INDUCTIVE PATTERN FORMATION

A Dissertation

by

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ABSTRACT

With the extended computational limits of algorithmic recursion, scientific investigation is transitioning away from computationally decidable problems and beginning to address computationally undecidable complexity. The analysis of deductive inference in structure-property models are yielding to the synthesis of inductive inference in process-structure simulations. Process-structure modeling has examined external order parameters of inductive pattern formation, but investigation of the internal order parameters of self-organization have been hampered by the lack of a mathematical formalism with the ability to quantitatively define a specific configuration of points.

This investigation addressed this issue of quantitative synthesis. Local space was developed by the Poincare inflation of a set of points to construct neighborhood intersections, defining topological distance and introducing situated Boolean topology as a local replacement for point-set topology. Parallel development of the local semi-metric topological space, the local semi-metric probability space, and the local metric space of a set of points provides a triangulation of connectivity measures to define the quantitative architectural identity of a configuration and structure independent axes of a structural configuration space. The recursive sequence of intersections constructs a probabilistic discrete space-time model of interacting fields to define the internal order parameters of self-organization, with order parameters external to the configuration modeled by adjusting the morphological parameters of individual neighborhoods and the interplay of excitatory and inhibitory point sets. The evolutionary trajectory of a configuration maps the development of specific hierarchical structure that is emergent from a specific set of initial conditions, with nested boundaries signaling the nonlinear properties of local causative configurations. This exploration of architectural configuration space concluded with initial process-structure-property models of deductive and inductive inference spaces.

In the computationally undecidable problem of human niche construction, an adaptive-inductive pattern formation model with predictive control organized the bipartite recursion between an information structure and its physical expression as hierarchical ensembles of artificial neural network-like structures. The union of architectural identity and bipartite recursion generates a predictive structural model of an evolutionary design process, offering an alternative to the limitations of cognitive descriptive modeling. The low computational complexity of these models enable them to be embedded in physical constructions to create the artificial life forms of a real-time autonomously adaptive human habitat.

for kaj

editor-in-chief
who often gives the impression of thinking this reporter has a good story

If not for you
The winter would hold no spring
Couldn't hear a robin sing
I just wouldn't have a clue, if not for you

(George Harrison, 1970)

thank you

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2017
College Station, Texas

To Gabriela, Mardelle, Jay, and Bob: thank you. I cannot adequately express my gratitude for your kindness, thoughtfulness, encouragement, patience and amazing grace when faced with my often inarticulate meanderings, seemingly lost with only the black maps of my intuition. My 'big six' now stands at an even ten.

Thank you also to Dean Tom Regan for offering a place to attempt this work. In over forty-five years of intermittent conversation, he always seemed to ask exactly the right question. Which, of course, was one for which I had no answer, prompting the next iteration of investigation.

And to Francis Downing. If scripted by Charles Webb, Calder Willingham, and Buck Henry, the synchronicity of our last conversation would have been this single short scene:

Dr. Downing: *I just want to say one word to you. . . . Just one word.*
Ray: *Yes, ma'am*.
Dr. Downing: *Are you listening?*
Ray: *Yes, I am.*
Dr. Downing: ***Situated.***
Ray: *Exactly how do you mean?*

One could envision a similar scene with Mark Clayton regarding the significance of proof and history. Thank you both.

Every day I give thanks for my family and friends: without feeling your support this marathon would not have been possible. And underpinning it all, embedded in the probabilistic adaptive information structures of inductive truth space, thank you to the *Deus ex Machina* for the synchronicities of *Ah-ha!* moment local phase transitions that have mapped the 67 year trajectory of this pattern formation process.

I've done my best, I know it wasn't much
I couldn't feel, so I learned to touch
I've told the truth, I didn't come here just to fool you
And even though it all went wrong
I'll stand right here before the Lord of Song
With nothing; nothing on my tongue but Hallelujah.

(Leonard Cohen, 1984)

1998
Blacksburg, Virginia

Olivio Ferrari was many things to us, to me. With his memory, the meaning of one word grows ever larger as the years advance: mentor. To us all. The fabric of his weavings often revealed two common threads -- develop, then trust, your intuition, and process is ultimately larger than product. Design, as well as life, seems to be about both.

Where are we?

> *Architects were then, as they are now, interested more in the appearance of buildings than in their functioning. . . . .*
>
> Witold Rybczynski

> *"Until we have an understanding of these complicated, changing interactions, our attempts to balance extraction of ecosystem resources against sustainability will remain at best naive, at worst, disastrous. We, as humans, have become so numerous that we perforce extensively modify ecological interactions, with only vague ideas of longer range effects. Yet our wellbeing, even our survival, depends on our ability to use these systems without destroying them."*
>
> John Holland

As architects we have lost or given up much of our process, as a result we have lost control of our product. We have reduced ourselves to being harbingers of fashion, with little understanding of design consequences to both the human inhabitants and a building's extended ecosystem. Our cumbersome performance modeling tools are based on the reductionist thinking of linear relationships and are inappropriate for the analysis of architecture's synergy. Worse, they are of little or no benefit to the *creation* of architecture.

Revolution!

> *Artificial life is the study of man-made systems that exhibit behaviors characteristic of natural living systems. . . . . a relatively new field employing a* synthetic *approach to the study of* life-as-it-could be*. It views life as a property of the organization of matter, rather than a property of the matter which is so organized.*
>
> Christopher Langton

Artificial life and the related study of complex adaptive systems [CAS] have risen from the power of computers to model the nonlinear relationships common in natural systems. Traditional science has placed emphasis on *analyzing* and explaining in terms of smallest parts; these are attempts to *synthesize* behavior and process. Self-organization and order emerge from simple-ruled, bottom-up constructions.

Towards A New Architecture

Within fifty to a hundred years, a new class of organisms is likely to emerge. These organisms will be artificial in the sense that they will originally be designed by humans. However, they will reproduce, and will evolve into something other than their initial form; they will be 'alive' under any reasonable definition of the word.

Doyne Farmer

2050: Scientific foundation of architecture. The tools of cellular automata and genetic algorithms are united with the symmetry operations of mathematical crystallography to evolve architectural morphology in computer time; to generate form in response to environmental forces - physical, social, cultural, economic. 2100: While not derived from carbon based 'organic' life as we know it, building assemblies will be alive in every sense -- growing, adapting, reproducing and dying in symbiosis with both inhabitants and environment.

Convergence

Artificial life, CAS: bottom-up, parallel processing, simple rules, emergent self-organization. Biology, economics, psychology, physics, anthropology, mathematics, art . . . Architecture. Déjà vu. Intuitively, we have been here before. It was called *design studio*.

To a giant, whose shoulders were broad enough that we may all aspire to stand. Thank you.

rcm, 1998

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Contributors

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TABLE OF CONTENTS

LIST OF FIGURES

1. INTRODUCTION

Some ideas are reeled into our mind wrapped up in facts; and some ideas burst upon us naked without the slightest evidence they could be true but with all the conviction they are. The ideas of the latter sort are the more difficult to displace.

(Kevin Kelly, 1994)

1.1. A SITUATED INVESTIGATOR

What are alternative hypotheses but competing narratives? Invent them as fancifully as you can. Sure, they ought to avoid explicit violations of reality (such as light acting like a particle when everyone knows it's a wave? [Einstein proposes photon particles, 1905])*, but censor those stories lightly. There is time for experiment—by you or others—to discover which story holds up better.*

(Roald Hoffmann, 2005)

Although we usually think about writing as a mode of 'telling' about the social world, writing is not just a mopping-up activity at the end of a research project. Writing is also a way of 'knowing' – a method of discovery and analysis.

(Laura Richardson, 1994)

. . . qualitative writing in essence [is] very different from quantitative writing. Qualitative writing becomes very much an unfolding story in which the writer gradually makes sense, not only of her data, but of the total experience of which it is an artifact.

(Adrian Holliday, 2002)

Along time ago in a galaxy far far away . . . 'The big three', Olivio Ferrari, Herbert Kramel, and Tom Regan, opened the door to a magical world of design, where '*everything is important and nothing is impossible'*.

Designers were specialists only in the process of synthesis, freeing them to be generalists in an increasingly specialized society. Encouragement was given to develop the criteria and boundaries for individual investigations, take risks, thoroughly document the process, and self-evaluate results: design was a self-organized system of learning how to learn. Faced with the complexities of design problems in a rapidly changing world, intuition was developed, checked-as-possible, and eventually trusted.

Modeling the process of synthesis was elevated in stature to be the equal of modeling the products of synthesis. It became explicit that a robust design process extended the range of synthesis beyond physical objects: the language of any discipline could be translated into the universal visual model structures of design. Synthesis was not a body of knowledge with established precedent that had to be mastered before participating, but individual modeling precedents emerged from self-directed study. Christopher Alexander's application of mathematical graph theory to design, *Notes on the Synthesis of Form*, (Alexander, 1964); Ludwig von Bertalanffy's *General System Theory* (von Bertalanffy, 1969); Herbert Simon's *The Sciences of the Artificial* (Simon, 1969); Norbert Weiner's *The Human Use of Human Beings* (Weiner, 1954); D'Arcy Thompson's *On Growth and Form* (Thompson, 1917) were instrumental in my development of a design process.

Projects often began by asking the question, "W*hat could it be?"*: an initial step of problem redefinition that made every investigation unique, dependent on sets of initial conditions and local interactions. By repeatedly playing the game of "W*hat if . . . ?",* the relationships of variables were investigated in a process of sequential visual simulations. Iterative *input-process-output-feedback* loops were followed until they achieved the critical threshold for an *Ah-ha!* moment: the emergence of a unified design concept. Once the design variables had achieved structural stasis, the process was repeated to develop the hierarchical levels of detail of the model.

In the design of human habitats, the design process partitioned space into structured patterns, giving the properties that define human comfort, environmental impact, and cultural meaning. Models were evaluated from an embedded 'inhabitant-eye-view': *be here now*. The time-ensemble documentation of discrete design iterations traced the evolutionary trajectories of design decisions in a configuration space, or search space, of possible solutions.

My big three expanded when ecologist Bob Giles explicitly formulated the concept that form determined properties of architectural objects, resulting in their environmental performance; crystallographer Gerry Gibbs opened the door to nonvisual mathematical modeling of form; and a summer in Carbondale with Technion architect Michael Burt introduced the concept of architectural search space as a morphological configuration space.

From 1968 to 1974, the newly formed College of Architecture at Virginia Tech was a unique place in a unique time: it placed the responsibility for education directly in the lap of the student and "allowed for the exception". I continue to believe in the pedagogy of that place in time, and remain a disciple of the collective structuralist wisdom of the 'big six'.

1.2. MOTIVATION

> *Each type of civilization has its own pattern of diseases.*
> (René Dubos, 1969)
>
> *We have met the enemy and he is us.*
> (Pogo, 1969)

The human population has flourished because of the high degree of fitness held by the pattern of human *niche constructions* of our pre-connected western culture. Current quantitative and qualitative changes in the connectivity structures of this pattern transcend the properties of individual artifacts to alter the systemic properties of our habitat. Many of the properties that have emerged from this transformation can be categorized as maladaptive -- to the inhabitants of the pattern as well as to the systems external to the pattern. Altering maladaptive properties necessitates a change in the pattern. Altering the pattern necessitates a change in the pattern formation process.

1.3. POSTULATES AND CONTEXT

> *architecture: The fundamental organization of a system, embodied in its components, their relationships to each other and the environment, and the principles governing its design and evolution.*
>
> (IEEE, 2000)

> *architecture, n. transf. or fig. Construction or structure generally; both abstr. and concr.*
>
> (OED, 2011)

> *Many of the basic notions of abstract algebra could be derived from the single idea of structure-preserving function, or, as it is now customarily known, morphism. Thus the attitude gradually emerged that the crucial characteristic of mathematical structures is not their internal constitution as set-theoretical entities but rather the relationships among them as embodied in the network of morphisms.*
>
> (John L. Bell, 1981)

Accepted scientific method and units of analysis are currently transitioning from deductive analytical investigations of atomistic units to inductive simulations of the structural connectivity of hierarchical units. While a transition to structural investigations is a recent development in many disciplines, it has remained the underlying paradigm of material science for nearly a century, and is most comprehensively expressed by G.B. Olson as a set theoretic model of four elements: *Process*; *Structure*; *Properties*; and *Performance*. (Olson, 1997a, 2001) Processes form structures, or patterns. Patterns hold properties. Properties can be evaluated for their fitness in meeting performance goals. The fundamental postulate of this investigation is the structuralist relationship of processes, patterns, properties, and performance.

The investigation is framed by the context of the connectionist properties of *complex systems*: inductive synthesis is a nonlinear phase transition process of local interactions without a central control mechanism. While an emergent pattern is dependent on the initial system state, its properties are not predictable from constituent parts but are grounded in the structure of their local connections. (Simon, 1962; Gell-Mann, 1992; Holland, 1992; 2006; Mitchell, 2006; Newman, 2011)

1.4. GOALS

> *Scientific progress may broadly be divided into two types: (1) an increase in factual knowledge, by the addition to the total amount of scientific observations; (2) an improvement in the body of theories, which is designed to explain the known facts and to predict the outcome of future observations. An especially important case of the second type is the replacement of an accepted theory (or body of theories) by a new theory (or body of theories) which is in some sense superior to it.*
>
> *The statements of the scientist are of two types: (1) Those reporting the results of his observations, or of the observations of other scientists that he is willing to accept. (2) Generalizations, hypotheses, general laws, etc., that he believes are correct. The former are his observational statements, the latter his theoretical statements.*
>
> (John Kemeny and Paul Oppenheim, 1956)

> *Theory is the keystone to understanding.*
>
> (Samuel Scheiner and Michael Willig, 2005)

The parameters of architectures of the human habitat are defined by the set of variables forming the intersection of three broad domains: the human animal, human culture, and the life-support system of the natural environment. Giving form to the human habitat can then be defined as a multi-variable optimization problem involving two parallel constructions: a habitat information structure and its physical expression as a set of finite partitions of two and three dimensional space. This is a problem of pattern formation.

The sheer number of variables and the complexity of their interactions overwhelm the art-based practice of design. By importing the historic precedents of habitat modelling into a computer environment, architecture has polished descriptive models to a gemstone gloss while ignoring several critical *inclusions*:

Modeling without predictive feedback of the consequences to human and environmental health.

Modelling incomplete parameters; omitting critical variables.

Modeling atomistic objects without regard to systemic properties of the larger aggregate.

Modeling limited morphologies of a very small region of architectural configuration space.

Modeling static habitat constructions unresponsive to dynamic contexts.

The intent is to address these issues by developing a model of a repeatable, verifiable design process: a scale invariant predictive model of comprehensive habitat parameters in a definable unbounded search space. The breadth of variables necessitates the model be domain invariant; the objective of real-time adaptation implies a level of simplicity necessary for the model to be distributively embedded throughout the materials of dynamic physical constructions.

The goal of this investigation is to develop a theoretical foundation that enables a science-based practice of architecture and construction of an autonomously adaptable human habitat.

1.5. METHOD

> *We're still stuck on that screw and the only way it's going to get unstuck is by abandoning further examination of the screw according to traditional scientific method. That won't work. What we have to do is examine traditional scientific method in light of that stuck screw. . . .*
>
> (Robert Persig, 1974)

> *We secure our mathematical knowledge by demonstrative reasoning, but we support our conjectures by plausible reasoning. A mathematical proof is demonstrative reasoning, but the inductive evidence of the physicist, the circumstantial evidence of the lawyer, the documentary evidence of the historian, and the statistical evidence of the economist belong to plausible reasoning. The difference between the two kinds of reasoning is great and manifold. Demonstrative reasoning is safe, beyond controversy, and final. Plausible reasoning is hazardous, controversial, and provisional. Demonstrative reasoning penetrates the sciences just as far as mathematics does, but it is in itself (as mathematics is in itself) incapable of yielding essentially new knowledge about the world around us. Anything new that we learn about the world involves plausible reasoning . . .*
>
> (George Pólya, 1954)

Studio structured academic preparation and the professional practice of architecture are founded on the art of inductive synthesis: a recursive cognitive process of multi-variable pattern formation. Modeling often assumes a situated point of view, placing architects and their clients in the space being modeled. Computer science recognizes this recursive process as the evolution of a-posteriori schema for unstructured and semi-structured data. It is familiar in the social sciences as the foundation of qualitative research methods.

Much of twentieth century science was dominated by deductive analytical external observer models of hypothesis verification: investigations of linear pattern-property relationships as independent and dependent variables of unstructured atomistic collections. With the advent of the computer, these investigations are giving way to structural examinations of pattern formation: simulation models of situated inductive synthesis are being used to investigate non-linear process-pattern relationships as multi-variable clustering of modular hierarchical structure.

The closing era of deductive science contributed little to the processes of the artist, yet the intuitive art of inductive synthesis was the common tunic covering the shoulders of Newton's giants. Twenty-first century science is now on the threshold of a reciprocal offering to the arts, where the art of architecture is ideally positioned to formalize the intuitive art of inductive synthesis with the science of multi-variable predictive models of inductive structural pattern formation.

The allure of predictive models of design raised the inevitable *What if?* to initiate in an examination of the causal relationship between the structures of process and pattern. The method of investigation was *meta-modeling*: in this case, using the recursive cognitive process of situated inductive synthesis of pattern formation to construct a recursive predictive model of the situated inductive synthesis of pattern formation. From my perspective as an outsider self-sentenced to solitary confinement in the grand library of specialization, this nonlinear inductive investigation of nonlinear inductive phenomena spanned multiple domains, constructing a cognitive case study of Searle's *Chinese Room Argument* of artificial intelligence. By tracing iterative *general systems theory* loops of *input-process-output-feedback* in a recursive a posteriori *threshold-cascade* structuring of unstructured and semi-structured data, the design process was used to design the design process.

1.6. STRUCTURE AND SCOPE

> *Inductive inferences are made by classifying events and the outcome of these events within suitable categories. Accuracy of inference is largely dependent upon how good the categories are. At even an elementary level of complexity, recognition of structural similarities and performance of substitutions become natural developments . . . At a slightly more complex level, relations, sets, and hierarchies of sets develop.*
>
> (Ray Solomonoff, 1957)

The inductive structure of the investigation followed the ***situated mathematics*** model of inductive pattern formation that the investigation produced. (Section 4) Unlike a Markov chain of successive system states constructed with each state having independent probabilities of existence, the hierarchical clustering of particulars by inductive inference constructs a situated conditional probability sequence of knowledge state spaces where the clustering structure of each state is dependent on the structure of relationships established in the preceding state.

A process of deductive inference establishes structure and scope prior to the investigation; the structure and scope of inductive inference is apparent only a posteriori. The top-down process of deductive reasoning descends from the central reserve of intellectual capital established by the accepted precedents of giants, where the linear linkage of categorical truth to a particular instance of investigation can be established with only a few key citations. Following the bottom-up nonlinear process of inductive reasoning, intellectual capital in this investigation is built by crowdsourcing from science's everyman, with the ascending hierarchy of categorization constructed without central control by the recursive abstraction of local relationships between particular instances. Keywords, phrases, sentences and paragraphs of individual citations define specific grains of truth, with populations of discrete grains providing input for the cognitive *quorum sensing* consensus of categorical clustering. As a non-deterministic process, the size and distribution of the input sample population is related to the validity of the output generalizations.

To date, the probabilistic relationships and recursive discontinuities of inductive reasoning present both problems of modeling and problems of methodological repeatability and verifiability. Inductive constructions of knowledge domain networks structured by connectivity of citations, authors, or keywords are unable to differentiate hierarchical boundary structures and reflect *cliques* of specialization. Generalizations are often obscured by the divisiveness of language: for the concept that *change in the connectivity structure of a set of elements results in a linearly unpredictable, discontinuous change in the properties of the set*, physics speaks of *critical states* and *phase transitions*; evolutionary biology of *punctuated equilibrium*; cognitive sciences of *threshold-cascades*; history of *conflicts*; creativity of *Ah-ha! moments*; science culture of *Kuhnian revolutions*; popular culture of *tipping points*. Discontinuity of the phase transition between successive knowledge state spaces in a process of inductive inference has resulted in traditional 'black box' models of categorization and pattern recognition in the sciences of biological and artificial intelligence, with only the inputs and outputs verifiable to an external observer.

To simplify documentation, the discrete granular particulars that form the input sample population of this investigation are considered to be individual citations. Pattern formation output, the set theoretic hierarchical structure of categorical generalizations, is presented in outline form. (Appendix A)

The initial use of accepted terms with domain specific meaning are shown in *italics*, but are (generally) used to express a structural concept having an *equivalence relation* that crosses academic boundaries. Proposed new structural concepts, to my limited knowledge not in accepted use, are shown in ***bold italics***.

The situated conditional chain of knowledge state spaces of the investigation was structured by a sequence of questions, each emerging from the preceding state of the investigation. This sequence of recursive meta-abstractions constructed a scope of six sets of categorical generalization:

(Model models): the parameters of model space
(Adaptation): situated inductive pattern formation
(Modularity): inductive transformation and hierarchical units of synthesis-analysis
(Proof and truth): existing precedents and boundaries of provable truth
(***Architectural identity***): *situated mathematics* of *configuration space*
(Life, the universe, and everything): situated meaning

1.6.1. Model models

The first recursive meta-loop of inquiry began in 1972 with Bob Giles, developing a simple *multi-criteria decision model* to optimize architectural form based on the limited parameters of climate, incident solar radiation, surface/volume ratio, structural stability, and net/gross ratio of habitable area. Dawning recognition of the design limitations of a finite library of candidate forms considerably slowed the pulse for a repeatable verifiable design process. Twenty-five years of diverse professional practice buried it under deadlines and dollars.

Twin defibrillating shocks were administered by the initial publication of *Artificial Life* (Levy, 1992) and *Complexity* (Waldrop, 1992), inducing an Ah-ha! moment of cognitive concept formation: evolution was a generative combinatorial optimization design process in an infinite library, replicable in computer-time to generate real-time architectural form. The resurrected inner Don Quixote posed a question:

> ***Can an evolutionary design process structure the foundation of a science of architecture?***

The inquiry reconvened by reviewing the precedents and structures of scientific modeling and examining the complex adaptive system characteristics of human factor, cultural, and environmental parameters of habitat design to define the domain of architecture as a complex adaptive system. The modeling domains of mathematics and science were themselves found to be complex adaptive systems, with peer review functioning as a *predictive control* structure. Across academic disciplines, the structural models of complex systems were found to conform to Olsen's set theoretic structural paradigm of material science: (*process* → *structure* → *properties* → *performance*). Structure = pattern. Design and evolution can be defined as nonlinear inductive pattern formation processes of complex adaptive systems.

In this phase of the investigation, the inductive process of a posteriori conceptual clustering hierarchically structured the specific data grains of individual citations into a union of five sets of categorical generalization:

(structural design case study)
(structures of pattern space)
(set theoretic structuralism)

(model structures)

(structure of knowledge domains)

1.6.2. Inductive-adaptive pattern formation

From the previous loop, with evidence of the human habitat as a complex adaptive system of inductive pattern formation and of Olsen's set theoretic structuralist paradigm underpinning complex system modeling across academic boundaries, the question surfaced:

> ***Are there invariant processes and structures of complex system pattern formation, independent of scale and academic boundaries, that encompass the range of variables that structure the human habitat?***

The second recursive meta-loop followed the precedent of Herbert Simon and John Holland (Simon, 1969; Holland, 1975) in a search for similarity in the structures of nonlinear inductive pattern formation of natural and artificial systems.

The emergence of commonality across genetic, cognitive, social group, and population scales of adaptive structural transformation lead to construction of a feedforward connectionist model, where pattern develops through a sequence of threshold-cascades reciprocating between information structures and their physical expression: a hierarchically self-similar parallel distributed processing model with predictive control, homomorphic to bipartite ensembles of 'black box' artificial neural networks. Theoretically capable of being embedded in the physical materials of habitat construction, this is an adaptive meta-model of autonomous nonlinear inductive pattern formation where the model itself evolves in response to the co-evolving relationships of internal and external variables. [Appendix B]

In this phase of the investigation, the inductive process of a posteriori conceptual clustering hierarchically structures the specific data grains of individual citations into a union of three sets of generalization:

(hierarchical structures of inductive-adaptive pattern formation)

(bipartite structures of inductive-adaptive pattern formation)

(invariant properties of inductive-adaptive pattern formation)

1.6.3. Hierarchical modular units of analysis, synthesis

The hierarchical bipartite parallel processing model of reciprocating genotypic-phenotypic development constructed in the previous loop raised the next broad question:

> ***What are the units of adaptation and the mechanism of their interaction?***

Defining boundaries of hierarchical structure has historically presented problems for atomistic models of units of analysis and synthesis. The adaptive interpretation of local information is a situated response within a level of a structural hierarchy, presenting the problem of adaptive units as one of uniquely defining an interacting configuration of elements and its boundary.

The third recursive meta-loop developed the topological operation of *Poincare inflation* of a point to form its *neighborhood*, effectively modeling a *field* surrounding an element. A set of elements undergoing simultaneous Poincare inflation structures a new approach to the *Boolean grain model* of the set, with continuous inflation constructing hierarchical sequence of *fundamental groups* of individual elements. (Appendix C) The hierarchical boundaries of local fundamental groups are defined by the qualitative mathematics of set theory, group theory, and topology, leading to a cross-disciplinary review of attempts to quantitatively define a specific configuration of points using the currently accepted mathematics from an external point-of-view.

In this step of the investigation, the inductive process of a posteriori conceptual clustering hierarchically structured the specific data grains of individual citations into a union of three sets of categorical generalization:

(individual element as a *local fundamental group*)
(hierarchical modular structure)
(quantitative configuration descriptors)

1.6.4. Limits of proof and truth

The previous loop evidenced the inhibition of inductive structural investigations across the disciplines of science, where current models of a configuration are limited to heuristic probabilistic descriptors of questionable validity and deterministic qualitative categorical descriptors. The emergent realization that the simple quantitative definition of a specific configuration of points remains an open problem in mathematics raised the question:

What are the cultural precedents and boundaries of provable truth structures?

The fourth recursive meta-loop investigated the structures and properties of deductive inference of an external observer, the reliance of predictive modeling on atomistic quantitative mathematics or its probabilistic approximation, and the structural problems of consistency and completeness in the foundations of mathematics.

The inductive process of a posteriori conceptual clustering hierarchically structured the specific data grains of individual citations into an incomplete union of four sets of categorical generalization:

(structures of proof)

(structures of provable truths)

(boundaries of provable truth structures)

(emerging situated point-of-view in science and mathematics)

1.6.5. *Architectural identity* of a configuration

With a knowledge state space constructed by the union of categorical generalizations from the preceding four meta-recursions, the emergent pattern connected the complications of structural investigation in science to the properties of mathematics structured from an external point-of-view:

external point-of-view mathematical proof of external point-of-view mathematics as a nonexclusive, nonsingular model of an external objective reality.

external point-of-view mathematical proof of the incompleteness and inconsistency of external point-of-view mathematics as a model of truth space.

external point-of-view mathematical proof of the possibility of alternative set theory(s).

foundational premise of external point-of-view probability theory and quantitative branches of mathematics as constructions of unstructured collections of atomistic elements.

emergent restructuring of external point-of-view mathematics into categorical configurations constructed by the local relationships of structure preserving functions.

emergent viewpoint in science that adaptive processes are grounded in the situated interpretation of local configurations of information.

emergent viewpoint in science that nonlinear processes of complex systems are grounded in the local interactions of configurations of elements.

emergent viewpoint in science that properties are grounded in the structure of configurations of elements.

Constraints on truth space imposed by the boundary structure of external point-of-view mathematics granted the freedom to simply attempt to design the local mathematics necessary to quantitatively model the hierarchical structures of inductive pattern formation. The categorical dissonance of current quantitative mathematical models, the prevailing view of current physics that we inhabit a stochastic universe, and my incorrect initial deterministic interpretation of the qualitative mathematics model of pattern formation developed in Section 1.5.3. prompted the question:

Is there a local probabilistic structure underlying the hierarchically local interaction of neighborhoods constructed by topological inflation in the Boolean grain model of pattern formation?

Fifty-seven iterations of trial-and-error resulted in a model of ***situated local probability***, where Poincare inflation structures the sequence of intersecting neighborhood spaces as *conditional sample spaces*, constructing a sequence of discrete local probability spaces in a ***situated conditional probability chain***. The concept of ***topological distance*** that emerged from the model constructed in section 1.5.3 was expanded into the concept of ***Boolean topology*** as a situated replacement of traditional point-set topology developed from the external point-of-view. In the sequential construction of a complete Boolean topology, simultaneous development of the situated metric, topological and probability spaces unique to a set of points provide a triangulation of connectivity measures that define a unique quantitative identity of a configuration and structure the parameters of configuration space. Each discrete step in the sequence constructs a hierarchical boundary definition, with the complete sequence tracing the developmental trajectory of a configuration in configuration space. These initial constructions of situated mathematics structure the formalism for a domain invariant predictive model of the nonlinear hierarchical structures of pattern formation.

In this phase of the investigation, the inductive process of a posteriori conceptual clustering hierarchically structured the specific data grains of individual citations into a single set of categorical generalizations that were used to inform situated local constructions:

(structures of external point-of-view mathematics)

1.6.6. Life, the universe, and everything

With the recursive process of situated inductive inference eventually constructing a simple space-time model of itself, the uncanny parallels to the journey of *Arthur Dent* (Adams, 1979) could prompt only one question:

What does it all mean?

Extrapolation of the trajectory of conceptual configurations that formed the penultimate knowledge state space constructed the ***incomplete union*** of four sets of situated categorical conjecture:

(structures and properties of the model)
(structures and properties of human niche constructions)
(structures and properties of science)
(structures and properties of provable truth)

2. ONCE AND FUTURE HABITAT

Architects were then, as they are now, interested more in the appearance of buildings than in their functioning. . .

(Witold Rybczynski, 1986)

Until we have an understanding of these complicated, changing interactions, our attempts to balance extraction of ecosystem resources against sustainability will remain at best naive, at worst, disastrous. We, as humans, have become so numerous that we perforce extensively modify ecological interactions, with only vague ideas of longer range effects. Yet our wellbeing, even our survival, depends on our ability to use these systems without destroying them.

(John Holland, 1992)

2.1. ONCE AND FUTURE STATES OF MODELING

2.1.1. The current state of modeling

Throughout history, two and three dimensional descriptive models have been the tools used to design objects that comprise human habitats. While the external physical representation of the object is traditionally thought of as the model, the modeling processes of variable attention, variable integration and the *AH-HA!* moment of pattern formation are internal cognitive processes of the modeler. This interplay of internal and external modeling is a process of *distributed cognition*. Developed by Edwin Hutchins in the late 1980's, distributed cognition extends the cognitive unit of analysis to include the social groups and physical artifacts of an individual's environment. (Hutchens, 1988)

Design, the art of synthesis of the human habitat, works in a distributed cognition descriptive modeling paradigm, severely restricted by the cognitive modeling process, the descriptive modeling process, and the selection of *disjoint* atomistic objects as the units of synthesis and analysis.

Cognitive modeling presents two critical limitations, the number of parameters included in the model, and the inference of properties from the model:

1. Modeling pattern parameters: Distributed cognition descriptive modeling of the human habitat is an iterative process of *pattern formation*. (Sloman, 1998; Thompson, 2000; Robertson, 2003; Palmeri, 2004; Barsalou, 2005; Uchida, 2006) As such, model development is limited by the individual modeler's cognitive load limit: the restricted ability to hold only three to seven *chunks* of information in working memory at one time. (Miller, 1957; Simon, 1974; Gobet, 2001) This creates a mismatch between the set of model parameters and the modeler's cognitive ability to integrate that set into a single cohesive pattern.

The design professions have attempted to overcome this problem by adopting parallel group processing (Rumelhart, 1987), but there is an upper limit of membership in an effective parallel process. As processing components increase, inter-component communication time reaches a threshold where it engulfs processing time and effectively halts computation. (Bhuyan, 1987; Dally, 1990; Kotsis, 1992, 1993; Blazewicz, 1993; Fraigniaud, 1994)

With a set of parameters exceeding the individual and collective cognitive load limits of a design team, a subset of parameters will always remain unaddressed by a descriptive distributed cognition model, often resulting in the untimely consequences of unforeseen interactions.

2. Modeling pattern properties: Were it possible to include the full set of pattern formation parameters in its development, an external descriptive model remains just a descriptive model: it is a visual description of a pattern, allowing no inference about the properties a pattern may hold, or inference of its performance in relation to design goals. The relationship of a pattern to its properties and performance resides in the internal cognitive model of the designer, with this cognitive coupling evidenced by the four states of a modeler's professional development:

> State 1. Academic mentoring: : With the assumption that students have yet to acquire the experiential memory of object-property relationships, academic evaluations traditionally consist of faculty speculation about possible property and performance implications inferred from students' external descriptive models.
>
> State 2. Professional mentoring: After five or six years of academic mentorship, up to 3 years of mentored professional apprenticeship is compulsory to qualify for state licensure examination in the United States. This represents the final step in the *oral tradition* of transferring pattern-property-performance knowledge across generations.
>
> State 3. Early to Mid-career specialization: A practicing architect begins to form cognitive relationships of patterns, properties and performance in individual objects by repetition of project-specific cycles of design *'AH-HA!'* pattern formation and experiential feedback of physical pattern construction. The cognitive load-limit for pattern formation and the extended duration of design-construction cycles suggest a causal relationship to the trend of specialization by functional typology.
>
> State 4. Late career categorical cascade: There is considerable anecdotal evidence that architects typically do their best work late in their careers, only after accumulating thirty or forty years of experiential memory. This is a hierarchical structure of cognitive pattern formation, where the accumulation of individual project-specific '*AH-*

HA!' pattern formation moments coalesce to form the meta-pattern '*AH-HA!*' of a categorical cascade.

Dependence on the cognitive coupling of pattern and properties from an experiential sample of historically homologous objects has restricted architectural synthesis to a narrow band of deviation from established morphological precedent. The attention given to typology and not topology has rendered architects to be conceptually incognizant of a search space. With the pioneering work of Michael Burt (Burt, 1972, 1974, 1984) being the seminal exception, architectural history is absent any exploration of the structure or boundaries of architectural search space as a morphological configuration space.

2.1.2. Current units of analysis

The modeling precedents in architecture that define the units of synthesis and analysis to be individual disjoint buildings are grounded in the cultural mores of ownership and legal precedents formulated under the concept of the world as a 2-dimensional flat surface. (George, 2006) The cultural concept of land ownership is the controlling parameter in pattern formation of the human habitat. With the global ascension and dominance of western culture, the commodity of land is defined by partitioning a 2-dimensional surface into closed regions that are projected both below and above that surface to create a discrete 3-dimensional boundary of individual ownership.

The cultural partitions of the earth's biosphere reduce architectural models to a single atomistic unit of synthesis and analysis, precluding any ability to examine the collective properties of groups of objects, in both static and changing relationships.

2.1.3. Situated hierarchical structure

Borrowing hierarchical concepts from physics that structure the relationships of objects, every object is type of *matryoshka doll*. Descending a hierarchical structure, every object is composed of a *set* of sub-objects, where each sub-object is composed of a set of sub-objects, and each of those sub-objects is composed of sub-objects . . . Likewise, ascending a hierarchical structure, each object is a member of a set that defines a larger object, with each larger object being a member of a set that defines an even larger object . . . In every level of a hierarchical structure, an object is a set, or *cluster*, of smaller objects, giving a unit of synthesis or analysis multiple definitions. Each level of hierarchical objects is defined by a discrete boundary, with that boundary defined by its local interaction with boundaries of adjacent objects. The definition of an object is a *situated* definition.

In a set of objects that form no physical connections, a discrete object is a readily definable unit of synthesis and analysis. But as Newton established in the 17^{th} century, objects absent a physical connection have other interactions. Relationships of objects, even if not physically connected, are defined by the union of the set of variables involving the relative location of objects and the set of variables involving intervals in time of object interactions. Properties are grounded in the structure of interactions, or connections, of sets of objects. The set of variables of temporal object interactions can be categorized as having ***non-binding***, ***quasi-binding***, and ***binding*** interactions.

With an atomistic object defined as a unit of synthesis or analysis, set structures of can be used to define the situated local locations and situated local interactions of collections of objects:

> A *configuration* is defined by the set of spatial location variables and the set of temporal interaction variables of a set of objects. The properties of a configuration are grounded in the structure of its sets of location and interaction variables. Variables of relative location and local interaction of objects define a configuration as a situated structure. The *initial configuration* of a set of objects is defined by the set of location variables that initiate the model. A *configuration space* is a structure on the set of all possible configurations formed from a set of objects: the set of all possible configurations includes an operation, or *morphism*, that transforms one configuration into another configuration. A morphism structures the neighborhood adjacency and accessibility distances of a configuration space.
>
> As the values of its variables change, a configuration will assume different *states*. A *configuration state* is the overall condition of a configuration at a specific instance in time: the sets of values for the sets of location and interaction variables of a set of objects at a specific instant in time. The *initial state* of a set of objects is defined by the set of values for the location variables and the set of values for the interaction variables that initiate the model. A *state space* is the set of all possible configuration states of a set of objects in a set of time: the sets of all possible values for the sets of location and interaction variables of a set of objects in a set of time. A temporal sequence of states traces a path in state space, defining the evolutionary trajectory, or *time-ensemble*, of a configuration.
>
> A *phase transition* is change in state space that results in a *discrete* morphism in configuration space. For a set of objects, a phase transition is a change in a configuration state that results in a discontinuous jump to a new configuration: change in the values of the location and interaction variables at a specific instant in time that yields a discrete transformation in the sets of location and/or interaction variables. A *phase space* is a mapping of a state space onto a configuration space. The variable

values of a configuration state entering a phase transition are the values of the initial state in the initial configuration that exits a phase transition.

2.1.3.1. properties of situated hierarchical structure

A full range of continuous values for the sets of location and interaction variables result in configurations of nonbinding interactions, having the properties of a gas.

As the range of continuous values for the set of location variables contract to a *threshold range* and the range of continuous values for the set of interaction variables contract to become discrete, a configuration with nonbinding interactions jumps to a configuration with quasi-binding interactions, having the properties of a liquid.

As the range of continuous values for the set of location variables contract to become discrete and the state values for the set of interaction variables remain discrete, a configuration with quasi-binding interactions jumps to a configuration of binding interactions, having the properties of a solid.

With set structures of discrete location and interaction variables, sets of objects form binding interactions, where boundaries of disjoint discrete objects are subsumed by clusters of objects with a common boundary. With properties grounded in hierarchical cluster-objects, hierarchical cluster-objects become the appropriate units of synthesis and analysis.

In large sets of objects where every object has multiple binding interactions, current 'objective' scientific modeling has difficulty defining discrete boundaries of hierarchical structure, making this a problem that consumes much of current scientific research. (Rohlf, 1970, 1974; Sokal, 1985; Bolker, 2000; Borgani, 2001; Schlosser, 2002; Grimmett, 2003, 2006; Newman, 2003; 2006, 2009; Spirin, 2003; Kosak, 2004; Danon, 2005; Kahn, 2005; Krause, 2005; Leydesdorff, 2006; Koseska, 2010; Landau, 2010; Everitt, 2011; Pivovarov, 2011; Molina, 2012; Qian, 2012; Arbelaitz, 2013; Bock, 2013) For structures of binding interactions, defining appropriate units of synthesis and analysis has become the open problem of the definition of discrete units of hierarchical structure.

Modeled from an external point-of-view, nested hierarchies of object clusters present no discrete boundaries, forming *near-infinite* strings, or *walks*, in a complex network of variables defining a hierarchical configuration. Following the work of Alfred North Whitehead and Bertrand Russell, (Whitehead, 1910) every model of a hierarchical configuration would then begin from a union of the sets of spatial location and temporal interaction variables of a set of Higgs particles and work its way up to the objects under investigation:

> *"Begin at the beginning," the King said, very gravely,*
> *"and go on till you come to the end: then stop."*
>
> (Lewis Carroll, 1865)

Evolution of adaptive and non-adaptive configurations is an *undecidable* problem. There is no central controller orchestrating the top-down formation of configurations in natural systems. A probabilistic universe is dependent on the situated probabilistic local interactions of objects for a bottom-up formation of configurations. External point-of-view mathematics can provide a quantitative description of structure, but natural systems perform no complex internal calculations to form structure. Adopting the situated point-of-view and situated logic of the configuration models the process of pattern formation.

The union of the set of situated location variables and the set of situated probabilistic temporal interaction variables of a set of objects reveal the discrete hierarchical boundaries of random hierarchical structure. The intervals of randomness accumulate to form discrete hierarchical structure: the *deep simplicity* of complex systems.

