 test both of the former, but test also the distributive property of multiplication over addition, heightened mastery of inverse operations, and, to a limited extent, factoring skills. Reemphasizing the importance of gold triangles, areas of Figure 4.1 that include, across many students, partially or fully gold boxes would be places in which the treatment and/or control populations either suffered or recovered from some type or types of conceptual misunderstanding. Because of the insight that it provides, Figure 4.1 will serve as our guidebook for navigating the analysis of students' cognitive changes; I identified four important themes across these results that I discuss in the subsequent sections.

Students' potential cognitive changes based on quantitative and qualitative data.
A combination of students' quantitative and qualitative data informed the researcher's following observations on cognitive changes which have been organized into four greater themes.

Metacognitive unidirectionality: likening game play to formal mathematics-doing.
Across the cognitive-focused interviews, students discussed the extent to which they felt

game play synchronized with formal equation-solving ideas. For the most part, student metacognition regarding the link between game play and formal equation solving was unidirectional; most observations made by students about game play expressed situations in which, while playing *Dragonbox*, they had called themselves back to ideas about formal mathematics doing. In general, there is little evidence showing that students might do the reverse (e.g. calling back to *Dragonbox* experiences while doing equation solving as a part of their algebra course), and no evidence that during the treatment, students considered the relationship bidirectionally. In the following transcript samples, John and Harold each discuss connections between the essence of the *Dragonbox* experience and the process of isolating variables. John's description provides an example of game play which he likens back towards formal mathematics for better cognitive maneuverability—he saw *Dragonbox* as a covert version of his algebra exercises. Harold's description provides a weak link between doing algebra in a formal classroom setting and drawing back to game play experiences.

[John Interview 1]

> [0:15-0:47] John: Like, at first, [players] wouldn't think [the game] would be like math 'cause there's no, like, "Oh, 2+2 is 4," but then you realize 'cause you have to get X by itself, and you have to like, what do you call it, like, um... I can't think of the word, but you have to get X by itself and that's like math, yeah.
>
> _____

[Harold Interview 2]
> [2:12-2:20] Harold: I see how DragonBox can relate to math, 'cause I remember [earlier this week in class] when I was first talking about [how] I need to isolate variables [to solve my problem,] that kind of reminded me of *Dragonbox*.

Perspectives like the one John put forth seemed to be dominant when speaking with the other two interviewees, Ivan and Greg. However, whereas John and Harold only elaborated on, arguably, the most obvious connection between game play and equation-solving, Ivan and Greg

each discussed deeper connections. Ivan spoke not only about recognizing that game play

strategies and goals aligned with equation-solving techniques, but also that sometimes it was

more convenient for him to perform in-game when he converted his problem into formal

mathematical notation—a resoundingly clear indication of his metacognitive perspective. Greg

simply noted further connections beyond just variable-isolation, pointing out in-game

implementations of like terms and negative numbers, as examples.

[Ivan Interview 1]

> [0:38-2:18] Ivan: I would say. . . sometimes [to beat a level] you have to actually think of [the game problem] as a math equation…like, you can just ignore those different little cubes[/tiles] and actually think of it as a math problem, like x and y, one and two and three…because if you actually look at the game, it has the two different sections of a work place, and that is almost symbolizing the equal sign, where you move it between them to change color…

_____

[Greg Interview 2]

> [0:38-1:09] Greg: [*Dragonbox* is] like an algebra game. The problems in the game are just like the ones we do in [class]. When you combine like terms, the symbols, numbers, and letters… if you bring stuff to the other side [of the screen] and you change the symbols…if you bring a negative star, like, whatever symbol, to the other side, it becomes positive.

However, while most interviewees indicated their calling-back of formal algebra-doing

for the sake of expediting their game play experiences, John's interview makes it clear that the

mapping of game play experiences to formal mathematics-doing is neither necessarily automatic

nor effortless when game play is never formally connected to the mathematics in one way or

another.

> [John Interview 2]

> [4:09-4:29] John: …like I said, *DragonBox* is different. It's not numbers. It's more like pictures and stuff to isolate the variable... I wouldn't think about this game while I'm doing math cause it's not like numbers.

Quantitative data could not be provided to parallel any of the here-stated qualitative data, as there

was no work produced on students' AGATEs that appears to refer to any in-game representations or modalities.

Dual-natured development of mathematical reflexes.
Some students reported that, because of game play, select mathematical processes became second-nature and automatic to them—here, the principal researcher adopts the term "reflexive" from an interview with student Ivan. Ivan discusses some circumstances in his formal algebra course in which he realized he was involuntarily calling upon his *Dragonbox* experiences. However, his mathematical reflexes—developed, in part, by *Dragonbox* mechanics that force certain actions—ended up leading him down the wrong path while solving a question in class on at least one occasion, demonstrating the negative potential of this attribute.

[Ivan Interview 1]

[3:20-4:03] Ivan: …so, for example, a few days ago when I was playing [the game], like, it's almost like... "reflex" when you're doing it. It's like the first step you automatically know, and then one time I remember on a math test, I actually just had it a few days ago, I've actually applied what I've just remembered, and kind of used the reflex that I've got off of this game for my math test.

[4:04-4:13] Researcher: Now, you're saying reflex. What do you mean by reflex? Is there a specific thing you meant?

[4:15-4:19] Ivan: Yes, it is a very simple [equation].

Ivan writes $\frac{x}{y} + a = b$ on the board, and indicates he needs to solve for x

[4:33-4:55] Ivan: Yeah. This. At first, I was confused on whether or not I should also multiply y to the a and multiply y to the b, and then after I watched the game, and kind of remembered what's going on in the game, I just automatically know that you just can't multiply [the y] to the a. So, this is almost like a reflex now where you see y you just multiply it to the other side.
Ivan writes $x + a = yb$ and concludes that $x = yb - a$

Recalling the description of the multiplicative property of equality's *Dragonbox* equivalent

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demonstrated in Chapter 3, one recognizes that the operation that Ivan describes would never have been allowed by *Dragonbox*'s in-game mechanics regulating operations involving multiplication. In this case, Ivan is misremembering a scenario from game play and internalizing it potentially because he recognizes the game's mechanics would not allow him to make an illegal movement. Although the AGATEs do not have any questions that line up precisely with the one Ivan presented ($\frac{x}{y} + a = b$), there is one that comes close: question 7, which appears as $\frac{z}{a} = 4 + (-d)$ [solving for z] on the AGATE 1, and as $\frac{z}{d} = 5 + (-c)$ [solving for z] on the AGATE 2. Figure 4.1 indicates that Ivan developed a conceptual misunderstanding related to this question type sometime between the AGATE 1 and AGATE 2 (meaning during the treatment phase).

In the AGATE 1 iteration, he is able to correctly multiply both sides of his equation by the denominator a—in particular, he is careful to indicate that he is multiplying the whole right-hand side of the equation by placing its original contents in brackets. Instead, on the AGATE 2 iteration, he effectively transfers his denominator to the right-hand side of the equation—even having just successfully solved a question, number 6, in which he clearly interacted differently with a fraction's denominator (Figure 4.2).

Figure 4.2: Ivan's Developed Conceptual Misunderstanding
Regarding Multiplying Both Sides of an Equation when
One Side has Multiple Terms

AGATE 1 Question 7
(Solving for z)

AGATE 2 Questions 6 and 7
(Solving for z)

Ivan

$\frac{z}{a} = 4 + (-d)$

$Z = a[4 + -d]$

Ivan

$\frac{z}{6} = 13$

$Z = 13 \cdot 6$

Ivan

$\frac{z}{d} = 5 + (-c)$

$Z = \frac{5 + -c}{d}$

Ivan's trust in the *Dragonbox* engine to prevent him from making algebraically incorrect moves is not unfounded and there are circumstances which highlight the potential positive aspects of the reflexivity attribute. Figure 4.1 shows, too, that each of Dan, Francine, and Ivan incorrectly responded to question 5 on the AGATE 1, and that, of these three who were following an incorrect conceptual framework, only Ivan was able to provide a correct solution to question 5 on the AGATE 2.

On the first exam, the equation was $y \times (-b) = a + 2$, and on the second exam, it was $y \times (-a) = 2 + b$; both cases asked to solve for y. Again, although game play did not feature a question that mirrored these precisely, it had one level that came close—the early-game Chapter 3-16, the in-game equivalent of $a \times x + b = d$ (Figure 4.3).

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Figure 4.3: A level reimagined by Ivan as a formal algebraic equation during his first interview

[Ivan Interview 1]

> [10:59-11:06] Researcher: … Can you try to solve this, but I would love it if you could explain every move that you're making, okay?

> [11:07-11:39] Ivan: Okay. So, in this one, the dot here kind of represents the multiply sign, so in normal math you would just divide the a, so I will just put it under, it goes out to all of it. You can just cancel it out…oh…actually, I have to reset this level…I would just do this…

> *Ivan has written the equivalent of $x + \frac{b}{a} = \frac{d}{a}$, but decides to scrap it when he sees an alternative method. *

> [11:40-11:41] Researcher: So again, tell me what you're doing.

> [11:42-11:47] Ivan: I was adding a negative [b to both sides] so that it could just cancel out into zero.

> *Now, Ivan has gotten the equivalent of $ax = d - b$. *

> [11:48-11:48] Researcher: Ah, I see.

> [11:49-12:02] Ivan: And then now I'm assuming I just have to divide by what's left, so a, and, uh, that's it.

In his solution to this question, Ivan works his way through the problem to a point that is very similar to question 5 on either AGATE with the main difference being the absence of a negative term multiplying onto the variable for which he is solving. However, in executing his final movement in the *Dragonbox* level, Ivan states he need only "divide by what's left," and the game engine only allows him to do just that. Comparing this to his work from the AGATE 1, it's clear that he has progressed and corrected an error in which he felt that division by a term changed the sign of that term—an error that Francine maintains across exams. Figure 4.4 compares the work of Dan, Francine, and Ivan on question 5 of the AGATEs 1 and 2.

Figure 4.4: Dan, Francine, and Ivan's Conceptual Misunderstandings in an Early AGATE Question

AGATE 1 Question 5 (Solving for y)	AGATE 2 Question 5 (Solving for y)
Dan	**Dan**
$y \cdot (-b) = a + 2$	$y \cdot (-a) = 2 + b$
$y = -b \cdot a + 2$	$x = 2 + b + a$
Francine	**Francine**
$y \cdot (-b) = a + 2$	$y \cdot (-a) = 2 + b$
$y = \frac{a+2}{b}$	$y = \frac{2+b}{a}$
Ivan	**Ivan**
$y \cdot (-b) = a + 2$	$y \cdot (-a) = 2 + b$
$y = \frac{a+2}{+b}$	$y = \frac{2+b}{-a}$

It's not entirely clear why Dan and Francine might not have corrected their misunderstandings as Ivan did from either game play experiences or regular algebra class sessions; it is important to note both that Chapter 3-16 was an early in-game level that all treatment students had cleared by the time of the AGATE 2, and 8 of the 11 control group students were able to answer question 5 correctly. However, there was one notable difference among the treatment students: Ivan discussed the level with the researcher during an interview, while the other students did not necessarily reflect on or discuss the level's content in a formal

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sense.

Challenges isolating variables included in fractions.
Students' AGATE 1 and AGATE 2 performances indicated that both treatment and

control group students struggled across the AGATEs with questions that involved fractional

multiplication and division; here, we review a variety of misconceptions related to these

questions that were present at either the start or end of the treatment. Some misconceptions

demonstrated at the beginning and end of the treatment phase appear unlinked in the sense that a

student may have had one misconception on the AGATE 1 and an entirely different

misconception on the AGATE 2. This diversity in demonstrated misunderstandings makes it

challenging to attribute any one aspect of game play to perpetuating a specific erroneous

concept. However, one interviewee was able to attribute game play to his mastery of the

multiplication and division properties of equality; following the review of misconceptions,

Greg's vignette demonstrating this is discussed.

Francine's performances stand out as particularly interesting cases because they bring to

light something curious: although Francine is completely unable to produce any correct answers

to questions 5 through 10 on the AGATE 2 (thus gaining points neither in the entire second

bucket of questions nor the tail of the first), she does go on to provide multiple correct answers in

the third bucket of questions which has several equations that do not utilize operations on

fractions. Francine is not alone in her challenges with the second bucket of questions, 6 through

10; Figure 4.1a shows that there was a large discrepancy between the groups' performances in

this section.

Figure 4.1a: Student Error Profiles from the AGATE 1 and AGATE 2
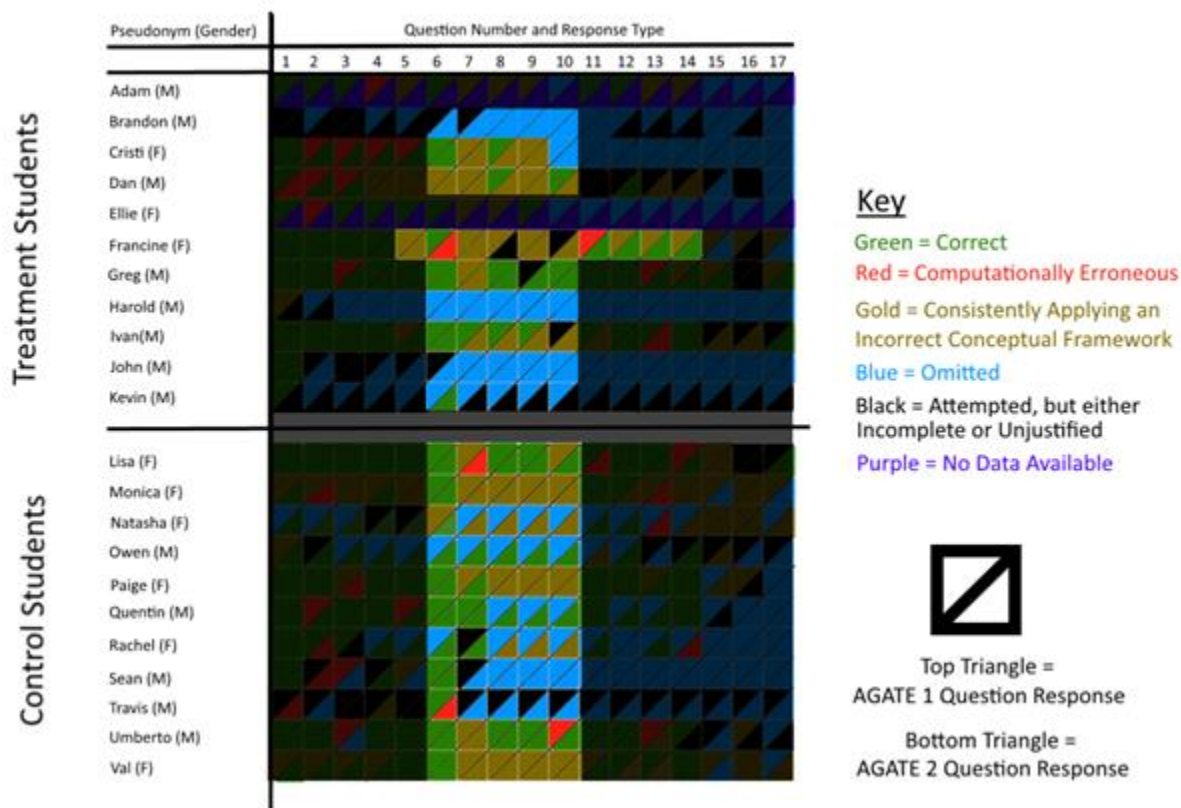
While control group students greatly increased the number of these questions they answered correctly on the AGATE 2 as compared to the AGATE 1, several treatment group students that answered questions correctly on the AGATE 1 answered the same question types *incorrectly* on the AGATE 2—look specifically to Cristi, Dan, and Ivan. Comparing the conceptual misunderstandings of students in both groups indicates a pervasive disclarity regarding the actual processes for isolating variables when the variables are a part of fractions. Figure 4.5 shows the work of Francine from the treatment group and Rachel from the control group demonstrating, respectively, misunderstanding of the multiplication property of equality, and misunderstanding of fractional multiplication including a variable.

Figure 4.5: A Comparison of Conceptual Misunderstandings
in Questions Involving Operations with Fractions
AGATE 2 Question 9 (Solving for a)

Francine

$$\frac{7}{a} = x + (-d) \cdot a$$

$$7 = x - d \cdot a$$
$$\rightarrow x \quad x$$
$$\overline{}$$
$$7 - x = -d \cdot a$$
$$+ d \quad + d$$
$$\overline{}$$
$$7 - x + d = a$$

Rachel

$$\frac{7}{a} = x + (-d)$$

$$\frac{1}{7}\left(\frac{7}{a}\right) = (x - d)\frac{1}{7}$$

$$a = \frac{x - d}{7}$$

Curiously, Rachel's exact error is replicated by several control group students, including

Paige, Sarah and Val, while some combination of both Francine and Rachel's errors are

replicated by Monica and Natasha; upon closer examination, more than a misunderstanding of

fractional multiplication, Rachel's conceptual misunderstanding in question 9, for example,

seems to be that $\frac{1}{7} \times \frac{7}{a}$ has the sevens "cancel," leaving only an a behind—presumably as the new

numerator, the old one having been erroneously "deleted."

Following up on this point, question 8 was correctly answered by Cristi, Dan, and Ivan on

the AGATE 1, but incorrectly answered by all three students on AGATE 2. However, each of

them presented unique errors on the AGATE 2, and only Ivan's lined up exactly with errors seen

by other students—namely, he has the same misunderstanding about fractional multiplication

that Paige, Rachel, Sarah, and Val do. Dan's misunderstanding is a variant that includes a sign

swapping. While Ivan and Dan consistently apply their erroneous misunderstandings in questions

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like 9 and 10, Cristi's misunderstanding seems unfounded on both exams; Figure 4.6 shows the

work of Cristi, Dan, and Ivan on question 8, plus Cristi's work on question 9. What is most

perplexing is that on the AGATE 1, both Cristi and Ivan's work on question 8 indicate some form

of interaction with fractional multiplication or division that was correct, but lost by the time of

the AGATE 2.

Figure 4.6: More Comparisons of Conceptual Misunderstandings in Questions Involving Operations with Fractions

AGATE 1 Questions (Solving for a)	AGATE 2 Questions (Solving for a)
Cristi, Question 8 $\frac{x}{a}=4$ $x \div 4 = a$	**Cristi, Question 8** $\frac{y}{a}=z$ $z \cdot y$
Cristi, Question 9 $\frac{x}{a}=4+(-d)$ $\dfrac{4+d}{x}$	**Cristi, Question 9** $\frac{y}{a}=x+(-d)$ $\dfrac{y}{x} - d = a$
Dan, Question 8 $\frac{x}{a}=4$ $\frac{x}{4}=a$	**Dan, Question 8** $\frac{y}{a}=z$ $a = \dfrac{z}{-y}$
Ivan, Question 8 $\frac{x}{a}=4$ $x = 4a$ $\frac{x}{4}=a$	**Ivan, Question 8** $\frac{y}{a}=z$ $a = \dfrac{z}{y}$

Having taken the previously examined misconceptions into account, it might seem unlikely that game play could serve as a good resource for correcting students' understandings of operations involving fractions with variables. However, in the following transcript sample, Greg explicitly discusses his correct understanding of the multiplication and division properties of

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equality which he derived from game play, indicating that such learning is indeed possible.

[Greg Interview 1]

[6:27-6:42] Greg: Well, the part in Chapter 3 when you have to take a number and put it to every side, like, made me think…when we have an algebra problem…and, like, say it's a variable that's a fraction, say $\frac{x}{3}$ plus…no, whatever, it doesn't matter… equals [21].

Although he has Chapter 8-17 open on screen, Greg clarifies that for what he wants to demonstrate, he is going to use, for simplicity, $\frac{x}{3} = 21$.

[6:43-7:03] Researcher: Here, can we put that on the board? That would be great if you wanted to show it to me. I know that marker's not the best, but... okay, so what have we got here?

[7:04-8:07] Greg: Um, so for this, I can multiply it by three…so, $\frac{x}{3} = 21$…you multiply by the reciprocal, wait um... yeah. So…$x = 63$…

Greg writes $\frac{x}{3} \div \frac{1}{3} = 21 \div \frac{1}{3}$ and evaluates it to get $\frac{x}{3} \times \frac{3}{1} = 21 \times \frac{3}{1}$, and ultimately $x = 63$.

[8:10-8:14] Researcher: Yeah, great. So, what about this is somehow related to the game play?

[8:15-8:22] Greg: So, you know the part where I said we take this [number] and then, like, you put it here? You had to [divide by $\frac{1}{3}$] everywhere.

[8:23-8:25] Researcher: When you say, "put it here," are you saying put it on the bottom? Yeah, why don't you just show me [in the game]?
[8:26-8:29] Greg: Like…and then you put it on everything here.

*Greg points to the 2 in $2x$ and indicates that, when multiplying by the reciprocal, the multiplication property of equality extends over all the terms in the equation; therefore, he places a 2 in the denominator of each term to signify multiplying by $\frac{1}{2}$ (Figure 4.7). *

[8:35-8:46] Researcher: So, the idea that when you're going to do some sort of, let's say, can we call it division… on a term in the equation, you need to do that division on all of the terms in the equation…is that what you're saying?

[8:47-8:47] Greg: Yeah.

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Figure 4.7: In the above shot, Greg demonstrates his intention to divide by 2. In the below shot, he expresses the division property of equality by dividing each term on both sides of the equation by 2.

Greg had the benefit of discussing game play that involved solving for variables that were

part of fractions with the principal researcher. It is also notable that Greg was one of only three students that completed all the game's content during the treatment phase—Cristi, Dan, and Ivan only completed 75%, 65%, and 85% of the sum content, respectively. Greg's ability to engage with many more levels that were visually similar to formal algebra content (e.g. Figure 4.7) may have helped him achieve understanding on this matter this his peers did not. However, Francine, the only other student who completed the AGATE 2 and the full game content, was shown to struggle in the second bucket of questions—although she did score highly on the AGATE 2 compared to her peers, overall.

Challenges with advanced content, especially factoring.

Because the final bucket of questions, 11 through 17, were very diverse from a mathematical perspective, students' results varied dramatically. Content tested in these questions provided students with opportunities to exercise heightened mastery over inverse operations, factoring, and the distributive property of multiplication over addition. In general, most students in both groups with non-zero scores managed to increase the number of questions they answered correctly in this section between the AGATEs 1 and 2. However, one question stands out: number 15 was answered correctly only by Greg during the AGATE 2 (and had been answered correctly by no student during the AGATE 1), as demonstrated in Figure 4.1b.

Figure 4.1b : Student Error Profiles from the AGATE 1 and AGATE 2
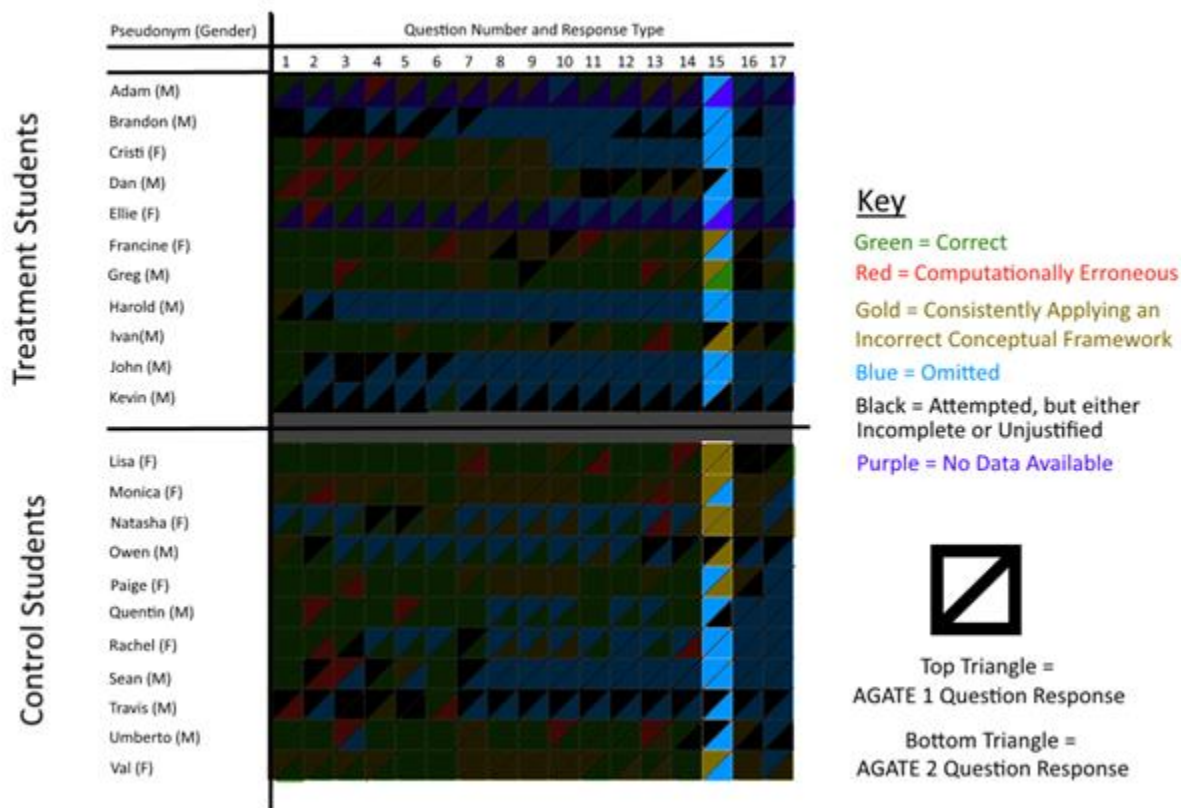
This question stands out as one of the best candidates throughout the exams for the use of factoring strategies, which interview data showed students had a challenging time understanding during game play. In an interview with Harold, it becomes clear that he understands the utility of the in-game representation of factoring but is entirely unable to articulate it as a parallel to algebraic factoring or prove that he understands the concept beyond being a game move achieved with trial and error.

[Harold Interview 2]
>*Harold loads Chapter 7-7, which is the equivalent of the equation $x + 2x + 3x = 3$. He subtracts 3 from both sides of the equation and *then* attempts to factor the left-hand side (Figure 4.8). *

>[13:21-13:44] Harold: Alright, so here's what I think is an alternative thing you can do. I want to get [everything] inside [a] bubble. ... Alright so now I'm a little bit confused, I'm trying to get the box up here [so that I can remove it from the bubble/parentheses], so I

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cancel this out.

[13:45-13:47] Researcher: So, let me just ask you, when you pull that box out [of the bubble/parentheses], why do you pull that box out?

[13:48-13:58] Harold: …so I can have it on the top, right here, so that I can remove these two boxes right here. All I need is one.

Figure 4.8: The left-hand side of the screen pictured above is, effectively, $x + (-3) + 2x + 3x$. Harold intends to factor the left-hand side of the level, but cannot do so, as not all terms share a common factor since he subtracted 3 from both sides of the initial setting.

In this vignette, Harold demonstrates an understanding that some combination of in-game movements will lead him to eliminate what he views as "extra copies" of his main variable; however, even though the level actually began with all of the left-hand side tiles set in a "factorable form," he is unable to recognize this, and assumes that *all* of the game tiles need to be present on the left-hand side in order to utilize a factoring technique.

Misunderstandings about in-game powers upon first reveal were not altogether uncommon, especially during later levels which introduced more advanced concepts—besides the representation of factoring, the in-game representations of parentheses, the enforcement of the distributive property, and the equivalence of 1 and $\frac{x}{x}$ (for non-zero x) all met some students

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with initial confusion. However, increased exposure to the content proved to be a useful way to grapple with these ideas for at least some student. Greg indicated that he received good practice with factoring exercises that tied directly into his coursework during some levels in the game's final chapter. For example, in Chapter 10-11, Greg attempted to solve the fully formally notated equation $b(e + x) + 2 = (-2)(x + (-3)) + (-4)$; to be clear, all of the numbers, variables, operations, and signs in the equation appeared exactly as written here (besides the parentheses instead being ice blocks or bubbles). He was able to explain the use of the distributive property and basic inverse operations to get the equation to $be + bx = (-2)x$, then was able to identify that, because he was solving for x, he would need to move all x-terms to one side of the equation to factor. This resulted in him getting $be = x((-2) + (-1)b)$, which he immediately changed to $\frac{be}{(-2)+(-1)b} = x$ by treating the parentheses next to x as a single term.

The experience that Greg received when working in *Dragonbox*'s later chapters seemed to help him score points on question 15 when encountering it at the treatment's conclusion, as he was able to both factor and then utilize the concept of polynomial division. Based on the work of the few other students that attempted the question in either group, it's not entirely clear that all students understood the concepts of the distributive property or factoring, and it's evident that many students still had minor confusion about utilizing inverse operations in novel situations. Figure 4.9 compares Greg's question 15 solution to the work of fellow treatment student Ivan, and the work of two control students, Natasha and Owen.

Figure 4.9: A Comparison of Student Work on a Question Necessitating Factoring and Polynomial Division

AGATE 2 Question 15 (Solving for c)

Greg

$(c \cdot a + d) + (b \cdot c + a) = b$

$ca + d + bc + a =$

$$\frac{b - a - d}{b + a}$$

c

Natasha

$(c \cdot a + d) + (b \cdot c + a) = b$

$$c = \frac{b - a - d}{c + c}$$

Ivan

$(c \cdot a + d) + (b \cdot c + a) = b$

$c \cdot a + d + b \cdot c + a = b$

$2ca + d + a = b$

$$c = \frac{b - d - a}{2a}$$

Owen

$(c \cdot a + d) + (b \cdot c + a) = b$

$$c = (b \div a - d)(c \div b - a)$$
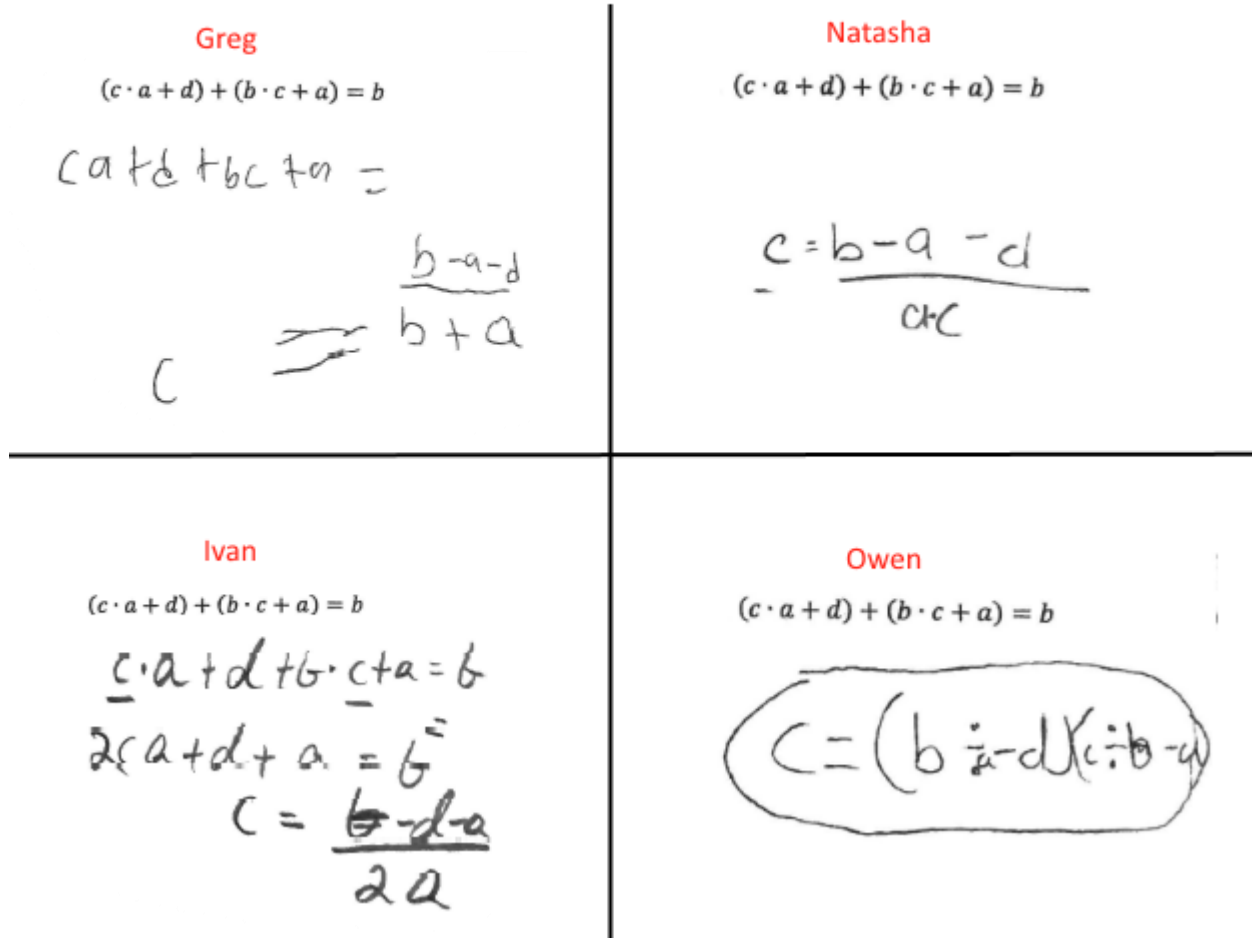
Ivan's work demonstrates a misunderstanding that equates $c \times a$ with $c + a$ to produce as their sum $2ca$; he also seems to mistakenly write a b term as an a term, but otherwise takes valid actions. Notably, no attempt at factoring is made. Natasha demonstrates an understanding that she may first subtract a and d from both sides of her equation, but presumably stumbles when dealing with a left-hand side of $c \times a + b \times c$, ultimately dividing both sides of her equation incorrectly as opposed to utilizing a factoring technique to help isolate c; however, Natasha fails to recognize that her final response includes a c on both sides of her equation. Owen, in attempting to isolate c, makes a poor attempt at replicating most of the left-hand side of the

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equation on the right-hand side, albeit with inverse operations; for instance, $b \times c + a$ becomes $c \div b - a$. Like Natasha's response, Owen's response also features a c on both sides of his final equation. These responses are representative of the types of answers students provided to question 15, and it quickly becomes clear that factoring is a technique that was entirely unapparent to students from their formal coursework. Greg's clear understanding of the process may link directly back to his articulated and demonstrated use of it in late-game *Dragonbox*.