2.1.4. Current configuration state of the human habitat

With cultural partitioning of space reducing human habitat models to atomistic units of analysis, habitat configurations readily admit to analysis by their set theoretic structure of interactions:

> The configuration of the human habitat is can be defined by the set of situated location variables and the set of situated interaction variables of a set of building-objects. The sets of location and interaction variables are globally ubiquitous to define all sets and subsets of building-objects as having the same configuration.
>
> Currently, and for any instant in time in Western culture, the values for the set of location variables of building-objects have been culturally contracted to become discrete. Constrained by fixed locations, values of the set of interaction variables between building-objects have also contracted to become discrete. Having a discrete values for both location and interaction variables historically gives the state of the configuration of building-objects the binding connection properties of a solid.
>
> With a non-hierarchical structure given by the partitions of land ownership and discrete horizontal boundaries above and below the surface, the configuration of Western human habitats can be categorized as a solid sheet or film of discretely varying thickness on the surface of the earth.
>
> Having the property of solid structure reveals the configuration state properties of the human habitat to be no longer grounded in disjoint building objects. Properties held by the physical human habit cannot be derived from examination of individual buildings any more than the properties of diamond can be derived from examination of single atoms

of carbon. In both cases, properties are grounded in the interaction structures, or configuration of connections, formed by the collections of objects.

Limiting habitat analysis to individual disjoint building-objects precludes analysis of their configuration of connections. The configuration of connections determines the properties of the configuration of human habitation.

New units of analysis are required to correctly evaluate the properties and performance of the configuration of human habitation.

2.1.5. Global phase transition of the human habitat

According to the World Bank's 2009 World Development Report, the year 2000 CE marked a global transition, with more than half of the human population living in cities and 85% of the developed world population living less than 1 hour from a city. While 95% of our population inhabits only 10% of earth's land area, 'remote' areas, defined as being at least 48 hours from a major city, have been reduced to just 10% of this surface. (World Bank, 2009)

This presents a mismatch between the current pattern of human habitation and the unit of synthesis and analysis accepted for modeling. With units of synthesis and analysis restricted to individual disjoint objects, architects are free to practice habitat design as art, unburdened of any responsibility for the combinatorial properties resultant from their individual object's interactions with other similar objects:

Each type of civilization has its own pattern of diseases.
(Rene Dubos, 1969)

The pandemics of obesity; depression; seasonal affective disorder; attention deficit hyperactivity disorder; regional urban and global scale climate modification; ecosystem fragmentation and instability are all properties inherent to our pattern of habitation.

Little house on the prairie is no longer an apt metaphor for our pattern of human habitation, nor is it appropriate as a singular unit of synthesis and analysis. To fully understand the human behavioral and environmental properties of our current pattern of habitation, units of synthesis and analysis need to extend beyond the disjoint object of an individual buildings to include their collective hierarchical interactions: immediate local neighborhood; a cluster of local neighborhoods or extended neighborhood; precinct; borough; city; county; region; sub-continent; continent; globe:

Does the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?
(Edward Lorenz, 1972)

The properties of hierarchical systems constructed from a large number of components, or *complex systems*, cannot be simply inferred from the properties of its individual component objects, but instead are a function of the relationships, or *connectivity*, of those objects:

> *Interactions at a lower scale emerge as objects expressing their own properties at a higher scale. Scaling is the key to emergence; emergent properties arise as new objects from one scale to the next.*
>
> (Joris Deguet, 2006)

As several objects in a local neighborhood interact to form a *configuration*, the properties originally grounded in an individual object become the collective properties of the configuration. As several configurations connect to form a larger configuration, and larger configurations connect to form even larger configurations, the properties at each level of the hierarchy become less dependent on the nature of the individual objects and become increasingly dominated by the connectivity of the configuration. Eventually, a collection of configurations interact to form a single large configuration, or a *giant component.*

The emergence of a giant component indicates a *phase transition* in a system, and a discontinuity in its properties. At this level of connectivity, properties are not derivable from the individual objects and bear no resemblance to the properties of the ascending hierarchy of configurations. Just prior to the formation of a giant component, the collection of large configurations achieve a *threshold* level of connectivity that defines a *critical state* preceding a phase transition. The critical state of a system might be inferred by one of its inhabitants only if that inhabitant had knowledge of a similar system experiencing a phase transition: with only an inhabitant-eye-view, a system in a critical state exhibits little evidence of a pending phase transition.

When a clustering process is confined to local interactions, the initial disjoint atomistic objects interact to form a set of local configurations, defining the local boundary and properties of the initial hierarchical level of a system. As the first set of configurations increasingly connect to form their own local giant components, this second set of local phase transitions defines the boundaries and properties of a second level of hierarchical structure. The third iteration, increasing the connectivity of second-order configurations, produces a third set of local giant components, where this set of local phase transitions defines the boundaries and properties of a third level of hierarchical structure. Hierarchical pattern formation is then defined by sequential sets of increasing local connectivity that result in ascending local phase transitions of local giant components.

On a finite 2-dimensional surface, such as a game board, disjoint objects can be divided into two groups and placed randomly on the board such that objects of each group form connections within their own group, but do not interact with objects of the other group. Grey objects interact only with Grey objects, and Green objects interact only with Green objects, and never the twain shall meet. The process of pattern formation for one group by itself would follow sequential stages of connectivity to eventually produce a single giant component that fills the entire board, with the board assuming the properties of the giant component.

However, simultaneous pattern formation of both groups becomes a *zero sum game*, with the connections formed within the Grey group blocking possible connections between Green members, and vice versa. If the Grey and Green sides make their connections at the same rate, the game typically results in a draw, with neither side achieving the single giant component that dominates the board. The final board is a mix of disjoint Grey and Green clusters, or *patches*, having similar properties.

But if one side connects more quickly, growing at a significantly faster rate, it will achieve a phase transition, forming a single giant component that dominates the board in spite of the opposing group. If Grey wins, a few isolated grey patches may remain unconnected, but the connectivity level predicates that the overall board will assume the giant component properties of Grey. Isolated disjoint Green patches remain also, but generally will not significantly alter the Grey properties of the board.

The most common variation of this game is played with one side opening play with established connections of a giant component and dominating the properties of the board, while the other side starts as randomly distributed isolated objects. In the alternating play of sides, each side may add objects to locations on the board adjacent to existing objects in their own group. This variation is also played with two additional connection rules:

> The first modifies the qualitative nature of connections, with the initial dominant pattern gradually constructing endurance connections with *slow-twitch polarity*, while the isolated objects of the opposite side are capable of quickly fabricating power connections of *fast-twitch polarity*.
>
> The second connection rule sets a minimum connectivity threshold for an isolated patch to remain intact. Below this threshold, new objects may not be added, and existing objects are removed on each turn to eventually leave a void on the game board.

The game is traditionally starts with Green as the dominant group on the board, and Grey opening play by adding objects to form several small local clusters. Green's traditional opening response is to eliminate as many isolated Grey objects as possible, and reduce the connectivity of the few initial Grey clusters. After only a few turns, Grey's speed and power break through the thicket of Greens' connections to link several of the Grey clusters and establish the second hierarchical level of pattern formation.

As Grey's population remains small in relation to Green's, there is a period of establishing and severing connections on both sides, producing a middle game pattern of thinly connected and disconnected clusters of Grey and Green. However, with Grey's increasing population and accelerated growth of fast-twitch connections, the middle game pattern of dynamic stasis is short-lived. Green's connectivity is rapidly overwhelmed by Grey's blitzkrieg as large clusters achieve the connectivity threshold of a Grey critical state.

The end game begins as Grey forms a single giant component, leaving Green broken into disconnected clusters, reversing their initial dominance and deference roles on the board. With a giant component achieved, Grey's metabolism slows, few additional Grey objects are added to the board, and the polarity of connections reverse. Grey's fast-twitch connections of conquest decelerate into the slow twitch links of maintenance. The smaller isolated Green clusters begin to drop below the minimum connectivity threshold and decompose to become voids on the board.

After several turns, the voids increase to achieve their own critical threshold, triggering a second set of cascades in both Green and Grey, initiating another role reversal. Grey's giant component begins to fragment, the population of Grey objects drops precipitously while voids erode Grey connections and isolate clusters from the central pattern. In the few remaining Green clusters, the process of adding objects accelerates to create new generation fast-twitch Green connections. Now the roles are fully reversed, and the slow-twitch Grey connections begin to collapse under the counterattack of a fast-twitch Green blitzkrieg. Green's rebound returns the board to the middle game pattern of thinly connected and disconnected clusters of Grey and Green.

The accumulation of probabilistic local interactions on the board of Grey and Green objects have now reached a bifurcation point, with the end game playing out a path in one of the following four groups of trajectories:

End game scenario 1:

A handful of Grey's remaining clusters stabilize above the connectivity threshold and preserve the accepted precedents for pattern formation. Grey's pattern retains its pre-collapse properties. Grey adapts to the recent catastrophe by introducing a central control mechanism to limit population growth and confine new pattern formation to strict boundaries. A middle game pattern of thinly connected and unconnected Grey and Green clusters form a long term dynamic stasis. The end-board is shared by Grey and Green.

End game scenario 2:

Green's counterattack is unrelenting as Grey's population continues to decline. The smaller isolated Grey clusters begin to drop below the minimum connectivity threshold and decompose to become voids on the board. Green's pattern formation continues unchecked, rapidly filling the voids left by Grey's collapse. Grey's larger clusters continue to contract, drop below the connectivity threshold, and become voids. Green's pattern achieves a phase transition, producing a second giant component. Grey's last cluster continues to deconstruct, but briefly holds out *On the Beach* before finally being overrun by Green. The end-board is Green.

End game scenario 3:

Grey's decimated population stabilizes into several disconnected clusters that remain above the connectivity threshold. Filling the voids of Grey's collapse, Green's pattern formation achieves a phase transition, producing a giant component that engulfs Grey's stabilized clusters. As Green's population stabilizes after the phase transition, its connections reverse polarity and revert to slow-twitch links. There is a brief period of stasis that ends as Grey's population again explodes.

The cascade of new objects and their new generation of fast-twitch connections follow Grey's accepted precedents of pattern formation to quickly unite its isolated clusters and fragment Green's giant component. The board returns to the middle game pattern of thinly connected and disconnected patches of Grey and Green.

Green's decimated population stabilizes into several disconnected clusters that remain above the connectivity threshold. Filling the voids of Green's collapse, Grey's pattern formation achieves a phase transition to produce a giant component that engulfs Green's stabilized clusters. As Grey's population stabilizes in its old pattern, its connections reverse polarity and revert to slow-twitch links. There is a brief period of stasis that ends as Greens population again explodes . . . The end-board becomes a *blinker*, oscillating between predominant Grey and Green states.

End game scenario 4:

Green's counterattack is unrelenting as Grey's population continues to decline. The smaller isolated Grey clusters begin to drop below the minimum connectivity threshold and decompose to become voids on the board. Green's pattern formation continues unchecked, rapidly filling the voids left by Grey's collapse. Grey's larger clusters continue to deconstruct, its population decimated, but a handful of disconnected clusters teeter on the connectivity threshold. Green's pattern formation achieves a phase transition, producing a giant component that engulfs Grey's quasi-stable threshold clusters.

As Grey continues to collapse, a succession of three independent quasi-stable clusters break from the old precedents of pattern formation, restructuring their connections into a new pattern. This pattern holds the new property of allowing Green's connections to pass uninterrupted through a Grey cluster. Grey's new pattern formation process no longer blocks Green's pattern formation. Grey and Green are no longer engaged in a zero sum game for the surface of the board.

In the real-world, the 2-dimensional game board is a sphere. The pattern formation of natural ecosystems is modeled by Green, while Grey models the current pattern formation of human habitation. 2009 World Bank data indicates the pattern of human habitats has likely achieved the connectivity threshold of a critical state, strongly suggesting it will form the giant component of a global phase transition in the 21st century.

This is a structural system state induced by a single parameter: the physical connectivity of the pattern of human habitation. The specific pattern of habitat connectivity is a function of the accepted cultural precedents of land and ownership. The current dominant global pattern of human habitation defines partitioning earth's biosphere as a zero sum game. In its current state of connectivity, the properties associated with this pattern of human habitation are no longer dependent on the properties of the individual objects that comprise the pattern.

Human habitats require new units of synthesis to enable formation of a new pattern; a new configuration.

2.2. THE CRITICAL THRESHOLD STATE

> *Artificial life is the study of man-made systems that exhibit behaviors characteristic of natural living systems. . . . . a relatively new field employing a* synthetic *approach to the study of* life-as-it-could be*. It views life as a property of the organization of matter, rather than a property of the matter which is so organized.*
>
> (Christopher Langton, 1989)

The history of mathematics and science both reveal a transition from the investigation of seemingly disparate objects to structural investigations of object connections. These investigations focus directly on the process-pattern and pattern-property relationships of systems. With design disciplines intrinsically involved in pattern formation of unrelated objects, this evolutionary trajectory may prove to be prophetic.

Prior to 1874, mathematics consisted of the investigation of a wide range of independent and seemingly unrelated mathematical objects. Cantor's development of Set Theory and its subsequent acceptance as the unifying foundation of mathematics in the early 1900's provided a structural basis for relating these disparate objects: (Kline, 1972; Eves, 1990, Bourbaki, 1994)

> *The reduction of mathematics to set-theory was the achievement of the epoch of Dedekind, Frege and Cantor, roughly between 1870 and 1895. As to the basic notion of set (to which that of function is essentially equivalent) there are two conflicting views: a set is considered either a collection of things (Cantor), or synonymous with a property (attribute, predicate) of things. In the latter case "x is a member of the set y" in the formula $x \in y$, means nothing but that 'x' has the property 'y'.*
>
> (Hermann Weyl, 1946)

With the publication of *General Theory of Natural Equivalences* in 1945, MacLane and Eilenberg introduced the beginnings of Category Theory: a relation of mathematical objects through processes, or *morphisms*. While this has generated ongoing debate about Category Theory displacing Set Theory as 'the' foundation of mathematics, it is clear they simply address different segments of the process–pattern–properties structure. Set Theory organizes mathematical objects as pattern-property couples, while Category Theory organizes mathematics in terms of process–pattern couples.

2.3. THE CASCADE: MODELING A BRAVE NEW WORLD

> *Within fifty to a hundred years, a new class of organisms is likely to emerge. These organisms will be artificial in the sense that they will originally be designed by humans. However, they will reproduce, and will evolve into something other than their initial form; they will be 'alive' under any reasonable definition of the word.*
>
> (Doyne Farmer, 1989)

> *We are as gods and might as well get good at it.*
>
> (Edmund Leach, 1968)

Unlike a blank canvas model of artistic synthesis, observations of synthesis in the natural world have produced a model of pattern formation in science founded on the concept of a *search space*: a theoretical pre-existing universe of all possible solutions to a design problem.

Architectural design initially follows a science model of synthesis, where a problem is defined by identifying the input of a pattern formation process: the set of variables that contribute to a solution.

After this, the currently accepted practices of pattern formation diverge, with architecture relying on the distributed cognition of descriptive models and science trusting the certainty of mathematical models. James Audubon may have been the one of the last great descriptive visualization modelers in science.

In comparison to architectural modeling, scientific modeling increases the number of variables able to be included in a model, extends the local exploration of a search space to become a global exploration of all possible solutions, and offers the predictive capability of quantitative mathematics. .

Once a set of all possible variables influencing a problem are identified, the search space of all possible combinations of all possible variables can be defined as a *power set*: a set of all of the possible subsets of a set. The power set of variables of a can be structured as a configuration space. The set and power set of variables and their associated configuration space can extend beyond limited finite constructions to become infinite discrete structures. There are no boundaries to an infinite configuration space, it simply structures the endlessly creative possibilities of design. The search space of a collective architectural form can be defined and structured as an infinite discrete morphological configuration space.

The pattern formation process of design then becomes a search to discover configurations, or patterns, having the properties to match performance goals. The obvious but pragmatically incomprehensible approach is the exhaustive process of generating and analyzing every configuration in the space. Traditional optimization strategies engage in variations of this process, but are limited to problems with manageably small finite configuration spaces. (Alexander 2001) Artists' intuitive search methods work with an infinite configuration space, but artists are restricted by their cognitive load limit to local explorations, and their pattern formation process is virtually impossible to repeat or reproduce.

Evolution offers a third search option: it is an adaptive pattern formation and evaluation process that models the movement of a population through a multi-variable infinite configuration space in a search for *fitness* to quickly arrive at near-optimum configurations holding the properties that correlate to performance goals. The path, or evolutionary trajectory, of a population examines a small finite subset of an infinite configuration space. (Wright, 1932, 1988; Mitchell, 1996).

The process of evolution is a member of a set of homologous adaptive processes of pattern formation, or *adaptive configuration formation.* The common structure of the adaptive process establishes a hierarchical *parallel distributed processing* model of adaptive configuration formation, connecting the scales of individual neuro-cognitive, clustered social group, and genetic-evolutionary population adaptation.

The interlocked scales of the adaptive process enable synthesis of hierarchical configurations in a complex multi-variable infinite search space. Adaptation is the universal process of synthesis of the living world: it is the process that forms patterns, structures, or configurations. Those patterns, structures, or configurations hold properties that can be evaluated for their situated performance in meeting goals that are specific to a context.

The process of adaptive configuration formation is defined by the relationships adaptive objects establish with each other as well as the relationships they establish with non-adaptive objects. An adaptive object holds the property of adaptation:

> *the ability of self-adjustment or self-modification in accordance with changing conditions of environment or structure.*
>
> (Aseltine, 1959)

2.3.1. Predictive control

Properties are grounded in configuration, or structure, making adaptive and non-adaptive objects structurally different: adaptive objects have an *adaptive control*, or *predictive control*, mechanism; (Aseltine, 1958; Widrow, 1964; Qin, 2003; Xu-Wilson, 2009; Lee, 2011) non-adaptive objects do not. Adaptive control is based on the semantic interpretation external and internal environments, or the interpretation of current external and internal configuration states, to guide formation of the next internal configuration state. The complexity of the adaptive control mechanism varies with the level of hierarchy in an adaptive configuration, with the most basic being a digital switch: if I sense 'this', I switch on, if not I do nothing; or if I sense 'this' I switch off, else I do nothing. In adaptive configurations of higher hierarchical levels, predictive control mechanisms are models of the configuration: they are models of *self*.

The process of adaptation can be quantitatively modeled in computer-time to generate hierarchical configurations in a complex multi-variable infinite search space. It has the property of prediction. It breaks the chunking limits cognitive pattern formation. Adaptive objects in every hierarchical level use this process to form the configuration of their habitats. The model of adaptation is an *architectural* modeling process of design:

> *. . . birds do it; bees do it . . ..Let's do it . . .*
>
> (Cole Porter, 1928)

The current interpretation that our universe is probabilistic at the lowest level of hierarchical structure encountered to date infers all ascending hierarchical levels of structure are also probabilistic. The indeterminacy of hierarchical structure infers that one or both of the parameters that form hierarchical structure are indeterminate: relative location and temporal local interaction.

In the 17th century deterministic model of the universe, a force is the result of an interaction, leading to Newton's formulation of laws of motion: object at rest will remain at rest unless an outside force acts on it, and an object . . . will remain in motion in a straight line unless acted upon by an outside force. The 19th century initiated the transition to a probabilistic model of the universe with Maxwell and Boltzmann applying randomness to the interactions of individual gas particles, followed by Gibbs models of entire particle fields, or ensembles of particles: the accumulation of temporal intervals of random events produced probability distributions of possible configuration states.

The application of probability theory to Newtonian mechanics results in a model of temporal intervals of randomness that interrupt an otherwise deterministic universe, with Newton's laws of motion inviolate between the random events of object interaction. Configuration modeling follows by defining the initial condition of a set of objects and structuring the subsequent relationships of random and/or deterministic interaction events in a set of objects.

2.3.2. Adaptive configurations

If the motion of adaptive objects is indeterminate, they form indeterminate location relationships with both adaptive and non-adaptive objects. Locations of adaptive objects are probabilistic, giving adaptive objects two intervals of indeterminacy: intervals of object location and time intervals of object interaction.

Adaptive objects engage in discrete probabilistic bidirectional local interactions with adaptive and non-adaptive objects to form *adaptive configurations*:

> An adaptive configuration is defined by the set of location variables and the set of interaction variables of a union of sets of adaptive objects and non-adaptive objects. The adaptive properties of an adaptive configuration are grounded in the structure of its sets of location and interaction variables. The *initial adaptive configuration* of a union of adaptive and non-adaptive sets is defined by the set of location variables initiating the model. An *adaptive configuration space* is a structure on the set of all possible adaptive configurations formed from a union of sets of adaptive objects and non-adaptive objects: the set of all possible adaptive configurations includes an operation, or *adaptive morphism*, that transforms one adaptive configuration into another adaptive configuration. An adaptive morphism structures the neighborhood adjacency and accessibility distances of an adaptive configuration space.
>
> As the values of its variables change, an adaptive configuration will assume different *adaptive states*. An *adaptive configuration state* is the overall condition of an adaptive configuration at a specific instance in time: the sets of values for the sets of location and interaction variables of a union of adaptive and non-adaptive sets of objects at a specific instant in time. The *initial adaptive state* of a union of adaptive and non-adaptive sets of objects is defined by the set of values for location variables and the set of values for location variables initiating the model.

An *adaptive state space* is the set of all possible adaptive configuration states in a set of time: the sets of all possible values for the sets of location and interaction variables of a union of adaptive and non-adaptive sets of objects in a set of time. A temporal sequence of adaptive states traces a path in adaptive state space, defining the evolutionary trajectory, or *adaptive time-ensemble*, of an adaptive configuration.

An *adaptive phase transition* is change in adaptive state space that results in a *discrete adaptive morphism* in adaptive configuration space. For a set of adaptive objects, an adaptive phase transition is a change in an adaptive configuration state that results in a discontinuous jump to a new adaptive configuration: change in the values of the location and interaction variables at a specific instant in time that yields a discrete transformation in the sets of location and/or interaction variables:

> A full range of continuous values for the sets of location and interaction variables result in adaptive configurations of nonbinding interactions, having the properties of a gas.
>
> As the range of continuous values for the set of location variables contract to a *threshold range* and the range of continuous values for the set of interaction variables contract to become discrete, an adaptive configuration with nonbinding interactions jumps to a configuration with quasi-binding interactions, having the properties of a liquid exhibited in the dynamic swarming, flocking, and herd behaviors of animals.
>
> As the range of continuous values for the set of location variables contract to become discrete and the adaptive state values for the set of interaction variables remain discrete, an adaptive configuration with quasi-binding interactions jumps to a configuration of binding interactions having the properties of a solid.

An *adaptive phase space* is a mapping of an adaptive state space onto an adaptive configuration space. The variable values of an adaptive configuration state entering a phase transition are the values of the initial adaptive state in the initial adaptive configuration that exits a phase transition.

2.3.3. Subjective fitness

A key feature in models of adaptive processes is a *fitness function*, or *cost function*: the set of rules or evaluative criteria that provide the basis for change of a configuration. A fitness function is an adaptive morphism initiated by a predictive control mechanism. The quantitative models of adaptation used in engineering, economics, and science rely on the definition of objective quantifiable fitness functions, (Bremermann, 1962; O'Donald, 1970; Lande, 1983) and are directly applicable to the quantitative parameters of human habitat configuration formation.

But they omit the important cultural parameters of configuration formation that establish the truths of the art of architecture: the subjective non-quantifiable design parameters Isamu Noguchi distilled to:

> *'make you feel better, feel happier to be there.'*
>
> (Isamu Noguchi, 1948)

Subjective non-quantifiable fitness functions have also been used successfully in adaptive configuration formation processes. (Frazer, 2002; Ibrahim and House, 2004, Mullican, 2005). The difference between purely quantitative and dual qualitative-quantitative fitness functions in adaptive modeling suggests the traditional structure of empirical research: the control group and the experimental group. In a qualitative-quantitative modeling process, the artist acts as a culturally biased *subjective predictive control* mechanism to periodically interrupt the quantitative model of adaptation and shepherd the evolutionary trajectory of habitat configurations.

In a world of quantitative modeling, subjective fitness functions save architecture's soul.

2.3.4. Memotype-phenotype interaction

That adaption occurs simultaneously in interactive hierarchical scales necessitates simultaneous modeling in multiple interactive hierarchical scales: a situated local phase transition in a hierarchical configuration produces the next hierarchical level of configuration. With genetic-evolutionary adaptation existing simultaneously in individual and population scales, a minimum adaptive modeling requirement would include both levels of hierarchy in a parallel model structure.

The subsets of adaptive physical clusters that compose the physical morphology of an adaptive object define it as a *phenotypic configuration*. The distributively embedded information structure used to construct and maintain a phenotypic configuration is composed of subsets of information clusters that define it as a *genotypic configuration*. As a cultural artifact in a human habitat, this embedded information structure would be re-defined by biologist Richard Dawkins as a *memotypic configuration*:

> *The new soup is the soup of human culture. We need a name for the new replicator, a noun that conveys the idea of a unit of cultural transmission, or a unit of imitation. 'Mimeme' comes from a suitable Greek root, but I want a monosyllable that sounds a bit like 'gene'. I hope my classicist friends will forgive me if I abbreviate mimeme to* meme.
>
> (Richard Dawkins, 1976)

2.4. IT'S ALIVE! (the zero-sum game continued)

In one of the quasi-stable threshold clusters approaching collapse, a monk in the Grey *Albertian Order of Leibowitz* experiences random electronic encounter on a still partially connected fragment of the old world's giant electronic component. Recovering from initial disbelief, she forwards the link to fellow oblates in two small remote Grey clusters housing the Order. Her dim display reveals a garbled but partially readable electronic file, *Situs identitatem arkhitektura et spatium conformation*, describing a process of pattern formation that allows Green's connections to pass uninterrupted through a Grey cluster:

> Conceptually treating 3-dimensional space as the simple projection of a closed boundary on a 2-dimensional surface produces a configuration with the properties of a solid. With the properties of a solid, human habitat configurations reduce the interaction with other ecosystem habitats to a zero sum game. Co-evolution requires a more complex partitioning of 3-dimensional space to form a different configuration.
>
> Disjoint building-objects partition 3-dimensional space into two discrete regions. Another configuration that partitions space into two discrete regions but has very different properties is expressed in the 3-dimensional labyrinth of solid and void exhibited by individual corrals in their formation of reef structures. The equivalent regular periodic structures of space partitioning were notably explored by inorganic chemist A.F. Wells, (Wells,1970,1977) with architect Michael Burt later defining the regular and semi-regular geometric surfaces of polyhedral configuration space. (Burt, 1967, 1996) Even more complex partitions of 3-dimensional space are possible, but unlike the surface boundaries of corral manifolds that partition space into two interpenetrating regions, the 2-dimensional surfaces of these configurations partition 3-dimensional space into at least seven interpenetrating bounded continuous regions. (Hyde, 2003)
>
> Deformed in response to local seasonal solar trajectories, the bounded regions of these 3-dimensional infinite weavings of partitioned continuous space can be used as the infrastructure for human habitation: some fully occupied, some partially occupied, and some unoccupied by human presence. Regions accessible to humans integrate the subsystems of enclosed human habitation, habitat service space, connective transportation, urban parks, and urban agriculture. Regions inaccessible to humans provide uninterrupted connectivity of natural systems, while continuous voids allow every occupied region to have direct surface contact with the local climate. With its extensive surface area collecting incident water and solar radiation, these configurations hold the inherent properties necessary to maintain a human population and its natural world life support system in a dynamic co-evolutionary state of equilibrium.

While not derived from carbon-based organic life as we know it, the enclosed region of human habitation will be alive in every sense; growing, adapting, dying, and perhaps even reproducing in symbiosis with human inhabitants and environment. Habitat constructions will become hierarchically interacting structures of fully autonomous discrete spatial partitions that are embedded with distributed systems of sensors and *predictive control* modules. Just as every physical living cell contains its own adaptive information structures, our physical habitat structures will transcend simple static physical form to contain its genotype, or more precisely, its *memotype*: the locally embedded distributed information structures that define 'self' and allow for a situated adaptive physical response to changing human behaviors, environmental forces, and cultural mores. Locally adaptive, self-regulating, self-healing, and potentially self-aware, this is a portrait of the house as a young dog.

It was a grand AH-WHOOM.

(Kurt Vonnegut, 1963)

As Karl Gauss first observed in 1847, quantitative mathematical models are the only models to hold the property of prediction. (Gauss, 1847)

3. SITUATED STRUCTURAL MODELING

We're off to see the wizard, The Wonderful Wizard of Oz
We hear he is a whiz of a wiz, if ever a wiz there was
If ever, oh ever a wiz there was, The Wizard of Oz is one because
Because, because, because, because, because
Because of the wonderful things he does

(H. Arlen, E.Y. Harburg, 1939)

Johnny, rosin up your bow and play your fiddle hard.
'Cause Hell's broke loose in Georgia and the Devil deals it hard.
And if you win you get this shiny fiddle made of gold,
But if you lose the devil gets your soul.

(Charlie Daniels, 1979)

Never trust anything that can think for itself if you can't see where it keeps its brain.

(J.K. Rowling, 1999)

What if, in search of prediction, design of the human habitat turned from its history of art-based descriptive models to adopt multi-variable quantitative models of pattern formation? What are the precedents for quantitative modeling? What are their origins? Their limitations?

To buy-in, do I have to sell out?

(John Careatti, 1984)

3.1. OZ AND THE MAN BEHIND THE CURTAIN: MODELING PRECEDENTS

Cum Deus calculat et cogitationem exercet, fit mundus.

-When God calculates and exercises thought, the world is made.-
(Gottfried Wilhelm von Leibniz, 1677)

All these things being considered, it seems probable to me, that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, moveable Particles, of such Sizes and Figures, and with such other Properties, and in such Proportion to Space, as most conduced to the End for which he form'd them.

(Isaac Newton, 1704)

3.1.1. 17th century: God's universal truth

With few exceptions, the precedents in mathematics and science established in the 17th century remained unquestioned through the end of the 20th century. Newton, Leibniz, Fermat, Pascal, Huygens, and Bernoulli, among others, abandoned Descartes' prevailing descriptive models of hierarchical structure to develop pattern-property models based on the quantitative linear relationships of atomistic non-hierarchical collections. Guided by the deeply held belief in a singular supreme creator, they assumed the viewpoint of an omniscient external observer to construct new mathematics on the foundations of ancient Greek culture: the atomistic philosophy of fundamental indivisible elements attributed to Leucippus and Democritus in the 5th century BCE and a top-down deductive method of inquiry dating to Aristotle in the 4th century BCE. (Kline, 1953; Boyer, 1968)

Continuing the Aristotelian tradition in his 1666 *Dissertatio de arte combinatorial*, Gottfried Leibniz took the first step toward his dream of a universal language for deductive reasoning, resuming in 1679 to begin outlining the basic concepts of *symbolic logic*. (Eves, 1958) In the preface of his 1678 *Treatise on Light*, Christiaan Huygens introduced the now ubiquitous *hypothetico-deductive method*, or at the time, simply 'the method of hypothesis':

> *One finds in this subject a kind of demonstration which does not carry with it so high a degree of certainty as that employed in geometry; and which differs distinctly from the method employed by geometers in that they prove their propositions by well-established and incontrovertible principles, while here principles are tested by the inferences which are derivable from them. The nature of the subject permits of no other treatment. It is possible, however, in this way to establish a probability which is little short of certainty. This is the case when the consequences of the assumed principles are in perfect accord with the observed phenomena, and especially when these verifications are numerous; but above all when one employs the hypothesis to predict new phenomena and finds his expectations realized.*
> (Christiaan Huygens, 1678)

3.1.1.1. continuous determinism

Taking up a range of geometry problems that had preoccupied mathematicians in the early part of the 17th century, Isaac Newton and Gottfried Leibniz independently recognized the calculation of area and the calculation of tangents were inverse operations, inaugurating concepts of continuous mathematics and establishing the basis of integral and differential calculus.

In letters to the Royal Society of London, Newton first publicly documented his thinking in 1676, and published portions of his work in his 1687 *Philosophiae Naturalis Principia Mathematica*. His manuscript *Method of Fluxions*, completed in 1671, was finally published in 1736. Unaware of Newton's effort, Leibniz began working on similar problems in 1672, publishing *Nova methodus pro maximis et minimis* in first German scientific journal, *Acta Eruditorum*, in 1684. Their results produced very different algorithms to deal with variable quantities, with Newton structuring change over time and Leibniz constructing differences in a sequence of infinitely close values. In conceptualizing an *infinitesimal*, Leibniz effectively shrunk discrete atomistic elements to their vanishing point, creating an unbroken continuum of real numbers, or points in a line. (Wilson, 1982)

Newton's coupling of continuous linear mathematics with the concept of continuous linear nature of time produced the first quantitative predictive models of physical reality, underpinning deterministic and stochastic models in science for nearly three centuries. As a model of causation, the prevailing dogma of theological determinism was challenged by the new quantitative 'truth' of mechanical determinism. The scientific belief in *catastrophism*, with its roots in the Judeo-Christian creation story, was displaced by another 'truth' of continuous mathematics: the inherent *gradualism* of incremental change.

3.1.1.2. chances of continuous determinism

Although the oral and written traditions of gambling found in all cultures reveal empirical origins of probability dating to at least 3500 BCE, a series of letters between Blaise Pascal and Pierre de Fermat in 1654 are generally acknowledged as inaugurating combinatorial probability as a branch of mathematics. (David, 1955, 1962; Kendall. 1956; Maistrov, 1974; Green, 1981;Bellhouse, 1993; Batanero, 2005)

Christiaan Huygens, a founding member and mentor of Leibniz at the Académie Royale des Sciences in Paris, published the first monograph on mathematical probability in 1656, with his *Van Rekeningh in Speelen van Geluck* remaining the definitive reference into the next century. (David, 1955, 1962; Kendall. 1956; Maistrov, 1974) Pascal's own treatise, *Traite du triangle arithmetique, avec quelques autres petits traits sur la meme matiere*, was completed in 1654, but not circulated until 1665. (Bernstein, 1996; Adamson, 2005; Cooke, 2005)

James Bernoulli began work on probability in 1684, with *Ars Conjectandi* published in 1713, eight years after his death, outlining a comprehensive foundation of modern probability theory. Bernoulli provided theorems and proofs to formalize the combinatorial work of Pascal and Huygens, and developed the concept of *the law of large numbers*: as the number of observations increases, the relative frequency of an empirical event will approach the 'truth' of the theoretical probability of its occurrence. The final section of *Ars Conjectandi* also introduced the new concept of *subjective probability*: a situated quantification of certainty-uncertainty of belief in a statement's truth. (Bernoulli, 1713)

While the 17th century produced the seeds that would later flower into accepted belief of a stochastic universe, the founders of mathematical probability retained a strictly deterministic worldview. (Sylla, 1998; Adamson, 2005) For them, probability was a comprehensive and consistent method of reasoning to be used when faced with insufficient information to formulate a deterministic model of events:

> *Given a certain position of a die, [its] velocity and distance from the board at the moment when it leaves the thrower's hand, it cannot fall otherwise than it actually does. … So these phenomena take place owing to their immediate causes with no lesser necessity than the phenomena of the eclipses follow from the movement of the heavenly bodies. And still, usually only the eclipses are ranked among necessary phenomena whereas the fall of a die and the future weather are thought to be contingent.*

> *The sole reason for this is that what is supposed to be known for determining future actions, and what indeed is such in nature, is not enough known. And, even had it been sufficiently known, geometrical and physical knowledge is inadequately developed for subjecting such phenomena to calculation in the same way as eclipses can be calculated beforehand and predicted by means of known astronomical principles. And, for the same reason, before astronomy achieved such perfection, the eclipses themselves had to be reckoned as future chance events.*
>
> (Jakob Bernoulli, 1713)

3.1.1.3. God's eye

> *For over 2000 years, the intellectual world, of which mathematics are a part, accepted Aristotle's logic. It is true that Descartes, who questioned all beliefs and doctrines, did raise the question of how we know that the principles of logic are correct. His answer was that God would not deceive us. The assurance we possess of the correctness of these principles was thereby justified to him.*
>
> (Morris Kline, 1980)

Until the latter part of the 20th century, historians modeled 17th century Europe as a culture of conflict, harmony, or intellectual segregation between non-overlapping secular and religious domains. (Shea, 2007) These longstanding models reflected the cultural bias of an external observer from the modern era, built on the implicit assumption that the cultural objects of 'science' and 'religion' are invariant with respect to time and place. The dichotomy is perfectly encapsulated by the title of Massimo Mazzotti's 2007 review, *The two Newtons and beyond*. (Mazzotti, 2007)

A situated investigation of history assumes a point of view from within a culture in time, with current research recognizing the Judeo-Christian roots of western mathematics and science:

> *Western science grew out of Christian theology. It is probably not an accident that modern science grew explosively in Christian Europe and left the rest of the world behind. A thousand years of theological disputes nurtured the habit of analytical thinking that could be applied to the analysis of natural phenomena. . . . The common root of modern science and Christian theology was Greek philosophy*.
>
> (Freeman Dyson, 1998)

Theological truth structured scientific investigation and by turn, new truths in mathematics and science were interpreted for theological significance. (Davis, 2002) Although there was unanimity in the dual threads of truth, weaving a fabric of belief exhibited considerable variation, as contrasted by Leibniz and Newton's differing theological implications of their mathematics.

A Christian believer, Gottfried Leibniz theorized God to be the most perfect rational being, with reason forming the common bond between God and man, differentiating man from the remainder of creation. For the divine creator-mathematician that calculated all possible worlds before selecting the one with maximum harmony, continuous intervention in the world was both unnecessary and unworthy. Yet the divine law embedded in a perfect creation was 'mostly not understood by the creatures within which it inheres', making God's intermittent intervention in the world vital. For Leibniz, exploring mathematics was working in the service of God; his universal harmony of *diversitas identitate compensate* parallels his concept of the Christian Trinity. (Garber, 1998; Antognazza, 2003; Breger, 2005; Caro, 2012)

For Isaac Newton, God was continually involved in the world, with divine causation acting through the structures of creation, such as gravity. Mathematics provided the tools to decipher the ideas of God as they were expressed in the laws of nature. Believers of Newtonian natural philosophy saw no conflict between unravelling God's message in the parallel truths of the Book of nature and the Bible, with both requiring devout study and insight (Hall, 1979; Force, 1981; Davis 2002; Shapiro, 2004; Markley, 2005):

> *Natural philosophy as such was a discipline and subject-area whose role and point was the study of God's creation and God's attributes. Thus, no-one ever undertook the practice of natural philosophy without having God in mind, and knowing that the study of God and God's creation — in a way different from that pursued by theology — was the point of the whole exercise.*
>
> (Andrew Cunningham, 1991)

Leibniz and Newton's new mathematics of continuous determinism and the development of probabilistic inference of continuous determinism marked an abandonment of the internal truth of mathematics as a platonic ideal unrealizable in physical reality. While the nascent calculus did lack the solid deductive foundation necessary for internal truth, mathematics was now developed by a culture of observation of the natural world, its truth confirmed by congruence to an external physical reality. (Kline, 1972) To a present day external observer, the 17th century empiricist confirmation of absolute mathematical truth presents a *logical woozle*:

> Mathematics is developed from observations of the natural world.
>
> As the unwritten word of God, the Book of nature is an absolute truth.
>
> If mathematics is isomorphic to the natural world, it is isomorphic to the absolute truth of God.
>
> Therefore, mathematics that is isomorphic to the natural world is an absolute truth.

Deviating from a church doctrine of God's direct causation, the consensus of 17th century scientists accepted Descartes' view of mechanistic causation while sharing his deeply held belief in a supreme creator. (Torrance, 1972; Smarr, 1985; Ramati, 2001) For Newton, Leibniz, Pascal, Fermat, Huygens, and Bernoulli, a singular omniscient being structured the universe with mathematics and continued to maintain, directly or indirectly, an integral position of omniscient central controller. By literally assuming a 'God's-eye view' as an external observer and deciphering the mathematics of the natural world, man may gain privilege in sharing the creator's divine vision:

> *And God said, Let us make man in our image, after our likeness: and let them have dominion over the fish of the sea, and over the fowl of the air, and over the cattle, and over all the earth, and over every creeping thing that creepeth upon the earth.*
>
> *So God created man in his own image, in the image of God created he him; male and female created he them.*
>
> (King James Bible, Genesis 1:26-27)

3.1.2. 18th century: return of discrete structure

Following his creation of continuous functions, Leibniz proposed an entirely different direction for mathematics in a 1679 letter to Huygens:

> *I believe we lack another analysis, properly geometric or linear, which expresses location directly as algebra expresses magnitude.*
>
> (Kline, 1972c)

Like his concept for a universal logic, Leibniz' proposed study of *geometria situs* or *analysis situs* was only partially developed and contained few examples, but he articulated a vision of a mathematics of structural properties more fundamental than those measurable by geometry. (Hopkins, 2004; Zaytsev, 2008)

In 1736, Leonhard Euler realized this vision with the publication of *Solutio problematis ad geometriam situs pertinentis*, or "The solution of a problem relating to the geometry of position", now widely known as the problem of the 7 bridges of Königsberg. It is ironic that in this single short paper of 13 pages, one of the most prolific mathematicians in history inaugurated the entirely new fields of graph theory and topology: qualitative models of discrete structure; the mathematics of pattern.

Graph theory builds a structural representation of discrete points connected, or not, by lines to show paired relationships of objects in their relative location. Topology defines the properties of structure or pattern that remain unaltered by continuous geometric distortion. Although the deformation processes under consideration are continuous and exclude 'tearing' a surface, the properties that remain invariant throughout a pattern deformation, such as the number of 'holes' in an object, are themselves discrete.

3.1.3. 19th century: indeterminism and discontinuity

3.1.3.1. evolution of indeterminate models of material structure

In the early 18th century, Jakob Bernoulli's posthumous publication of *Ars Conjectandi* and Abraham de Moivre's *The Doctrine of Chances* outlined a sound mathematical basis of probability. Statistics in that time was unrelated to probability and consisted largely of ad hoc treatments of data collected by city--states, bereft any predictive ability. (Kendall, 1956; Bernstein, 1996)

With an 1812 publication that remained influential until the beginning of the 20th century, Pierre Simon Laplace' *Théorie analytique des probabilities* consolidated these separate endeavors by introducing probability as a rigorous foundation for statistical inference. (Molina, 1930; David, 1955) Despite 100 years of established mathematical foundations of probability and his broad application of statistical models, Laplace retained a firm belief in Descartes' deterministic universe:

> *We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated, . . . it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.*
>
> (Pierre Simon Laplace, 1814)

For Laplace, probability and statistics were simply methods to approximate what is not known but presumed to be knowable. (David, 1962; Cooke, 2005; Pepin, 2012) It would take almost another 100 years and the development of comprehensive probabilistic models of physical structure for scientific determinism to be seriously challenged.

Shortly after the publication of *Ars Conjectandi*, Jakob's nephew, Daniel Bernoulli, outlined the basis of the kinetic theory of gasses: the observable physical properties of a body of gas, such as temperature and pressure, were the result of the disorderly movement of discrete microscopic particles. (Stigler, 1990) Even with the belief that, at equilibrium, all of the particles moved at the same speed, the independence of individual particles presented an insurmountable problem for the accepted deterministic Newtonian models of motion, inhibiting the development of quantitative models of kinetic gas theory as well as broader efforts investigating the relationship between the patterns and properties of materials.

In 1856, countering the prevailing astronomical belief that the rings of Saturn were a fluid, James Clerk Maxwell published a model showing how the spinning motion of discrete bodies could appear visually united as fluid flow. (Maxwell, 1855) This lead to his interest in the thermodynamic problems presented by the kinetic theory of gases, and the unconventional realization that, as a result of their collisions, not all particles in a volume of gas at equilibrium moved at the same speed/velocity.

From his familiarity with Laplace' writings, *Théorie analytique des probabilities*, Maxwell recognized the success of probability theory in investigating the properties of unstructured collections of a large number of similar objects, and considered this radically different approach to the variation of individual particle velocities in kinetic gas theory. With his 1860 publication of *Illustrations of the Dynamical theory of gasses*, he presented the first non-deterministic model of material structure: a probability distribution of the varying particle velocities in an ideal gas in thermal equilibrium. (Maxwell, 1860)

In 1866, Ludwig Boltzman published a Newtonian model of kinetic gas theory. After reading criticisms of Maxwell's 1860 paper, Boltzmann responded with a mathematically sound derivation of his own that confirmed Maxwell's probability distribution of particle speeds. (Porter, 1986; Renn, 2008) He then continued with an extensive series of papers, extending the probabilistic gas model beyond the ideal state of mono-atomic particles considered by Maxwell and constructing a probabilistic basis of the second law of thermodynamics: systems always evolve toward thermodynamic equilibrium, the state of maximum disorder, or maximum entropy. With his work, equilibrium is now conceived of as the most probable state of a system and no longer seen as a stationary state. (Renn, 2008; Uffink, 2006)

After a prolific career establishing that systems naturally evolve from less probable to more probable states, Boltzmann somehow retained a core belief in determinism. His lectures and writings toward the end of his life are surprisingly explicit:

> *. . . present day atomism is a perfectly apt picture of all mechanical phenomena, and given the closed nature of this domain we can hardly expect it to throw up further phenomena that would fail to fit into that framework. Indeed, the picture includes thermal phenomena: that this is not so readily proved is due merely to the difficulty of computing molecular motions.*
>
> (Boltzman, 1897)

> *. . . I am the only one left who still grasped the old doctrines with unreserved enthusiasm – at any rate I am the only one who still fights for them as far as he can. I regard as my life's task to help to ensure, by as clear and logically ordered an elaboration as I can give of the results of classical theory . . .* (Boltzmann, 1899)

> *In nature and art the all-powerful science of mechanics is thus ruler, . . . The god by whose grace kings rule is the fundamental law of mechanics.*
>
> (Boltzmann, 1900)

Boltzmann, along with 200 years of scientific determinism, did not go gentle into that good night.

In 1873, J. Willard Gibbs published two papers, developing both 2 and 3 dimensional phase diagrams: graphical models of the relationship between the thermodynamic properties and patterns of material structure. He followed this with the 1876-1878 publication of *On the Equilibrium of Heterogeneous Substances*, effectively defining the field of classical thermodynamics with a treatment that remains little modified to this day. This work delineates his theories of the phases of matter and introduced the concept of a phase transition: each individual substance is a component of a system, each equilibrium state of a system is a different phase, and a phase transition is a discontinuous transformation between the equilibrium states of a system. (Maistrov, 1967; Kadanoff, 2009)

In 1884 he published a paper that initiated the term *statistical mechanics*, differentiating the probabilistic models of Maxwell and Boltzmann from the traditional deterministic model of Newtonian mechanics. By 1892 he was working on a comprehensive treatment of this new field:

> *Just now I am trying to get ready for publication something on thermodynamics from the a priori point of view, or rather on 'statistical mechanics' [...] I do not know that I shall have anything particularly new in substance, but shall be contented if I can so choose my standpoint (as seems to me possible) as to get a simpler view of the subject.*
>
> (Gibbs, 1892)

Maxwell and Boltzmann limited the applicability of their results to dilute gasses by modeling the probabilities of independent particles: solid and liquid states of matter are comprised of strongly interacting particles that cannot be regarded as statistically independent. Gibbs generalized their work and extended it to all states of matter by developing a probabilistic model based not on the state of a particle, but on the state of the entire system of particles. (Kadanoff, 2014) This work culminated with the 1902 publication of *Elementary Principles of Statistical Mechanics*:

> *We may imagine a great number of systems of the same nature, but differing in the configurations and velocities which they have at a given instant, and differing not merely infinitesimaily, but it may be so as to embrace every conceivable combination of configuration and velocities. And here we may set the problem, not to follow a particular system through its succession of configurations, but to determine how the whole number of systems will be distributed among the various conceivable configurations . . .*
>
> (J. Willard Gibbs, 1902)

Based on his previous work of 1876-1878 that deduced a system of particles in equilibrium could be in a number of different states, Gibbs developed the concept of an *ensemble*: a set of system states containing each possible state that a particle system might assume. An ensemble is the micro-state space, or the configuration space of a system. Keeping the number of particles in the model constant by the selection of one or more extensive properties, he calculated the probability that a system might assume each of these possible states, constructing a *statistical ensemble* as the probability distribution for the configuration space of a system in equilibrium. (Gibbs, 1902)

3.1.3.2. evolution of discontinuous mathematical structure

The development of probability and statistics and their subsequent incorporation into physics, with probabilistic models of material structure reaching a critical threshold of at the end of the 19th century, is paralleled by the evolution of continuous mathematics. The synergy of developments in mathematics and physics initiated by Newton and Leibniz also achieved a critical threshold at the end of the 19th century with the inclusion of discontinuity in continuous mathematical structure.

Colin Maclaurin was appointed to the chair of mathematics at the University of Edinburgh on Newton's letter of recommendation of 1726 and published the first systematic treatment of Newton's calculus in 1742. In his *Treatise of fluxions*, Maclaurin introduced several mathematically sound derivations and demonstrated a diversity of applications with the construction of mathematical models employing empirical data.

The gradualist concept of continuity stemming from Newton's fluxions is apparent in the work of James Hutton, a student of Maclaurin's at Edinburgh and now considered the founder of modern geology. Hutton's 1785 lectures *Concerning the System of the Earth, Its Duration, and Stability* to the Royal Society of Edinburgh refuted the accepted estimates of the age of the earth derived from theology: the observable geological formations of the present were the result of the slow accumulation of incremental change over incredibly long periods of time. Niles Eldredge succinctly summarized geological gradualism in 2006:

> *. . . however joltingly abrupt any earthquake may be, it takes a lot of them regularly interspersed through lots of geological time to make a mountain chain out of a seafloor.*
>
> (Niles Eldredge, 2006)

In the early 1830's, Hutton's gradualism was incorporated by Charles Lyell in his influential *Principles of Geology*, which in turn was instrumental in Charles Darwin's 1859 conception of gradual incremental change in the evolution of life forms: *The Origin of Species*.

While the gradualist concept of incremental continuity embedded in the calculus of Leibniz and Newton spread to other disciplines, the concept of continuity itself was changing within mathematics as the concept of a mathematical function evolved.

With Leonhard Euler's 1748 publication of *Introductio in analysin infinitorum*, the interrelated concepts of function and continuity became a central focus in the development mathematics. In 1755, Euler followed with *Institutiones calculi differentialis*, giving a modern general definition of a function as a one-to-one correspondence:

> *If some quantities so depend on other quantities that if the latter are changed the former undergoes change, then the former quantities are called functions of the latter. This definition applies rather widely and includes all ways in which one quantity could be determined by other. If, therefore, x denotes a variable quantity, then all quantities which depend upon x in any way, or are determined by it, are called functions of x.*

He restricted his definition by examining only functions of different types of well-established mathematical operations but retained a global concept of continuity not dissimilar to that of Leibniz and Newton: continuity was a property of the entirety of a function of a single analytical expression. Euler's identification of a function and its global continuity with an analytical expression remained largely intact until the early 19th century when the problems encountered in the physics of particle mechanics, equilibrium states, and material properties stimulated successive expansions.

Bernard Bolzano's 1817 *Rein analytischer Beweis des Lehrsatzes, dass zwischen je zwey Werthen, die ein entgegengesetzes Resultat gewähren, wenigstens eine reele Wurzel der Gleichung liege* and Augustin-Louis Cauchy's 1821 *Cours d'Analyse de l'École Royale Polytechnique* departed from Euler's interpretation of continuity as a whole-function global property to define continuity as a local property specific to an interval of a function. Cauchy's work extended the operation of integration to include functions having a finite number of discontinuities on a bounded interval, giving a definition of continuity that conforms to the generally accepted meanings of continuous and discontinuous used today:

> *In other words, the function f(x) remains continuous with respect to x in a given interval, if an infinitesimal increase in the variable within this interval always produces an infinitesimal increase in the function itself.*

Cauchy's work of 1821 also introduces a key refinement in the definition of a function that would prove to have a lasting effect on mathematical modeling:

> *When variable quantities are related so that, given the value of one of them, one can infer those of the others, we normally consider that the quantities are all expressed in terms of one of them, which is called the independent variable, while the others are called dependent variables.*

Bernhard Riemann's 1854 lecture *Ueber die Darstellbarkeit einer Function durch eine trigonometrische Reihe* extended the process of integration to include functions having an infinite number of discontinuities, thereby reducing the local measure of discontinuity from Cuachy's intervals to sets of points and precipitating the modern term of *pointwise* connectivity. Riemann's work stands as the first systematic investigation of discontinuous functions.

Throughout the 19th century the mathematical concepts of continuous and discontinuous reflected the expanding abilities to calculate derivatives and integrals of functions, evolving to tame discontinuity with the expanding apparatus of continuous mathematics.

The end of the century is marked by Henri Lebesgue's work to establish the most stringent manifestation of continuity, *absolutely continuous,* where a function is the integral of its derivative and Rene-Louis Baire's extending the examination of continuity and discontinuity to the convergence of a sequence of functions.

In the 1904 publication of *Leçons sur l'intégration et la recherche des fonctions primitives*, Lebesque writes:

> *If we wished to limit ourselves always to these good [that is, smooth] functions, we would have to give up on the solution of a number of easily stated problems that have been open for a long time. It was the solution of these problems, rather than a love of complications, that caused me to introduce in this book a definition of the integral that is more general than that of Riemann and contains the latter as a special case.*

Baire follows in 1905 in his *Lecons sur les functions discontinues*:

> *Is it not the duty of the mathematician to begin by studying in the abstract the relations between these two concepts of continuity and discontinuity, which, while mutually opposite, are intimately connected?*

3.1.4. 20th century: the modern synthesis; properties of modeling precedents

Mathematics and science themselves are simply large patterns of information that conform to the structuralist subsets of process-pattern-properties-performance and exhibit the characteristics of complex systems: each are sensitive to initial conditions or assumptions, lack a central controlling agent, and are predicated on local interactions percolating to a critical threshold to produce emergent systemic change. While they are not Gibbs closed systems of equilibrium states, undergoing phase transitions, they are the open systems of von Bertalanaffy in homeostatic states experiencing phase transitions.

In this light, the late 17th century can be seen as a phase transition in mathematics and science initiated by the predictive ability of the continuous mathematics introduced by Newton and Leibniz. The 18th-20th centuries is then seen as a period of stasis, where advancements were largely incorporated into the established framework, and challenges to accepted precedents, the most notable being indeterminacy, remained below a critical threshold. However, the end of the 20th century is witness to a change of focus in science, with attention shifting from the properties of objects examined as linear relationships, to properties resulting from the nonlinear relationships between objects: networks and complex systems; patterns, or configurations. The investigation of relationship configurations presents new set of modeling problems that, to date, have proven to be not amenable to solution. The grounding of properties in the nonlinear relationships of objects and not in the objects themselves represents a serious challenge to the accepted precedents of current mathematics and science and suggests an impending 21st century phase transition.

The current state of math and science, based on precedents established from the 17th through 19th centuries, are evident in the 3 dimensions of accepted 20th century mathematical modeling:

continuous – discrete

quantitative – qualitative/categorical-qualitative

deterministic – stochastic

While these dimensions are often used specifically to categorize scientific models, they also provide a useful framework to examine the underlying mathematics used to construct models. They are properties of mathematical structures that can be used to partition the body of current mathematics into three sets: continuous, discrete, and stochastic.

The discontinuities between continuous, discrete, and stochastic mathematics have resulted in a mélange of approaches to 20th century scientific modeling. Continuous and stochastic mathematics define change, thereby enabling prediction, but remain unmindful of pattern. Discrete mathematics defines pattern, but remains unmindful of change, thereby precluding prediction.

Bridging the discontinuities between continuous, discrete, and stochastic mathematics and deeply embedded in scientific modeling, two precedents that were effectively handed down from God in the 17th century have remained sacrosanct in the 20th: the objective truth of mathematics and the omniscient objective external observer.

3.2. A PROMISE OF PARADISE: STRUCTURING UNIVERSAL TRUTH

> *No one shall be able to drive us from the paradise that Cantor created for us.*
>
> (David Hilbert, 1926)

3.2.1. Logical foundations of universal truth

Georg Cantor's 1874 set theory presented a unifying structure that encompassed the previously unrelated branches of mathematics to provide a foundation for mathematics that remained unquestioned for nearly a century. (citation) Using Cantor's set theory as a blueprint, Gottleib Frege resurrected Leibniz' unrealized idea of a universal logic with his attempt to derive a complete set theoretic restructuring of mathematics using only constructions of formal logic. (Kline, 1980; Kline, Vol.3, 1982; Stillwell, 2010) In 1902, with Volume 1 of his *Grundgesetze der Arithmetik* already in print and Volume 2 delivered to the publisher, Frege famously received a letter from Bertrand Russell revealing a logical inconsistency in the fundamental structure of set theory, now known as Russell's paradox (Stewart, 1975; Davis, 1981; Weisstein, 2015):

> *Let* $\boldsymbol{R}$ *be the set of all sets which are not members of themselves. Then* $\boldsymbol{R}$ *is neither a member of itself nor not a member of itself.*
>
> Russell often used a story about a village barber to put the structural problem in popular form (Eves, 1958; Kline, Vol.3, 1982):
>
> *A village barber states that he shaves all in the village who do not shave themselves. Who shaves the barber?*

This, and other emerging contradictions, prompted Russell's exhaustive work with Alfred Whitehead from 1910-1913 to produce a three volume landmark in propositional logic to structure mathematics: *Principia Mathematica*.

> *I wanted certainty in the kind of way in which people want religious faith.*
> (Russell, 1956)

3.2.2. Axiomatic foundations of universal truth

Understanding that contradictions were to be expected in a young and evolving discipline, David Hilbert suspected the inconsistencies were rooted in the logicist approach to mathematical unification, where a truth value is structured as one-to-one relationship between a mathematical object and a concept. (Giaquinto, 1983; Cory, 1997; Cantini, 2009) Hilbert understood truth to present a more complex structure:

> *. . . a concept is really fixed in a net with other concepts and this net is specified by the axioms; so only the consistency of the axioms that define the concept grants the legitimacy of [a mathematical concept].*
> (Cantini, 2009)

Using Cantor's blueprint and treating a proof itself as a mathematical object to be studied by mathematical methods, Hilbert began an extensive research program in the early 1920's to restructure mathematics using the formal axiomatic techniques of mathematics, conceiving *metamathematics*:

> *The chief requirement of the theory of axioms must go farther [than merely avoiding known paradoxes], namely, to show that within every field of knowledge contradictions based on the underlying axiom-system are absolutely impossible.*
> (David Hilbert, 1918)

> *For logical deduction to be certain, we must be able to see every aspect of these [extralogical concrete] objects, and their properties, differences, sequences, and contiquities must be given, together with the objects themselves, as something which cannot be reduced to something else and which requires no reduction. This is the basic philosophy which I find necessary, not just for mathematics, but for all scientific thinking, understanding, and communicating. The subject matter of mathematics is, in accordance with this theory, the concrete symbols themselves whose structure is immediately clear and recognizable.*
> (David Hilbert, 1925)

Hilbert's program initiated the transition in mathematics from the external verification of truth back to an internal verification: the completeness and consistency of the formal axiomatic system. Internal consistency requires that no logical contradiction can be deduced from the axiomatic system: a proposition A and its negation $not\,A$ cannot both be derived from the same set of axioms. (Kleene,

1952) Internal completeness requires that all possible assertions that can be deduced from a set of axioms are provable by that set, an additional independent axiom cannot be added that is consistent with the set. (Kleene, 1952; Kline, 1972c)

The precedents of consistency and completeness remain the accepted standards for mathematical truth, with accepted truth in empirical science a close parallel to mathematics: consistency translates to empirical repeatability and reproducibility while completeness translates to empirical causality and correlation.

> *The goal of my theory is to establish one and for all the certitude of mathematical methods.*
>
> *. . . Insight is procured for us by a discipline which comes closer to a general philosophical way of thinking . . . This discipline, created by Georg Cantor, is set theory. . . . the finest product of mathematical genius and one of the supreme achievements of purely intellectual human activity.*
>
> *. . . Admittedly, the present state of affairs where we run up against the paradoxes is intolerable. Just think, the definitions and deductive methods which everyone learns, teaches, and uses in mathematics, the paragon of truth and certitude, lead to absurdities! If mathematical thinking is defective, where are we to find truth and certitude?*
>
> (David Hilbert, 1925)

3.3. PARADISE LOST: THE EMERGENCE OF LOCAL MATHEMATICS

> *It is to the writer's continuing amazement that ten years after Godel's remarkable achievement current views on the nature of mathematics are thereby affected only to the point of seeing the need of many formal systems, instead of a universal one. Rather has it seemed to us to be inevitable that these developments will result in a reversal of the entire axiomatic trend of the late 19th and 20th centuries, with a return to meaning and truth.*
>
> (Emil Post, 1941)

3.3.1. Multiple truths: non-Euclidean geometries

For more than 2000 years of western civilization, mathematics was believed to be the accurate description of physical phenomena that illuminated the underlying design of the universe. Mathematics was held as the embodiment of absolute truth about the external natural world. (Gray, 2006)

Secure in this belief, mathematics in the 17th and 18th centuries all but neglected the early Greek methods of deductive proof and continued develop on pragmatic and empirical grounds. Euclid's axioms outlining the properties of physical space were held as a triumphal exemplar of the attainment of absolute truth, with the defining axiom eventually revealed to be Euclid's 5th, or parallel, postulate: given a straight line and a point not on that line, one and only one straight line can be constructed through that point that never intersects the first line, even if they are infinitely extended.

The first strike against mathematics and logic embodiment of absolute truth was delivered between 1812 and 1823 by the formulation of alternative, non-Euclidean, geometries. By the mid-18th century, a handful of mathematicians had come to the conclusion that the parallel postulate could not be deduced from the other axioms of geometry – that it was independent of Euclid's remaining axioms -- with Karl Gauss the first to recognize that Euclidean geometry held no privileged claim on truth. In his personal writings between 1799 and 1829, Gauss became convinced of the independence of the 5th postulate, that a non-Euclidean geometry was logically consistent, and that it might be applicable to the representation of physical space. (Knoebel, 2007; Stillwell, 2010)

Publishing independently in 1823, Nikolai Lobachevsky and Janos Bolyai are generally credited with establishing non-Euclidean geometry: they each derived fully formed systems of geometry based on the logical consequences of accepting an initial axiom that contradicts Euclids' parallel postulate. (Kline, 1972c; Kline, 1980; Knoebel, 2007)

The representation of physical space by multiple geometries, each consistent within itself, but contradictory with each other, was the first realization that mathematics did not represent a singular universal/absolute truth about the external natural world. (Kline, 1980) Having surrendered the tenet of universal external truth to non-Euclidean geometry, the premise of universal internal truth remained: mathematics was surmised to be both internally complete and internally consistent.

3.3.2. Incomplete truths: Godel's incompleteness theorems

The second strike against mathematics and logic as an objective and absolute truth was delivered in 1931 with Kurt Godel's proof of two theorems that establish the inherent limitations of proof in formal axiomatic systems. (Boyer, 1968)

His first incompleteness theorem shows that in any consistent axiomatic system strong enough to develop simple integer arithmetic, there will always be truths that can be expressed/ derived from its axioms that can be neither proved nor disproved based on those axioms. (Devlin, 1999) Using the logical structures outlined in Whitehead and Russell's *Principia Mathematica*, Godel constructed a statement equivalent to "This sentence is not provable." Of course, if this statement is true, then it is not provable. However, if it is false, and provable, then the axiomatic system can be used to prove a statement that is not true in that system. Later developments extend these results to any formal axiomatic system: if logic and mathematics are consistent, they cannot be complete. (Stillwell, 2010)

Godel's second incompleteness theorem extends the unprovable statements of his first incompleteness theorem to include statements of an axiom systems' own consistency: any formal system robust enough to formulate its own consistency can prove its axiomatic consistency if and only if it is inconsistent. (Stillwell, 2010) Truth does not reside exclusively in the province of proof.

3.3.3. Local truths: Cohen's independence proofs

The third strike against mathematics and logic as a representation of absolute truth was delivered in 1963 with Paul Cohen's re-examination of the relationship/ roles of the continuum hypothesis and the axiom of choice to the consistency of set theory.

Prior to 1873, Aristotle's potential infinity was recognized in mathematics primarily as something to be avoided. Actual infinity was devoid of structure or measurement, consequently a formal definition remained beyond the capabilities of mathematics. Georg Cantor's seminal construction of set theory included infinite sets, thereby giving measure and definition to actual infinity as a mathematical object. (Kline, 1953; Maor, 1987; Kaplan, 2003) Cantor presented proof that set of whole numbers had fewer elements in it than the set comprised of the continuum of real numbers: there are 'lesser' and 'greater' sizes of actual infinity. This lead to his formulation of the *continuum hypothesis* in 1884/88?: there is no infinite set having a number of elements between the lesser infinite set of whole numbers and the greater infinite set of real numbers. (Dauben, 1979) In his famous lecture delivered to the International Congress of Mathematicians in 1900 outlining the significant outstanding problems in mathematics, Hilbert listed the continuum hypothesis as number one. (Elwes, 2011)

In 1904, to resolve the contradictions/paradoxes inherent in Cantors initial constructions of set theory, Ernst Zermelo formulated the *axiom of choice* to prove a basic theorem in his initial axiomatic formulation/treatment of set theory. (Wilder, 1952) This axiom formally states a deceptively simple and intuitive concept: for any collection of unrelated sets, where each set contains at least one object, it is possible to select one object from each set to form a new set, a *choice set*. (Mac Lane, 1986) In 1928, Abraham Fraenkel amended / extended Zermelo's axioms to be more inclusive/ comprehensive, resulting in their widespread acceptance as the unifying foundation of mathematics: Zermelo-Fraenkel set theory with the axiom of choice. (Luce, 1977; Dieudonne, 1992)

In 1940, Godel offered a proof that both the axiom of choice and the continuum hypothesis were consistent with set theory, provided the axioms of Zermelo-Fraenkel were themselves consistent: on the basis of Zermelo-Fraenkel, the axiom of choice could not be disproved, nor could the existence of an infinite set between the greater and lesser infinite sets defined by Cantor. (Kline, 1972c; Johnson, 1972; Dawson, 1984; Uspensky, 1994; Devlin, 2002; Goldstein, 2006)

Thirteen years after Godels proof of consistency, Paul Cohen conceived of a new approach that resulted in the proof of inconsistency: neither the axiom of choice nor the continuum hypothesis can be proved from the axioms of Zermelo-Fraenkel, provided set theory is itself consistent. Further, Cohen showed that Zermelo-Fraenkel set theory broadened by the inclusion of the axiom of choice is also incapable of proving the continuum hypothesis. Godel proved the *absence* of an infinite set between the set of whole numbers and the set of rational numbers was consistent with set theory. Cohens' later proofs showed the *existence* of an infinite set between the set of whole numbers and the set of rational numbers was also consistent with set theory. (Stewart, 1995; Stillwell, 2010)

Just as proof of the independence of Euclid's parallel postulate gave rise to consistent and complete non-Euclidean geometries, the proofs of Godel and Cohen reveal the continuum hypothesis and the axiom of choice to be independent of the axioms of Zermelo-Fraenkel, and allow the potential formulation of equally valid alternative set theories. (Eves, 1990; Stillwell, 2010) Set theory, accepted as the unifying foundation of mathematics for nearly a century, has no more claim to universal truth than did Euclidean geometry before it, resurrecting the debate over the foundation of mathematics. What became clear in the aftermath of Cohen's work was that the provable truths of mathematics are dependent on the initial postulates. (Eves, 1990; Chaitin, 2002; Davis, 2005)

The vision of mathematics as a model of absolute truth has been proven to be little more than mirage. Lobachefsky and Biarani's independent axomizations of non-Euclidean geometries in 1923, Godels incompleteness proofs of 1931, and Cohens independence proofs of 1963 have resulted in the reluctant acceptance that universal/absolute truth is unattainable within the known framework of mathematics and logic. Mathematics is a cultural construct that has achieved neither a standard of universal objective external representation nor a standard of internal completeness and consistency:

> *The questions of the ultimate foundations and the the ultimate meaning of mathematics remains open; we do not know in what direction it will find its final solution or even whether a final objective answer can be expected at all. 'Mathematizing' may well be a creative activity of man, like language or music, of primary originality, whose historical decisions defy complete objective rationalization.*
>
> (Hermann Weyl, 1946)

Truths provable by mathematics and logic endure, but it must be recognized that these truths are constrained by their context and initial postulates to remain relative and local. Mathematical properties do not reside in disparate mathematical objects, but are defined by the pattern of local relationships of mathematical objects. Thus spake category theory: provable truth is *situated*.

3.4. LOCAL TRUTH AND CONSEQUENCES: SITUATED MODELING

> *One of the profoundest instances of this phenomenon arose in the transition from classical (Newtonian) to relativistic physics, when physical concepts such as simultaneity of events and mass of a body formerly ascribed an absolute meaning were seen to possess meaning only in relation to local coordinate systems.*
>
> *There is an evident analogy between local mathematical frameworks and the local coordinate systems of relativity theory: each serve as the appropriate reference frames for fixing the meaning of mathematical or physical concepts respectively.*
>
> (John L. Bell, 1986)

3.4.1. Incompleteness of knowledge

In 2008, David Wolpert followed Godel's proof of the incompleteness of axiomatic systems to construct a proof of the incompleteness of any method to acquire knowledge of a physical system: any device embedded in a physical system that performs observations, predictions, recollections, or inference, about that system is incapable of obtaining complete knowledge of that physical system. (Wolpert, 2008) Provable truth is incomplete, absolute truth is unobtainable.

In the absence of absolute truth, the position of the external objective observer also becomes unattainable. The observer of local truth is situated, defined by the observer's context and initial postulates. The properties of an observer are not defined by a disparate 'observer-as-object', but are defined by the local pattern of interactions 'observer-as-constituent'.

3.4.2. Naturalistic inquiry

From their divergent points of inception, the collection of investigative methods grouped under the umbrella of *qualitative research*, or *naturalistic inquiry*, have followed the relativistic revolution of early 20th century physics to accept a situated observer as a core tenant: research reflects the embedded nature of the investigator in the context of the local universe in which observations are made as well as the unstated or often unrecognized internal postulates of the investigator. (Lincoln, 1985) The 'Godelian truths' that hide in the interstitial spaces of provability suggest a possible role for the continued development of naturalistic methods in science: the verification of non-provable truths.

3.4.3. Modeling situated configurations

The emergence of the locally situated observer as an alternative to the external 'objective' observer establishes? an investigators point-of-view as another dimension of scientific modeling: external – situated. An 'external' model assumes an omniscient 'God's-eye view' perspective of phenomena. A situated model assumes a locally embedded 'particle-eye view' perspective of phenomena.

To date, the situated models of naturalistic inquiry are descriptive models, generally derided by the scientific community for their lack of rigor. It's not their fault: all of existing mathematics begins and ends with the external point-of-view. The mathematical foundation for the accepted dimensions of modeling in 20th century science can be defined by the following subsets of properties:

> (continuous, quantitative, deterministic, external)
> (continuous, quantitative, stochastic, external)
> (discrete, quantitative, stochastic, external)
> (discrete, qualitative, external)

That the mathematics from an external point-of-view has revealed itself to be situated, the obvious question is then of the possible subsets of mathematics created by the substitution of the characteristic *situated* for that of *external*. This question rises not from the 'Because it's there.' raison d'être lamented by Morris Kline as 'mathematics as the study of arbitrary structures' (Kline, 1980), but rises from the spirits of the 19th century and earlier, when the hard line between mathematician and scientist had not yet been drawn, and new mathematics was often a response to the call of new problems in the scientific exploration of the natural world.

Today much of science continues to hold to the precedents of the 20th century and accept atomistic objects as the unit of analysis on the assumption that properties are grounded in objects. But virtually every discipline now has an expanding faction that views their subject from a connectionist perspective: investigation converges on the relationships between objects, with a growing realization that properties are grounded in these relationships. Many of these diverse investigations share the vocabulary of networks and complex systems, but they are perhaps more expediently defined as structures, patterns, or configurations.

While this transition is underway, most disciplines remain incognizant that their work can be defined by the set of categorical elements in structuralist philosophy accepted by material science nearly a century ago: (Phillips, 2001)

> (process, pattern, properties, performance).

Processes define pattern, patterns define properties, properties define performance. The issue of structure, pattern, or configuration, has become a common problem throughout science.

Pattern is where science intersects the art of architecture: distilled to its abstract essence, the profession of architecture is a discipline of patterns and pattern formation of the human habitat. Architectural modeling is currently a process of distributed cognition employing descriptive models. With a history at least as long, if not longer, than mathematics, and considerably longer than science, the seminal precedent for architectural modeling is the concept of atomism: the human habitat is conceived as a collection of disparate objects, buildings, with properties grounded in the object. In that sense, the art of architecture and the accepted precedent for much of 20th century scientific modeling share the same foundation. The limitations inherent in this assumption are revealed by Boltzmann's atomistic modeling of material properties: it is a model applicable only to weakly interacting systems of particles; quasi-binding and non-binding interactions, having properties of dilute gases.

Modeling systems of objects with binding interactions is the science of structure, pattern, or configuration, and can be distilled into two discrete problems: pattern identity and pattern formation. Both of these problems appear to reside in the discontinuities between the existing objective frameworks of continuous, discrete, and stochastic mathematics.

A pattern, or configuration, is a definition of the connections between discrete objects, indicating a discrete model. Categorical definitions of pattern lack the specificity to rigorously define a single configuration that uniquely differentiates it from other similar configurations. The problem of configuration identity is non-categorical, implying a quantitative model. That configuration identity eludes the extensive body of external quantitative mathematics suggests a situated model. These prerequisites combine to form the set of characteristics of a mathematical model of pattern identity as:

(discrete, quantitative, situated)

Pattern formation is the evolution of the connections between discrete objects, requiring a discrete model. The local interactions at the heart of pattern formation suggest an 'object-eye view', or situated model that answers the question: what did the object know and when did it know it? Local interactions at all scales in the physical and life sciences have some degree of indeterminacy, entailing a stochastic model. These prerequisites combine to form the set of characteristics of a mathematical model of pattern formation as:

(discrete, quantitative, stochastic, situated)

A discrete quantitative definition of configuration identity inherently gives a measurable degree of configuration similarity or dissimilarity, or a distance metric for configurations. A discrete quantitative definition configuration formation gives a transformation operation that changes one discrete configuration identity into another; a discrete configuration morphism.

The combination of distance metric and morphism define the configuration space of pattern, enabling the definition of evolutionary trajectories of patterns and allowing the operations of discrete configuration interpolation and extrapolation. A discrete quantitative model of configuration space has the property of prediction.

3.5. SITUATED STRUCTURE: CONFIGURATION UNITS OF ANALYSIS

> *The local interpretation of mathematical concepts, based as it is on category theory, has an essentially relational character. According to the local interpretation, the reference of a mathematical concept, insofar as it can be construed as an entity, is no longer to be regarded as being a thing in itself, whose nature is independent of other things, and whose characteristic properties are entirely intrinsic to it. On the contrary, the properties of a mathematical entity are now determined by, and indeed only have meaning in terms of, the totality of its relationships with other entities.*
>
> (John L. Bell, 1986)

3.5.1. Loss of individual identity

Adaptive systems have a history of individuals presenting evidence for properties that have blurred the definition of an object to expand the unit of analysis. With the interpretation of local information forming the basis of an adaptive process (Battail, 1997; Bookheimer, 2002; Jablonka, 2002; Friederici, 2003; Khallad, 2004; Maguire, 2004; Noppeney, 2004; Binder, 2005; Gupta, 2006; Schultz, 2006), this blurring extends across the hierarchy of adaptive scales; cellular-genetic, neural-cognitive, and social group:

In 1896, psychologist James Baldwin introduced the concept of evolution now known as the *Baldwin effect*: adaptive learning at the scale of an individual accelerates genetic evolution of a phenotype. (Baldwin, 1896)

In 1941 biologist Conrad Waddington proposed the concept of *epigenetics*: initially defined as inclusive of all of the factors in an organism's development, the concept is now restricted to factors of heritable change that do not involve modification of DNA structure. (Waddington, 1941)

In 1947, Georgy Gause proposed the concept of *phenotypic plasticity*: the ability of a single genotype to produce differing phenotypes in response to different environments. (Gause, 1947)

In 1951, biologist Victor Freeman reported the first observed instance of *horizontal gene transfer*: the transfer of genes between organisms, often of different species, through local interactions completely unrelated to a reproductive process. (Freeman, 1951)

In 1967, biologist Lynn Margolis presented evidence for *endosymbiosis*: one or more organisms living inside another organism that define the properties of a single individual. (Margolis. 1967)

In 1978, biologist Richard Dawkins proposed the concept of the *extended phenotype*: the redefinition of a phenotype beyond an individual to include the effects of an individual's genes on the environment. (Dawkins, 1978)

In 1982 biologist Richard Lewontin proposed the concept of *niche construction*: the environmental modification processes of organisms that alter the evolution their own species as well as others. (Lewontin, 1982)

In 1987, anthropologist Lucy Suchman proposed the concept of *situated-embedded cognition*: the examination of cognition and learning in its natural physical, environmental and social context. (Suchman, 1987)

In 1988, cognitive scientist Edwin Hutchens proposed the concept of *distributed cognition*: extension of the cognitive unit of analysis to include the social groups and physical artifacts of an individual's environment. (Hutchens, 1988)

While initially met with skepticism, these concepts have all been incorporated into their respective branches of science as areas of active research. In adaptive systems, units of analysis no longer conform to discrete boundaries of objects, but extend to the sum of the object and its local interactions. This set, or *cluster*, of objects forms a *nonlinear* local group structure: a *situated pattern*, *a* ***situated configuration***.

> *Systems biology can be characterised as an approach to the understanding of life through the study of how the properties of biological systems arise through interactions between components, as these are situated and organized within the system. What is regarded as the 'system' is, inevitably, bounded and is itself situated within some environment, with which the system (through its components) may have relevant interactions. It is an open system, whose boundary is to some extent drawn at our discretion.*
> (Tom Melham, 2013)

3.5.2. Situated units of analysis: problem definition

Expanding the unit of analysis beyond an individual object to form a situated configuration creates two problems:

1. Definition of the situated configuration boundary.
2. Definition of the situated configuration internal to the boundary.

The connectivity structures and connectivity metrics of topology and graph theory can be used to examine both problems.

3.5.3. Configuration units: boundary structure definition

In an atomistic model, the boundary of the unit of analysis has been defined by the property of individual objects to be physically disjoint, but that disjoint property boundary is being called into question by the extended grounding of properties found in hierarchical structures of situated-embedded adaptive systems. A unit of analysis has no meaning without a definable boundary.

Expanding the unit of analysis from a single individual object to include other objects in the individual's neighborhood can be visualized in 2 dimensions as the inflation of a point into a circle: the initial inflation transforms the point into an infinitesimally small circle, opening an infinitesimally small space inside the point. This initial inflation partitions space into two regions, inside the circle and outside the circle, with the line of the circle defining the boundary of the partition. The circle can then continue to inflate or deflate to any size without changing its boundary property of dividing space into two parts.

Of course, the space partitioning property is not unique to the shape of the circle; any number of shapes made with a single continuous closed line will also separate a 2-dimensional space into two regions. But there are more complex shapes that also divide space into two regions that have very different boundary properties than a simple closed line. The connectivity of a boundary is a *topological* property that remains invariant with a continuous change from one shape into another. Topological categories of boundary connectivity can be defined by the *fundamental group* of paths that start from an arbitrary point inside the boundary, loop through the interior space, and return to the original point. The loops from an arbitrary point inside a circle are all equivalent, defining the boundary as *1-group-connected* or *simply connected*, topologically categorizing the boundary as fundamental group [1]. All space partitioning boundaries of fundamental group [1] can be generated from a point and reduced back to a point with the topological morphisms of ***Poincare inflation*** and ***Poincare deflation***.

Expanding the unit of analysis from a disjoint atomistic object to include its relationship to neighboring objects can be modeled as a Poincare inflation of a point-object. The object-eye-view is that of the object inflating to engulf its neighboring point-objects. However, the object-eye-view of a neighbor is also one of inflation to engulf its own set of neighbors. The inflating boundaries between two disjoint neighboring point-objects initially touch then infinitesimally overlap to form the union of a single boundary. The objects are now related, and the properties attributable to a single object are no longer distinguishable. This transfers the unit of analysis from a single object to the group of two objects. Any loop traveled from an arbitrary point inside this new boundary can include the object-points, so in the case of the union of two circles, the topological boundary categorization remains unchanged as fundamental group [1].

In an example of a static random field of point-objects undergoing Poincare inflation, the boundary of one object may merge with the boundary of a second, followed by a third joining the initial pair, then perhaps a fourth and fifth merging with the first three simultaneously. These sequential unions result in a hierarchy of boundaries that define a discrete hierarchical structure for situated pattern units of analysis.

As multiple objects are joined, the boundary union can become more complex, potentially resulting in a cluster boundary having 'holes' that interrupt the interior space. This can be visualized in three dimensions as a tunnel through a sphere: the walls of the tunnel join the surface of the sphere, forming an unbroken boundary to contain the space inside the sphere, yet the space outside the sphere can flow through the tunnel: like a simple sphere, the tunnel-through-the-sphere construction partitions space into two regions. However, the boundaries of the two shapes are fundamentally different, with the tunnel-through-the-sphere boundary being topologically equivalent to a donut, or torus, where the interior loop-paths have to contend with the tunnel, resulting in two different kinds of loops. One group of loops does not circumnavigate the obstructing tunnel, remaining in the undifferentiated space adjacent to the tunnel. These loops are of the same nature as those inside the undifferentiated space of a sphere or circle. The second group of loops does circumnavigate the tunnel to return to their points of origin. The boundary of the tunnel-through-the-sphere is then said to be 2-group-connected, topologically categorized as the fundamental group [2].