On Affect

Preliminary observations.

The Affect-focused interview protocols were administered to four treatment group students[12] three times each during the treatment phase: once during the first week, once at the treatment's half-way mark, and once during the last week of the treatment. Interviews were transcribed, and transcriptions were reviewed by the principal researcher to identify convergences and divergences across students' viewpoints to identify and elaborate upon generally occurring themes.

Because students' responses were preceded by Likert scale ratings (e.g. five positions from Strongly Disagree to Strongly Agree), data analysis began by combing responses to identify prompts that elicited convergent and divergent ideas across the protocols (Charts 4.5a and 4.5b). Prompt responses demonstrating group convergences originally elicited disparate responses (e.g. at least two different Likert scale positions that were at least 2 steps apart) among students during the first and/or second round of interviews, but elicited comparable responses (e.g. either a unanimous Likert scale position, or 2 positions no more than 1 step apart) among students during the last round of interviews; conversely, prompts demonstrating group divergences originally

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[12] As mentioned earlier, these were not the exact same set of four students as in the Cognition-focused interview protocols; however, there was an overlap of two students.

elicited comparable responses among students during the first and/or second rounds of interviews, but elicited disparate responses among students during the last round of interviews. Prompts 4, 5, 7, and 14 were identified as convergent prompts, while prompt 13 was identified as the only divergent prompt.

Chart 4.5a: Students' Initial Responses to a Selection of Affect-Related Prompts

Chart 4.5b: Students' Final Responses to a Selection of Affect-Related Prompts
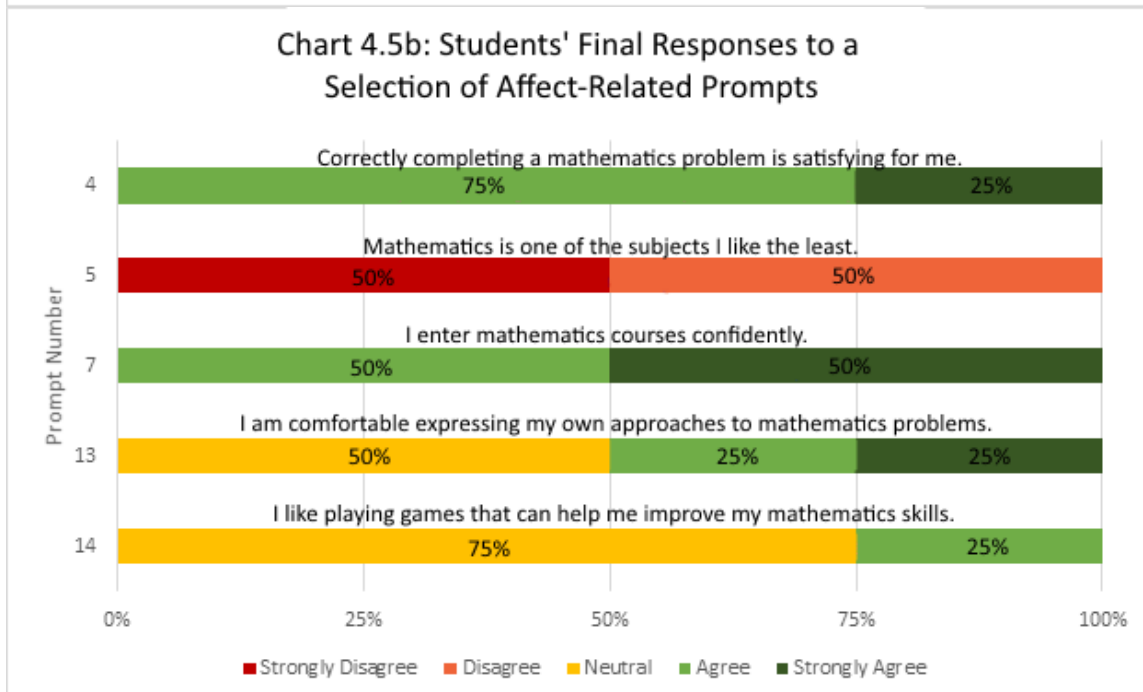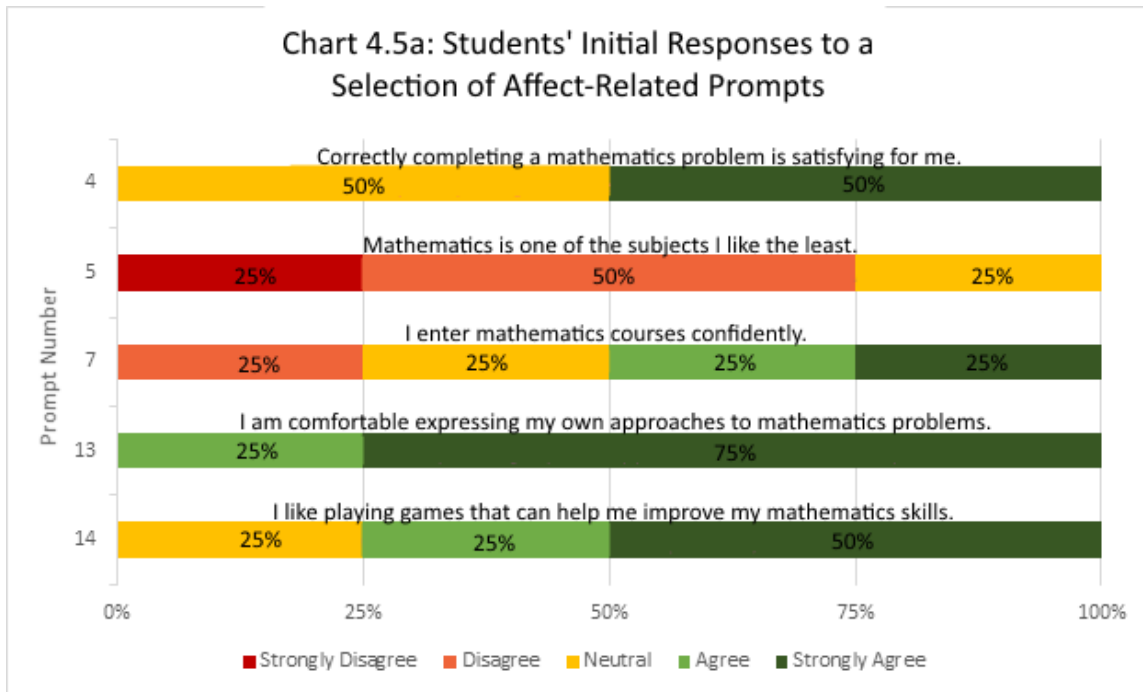
Students' potential affective changes based on qualitative data and researcher observations.

Student responses to prompts 4, 5, 7, 13, and 14 elucidated five themes related to the

algebra game play experience which are examined in this section.

Engagement with a sense of completion and understanding the importance of "working at one's level."

By the third interview protocol, all four interviewees had decided that they (minimally)

agreed that completing a mathematics problem was generally a satisfying experience; however,

justifications for the satisfaction varied slightly from student to student, and qualifiers were

sometimes attached to the statement. Two students, Ivan and Dan, spoke about a sense of

"wholeness" or "completeness" that overtook them at the end of a mathematical problem-solving

experience when they felt the problem was at their level of workability and closely related to

their existing pool of knowledge; during interview two, Ivan was able to elaborate on the

exploratory nature of mathematics-doing as experienced through *Dragonbox* game play.

[Ivan Interview 2]

> [2:43-2:49] Researcher: [Since you emphasize satisfaction when doing work "at your level,"] can you give me an example of maybe a good mathematics problem you completed recently, maybe in your current algebra course or elsewhere?

> [2:50-2:50] Ivan: That I completed?

> [2:51-2:51] Researcher: Yeah.

> [2:52-2:52] Ivan: Successfully?

> [2:53-2:56] Researcher: Yup and tell me about why [you had] a satisfying feeling.

> [2:57-3:08] Researcher: I don't know, like today [in *Dragonbox*]? Because um, the way, when you're [solving the equation], you learn more stuff on the way of trying to solve it… [you sometimes have to experiment] until you feel like something is complete, and that's pretty much [what is satisfying].

> *For clarity, Ivan did not reference a specific problem. *

> [3:09-3:18] Researcher: So, you work until you feel something is complete. Interesting.

He raises a point about the exploratory nature of mathematics that is mirrored in game play. In one sense, *Dragonbox* game play is a slightly more confined and constricted version of the larger mathematical/algebraic exploratory experience, since it is constrained by a system of preset in-game mechanics. However, being able to grapple with new ideas and new strategies to try to achieve a specific goal can be rewarding and provide the solver with a sense of closure upon a successful completion. Rather than constraints, in-game mechanics could also be viewed as scaffolds—instead of having infinitely many tools and possibilities to consider when exploring equation-solving, *Dragonbox* game play usually requires the player to navigate each chapter using a specific set of pre-designated tiles and in-game powers. It gives players some sense of direction on their quest for completion, which several students picked up on and identified, as Ivan demonstrates. However, in his third interview, Harold brings up an important point that shows some potential danger of working within a game-based system.

[Harold Interview 3]

> [2:31-2:37] Harold: If I solve a problem without really knowing what I was really doing, it wouldn't feel very satisfying it would just be like "Oh wow, finished." You know? "I did that equation."

> [2:38-2:42] Researcher: Can you give me an example about what… sort of situation that might be?

> [2:43-2:53] Harold: What sort of situation? I can't really give an example, but [there have] been multiple cases with, you know, me, solving a problem in [Algebra Teacher's] class or [*Dragonbox*] with me having no idea how to do it.

> [2:54-2:54] Researcher: But you were able to solve it [each time]?

> [2:55-2:58] Harold: Yeah. I was able to solve it. I had no idea what I was doing though, so it was mainly luck.

> [2:59-3:01] Researcher: I see. So, you were sort of just… maybe, following a procedure, or something like that?

[3:02-3:02] Harold: Yeah.

[3:03-3:03] Researcher: But you didn't understand the procedure.

[3:04-3:05] Harold: I didn't understand the process of how to do it.

[3:06-3:12] Researcher: Got it. But are you saying, then, that you do derive satisfaction in those cases where…

[3:13-3:29] Harold: Well, when I know the content very well and I'm very comfortable with it, if I solve it, then that would give me a feeling of satisfaction.

Harold's discussion here indicates that a meaningful sense of completion is only achievable when the solver has a sense of rightness and resoluteness about the actions he or she carries out to solve a problem. Harold demonstrates here that, in his *Dragonbox* experiences (perhaps notably in his challenges with factoring discussed earlier), there is the danger of being able to complete a level with experimentation among finite options and, not even understanding the sum problem, feel no major impetus to revisit it. Although Harold doesn't explicitly indicate it in his discussion, such experiences may have adverse effects on affect, potentially disillusioning students with content that might be considered "over their level."

Improved outlook on mathematics and, specifically, algebra.
At the time of the final interview, all interviewees had begun more strongly rejecting the idea of mathematics being a least favorite subject of theirs. Adam's meditation on this issue showed growth in a very particular direction: enjoyment of mathematics was enhanced when learning was structured in a puzzle-like way, as the *Dragonbox* game play experience attempts.

[Adam Interview 1]

[3:57-4:04] Researcher… Okay, let's go on to the next question. So, "Mathematics is one of the subjects I like the least."

[4:05-4:06] Adam: Disagree.

[4:06-4:10] Researcher: Why is that? So, you do like it…not the least, right?

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[4:10-4:18] Adam: Yea. It's probably in, like, my half-er classes that I dislike the most.

[4:20-3:21] Researcher: So, it's in the bottom half?

[4:21-4:22] Adam: Yea.

[4:21-4:22] Researcher: Okay.

[4:22-4:37] Adam: But still, it's just a really fun class, 'cause now we're learning about new things. When last year, it was, kind of, just, like, mostly just [simple algebra], over and over again…but in different forms…but now we're really learning about new rules and stuff like that. [It was an Honors Mathematics course.]

[4:46-4:53] Researcher: Honors Math. So, it was probably, like, a pre-Algebra course that was sort of getting at some ideas of Algebra. So, you weren't crazy about that course, but you like your current Algebra course a lot more?

[4:53-4:54] Adam: Yes.

[4:54-4:56] Researcher: And that's because of the variety of the topics?

[4:56-4:57] Adam: Yes.

_____

[Adam Interview 3]

[4:00-4:14] Researcher: So, this question is asking, "mathematics is one of the subjects I like the least," and you said you disagree, right? So, it must not be one of the subjects you like the least, so there must be subjects you like less than this. So, tell me about how math gets this position compared to the other subjects.

[4:15-4:23] Adam: I don't really…hate any of the subjects, they're just kind of like, which is more, I don't know, like, to my liking, I guess.

[4:26-4:28] Researcher: What like aspects of math do you like?

[4:29-4:36] Adam: I don't know. With, like, algebra it's pretty fun, like, trying to figure out "What's this? What's this?" based on other information you're given.

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[Adam Interview 3]

[15:57-16:00] Adam: I don't really play any of those [games designed to improve your mathematics skills,] but this one's pretty fun.
[16:01-16:01] Researcher: You're talking about *Dragonbox*?

[16:02-16:02] Adam: Yeah.
[16:03-16:03] Researcher: Great, and you think it's really helpful [for algebra learning]?
[16:04-16:04] Adam: Yeah.

Adam shows that he initially liked mathematics study because of the variety of topics he could encounter during the learning process; he later emphasizes the nature of algebra specifically as offering problem solvers systems of knowns and unknowns, which he finds enjoyable as an alternative to raw arithmetical computations and regards his game play experiences as being helpful facilitators for that work. Based on this description, it seems that a game like *Dragonbox*, which offers a more exploratory realm for learning about mathematics, could be a good tool for sharing exciting aspects of certain mathematical learning processes as opposed to, for example, games whose archetypes could be reduced to the "drill-and-kill" philosophy. In its earliest levels, *Dragonbox* did not aim to present itself to the player as an "algebra-learning game;" instead, it was focused on appealing to players as a puzzle-game of sorts in which a collection of pieces from a closed system must be manipulated for the player to progress (e.g. help his or her dragon grow). Only later does it reveal itself to be a game for encouraging algebra-learning, at which point the player might already have come to equate the equation-solving process with a sort of puzzle-completing journey.

Newfound self-confidence as students in mathematics courses and the power of destressing.

By the third interview, all interviewees had (minimally) agreed that they felt they would enter mathematics courses confidently. The responses to this prompt were enlightening when considering the extent to which young mathematics students discussed the pressures and stressors of mathematics learning. Only Ivan expressed absolute confidence in his own abilities regarding new mathematics courses from start to finish, spurred on by his view of mathematics being a one-solution world. Comparatively, Adam and Dan initially expressed reservations about

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diving into new mathematics courses; Adam said he felt neutral about entering courses confidently, while Dan said he disagreed about being confident, usually. Later, Adam meditated on the uniqueness of his algebra course's fresh take on equation solving compared to his pre-algebra studies, citing *Dragonbox* game play as a positive learning experience related to algebra content (as mentioned earlier, but reincluded ahead). Dan, having flipped from claiming to always be confused to claiming to almost never be confused, reflected further on the pressures of mathematics courses in general, and clarified how helpful doing mathematics in a destressed environment—like a game space—can be.

[Adam Interview 3]

> [5:33-5:42] Researcher: … let's go onto the next one. "I enter mathematics courses confidently."
>
> [5:43-5:45, 5:48-5:51] Adam: Agree… I try to … have the highest expectations for myself.
>
> [6:11-6:15] Researcher: ... Has it always been like this?
>
> [6:16-6:17] Adam: No, not really.
>
> [6:18-6:24] Researcher: It hasn't. Can you tell me about an experience when, or maybe a time when you weren't entering mathematics courses confidently?
>
> [6:25-6:35] Adam: When I was in like second to sixth grade, I didn't really like mathematics, but now I like it much more than I used to.
>
> [6:36-6:36] Researcher: What changed?
>
> [6:37-6:44] Adam: I don't really know. I guess [learning algebra this year feels new] rather than basically just the same equations [in] different formats.
>
> _____

[Adam Interview 3]

> [15:57-16:00] Adam: I don't really play any of those [games designed to improve your mathematics skills,] but this one's pretty fun.
> [16:01-16:01] Researcher: You're talking about *Dragonbox*?
> [16:02-16:02] Adam: Yeah.

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[16:03-16:03] Researcher: Great, and you think it's really helpful [for algebra learning]?
[16:04-16:04] Adam: Yeah.

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[Dan Interview 3]

[0:48-0:52] Researcher: Terrific. Okay, second, "I like playing games."
[0:53-0:53] Dan: Strongly agree.
[0:54-0:54] Researcher: Tell me about it.
[0:55-1:10] Dan: It's fun, and you know, it separates you from [stressful experiences]…you can enjoy your time when you're playing games and everything, and you get a good experience out of it, so I enjoy it, whether it's like [*Dragonbox*] or [a sports game].

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[Dan Interview 3]

[4:32-4:39] Researcher: …Okay here's the next question, "I enter mathematics courses confidently."

[4:40-5:08] Dan: Agree. Usually, I don't feel too confused about a mathematics course, and if you're doing good with the classwork that you've done, then moving into a new course will be, I would say, easier, because [you don't have unresolved learning issues creating lingering sources of stress].

Based on Adam's earlier cited description of his algebra game play experience as having utility, it is likely that the diversity and freshness Adam references regarding his course work was impacted positively by his game play experience; these factors likely contributed to an improvement in his self-view as a mathematics student. Dan, having entirely flipped his self-view as a mathematics student from a position of uncertainty to a position of confidence, emphasizes how easily stressors can build up and inhibit progress during mathematics learning— and shows that when students have relatively stress-free experiences, their confidence levels can quickly improve. Dan mentions that, in his case, the type of game play for stress relief doesn't matter so long as the game being played provides an enjoyable experience; since all students reported enjoying *Dragonbox* game play, it seems to be a good candidate for destressing the algebra-learning process.

Increased self-consciousness slightly weakening expressiveness and interest in reflection.

At the beginning of the treatment, all students agreed that they were comfortable expressing their mathematical ideas to other people, and all but Adam indicated they strongly agreed. However, in the final interview set, opinions had diverged, with Adam and Harold neutral about this point, Dan agreeing, and only Ivan yet strongly agreeing. The most prevalent concerns among interviewees when discussing this topic were all related to fears of failure, embarrassment, and insufficiency; for example, in his second interview, Dan expressed his paranoia about his peers discovering his incorrect work (or his inability to produce correct work) and poking fun at him for it. However, an equally deep issue came to light: it seemed that in some cases, the increased complexity of game play content mitigated students' interests in attempting reflection on their work during the treatment phase, which may have internalized an approach to mathematics that encourages dismissiveness of complex ideas. Harold's statements from the third round of cognition-focused interviews, addressed earlier, provides insight into the impact that game play experiences had in this regard. Here, that text is preceded by a transcript sample from his second affective-focused interview to demonstrate his views on approaching "something hard."

[Harold Interview 2]

> [15:19-15:27] Researcher: …Let's go onto the next question. "I am comfortable expressing my own approaches to mathematics problems."
>
> [15:35-15:59] Harold: Neutral. I chose neutral because there's some things that, like I said, that are easy, and hard. The things that are easy I have no problem explaining what my thinking behind it is. For something hard, my only goal would be to do it however I'm shown, so that's why I would say that.
>
> _____

[Harold Cognition-Focused Interview 3]

> [2:43-2:53] Harold: What sort of situation? I can't really give an example, but [there

have] been multiple cases with, you know, me, solving a problem in [Algebra Teacher's] class [and] [*Dragonbox*] with me having no idea how to do it.

[2:54-2:54] Researcher: But you were able to solve it [each time]?

[2:55-2:58] Harold: Yeah. I was able to solve it. I had no idea what I was doing though, so it was mainly luck.

Harold comments quite specifically in the first sample that for content that he feels he doesn't understand, he sees mimicry as a better option than actively seeking remediation; in the second sample, he openly discusses moving through the content of both his formal algebra coursework and *Dragonbox* game play without taking the time to pause for reflection and consideration, instead, attributing his content completion to the aforementioned mimicry and "mainly luck."

Adam, the other student who felt neutral about explaining his work during the third interview, provided insight into the emotional challenges present when trying to explain content to others while not being entirely clear about it.

[Adam Interview 3]

> [14:17-14:27] Researcher: So, do you think if I brought you a question [that] you got…right, would you be comfortable explaining… what you did…the steps that you did in that question?
>
> [14:28-14:28] Adam: Yes.
>
> [14:29-14:32] Researcher: What about if you got it wrong, would you be comfortable explaining what you did?
>
> [14:33-14:34] Adam: No.
>
> [14:35-14:36] Researcher: You wouldn't. So, what's the difference between the two?
>
> [14:37-14:44] Adam: I don't know I guess it's just … embarrassing, I guess, like when you're [explaining something wrong to your] friends and stuff.

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[14:45-15:03] Researcher: Interesting. So, there's like this sense of embarrassment, you said, when around your friends. Is that because they're on the same level as you?

[15:04-15:10] Adam: Yes…

Here, Adam indicates that he would generally be worried about expressing his mathematical errors to a peer out of embarrassment that he would be viewed as performing worse than his expected outcome. In this study, because students completed game play sessions in close physical proximity to one another, it's possible that some of these feelings were exacerbated by the setting. To elaborate on this, although data on peer-to-peer interactions were not explored during interviews, they were noted by the researcher on several occasions. In over 50% of sessions, some form of peer-to-peer commentary expressing the heightened position of one student over another would occur; an example comment might be "What, you're still on Chapter 3? I'm already in Chapter 4," or "how can you get stuck on that one? It's so easy!" Although students' emotional and psychological profiles were not investigated with reference to peer-to-peer interactions, there is clear potential for feelings of inferiority or embarrassment to set in if one student feels he is struggling while his peer is clearly excelling. Additionally, while such a social setting could also potentially enable student collaboration, Harold and Adam's interviews indicate that collaborative discussion is not universally the norm, and indeed, the researcher noted instances of collaboration in fewer than 25% of sessions.

Tempered interest in the use of games as tools for mathematics learning.
Although most of the students initially were high-energy and optimistic when discussing the prospects of games for mathematical learning, by the time of the third interview, they seemed somewhat fatigued by the treatment process; although they consistently expressed an affinity for the game used in the study in nearly every interview, as time went on, their discussions moved towards drawing deeper connections between commercial games and recreational tools rather

than between educational games and pedagogical tools. They also somewhat differed and wavered on their views of whether mathematics learning and doing had a place—passively or intentionally—in commercial recreational games. To be blunt, after eight weeks of game play sessions, it seemed that the novelty of the game play experience had worn off.

In his third interview, Adam emphatically stated that, due to his enjoyable experience with *Dragonbox*, he would be interested in looking at other games that presented mathematics in interesting or novel ways. However, all other students felt neutral on this prospect. In his second interview, Ivan seemed to be in stark support of his algebra game play experiences as worthwhile means of learning mathematics in alternative ways—"math logic" would stand apart from "game logic" and parallels could be drawn. However, by the time of his third interview, he felt differently.

[Ivan Interview 2]

> [16:37-16:43] Researcher: Interesting. Okay great, next question, "I like playing games that can help me improve my mathematic skills."
>
> [16:44-16:45] Ivan: Strongly agree.
>
> [16:46-16:48] Researcher: Can you tell me a little bit about that?
>
> [16:49-17:09] Ivan: First of all, when you play games [like *Dragonbox*], it helps you interpret things in a different way where you can apply that game logic to real life, just like math. Yeah and...
>
> [17:10-17:12] Researcher: What is game logic? I like that, that's an interesting term.
>
> [17:13-17:25] Ivan: It's almost like…which one is a … better solution to something. What is the not so good solution? Uh, yeah...
>
> [17:26-17:29] Researcher: So, you judge the quality of solutions also.
>
> [17:30-17:30] Ivan: Yes.

[17:31-17:32] Researcher: And you think that playing games helps you do that?

[17:33-17:33] Ivan: Yes.
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[Ivan Interview 3]

[10:50-10:57] Researcher: … "I like playing games that help me improve my mathematics skills."

[10:58-11:02] Ivan: Neutral.

[11:03-11:03] Researcher: Okay.

[11:04-11:15] Ivan: Because usually you don't really have to play games in order to understand something, cause usually when you just do math, you just know math. Yeah.

[11:16-11:19] Researcher: If you do math, you know math? You don't think that…

[11:20-11:34] Ivan: It's like, as long as you're practicing with it [in a class], you always come up with... you'll... math will sink in, and when you get really good with that method, then you'll start manipulating that method into something you can use however you want.

[11:35-11:42] Researcher: And are you qualifying games that improve your mathematical skills as part of that reinforcement?

[11:43-11:47] Ivan: Yeah, I mean I'm not against playing games just to learn math, it's...

[11:48-11:49] Researcher: Got it. You just sometimes have other reasons to play games.

[11:50-11:50] Ivan: Yeah.

During his third interview, Ivan seems to have completely sidelined his notions about the

importance of utilizing the logical systems inherent in game play for mathematics learning and

instead expresses an affinity for just practicing mathematics in a more traditional manner.

However, he does briefly discuss how mathematics can appear in commercial games, usually in

the form of arithmetical calculations for things like prices. All other students also agreed with

this idea at points in their second and third interviews, clarifying in general that they recognize

the presence of mathematical ideas in commercial games that are not designed to treat those

concepts intimately. In his final interview remarks, Dan expressed his views on the place of

educational games for mathematics learning relative to the place of commercial games.

[Dan Interview 3]

> [13:36-13:41] Researcher: Okay, very good. Next question, "I like playing games that can help me improve my mathematical skills."

> [13:42-14:14] Dan: Neutral because now, like, everybody plays different types of games and, like, different styles of games, and it doesn't really focus on the category of math. Games like *DragonBox2*, I would like [them] to help me out, and [*Dragonbox*] is helping me out in ways, but, like, if we're talking about games in general that are mainly focused on [the current commercial market], I don't think it would, like, correspond back to math in any way…

Although Dan clarifies that he does feel that his *Dragonbox* experience was useful and

productive, and that he's thankful for the opportunity to engage in the game's algebra learning

experience, educational games for mathematics just seem outside of the norm.

On Retention

Preliminary observations.

The AGATE 3 was administered on the first day of regular classes to 21 participants

following the host school's usual winter recess to determine the extent to which an understanding

of algebraic equation solving skills was retained by students over a one-month period in which

they would not have access to their usual algebra schooling; as discussed earlier, one treatment

group student was unable to complete the AGATE 3 prior to engaging in formal algebra

coursework, so his data were not collected. While school was in recess, all treatment group

students had access to their own copies of the *Dragonbox* game, provided by the researcher; no

student reported playing the game during this period, although the one treatment student who

returned late was not able to be consulted on this.

A selection of the measures of central tendency and dispersion (e.g. simple range, median, and interquartile range) were calculated within and across each of the treatment and control groups' results to assess students' content masteries following the recess. As before, use of these measures was selected over the use of alternatives (e.g. mean and standard deviation) to make the data more robust, as the small population sizes in question may otherwise be more susceptible to outlier data points.

For the AGATE 3, ranges favored the control group: the treatment group floor was 0%, while the ceiling was 76.47%; the control group floor was 17.65%, while the ceiling was 82.35%. However, the treatment and control medians were identically 41.18%. The IQR of the treatment group was 47.06%, while the IQR of the control group was 29.41%, continuing the trend seen when shifting from AGATE 1 to the AGATE 2 of the treatment IQR growing while the control IQR diminished. This distribution of scores is displayed in Chart 4.6.

Chart 4.6: Comparison of Number of Questions Answered Correctly by Control and Treatment Students on AGATE 2 (+/- 1 Standard Deviation)
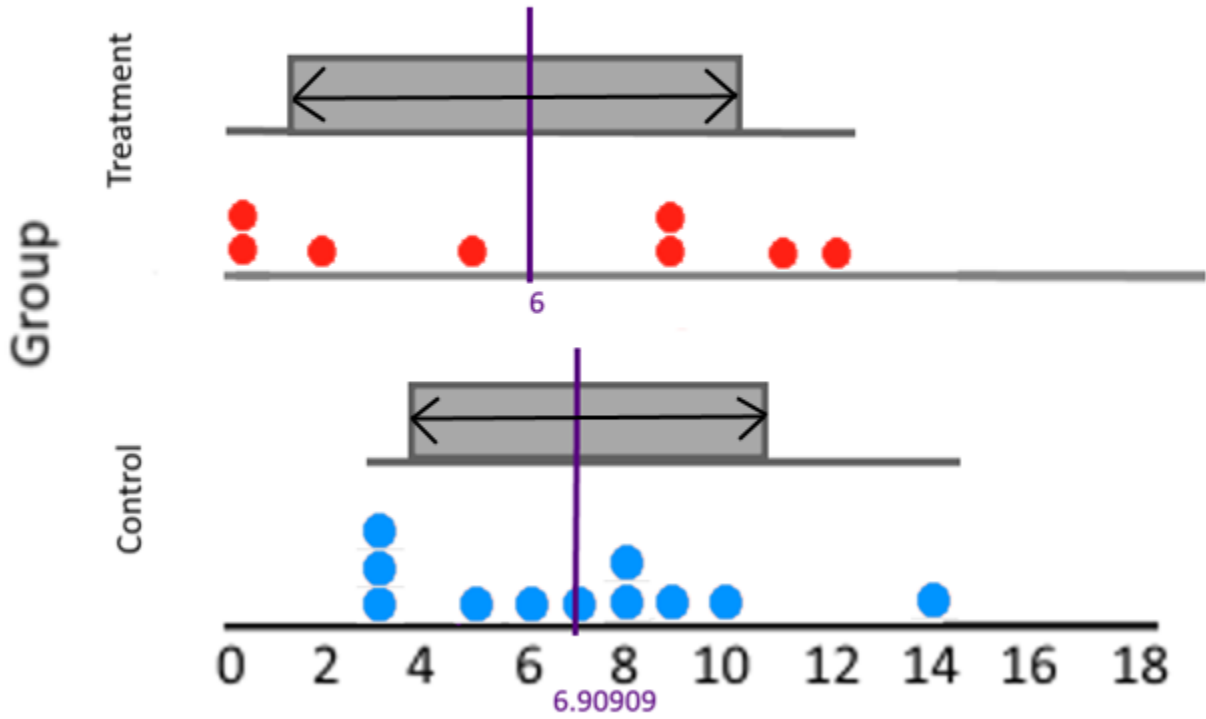
Number of AGATE 3 Questions Answered Correctly (of 17)

Chart 4.7 summarizes each student's results on the AGATE 2 and AGATE 3 and Table 4.2 offers a summary of the measures of central tendency and dispersion from both groups' results across both examinations, including revised values for the AGATE 2 treatment population which reflect the removal of the data of the student that could not complete the AGATE 3.