The different equivalency groups of possible loop-paths inside a boundary having to contend with the number of tunnels passing through the enclosed space gives topological measure to the nature of the boundary of the space. In answer to the situated question 'What did the object know?', the object-eye-view of an arbitrary point-object inside the space partitioning boundary 'knows' the nature of the boundary from the different types of loops it can travel in the bounded region of space. An object situated inside the enclosed space 'knows' the discrete sequential evolution of boundary states

Boundaries and units of analysis are then not drawn at the investigators discretion; they are selected at the investigators' discretion in recognition of the situated pattern of relationships specific to the phenomena being investigated and the hierarchical boundaries defined by those relationships.

3.5.4. Configuration units: internal structure definition

The topological morphism that developed the boundary of a collection of objects also models the pattern of object-points inside the boundary: as the inflating boundaries of two adjoining objects intersect form a union, the relationship between the objects can be established, modeled as a line connecting the two points. When applied to a point field of objects, Poincare inflation defines a type of *Boolean grain model*, or *germ grain model*, first comprehensively developed by Georges Matheron in his 1975 *Random Sets and Integral Geometry*. The relationship structures delineated in a Boolean grain model are defined by graph theory, with objects modeled as points, or *vertices*, and their relationships as lines, or *edges*.

Directed graphs, or *digraphs*, have been extensively used to model structure, where edges are expressed as directional arrows to represent a one-way relationship. A situated pattern unit of analysis is based on the interaction of objects forming two-way relationships, modeled as an *undirected graph*. Some relationships between objects are stronger than others, often modeled as numerical weights assigned to edges in a *weighted graph*. This raises the subtle difference between structure and configuration: a configuration is a spatial structure. Strengths of relationships in a configuration are modeled by distance: edge length expresses the relative intensities of relationships in a *spatial graph*. A spatial graph defines an object's location relative to other objects, the critical parameter of situated local interactions, and allows the construction of sub-graph boundary partitions to reveal the covert hierarchical structure of a point field.

A ***configuration graph*** is special case of a spatial graph that contains the hierarchical sets of boundary partitions. The situated interactions of objects can be modeled as hierarchically nested pattern sub-graphs.

3.5.4.1. situated sub-graph structures

One category of sub-graph structure that can define a situated configuration unit of analysis is a *neighborhood*: a *closed neighborhood* of a vertex *v* is defined as the set of all vertices adjacent to *v* and including *v*, joined with the set of all of the edges connecting those vertices.

If one object in a situated configuration unit of analysis has a relationship with every other object in the configuration, it forms a neighborhood unit of analysis. If every object in a situated configuration sub-graph has a relationship with every other object in the sub-graph, it forms a *complete sub-graph*, or a *complete neighborhood* unit of analysis. What are encountered frequently in empirical network models are *hubs*: objects with a higher number of relationships than typically found in the remaining structure, modeled as vertices with a 'high' number of edges.

If no object in a situated configuration pattern-object of analysis has a complete set of relationships with every other object, it is no longer a neighborhood structure, but can be categorized as a *ring* unit of analysis. The structure of a *Boolean ring* is defined when every possible union of two subsets of objects is also a set, and these unions and their relative complements are all within the unit of analysis. A Boolean grain model partitions a point field into hierarchical sets of Boolean rings. A Boolean ring defines the minimum standard for a collection of things to be considered a unit of analysis.

The definition of a hub can also be used to categorically differentiate sub-graph ring structures, as being a *simple ring*, ***single-hub ring***, or ***multi-hub ring***. More specifically, a multi-hub ring containing 2 hubs would be a ***2-hub ring***.

3.5.4.2. situated sub-graph connectivity

Neighborhood and ring structures can further be categorized by their connectivity. The connectivity of a graph or sub-graph is given as the number of vertices that must be removed from a graph for it to become disconnected, defined as total number of vertices in the graph minus 1. In a pattern of 6 objects, 5 have to be removed for it to become disconnected: it is *5-vertex-connected*.

Vertex degree is the measure of connectedness of an individual object in a configuration, given in graph theory as the number of edges that connect to a specific vertex. An object forming a closed neighborhood with 4 other objects has a vertex degree of 4. A hub has a 'high' vertex degree.

The concept of topological connectivity can also be applied to a graph structure, with the number of different loop-paths from an arbitrary vertex in the graph giving measure to the *fundamental group of a* *graph*, where loops, called *cycles* or *closed walks*, are restricted to the vertex-edge structure of the graph. A graph having five equivalence groups of cycles is ***5-cycle-connected***. This measure is based on the definition of a graph being embedded in an undifferentiated topological space, where a cycle bounding a region of space that is not divided by a graph edge produces a 'hole' in the space. However, a situated configuration sub-graph resides in a finite region of topological space already partitioned by a boundary.

The spatial positions of independent point-objects define the boundary that can be topologically categorized by a fundamental group, establishing the independence of the boundary from the internal relationship structure. A situated pattern sub-graph is embedded in a bounded region of topological space, with the boundary defining the 'holes' in the space. The equivalences of closed walks are then restricted by the boundary definition of the enclosed region so the ***situated fundamental sub-group of the graph*** is equal to the fundamental group of the boundary of the sub-graph.

The measurable reduction of the fundamental group of a graph to the situated fundamental group of a sub-graph is an additional categorization of the connectivity of internal structure, measuring the boundary effect on the connectivity of the internal pattern. A 5-cycle-connected sub-graph whose connectivity is reduced by its boundary topology to be 2-cycle-connected is ***3-cycle-disconnected***.

The partitioning of a large graph into manageable sub-graphs suggests using connection topology to formulate a definition of individual vertex connectivity in terms its contribution to the fundamental group of a graph, thereby giving a discrete measurable definition of a hub vertex. The removal of a vertex from a graph can reduce the number of equivalency groups of cycles in the graph: if a 5-cycle-connected graph is reduced to a 3-cycle-connected graph by the removal of a vertex, that vertex can be defined as being a ***2-cycle-connected vertex***.

The removal of some vertices may have little or no impact on the overall connectivity of the graph, while the removal of others may result in a reduction of topological connectivity to become the fundamental group of a graph [1]. This suggests the minimum definition of a hub to be a vertex whose removal reduces the fundamental group of a graph or sub-graph, or an ***internal-hub***.

A more critical case is where the removal of a vertex alters the situated fundamental group of the sub-graph. In this case, vertex removal has not only altered the topological connectivity of the internal configuration, it has altered the topological boundary definition of the configuration. A vertex in this case may be defined as a ***boundary-hub***.

This leads to the general differentiation in larger patterns between an object-point whose boundary contributes to the union of boundaries defining an object-pattern as a ***boundary-vertex*** and every other object-point in the pattern as an ***interior-vertex***.

3.5.5. Situated variable space

A hub neighborhood structure is a graph theoretic expression of the relationship independent and dependent variables. Deterministic independence-dependence relationships in continuous mathematics can be modeled as a digraph hub neighborhood, while the strength/degree of the bi-directional relationships of a hub in an undirected graph can be examined as partial correlation measures in stochastic mathematics. While introduced as an analysis tool of structure, it can also be used for synthesis of structure: partial correlation measures can be translated into spatial distance between points to establish the initial conditions of a hub neighborhood.

If the initial conditions of a point field are known and the degree of independence of variables can be approximated, the distance between two variables can be proportioned as the degree of independence-dependence. As this process is repeated around an 'independent' hub vertex-variable to include its 'dependent' neighboring vertex-variables, the asymmetric boundary resulting from a Poincare inflation of a point can be defined. This implies both differences in the directional inflation rates of a point and differences in the inflation rates of different points. Poincare inflation then becomes a situated topological morphism, dependent on both the strength of local interactions with its neighbors and the initial condition of the spatial location of a neighborhood.

3.5.5.1. Boolean ring as synergistic structure

Empirical models have demonstrated the importance of hubs in the structure of networks, but those same models reveal the relative infrequency of their occurrence. Much of the bounded subset structure in spatial graphs is defined as a simple Boolean ring: a discrete sub-graph representing the varying degrees of cooperative interaction in a set of variables that contains no independent variable. It is this synergistic structure of variables that restricts modeling applicability of objective continuous and stochastic mathematics to small subsets of their extensive body of provable truths.

However, following their application to network hubs, partial correlation measures can be methodically employed to translate relationships of non-dominant variables into a spatial point field. This might be accomplished as a situated process of iteration, starting with one variable and examining its related variables to form the adjacency structure of an initial neighborhood, followed by this first generation neighborhood producing the second, the second producing the third . . . Fortunately, a pattern graph developed from a Boolean grain model is generated from situated local interactions, so the model need extend only beyond the boundaries of a unit of analysis.

Situated pattern modeling begins by structuring a portion of variable space and follows by the hierarchical partitioning this spatial structure prior to the selection of a pattern-unit of analysis. While pattern graphs were previously introduced with the aid of two and three dimensional visualizations, independent variables are traditionally modeled as having a *degree of freedom*, indicating the graph structures of configuration-units of analysis are likely constructed in higher dimensional spaces.

3.5.6. Situated indeterminacy

That our universe is understood to be probabilistic at the very smallest scale implies indeterminacy is pervasive in our observable reality. It is the accumulation of randomness that gives rise to the properties of pattern formation: the situated locally controlled interactions of objects that are dependent on their initial state to precipitate the emergent local phase transitions of hierarchical structure. The accumulation of random events defines both the evolutionary trajectory and devolutionary trajectory of a system, with the threshold-cascade rhythms of punctuated equilibrium being ubiquitous to empirical observation.

A series of small random events insufficient to alter a pattern's connectivity results in a period of apparent equilibrium. To an embedded object that 'knows' only this period of stasis, the object-eye-view might best be expressed by Frank Zappa: *It can't happen here*. (Zappa, 1966)

As a buildup of random events continues, the connectivity of a pattern approaches a critical threshold where a single random event, or the proverbial 'straw-that-broke-the-camel's-back', results in a systemic structural transformation. Even in a non-critical state, a single random event can induce systemic structural change when it alters the connectivity of a critical hub component in a hierarchical pattern. In each of these cases, the object-eye-view of an unanticipated devolution might best be expressed by Walter Jacobs: *Boom, boom, out go the lights*. (Lewis, 1957) When either a single random event interacts with a critical component, or randomness accumulates to a critical threshold, there is a discontinuous change in a pattern, or a phase transition:

> *To make this [criticality] less abstract, consider the scenario of a child at the beach letting sand trickle down to form a pile. In the beginning the pile is flat, and the individual grains remain close to where they land. Their motion can be understood in terms of their physical properties. As the process continues, the pile becomes steeper, and there will be little sand slides. As time goes on, the sand slides become bigger and bigger. Eventually, some of the sand slides may even span all or most of the pile. At that point the system is far out of balance, and its behavior can no longer be understood in terms of the behavior of the individual grains. The avalanches form a dynamic of their own, which can be understood only from a holistic description of the properties of the entire pile rather than from a reductionist description of individual sand grains: the sandpile is a complex system.*
>
> (Per Bak, 1996)

The degree of randomness is measured over intervals: the frequency of probabilistic events in time, located in space. If randomness is independent of space and time, any two objects in the universe of objects have an equal probability of interacting over any distance and any time interval. This is a universe of absolute randomness, with no apparent structure, often modeled as an Erdos-Renyi random graph.

In a structured universe, randomness is applied uniformly across space and time, with the probability of interaction becoming a function of the interval of indeterminacy between two objects. Indeterminacy in a structured universe is then a function of the density of a set of objects: the relative location of two objects in space and time constrains the degree of randomness of their interaction. Randomness is situated.

Intervals of spatial randomness are expressed as the relative location of points in a point field, defining the situated initial conditions of a model. 'Tuning' the parameter of spatial indeterminacy models sets of objects in different phases, or states:

> Intervals of spatial randomness defined by a continuous random variable produce the initial condition of random dynamic point fields that model the properties of a gas or a liquid.
>
> Intervals of spatial randomness defined by a discrete random variable produce the initial condition of random static point fields that model the properties of amorphous solids.
>
> Intervals of spatial randomness defined by a discrete random constant produce the initial condition of static point fields that model the properties of crystalline solids.

An interaction between two objects can be defined by its location in space and duration in time. With intervals of spatial randomness necessary to specify an initial set of objects, the intervals of temporal randomness are sufficient to specify the interaction of objects in that set. Intervals of temporal randomness are expressed as pattern by the set structures of the interactions of objects and their boundaries:

> If nonintersecting sets of boundaries have nonintersecting sets of their interior elements, patterns of interaction are created that model the systemic properties of a gas.
>
> If intersecting sets of boundaries that form a union have nonintersecting sets of their interior elements, patterns of interaction are created that model the systemic properties of a liquid.
>
> If a 'small' number, or *cluster*, of intersecting sets of boundaries that form a union have intersecting sets of their interior elements that also form a union, patterns of interaction are created that model the systemic properties of a liquid.
>
> If a 'large' number of intersecting sets of boundaries that form a union have intersecting sets of their interior elements that also form a union, patterns of interaction are created that model the properties of a solid.

In sets of objects where interior elements do not form unions, the temporal interval of randomness, expressed as the probability of local interaction, is 0. This leaves a model with one indeterminate parameter: the dynamic interval of spatial randomness. The non-intersecting set structure of temporal indeterminacy in systems exhibiting the properties of gasses or liquids allows the unit of analysis to be disjoint objects and their boundaries. These units are appropriately modeled by the atomistic properties of objective stochastic mathematics with one parameter of indeterminacy; their periods of dynamic stasis can be modeled by objective continuous mathematics.

In sets of objects where interior elements form unions, both parameters of indeterminacy are expressed: the interval of temporal randomness is constrained by the interval of spatial randomness to produce hierarchical structure. Fully expressed randomness is hierarchical. The union set structures of temporal indeterminacy in systems exhibiting the properties of solids necessitate the object-unit of analysis and its boundaries expand to become a pattern-unit of analysis and its boundaries. Hierarchical pattern-units of analysis appropriate to indeterminate hierarchical structures preclude the modeling abilities of much objective stochastic and continuous mathematics.

Intervals of temporal randomness are expressed by the interactions of objects constrained by the initial conditions of their interval of spatial indeterminacy. A point field models the initial condition of intervals of spatial indeterminacy of situated objects, the probability of edge formation in a point field models the local interaction of intervals of temporal indeterminacy of situated objects. In a fully indeterminate Boolean grain model, the pattern sub-graphs and their boundaries are formed by the interaction of the intervals of spatial and temporal indeterminacy. While indeterminacy is not a property of the boundary itself, a boundary separates discrete levels of fully indeterminate units of structure, expressing the internal set of probabilistic connections as a single measurement of situated discrete probability.

Derived primarily with objective discrete mathematics, the model of hierarchical pattern formation and bounded pattern-units of analysis presented thus far is not fully indeterminate, having the set of properties:

(discrete, quantitative, stochastic, situated)

3.5.7. Configuration units of analysis: partial definition

The approach outlined above achieves the level of specificity necessary for a model of object interactions to reveal hierarchical structures that are appropriate for situated pattern-units of analysis, but falls short of the level of specificity required for their precise definition.

This is a situated model of pattern formation that corresponds to the situated interactions empirically observed in complex systems, dependent on the initial condition of objects and evolving by their local interactions. The model is developed from an 'object-eye-view' of each of the objects inside the model, with each object iteratively answering two questions as its relationships to other objects evolve:

1. What do I ‘know’?
2. When did I ‘know’ it?

To provide an appropriate language to answer those questions, the almost-global provable truths of external viewpoint mathematics were recast to reflect the situated viewpoint of an object, resulting in some mathematical structures that may make little sense or become provable untruths if extended beyond the object’s point of view. That this local language was expressive of an object’s situation suggests the possibility of a 21st century extension of external viewpoint mathematics to define situated mathematical structure that parallels the 19th century extension of continuous mathematics to define discontinuity.

Reframing the perspective from an external point of view mathematics to a situated point of view expands the boundaries of provable truth, and provides narrower categories for the definition of mathematical objects. But in the end, current objective mathematical structures and their situated extensions are limited to produce only a categorical definition of a pattern-object of analysis.

The properties of repeatability and reproducibility are the cornerstones of scientific inference and induction, providing the grounds to approach provable truth. A categorical definition of a unit of analysis provides neither property, making it imperative that a configuration-unit of analysis has a uniquely measurable identity.

The measurable identity of the interior structure of a configuration is a problem of the definition of a *graph identity*. The boundary of the interior structure describes its external morphology, with the measurable identity of both boundary and interior structure being a problem of the definition of a morphological identity, or an ***architectural identity***.

4. ARCHITECTURAL IDENTITY AND CONFIGURATION SPACE

This intuition of two-oneness, the basal intuition of mathematics, creates not only the numbers one and two, but also all finite ordinal numbers, inasmuch as one of the elements of the two-oneness may be thought of as a new two-oneness, which process may be repeated indefinitely . . . therefore all mathematical sets of units which are entitled to the name can be developed out of the basal intuition, and . . . any previously constructed set or any previously performed constructive operation may be taken as a unit.

(L.E.J. Brouwer, 1913)

4.1. SITUATED BOOLEAN STRUCTURES OF TOPOLOGICAL SPACE

4.1.1. Situated Boolean topology of a set

The topological morphism of continuous Poincare inflation of the boundary of an isolated point can be used to partition the undifferentiated local space around the point, defining a topological *neighborhood*. While boundary inflation can assume any shape having an invariant topological characteristic of fundamental group [1], the 2-dimensional models in this investigation are restricted to the uniform omnidirectional inflation of circular neighborhoods. (Appendix C)

As the neighborhoods of two adjacent points inflate to their initial intersection *limit point*, the two topologically equivalent local spaces are connected. Inflation beyond the limit point produces an 'overlap' of the two connected neighborhoods to form a third topologically inequivalent local space: a ***limit point neighborhood***. Topological equivalence of different neighborhoods is a function of the number of local spaces that have connected in forming a neighborhood: the local space of a singleton neighborhood connected only to itself, a ***1-connected neighborhood***, is inequivalent to the space formed by the intersection of three singleton neighborhoods, a ***3-connected neighborhood***.

Simultaneous and continuous Poincare boundary inflation of a set of points develops a sequence of neighborhood intersection limit points to connect local regions of finite topological space, delineating the discrete hierarchical subset structures that define a ***situated Boolean topology*** of the set: the complete set of inequivalent topological neighborhoods developed by a Poincare inflation of a set of points. [Figures 4.1A. - 4.1B.]

4.1.2. Locally complete topological space

The Boolean topology of a set of points provides a measure of the *completeness* of the local topological space. Inflating neighborhood intersections connect previously disjoint local regions of topological space to form a local cluster: a *union* of a set of neighborhoods. When the all the neighborhoods of a union connect with a single inclusive intersection, the neighborhoods form a ***completely connected topological space***: a ***complete union*** of neighborhoods. A sequence of inflating neighborhood intersections partitions an increasing sequence of finite topological spaces into incompletely and completely connected regions: nested sets of ***locally incomplete*** and ***locally complete*** unions of neighborhoods.

In a set of three points, a, b, and c, an initial Poincare inflation produces three singleton neighborhoods, a', b', and c'. If continued inflation results in the neighborhood intersection $a'b'$, then a' and b' are completely connected local topological spaces and form a complete union $a'b'$. If the subsequent step in an inflation sequence results in the intersection $a'c'$, forming the union $a'b'c'$, the neighborhoods a', b', and c' are connected but not completely connected: $a'b'c'$ is an incomplete union, with the subsets of locally complete unions $a'b'$ and $a'c'$. When the set of three neighborhoods inflate to form the ***locally complete intersection*** $a'b'c'$, the union of finite topological spaces has become completely connected to define the locally complete union of neighborhoods and the ***complete Boolean topology*** of the set of points.

Not all possible subsets of a set of points are spatially realized in a complete Boolean topology. The complete Boolean topology shown in Figure 4.1B.(h) develops four subsets of equivalently connected neighborhood structure:

four 1-connected neighborhoods: (a', b', c', d')
four 2-connected neighborhoods: $(a'b', b'c', b'd', c'd')$
three 3-connected neighborhoods: $(a'b'c', a'b'd', b'c'd')$
one 4-connected neighborhood: $(a'b'c'd')$

Limit point neighborhood $a'd'$ is not formed; the intersection of neighborhoods a' and d' occurs within the boundary of b' to give the 3-connected neighborhood $a'b'd'$. [Figure 4.1A.(d)] Likewise, neighborhood $a'c'$ is not formed, with a' connecting to c' in the 3-connected neighborhood $a'b'c'$ [Figure 4.1b.(g)] and neighborhood $a'c'd'$ is not formed, subsumed by the 4-connected neighborhood $a'b'c'd'$. [Figure 4.1B.(h)]

Twelve of the fifteen possible Boolean neighborhood subsets are formed by the complete union $a'b'c'd'$, to structure a complete Boolean topology on the set of points (a,b,c,d). The degree of completeness of an incomplete union formed in a step of the developmental sequence can be determined retrospectively, after a complete Boolean topology has been developed. Completeness of a union of neighborhoods can be expressed as the ratio of the number of Boolean neighborhood subsets formed by the union in a given step and the number of neighborhoods formed by the complete Boolean topology of the set.

4.1.3. Situated connectivity of topological space

With the development of inequivalent subsets of interior space, the full measure of connectivity of a finite topological space needs to consider invariant boundary characteristics as well as invariant characteristics of the interior partition structure.

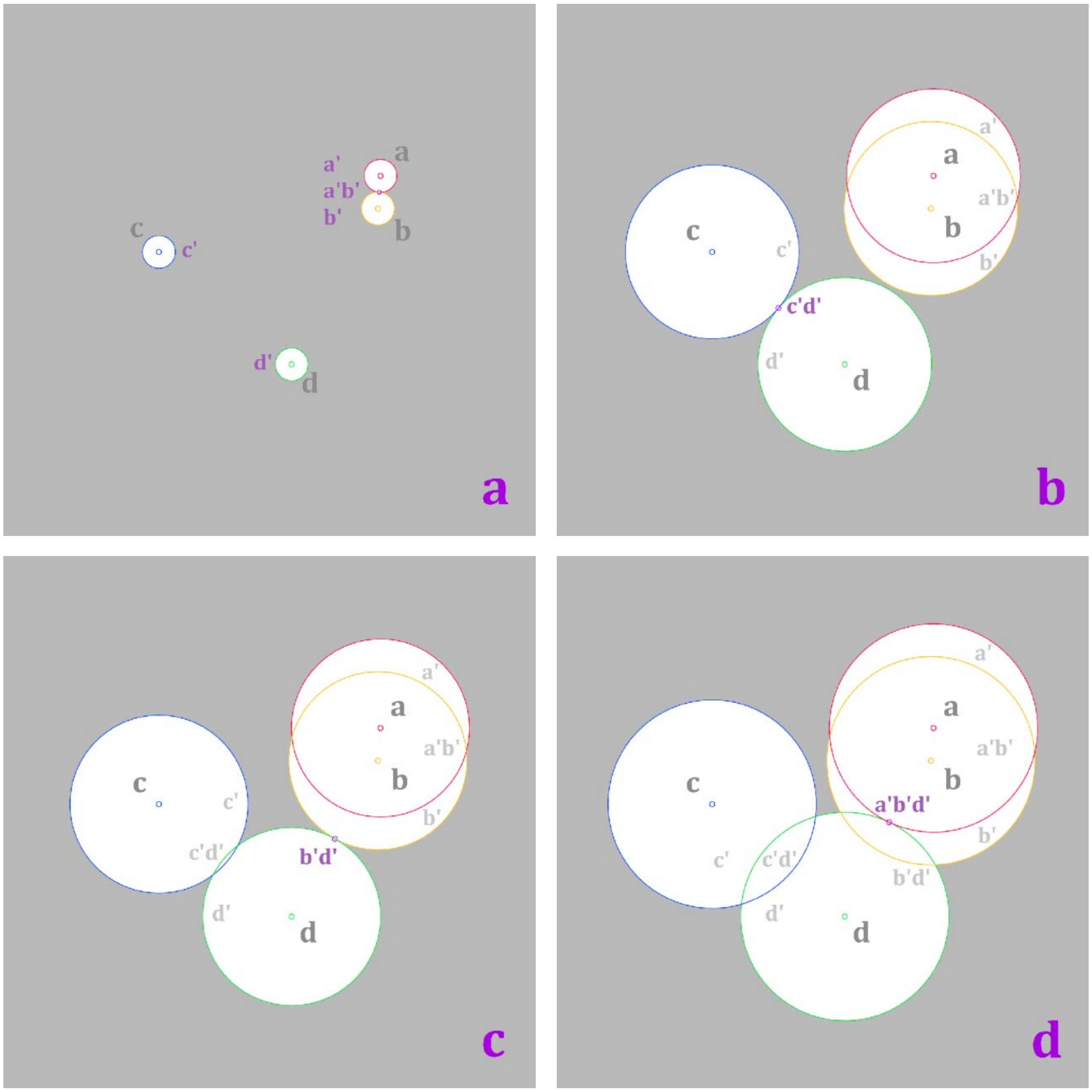

Figure 1A. Discrete developmental sequence of a situated Boolean topology from a set of random points. Local space states a-d, showing formation of neighborhood intersection limit points and topological partitioning.

4.1.3.1. local fundamental groups

In reviewing existing mathematical structures that might be applied to develop a situated unit of analysis, Section 3.5.3. presented a brief overview of topological fundamental groups as a measure of the number of 'holes' in a bounded finite region of space. This is a mathematical structure that transitions readily from its external viewpoint precedent to a situated viewpoint from within a space.

A loop is a closed path in the interior of a finite topological space that starts and ends at the same arbitrary point,

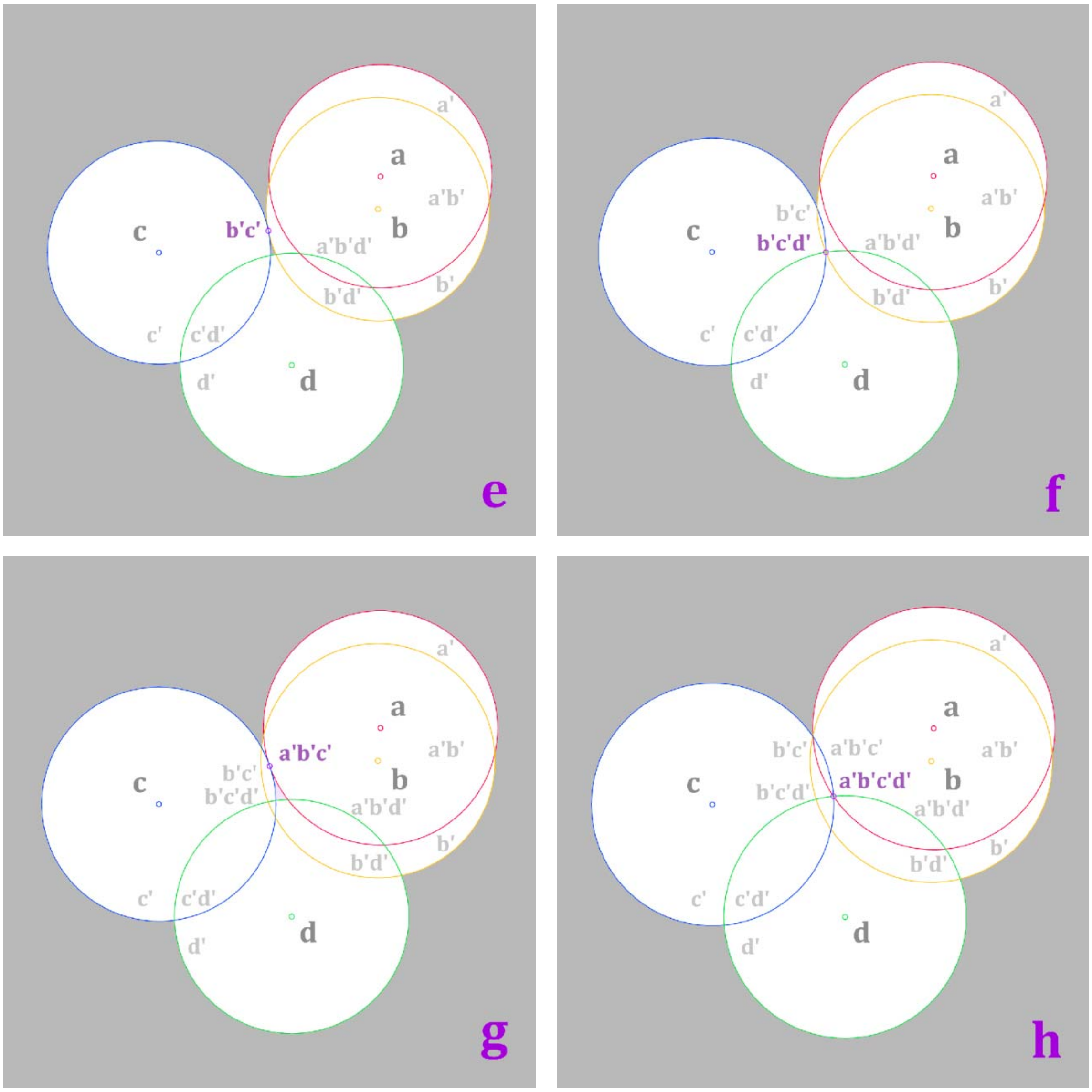

Figure 1B. Locally complete Boolean topology terminates sequence. Local space states e-h, showing formation of neighborhood intersection limit points and topological partitioning.

or *base point*. A set of loops that can be continuously deformed into each other are topologically equivalent: a loop that encircles a 'hole' cannot be continuously deformed into a loop that does not encircle a 'hole' without exiting the topological space by passing through the boundary of the hole, making the two paths topologically inequivalent.

A set of topologically equivalent closed paths structure a mathematical *group*, where the number of inequivalent groups of paths in a space becomes a function of the number of boundary 'holes' that can be encircled. The number path groups in a space give an invariant topological measure of its boundary connectivity: the *fundamental group* of a space.

In a space with no 'holes', all loops are equivalent to form a single group, fundamental group [1]. With one 'hole' in a space, the set of possible paths form a group that encircles the hole, and a group that does not, giving a fundamental group [2]. The union of neighborhoods in Figure 1B.(e) presents a local topological space with a 'hole' in the boundary between the neighborhoods a', c', and d', categorizing the union a'b'c'd' as fundamental group [2].

4.1.3.2. local Boolean neighborhood groups

The loop constructions used to determine the boundary connectivity of a union of connected neighborhoods can also be used to determine invariant topological connectivity of the interior space partitions formed by the neighborhoods. Subsets of loops in a fundamental group connect topologically inequivalent subsets of a union of neighborhoods to define the *homology groups* of a finite topological space.

In a local Boolean neighborhood structure, the number of topologically inequivalent groups of loops is a function of the combinatorial possibilities inequivalent base points and path lengths. Closed loops that start and stop in topologically inequivalent neighborhoods have inequivalent base points: a base point in a 2-connected neighborhood is inequivalent to a base point in a 3-connected neighborhood. Path length is measured by the number of inequivalent neighborhoods traversed before a loop returns to its base point; a path traversing a single 2-connected neighborhood is inequivalent to a path traversing both a 2-connected neighborhood and a 3-connected neighborhood. The combinatorial problem of base points and path length is simplified by the odd-even adjacency pattern of Boolean neighborhoods: a 2-connected neighborhood is always bounded by 1-connected and/or 3-connected neighborhoods; a 3-connected neighborhood is always bounded by 2-connected and/or 4-connected neighborhoods.

With the diameter of the union limiting a path length to eight inequivalent neighborhoods, the complete Boolean topology in Figure 1B.(h) constructs sixty-three local homology groups, or ***Boolean neighborhood groups***:

4 groups traversing a single neighborhood,
3 groups traversing two adjacent neighborhoods,
7 groups traversing three adjacent neighborhoods,
5 groups traversing four adjacent neighborhoods,
12 groups traversing five adjacent neighborhoods,
7 groups traversing six adjacent neighborhoods,
16 groups traversing seven adjacent neighborhoods,
9 groups traversing eight adjacent neighborhoods.

4.1.3.3. compound connectivity groups

The fundamental group categorizing boundary connectivity and the neighborhood subgroup categorizing interior spatial partition connectivity are both necessary to fully categorize the connectivity of a structured finite topological space, suggesting a compound notation:

$$[\textit{fundamental group} \,\textbf{.}\, \textit{neighborhood subgroup}]$$

The union of neighborhoods in Figure 1B.(e) forming fundamental group [2] and neighborhood subgroup [23] can be expressed as the ***compound connectivity groups* [2.23]**.

4.2. TOPOLOGICAL DEVELOPMENT OF SITUATED LOCAL PROBABILITY SPACE

A situated point of view from within the boundary of a finite topological space limits the probability of an interaction event to the population of the 'known universe' within the space: the boundary of the topological space creates the boundary of a local probability sample space. Disjoint finite topological spaces create a structure of independent probabilistic sample spaces, giving no probability of an interaction event between the spaces. In a Poincare inflation producing two disjoint unions of topological neighborhoods, situated local event probabilities develop independently in each union until their inflating boundaries intersect to form a single connected sample space.

Unlike a Markov chain of independent probabilities of successive system states, a Poincare inflation process structures a sequence of intersecting neighborhood spaces where the situated probability of a local interaction event for any step of the sequence is dependent on the probabilistic interactions of the preceding step. The sequential development of a complete Boolean topology from a set of points generates a sequence of discrete local probability spaces that form a ***situated conditional probability chain***. Probabilistic events are a function of spatial proximity, where neighborhoods or points of closer adjacency have higher probabilities of interaction that those that are more remote.

4.2.1. Situated probabilistic neighborhood structure

The situated probability of a neighborhood intersection is function of the set of neighborhoods that form a union of locally connected topological spaces where the ***situated local sample space*** of possible intersections is number of possible discrete Boolean subsets that can be formed by the number of neighborhoods in the union.

A disjoint singleton neighborhood has a situated local sample space of possible intersections of 0.

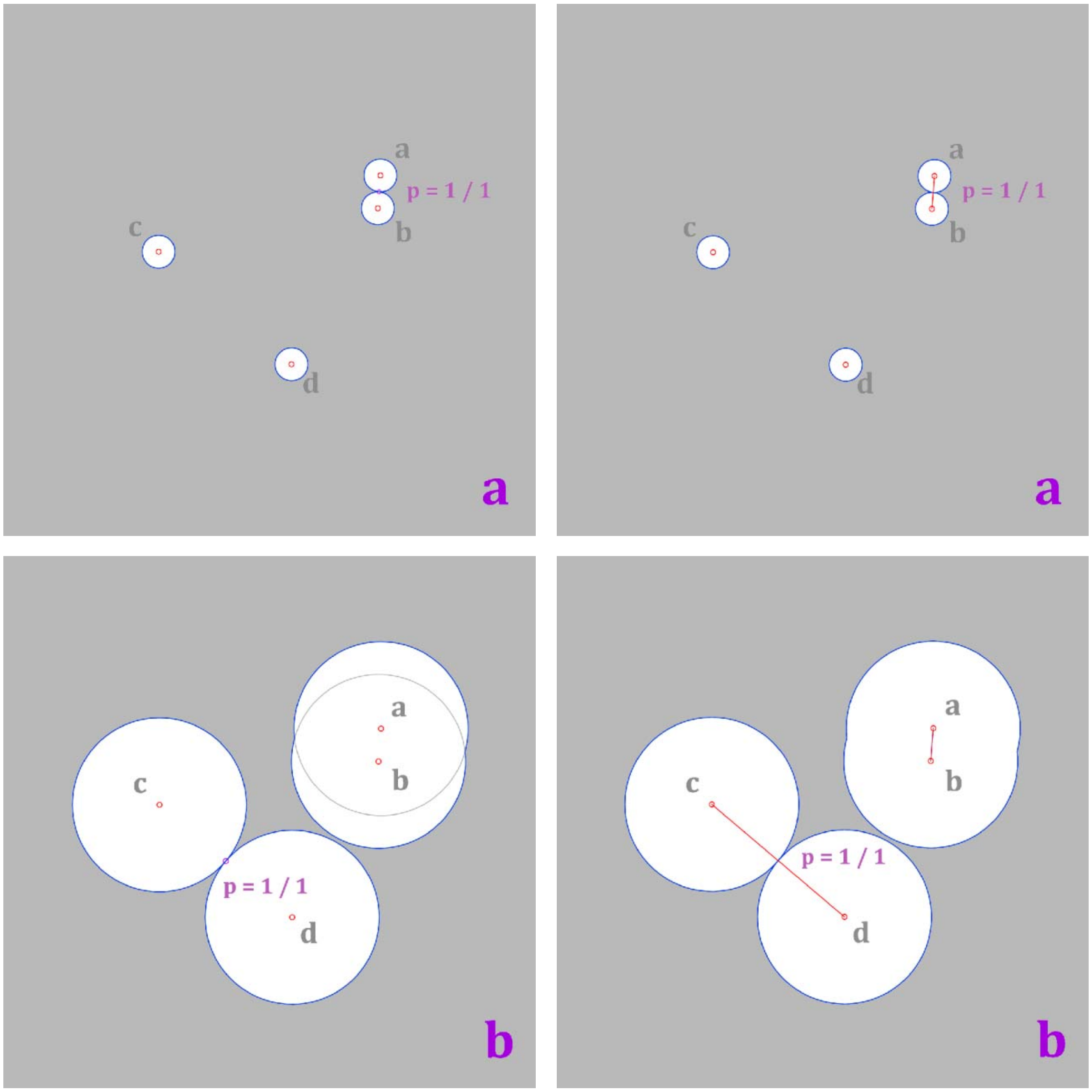

Figure 2A. Situated local probability space. Sequential development of conditional sample space states a-b. Left: neighborhood structure. Right: probability graph structure.

A sequence of discrete neighborhood intersections generated by Poincare inflation produces a sequence of *conditional sample spaces*: the number of possible intersections in each step of the sequence is the difference between the number of possible Boolean subsets in the current step and the number of subsets formed by the previous step. Sequential steps that add previously disjoint neighborhoods to an existing union increase the local sample space, steps that form a neighborhood intersection within the existing union decrease the local sample space. [Figures 2A. – 2D. left column]

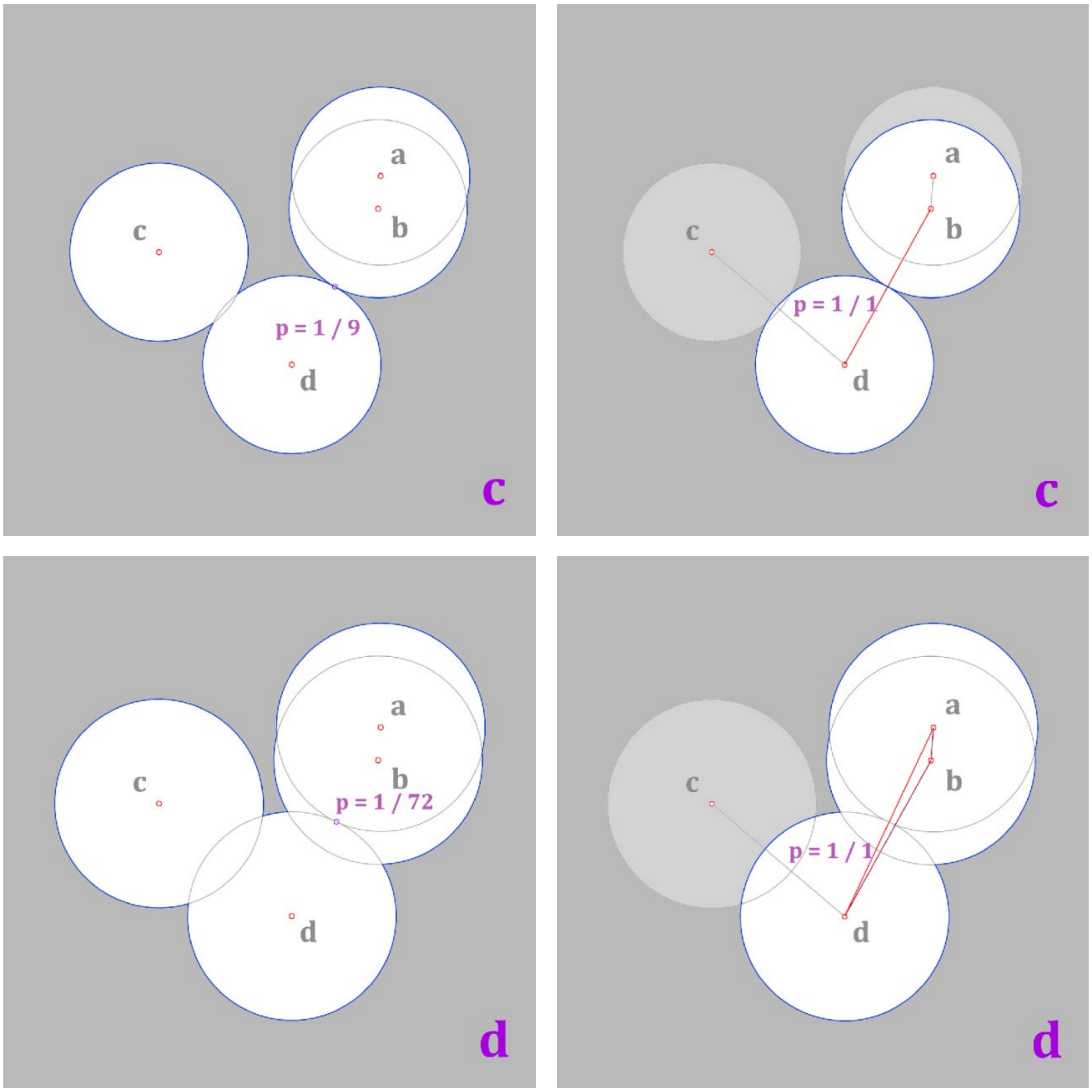

Figure 2B. Sequence of local conditional sample space states c-d. Left: neighborhood structure. Right: probability graph structure.