Chart 4.7: Comparison of Students' Results on the AGATE 2 and AGATE 3 Across Exams and Groups
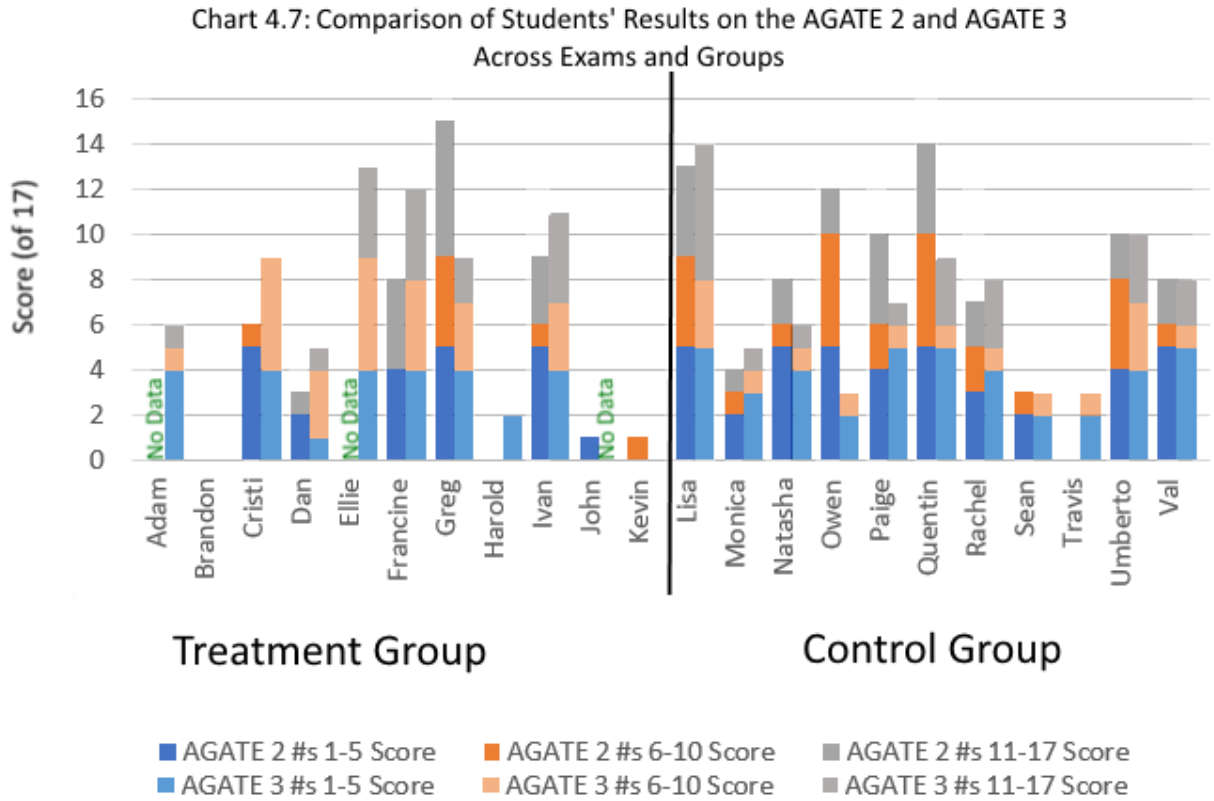
Table 4.2: A Comparison of the Measures of Central Tendency and Dispersion of Students' Results on the AGATE 2 and AGATE 3 Across Exams and Groups				
Data Type	AGATE 2 Treatment ($n = 8$)	AGATE 2 Control ($n = 11$)	AGATE 3 Treatment ($n = 8$)	AGATE 3 Control ($n = 11$)
Score Floor/Ceiling	0%/88.24%	0%/82.35%	0%/76.47%	17.65%/82.35%
Median	26.47%	47.06%	41.18%	41.18%
IQR	44.12%	35.30%	47.06%	29.41%

As discussed in Chapter 3, the AGATE 3 results were also analyzed twice via an analysis of covariates (ANCOVA) in which the primary covariate was students' AGATE 2 results. The following formula was used for the first analysis:

$$AGATE\ 3\ Results = (B_0 + B_1 * AGATE\ 2\ Results + B_2 * Gender + B_3 * Group).$$

This ANCOVA detected that AGATE 2 scores served as statistically significant predictors of AGATE 3 results ($f = 4.632, p = 0.0003$) when controlling for gender and group assignment;

however, gender was also found to be a statistically significant predictor of AGATE 3 results $(f = -2.566, p = 0.022)$. A second ANCOVA was computed using the following formula:

$$AGATE\ 3\ Results$$
$$= (B_0 + B_1 * AGATE\ 2\ Results + B_2 * Gender + B_3 * Game\ Chapters\ Attempted).$$

This ANCOVA detected that AGATE 2 scores served as statistically significant predictors of AGATE 3 results $(f = 3.821, p = 0.002)$ when controlling for gender and considering the explanatory variable for the number of game chapters attempted by students; gender was again found to be a statistically significant predictor of AGATE 3 results $(f = -2.385, p = 0.033)$. Based on these two ANCOVA results, two claims can be made: first, that students' understanding of equation-solving techniques at the end of the treatment phase was the best predictor of students' results following the one-month recess; second, that males appear to have retained less content knowledge than did females. However, the second claim is potentially overly-influenced by unaccounted factors of the control group as, of 8 female students who contributed data to these ANCOVA, only 2 were from the treatment group. The following Chart 4.8 demonstrates the relationship between the percentage of game chapters attempted by treatment group students and each student's AGATE 3 performance for the 8 treatment group students who completed the AGATE 2 and 3.

Chart 4.8: Comparison of Treatment Students' AGATE 3 Scores and Game Chapters Attempted
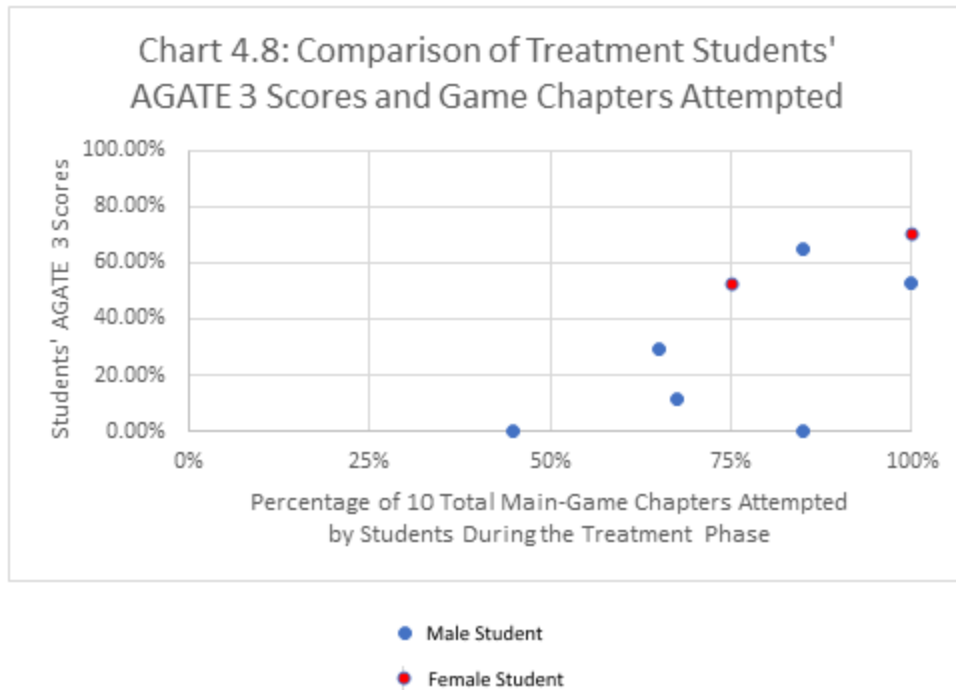
As with the comparisons between the AGATE 1 and AGATE 2, additional quantitative data were collected on the number of error types students across the groups made on the AGATE 2 and AGATE 3. The principal researcher again coded responses as either being correct or falling into one of the following error categories: 1) Computationally Erroneous; 2) Consistently Applying an Incorrect Conceptual Framework; 3) Omitted; 4) Attempted, but either Incomplete or Unjustified.

For the AGATE 3, the treatment group gave 170 responses in total[13]. Sixty-eight responses were correct, 15 were computationally erroneous, 22 displayed a conceptual misunderstanding, 55 were omitted, and 10 were attempted, but either incomplete or unjustified. For the AGATE 3, the control group gave 187 responses in total[14]. Seventy-six responses were correct, 14 were computationally erroneous, 53 displayed a conceptual misunderstanding, 33

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[13] Because one student was absent for the AGATE 3, each of 10 students answered the same 17 questions.
[14] Each of 11 students answered the same 17 questions.

were omitted, and 11 were attempted, but either incomplete or unjustified. In figure 4.10, these

data were compared with students' AGATE 2 data to understand changes to student

understanding over the recess period, as well as check for potential content retention.

Figure 4.10: Student Error Profiles from the AGATE 2 and AGATE 3
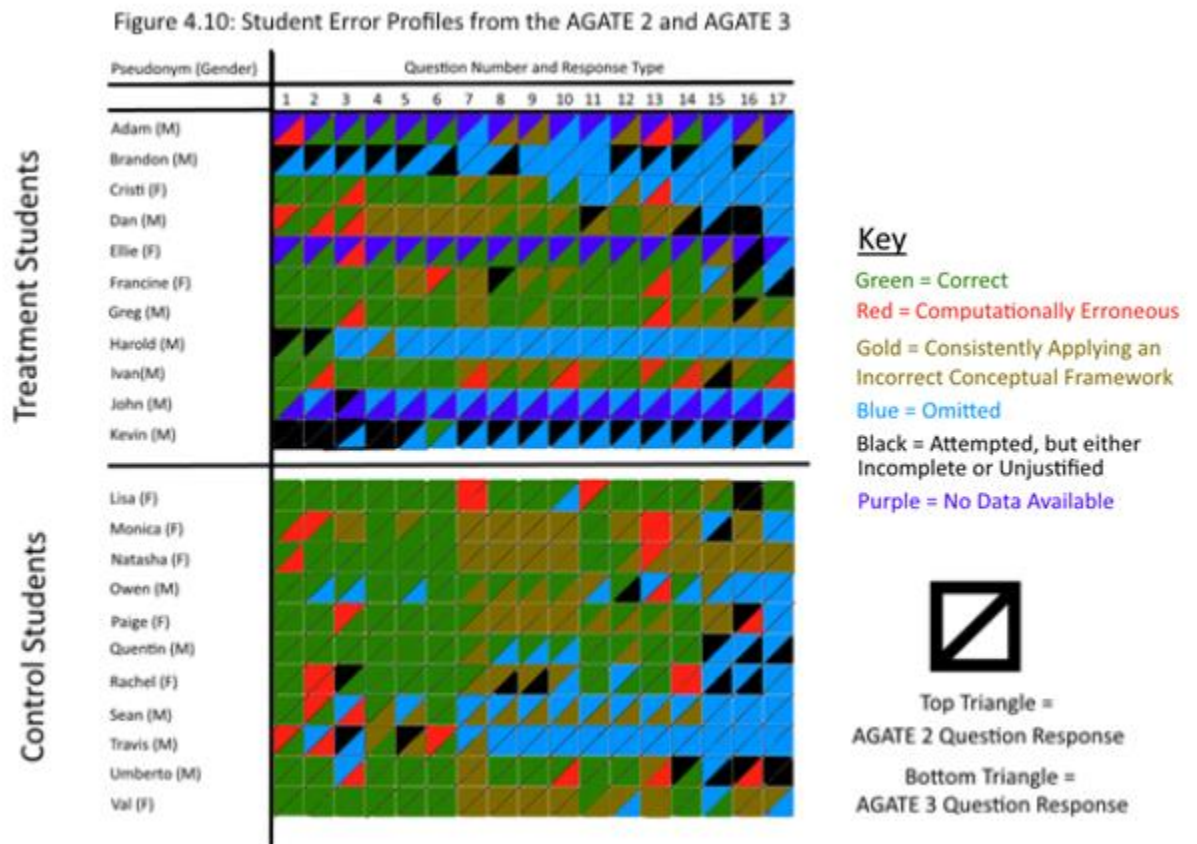
Figure 4.10, like Figure 4.1, provides many directions for examination, but can be hard to

utilize optimally at a glance. In the case of retention, the best boxes for examination would be

boxes that consistently held their color from exam to exam (e.g. Monica, Question 8), indicating

that a student's view was perhaps correct on both the AGATE 2 and 3, or that the student may

have maintained a misconception over the recess period; however, it should be noted that a

student's box could be singularly colored even if the misconceptions demonstrated for a question

on either exam differed. Boxes that show a shift from green to gold (e.g. Greg, Question 15)

indicate that a student may have forgotten some aspect of equation-solving that was tested in a question; these can be useful for pointing out problems in which content information was not recalled by students over time. Finally, one other collection of boxes must be addressed: boxes which shift from being incorrect on the AGATE 2 to correct on the AGATE 3 (e.g. Dan, Question 8). Because students were expected to not have access to formal mathematics work during the winter recess, it is unlikely (though possible) that they would be able to independently correct conceptual misunderstandings up to their own resources. Therefore, Figure 4.10 helps explain a seeming anomaly demonstrated in table 4.2: in the data presented tracking group progress from the AGATE 2 to the AGATE 3, the treatment group's median rose, while the control group's median fell. In the case of checking for content retention, only the latter case is to be expected. However, certain conditions need to be recognized as to why a median that rose may still actually be valuable for assessment in terms of content retention. First, students as test-takers may sometimes feel, for example, time-pressure, which may cause them to omit questions they might know how to solve (e.g. Val, Question 15; Owen, Question 14). Second, students may carelessly make procedural errors for which they cannot be awarded credit, even though they may understand the content they are responding to, in general, and could prove as such on a different occasion (e.g. Dan, Question 1; Ivan, Question 14). Third, because medians range over all students' performances, they can be unreliable in terms of portraying the reality in terms of content retention; if two students had, together, answered the same number of questions correctly on two separate exams, their performances across exams would be equal in terms of credit received, but their individualized results where work was shown might demonstrate differences. Therefore, using Figure 4.10 as a guidebook, the following review of potential game play-attributed content retention will assess student work on a question-by-question basis, as

supported by the qualitative content appearing in students' questionnaires.

Students' potential retentive changes based on quantitative and qualitative data.
Comparing each group's AGATE 3 results to their AGATE 2 scores provides insight into

the potential retentive benefits of game play sessions. First, of all the questions answered

correctly on the AGATE 2, the treatment group retained correct answers on the AGATE 3's

equivalent questions in 73% of cases, while the control group retained correct answers on the

AGATE 3's equivalent questions in 71% of cases. However, of the no-credit equivalent AGATE

3 questions, treatment group responses showed that only 9.8% featured conceptual

misunderstandings and 17% featured procedural errors (of 27%), while control group responses

showed that 15.73% featured conceptual misunderstandings and only 3.3% featured procedural

errors (of 29%). Therefore, the treatment group potentially remembered algebraic concepts more

effectively than the control group, although they carried out far more procedural errors. Several

students discussed game play as a factor contributing to their content retention levels. Figure

4.11 contains a sample list of student responses citing some reasons students provided for

potentially retaining content over time, including considering parallels between formal algebra

learning and game play content and the notion of having interactivity with algebra-learning via a

highly responsive and visualized system.

Figure 4.11: Sample Responses of Treatment Group Students
Discussing Potential Benefits of Game Play for Content Retention

Dan

1. In your studies, what have you found to be the most memorable aspects of mathematics courses?

Getting to higher ranks and levels in the game which becomes harder and corresponds to mathematics even more which becomes challenging and fun.

Ellie

3. What learning experiences in your algebra course have you found most memorable? Why?

Being able to apply what I learned in the game to my class.

Harold

3. What learning experiences in your algebra course have you found most memorable? Why?

Playing dragon box 2 was memorable because it sort of helped by mathematical skills.

Ivan

5. Do you feel that any aspect of your algebra course contributed to "improved content retention?" That is, do you feel that anything in your course helped you remember the course's mathematical ideas better? If so, explain using specific examples.

it is easier to remember content visually e.i. you will always remember what your house look like in this case, a game is better than worksheets. However, you can't remember a ISBN number.