4.2.2. Situated probabilistic graph structure

Intuitively, following the logic of probabilistic neighborhood intersections, it may first appear that the probability of an edge forming between two points in a finite topological space would also be a function of the union of neighborhoods that structure the space. Yet when a Poincare inflation creates a union of neighborhoods to form a locally incomplete topological space where some neighborhood intersections have not been formed, the situated probability space of the union is structured as a collection of disjoint subsets of situated sample spaces. This structure of independent local sample spaces restricts the

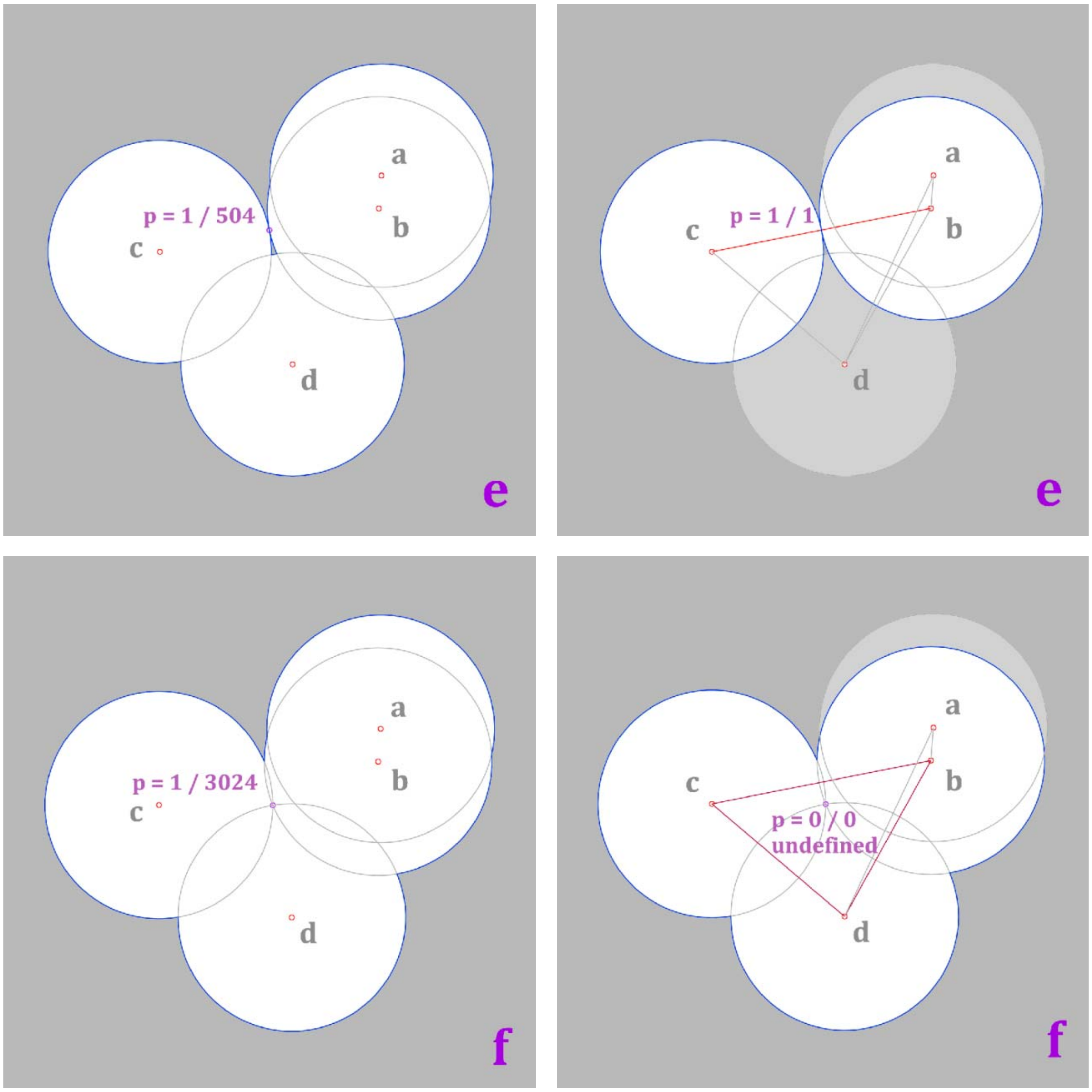

Figure 2C. Sequence of local conditional sample space states e-f.

situated probability of edge formation in a union of neighborhoods to the Boolean subsets of locally complete topological space. [Figures 2A. – 2D. right column]

In an inflation sequence of a union of neighborhoods, a single step that produces a subset of two intersecting neighborhoods presents only one possible edge to be formed, defining the subset as a local probability sample space of 1. A single step in an inflation sequence that produces a subset of more than two intersecting neighborhoods presents the possibility of multiple edges forming, but in all cases, edges formed earlier in the sequence leave only one possible edge remaining to be formed, defining the topological subset formed in the current step as a local probability sample space of 1.

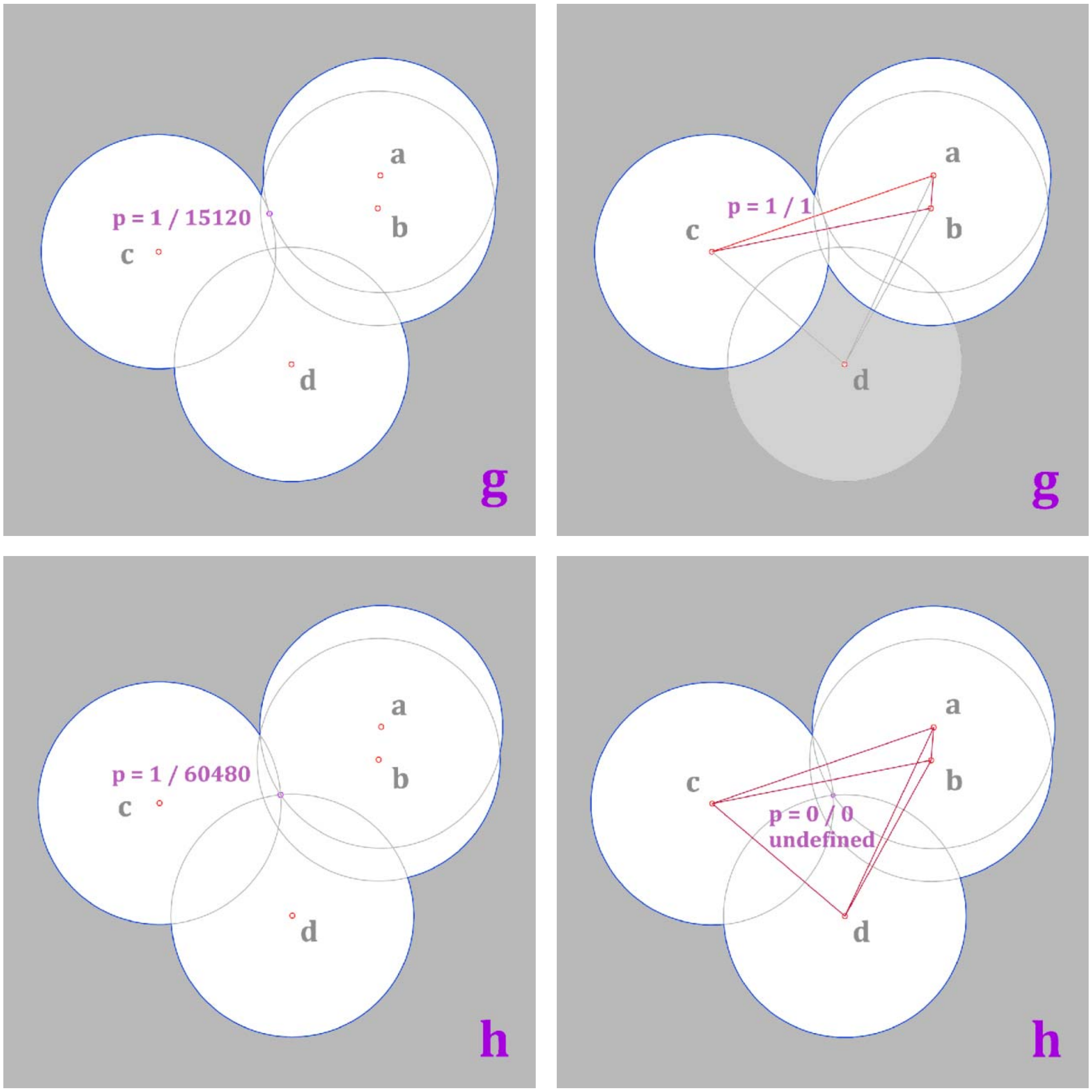

Figure 2D. Sequence of local conditional sample space states g-h. Locally complete Boolean topology(L) and complete graph (R) terminates sequence.

In a set of random points with no two points equidistant from a third, steps in an inflation sequence of a union of neighborhoods produce intersecting neighborhood subsets that define local probability edge formation sample spaces of either 0 or 1. Inflations yielding edge formation sample space subsets of 0 close ‘holes’ in the topological boundary to reduce the fundamental group of the union. Inflations yielding edge formation sample space subsets of 1 return a 100% probability of edge formation in the corresponding subset of locally complete topological space.

If a Boolean neighborhood $a'b'$ defines a local edge formation sample space of 1, the probability of edge formation between the points a and b in a union of neighborhoods containing $a'b'$ is equal to the probability of neighborhoods a' and b' intersecting to form the neighborhood $a'b'$.

The final step in an inflation sequence of a union of neighborhoods forms a locally complete intersection of the union to define both a locally complete topological space and a ***locally complete probability space***. With this step, all possible edges have been sequentially formed to probabilistically construct a *locally complete graph* of the points in the space.

The sequential formation of edges by Poincare neighborhood inflation establishes a relative distance metric between points to structure a *local metric space*, where formation of a locally complete graph defines a ***locally complete metric space***. The relationship of topologic, probabilistic, and metric distance is introduced in Section 4.3.

4.2.3. Situated probabilistic hierarchical structure

In an infinite set of random points, the initial steps in a neighborhood inflation sequence form small local unions of neighborhoods, which in turn, intersect with each other to locally structure ever larger regions of finite topological space. With indefinite inflation, these probabilistic constructions develop a countably infinite hierarchy of local set structures: hierarchical unions in locally complete and incomplete topological, probability and metric spaces.

4.3. TOPOLOGICALLY INFERRED AND CONSTRUCTED METRIC SPACES

A metric can be defined for a topological space provided that points $a, b,$ and c in the space meet the following conditions (Bing, 1951):

1. The distance from $a \rightarrow b$ is a nonnegative real number, and the distance from $a \rightarrow a$ is zero. (positivity)
2. The distance from $a \rightarrow b$ equals the distance from $b \rightarrow a$. (symmetry)
3. The sum of the distance $a \rightarrow b$ and the distance $b \rightarrow c$ is greater than or equal to the distance $a \rightarrow c$. (triangle inequality)

A *semi-metric* can be defined for a topological space provided that points *a*, *b*, and *c* in the space meet the following conditions (Wilson, 1931):

1. The distance from $a \rightarrow b$ is a nonnegative real number, and the distance from $a \rightarrow a$ is zero. (positivity)
2. The distance from $a \rightarrow b$ equals the distance from $b \rightarrow a$. (symmetry)

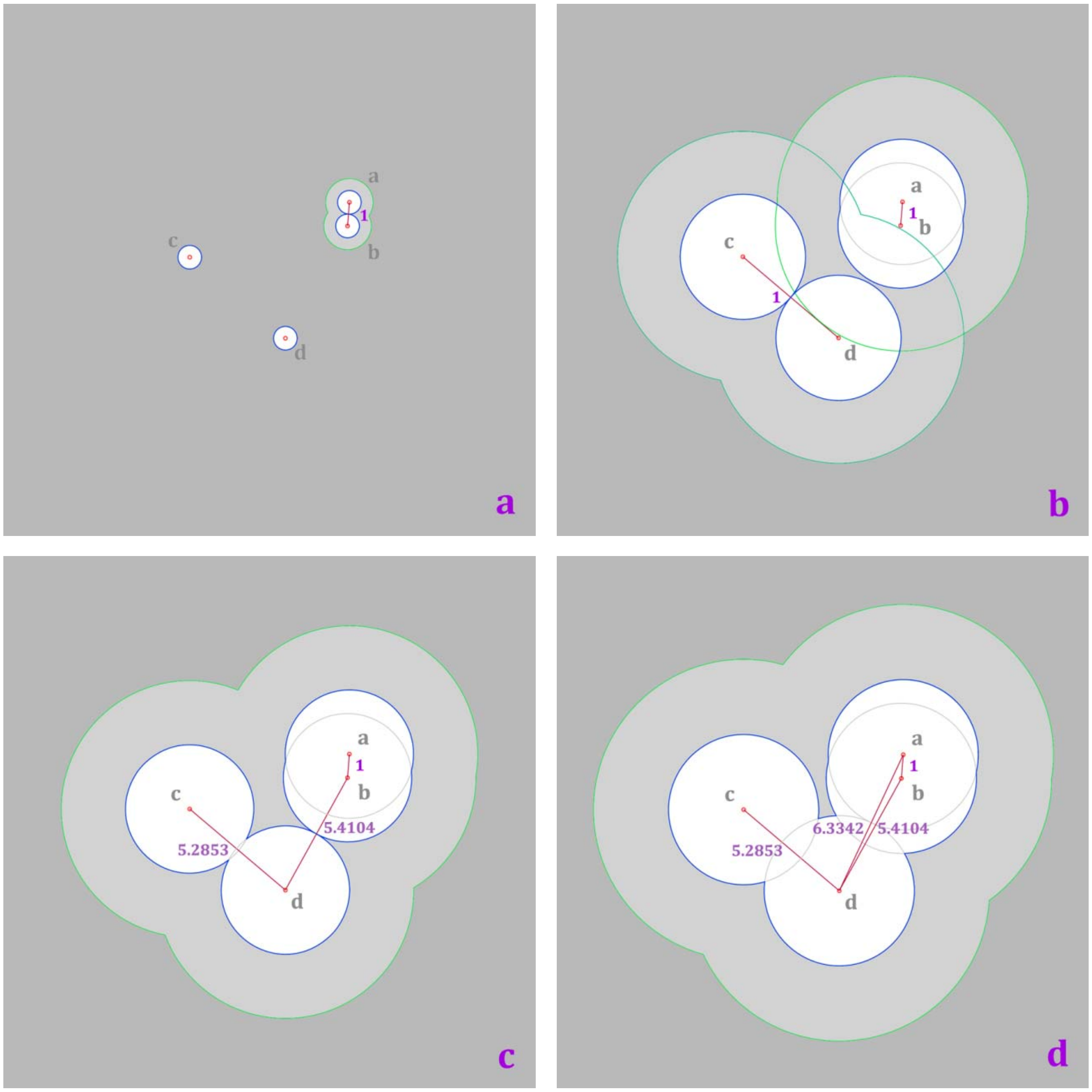

Figure 3. Development of topologically inferred metric space, states a-d.

A metric space and a semi-metric space are each sets where every edge length is defined in the complete graph of the set to conform to the above conditions.

4.3.1. Metric distance

Throughout history, from cubits, to yards, to one minute of arc of a great circle of the Earth, distance metrics have been established by cultural precedent and applied by an external observer to a finite region of space. (NIST, 1974) The cultural precedent of the external viewpoint in mathematics has resulted in a convoluted intersection of less restrictive topological space being developed from a more restrictive metric space, producing a *metric topology* having topological properties that are a function of an externally

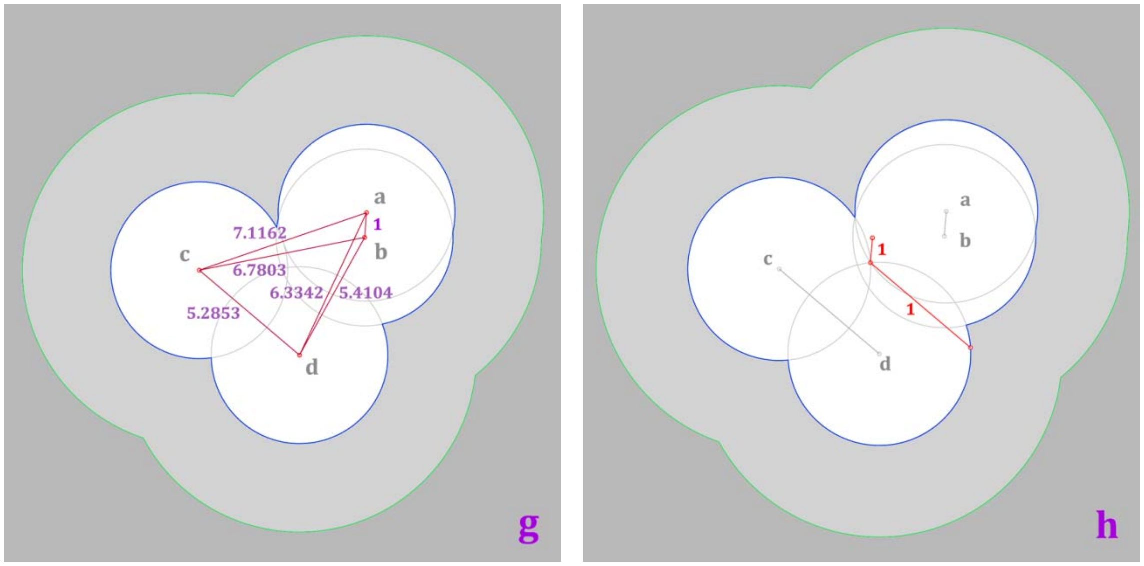

Figure 4. Local metric distance and coordinate axes inferred from topological space.

applied metric space. A topological space is *metrizable* if a metric can be defined that induces a topology on the space.

From a situated viewpoint from within a local topological space, metrics can be inferred from the structure of the space itself. However, no metrics can be inferred from within a singleton neighborhood. The intersection of two neighborhoods provides the initial basis for situated metric inference by establishing a referential a unit of measure between the two points. That additional neighborhoods have not been encountered, and assuming the known neighborhood inflation rate is universal, a situated inference can also be made to extend the reference metric to the unknown region of unstructured space external to the neighborhood boundary. [Figure 3.]

While situated inference can generate a local metric space that covers holes and extends beyond the boundaries of its precursor topological space, Section 4.2 presented a construction sequence of local metric spaces as a determinate of the probabilistic intersections of inflating topological neighborhoods. A ***constructed metric space*** is defined by the boundary of the underlying local union of topological neighborhoods of a set of points.

A locally incomplete topological union of neighborhoods defines the boundary of a ***locally incomplete metric space***, with the boundary of a ***locally complete metric space*** defined by the boundary of the underlying locally complete topological and probability spaces.

Following Auguste Bravais' logic in his 1850 delineation of coordinate axes forming the five possible plane lattices and fourteen possible space lattices in Euclidean metric space (Buerger, 1956; Bloss, 1971; Giacovazzo, 1992), local coordinate axes can be inferred from a constructed metric space.

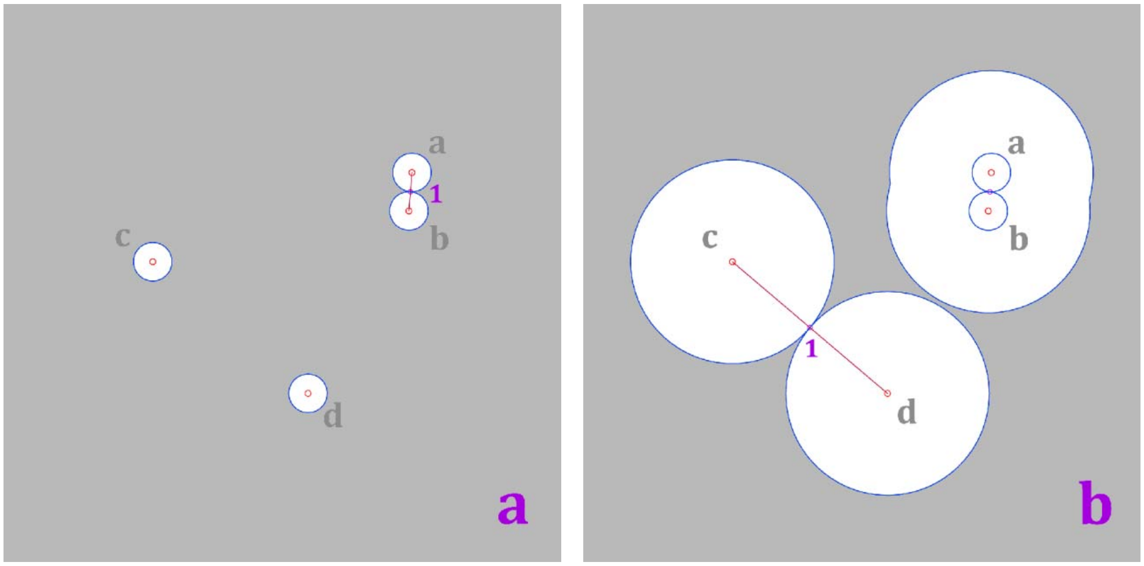

Figure 5A. Development topological distance and semi-metric topological space. Discrete neighborhood intersection sequence, states a-b.

In a Poincare inflation sequence, the limit point of the final intersection constructing a locally complete union of neighborhoods presents an appropriate origin for a local coordinate metric on locally complete metric space. In two dimensions, the first two steps of an inflation sequence construct the two shortest edges in the graph of a local union of neighborhoods. Assuming they are nonparallel, each edge can be translated to map an endpoint on to the selected origin point, defining the angular relationship and unit lengths of two local coordinate axes. [Figure 4.(h)]

4.3.2. Topologic distance

The unit interval of structural change in the development of a Boolean topology of a set of points is marked by the formation of a limit point of a neighborhood intersection. In a locally connected neighborhood cluster containing two points *a* and *b*, the topological distance between *a* and *b* can then be defined as the number of limit point intervals developed by the Boolean topology to produce an intersection of the neighborhoods *a'* and *b'* to form a locally complete topological space. [Figure 5A.(a)]

In simple cases, the spatial pattern of intersecting neighborhoods can be examined graphically in a manner similar to a dendrochronology investigation of the growth rings of trees: topological distance can be determined by counting the number of neighborhood boundary rings that have formed between adjacent points.

The sequence of limit points generated by intersecting neighborhoods forms a path of equal edge lengths, each with the topological unit distance 1. [Figure 6.(i)] The sequence of edges generated by

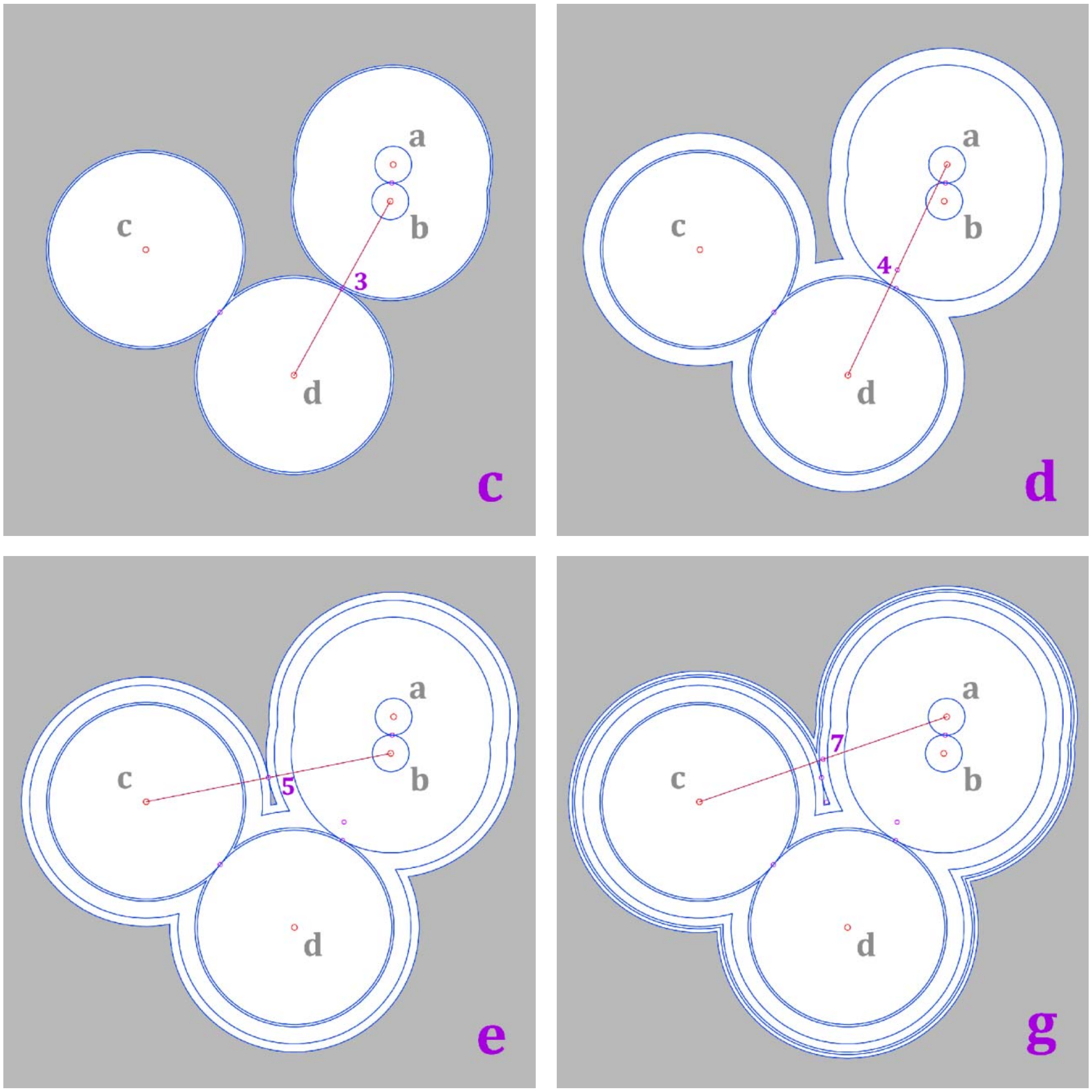

Figure 5B. Topological distance and semi-metric topological space. Discrete neighborhood intersection sequence, states c-g.

intersecting neighborhoods form a *topological graph*, with the edge lengths revealing a complete Boolean topology to be a semi-metric space. [Figure 6.(h)]

The early discrete steps in a developing Boolean topology that have no 'holes', a local boundary connectivity of fundamental group [0], and the first discrete step that yields at least one boundary 'hole', fundamental group [>0], define edge lengths in a topological graph that meet the prerequisites of a metric space. In these cases, metric space and topological space are homeomorphic: a continuous function and its continuous inverse exist between the two spaces. [Figure 7.(e)]

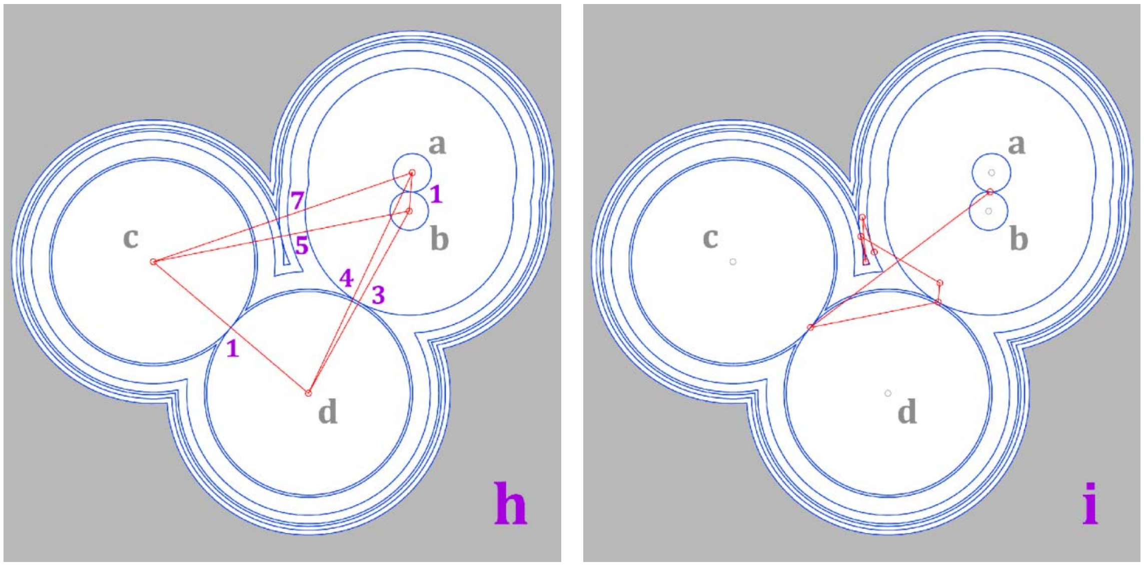

Figure 6. Locally complete semi-metric topological space. (h): Discrete sequence of inflating neighborhood intersections (blue) constructs a locally complete union of neighborhoods and a complete graph (red), defining topological distance between vertices. (i): Path of limit points trace a neighborhood intersection sequence to form topological graph of uniform edge length 1.

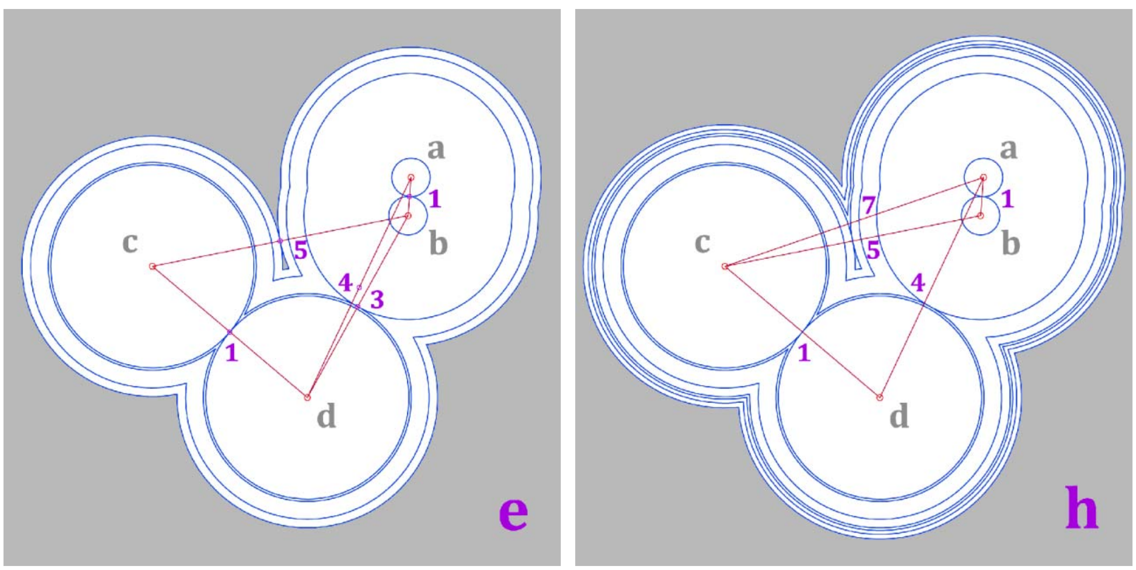

Figure 7. Local topological-metric space phase transition, space states e and h.
(e): Homeomorphic critical state of local topological and metric space. (h): Discontinuity between semi-metric local topological space and local metric space.

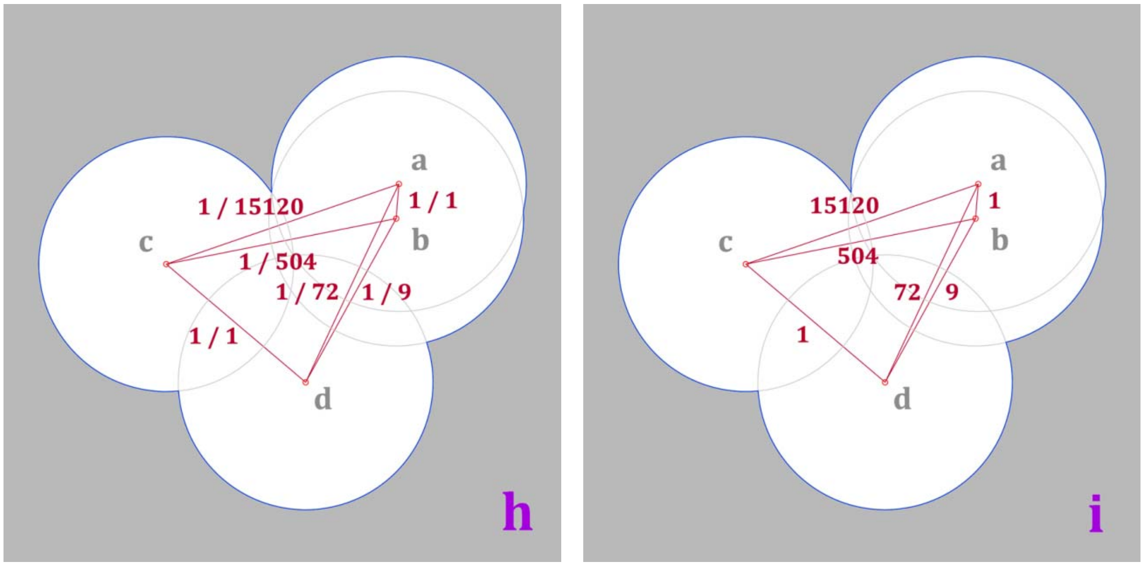

Figure 8. Situated probability distance.

After an initial 'hole' closes to change boundary connectivity from fundamental group [n] to fundamental group [n-1], the next sequential edge formed by neighborhood inflation alters the graph structure to no longer fulfill the conditions of triangle inequality: topological space jumps from a metric space to a semi-metric space. Development of a locally complete Boolean topology introduces a discontinuity between topological and metric spaces, with continuous mappings between the two no longer possible. [Figure 7.(h)]

4.3.3. Probabilistic distance

The probabilistic formation of neighborhood intersections from a set of points establishes a situated local probability space, where the probability of an edge forming between two points can be interpreted as a distance between those points. Counter to an intuition that greater distances correspond to increasing metric values, increasing distances in probability space are measured as decreasing probabilities. [Figure 8.]

Edge lengths formed in the sequential development of a complete Boolean topology of an infinite set of points construct a one to one mapping between a decreasing probability sequence that converges to the limit 0 and an increasing metric sequence that converges at the limit infinity. The probability of points interacting in space is inversely related to their metric distance.

An interpretation of probabilistic distance as the reciprocal function of intersection probability, $1/p$, allows edge formation in a probabilistic graph of an infinite number of points to generate an increasing sequence that converges to the limit infinity. This graph structure reveals situated probability space to be a semi-metric space, homeomorphic to situated topological space.

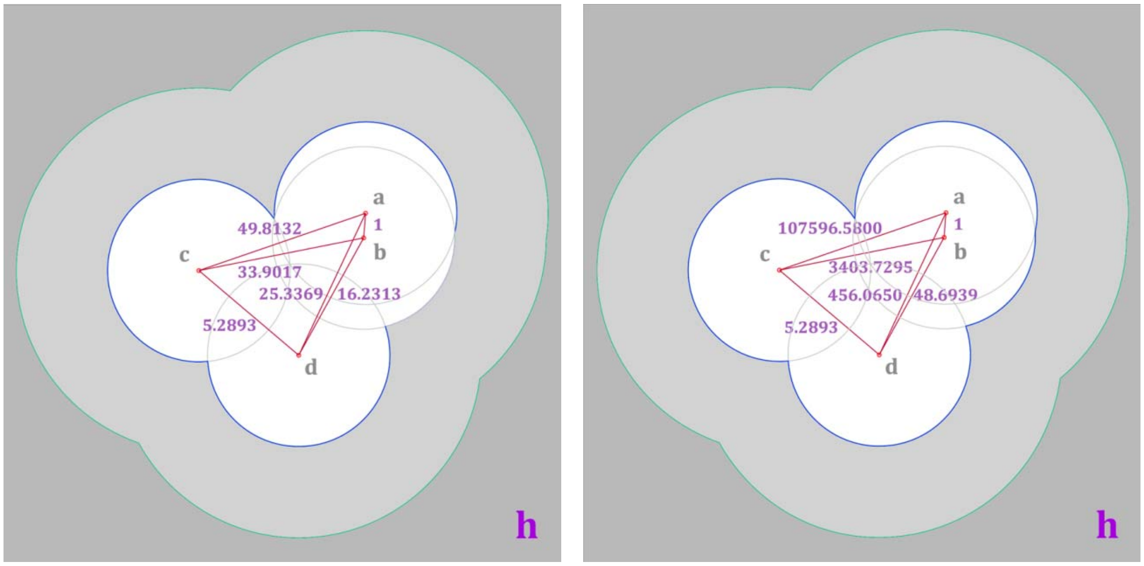

Figure 9. Locally complete composite spaces, state h. (Left) Topologic-metric composite space. (Right): Probabilistic-metric composite space.

4.4. BRIDGING TOPOLOGICAL – METRIC SPACE DISCONTINUITY

The morphism of Poincare neighborhood inflation can be interpreted as a multidimensional vector, or *tensor*, that establishes the local space interior to a union of connected neighborhoods as a topologic vector space, a probabilistic vector space, and metric vector space. This commonality of vector spaces suggests a method of bridging the discontinuity between the structures of metric space and the semi-metric structures of probabilistic and topological spaces. The edges of a graph formed by neighborhood tensor inflation define collinear metric, probabilistic, and/or topological vectors. The product of a pair of collinear metric and semi-metric vectors yields a scalar distance to structure a ***composite space***.

4.4.1. Topologic-metric index

The product of the topological vector and the collinear metric vector between two points in a locally structured Boolean space produces a scalar topological-metric distance between the points: a ***topologic-metric index*** value. The topologic-metric index graph reveals the composite topological-metric space to be semi-metric. [Figure 9.]

4.4.2. Probabilistic-metric index

The product of the probability vector and the collinear metric vector between two points in a locally structured boolean space produces a scalar probability-metric distance between the points: a ***probability-metric index*** value. The probability-metric index graph reveals the composite probabilistic-metric space to be semi-metric. [Figure 9.]