Examining the three buckets of questions also provides some insight into the nature of content retention as affected by game play. From questions 1 through 5, almost no students in either group that answered a question correctly on the AGATE 2 answered the question's AGATE 3 counterpart incorrectly while demonstrating a conceptual misunderstanding—the only such case was control group student Sean's response to question 4. Since this section was primarily dealing with the most elementary aspects of equation solving, this is not a surprising outcome.

For questions 6 through 10, results were quite different. In this set of questions, Greg's work on question 9 demonstrated a conceptual misunderstanding he had not shown in his correct answer to question 9 on the AGATE 2—namely, misuse of the multiplication property of equality. Every other treatment group student responded to the AGATE 3 counterpart of a question he or she had answered correctly on the AGATE 2 with a correct response. Four control

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group students responded to question 7 on the AGATE 3 demonstrating a conceptual misunderstanding, and one student, Owen, demonstrated conceptual misunderstandings in all of questions 7 through 10, whose counterparts he had answered wholly correctly on the AGATE 2. As mentioned earlier, these questions primarily dealt with multiplication and division when the variable being solved for was part of a fraction.

The final bucket of questions—numbers 11 through 17—had a similar pattern to that of the second bucket. Here, of the four treatment group students that had answered questions correctly on the AGATE 2, only one, Greg, answered such questions' counterparts on the AGATE 3 demonstrating conceptual misunderstandings. From the control group, nine students had been able to answer at least one question of 11 through 17 correctly on the AGATE 2, but four answered those questions' counterparts on the AGATE 3 demonstrating conceptual misunderstandings (or, in the case of a fifth student, Owen, omitting or failing to complete the question). Notably, this section of questions tested further mastery of the properties of equality, plus some finer points such as the distributive property and potentially factoring techniques; therefore, it is unsurprising that the results of this bucket would be in line with the results of the second bucket.

In Figure 4.12, a sample of treatment group students note that game play helped them recall strategies for solving algebraic equations—Francine comments on how *Dragonbox* mechanics automatically enforced the multiplication and division properties of equality, while John and Ellie comment about how the game's "special powers" (effectively, introductions to in-game parallels of formal mathematical concepts) made recalling equation-solving techniques simpler to recall and later apply. These comments may give some insight as to why a much larger number of control group students than treatment group students seemed to forget equation

solving techniques and mathematical properties necessary for solving the second bucket of

questions and making progress through the third bucket of questions.

Figure 4.12: Students' Comments about Recalling Game Play
Ideas for Formal Mathematics Doing

5. Do you feel that any aspect of your algebra course contributed to "improved content retention?" That is, do you feel that anything in your course helped you remember the course's mathematical ideas better? If so, explain using specific examples.

Ellie

Yes. The processes of the game were easy to memorize and apply.

5. Do you feel that any aspect of your algebra course contributed to "improved content retention?" That is, do you feel that anything in your course helped you remember the course's mathematical ideas better? If so, explain using specific examples.

Francine

It sort of did because it won't let you plug in a number without doing it to the rest of the numbers.

3. What learning experiences in your algebra course have you found most memorable? Why?

John

In the study I remember most of the powerups. I remember them because they were simple and easy.

Chapter 5: Conclusion

Summary

Thirty 8th-grade algebra students from a K-12 independent school in a large city on the eastern coast of the United States were invited to participate in this research study investigating what effects arose from first-time algebra learners spending a portion of their regularly allotted algebra class time playing a mathematical video game intentionally designed to help students acquire techniques for solving algebraic equations. Research questions guiding this study addressed three types of outcomes that impacted students' algebra learning experiences: cognitive learning outcomes, affective outcomes, and content-retentive outcomes.

A total of 22 students participated in the study; 11 students served as a control group and studied their traditional algebra curriculum, while the other 11 students served as a treatment group and played the mathematical video game *Dragonbox Algebra 12+* twice a week in 20-minute sessions for eight weeks during time typically allotted for their traditional algebra curriculum, studying their traditional algebra curriculum otherwise. During the treatment phase, data were collected on students' cognitive baselines and outcomes as related to algebra equation solving content, as well as on students' affective baselines and outcomes as related to views on mathematics, algebra, and identities as mathematics doers using a variety of quantitative and qualitative instruments. Following the 8-week treatment phase, participants had a 4-week winter recess from school in which they were not expected to engage with formal mathematical learning; data on content retention was collected the first two days following this recess period via an additional set of quantitative and qualitative instruments.

Students' results on two cognitive-focused tests (AGATE 1, AGATE 2) and one content-

retention-focused test (AGATE 3), together with data collected from cognitive- and affective-focused interviews (conducted only with the treatment group) and one content-retention-focused questionnaire (also conducted only with the treatment group) were the primary data sources used to answer this study's research questions.

Some additional data were collected on treatment group students' game play experiences, such as the percent of game content completed by students during the treatment period and peer-to-peer interactions during game play.

To deeply understand the impact that mathematical game play may have in a course for new algebra learners, it is necessary to evaluate the multidimensionality of the student experience. This includes the following aspects: 1) the impact of mathematical game play on students' cognitive outcomes as related to algebra doing; 2) the impact of mathematical game play on students' affective outcomes as related to mathematics in general and algebra specifically; 3) the impact of mathematical game play on students' content retention as related to algebra content knowledge. Conclusions for this study were motivated by analysis of quantitative data supplemented by analysis of qualitative data when possible; in some cases, certain conclusions were drawn strictly from analysis of qualitative data without the use of quantitative data (especially as related to affective change).

Conclusions

Question 1: How does integrating mathematical game play into a traditional eighth grade algebra curriculum impact students' cognitive learning outcomes in elementary algebra?

Responses from the harvested data—most notably the study pretest (AGATE 1), the study posttest (AGATE 2), and the cognition-focused interviews—suggest that the integration of

mathematical game play impacted students' cognitive learning outcomes in three ways: 1) on average, game-playing students did not improve their cognitive reasoning with regard to algebraic equation solving as significantly as did non-game-playing students; 2) game-playing students were able to recognize that game play was explicitly modeled around the solving of algebraic equations and, in some cases, made attempts (some productive, others unproductive or detrimental) to internalize experiences from game play for the sake of improving their mathematics content knowledge; 3) game-playing students had greater payoffs from game play in terms of improved cognitive reasoning with regard to algebraic equation solving when they were already strong mathematics students.

Both game-playing students and non-game playing students performed very poorly (e.g. median scores below 25%) on the study's quantitative pretest checking for cognitive reasoning regarding algebraic equation solving. More surprisingly, on the study's quantitative posttest, both student groups continued to perform poorly (median scores below 50%), but non-game-playing students were able to significantly improve upon their pretest median score, while game-playing students maintained their pretest median score; this may have been in part due to student game players trading off some class time (usually lecture-focused) for game play time. However, since game play sessions only occurred twice a week during the treatment phase of the study, and since, on game play days, treatment students rejoined control students during the second half of class periods, approximately 80% of the instruction received by all students was identical.

However, the raw quantitative evidence belies the full extent of cognitive changes undergone by game-playing students; it became clear from discussion with all interviewees that game-playing students found deep parallels and similarities between the game play of *Dragonbox Algebra 12+* and the process of formally solving algebraic equations. In most cases,

this manifested as an observed "unidirectionality of metacognition" in which students would reimagine their game play content as formal algebra content to facilitate its completion; few cases were observed of the reverse scenario, and no cases were observed of any sort of bidirectional likening. Based on interview data, it seemed that students who might have initially been considered weaker than their peers were slower to create this linkage, providing evidence that the connections between the algebra game play and actual formal algebra doing were nonobvious and required significant exposure to both game play and algebra content to see direct parallels. When students did see these deeper parallels, they made efforts to capitalize on them. Some students were able to describe the development of "mathematical reflexes" that prompted them to make certain decisions during the solving of algebraic equations based on newly constructed instincts arising from algebra game play. However, these new instincts served as double-edged swords; because the *Dragonbox Algebra 12+* game mechanics would never allow a student to carry out an in-game move that paralleled an illegal operation in terms of equation solving, students developed a significant trust in their game play experiences to map directly back to their formal content. This meant that if students thought that a certain mathematical idea—correct or incorrect—paralleled something that they had done in game play, they would instinctually repeat that movement whenever possible. In one interview, an example was shown in which a student misattributed his understanding of an incorrect variant of the multiplication property of equality to a game play experience, although no equivalent to the procedure he described would ever have been able to appear during game play. However, other students (almost always those with strong pretest scores) were able to correctly attribute their masteries of certain mathematical ideas to correctly corresponding game play experiences. In at least one interview, a student discussed his mastery of isolating variables included in fractions as

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stemming from game play, although this result was not widespread; that same student also recognized equivalents of factoring procedures in game play and replicated their formal algebraic variants when solving equations on the study posttest. The latter point is notable as this student was the only student across both the treatment and control groups to correctly answer a question that required factoring, giving some indication that factoring strategies were not taught in the traditional algebra course at the time of the posttest; other interviewees (commonly those with poorer pretest scores) struggled to describe and conceptualize the game play equivalents of factoring techniques as formal equation solving processes. These data made it clear that students who were more active about drawing parallels between game play and formal algebra equation solving techniques were those students who had begun the treatment with relatively strong pretest scores as compared to their peers and who had ended the treatment with relatively strong posttest scores as compared to their peers. Therefore, having a natural inclination towards or interest in mathematics—or perhaps succinctly phrased, "being a strong mathematics student"— seemed to correlate with making greater gains in cognition when independently exploring *Dragonbox Algebra 12+* game play in a co-instructional setting.

Question 2: How does integrating mathematical game play into a traditional eighth grade algebra curriculum impact students' affective outcomes about both mathematics in general and algebra specifically?

Responses from the harvested data—most notably the affective-focused interviews—suggest that the integration of mathematical game play impacted students' affective outcomes in four ways: 1) students adopted an improved outlook on mathematics and specifically algebra; 2)

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students gained self-confidence as learners in mathematics courses; 3) students became more self-conscious about their mathematical abilities relative to their peers' abilities; and 4) students acquired a tempered interest in the use of games as tools for learning mathematics. It should be noted that interviews ranged across only four male treatment group students; these data might be fair descriptors of affective changes that occurred for treatment group students in this study but are not necessarily widely applicable.

Across the three affective-focused interview sessions, interviewees consistently became more and more confident that they did not consider mathematics a least favorite subject of theirs. Many students became more capable of describing a sense of completion and "wholeness" that they felt when correctly solving mathematics problems; game play experiences helped students articulate, specifically, that mathematics doing—either in the game space or in a formal classroom setting—was only rewarding when students were working at a level that they personally felt was appropriate for their mathematics study. Students discussed how game play content that was too easy felt empty, but how appropriately challenging game play content (or formally posed algebraic equations) could reward the solver with great satisfaction. Notably, students also echoed a point raised by Bragg (2007) regarding overly-challenging or complicated game play: game players (especially players of educationally-driven games) can be alienated from content and accordingly lose motivation to complete said content if the content's presentation seems too opaque, insurmountable, or unwieldy. In this study, playing *Dragonbox Algebra 12+* had a generally positive effect on students' motivations; at least one student pointed out that formally solving algebraic equations became more enjoyable because he began seeing algebra questions as sorts of puzzles, based on his corresponding experiences with *Dragonbox* which did present itself as a typical puzzle-solving game. The sense of fulfilment and enjoyment

that students derived from their game play experiences significantly altered their views on mathematics study as potentially being a source of great stress. Many students explained that the pressures inherent in formal mathematics courses split between the responsibilities of continuously learning new content and the duties of having to prove knowledge of said content for a recorded grade that potentially carried real-world repercussions were fatiguing and persistent sources of stress. Game playing in general had consistently been described by all interviewees as an outlet for stress relief during their leisure time; by incorporating mathematics learning into game playing, students found that they were less stressed and more relaxed when thinking about algebra during their course, which directly improved their self-confidence levels and self-images as learners in mathematics courses. It is notable that most but not all aspects of the game playing experience of participants in this study were conducive to improvements in affect, however; among participants, there was an overall increase in self-consciousness regarding mathematics ability, which weakened students' expressiveness and interest in extensive mathematical reflection. Playing a game that could provide implications for one's mathematical abilities in the same physical space as peers who might potentially be perceived as more capable mathematics doers welled up fears of failure, embarrassment, and insufficiency as algebra learners for some students. Some students expressed paranoia about peers discovering their incorrect work either in game or on, for instance, a returned formal examination. Lastly, it became clear by the end of the treatment phase that although students still very much had an interest in both commercial and educational games, they had grown worn out of their *Dragonbox* experiences; what had at the study's start seemed like an exciting and novel opportunity had later been viewed as an enjoyable short-term experience, but ultimately detached from students' internalized "norms" of mathematics study.

Question 3: How does integrating mathematical game play into a traditional eighth grade

algebra curriculum impact students' content retention in elementary algebra?

Responses from the harvested data—most notably from the post-treatment, post-recess test checking for content retention (AGATE 3) and the retention-focused questionnaire—suggest that the integration of mathematical game play impacted students' content retention in elementary algebra in two ways: 1) students with correct conceptual frameworks for algebraic equation solving maintained those frameworks slightly more frequently if they participated in the game playing experience as opposed to peers who did not; 2) students regarded game play experiences as forging powerful memories related to algebra learning even when not specifically prompted to make reference to game play.

By comparing results from the posttest AGATE 2 and the post-recess test AGATE 3, it was determined that game playing students retained roughly 73% of their correct conceptual frameworks between exams, and non-game playing students retained roughly 71% of their correct conceptual frameworks between exams. More curious was the result that, of the respective incorrectly answered 27% and 29% of AGATE 3 content equivalent to the correctly answered AGATE 2 content, errors in conceptual frameworks were detected in roughly a third of the treatment group's responses, while they were detected in slightly more than half of the control group's responses. The treatment group seemed to retain content better than the control group when dealing with procedures for isolating variables that were contained within fractions; treatment students also retained more content than did control students—though to a lesser extent—when dealing with more advanced algebra equation-solving content, such as uses of the

distributive property, factoring techniques, and more complicated applications requiring the use of inverse operations. Across all these results, however, it should be noted that control students, having performed better than treatment students in general on the posttest, had a slightly broader base of content that could potentially be retained/forgotten. Even accounting for this, strong evidence was provided via the post-recess questionnaire that game play created objectively memorable experiences linked to algebra learning for many students. When prompted only to discuss memorable aspects of mathematics experiences in their current and previous courses, treatment students made several references to their game play experiences. Some students discussed opportunities they'd had to apply concepts learned during game play to their formal algebra study. Others commented on the goal-based structure of game play that provided an impetus to master new content for the sake of progression. Still others commented on the visual nature of game play as being superior to, for example, learning via drill-focused worksheets. Although no treatment student reported playing the game (to which they all had access) during the recess period, more than half of all treatment students were able to describe some way in which *Dragonbox Algebra 12+* game play had created memorable learning experiences that helped in retaining algebra equation solving content.

The theoretical mapping between necessarily imperfect representations of mathematics in game play and formal mathematical ideas.

This research demonstrates that a tension exists in a student game player's theoretical mapping that binds together the space in which formal mathematics is done and the space in which mathematical game play occurs. For each mathematical game, and for each student game player, this mapping will be different. However, the findings demonstrated in this research suggest that students need significant guidance in order to successfully bridge the gap between game play and formal mathematics, or else they risk cognitive disconnects which could lead to

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conceptual misunderstandings.

Formal mathematics, in the way it is typically presented and taught in educational institutions worldwide, is not described or classified as being a game; accordingly, any game that claims to offer a new means of learning formal mathematics must contain some content that does not directly parallel some aspect(s) of formal mathematics doing. If all a game's content was comprised simply of things that one could do within a typical course of study—perhaps as read from a textbook or as presented in a lecture—or if, for instance, the game's content was designed to explicitly spam game players with mathematical drills, the game likely wouldn't fit the definition of being a mathematical game as described in this text. Therefore, we must recognize that any game selected for research of this nature must be a necessarily imperfect representation of formal mathematics with structural limitations influenced by design choices.

The imperfect representation of mathematics present within *Dragonbox Algebra 12+* is not a proper subset of formal mathematics doing (and no such game-based representation can be); there are necessarily elements inherent to the game's endogenous fantasy that have no place or parallel in formal mathematics. Instead, we should qualify this connection as a mapping that exists between the content presented in game play and some equivalent extant content of formal mathematics, recognizing that some elements of game play do not map to formal mathematics, and that some elements of formal mathematics do not map to game play. Herein lies something fairly problematic: students who are co-instructionally learning new mathematical ideas while simultaneously learning about ideas recurrent in game play must ably navigate the realms of both types of ideas. As the conclusions stated earlier demonstrate, students will often need support in spanning the gap between these realms in order to achieve cognitive growth. However, for students who do bridge the gap, it might be worthwhile; not only can they achieve growth in

cognition, but they can also potentially improve their affect regarding mathematics and their retention of learned content, as demonstrated in this study. As discussed in Chapter 2, navigating the reimagined sort of mathematics found in game play may have its own benefits up to each game's design; in *Dragonbox*, one recognizes, as examples, that the in-game equivalents of many algebraic ideas are spiraled and revisited, that certain abstract aspects of algebra are given a concretized representation, and that the player has infinitely many chances to revise or reattempt work in pursuit of a high(er)-quality correct response. While these things could all be done within a classroom and by hand, packaging them within an endogenous fantasy helps draw students' attention and maintain students' interest (for a finite amount of time, as demonstrated), and also situates the learning in a space where it is automatically valuable to the student game player. Ultimately, we must recall that game play shouldn't be considered a replacement for formal mathematics instruction, but a tool used to supplement a traditional learning process as seen in the classroom.

When the imperfect mapping is utilized and implemented correctly, students rightly recognize different concepts found in mathematics game play and formal mathematics content as being parallel equivalents; they recognize, too, that there are several aspects of formal mathematics content that might not be represented in game play, and several aspects of mathematics game play that might not necessarily be related to formal mathematics content. The intended cognitive mapping that game designers and content instructors want game players to acquire is visualized in Figure 5.1. In that figure, connections are represented unidirectionally from game play content to formal mathematics content, as this was the predominant reasoning scheme utilized by game playing students in this study; textually, one connection might read "'game playing content *a*' is related to 'formal mathematical content *1*.'"

Figure 5.1: The Intended Cognitive Mapping of Game Players
Linking Game Play Content and Formal Mathematics Content

Game Players'
Interspatial Cognitive
Connections

Key:

◇ = "node" representing
some sort of content

▬ = "cognitive connection"
correctly linking content
from game play to its
formal mathematical
counterpart

All Content Featured
in Mathematical Game Play

All Content Considered
a Part of Formal Mathematics
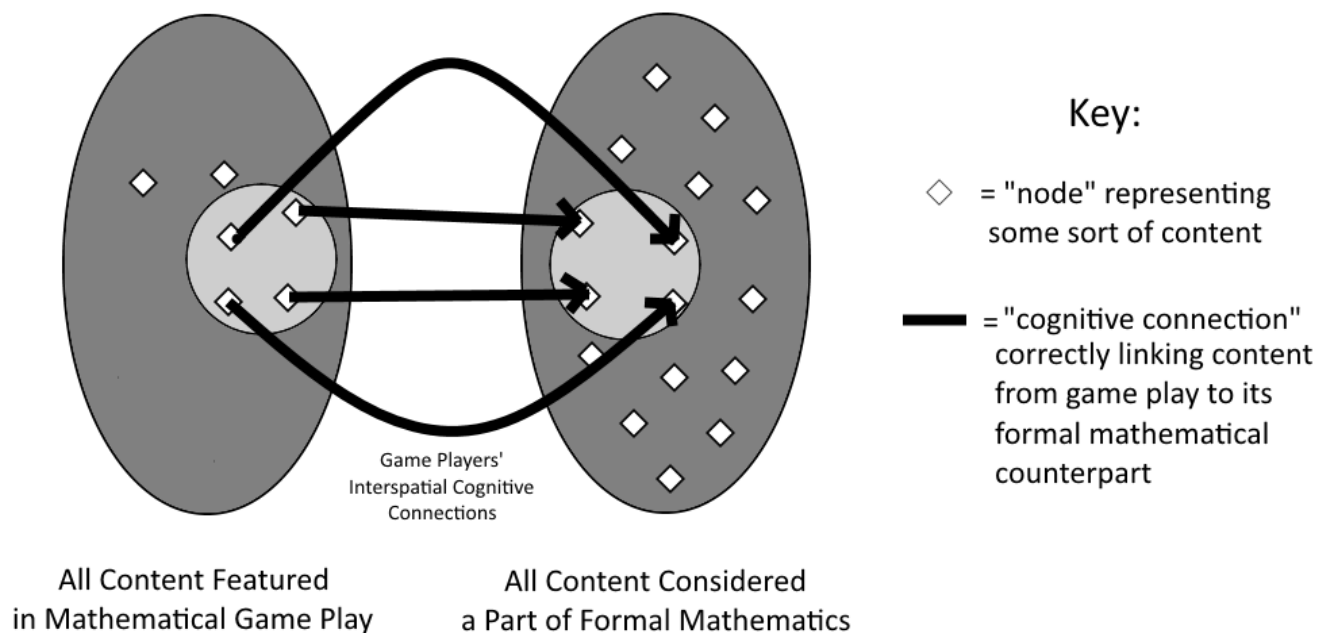
However, as demonstrated in this study, it is not uncommon for students (especially students who might have weaker conceptual frameworks for considering formal mathematics) to form different connections than those intended by the *Dragonbox* game designers. I will discuss, as examples, two game design decisions that led to cognitive confusion on the parts of some students and explain how this confusion might be pictorially visualized.

First, one design implementation that caused some cognitive confusion was the "pre-emptively corrective" mechanic, which prevented students from making incorrect moves when trying to utilize one of the properties of equality; for example, as discussed in Chapter 3, when a player tries to "add" a term to one side of an "equation" in *Dragonbox*, he or she cannot make any additional moves before adding a copy of the same term to the opposite side of the equation—denoted by a graphically striking "groove" that must be filled by a game play tile. As shown in interviews, this mechanic played some part in causing a cognitive disconnect with one

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student, who began incorrectly applying the multiplication property of equality outside of game play (in a realm bereft of the pre-emptively corrective mechanic). This shows a linkage—albeit an incorrectly formed one—between a game play mechanic that cannot be automatically enforced for mathematics practitioners in the real world and a concept of algebraic equation solving that was targeted for learning by *Dragonbox*'s game designers.

A second design implementation that caused some cognitive disconnect among students was the in-game representation of parentheses as sometimes being "ice blocks" and sometimes being "bubbles." To an individual knowledgeable about formal mathematics, it might quickly become apparent that ice blocks were used when a parenthesis had a coefficient other than 1, and bubbles were used when a parenthesis had a coefficient of 1 (which would usually not be indicated in-game). However, no student interviewed was able to articulate a meaningful difference between the two types of parentheses and all were confused about what the need was for a representational difference (e.g. "Could one be parentheses, and the other, brackets?"). Here, the game designers intended for students to form a cognitive linkage joining the representation of parentheses in-game to, say, a potentially better recognition of the distributive property of multiplication over addition. Instead, no productive mathematical connection was formed by students.

In Figure 5.2., a sample diagram illustrating two types of cognitive disconnects that were observed in this study are shown; however, more could potentially exist, even if not witnessed in this study. In the two disconnects demonstrated, we recognize the pre-emptively corrective mechanic as game developers' attempt to teach students the properties of equality, but which ended up causing conceptual misunderstanding (red arrow); we recognize the bubbles/ice block conflict as game developers' attempt to teach students about how coefficients work with

parentheses, but which ended up going unlinked to formal mathematics content by all students.

Figure 5.2: One Potential Unintended Cognitive Mapping of Game Players Linking Game Play Content and Formal Mathematics Content

Key:

◇ = "node" representing some sort of content

■ = "cognitive connection" correctly linking content from game play to its formal mathematical counterpart

■ = "cognitive connection" incorrectly linking content from game play to its formal mathematical counterpart

◆ = a "node" representing content designed to map to a formal mathematical counterpart, but didn't

Game Players' Interspatial Cognitive Connections

All Content Featured in Mathematical Game Play

All Content Considered a Part of Formal Mathematics

In addition to the discussed unintended, incorrect linkages observed in some students' cognitive mappings in this study, other linkages might be imagined: 1) one could incorrectly link an aspect of game play with no formal mathematical equivalent to a formal mathematical idea targeted by game designers; 2) one could incorrectly link an aspect of game play with no formal equivalent to a formal mathematical idea not targeted by game designers; 3) one could incorrectly link an aspect of game play with a formal mathematical equivalent to a formal mathematical idea not targeted by game designers.

Limitations and Recommendations

As described in the previous section, the central limitation of this study stems from the choice of *Dragonbox Algebra 12+* as candidate for a mathematical game while investigating the

stated research questions. Therefore, this study's findings and results are subject to the representation of mathematics contained uniquely within that game. While the principal researcher still considers *Dragonbox Algebra 12+* to be a good representative of a mathematical game, several aspects of this study have highlighted instances and places in game play where instructor guidance is strongly recommended for new algebra content learners.

Additional limitations.

For future studies that might be structured similarly to this one, a few recommendations are made, in relation to limitations, that the principal researcher advises should be addressed. First, population choice is of paramount importance. Although the initial population of 30 prospective students was well-mixed in terms of prior knowledge, gender, race, socioeconomic status, and additional attributes, there was some potential for attribute skewing in the formation of this study's treatment and control groups; this is to say that some biases in the treatment student selection process may exist because the set of students agreeing to participate in the study may have begun the study with, as one example attribute, heightened interest in the algebra game play experience. Although the principal researcher randomly selected 11 students of the 12 prospective participants with interest in joining the treatment group, the final treatment group of 11 may not have been wholly representative of the entire population of 30 students. The same can be inferred about the control group, which was constructed by blindly sourcing 11 of the 19 remaining prospective participants. In general, a larger population size could be utilized to help dilute potentially impactful attribute skewing. More specifically, a pre-study questionnaire might be utilized to purposely identify study participants who could form a diverse population. Collaboration across many potential study sites could improve population sourcing, as could multiple-researcher coordination. This would, additionally, counteract findings representative

only within unique populations and improve the strength and validity of the stated statistical analyses.

The instruments used in this study might also be examined as places for potential adaptations. In the case of the AGATEs, questions might need to be reselected or rescaled based on the content that a specific course intends to cover; in the case of this study, although collaborative efforts between the course instructor and principal researcher were made in designing the AGATEs, certain question types (e.g. those involving factoring) seemed like they had never been discussed formally with students during the usual algebra course. Being able to closely align course content with game play content is essential in optimally collecting data specifically on students' cognitive changes during a course of study, and improving students' odds of forming correct cognitive connections; additional specifications could be made to game play levels that are covered or potentially assigned for even further alignment. With regard to the affect-focused interview protocols, additional questions could be included from Tapia and Marsh's (2004) original ATMI—although many of the questions from that instrument were adapted or directly quoted for use in this study's interview protocols, if a new study's time affordances permitted, additional questions could be investigated. A general note regarding all interview protocols is that a larger research team could likely generate more interviews than did the principal researcher in this study; more interviews would make the qualitative data more robust and could potentially offer new perspectives not voiced by the interviewees from this study. As one limitation, all interviewees in this study were male, so it would be interesting to see if there were any differences in the treatment from the female perspective.

Additionally, the timing of the implementation of the *Dragonbox* game as a learning tool might be reconsidered. In this study, game play sessions most often cut into treatment students'

lecture time. However, it's possible that game play could have been used in place of traditional classroom practice. This might require an extensive review of content being taught within students' course to ensure content alignment with appropriate in-game equivalents.

Classroom Applications.

The results of this study offer lessons for all professional educators, but most especially for those interested in utilizing technological innovations or specifically implementing game-based learning innovations for their students. Several characteristics were identified in this study that highlighted positive, neutral, and negative developments arising from prolonged game play; using these data, efforts should be made to offset potential negative effects of game-based learning scenarios, which may also manifest with other types of instruction.

One important quality of classroom instruction informed by the results of this study is the need for well-differentiated content within a given curriculum. In this study, it was observed that student game players completed different amounts of game play content, and that while some students barely reached the main game's half-way mark, other students managed to entirely clear all available content. Because students have different strengths and weaknesses, it's important that students who excel at specific content can continuously encounter more and more challenging ideas and push their understanding; for students who might struggle, it's equally important that they are supported to grasp the essential ideas of mathematics content and, ideally, achieve holistic understanding.

In this study, it might have been prudent to implement a system in which a "classroom pace" was set; this may have prevented students from falling too far behind or getting too far ahead, while still feeling that they had agency and control over their mathematical learning. This consideration might help reduce students' tendencies to become self-conscious about their abilities when learning new content, as was witnessed. Constructing individual/team-based

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competitive environment(s) that could potentially motivate students in healthy ways during game play could also offset burgeoning self-conscious emotional reactions to difficulties. Such competition could be based around, in this case of game play, in-game performance, but also on, potentially, a presentation of formal mathematical concepts related to corresponding in-game content for which each student or student group would be responsible. As shown in this study, sometimes students would clear a level in *Dragonbox* explicitly for the sake of moving forward, taking no time to reflect on the mathematical concepts embedded within; reflection is important, as it allows students to better internalize the complex ideas that they may encounter. Reflection and presentation as a classroom unit encourages students to connect with peers and communally address the mathematics at hand.

Additionally, this study's results seemed to indicate that students did not always instantaneously recognize connections between game play and formal mathematics, which often created conceptual misunderstandings. It is essential that students are supported during any attempt to learn mathematics content in order to form correct cognitive connections linking formal mathematical content to any other analogous representation of such ideas. In the case of *Dragonbox* game play, rather than having students play the game individually, students can be broken into groups to complete and discuss levels together. Students or groups of students might present their solutions on certain levels to the class, which could catalyze whole-class discussions of parallels between game concepts and formal equation solving concepts. The greater the number of opportunities that students have to discuss and exhibit their understanding of conceptual ideas, the more likely it is that erroneous ideas will be flushed out and corrected.

Lastly, especially with games and other more-general learning utilities that contain finite and reiterative content, it is worthwhile to create pacing that prevents students from fatiguing and

losing interest in their non-traditional learning source. Solutions for this will vary up to the resource being utilized but keeping students genuinely interested and excited in content will only benefit them during their learning experiences.

Further research.

There are also many aspects of the study which can inform further research. With consistently evolving technological innovations, the integration of new learning utilities into mathematics classrooms is key to working towards improved mathematics education. Therefore, further investigations must be undertaken to create a more essential picture of the best means for general technological and specifically game-based implementations.

To achieve a better understanding of how game-based learning technologies may be useful to improving mathematics education, a series of parallel studies might be conducted across several educational strata (i.e. elementary school level, junior high school level, high school level, university level, etc.). This study allowed junior high school students to independently and co-instructionally explore the connections between game play and formal mathematical concepts, which might not be possible with younger students who have had less cognitive development. However, there is potential for use of this design choice with older students similarly to what was done in this study. It would be worthwhile to investigate other titles in the *Dragonbox* series to see if they similarly qualify as strong representatives of mathematical games that may function as pedagogical tools based on the literature review accompanying this study; other mathematical disciplines besides algebra, such as number theory or geometry, may also be suitable for this type of game-based exploration. Alternatively, a design like the one used in this study might be implemented using games outside of the *Dragonbox* family. Several of this study's design variables may be altered to check their impact on the

overall effects of game-based learning, including the following: the subject content examined, the game utilized corresponding to the subject content, the type of structure and support provided to game-players, the type of learning experience for students as being independently driven or group driven, the amount of time spent playing the game, etc. However, as stated in earlier chapters, there are already a wealth of studies with combinations aligning with some of the stated design choices, so it would be prudent to ensure the collection of data from new study variants or to meaningfully revisit study variants which provide avenues for further investigation.

Reflecting on the three conclusions drawn describing game-playing students' cognitive changes, it becomes clear that connecting game play experiences to mathematics experiences was not a trivial, automatic occurrence on students' parts in this study; when students did individually form a connection, the quality of the connection was variable (correlated, generally, with the quality of each student's pre-existing cognitive framework of equation solving practices), and students' attempts to translate information between mediums sometimes led to misunderstanding of formal mathematics content. Evidence exists to suggest that while reasonably strong mathematics students may benefit from exploratory, self-guided game play experiences, this will not be a universal norm; students with less developed cognitive frameworks for mathematics at the time of game play introduction may not make high-quality connections between game play and formal mathematics doing. Therefore, it is recommended that researchers intending to explore the uses of mathematical games as pedagogical tools for cognitive growth utilize the mathematical game of their choosing with (primarily) a guided approach (as opposed to an individualized exploratory approach); that is, researchers should enable course instructors to explicitly draw connections between game content and formal mathematics content for students at regular intervals during the learning experience. This should

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not be viewed as wholly decrying the utility of individualized game based exploration; instead, a guided approach should be used to provide all students with a "safety net" of sorts in order to ensure that they are making the correct types of mathematical connections at each step of the learning process, thus preventing foundational conceptual misunderstandings potentially acquired through game play from concretizing in their mental schemas. Within the classroom, this might be implemented by splitting a class period into halves, allotting the first half for students' individual game-based exploration, and the second half for guided lecture or guided group discussion that makes clear the connections between game content and formal mathematics content. Alternatively, game play could be assigned as an out-of-classroom activity and discussed within the classroom to provide examples of or parallels to formal mathematical ideas. The variety of ways that a guided approach may be implemented and utilized to mitigate the weaknesses of the individualized exploratory approach offers many avenues of investigation.