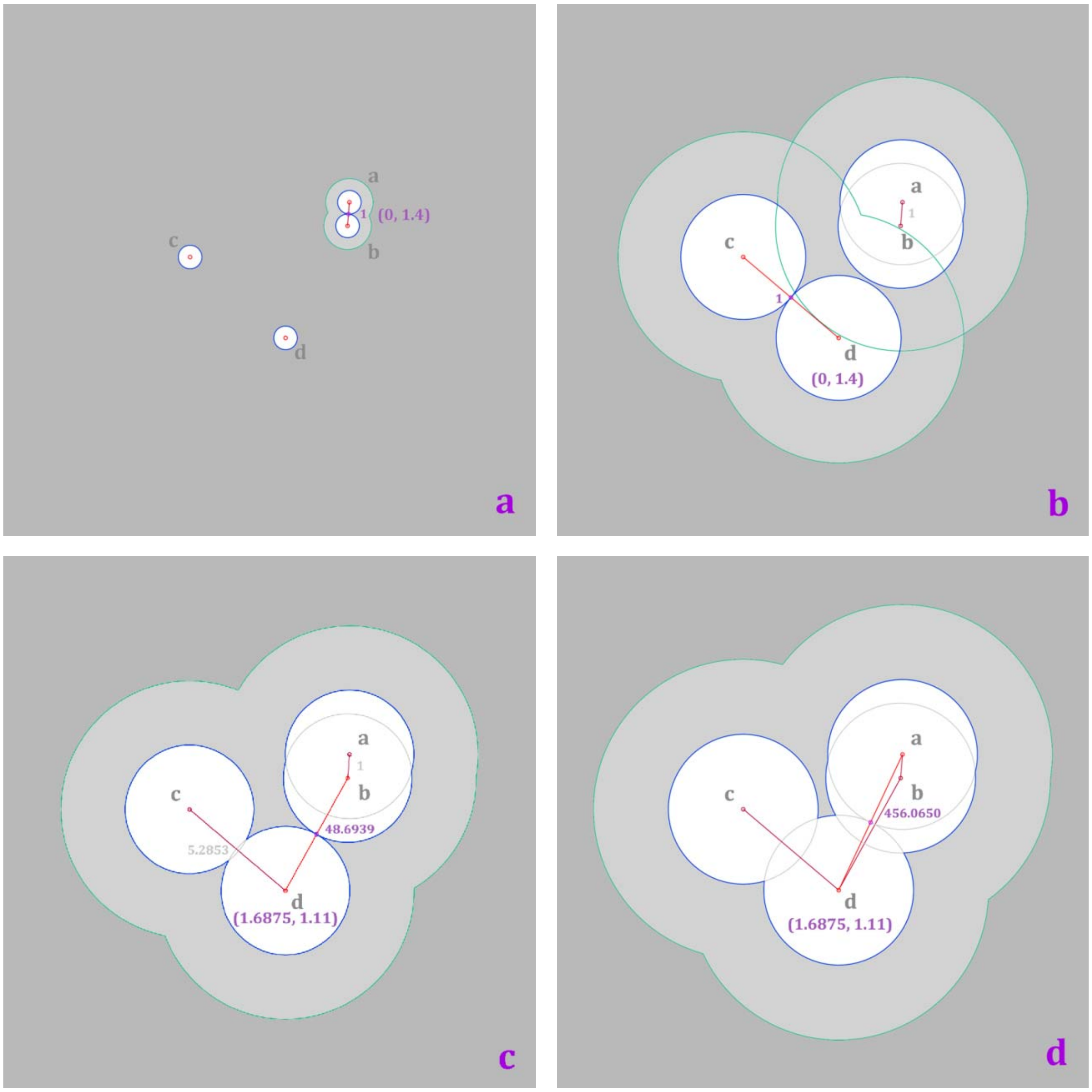

Figure 10A. Architectural identity. Initial development of architectural descriptors leading to the architectural identity of a locally complete 3-composite topologic-probabilistic-metric space. Discrete neighborhood intersection sequence states a-d.

4.5. ARCHITECTURAL IDENTITY

A unique numerical identity defining of a set of points can be obtained by the combination of situated connectivity measures of local metric, probabilistic, and topological spaces developed by the complete Boolean topology of the set:

> The situated distance metric inferred from a Boolean topology defines a connectivity triangulation structure: the relative positional distances specific to a set of points.

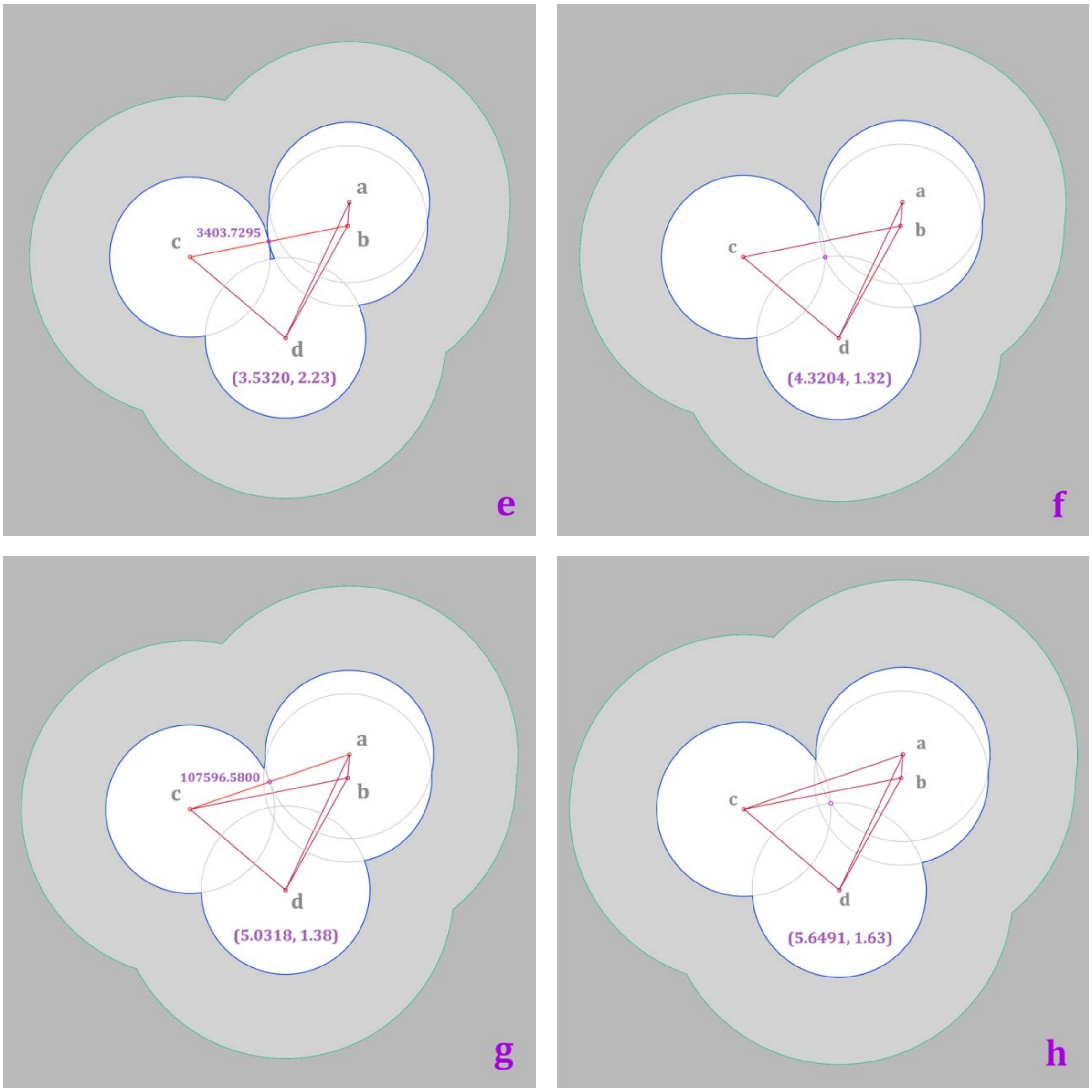

Figure 10B. Discrete sequence states e-h, terminating in the locally complete Boolean topology and unique architectural identity of the situated configuration (a,b,c,d).

> The situated probability sequence of neighborhood intersections developed by a Boolean topology defines a temporal semi-metric connectivity structure: the assembly sequence specific to a set of points.
>
> Situated topology defines both boundary and internal connectivity structures of local space: the spatial partitioning specific to a set of points.

Taken together, these three situated connectivity measures of a locally complete Boolean topology form a unique numerical ***architectural identity*** to define a specific set of points.

4.5.1. Ordered pair identity

To bridge the discontinuity between semi-metric and metric spaces, Section 3.4 introduced the combined the probabilistic and metric components of an architectural identity, the probabilistic-metric index, to create a distance function in a composite probabilistic-metric space.

That a sequence of connection probabilities rapidly descends several orders of magnitude for even a very small set of points, probabilistic-metric index values have been expressed as the base 10 logarithm of the product of the probability and metric vectors.

By using the probabilistic-metric index, the architectural identity of the ***completely developed*** finite space of a set of points can then be reduced to two terms, expressed as an ordered pair [Figures 10A. – 10B.]:

$$(\textit{probabilistic-metric index, compound connectivity group}).$$

4.5.2. Architectural descriptor

In an incomplete union of neighborhoods, the ordered pair $(\textit{probabilistic-metric, compound connectivity})$ may represent a range of similar but inequivalent point sets, yielding a non-unique ***architectural descriptor***. In the developmental spaces of a large set of points that form disjoint clusters structured as incomplete unions of neighborhoods, a subset cluster can be extracted from the set and developed in isolation to reveal its specific architectural identity.

4.5.3. Semi-metric space triangulation

The triangulation of three points defines fundamental geometric units of structural stability (Williams, 1972; Fuller, 1982; Wester, 2002) and unique positional information (Davenport, 2013) in a metric space. The construction of a metric space by the probabilistic interaction of local topological spaces induces the concepts of a ***topological triangulation***, and a ***probabilistic triangulation***.

A ***locally triangulated union***, or structurally, a ***locally stable union***, is formed when every neighborhood in a union of local spaces is part of a 3-connected intersection. This marks a critical threshold in an incomplete union of neighborhoods, where an architectural descriptor undergoes a discrete transition to become a unique architectural identity of a set of points. This writing has developed the concept of a unique architectural identity as a dependent on the formation of a locally complete space. Other than a difference in computation time, the implications of accepting a locally triangulated incomplete space as the standard for architectural identity are currently unclear.

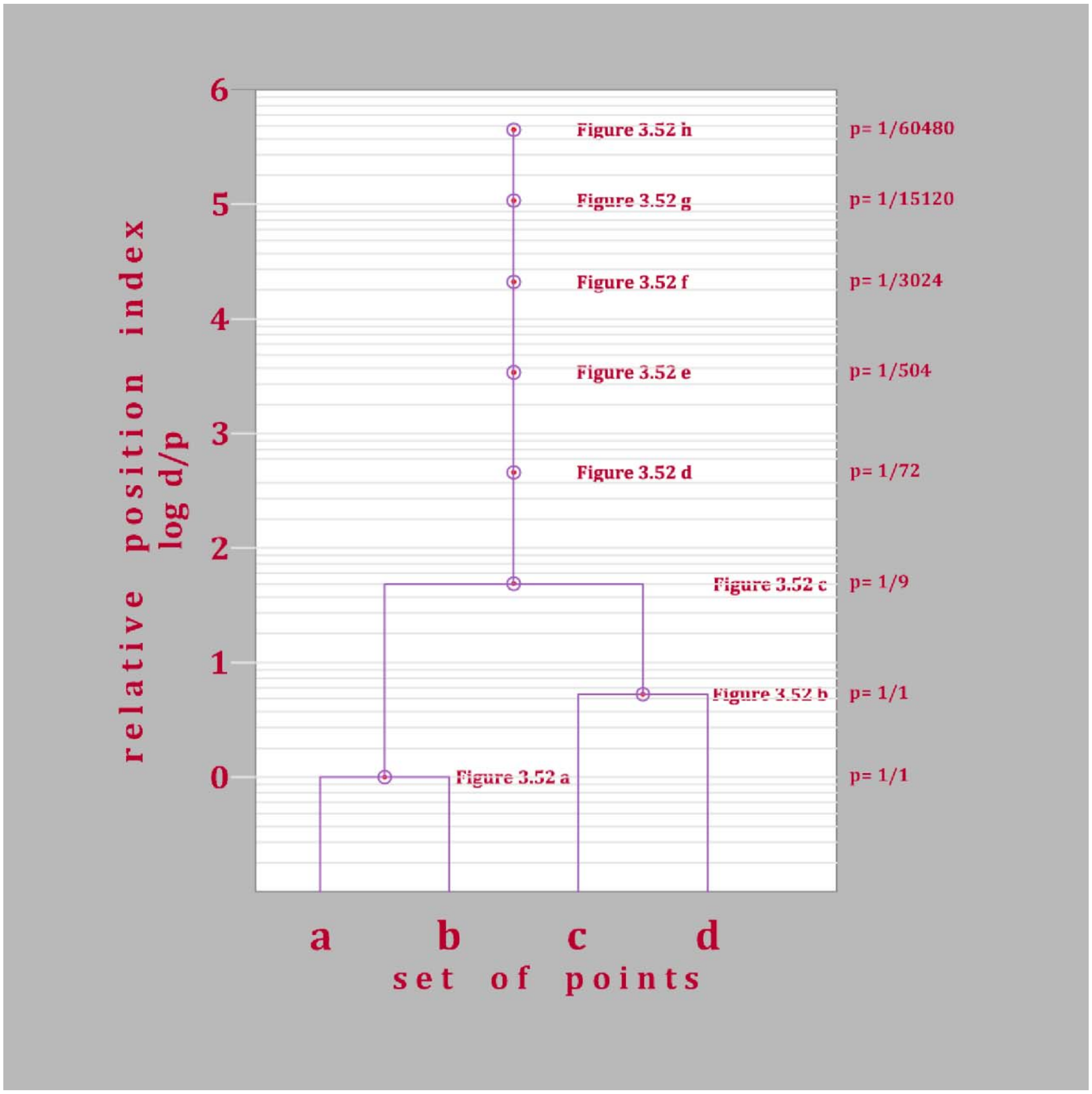

Figure 11. Configuration dendrogram. Hierarchical neighborhood intersections, states a-h.

4.6. ARCHITECTURAL CONFIGURATION SPACE

Dendrograms are graph theoretic structures commonly employed to model the relationships and discrete developmental sequences of a set of elements (Posada, 2001; Rosenberg, 2002; Butts, 2004; Li, 2004; Baum, 2005; Marks, 2006). While they offer a clear visual expression of the discrete subsets formed by the hierarchical unions of elements observed in structural development, dendrograms are limited by their inability to delineate structural changes occurring within the boundary of a developing union. [Figure 11.]

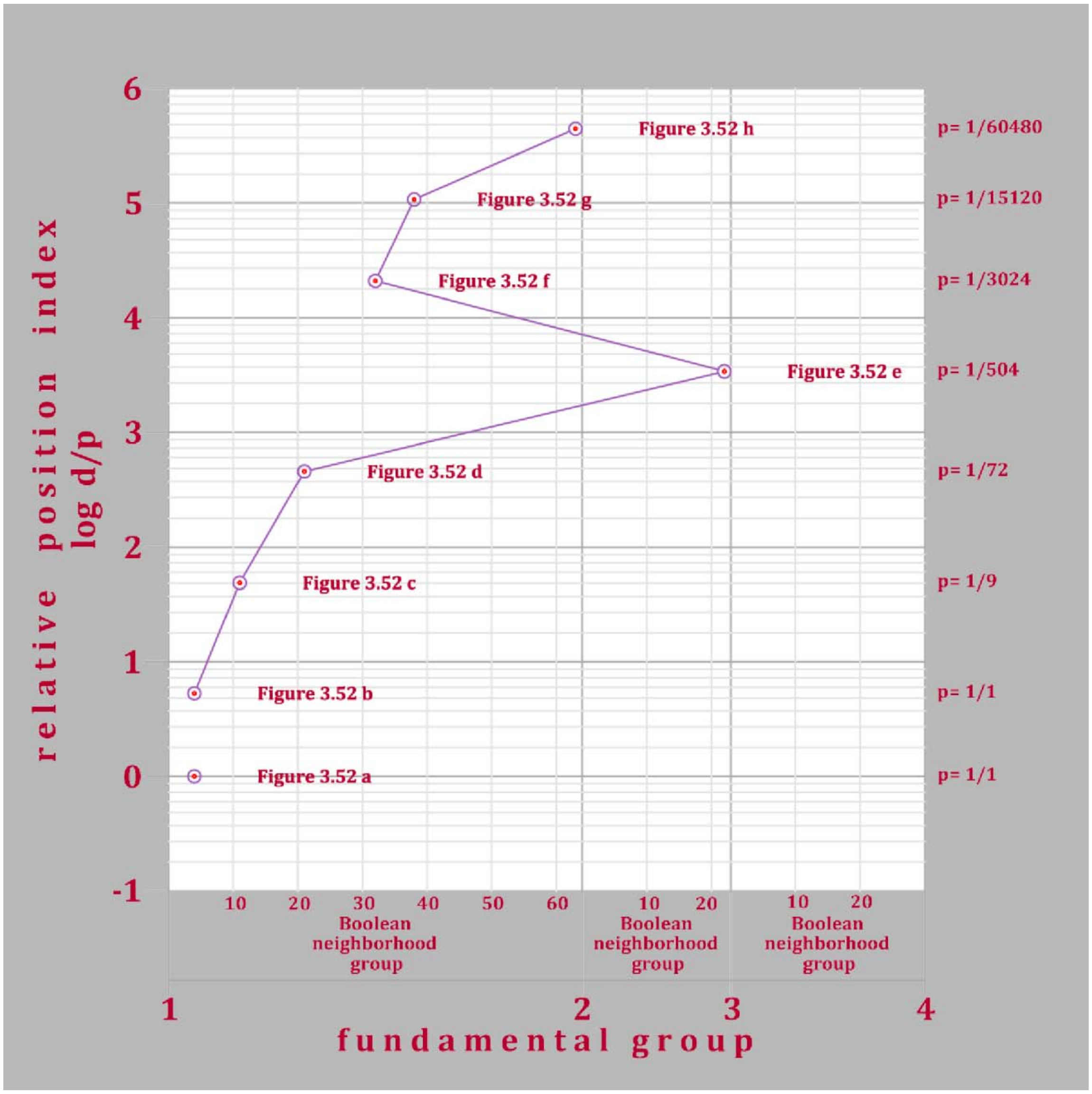

Figure 12. Architectural configuration space. Discrete architectural trajectory of locally complete Boolean topology, states a-h. Discrete developmental sequence constructs the unique architectural identity of the situated configuration (a,b,c,d).

In 1932 geneticist Sewall Wright introduced the concept of a *configuration space* to delineate the range of structural possibilities available to individuals in an adaptive population. Mapping differences in boundary and internal structure, a configuration space is comprised of two elements (Wright, 1932, 1988; Stadler, 2002; Vassilev, 2003):

1. A *search space*: the set of all combinatorial possibilities of a set of elements.
2. A morphism: an operation that transforms one configuration into another and structures adjacency and the accessibility distances between configurations.

Structuring a configuration space presents two open problems:

1. the quantitative definition of the specific set of points that comprise a configuration
2. the derivation of a quantitative similarity-dissimilarity measure differentiating two discrete configurations.

The problems of discrete configuration identification and differentiation are currently encountered by every branch of science in the search for non-linear relationships in data: finding clusters, groups, patterns, or classifications that correlate to properties. (Aldenderfer, 1984; Jain, 1988; Vassilev, 2003; Gaertler, 2005; Fielding, 2007; Landau, 2010) Performed by an assortment of ad-hoc discipline-specific algorithms developed without a unifying mathematical formalism, current searches for non-linear structures uniformly return results of questionable validity. (Bailey, 1982; Riesen, 2010; Evertt, 2011; Arbelaitz, 2013) The inability to define a unique set of points structuring a configuration as an individual discrete mathematical object has curtailed scientific application of Wright's concept. (Whitlock, 1995; Reidys, 2002; Skipper, 2004; Borenstein, 2005; Jin, 2005)

4.6.1. Structure of configuration space

Discontinuities induced by the external point of view in mathematics and statistics disclose a Godelian truth: differences between two discrete set of points can be readily observed and described, but cannot be precisely defined. With a situated point of view producing an architectural identity that gives an exact definition to a specific set of points, an unstructured search space of combinatorial configurations can be structured as a metric space: an ***architectural configuration space***. The Euclidean distance between two points in an architectural configuration space quantifies the structural similarity-dissimilarity between two discrete configurations.

The parameters outlined in Section 4.5 enable construction of a three dimensional architectural configuration space structured by metric, probabilistic, and topological coordinate axes. The introduction of a probability-metric composite space in Section 3.4 permits a structure of two axes, with the coordinates of a point in an architectural configuration space defined by the ordered pair of an architectural descriptor or an architectural identity [Figure 12.]:

$(\textit{probability-metric index, compound connectivity groups})$.

4.6.2. Trajectories in configuration space

A sequence of architectural descriptors that concludes with an architectural identity forms the ordered ***architectural set*** of a configuration of points. An architectural set traces a discrete path or tree graph in configuration space to form an ***architectural trajectory***: the ***architectural graph*** unique to that set of points.

5. SITUATED CONCLUSIONS

Now this is not the end. It is not even the beginning of the end. But it is, perhaps, the end of the beginning.

(Winston Churchill, 1942)

5.1. STRUCTURES AND PROPERTIES OF THE MODEL

Using a term like nonlinear science is like referring to the bulk of zoology as the study of non-elephant animals.

(Stanislaw Ulam, in Campbell, 2004)

5.1.1. Space-time

In the simultaneous development of local metric space and local semi-metric topological-probability space from a set of points by Poincare inflation, the initial neighborhood intersection of 2 points establishes a common unit interval 1 for metric, topological, and probability distances specific to the local space of the set. With the concept of topological distance measured by discrete neighborhood intersection increments of time, development of a complete Boolean topology of a local space constructs a probabilistic discrete space-time model of interacting fields.

From an initial condition of a random set of points, hierarchical structure is constructed by recursion of the locally probabilistic intersections of neighborhoods. The sequence of neighborhood intersections traces the space-time developmental trajectory of a set of interacting fields in configuration space to construct a quantitative definition of hierarchical boundaries: a scale-independent predictive model of the nonlinear processes and structures of inductive pattern formation.

5.1.2. In absentia

Following models of classical physics where temperature and pressure of the environmental container determine the states and properties of matter within the container, *in situ*, *in vivo*, and *in vitro* experiments in science have been followed by *in silico* heuristic models that construct external order parameters as the determinants of phase transitions of pattern formation. From the *ADMET* (absorption, distribution, metabolism, excretion, toxicity) interactions of pharmaceuticals to the phase transitions of physics, attempting to define the properties of a configuration from topological and graph theoretic structural models of invariant internal order parameters has been an open problem.

In a balloon-like Poincare inflation of the boundary of a point into its neighborhood, topological properties remain unaltered. With recursion of this topological *isomorphism*, the sequence of local interactions within a set of elements constructs an *in absentia* model of invariant structural parameters of probabilistic self-organization: the internal order parameter of a system, independent of the properties of an external container. Pattern formation in this model is dependent solely on the initial relative spatial position of elements, structuring a *base case* local probabilistic model of self-assembly without central control mechanisms. The influences of external order modifiers can be modeled by combinations of varying initial grain size, synchronous and asynchronous inflation rates, interactions of excitatory and/or inhibitory point sets, and threshold limits of intersections. (Appendix C)

5.1.3. Causation

The multiplicity of interacting elements within a hierarchical boundary of a configuration construct nonlinear properties of *parallel causation*, modeled by the Boolean ring and the situated Boolean topology structures of a ***causative configuration***.

5.1.4. Coherence

The predictive performance of structural modeling is dependent on structural coherence across four model-phenomena order parameters: central v. local control, unstructured aggregate v. hierarchically local specificity, pattern v. process taxonomy, and deductive v. inductive epistemology.

5.1.4.1. control coherence

Alan Turing's seminal paper *The Chemical Basis of Morphogenesis* (Turing, 1952) established two precedents in modeling self-regulated pattern formation, now known as reaction-diffusion models. The reaction between Turing's inhibitory and excitatory theoretical morphogens has been widely confirmed, most notably in the neural connectivity patterns of the brain. (Schwartzkroin, 1980; Isaacson, 2011; Ben-Ari, 2012) However, the diffusion process of the model presents a problem of local positional information available to an individual morphogen, inhibiting widespread acceptance by experimental biologists. (Wolpert, 1969, 1971, 1996, 2009, 2011; Kerszberg, 2007; Kondo, 2010)

A diffusion process models pattern formation using the continuous mathematics of concentration gradients: changing density functions of the number of elements per unit of area or volume. This information is not available to an individual embedded element but is derivable by an external observer or control mechanism with the oversight ability to count elements and calculate area or volume from defined spatial boundaries. This is readily accomplished by a control mechanism integral to the system or the divine external control agent envisioned by Newton and Leibniz in their development of continuous mathematics. If the elements of pattern formation being modeled have no physical expression of an integral control structure, then a density gradient is a structure artificially imposed by the mathematics of the model.

Patterns reproducible by reaction-diffusion models have also been generated by the situated local connectivity models of *cellular automata*. (Gardner, 1970, 1971; Langton, 1984, 1990; Toffoli, 1987; Bays, 1987, 2012; Rucker, 1989; Wolfram, 2002) A situated point-of-view structures the internal mathematics of an individual element embedded in a gradient as a computation of locally available information. From the element's point-of-view, the universe that is external to electro-magnetic, gravitational, or other sensory fields simply does not exist. In a situated model, an individual element only 'knows' its local neighbors – the situated intersections of its local field. With no oversight control mechanism, it is the parallel processing of local information by a configuration of elements that constructs the emergent systemic structure and properties of the configuration.

5.1.4.2. specificity coherence

Descendants of the seminal model ontologies of discontinuous inductive pattern formation, *statistical mechanics* (Gibbs, 1902), *quantum mechanics* (Born, 1924), *percolation theory* (Broadbent and Hammersley, 1957), *discrete-event simulation* (Lackner, 1964), *catastrophe theory* (Thom, 1972), *Boolean-grain random sets* (Matheron, 1975), and *connectionism* (Rumelhart and McClelland, 1986), have evolved as structure-property constructions of external point-of-view probability and continuous mathematics, effectively limiting the optical resolution of predictive models to an aggregate-population level of specificity. These linear mathematical constructions of unstructured collections create structural dissonance when applied to the hierarchically local interactions of pattern formation.

In contrast, the discrete development of a situated Boolean topology constructs a probabilistic process-structure model that unites connectionist, discrete-event, Boolean-grain, and percolation ontologies with the optical resolution of atomistic local specificity: a recursive sequence of specific local connections constructs the unique architectural identity of a configuration, revealing the systemic structure and properties emergent from a specific set of initial conditions.

In configuration space, with structural similarity-dissimilarity measured by the Euclidian distance between unique architectural identities, similar architectural identities will form structure-property clusters to display the probabilistic taxonomy of categorical patterns. These configurations of architectural identities, or configurations of configurations, form peaks and valleys in a probabilistic fitness landscape of the internal order parameters of pattern formation.

5.1.4.3. taxonomy coherence

The success of scientific modeling over the past three centuries has centered on the examination of pattern-property relationships, developing external point-of-view models that construct a high fidelity reproduction of pattern: the discrete qualitative mathematical models of binding configurations and the continuous quantitative models of quasi-binding and non-binding configurations.

However, there is no evidence to suggest these are calculations performed internally by the elements of a configuration in a self-regulated process of pattern formation. Analogous to the concentration gradients of reaction-diffusion models of pattern formation, the point and space group symmetry operations of mathematical crystallography faithfully reproduce the 3-dimensional structures found in crystalline solids, but in the process of mineralization, atoms do not physically rotate, mirror, or translate across spatial distance to construct a pattern. Both of these examples accurately replicate structure but not process: external point-of-view mathematics constructs a descriptive model of a pattern. Continuous mathematics provides a quantitative documentary of change in non-hierarchical pattern over time but does not reproduce the causative internal computations of a self-organizing process of pattern formation.

Process-pattern models require a modeling taxonomy that departs from the precedents established by pattern-property models. Parallel processing of local neighborhood information by the individual elements of a configuration structures the nonlinear self-assembly process of inductive pattern formation. A situated point-of-view constructs the necessary structural coherence between model and phenomena.

5.1.4.4. epistemology coherence

The issues of structural dissonance reflect the underlying philosophical inconsistency of attempting to model the inductive nonlinear processes of phenomena with a deductive linear process of investigation.

The external observer's statistical approximations and deterministic certainty of deductive inference that evolved within the computational limits of pencil and paper are restricted to problems that are computationally *decidable*. While these precedents are embedded in the information structures and processes of Western culture, it is the probabilistic certainty of situated inductive inference that is embedded in the information structures and processes of natural systems.

Although much of inductive pattern formation appears to be computationally *undecidable*, it can now be addressed with the electronically extended computational limits of parallel algorithmic recursion. The parallel recursive development of an architectural identity quantitatively traces the evolutionary space-time trajectory of a configuration of intersecting fields. This predictive process-pattern model offers structural coherence with the discontinuities of nonlinear hierarchical inductive pattern formation of natural and artificial systems.

5.1.5. Construction

In the context of accelerating advances in *DNA computing* and *computational nanotechnology*, the low computational complexity of the model enables it to be embedded in the physical materials of human niche constructions, creating the artificial life forms of an adaptive human habitat. Undergoing the parallel distributed processes of situated inductive pattern formation, bipartite recursion of information structures and their physical expression constructs the evolutionary trajectory of hierarchically local natural and artificial adaptive configurations. (Appendix B)

5.2. STRUCTURE AND PROPERTIES OF HUMAN NICHE CONSTRUCTIONS

> *There are certain moments in any mathematical discovery when the resolution of a problem takes place at such a subconscious level that, in retrospect, it seems impossible to dissect it and explain its origin. Rather, the entire idea presents itself at once, often perhaps in a vague form, but gradually becomes more precise.*
>
> (Paul Cohen, 2002)

With variables spanning the domains of man, culture, and environment, there is little, if anything, that does not impinge, directly or indirectly, on the design and fabrication of the human habitat. Human niche constructions are an extension of the human phenotype: the physical expression of the combined information structures of the human genotype and the extended genotype, the memetic information structures of human culture. Pattern and properties of the extended phenotype are shaped by the configuration of local interactions of the human phenotype, human genotype, cultural memotype, and the genotype-phenotypes of their collective environmental container.

5.2.1. Critical state of the environmental container

Reinforced by the few inclusions of nature in urban crystalline structure, the precedents of Judeo-Christian theology and external-observer science have established a western *collective consciousness* of man-apart-from-nature. Nature is authoring the dissenting opinion.

With global climate change being just one parameter of a changing biosphere, there is rapidly mounting evidence of an imminent planetary phase transition: the sixth period of mass extinction in earth's 4.5 billion year history. Conforming to the top-down process of hierarchical disordered phase transitions, evidence specific to the extinction of mammal and primate populations has direct bearing on the primate species holding the apex position in the top trophic level of the global food web.

These structural changes reflect a reduction in connectivity of local intra and inter-species adaptive configurations undergoing local disordered phase transitions, with the increasing number of local phase transitions intersecting to structure the threshold critical state preceding a systemic disordered phase transition. Paralleling deconstruction, clusters of species in disordered phases begin forming the local connections of new adaptive configurations in the initial construction of new hierarchical order.

In its current state, the global connectivity of science is able to provide the uniquely situated real-time documentation for a case study of Lovelock's *Gaia* hypothesis: the inductive pattern formation process of local configurations structuring the adaptive fundamental group of planet earth.

5.2.2. Critical state of the human genotype-phenotype

Newton's seminal model of gravitational fields dispelled the apparent magic of noncontact interaction-at-a-distance between inanimate objects. The current sciences of adaptive systems remain in an analogous pre-Newtonian state, with the magic of adaptive interaction not conducive to examination by models of linear causality.

Macroscopic sensory fields of the human phenotype are well studied with quantitative models of auditory and visual fields, but little research has taken up the pioneering map sketched by John Flynn, relating the structure of local sensory information to behavioral properties and performance. The macroscopic collective perceptual fields of social groups and the microscopic cellular-genetic fields of individuals are in the initial stages of examination, while subliminal perceptual fields and the enteric nervous system have only recently re-entered the periphery of science's attention.

Conforming to endosymbiotic theory, research of the human microbiome has resulted in the emergent definition of the human phenotype as a local fundamental group of interacting species. The union of micro and macroscopic fields in a single fundamental group constructs the configuration of an ***adaptive field***. The set of elements of the local environmental container that fall within the boundaries of an adaptive field form a union with the adaptive field to structure a second fundamental group, an adaptive configuration. Unions of adaptive fields construct the hierarchical fundamental group boundaries of genetic-cellular, neuro-cognitive, social group, and evolutionary-population adaptive configurations. Epigenetics and horizontal gene transfer are just two of the mechanisms that define the bi-directional relationship between information structures of adaptive fields positioned in the same adaptive configuration. With the interpretation of local information following a process of inductive pattern formation, the properties of an adaptive field are grounded in the structure of its adaptive configuration.

In the human species, light exposure, contact with soil microorganisms, and visual fields of natural greenscapes are known to interact with neurotransmitter chemistry, altering the structure of neural connectivity and triggering changes in the hormonal structure and behavioral properties of the phenotype. (Ulrich, 1984, 1991; Brainard, 1991, 2005; Beauchemin, 1996; Benedetti, 2001; Jasser, 2006; Stevens, 2007; Rook, 2008, 2012; Azmitia, 2010; Freeman, 2010; Raison, 2010; Berman, 2012; Roe, 2013; Reklaitiene, 2014; Kardan, 2015) The global pattern of increasing urbanization is changing the structure of human adaptive configurations by introducing the pervasive overcast cloud cover of urban heat islands, low interior light levels, impermeable surfaces, and a greyscape lacking green inclusions. These and other broad changes to connectivity of human adaptive configurations translate to the well documented changes in the properties of individual genotype-phenotype adaptive fields now confronting western medicine and society, conforming to Dubos' observation of patterns of disease specific to a civilization.

The changes induced by the physical configuration pale in comparison to the unprecedented change in the structure of the genotypic-phenotypic adaptive field itself: the extension of real-time electronic connectivity to include information well outside the boundaries of local space defined over evolutionary time. With the limits of information processing imposed by the cognitive load limit, the increased non-local connectivity of an adaptive configuration alters the structure and properties of the adaptive field, limiting attention to local spatial information and effectively situating neuro-cognitive, social group, and population information structures in the electronic artifacts of their own niche construction. The evolving structure and properties of the adaptive field conform to models of *information overload* in cognitive and social sciences that evidence a relationship to performance: a reduction in decision quality, translating to a reduction in fitness of the genotype-phenotype.

Recent research reveals an average American human adaptive field spends 87% of its time in enclosed buildings and 6% of its time in enclosed vehicles: 93% of its life physically encapsulated by its own niche constructions. (Klepeis, 2001) By excluding the environmental container of natural systems, the human genotype-phenotype remains embedded only in the extended genotype-phenotype of its own construction, structuring a *closed loop* of *adaptive positive feedback*. Positive feedback constructions are well modeled in science and engineering, where small structural changes are compounded by recursion, accelerating destabilization of the initial system to terminate in a state of systemic collapse and/or a phase transition to a new state of stability.

5.2.3. Critical state of the extended human genotype

Following from the precedents of its mathematics and science, western culture has evolved on a premise of objectivity and universal truth attainable by deductive inference of an external omniscient observer. From jurisprudence, to economics, to sports, the authority of objective truth has permeated the structures of western culture.

But developments in mathematics from 1826-1964 exposed the provable truths of accepted mathematics as having constructed an incomplete and inconsistent truth space, providing a nonexclusive model of objective reality. These revelations, coupled with the introduction of the principle relativity in physics (1907-1915), triggered the erosion of objectivity and truth in culture's collective consciousness:

> In mathematics, the universal foundation of set theory initiated by Canter in 1874 is yielding to the local relativistic foundation of category theory, introduced by Eilenberg and MacLane in 1945.

> In science, agent based modeling, emerging from von Neumann in 1948 and Conway in 1970, and qualitative research methods, seminally outlined by Lincoln and Guba in 1985, assume a local situated perspective in an inductive pursuit of truth.
>
> In popular culture, the World Wide Web, initiated as a homogeneous structure to increase connectivity among researchers of objective truths, has ironically spawned the emergence of local electronic cliques of visceral experience. Dissonance between the collective conscious and the perception of local situated realities, *in silico* and *in situ*, have led to an increased questioning of authority, culminating with the conceptual emergence of *alternative facts*.

The fragmentation of a homogeneous prevailing collective conscious by spatially isolated pockets of dissent is a change in the connectivity pattern of cultural information structures undergoing local disordered phase transitions. With electronic connectivity accelerating the intersection of local phase transitions, clusters of local dissent are currently approaching the *percolation threshold* of a critical state, where an incremental increase in connectivity spans the culture in a phase transition to a new cultural memome.

Interaction of the cognitive load limit and the information overload of adaptive fields introduced a second change in the connectivity of cultural information structures: specialization. With increasing rates of urbanization encapsulating more than half of the global human population in 2009 and expected to rise to two thirds by 2050 (UN, 2014), the cultural information structures constructed from pre-agricultural and agricultural adaptive configurations no longer reside in the collective conscious of a majority of the human population. This change in the connectivity of the memome limits the adaptive properties of the population, bringing to mind the words of Churchill: *so much owed by so many to so few*.

Initially residing solely in the neural structure of individual adaptive fields, the collective conscious of Western cultural information has now been uploaded to the neural circuits of the World Wide Web. The eyu6tgfcvmergence of central repositories of cultural information in the artifacts of human niche construction structure a third change in connectivity, altering the robustness-stability properties of the memome to be dependent on the global robustness and stability of the physical structures of culture.

5.2.4. Critical state of the extended human phenotype

The evolutionary trajectory of the configuration of human niche construction reveals changes in space-time connectivity and discontinuous changes in properties that are indicative of structural phase transitions.

The initial habitats of the human population exhibited the non-binding connectivity pattern of a gaseous phase: climate induced expansion and contraction of low density compressible random configurations moving past one another in a diffusion process, filling the container in a roughly uniform distribution having large amounts of free space.

The emergence of agricultural partitioning structured an initial phase transition, with human habitats exhibiting a predominant quasi-binding connectivity pattern of a liquid phase: the seasonal ebb and flow of crop rotations forming non-compressible irregular configurations that conform to the shape of the local container with little interstitial space. Limited by local topographic-climatic boundaries and coinciding with emergent cultural specialization in local town centers, the global state of human niche constructions consisted of coexistent non-binding, quasi-binding and binding phases.

With the industrial revolution, the emergence of a predominant urban-suburban structure signaled the beginning of a second phase transition, with human habitats exhibiting the binding connectivity pattern of a solid phase: rigid close-packed constructions of non-compressible regular configurations of independent fixed shape and little interstitial space. Unlike earlier states of human niche construction, an urban solid-state is not an independently sustainable pattern, remaining dependent on the free spaces of the initial phase and the partitions of the agricultural phase.

On a container of finite surface area, growth of the solid phase is directly proportional to reductions in the areas of remaining initial and agricultural phases, structuring competition for space and resources as a zero-sum game between human and non-human habitats. In the United States, the accelerating process of urban substitution has been directly observable over the past half century, with current documentation of the urban-suburban solid extending from Charlotte, NC to Boston, MA, covering more than half of the eastern coast. The dwindling areas that have retained pre-urban phases of connectivity have been further subdivided by the connective tissue of human niche constructions. Current evidence reveals roadway structure divides the global land surface into 600,000 discrete fragments, only a third of which remain unaffected by human niche constructions. (Ibisch, 2016) With an enclosed area less than a square kilometer, half of the fragments are considered too small to support significant wildlife.

With the accelerating change from agricultural to urban connectivity, the previously disjoint local ordered phase transitions of urban adaptive configurations have intersected to form large components of a critical state on the threshold of a systemic global ordered phase transition. The increasing global connectivity of human configurations is directly related to the decreasing global connectivity of non-human configurations on the critical threshold of a global disordered phase transition, precipitating a discontinuous change in the properties of both configurations.

5.3. STRUCTURES AND PROPERTIES OF SCIENCE

When we make a new tool, we see a new cosmos.

(Freeman Dyson, 1988)

The familiar quill of deductive analysis employed by Newton and Leibniz is being supplanted by the recursive algorithms of inductive synthesis envisioned by Turing and von Neumann to render the entire class of mathematically undecidable problems accessible to scientific inquiry. Deductive linear interpretations of the pattern-property relationships of atomistic elements are yielding to the nonlinear inductive inference of the process-pattern relationships of discrete clustering in hierarchical structure. Methodologies of a situated investigator long advocated by Lincoln and Guba have been embraced by the social sciences to parallel the a-posteriori structuring of unstructured data and the algorithms of autonomous agents in computer science: systemic models are inductively developed from local situated perspectives. The deductive proofs of Godel and Cohen reveal the external point-of-view in mathematics is not omniscient, but situated by initial postulates and context.

Category theory's local relativistic restructuring of mathematics and the proliferation of domain-specific inductive structural models in science are both indications of complex adaptive systems that have reached penultimate critical thresholds at the turn of the 20th century. The inability of current mathematics to define the unique architectural identity of a configuration of elements has produced a science populated by disjoint clusters of heuristic inductive structural models lacking a common formalism. The point-of-view of an external observer in a metric space acts as a common inhibitory morphogen to prevent these clusters from joining together to structure the giant component of a systemic Kuhnian phase transition.

5.3.1. Structure of non-spooky action at a distance

5.3.1.1. semi-metric, metric discontinuity

In a developing Boolean topology of a random static set, it is only under conditions of extreme locality and simplicity, limited to a small number of interacting elements, that the locally developed metric space of a configuration is homeomorphic to its locally developed topological and probability spaces. With continued development beyond the formation of an initial boundary 'hole', topologically categorized as local fundamental group [2], topological and probability distances construct semi-metric spaces that are no longer homeomorphic to their corresponding metric space. This initial 'hole' is the signature of a critical threshold state of the structural phase transition from metric to semi-metric space, when continuous mappings between the structure of metric space and the semi-metric structures of topological and probability spaces are no longer possible. Topological space and probability space appear to remain homeomorphic throughout their development.

5.3.1.2. semi-metric nonlinear relationships

The discontinuities between metric space and topological and probabilistic semi-metric spaces structure the pervasive nonlinear metric space relationships now being ubiquitously described as *complex systems*: the fields surrounding individual elements construct semi-metric situated probability spaces of interaction. From a viewpoint situated in a metric space, the probabilistic relationship structures of semi-metric space can present a perplexing apparition of interaction at a distance.

5.3.1.3. non-primacy of metric space

The primacy of metric space is implicit in the quantification imperative of modern scientific inquisitions, yet a situated development of the local space of a random set of points reveals metric space forms discontinuous ***local patches*** of *covering maps* of a base topological-probability space. Temporal duration and topological-probabilistic spatial proximity frame the parameters of local interactions of a probabilistic universe.

5.3.2. Structure of nonlinear configurations

Initially developed for modelling the change in non-binding interaction over time, the independent-dependent constructions and linear relationships of continuous mathematics are metric space structures of implicit non-hierarchical atomism. In his seminal work of modern statistics, Fisher detailed experimental methods to minimize the influence of 'extraneous' spatial variation across unstructured sets of atomistic elements in metric space models of randomness. Historically, atomistic structures of metric space have been the only predictive models available to science.

The limited intensity and duration of local interactions of fields in non-binding and quasi-binding structure preclude spatial neighborhood models from developing a discontinuity between semi-metric topological-probability spaces and metric space. But in the hierarchical universe structured by locally binding spatial-temporal interactions of fields, topological-probabilistic semi-metric neighborhood connectivity creates nonlinear causative clustering.

Evidenced by hierarchically local phase transitions, multi-variable causation by synergistic interdependent fields can be modeled by the situated topological structures of local fundamental and Boolean neighborhood groups and the situated algebraic structures of local Boolean rings found in the situated set theoretic structures of locally complete and incomplete hierarchical unions of neighborhoods.

The relationship of the local semi-metric topological spaces, local semi-metric probability spaces and local metric spaces developed by Poincare neighborhood inflation of a set of points can be used to define a unique quantitative identity of a configuration of elements.

5.4. STRUCTURES AND PROPERTIES OF PROVABLE TRUTH

5.4.1. Structure of deductive inference

5.4.1.1. boundary structure of deductive inference space

Historically, scholars in western culture have extrapolated from a set of known deductive truths to inductively infer the existence of a single unbounded universal truth space structured by *everywhere continuous* deductive certainty.

By the close of the 17th century, the work of Leibniz and Newton cemented the belief that the continuous certainty of a universal truth space was modeled by the mathematics of an external point of view: the conceptual relationship between unbounded universal truth space and deductive inference space was structured as a *bijective mapping*.

The dawn of the 20th century brought the common assumption that Cantor's still evolving set theory held the properties of consistency and completeness necessary to finally realize the mathematical structure of universal truth.

Brouwer's intuitionist constructivism, Russel and Whitehead's logicism, and Hilbert's axiomatic formalism had all achieved some initial success as mathematical models of truth space when, in 1932, Godel presented a proof that the structure of deductive certainty was not everywhere continuous, but incomplete: within any system of provable truth, statements can be made that are true but are not provable. The can't-get-there-from-here property of Godelian truths constructs the deductive inference space of provable truths as a subset of a larger truth space, where the subset is modeled as connected infinite single topological space perforated by holes. The internal discontinuities in deductive certainty define the connected structure of a universal deductive inference space as a topological fundamental group [>1].

Just as non-Euclidean geometries were developed from the independence of Euclid's fifth axiom, Cohen's 1963 proof of the independence of the axiom of choice opens the door for the development of alternative set theories and forever fractures the long-held intuitive model of a single universal deductive inference space. Cohen's proof establishes the existence of an external boundary of deductive inference space that is dependent on its initial postulates and context, dividing deductive provable truth into a set of categorically situated inference spaces modeled as a set of independent finite topological spaces.

An unbounded universal truth space may exist, but the path-connected structure of provable certainty has limits: deductive inference space is constructed as a disjoint collection of categorical truths, where each categorical inference space is modeled as a finite topological space having a perforated boundary structure of fundamental group [>1]. This model is reflected in the current restructuring of mathematics, a transition from the foundation structured by Cantor's 1884 set theory to a foundation constructed on the situated morphisms of MacLane and Eilenberg's 1945 category theory.

5.4.1.2. internal structure of deductive inference space

A process of deductive inference constructs a linear structure of truth: each valid deductive step is a *directed equivalence relation* of unit interval 1, connecting the initial premise of an accepted categorical truth through a sequence of logical statements to reach the conclusion of a particular truth.

Discrete statements connected by continuous certainty structures the linear process of deductive inference as the vertex-edge relationship of a ***certainty path graph***. The non-cyclic structure of the union of possible certainty path graphs rooted in a single premise of categorical truth suggests a *tree lattice* or *semi-lattice* structure of a ***deductive inference graph***. The union of possible certainty path graphs derivable from a closed set of categorical truths suggests a *multi-rooted tree lattice* or *multi-rooted semi-lattice* as the internal structure of a ***categorical deductive inference space.***

A single certainty path graph of deductive inference forms finite connected topological space without holes having a boundary structure of fundamental group [1]. The directed equivalence relation of unit interval 1 develops a deductive inference space as a metric space that is isomorphic to its topological space. Every interior vertex of a certainty path graph is structurally equivalent, having a vertex degree of 2, to define the process of deductive inference as constructing a *homogeneous* structure of truth. Path-connected certainty, ensuring the truth of a conclusion from truth of its premises, defines deductive inference as a deterministic model of truth.

The acyclic process of deductive inference constructs a locally deterministic model of truth as a homogeneous linear metric structure embedded in an incompletely connected topological space.

5.4.2. Structure of inductive inference

By reversing the direction of certainty in a deductive inference graph, a linear model of inductive inference produces discontinuities in each certainty path from an initial set of particular truths to a conclusion of a general categorical truth to define inductive inference as a non-deterministic truth space. A discontinuity maps the signature non-linear *AH-HA!* moment of cognitive categorical-conceptual pattern formation structured by a physical phase transition in neural connectivity: a set of disjoint configurations of locally connected neurons join together in the formation of a giant connected component.

This conceptual leap from a particular to a categorical truth occurs without enumeration of every particular instance to mirror the model of adaptation as a multi-variable combinatorial optimization process that is not dependent on the examination of every possible combination of a configuration space.

Both physical and conceptual local phase transitions of inductive pattern formation can be modeled by the situated mathematical structures derivable from the spatial development of a Boolean topology. In a nonlinear threshold-cascade of process of inductive inference, a discrete kernel, or *germ*, of a specific truth is abstracted to structure a neighborhood, field, or *grain*, of a specific truth. With increasing abstraction constructing local intersections of structural commonality, recursive topological grain inflation models ascending generalizations to develop an inductive inference space as a probabilistic hierarchical model of truth.

5.4.2.1. boundary structure of inductive inference space

The process of recursive abstraction results in a changing structure of inductive inference space, modeled as the developmental trajectory of a configuration of truth grains in an architectural configuration space.

With an initial condition of a finite stationary set of specific germs, hierarchical abstraction of particulars to a categorical generalization is computationally decidable, requiring a finite number of recursions equal to the number of edges of the complete graph of germs. Terminal recursion structures the configuration of a complete Boolean topology having a boundary of fundamental group [1], constructing a complete truth space.

An infinite set of situated truth germs undergoing infinite parallel recursion constructs a single infinite complete Boolean topology: an unbounded completely connected infinite truth space.

5.4.2.2. internal structure of inductive inference space

The recursive parallel process of inductive inference constructs a locally probabilistic model of truth as a hierarchical nonlinear semi-metric structure embedded in a completely connected topological space.

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APPENDIX A

A POSTERIORI HIERARCHICAL DATA STRUCTURE

To simplify documentation, the discrete granular particulars that form the input sample population of this investigation are considered to be individual citations. Pattern formation output, the set theoretic hierarchical structure of categorical generalizations, is presented in outline form.

The situated conditional chain of knowledge state spaces of the investigation was structured by a sequence of questions, each emerging from the preceding stage of the investigation. This sequence of recursive meta-abstractions constructed a scope of five sets of categorical generalization:

(model models): the parameters of model space
(adaptation): situated inductive pattern formation
(modularity): inductive morphism and hierarchical units of synthesis-analysis
(proof and truth): existing precedents and boundaries of provable truth
(***architectural identity***): *situated mathematics* of configuration space

A1. APPENDIX A CONTENTS

A2. MODEL MODELS

Can an evolutionary design process structure the foundation of a science of architecture?

In the initial knowledge state space of the investigation, the inductive process of a posteriori conceptual clustering hierarchically structured the specific data grains of individual citations into a union of five sets of generalization:

(structural design case study)

(structures of pattern space)

(set theoretic structuralism)

(model structures)

(structure of knowledge domains)

A2.1. Structural design case study: pharmaceutical QSAR and SAR models

structural design process: (Randic, 1974, 1992; 2001a, 2001b, 2002; Basak, 1988, 1990, 1991; Grossman, 1991; Dubos, 1992; Bugg, 1993; Kier, 1993; Goel, 1995; Gutman, 2000; Bonchev, 2001; Estrada, 2001, 2003; Hall, 2001; Macina, 2001; Churchwell, 2004; Balaban, 2005; Hillier, 2005; Ban, 2003; Churchwell, 2004; Clercq, 2004; Skvortsova, 2004; Balaban, 2005; Gilardoni, 2005; Jhoti, 2007; Consonni, 2010; Bekker, 2011; Afshar, 2012; Cherkasov, 2014)

combinatorial database / configuration library: (Shen, 2004; Landman, 2005)

graph theoretic predictive modeling: (Wiener, 1947; Randic, 1974, 2001; Basak, 1990; Dubos, 1992; Kier, 1993; Bonchev, 2001; Hall, 2001; Macina, 2001; Estrada, 2001, 2003; Ban, 2003; Churchwell, 2004; Skvortsova, 2004; Balaban, 2005; Hu, 2005;

artificial neural network, genetic algorithm predictive modeling: (Gakh, 1994; Kireev, 1995; Kovesdi, 1999; Agatonovic-Kustrin, 2000; Schneider, 2000; Ichikawa, 2003; Bayram, 2004; Winkler, 2004)

A2.2. Structures of pattern space:

hierarchical self-similar: (Mandelbrot, 1982, 1989; Rossler, 1986; Barnsley, 1987, 2010a, 2010b, 2012, 2013; Röhricht, 1987; Greifswald, 1989; Kahng, 1989; Perreau, 1989; Bandt, 1992; Falconer, 1995; Kenyon, 1996; Kittel, 1997; Conway, 1998; Majewski, 1998; Sole, 1999; West, 1999; Dodds, 2000; Abraham, 2001; Fathauer, 2001; Tarafdar, 2001; Gutierrez, 2002; Rani, 2004, 2009, 2013; Srinivasan, 2004; Chung, 2005; Freiberg, 2005; Zhou, 2005; Bourke, 2006; Landreneau, 2006; Devaney, 2007; Mureika, 2007; Bunde, 2009; Singh, 2012; Vejnar, 2012; Goel, 2013; Capriani, 2014)

periodic: (Bragdon, 1913; Stuart, 1963; Burt, 1966, 1972, 1996, 2007, 2011; Emmerich, 1967, 1990; Wood, 1967, 1968; Critchlow, 1969; Guran, 1969; Rittell, 1970;

Williams, 1972; Wachman, 1974; Lalvani, 1977, 1981, 1996; Huybers, 1979, 1995, 1996; Hyde, 1984, 1989; Fischer, 1987, 1996; Haase, 1987; Andersson, 1988; Smith, 1988; Hyde, 1994, 2003; Wester, 1994; Gabriel, 1997; Liu, 1998; Friedrichs, 1999; Leoni, 2000, 2003; Kleinmann, 2001; Nesper, 2001; Carlucci, 2002; Giesen, 2002; Sowa, 2002, 2003, 2005; Thompson, 2002; Korren, 2003; Schroder, 2003; Piotto, 2004; Robins, 2004; Aste, 2005; Mittemeijer, 2010; Thomas, 2012)

quasi-periodic: (Penrose, 1974; Shechtman, 1984; Duneau, 1985; Cahn, 1986; Levine, 1986; de Boissieu, 1990; Goldman, 1991; Baranidharan, 1994; Gopal, 1994; Caspar, 1996; Peterson, 1996, 1999; Vainshtein, 1996; Polyakov, 1997; Ranganathan, 1997; Steinhardt, 1997, 2011; Hof, 1998; Makowski, 1998; Goodman-Strauss, 1999; Radin, 1999; Hu, 2000; Lagarias, 2000; Lück, 2000; Baake, 2002; de Prunele, 2002; Abe, 2004; Delvenne, 2004; Steurer, 2004; Boroczky, 2006; Macia, 2006; Zhou, 2006; Bohannon, 2007; Lu, 2007; Ben-Abraham, 2011; Bindi, 2012; Bellos, 2015)

quasi-random*:* (Watts, 1998; Barabasi, 1999a, 1999b, 1999c; Newman, 1999; Amaral, 2000; Barrat, 2000; Jeong, 2000; Bagnoli, 2001; Mathias, 2001; Wagner, 2001; Wuchy, 2001; Kim, 2002; Willinger, 2002; Wolf, 2002; Piel, 2003; White, 2003; Wuchty, 2003; Dekker, 2004; Sporns, 2004a, 2004b, 2006; Stam, 2004; Albert, 2005, 2006; Doye, 2005; Dunne, 2005; Lee, 2005; Li, 2005, 2006; Lin, 2005; Marquet, 2005; Schilling, 2005; Stumpf, 2005; Vazquez, 2005; Zhou, 2005; Barriere, 2006; Goyal, 2006; Kalisky, 2006; Li, 2006; Louzoun, 2006; Mathias, 2006; Sarshar, 2006; Zhang, 2006a, 2006b)

random: (Erdos, 1959; Gilbert, 1961; Dacey, 1963; Fortuin, 1972a, 1972b, 1972c; Chaitin, 1975; Braddeley, 1980; Serra, 1980; Rivier, 1987; Browne, 1993; Luczak, 1994; Grimmett, 1995, 2006; Stahl, 1995; Bollobas, 1996; Cahill, 1996, 1997, 1998, 2005; Barrett, 1999; Claude, 1999; Stoyan, 2000; Hayes, 2001; Newman, 2001, 2006, 2009; Robins, 2001; Aldana, 2002; Calude, 2002; Crutchfield, 2002; Passy, 2002; Peterson, 2002, 2003; Sosa, 2002; Watts, 2002; Cannings, 2003; Kali, 2003; Venter, 2003; Bentley, 2004, 2005; Gunduz, 2004; Ioannides, 2004; Mayer, 2004; Mossel, 2004; Park, 2004; Sood, 2004; Fronczak, 2005; Griffith, 2005; Malescio, 2005; Abe, 2006; Palla, 2007; Matzutt, 2008; D'Souza, 2009; Florens, 2009; van Lieshout, 2012; Brooks, 2015a, 2015b; Holmes, 2015)

A2.3. Set theoretic structuralism: (process, structure, properties, performance)

canalization; structural stability, robustness: (Agrawal, 1984; Lady, 1995; Tononi, 1999; Callaway, 2000; Gibson, 2000; Long, 2001; Krivelevich, 2002; Stang, 2003; Jonckheere, 2004; Kitano, 2004; Wilson, 2004; Klau, 2005; Criado, 2006; Ay, 2007; Lesne, 2008, 2011; Srinivasan, 2011; Demongeot, 2012a, 2012b, 2012c, 2012d)

structure-properties: (Barnes, 1969; England, 1971; Basak, 1991; Cohen, 1991; Gakh, 1994; Sumpter, 1994; Lorch, 1999; Jeong, 2000; Skolnick, 2000; Hasty, 2001; Lengauer, 2001; Dunne, 2002, 2005; Girvan, 2002; Otten, 2002, 2006; Smith, 2002; Torquato, 2002; Bu, 2003; Dodds, 2003; Gershell, 2003; Pržulj, 2003; Selassie, 2003; Schultz, 2003; Turner, 2003; Whisstock, 2003; Dubois, 2004; Hood, 2004; Jordan, 2004; Middendorf, 2004; Seepersad, 2004; Vazquez, 2004;

Veith, 2004; Wong, 2004; Yu, 2004; Arita, 2005; Borgwardt, 2005; Brinda, 2005; Dekker, 2005; Doyle, 2005; Ferrarini, 2005; Gore, 2005; Guimera, 2005; Haddon, 2005; Huson, 2005; Ma'ayan, 2005; Stewart, 2005; Wunderlich, 2005; Aggarwal, 2006; Phillips, 2006; Ruano, 2006)

infomatics: (Wu, 1990, 1992; Emmett, 2000; Hagen, 2000; Paton, 2000; Cohen, 2004; Maojo, 2004; Savchuk, 2004; Winkler, 2004; Benoit, 2005; Bradford, 2005; Rajagopalan, 2005; Ezziane, 2006; Ferhatosmanoglu, 2006; Jones, 2006)

configuration space: (Landau, 1930; Hakim, 1966; Eichinger, 1977; Lozano-Perez, 1983; Kirkpatrick, 1985; Fontana, 1993; Latombe, 1999; Shpak, 2000; Gewaltig, 2001; Stadler, 2001; Bastert, 2002; Aldana, 2003; Casetti, 2003; Moraglio, 2004; Pereira, 2005; Thurner, 2006; Gfeller, 2007; Berdahl, 2008; Kastner, 2008)

multi-criteria combinatorial optimization: (Wilcoxon, 1945; Schuyler, 1948; Bradley, 1954; Bendig, 1956; Matthews, 1966; Bruk, 1972; Farquhar, 1974; Barnett, 1976; Morse, 1977; Clark, 1978; Eliashberg, 1980; Eshragh, 1980; Stewart, 1981, 1996; Horsky, 1984; French, 1984, 1985; Weber, 1985, 1997; Allett, 1986; Eiselt, 1991; Young, 1995; Henig, 1996; Lipovetsky, 1996; Belton, 1997; Hamalainen, 1997; Salo, 1997; Buchanan, 1998; Klein, 1998; Elomaa, 1999; Kasprzak, 2000; LePelley, 2000; Valls, 2000; Forman, 2001; Levitin, 2003; DeWeck, 2004; Kim, 2005; Scott, 2005)

inductive combinatorial structural design: (Radford, 1980; Hopfinger, 1984; Bull, 1989; Kier, 1993; Macready, 1995; Sepetov, 1995; Richon, 1997; Kauffman, 2000; Mann, 2000; Apic, 2001; Dagani, 2001; Debnath, 2001; Shai, 2001; Augen, 2002; Darvas, 2002; Güner, 2002; Hasty, 2002; Katritzky, 2002; Selassie, 2002; Fermeglia, 2003; Aguilar, 2004; Gusfield, 2004; Seepersad, 2004; Walsh, 2004; Hatzimanikatis, 2005; Banville, 2006; Merks, 2006; Richardson, 2006; van der Velde, 2006; Kennedy, 2008)

set theoretic structuralism: (Olson, 1994, 1998, 2000; Bensaude-Vincent, 2001, 2004; Campbell, 2001; Desiraju, 2001; Flemings, 2001; Edling, 2002; McDowell, 2004, 2007; Seepersad, 2004; Fine, 2005; Barat, 2006; Schroder, 2006; Seppelt, 2006; Battaile, 2009; Boettiger, 2013; Rahman, 2014)

A2.4. Model structures

A2.4.1. modeling ontologies

reductionism: (Oppenheim, 1956; Schaffner, 1967; Anderson, 2001; Van Regenmortel, 2004)

atomism: (Post, 1975; Shrader-Frechette, 1977; Cushing, 1982; Bell, 1995; Mayberry, 2000; Schuster, 2006; Frohlich,2007; Sternheimer, 2007; Harris, 2008; Anapolitanos, 2009; Izquierdo-Aymerich, 2009; Matthews, 2009)

creativity, innovation: (Simon, 1982, 1983, 2001; Thagard, 1987; Bernstein, 1989; Horrobin, 1990; Vaughn, 1996; Zucker, 1996; Bhatta, 1997; Gero, 1998; Colton, 1999; Goldenberg, 1999; Stepanek, 1999; Carayol, 2000, 2005; Rajan, 2000; Damsker, 2001; Klahr, 2001; Wolff, 2001; Bhattacharya, 2002, 2005;

Cohen, 2002; Taylor, 2002; Temel, 2003; Cosgrove, 2004; Cowan, 2004, 2006; Laszlo, 2004; Rank, 2004; Pittaway, 2004; Runco, 2004; Sim, 2004; Sonntag, 2004; Barabási, 2005; Breschi, 2005; folley, 2005; Gabora, 2005; Guimera, 2005; Lane, 2005; Miller, 2005; Muller, 2005; Schiffer, 2005; Swan, 2005; Uzzi, 2005; Wilczek, 2005; Brown, 2006; Bunk, 2006; Chia, 2006; Clemmitt, 2006; Creedon, 2006; Jewett, 2006; Klein, 2006; Nettle, 2006a, 2006b; Simonton, 2006; Bilalic, 2008; Hartmann-Sonntag, 2009)

systematics, cladistics: (Goodman, 1971; Penny, 1982; Seberg, 1989; Mishler, 1994; Brower, 1996; Mayr, 1998, 2002; Pagel, 1999; Strimmer, 2000; Congdon, 2001; Grandcolas, 2001; Ebach, 2002; Jobling, 2003; Willmann, 2003; Agatha, 2004; Henz, 2004; Kraus, 2004; Baum, 2005: Hey, 2005; Johnson, 2005; Roth, 2005; Bothwell, 2006; Makarenkov, 2006; Rutschmann, 2006)

game theory: (Smith, 1974, 1976, 1979; Axelrod, 1981; Leonard, 1995; Sigmund, 1999; Arrow, 2003; Nowak, 2004, 2005; Ohtsuki, 2006; McGill, 2007; Roca, 2009)

general systems theory: (von Bertalanffy, 1950, 1968, 1972; Boulding, 1956; Simon, 1962; Caws, 1963; Henning, 1963; Sprague, 1963; Winthrop, 1963; Blachowicz, 1971; Becht, 1974; Ball, 1978; Rodin, 1978; Salmon, 1978; Denenberg, 1980; Skyttner, 1996, 2005; Barros, 1997; Bogusch, 1997; Ideker, 2001; Gulyaev, 2002; Hatfield, 2002; Kitano, 2002; Uso-Domenech, 2002; Bernard, 2005; Stoyanov, 2005; Schneider, 2006; Arrell, 2010; Seising, 2010; Wolkenhauer, 2012; Adams, 2014)

complexity theory: (Levy, 1985; Emmeche, 1994; Blakeslee, 1995; Howard, 1997; Levin, 1997; Mitchell, 1998; Brooks, 2001; Grand, 2001; Strogatz, 2001; Ziemelis, 2001; Casti, 2002; Giles, 2002; Hayes, 2002; Kitano, 2002; Katagiri, 2003; Gambardella, 2004; Crawford, 2005; Bullock, 2006)

theory structures: (Ziman, 1965; Ayala, 1989; Gopen, 1990; Collier, 1992; Levin, 1997; Benyon, 1999; Bokulich, 2001; Damper, 2001; Gardner, 2001; Pfenninger, 2001; Konopka, 2002; Nordgren, 2003; Rhyne, 2003, 2004; Johnson, 2004; Nerlich, 2004; Poon, 2004, 2005; Baum, 2005; Cardelli, 2005; Goguen, 2005; Neuman, 2005; Roth, 2005; Turney, 2005; Caminati, 2006; Ratto, 2006)

A2.4.2. modeling precedents

discrete v. continuous structure: (Elman, 1990; Olurotimi, 1991; Wang, 1992; Scott, 1995; White, 1996; Zeigler, 1998; Izard, 2000; Shnerb, 2000; Koopman, 2001; Doherty, 2003; Fishwick, 2004; Pelanek, 2004; Bonchev, 2005; Fayez, 2005; Huyck, 2005; Fingelkurts, 2006; Freeman, 2006; Kaznessis, 2006)

deterministic v. stochastic process: (Fisher, 1922; Matheron, 1963; Gray, 1967; Chayes, 1972; Hampton, 1973; Stewart, 1986; Stigler, 1986, 2000; Cohen, 1987; Good, 1987, 1988; Hacking, 1987; Stone, 1987; Wise, 1987; Cohen, 1988; Clevenson, 1991; Fienberg, 1991; Schneider, 1991; Bozda, 1999; Mumford, 2000; Stoyan, 2000; Swift, 2000; Breiman, 2001; Ball, 2002; Bridgeman, 2003; Atkinson, 2006; Clark, 2006; Callaway, 2008; Lyons, 2008; D'Souza, 2009; Wilkinson, 2009; Simeonov, 2010; Augustin, 2011; Parascandola, 2011; Melham, 2012; Pépin, 2012; Velasco, 2012; Brooks, 2015a, 2015b; Holmes, 2015; Nuzzo, 2015) *cellular automata, artificial life models:* (Von Neumann, 1951, 1966; Burks, 1957a, 1957b; Schrant, 1960, 1967; Ulam, 1962; Gardner, 1970; Baer, 1974; Wolfram, 1984; Couclelis, 1985; Levy, 1985; Bays, 1987; Hogeweg, 1988; Bak, 1989; Li, 1990; Ermentrout, 1993; Crutchfield, 1995; Lohn, 1997; Barrett, 1999; Blok, 1999; Hayes, 1999; Sloot, 1999; Sarkar, 2000; Semboloni, 2000; Bedau, 2000, 2003, 2005; Bandini, 2001; Dormann, 2001; Gordon, 2001; Kundu, 2001; Torrens, 2001; Feick, 2002; Tomita, 2002; Boots, 2003; Adami, 2004; Edlund, 2004; Holmes, 2005; Kari, 2005; Lenaerts, 2005; Kim, 2006; Smith, 2011)

petri net models: (Petri, 1962; Peterson, 1977; Peleg, 2002; Pinney, 2003, 2006; Errampalli, 2004; Gašević, 2004; Langley, 2006)

discrete event system models: (Geoff, 1972; Austin, 1985; Fujimoto, 1989; Ho, 1989; Singh, 1989; Zeigler, 1989, 1991, 1992, 1993, 1997, 2003; Heileman, 1992; Passino, 1992; Schruben, 1993, 2000; Buss, 1995, 1996; Flood, 1995; Lin, 1996; Minar, 1996; Bullnheimer, 1997; Righter, 1998; Uhrmacher, 2000, 2001, 2005; Deutsch, 2001; Tropper, 2001; Willinger, 2002; Makino, 2003; Lobb, 2005; Miller, 2005; Rohl, 2005; Scheiner, 2005)

simulation: (Hogeweg, 1980; Shannon, 1998; Ingalls, 2002; Grimm, 2005; Railsback, 2006; Brown, 2006; Aumann, 2007; Gurney, 2007; Kramer-Schadt, 2007; Prescott, 2007)

phase diagram: (Bridgman, 1937; Doolittle, 1938; Nishizawa, 1992; Kattner, 1997; Lobban, 1998; Slyusarenko, 1999; Kosyakov, 2000; Pogliani, 2003; Epps, 2004; Miura, 2006; Zhao, 2006; Reatto, 2007)

graph theoretic phase transition models: (Luczak, 1994; Kalapala, 1998; Krishnamachari, 2001, 2003; Krivelevich, 2002; Aldana, 2003; Ishihara, 2005)

topological phase transition models: (Franzosi, 2000; Fiałkowski, 2002; Casetti, 2003; Khaldoyanidi, 2003; Derenyi, 2004; Palla, 2004; Teixeira, 2004; Angelani, 2005; Baroni, 2006; Kastner, 2006, 2008; Risau-Gusman, 2006; Chen, 2012)

phase transition process: (Stanley, 1971; Langton, 1990; Back, 1995; Ormerod, 2006; Sole, 1996; Matsushita, 1998; Newman, 1999; Goldstein, 1999; Sachdev, 1999; Araki, 2001; Enomoto, 2001; Fiałkowski, 2002; Watts, 2002; Angelani, 2003, 2005; Casetti, 2003; Kalapala, 2005; Deguet, 2006; Kinouchi, 2006; Fronczak, 2007; Buchanan, 2008; Dorogovtsev, 2008; Lizier, 2008; Nishikawa, 2011)

graph theoretic structure: (Harary, 1960, 1962; Nystuen, 1961; Ramamoorthy, 1966; Wirth, 1966; Foulkes, 1967; Essam, 1970; Cartwright, 1975; Randic, 1975; Scrensen,1978; Hage, 1979; Kruger, 1979; Burt, 1980; Zubkov, 1980; McKee, 1985; Hopkins, 1987; Scott, 1991; Tjur, 1991; Osman, 1996; Rabino, 1996; Keitt, 1997; Kim, 1999; Remolina, 1999; Shai, 1999a, 1999b; Salingaros, 2000; Du, 2001; Modarres, 2002; Redner, 2002; Shmulevich, 2002; Tangmunarunkit, 2002; Bafna, 2003; Chowell, 2003; Newman, 2003; Penn, 2003; Washio, 2003; Barabasi, 2004a, 2004b; Bera, 2004; Haus, 2004; Jiang, 2004; Kakade, 2004; Middendorf, 2004; Pržulj, 2004; Torres, 2004; Watts, 2004; Xia, 2004; Dekker, 2005; Fronczak, 2005; Hillier, 2005; Sarshar, 2005; Wilkins, 2005; Wilkinson, 2005; Cardillo, 2006; Jain, 2006; Klamt, 2006; Miura, 2006; Porta, 2006a, 2006b; Costa, 2007) *topological structure:* (Phillips, 1971; Brantingham, 1978; Mermin, 1979; Melott, 1990; oelzeman, 1994; Archdeacon, 1996; Fleck, 1996; Sahni, 1998; Steadman, 2001; Robins, 2002; Gero, 2003; Jupp, 2003; Claramunt, 2004; Crane, 2004; Reitsma, 2004; Kotschick, 2006; Mackenzie, 2006; Mendoza, 2006; Overbye, 2006; Hitchin, 2007; Moore, 2010)

A2.5. Structure of knowledge domains

A2.5.1. architectural order parameters as complex adaptive systems

human factors: (Zeki, 1998; Moore, 2000; Lansing, 2003; Gunduz, 2004; Nolfi, 2004; Thadakamalla, 2004; Blanchard, 2005; Lewis, 2005; Lledo, 2005; Percha, 2005; Bathellier, 2006; Chialvo, 2006; Roska, 2006; Kurakin, 2007; Motluk, 2007; Werner, 2007; Davis, 2008; Grigolini, 2009; Ananthaswamy, 2010; Durstewitz, 2010; Sadaghianil, 2010; Palombo, 2013)

culture: (Maranda, 1972: Mitchell, 1974; Schmidt, 1975; Vastokas, 1976; Beeby, 1980; Leach, 1983; Buchanan, 1984; Egenter, 1987; Gilman, 1987; Goss, 1988; Croome, 1991; Scott, 1991; Abel, 1992; Boden, 1992; Cook, 1992; Goodstein, 1992; Szczepanski, 1994; Biddulph, 1995; Senft, 1995; Spreckelmeyer, 1995; Coleman, 1996; Jones, 1996; Low, 1996; Rendell, 1996; Simonsen, 1996; Steadman, 1996; Harper, 1998; Mohr, 1998; Ramadier, 1998; Xu, 1998; Birmingham, 1999; Herman, 1999; Kauffman, 1999; Satler, 1999; Walker, 1999; Broder, 2000; Carnes-McNaughton, 2000; Gieryn, 2000; Wilkins, 2000; Conzen, 2001; Kranton, 2001; Krishnamachari, 2001; Pellow, 2001; Rapoport, 2001; Saleh, 2001; Sole, 2001; Hwangbo, 2002; Loukaitou-Sideris, 2002; Marzot, 2002; Mills, 2002; Reed, 2002; Watkins, 2002; Aroche-Reyes, 2003; Ball, 2003; Bremner, 2003; Lansing, 2003; Auge, 2004; Chiu, 2004; Mallett, 2004; Moore, 2004; Ronnes, 2004; Brain, 2005; Eerkens, 2005; Forgan, 2005; Jeffares, 2005; McNeill, 2005; Mesoudi, 2005; Pauketat, 2005; Samadhi, 2005; Sklair, 2005; Anderson, 2006; Fragaszy, 2006; Johnston, 2006; Jones, 2006; Sadler, 2006; Sagsoz, 2006; Steels, 2006; Sterelny, 2006; Whitehand, 2006; Castellano, 2007; Fronczak, 2007; Tierney, 2007; Garreau, 2008; Rupert, 2008; Sudjic, 2008; Bolourian, 2009; Heaney, 2009; Cho, 2013; Wainwright, 2013)

environment: (Oster, 1971; Tainaka, 1991; Dietrich, 1992; Ito, 1995; Linehan, 1995; Heylighen, 1997; Levin, 1998; Fath, 1999; Pace, 1999; Bacon, 2000; Beekman, 2001; Urban, 2001; Bascompte, 2003; Mossel, 2003; Ovaskainen, 2003; Vermeij, 2004; Baranyai, 2005; Farina, 2005, 2006; Green, 2005; Eilperin, 2006; Hansen, 2006; Schmitz, 2006; Balter, 2007; Duger, 2007; Wicks, 2007; Wimberley, 2007; Grimm, 2008; Armstrong, 2010; Drake, 2010; Lenton, 2011; Seto, 2011, 2012; Carrington, 2014)

A2.5.2. science and mathematics as complex adaptive systems

meta-process of science: (Kuhn, 1962; Simon, 1962; Gell-Mann, 1992; Holland, 1992, 2005; Chu, 2003; Sengupta, 2003; Amaral, 2004; Arevalo, 2008; Boccaletti, 2009; Wang, 2009; Stenholm, 2010; Newman, 2011)

meta-process of mathematics: (Crowe, 1975; Dieudonne, 1978; Goodman, 1984; Rockafeller, 1988; Feferman, 1992, 1999, 2000; McLarty, 1993; Corry, 1997;

Devlin, 1997; Hoare, 1999; Corfield, 2001; Levin, 2002; de Villiers, 2004; Greer, 2004; Crossley, 2005; Chaitin, 2007)

A3. ADAPTIVE PATTERN FORMATION

Are there invariant processes and structures of complex system pattern formation, independent of scale and academic boundaries, that encompass the range of variables that structure the human habitat?

In the second knowledge state space of the investigation, the inductive process of a posteriori conceptual clustering hierarchically structures the specific data grains of individual citations into a union of three sets of generalization:

(hierarchical structures of adaptive pattern formation)

(bipartite structures of inductive pattern formation)

(invariant properties of adaptive / inductive pattern formation)

A3.1. Bipartite structure of inductive pattern formation

A3.1.1. information structures: genotypes

multi-sensor data fusion: (Bar-Shalom, 1978, Luo, 1987; Fincher, 1990; Ruck, 1990a, 1990b; Wang, 1994; Hall, 1997; Chen, 1998; Cox, 1998; Mazor, 1998; Swanson, 1998; Bowyer, 1999; Celinski, 1999; Mahotra, 1999; Dubois, 2000; Farooq, 2000; Gandetto, 2003; Ross, 2003; Mahler, 2004; Fiocco, 2005; Pulford, 2005; Huang, 2006; McGeorge, 2006; Starzyk, 2006; Whitehouse, 2006; Xiang, 2006)

memes: (Gabora, 1995, 2004; Feldman, 1996; Tanaka, 1996, 2002, 2004a, 2004b, 2005; Flynn, 1997; Best, 1999; Wimsatt, 1999; Bull, 2000; Higgs, 2000; Hirschberg, 2000; Jeffreys, 2000; Moore, 2001; Barnett, 2002; Salingaros, 2002; Bentley, 2003; Castro, 2004; Deacon, 2004; Danchin, 2004; Blute, 2005; Shermer, 2005; Wilkins, 2005; Sterelny, 2006)

a priori data structure: (Smardzija, 1990; Aurenhammer, 1991; Mayr, 1994, 1995; Graves, 1995; Nielsen, 1996; Agrawal, 1997; Hubbard, 2002; Schneider, 2002; Clamp, 2003; Rose, 2003; Stoll, 2003; Green, 2004; Shen, 2004; Weaver, 2004; Patrinos, 2005; Frehner, 2006)

citation networks: (Peritz, 1992; Yaru, 1997; Ding, 1998a, 1998b; Leydesdorff, 1998, 2004, 2005a, 2005b, 2007; Allen, 2005; Scharnhorst, 2005; Synnestvedt, 2005)

graph theoretic data structure: (Aurenhammer, 1991; Graves, 1994, 1995, 2012; Gyssens, 1994; Erwig, 1994; Zurawski, 1994; Ellis, 1995; Harel, 1998; Kolpakov, 1998; Lee, 1999; Mitra, 2000; Novak, 2002; Muezzinoglu, 2004; Weaver, 2004; Ray, 2005)

a posteriori self-structuring data: (Kohonen, 1990, 1996; Ruppin, 1990; Ichiki, 2001; Lee, 2001; Grim, 2002, 2004a, 2004b; Ayad, 2003; McClelland, 2003; Pomi, 2004; Rogers, 2004; Smigiel, 2004; Barb, 2005; Shanmuganathan, 2006; Steyvers, 2005) *meta-data structures:* (Mark, 1990; Snaprud, 1992; Guting, 1994; Lu, 1995; Iannella, 1998; Miller, 1998; Corby, 2000; Medeiros, 2000; Cannataro, 2002; Sonneck, 2003; Suleman, 2003; Fujima, 2004; Hayes, 2004; Sayers, 2004; Devillers, 2005; Gergatsoulis, 2005; Armstrong, 2006; Christiansen, 2006)

genome: (Moss, 1992; D'haeseleer, 2000; Kolchanov, 2000; Sterelny, 2000; Reidys, 2001; Tseng, 2001; Brazhnik, 2002; Kholodenko, 2002; van Someren, 2002; Chen, 2003; Fawcett, 2003; Miller, 2003; Sole, 2003; Zhao, 2003; Harju, 2004; Kosak, 2004; Pennisi, 2004; Blute, 2005; Freudenberg, 2005; Sanchez, 2005; Holmes, 2006; Nadeau, 2006; Salzberg, 2006)

bio-information theory: (Rashevsky, 1955; Shimbel, 1965; Eigen, 1973, 1993; Battail, 1997; Smith, 1999; Sterelny, 2000; Winnie, 2000; Ricard, 2001; Jablonka, 2002; Yockey, 2002; Barbieri, 2003, 2004, 2006; Segal, 2003; Meyer, 2004; Farnsworth, 2013)

information theory: (Shannon, 1949; Shimbel, 1965; Mowshowitz, 1968; Cole, 1993; Kay, 1995; Battail, 1997, 2006; Smith, 1999, 2000; Ohya, 2000; Sterelny, 2000; Weiss, 2000; Wilbur, 2000; Shimogawa, 2001; Jablonka, 2002; Segal, 2003; Boniolo, 2003; Frappat, 2003; Linstone, 2003; Segal, 2003; Sullivan, 2003; Cornelius, 2004; Furner, 2004; Gherardi, 2004; Sole, 2004; Williams, 2004; Bates, 2005; Chang, 2005; Dall, 2005; Stegmann, 2005; Yoo, 2005; Braman, 2006; Katare, 2006; Schneider, 2006; Zvarova, 2006)

A3.1.2. physical expression of information structures: phenotypes

sensor nets: (Pin, 1991; Biel, 2000; Goldberg, 2000; Petriu, 2000; Akyildiz, 2002; Stankovic, 2003; Hedley, 2004; Kim, 2005; Singh, 2005; Barros, 2006; Moreno, 2006)

evo-devo: (Takahashi, 2001; Arthur, 2002; Passy, 2002; Griffin, 2003; Hall, 2003a, 2003b; Johnson, 2003; Vergara-Silva, 2003; Barabé, 2004; Badyaev, 2005; Griffiths, 2005; Love, 2005; Medina, 2005; Muller, 2005; Rutishauser, 2005; Wilkins, 2005; Furusawa, 2006; Mabee, 2006; Rosenfield, 2006; Salazar-Ciudad, 2004, 2006; Sheldrake, 2006; Young, 2006; Le Page, 2007; Othmer, 2009)

phenotypic plasticity: (Waddington, 1959; Bradshaw, 1965, 1973; Schmid, 1992; Roff, 1996, 2003; Wagner, 1996; Ramirez, 1999; Amzallag, 2000; Debat, 2001; Alpert, 2002; Igel, 2002; Pandolfi, 2002; Callaway, 2003; Suzuki, 2003; Tang, 2003; Hartman, 2004; Li, 2004; Scriver, 2004; Strand, 2004; van Kleunen, 2004; Biesecker, 2005; Borges, 2005; Promislow, 2005; Fordyce, 2006; Jablonka, 2006; Karmiloff-Smith, 2006; Phillips, 2006; Pigliucci, 2006)

phenome: (Mahner, 1997; Lyubich, 2001; Takahashi, 2001; Bochner, 2003; Freimer, 2003; Crampin, 2004; Scriver, 2004; Burggren, 2005; Jones, 2005; Nochomovitz, 2006)

allometric scaling: (Berntson, 1997; Bono, 1997; Lewin, 1999; Brown, 2004a, 2004b, 2004c, 2005; Carpinteri, 2005; Cudennec, 2005; Farrell-Grey, 2005; Gillooly, 2005; Glazier, 2005; Kaspari, 2005; Kerkhoff, 2005; Li, 2005, 2006; Makarieva, 2005; Marquet, 2005; van Nimwegen, 2005; West, 2005; Woodward, 2005;