Additionally, the two conclusions drawn describing game-playing students' changes in content retention also stand out as offering leads in studying the utility of game-based learning; however, because content retention would necessarily be examined following a treatment phase, it is important to reiterate the suggestion that future studies utilize primarily a guided approach. In this study, content retention was checked for by utilizing a natural 1-month gap in the formal algebra curriculum, but it would be easy to iterate on this using both shorter scales (e.g. a 1-week winter recess) and longer scales (e.g. a 3-month summer recess). Although some information was collected in this study regarding what aspects of the *Dragonbox* game play experience aided students in content retention, responses were reasonably varied over a relatively small sample size. Investigations using primarily a guided approach should be done to further examine these results; researchers could potentially identify which themes among those presented by the

students in this study are recurring, and how the presence of those themes (or new themes) can be amplified during game-based learning experiences. It would also be worthwhile to check for content retention with the *Dragonbox Algebra 12+* game playing experience with additional variants of this study's design to determine further improvements that can be made in this regard.

Closing.

Because of their inviting nature, mathematical games have the potential to function as exciting pedagogical tools, but a firm understanding of how they work, how they should be used, and what effects they may have on students in terms of cognition, affect, and content retention is essential to improving their utility for students. From this study, an understanding of how the introduction of one mathematical game into the curriculum of an 8th grade class of new algebra learners has impacted those learners' cognitive, affective, and content retentive outcomes has been gained. From the perspective of educators consistently seeking technological innovations, especially game-based innovations, for mathematical learning, insight into the use of mathematical games as pedagogical tools has come from this study, offering substantial conclusions and opening additional pathways for future exploration in the field.

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Appendix

Instruments for Investigating Cognitive Changes

AGATE 1 [2 pages].

Name:

Directions: Complete all problems by solving for a specified variable in the time provided. Please use one new line for every step and show all work. Indicate your final answer to a problem by boxing or circling it. If you use any scrap paper, please submit it. Good luck!

I. Solve for x.

1) $x + 3 = c$

2) $(-2) + x = (-c) + d$

3) $x + 4 + (-z) = (-a) + 2$

II. Solve for y.

4) $a \cdot y = b$

5) $y \cdot (-b) = a + 2$

III. Solve for z.

6) $\frac{z}{4} = d$

7) $\frac{z}{a} = 4 + (-d)$

IV. Solve for a.

8) $\frac{x}{a} = 4$

9) $\frac{x}{a} = 4 + (-d)$

10) $\frac{x}{a} + \frac{x}{a} = y$

V. Solve for b.

11) $\frac{b}{(-1)} = y$

12) $\frac{6 \cdot b}{2} = d$

13) $(-a) \cdot (-b) = y$

VI. Solve for c.

14) $b(c + a) = d$

15) $(a \cdot c + b) + (b \cdot c + a) = d$

VII. Solve for d.

16) $\frac{d + 4}{d} = a$

17) $\frac{2}{3} \cdot \left(\frac{d \cdot a}{4} + y \right) = \frac{-x}{((-1) \cdot x)}$

AGATE 2 [2 pages].

Name:

Directions: Complete all problems by solving for a specified variable in the time provided. Please use one new line for every step and show all work. Indicate your final answer to a problem by boxing or circling it. If you use any scrap paper, please submit it. Good luck!

I. Solve for x.

1) $4 + x = d$

2) $7 + x = (-b) + 1$

3) $y + x + (-3) = (-a) + 2$

II. Solve for y.

4) $y \cdot 2 = b$

5) $y \cdot (-a) = 2 + b$

III. Solve for z.

6) $\frac{z}{6} = 13$

7) $\frac{z}{d} = 5 + (-c)$

IV. Solve for a.

8) $\frac{y}{a} = z$ 9) $\frac{7}{a} = x + (-d)$ 10) $\frac{3}{a} + \frac{1}{a} = x$

V. Solve for b.

11) $\frac{b}{(-1)} = a \cdot d$ 12) $\frac{2 \cdot b}{4} = d$ 13) $a \cdot (-b) = (-y)$

VI. Solve for c.

14) $d\,(b + c) = a \atop b$ 15) $(c \cdot a + d) + (b \cdot c + a) =$

VII. Solve for d.

16) $\frac{d+2}{d} = 4a$ 17) $\frac{1}{6} \cdot \left(\frac{d \cdot b}{3} + z \right) = \frac{x}{((-1)\cdot(-x))}$

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Cognition-focused interview protocol 1 [1 page].

Cognition-Focused Interview Protocol, Round 1:

1) How would you describe the game play of *Dragonbox* to somebody who's never played it before? [Probing to see if students feel that the game play models a system appropriate for their Algebra course]

2) Do you only play *Dragonbox* during school, or have you played it elsewhere? [Probing for student-use of the game]

3) Do you think that playing *Dragonbox* has changed the way you understand your regular Algebra course content, for better or worse? Why or why not? [Probing for students' perceived mathematical connections, or lack thereof]

4) Did any specific game play experiences impact the way you approached or solved a question in class? If so, can you explain which part of the game influenced you? Feel free to draw images or directly show information using the game itself. [Probing for explicit connections between students' algebraic reasoning and game play]

5) Is there content that appears in *Dragonbox* that doesn't make any sense to you, or that confuses you in the way it's presented? [Probing for dissonance between algebraic ideas encountered in game play and in regular class]

6) Does everything you encounter in *Dragonbox* feel like it has a mathematical parallel from your Algebra course? Do you ever find something in game play that you feel you didn't understand or encounter in your regular class sessions? [Probing for perceived limitations of algebra learning, if any]

7) [REQUIRES SET-UP] I've picked a level of *Dragonbox* that I'd like you to walk me through, if possible. Take your time, but explain each move that you make. When you're done, could you try to model the original situation using an algebraic expression or equation? [Probing for the way that students express their algebraic thinking and connect game play ideas with formal algebraic ideas]

Cognition-focused interview protocol 2 [1 page].

Cognition-Focused Interview Protocol, Round 2:

1) In our last interview session, you said you would describe the game play of *Dragonbox* to new players as follows: [RESPONSE]. Is this still consistent with the way you view the game, or would you now describe it differently? [Probing to see if students feel that the game play models a system appropriate for their Algebra course]

2) Previously, you mentioned that you played *Dragonbox* [RESPONSE]. Has that changed? [Probing for student-use of the game]

3) Revisiting a question from the last interview, do you think that playing *Dragonbox* has changed the way you understand your regular Algebra course content, for better or worse? Why or why not? [Probing for students' perceived mathematical connections, or lack thereof]

4) Since the previous interview, did any specific game play experiences impact the way you approached or solved a question in class? If so, can you explain which part of the game influenced you? Feel free to draw images or directly show information using the game itself. [Probing for explicit connections between students' algebraic reasoning and game play]

5) Since the previous interview, has there been content that appeared in *Dragonbox* that didn't make any sense to you, or that confused you in the way it was presented? [Probing for dissonance between algebraic ideas encountered in game play and in regular class]

6) Does everything you encounter in *Dragonbox* feel like it has a mathematical parallel from your Algebra course? Do you ever find something in game play that you feel you didn't understand or encounter in your regular class sessions? [Probing for perceived limitations of algebra learning, if any]

7) [REQUIRES SET-UP] I've picked a level of *Dragonbox* that I'd like you to walk me through, if possible. Take your time, but explain each move that you make. When you're done, could you try to model the original situation using an algebraic expression or equation? [Probing for the way that students express their algebraic thinking and connect game play ideas with formal algebraic ideas]

Instruments for Investigating Affective Changes
Affect-focused interview protocol 1 [2 pages].

Affect-Focused Interview Protocol, Round 1:

For this interview, I'm going to ask you some questions about how you feel about things related to mathematics and your Algebra course. Try to respond initially by saying you either Strongly Disagree, Disagree, feel Neutral, Agree, or Strongly Agree. Then, explain as best you can why you feel the way that you do.

1) I think it's useful that I study mathematics in school.** [UTILITY OF MATH]

2) I like playing games. [AFFECT RE: GAMES]

3) I want to develop my mathematical skills.* [DRIVE IN MATH]

4) Correctly completing a mathematics problem is satisfying for me.** [SATISFACTION IN MATH]

5) Mathematics is one of the subjects I like the least.** [NEGATIVITY WITH MATH]

6) Studying mathematics makes me feel nervous or stressed.** [DISCOMFORT WITH MATH/MATH INSTRUCTION]

7) I enter mathematics courses confidently.** [OPTIMISM TOWARDS MATH INSTRUCTION]

8) I am often confused when doing mathematics.** [CONFUSION WITH MATH CONCEPTS]

9) Mathematics is easy to learn.** [OUTLOOK ON MATH/MATH INSTRUCTION]

10) I enjoy solving new problems in mathematics courses.** [ATTITUDE TOWARDS EXPLORATION IN MATH]

11) I like the challenges I face in mathematics courses.**[ATTITUDE TOWARDS EXPLORATION/TOIL IN MATH]

12) I believe that studying math helps me with problem solving in other areas.* [UTILITY OF MATH]

13) I am comfortable expressing my own approaches to mathematics problems.** [CONFIDENCE IN MATH]

14) I like playing games that can help me improve my mathematics skills. [ENJOYMENT IN MATH GAME PLAY]

15) There's no mathematics involved in playing games. [OUTLOOK ON MATH GAME PLAY]

16) I would prefer to hear multiple explanations of ideas in mathematics. [OPENNESS TO ALTERNATIVE APPROACHES TO LEARNING]

[* indicates direct text from ATMI; ** is adapted]

Affect-focused interview protocol 2 [2 pages].

Affect-Focused Interview Protocol, Round 2:

For this interview, I'm going to ask you some questions about how you feel about things related to mathematics and your Algebra course. Try to respond initially by saying you either Strongly Disagree, Disagree, feel Neutral, Agree, or Strongly Agree. Then, explain as best you can why you feel the way that you do.

1) I think it's useful that I study mathematics in school.** [UTILITY OF MATH]

2) I like playing games. [AFFECT RE: GAMES]

3) I want to develop my mathematical skills.* [DRIVE IN MATH]

4) Correctly completing a mathematics problem is satisfying for me.** [SATISFACTION IN MATH]

5) Mathematics is one of the subjects I like the least.** [NEGATIVITY WITH MATH]

6) Studying mathematics makes me feel nervous or stressed.** [DISCOMFORT WITH MATH/MATH INSTRUCTION]

7) I enter mathematics courses confidently.** [OPTIMISM TOWARDS MATH INSTRUCTION]

8) I am often confused when doing mathematics.** [CONFUSION WITH MATH CONCEPTS]

9) Mathematics is easy to learn.** [OUTLOOK ON MATH/MATH INSTRUCTION]

10) I enjoy solving new problems in mathematics courses.** [ATTITUDE TOWARDS EXPLORATION IN MATH]

11) I like the challenges I face in mathematics courses.**[ATTITUDE TOWARDS EXPLORATION/TOIL IN MATH]

12) I believe that studying math helps me with problem solving in other areas.* [UTILITY OF MATH]

13) I am comfortable expressing my own approaches to mathematics problems.** [CONFIDENCE IN MATH]

14) I like playing games that can help me improve my mathematics skills. [ENJOYMENT IN MATH GAME PLAY]

15) There's no mathematics involved in playing games. [OUTLOOK ON MATH GAME PLAY]

16) I would prefer to hear multiple explanations of ideas in mathematics. [OPENNESS TO ALTERNATIVE APPROACHES TO LEARNING]

[* indicates direct text from ATMI; ** is adapted]

Affect-focused interview protocol 3 [2 pages].

Affect-Focused Interview Protocol, Round 3:

For this interview, I'm going to ask you some questions about how you feel about things related to mathematics and your Algebra course. Try to respond initially by saying you either Strongly Disagree, Disagree, feel Neutral, Agree, or Strongly Agree. Then, explain as best you can why you feel the way that you do.

1) I think it's useful that I study mathematics in school.** [UTILITY OF MATH]

2) I like playing games. [AFFECT RE: GAMES]

3) I want to develop my mathematical skills.* [DRIVE IN MATH]

4) Correctly completing a mathematics problem is satisfying for me.** [SATISFACTION IN MATH]

5) Mathematics is one of the subjects I like the least.** [NEGATIVITY WITH MATH]

6) Studying mathematics makes me feel nervous or stressed.** [DISCOMFORT WITH MATH/MATH INSTRUCTION]

7) I enter mathematics courses confidently.** [OPTIMISM TOWARDS MATH INSTRUCTION]

8) I am often confused when doing mathematics.** [CONFUSION WITH MATH CONCEPTS]

9) Mathematics is easy to learn.** [OUTLOOK ON MATH/MATH INSTRUCTION]

10) I enjoy solving new problems in mathematics courses.** [ATTITUDE TOWARDS EXPLORATION IN MATH]

11) I like the challenges I face in mathematics courses.**[ATTITUDE TOWARDS EXPLORATION/TOIL IN MATH]

12) I believe that studying math helps me with problem solving in other areas.* [UTILITY OF MATH]

13) I am comfortable expressing my own approaches to mathematics problems.** [CONFIDENCE IN MATH]

14) I like playing games that can help me improve my mathematics skills. [ENJOYMENT IN MATH GAME PLAY]

15) There's no mathematics involved in playing games. [OUTLOOK ON MATH GAME PLAY]

16) I would prefer to hear multiple explanations of ideas in mathematics. [OPENNESS TO ALTERNATIVE APPROACHES TO LEARNING]

[* indicates direct text from ATMI; ** is adapted]

Instruments for Investigating Changes in Students' Content Retention AGATE 3 [2 pages].

Name:

Directions: Complete all problems by solving for a specified variable in the time provided. Please use one new line for every step and show all work. Indicate your final answer to a problem by boxing or circling it. If you use any scrap paper, please submit it. Good luck!

I. Solve for x.

1) $8 + x = a$

2) $2 + x = (-d) + 3$

3) $a + (-2) + x = (-b) + 2$

II. Solve for y.

4) $y \cdot 12 = c$

5) $y \cdot (-b) = 2 + z$

III. Solve for z.

6) $\frac{z}{a} = 4$

7) $\frac{z}{d} = c + (-5)$

IV. Solve for a.

8) $\frac{4}{a} = y \cdot z$

9) $\frac{1}{a} = y + (-6)$

10) $\frac{3}{a} + \frac{b}{a} = y$

V. Solve for b.

11) $\frac{b}{(-1)} = 17 \cdot d$
$(-c)$

12) $\frac{4 \cdot b}{5} = a$

13) $(-b) \cdot (-a) =$

VI. Solve for c.

14) $b\,(c + d) = a$

15) $(c \cdot b + d) + (x \cdot c + a) = b$

VII. Solve for d.

16) $\frac{d + (-5)}{d} = 9$

17) $\frac{4}{2} \cdot \left(\frac{d \cdot b}{3} + z \right) = \frac{(-x)}{((-1) \cdot (-x))}$

Content-retention-focused questionnaire [1 page].

Effects of Mathematical Game Play Study Questionnaire

Please answer the following questions in your own words as best you can. Feel free to ask clarifying questions!

1. In your studies, what have you found to be the most memorable aspects of mathematics courses?

2. What content in your algebra course have you found most memorable? Why?

3. What learning experiences in your algebra course have you found most memorable? Why?

4. Do you feel that your algebra course has had more memorable learning experiences or fewer memorable learning experiences than did your previously attended mathematics courses? Explain your position.

5. Do you feel that any aspect of your algebra course contributed to "improved content retention?" That is, do you feel that anything in your course helped you remember the course's mathematical ideas better? If so, explain using specific examples.