Hedin, 2006; Loeuille, 2006; Phillips, 2006; Reich, 2006; Vladar, 2006; Holmes, 2007; Whitfield, 2007)

morphology: (Thompson, 1917; Raup, 1965; Stoddart, 1969; Seilacher, 1974; Stebbins, 1974; Stevens, 1974; Chappell, 1980; Caldwell, 1986; de Kroon, 1994; Kaandorp, 1996; Marquis, 1996; Stone, 1997; McGhee, 1999; Eble, 2000; Kobayashi, 2001; Rasskin-Gutman, 2001; Meroz, 2002; Loya, 2001; Stone, 2003; McHenry, 2004; Ubukata, 2005; Honeycutt, 2008; Kerry, 2012)

A3.1.3. bipartite structure

ambient intelligence; intelligent space; pervasive computing; mixed reality: (Sharma, 1998; Lee, 2002; Strohbach, 2002; Bove, 2003; Eng, 2003; Fogli, 2003; Knuth, 2003; Lifton, 2003a, 2003b; Riva, 2003; Yamaguchi, 2003; Briscoe, 2004; Bull, 2004; Callaghan, 2004; Paradiso, 2004; Wright, 2004; Broxton, 2005a, 2005b; Lektauers, 2005; Manzolli, 2005; Remagnino, 2005; Rutishauser, 2005; Chakravarti, 2006)

evolvable, self-reconfigurable hardware: (Hikage, 1996; Manderick, 1996; Sipper, 1997, 1999a, 1999b; Zebulum, 1997; Higuchi, 1999; Yao, 1999; Lahoz-Beltra, 2001; Chen, 2002; Sukhatme, 2002; Yoshida, 2002; Lungarella, 2003; Teuscher, 2003; Glackin, 2004; Lee, 2004; Murata, 2004; Tan, 2004; Torresen, 2004; Blank, 2005; Demaine, 2005; Griffith, 2005; Kamimura, 2005; Wu, 2005; Xu, 2005; Zykov, 2005)

genotype-phenotype mapping: (Smogi, 1997; Schena, 1998; Marchant, 2000; Mattick, 2001; Patton, 2001; Reinke, 2001; Southern, 2001; Cooper, 2002; Stoeckert, 2002; Storz, 2002; Trent, 2002; Cavalli-Sforza, 2003; Furusawa, 2003; Gibson, 2003; Hansen, 2003; Miller, 2003; Nagy, 2003; Roses, 2003; Sato, 2003; Venter, 2003; Wolfe, 2003; Brewster, 2004; Friedman, 2004; Gomes, 2004; Jasney, 2004; Ochiai, 2004; Brors, 2005; DiPetrillo, 2005; Ganley, 2005; Jones, 2005; Moulton, 2005; Repsilber, 2005; Rodriguez-Caso, 2005; Butte, 2006; Meynert, 2006; Phillips, 2006; Rigoutsos, 2006; Shai, 2006)

inhibitory-excitatory process: (Turing, 1952; Wardlaw, 1953; Nelson, 1963; Rosen, 1968; Goodwin, 1985, 1993; Harrison, 1987; Amato, 1990; Murray, 1990; Ives, 1991; Swinney, 1991; Ermentrout, 1993; Dillon, 1994; Mecke, 1996; Bonabeau, 1997; Schiffmann, 1997, 2005; Dormann, 2001; Liaw, 2001; Crampin, 2002; Theraulaz, 2002; Weimar, 2002; Cho, 2004; Cickovski, 2005; Genieys, 2006; Karmarkar, 2006; Liu, 2006; Roska, 2006; Colizza, 2007; Wolpert, 2009; Miyazawa, 2010; Saey, 2010; Short, 2010; Cerveny, 2012; Chen, 2012; Tompkins, 2014)

bipartite network models: (Von Foerster, 1988; Friedler, 1998; Merker, 2003; Caldarelli, 2004; McIlraith, 2004; Ramasco, 2004; Robins, 2004; Toyoda, 2004; Borner, 2005; Doslic, 2005; Huang, 2005; Lambiotte, 2005; Fortuna, 2006; Guillaume, 2006; Lewinsohn, 2006; Zhang, 2006)

bipartite graph theory: (Eades, 1994; Balbuena, 2001; Estrada, 2005; Morris, 2005; Ohkubo, 2005)

A3.2. hierarchical structures of adaptive pattern formation

A3.2.1. genetic-cellular adaptation

transposable genes, jumping genes:

genetic adaption: (Walbot, 1996; Frank, 1997; Moore, 2001; Elena, 2003; Beltman, 2004, 2005a, 2005b; Shea, 2004; Zheng, 2004; Bijlsma, 2005; Dekel, 2005; Feder, 2005; Frankham, 2005; Howe, 2005; Lexer, 2005; Zhuravel, 2005; Angilletta, 2006; Grennan, 2006; Lortie, 2006; Shinar, 2006)

network models: (Bray, 1990; Marijuan, 1991; Lowndes, 1992; Thomas, 1993; Graves, 1994, 1995; Somogyi, 1997; Kasabov, 1998, 2002, 2004; 2005; Scheetz, 1998; Weng, 1999; D'haeseleer, 2000; Guarente, 2000; Kolchanov, 2000; Mattick, 2001; Reidys, 2001; Brazhnik, 2002; Forst, 2002; Kholodenko, 2002; van Someren, 2002; Chen, 2003; Falk, 2003; Krul, 2003; Sole, 2003; Stetter, 2003; Deckard, 2004; Harju, 2004; Kosak, 2004; Sauro, 2004; Rudge, 2005; Sanchez, 2005; Wood, 2005; Dicke, 2006; Kell, 2006; Kholodenko, 2006; Klamt, 2006; Salzberg, 2006; Macía, 2012)

A3.2.2. neural-cognitive adaptation

attention; perception: (Bundesen, 1987; Field, 1994; Makeig, 1997; Barsalou, 1999, 2003; Robertson, 2003; VanRullen, 2003, 2005, 2006; Arnott, 2004; Cichocki, 2004; Muller, 2004; Bartels, 2004, 2005; Choi, 2005; Gupta, 2005; Millar, 2005; Palva, 2005; Humphries, 2006; Raz, 2006; Uchida, 2006; Cardin, 2007; Dhamala, 2007; Tosh, 2007)

classification; category learning: (Clancey, 1985; Redington, 1998; Roy, 2002; Rehder, 2003a, 2003b, 2004; Ashby, 2001, 2004, 2005; Huyck, 2005; Pothos, 2006; Spratling, 2006; Sun, 2006)

concept formation: (Medin, 1984; Hori, 1994; Bhatta, 1997; Schena, 1998; Sloman, 1998; Sinha, 2000; Thompson, 2000; Richards, 2001; Dogil, 2002; Barsalou, 2003, 2005, 2009; Griffiths, 2003c, 2009; McClelland, 2003; Cela-Conde, 2004; Härle, 2004; Kawabata, 2004; Khallad, 2004; Lin, 2004; Mandler, 2004; Maguire, 2004; Nelson, 2004; Tapp, 2004; Binder, 2005; Goguen, 2005; Heit, 2005; Hutchins, 2005; Inselberg, 2005; Wermter, 2005; de Kamps, 2006; McCray, 2006; Tenenbaum, 2011)

concept integration: (Fisher, 1987; Fauconnier, 1998; Chiappe, 2000; Grady, 2000; Kim, 2000; Smith, 2000; Teng, 2000; Pereira, 2002; Ritchie, 2004)

concept graph: (Vaughan, 1974; Lehmann, 1992; Bovasso, 1993; Carley, 1993; Young, 1996; Hartley, 1997; Mohr, 1998; Wang, 1998; Resnik, 1999; Berners-Lee, 2002; Feng, 2002; Rindflesch, 2003; Heesch, 2004; McRae, 2004; Pelekis, 2004; Dieng-Kuntz, 2005; Mazard, 2005; Menczer, 2005; Miller, 2005; Bales, 2006; Guo, 2006; Kay, 2006; Sowa, 2006)

cognition: (Hinton, 1989; Brown, 1990; Muller, 1996; Stewart, 1996; Song, 2000; Newell, 2002; Zeigler, 2002; Martin, 2003; Munakata, 2003, 2004; Makeig, 2004; Siegler, 2004; Ridderinkhof, 2005; Segura, 2005; Shanks, 2005; Sol, 2005;

Barrett, 2006; Basar, 2006; Elman, 2006; Karakaş, 2006; Shaw, 2006; Westermann, 2006; Waber, 2007; Khundrakpam, 2014)

memory: (Schneider, 1993; Raffone, 2001; Noppeney, 2004; Fenker, 2005; Malin, 2006; Yago, 2006; Fedulov, 2007)

consciousness: (Greenwald, 1992; Zeki, 1998; Coward, 1999; Young, 1999; Atkinson, 2000; Brust, 2000; John, 2002; Rees, 2002; Thier, 2002; Crick, 2003; Harnad, 2003; Holland, 2003, 2004; Tunney, 2003; Fell, 2004; Tononi, 2004, 2005; Aleksander, 2005; Cleeremans, 2005; Maia, 2005; Seth, 2005, 2006; Broks, 2006; Bower, 2007; Buchanan, 2007; Deutsch, 2007; Fox, 2007; Henig, 2007; Shadlen, 2007; Anthes, 2013)

self-consciousness: (Kao, 1996; Lee, 1998; Posner, 1998, 2005; Hauber, 2001; Churchland, 2002; Craig, 2002; Johnson, 2002; Baars, 2003; LeDoux, 2003; Thom, 2004; Gusnard, 2005; Hobson, 2005; Macinnes, 2005; Bongard, 2006; Decety, 2006; Morin, 2006; Northoff, 2006; Pérez, 2006; Plotnik, 2006; Raichle, 2006; Ross, 2007; Sedikides, 2007; Kaipa, 2010)

neural structure: (Brown, 1990; Bressler, 1995, 2006; Muller, 1996, 2004; Bower, 1998, 1999, 2000, 2004, 2007; Mckeown, 1998; Omurtag, 2000; Page, 2000; Richter, 2000; Sporns, 2000, 2004a, 2004b, 2005, 2011; Baillet, 2001; Bi, 2001; Jung, 2001; Bhattacharya, 2002, 2005; Makeig, 2002; Burrone, 2003; Glassman, 2003; Siegelmann, 2003; Bartels, 2004a, 2004b, 2005; Buchel, 2004; Chialvo, 2004; Croxson, 2005; Eguiluz, 2005, Esposito, 2005; Gupta, 2005; Hoke, 2005; Knoch, 2005; Segura, 2005; Stam, 2005; Zeki, 2005; Freeman, 2006; Karmarkar, 2006; O'Brien, 2006; Abbot, 2007; Livet, 2007; Lichtman, 2008; Bullmore, 2009, 2012; Stam, 2010; Boersma, 2011; Alexander-Bloch, 2012)

artificial neural networks: (McCulloch, 1943, 1949; Pitts, 1947; Rosenblatt, 1958; Block, 1962a, 1962b; Peretto, 1984; Chalmers, 1990; Gustin, 1990; Widrow, 1990; Hassoun, 1991; Lin, 1991; Bishop, 1992; Chakrabarti, 1992; Freund, 1992; Hunt, 1992; Yoon, 1992; Fritsch, 1993; Hammerstrom, 1993; Honavar, 1993; MacLennan, 1993; Zwietering, 1993; Fiesler, 1994, 1996; Wang, 1994; Chakrabarti, 1995; Arena, 1996; Mahapatra, 1996; Parisi, 1996; Sumpter, 1996; Xu, 1996; Ghazanfari, 1997; Lyon, 1997; Maass, 1997; Michel, 1997; Portas, 1997; Roadknight, 1997; Cooper, 1998; Daqi, 1998; De Falco, 1998; Kung, 1998, 1999; Papik, 1998; Sabuncuoglu, 1998; Tsoi, 1998; Jedra, 1999; Kurfess, 1999; Lek, 1999; Vellido, 1999; Biel, 2000; Bojkovic, 2000; Kartam, 2000; McLaren, 2000; Paik, 2000; Patterson, 2000; Peterson, 2000; Roy, 2000; Smith, 2000; Zhang, 2000; Adeli, 2001; Bailer-Jones, 2001; Bayro-Corrochano, 2001; Recknagel, 2001; Rosa, 2001; Bien, 2002; Cardot, 2002; Chen, 2002; Hanson, 2002; Natarajan, 2002; Poulton, 2002; Tijsseling, 2002; Xu, 2002; Harnad, 2003; Knuth, 2003; Meireles, 2003; Merlo, 2003; Sima, 2003; Stankovic, 2003; Wakeling, 2003; Bedaux, 2004; Kasabov, 2004; Lin, 2004; Narayanan, 2004; Robert, 2004; Seiffert, 2004; Černá, 2005; Chortaras, 2005; Ferentinos, 2005; Kinbara, 2005; Bianchini, 2006; Gruning, 2006; Li, 2006; Muselli, 2006; Wang, 2006)

multi-layer artificial neural networks: (Benedict, 1988; Mavrovoijniotis, 1992; Sontag, 1992; Bose, 1993; Funabiki, 1997; Rudolph, 1997; Svozil, 1997; Egmont-

Petersen, 1998; Vaughn, 1999; Pal, 2001; Utgoff, 2002; Krawczak, 2005; Park, 2006a, 2006b)

A3.2.3. social group adaptation

quorum sensing: (Kempner, 1968; Nealson, 1979; Bassler, 1999; Miller, 2001; Nakagaki, 2004; Amos, 2005; Terrazas, 2005; Waters, 2005; Fricker, 2008; Nadell, 2008; Takamatsu, 2009; Dussutour, 2010; Niizato, 2010; Tero, 2010; Dandekar, 2012; Adamatzky, 2011; Reid, 2012)

swarm, flocking: (Hamilton, 1971; Pulliam, 1973; Eriksson, 2010;

slime mold: (

formal, informal networks; invisible colleges: (Crane, 1969; Van Rossum, 1973; Boorman, 1975; Kronin, 1982; Todorov, 1986; Hartman, 1990; Lievrouw, 1990; Gould, 1991; Morris, 1994; Falkenberg, 1995; Morand, 1995; Lievrouw, 1996; Walsh, 1996; Kautz, 1997; Baker, 1999; Moenaert, 2000; Stiglitz, 2000; Jirojwong, 2001; Kronick, 2001; Tuire, 2001; Priss, 2002; Wilkinson, 2003; Owen-Smith, 2004; Heisenberg, 2005; Muller, 2005; de Mesquita, 2006; Fry, 2006; Kossinets, 2006; Nunes, 2006; Zuccala, 2006)

animal culture: (Laland, 2003; Sumpter, 2006

plant communication: (Trewavas, 2005a, 2005b; Callaway, 2007;

community genetics: (Antonovics, 2003; Chase, 2003; Collins, 2003; Neuhauser, 2003; Rickleps, 2003; Whitham, 2003, 2005, 2006, 2008; Biernaskie, 2005; Turegon, 2005; Vellend, 2005; Crutsinger, 2006; Fordyce, 2006; Garrett, 2006; Shuster, 2006; Urban, 2006; Silvertown, 2009; Leo, 2010; Hersch-Green, 2011; Tack, 2012)

artificial neural network ensembles: (Hansen, 1990; Opitz, 1999; Ghosh, 2002; Valentini, 2002; Zhou, 2002; Kolter, 2003; Fernández-Redondo, 2004; Torres-Sospedra, 2005; Hernández-Espinosa, 2005; Garcez, 2007)

interconnection nets: (Bhuyan, 1987; Reed, 1987; Akers, 1989; Cooperman, 1991; Hatz, 1992; Stiihr, 1991; Kotsis, 1992; Schibell, 1992; Blazewicz, 1993; Harary, 1993; Barth, 1994; Fraigniaud, 1994; Keller, 1994; Tzeng, 1994; Wong, 1995; Bhatt, 1996, 1998; Vadapalli, 1996; Hasunuma, 1997; Agrawal, 1998; Bermond, 1998; Huang, 1998; Wei, 1999; Taghiyareh, 2000; Hwang, 2001; Comellas, 2003; Ziavras, 2003; Jan, 2004; Xiao, 2005; Wagh, 2006)

community parallel distributed processing: (Noy-Meir, 1987; Findler, 1992; Nordhaus, 1992; Gordon, 1996; Hirsh, 2001; Hubbell, 2001; Bijlsma, 2005; Brookfield, 2005; Chase, 2005; Monzeglio, 2005; Friedmann, 2005; Harnad, 2005, 2006; Gravel, 2006)

A3.2.4. evolutionary-population adaptation

population genetics: (Dawkins, 1976; Lewin, 1982; Wade, 1998; Skyrms, 2000; Singh, 2003; Buss, 2004; Dyer, 2004; Wilke, 2005; Song, 2006)

evolution: (Smith, 1978; Smith, 1983; Delsol, 1991; Sereno, 1991; Mayr, 1992, 1993; 1999, 2000, 2005; Pocklington, 1997; Channon, 1998; Lyman, 1998; Kauffman, 1999; Sole, 1999; Wallace, 1999; Ben-Ari, 2000; Jeffreys, 2000; Nowak, 2000; Brookfield, 2001, 2004, 2009; Hodgson, 2001; Hoenigsberg, 2002; Padian, 2003; Bentley, 2003, 2004, 2005; Borges, 2005; Eerkens, 2005; Hey, 2005; Jeffares, 2005; Reynolds, 2005; Wilkins, 2005; Mesoudi, 2006; Sterelny, 2006; Lieberman, 2007; Prosser, 2007; Spinney, 2007; Atkinson, 2008; Gross, 2008; Rogers, 2008; Laland, 2010)

fitness landscape: (Jones, 1987; Kauffman, 1991; Bak, 1992, 1997; Mangel, 1992; Stadler, 1995, 1996, 1999, 2002; Whitlock, 1995; Westhoff, 1996; Bagnoli, 1997; Sloman, 1998; Weber, 1998; Imada, 1999; Skipper, 2001, 2004; Wilke, 2001; Yu, 2001; Reidys, 2002; Blackburne, 2005; Borenstein, 2005; Doye, 2005; Iguchi, 2005; Jain, 2005, 2007; Ochoa, 2008)

ecosystem connectionism: (Keitt, 1997; Urban, 2001; Jordan, 2004; Green, 2005; Brooks, 2006; Memmett, 2006; Pascual-Hortal, 2006;

graph theory giant component: (Newman, 2001;

A3.3. invariant properties of adaptive inductive pattern formation

A3.3.1. interpretive information processing

mimicry: (Bates, 1862; Thayer, 1918; Dafni, 1984; Oldstone, 1987, 1998, 2005; Behrens, 1988; Albert, 1999; Stebbins, 2001; Sherratt, 2005; Yuki, 2005; Hanlon, 2007; Stevens, 2007, 2009; Skelhorn, 2010; Scott-Samuel, 2011; Cusick, 2012; Heliconius Genome Consortium, 2012; Hoffman, 2012; von Beeren, 2012)

positional information: (Wolpert, 1969, 1989, 1994, 1996, 1971, 1981, 2009, 2011; Lewis, 1977; Maden, 1977; Jeong, 2001; Kerszberg, 2007; Richardson, 2009)

cybernetics: (Wiener, 1954; Miller, 1956; Rapoport, 1956; Trucco, 1956; Cowan, 1965; Johnson, 1970; Thomas, 1995; Battail, 1997; Heylighen, 2001; Riegler, 2005; Gupta, 2006; Wang, 2008; Denning, 2012)

semantic web:

semiotic; semantic systems: (Morris, 1938; Yates, 1985; Hoffmeyer, 1997; Kao, 1996; Battail, 1997; Posner, 1998; Kirschvink, 2001; Bookheimer, 2002; Chandler, 2002; Craig, 2002; Jablonka, 2002, 2009; Friederici, 2003, 2009; Ben Jacob, 2004; Khallad, 2004; Maguire, 2004; Mateo, 2004; Noppeney, 2004; Binder, 2005; Macinnes, 2005; Gupta, 2006; Schultz, 2006; Thom, 2006; Milius, 2009; Saey, 2009; Atkin, 2010)

A3.3.2. distributed local control, local phase transition processes

self-assembly; self-organization: (Crane, 1950; Ashby, 1962; Caspar, 1962, 1963; Eigen, 1971, 1977; Klug, 1972; Bak, 1987, 1988, 1990; Tang, 1987; Constance, 1991; Farmer, 1992; Sipper, 1999; Lee, 2000, 2002; Anderson, 2002; Banzhaf, 2002; Pascual, 2002; Whitesides, 2002; Lifton, 2003; Skår, 2003; Van Orden, 2003; Abrescia, 2004; Guerin, 2004; Pfeifer, 2004; Tan, 2004; Costa, 2005, Krishnamurthy, 2005; Remagnino, 2005; Rutishauser, 2005; Årzén, 2006; Halley, 2006, 2008; Edelmann, 2007; Garlaschelli, 2007; Keller, 2007; Werner, 2007; Dressler, 2008; Karsenti, 2008; Kurdi, 2008; Rusu, 2008; Keller, 2009; Prokopenko, 2013;

connectionism, parallel distributed processing: (Rumelhart, 1986; McClelland, 1988; Parunak, 1988; Elman, 1990; Mjolsness, 1991; Raghupathi, 1991; Rasmussen, 1992 ; Bray, 1993; Fu, 1993,1995; James, 1996; Coward, 1997; Merigot, 1997; Somogyt, 1997; Machado, 1998; van der Vet, 1998; O'Brien, 1999; Browne, 2000; Fletcher, 2000,2001; Page, 2000; Haberly, 2001; Kang, 2001; Kremer, 2001; Wirth, 2001; Seilewiesiuk, 2002; Paton, 2003; Kasabov, 2004; Lobb, 2005; Sander, 2005; Shapiro, 2005; Reza, 2006; Roth, 2006; Rogers, 2014)

morphology of porous media: (Roberts, 1997; Perret, 1999; Prasher, 1999; Suding, 1999; Ioannidis, 2000; Liang, 2000; Hilpert, 2003; Hillmyer, 2005; Hunt, 2009)

first passage time: (Weiss, 1967; Ricciardi, 1999; Condamin, 2005, 2007; Shlesinger, 2007; Ma, 2009; Oshanin, 2009; Mejía-Monasterio, 2011; Mattos, 2012;

percolation: (Broadbent, 1957; Shante, 1971; Kirkpatrick, 1973; Essam, 1980; Wierman, 1982; Kesten, 1987; Yonezawa, 1989; Winsor, 1995; Galam, 1997; Prea, 1997; Stauffer, 1997; van der Marck, 1997; Berkowitz, 1998; d'Iribarne, 1999; Callaway, 2000; Moore, 2000; Blanchard, 2002; Cohen, 2002; Wierman, 2002; Banaei-Kashani, 2003; Gamba, 2003; Martins, 2003; Arns, 2004; Krapivsky, 2004; Beffara, 2005, 2008; Christensen, 2005; Gandolfo, 2005; Kozma, 2005; Perez, 2005; Wierman, 2005; Bollobás, 2006a, 2006b; Camia, 2006a, 2006b; Giménez, 2006; Moukarzel, 2006; Sarshar, 2006; Acin, 2007; Balter, 2007; Henry, 2007; Motluk, 2007; Davis, 2008; Haenggi, 2009; Hunt, 2009; Son, 2012)

catastrophy theory bifurcation: **(**Zeeman, 1971, 1976, 1977, 1988; Thom, 1975, 1976; Bellairs, 1977; Poston, 1978; Sibatani, 1978; Stewart, 1977, 1981,1982; Saunders, 1980; Wilson, 1981; Oster, 1982; Arnol'd, 1984; Silvi, 1994; Kappos, 1995; Krokidis, 1997; Marx, 1997; Priest, 1997; Borisuk, 1998; Brown, 1999; Margalef-Roig, 2000; Tyson, 2001, 2007; Garliauskas, 2003; Stark, 2003; Maclean, 2005; Rosser, 2007; Kitajima, 2009; Lundstedt, 2009; Strogatz, 2012)

biological bifurcation: (Maynard Smith, 1985; Brown, 1988; Goodwin, 1990; Gilbert, 1991; Thatcher, 1992; Koch, 1994; Affolter, 2003; Cinquin, 2005; Foster, 2009)

speciation: (Barton, 1989; Weitzman, 2003; Beltmanan, 2004, 2005a, 2005b; Atkinson, 2005; Bull, 2005; Butlin, 2005, 2006; Lexer, 2005; Sadedin, 2005; West-Eberhard, 2005; Nosil, 2008)

A4. HIERARCHICAL MODULAR UNITS OF ANALYSIS, SYNTHESIS

What are the units of adaptation and the mechanism of their interaction?

In the third knowledge state space of the investigation, the inductive process of a posteriori conceptual clustering hierarchically structured the specific data grains of individual citations into a union of three sets of generalization:

(individual element as a *local fundamental group*)

(hierarchical modular structure)

(quantitative configuration descriptors)

A4.1. individual element defined as a *local fundamental group*

human microbiome: (Cossins, 2016;

extended phenotype: (Dawkins. 1978, 2004; Blyth, 1994; LaBarbera, 2000; Schuck-Paim, 2000; Sterelny, 2000; Miller, W., 2002; Miller, M., 2003; Turner, 2002, 2003, 2004; Jablonka, 2004; Laland, 2004; Stake, 2004; Lambrechts, 2006; Hoover, 2011)

niche construction: (Odling-Smee, 1988, 1996a, 1996b; Jones, 1994, 1997; Layland, 1999, 2000, 2001, 2006; Smith, 2002; Day, 2003; Keller, 2003; Robert, 2003; Stone, 2003; Cintas, 2004; Donohue, 2004; Taylor, 2004; Vandermeer, 2004; Brodie, 2005; Jones, 2005; Shorthouse, 2005; Sterelny, 2005; Sultan, 2005; Suzuki, 2005; Borenstein, 2006; Wright, 2006)

distributed cognition: (Hutchins, 1989, 1995, 1999, 2000, 2014; Rogers, 1994; Zhang, 1994; Derry, 1998; Moore, 1998; Hollan, 2000, 2009; Parsons, 2014)

interspecies horizontal gene transfer: (

epistatic clustering: (Bateson, 1907; Wright, 1935; Fraser, 1960; Kojima, 1964; Spassky, 1965; Jinks, 1973; Perkins, 1973; Orr, 1996; Bateson, 2002; Cordell, 2002, 2009; Phillips, 2002, 2008; Nei, 2003; Pepper, 2003; Carlborg, 2004; Segre, 2004; Battle, 2005; Moore, 2005a, 2005b, 2006, 2009; Van Driessche, 2005; Weinreich, 2005; Wong, 2005; Bershtein, 2006; Gjuvsland, 2007; Oti, 2007; Taylor, 2007; Hintze, 2008; Shao, 2008; Ma, 2008; Tang, 2009; Tyler, 2009; Greene, 2010; Lunzer, 2010; VanderWeele, 2010; Lehner, 2011; Ostman, 2011; Breen, 2012; Huang, 2012; Valverde, 2012; Xie, 2012; Zhang, 2013; Sun, 2014; Turner, 2016)

epigenetics; Baldwin effect: (Waddington, 1953; Jablonka, 1989, 1994, 1998a, 1998b, 2002, 2009; French, 1994; Carley, 1997; Turney, 1997; Reik, 2001; Gershenson, 2002; Griesemer, 2002; Van De Vijver; 2002; Van Speybroeck, 2002a, 2002b; Jaenisch, 2003; Mameli, 2004, 2005; Yong-hui Jiang, 2004; Grant-Downton, 2005; Berthouze, 2005; Harper, 2005; Prince, 2005; Balter, 2006; Hogenesch, 2015; Ropars, 2015) *endosymbiosis:* (Margolis-Sagan, 1967; Travis, 1998; Husnik, 2011, 2013; Lawrence, 2012)

hierarchical multi-species symbiosis: (Hooper, 2001; Willis-Karp, 2001; Brogaard, 2004; Lowery, 2007; Rook, 2008; Blaser, 2009, 2012, 2013; Nikoh, 2009; Rajagopal, 2009; Hehemann, 2010; Hellman, 2010; Jaenike, 2010; Meadows, 2010; Ochman, 2010; Feldhaar, 2011; Harder, 2011; Anthony, 2014; Bultman, 2014)

fundamental group topology: (Croom, 1978; Mermin, 1979; Simon, 2008; Dooley, 2011)

A4.2. hierarchical modular structure

modularity: (Caspar, 1963; Parnas, 1984; van der Linden,1995; Auda, 1996; Wagner, 1996; Draye, 1997; Sharkey, 1997; Calabretta, 1998; Minelli, 1998; Coltheart, 1999; Husken, 2002; Langlois, 2002; Lipson, 2002, 2007; Michl, 2002; Ravasz, 2002; Sole, 2002, 2003, 2006, 2008; Inbar, 2003; Peretz, 2003; Sole, 2003; Ethiraj, 2004; Guimera, 2004; Han, 2004; Ma, 2004; Variano, 2004; Danon, 2005; Guimera, 2005; Hughes, 2005; Irizarry, 2005; Pawson, 2005; Prill, 2005; Sadedin, 2005; Arney, 2006; Barrett, 2006; Gavin, 2006; Newman, 2006; Barrett, 2006; Karmiloff-Smith, 2006; Pan, 2007; Wagner, 2007; Alexander-Bloch, 2010; Meunier, 2010; Clune, 2013)

cognitive chunking: (Miller, 1956; Simon, 1974; Gobet, 1998, 2001, 2004; Sweller, 1998; Cowan, 2000; Pollock, 2002; Glassman, 2003; Freudenthal, 2005; van Merrienboer, 2005; Hockey, 2006; O'Reilly, 2006; Uchida, 2006; Stevens, 2012;

graph motifs: (Milo, 2002, 2004; Berg, 2004; Kashtan, 2004, 2005; Sporns, 2004; Tanay, 2004; Arenas, 2008; Gherardini, 2010)

A4.3. quantitative configuration descriptors

ordination: (Goff, 1972; Dale, 1975; Austin, 1985; Kent, 1988)

statistical physics: (Maxwell, 1856; Boltzmann, 1897 (1974); Gibbs, 1902; Hastings, 1909; North, 1970; Suffritti, 1987; Brush, 1994; Gilbert, 1995; Flamm, 1997; Shafer, 1998; Ball, 2002a, 2002b; Cecconi, 2005; Thurner, 2006; Uffink, 2006; Oestreicher, 2007; Ny, 2008; Kadanoff, 2009a, 2009b, 2014; Dembo, 2009; Mussardo, 2010; de Vladar, 2011)

spatial statistics: (Gould, 1970; Jumars, 1997; Baddeley, 1980; Wiencek, 1993; Kerscher, 1998; Stoyan, 2000; Diaz, 2001; Shimatani, 2001; Mayer, 2004; Cliff, 2009; Lucas, 2013; Stojanova, 2013)

morphometrics: (Prusinkiewicz, 1997; Laffont, 2011)

mathematical morphology: (Maragos, 1986; Vincent, 1989; Vincent, 1991; Michielsen, 2000, 2001, 2002; Eckhardt, 2003)

fragmentation of homogeneous connectivity in the prevailing collective conscious by spatially isolated clusters of dissent *minkowski functionals:* (Kellerer, 1984; Mecke, 1993, 1997, 2000, 2005; Burchert, 1994; Kerscher, 1995, 2000; Platzoder, 1995; Schmaltzing, 1995, 1997, 1999; Mecke, 1997, 1998, 2000, 2002, 2003; Sahni, 1997, 1998; Schmalzing, 1997, 1999; Michielsen, 2000; Dominguez, 2001; Arns, 2002; Beisbert, 2002; Hilfer, 2002; Karimova, 2003; Sheth, 2003, 2005; Agterberg, 2004; Sych, 2004; Platzoder, 2005; Kjeldsen, 2008)

boolean grain models: (Wicksell, 1925; Solomon, 1953; Serra, 1980; Lutwak, 1986; Weil, 2000; Agterberg, 2004; Stoyan, 2005a, 2005b; Moller, 2010; Chiu, 2013)

graph theoretic descriptors: (Hasegawa, 1976; von Schnering, 1987; Liu, 1998; Fernandez, 2005; Conte, 2007; Gfeller, 2007; Barthelemy, 2011; Goel, 2013)

topological descriptors: (Hyde, 1987, 1989; Boxer, 1999; Melott, 2000; Svensson, 2003)

A5. LIMITS OF PROOF AND TRUTH

What are the cultural precedents and boundaries of provable truth structures?

The fourth knowledge state space of the investigation structured the specific data grains of individual citations into a union of four sets of generalizations:

(structures of proof)

(structures of provable truths)

(boundaries of provable truth structures)

(emerging situated point-of-view in science and mathematics)

A5.1. structures of proof

deductive proof: (Wang, 1960; Tarski, 1969; Troelstra, 1977; Feferman, 1979; Kleiner, 1991, 1997; Horgan, 1993; Andrews, 1994; Thurston, 1994; Devlin, 1997, 2002, 2003; Barker, 2000; Kvasz, 2000; Van Bendegem, 2000, 2005, 2009; Mancosu, 2001; Lee, 2002; Barendregt, 2005; Cohen, 2005; Macintyre, 2005; MacKenzie, 2006; Khamsi, 2006; Hales, 2008; Harrison, 2008; Van Kerkhove, 2008; Wiedijk, 2008; Crosley, 2011)

inductive proof: (Bussey, 1917; Morris, 1938; Wang, 1960; Aubin, 1979; Bundy, 1988; Kounalis, 1990; Paulson, 1990; Parigot, 1992; Merritt, 1997; Bouhoula, 2001; Lange, 2001; Urso, 2004; Hutter, 2005; Polycarpou, 2008; Paenke, 2009; Blanchette, 2010; Palla, 2011; Roychoudhury, 2012; Moller, 2013)

visual proof: (Epstein, 1991; Brown, 1997; Goldstein, 1998; Casselman, 2000; Faris, 2000; Thornton, 2001; de Villiers, 2003; Mancosu, 2005; Hanna, 2007)

A5.2. structures of provable truths

set theoretic foundation of mathematics: (Weyl, 1946; Tucker, 1963; Cohen, 1967; Erdos, 1967; Kreisel, 1967; Monk, 1970; Johnson, 1972; Bell, 1975; Weston, 1976; Dauben, 1978, 1983; Hegner, 1978; Moore, 1978; Dauben, 1979; Maddy, 1980; Burgess, 1984; Smorynski, 1987; Dale, 1990; Anellis, 1991; Hallett, 1991; Roitman, 1992, 2011; Potter, 1993; Franzosi, 1994; Kanamori, 1996, 2004; Dreben, 1997; Allen, 2000; Marek, 2001; Muller, 2001, 2004, 2011; Hardegree, 2003; Burgess, 2004; Ferreiros, 2004; Shapiro, 2004, 2005; Graham, 2006; Holmes, 2006; Kerkhove, 2006; Bell, 2007; Kanamori, 2008; Shulman, 2008; Strogatz, 2010; Ferreiros, 2011; Roitman, 2011; Dasgupta, 2014)

category theoretic foundation of mathematics: (Eilenberg, 1945; Lawnere, 1964, 1999; Bell, 1981, 1982, 1986, 2001, 2005a, 2005b; McLarty, 1987, 1993, 1998; 2004, 2005, 2007; Lambek, 1989; Adamek, 1990; Goguen, 1991; Fokkinga, 1994; Marquis, 1995; Awodey, 1996, 2002, 2004, 2005, 2006, 2007; Isham, 1996; Martini, 1996; Corfield, 2002; Shapiro, 2005; Zafiris, 2005; Hellman, 2006a, 2006b; Landry, 2005, 2006; Pedroso, 2008; Linnebo, 2011) *probability theory:* (Laplace, 1902; Fisher, 1922; Molina, 1930; Matheron, 1963; Gray, 1967; Chayes, 1972; Hampton, 1973; Hacking, 1975, 1987, 1990a, 1990b, 1990c; Stewart, 1986; Cohen, 1987, 1988; Kruger, 1987; Stone, 1987; Wise, 1987; Oberschall, 1989; Shafer, 1989; Clevenson, 1991; Fienberg, 1991; Schneider, 1991; Bogza, 1999; Hacking, 2000; Mumford, 2000; Swift, 2000; McCullagh, 2002; Hodgson, 2004; Atkinson, 2006; Callaway, 2008; Wilkinson, 2009; Augustin, 2011; Parascandola, 2011; Pépin, 2012; Nuzzo, 2015)

philosophy of mathematics: (Brouwer, 1913; Hilbert, 1923; Weyl, 1946, 1953; Grabiner, 1974; Heyting, 1974; Putnam, 1975; Beeson, 1980; Mac Lane, 1980; Maddy, 1980, 2001, 2008; Giaquinto, 1983; Machover, 1983; Detlefsen, 1990; McLarity, 1993, 1997, 2007; Gauthier, 1994; Lambek, 1994; Nagorny, 1994; Tieszen, 1994; Corry, 1997; Mancosu, 1997; Feferman, 1999, 2000a, 2000b; Folina, 2000; Reck, 2000; Reed, 2000; Steen, 2000; Muller, 2001; Devlin, 2002; Lomas, 2002; Raatikainen, 2003; Bell, 2004; Shapiro, 2004; Schlimm, 2005; Zach, 2005; Chaitin, 2006; van Kerkhove, 2006; Carter, 2008; Cantini, 2009, 2010; Kreinovich, 2011; Tait, 2011; Liston, 2012; Aron, 2013; Hartimo, 2013; Pollard, 2013)

A5.3. boundaries of provable truth structures

theological precedent of an external point-of-view: (Huygens, 1678; Bernoulli, 1713; Bayes, 1763; David, 1955, 1962; Kendall, 1956; Torrance, 1972; Feyerabend, 1975; Hall, 1979; Force, 1981; Green, 1981; Dolby, 1987; Bellhouse, 1993; Fuller, 1997; Osler, 1997; Garber, 1998; Sylla, 1998; Zabell, 1998; Ramati, 2001; Antognazza, 2003; Guicciardini, 2004; Shapiro, 2004; Adamson, 2005; Batanero, 2005; Breger, 2005; Cook, 2005; Koetsier, 2005; Markley, 2005; Rudiger, 2005; Krants, 2006; Oestreicher, 2007; Mazzotti, 2007; Shea, 2007; Hannabuss, 2009; Bradley, 2011)

atomistic continuity, discontinuity: (Piaggio, 1951; Youschkevitch, 1976; Waesberghe, 1982; Wilson, 1982; Dunham, 1990; Ponte, 1992; Watson, 1993; Brace, 1998; Malet, 1996a, 1996b; Baker, 1998; Garber, 1998; Laugwiz, 1999; Palmer, 1999; Rosser, 2000; Thomson, 2001; Ausin, 2005; Cooke, 2005; Koutsoukos, 2005; Rusnock, 2005; Dahan-Dalmedico, 2009; Rosser, 2009, 2011; Stillwell, 2010; Drago, 2011; Kleinert, 2011; Velupillai, 2011; Bell, 2014)

incompleteness; inconsistency: (Cohen, 1967; Davis, 1982, 2005, 2006; Dawson, 1991; Uspensky, 1994; Stillwell, 1998, 2002; Feferman, 1999, 2000, 2006, 2012; Chaitin, 2002; Devlin, 2002; Davis, 2003; Goldstein, 2006; Grattan-Guinness, 2006; Hellman, 2006; Zach, 2006; Binder, 2008; Kanamori, 2008; Wolpert, 2008, 2010; Collins, 2009; Elwes, 2010, 2011; Plisko, 2011; Tait, 2011; Critchley, 2014)

computability; computational complexity: (Rosen, 1962; Steen, 1975; Lewis, 1977; Kleene, 1981; Davis, 1982; Cipra, 1989; Hartmanis, 1989; Stewart, 1991; Sipser, 1992, 1996; Fenstad, 1993, 2004; Feferman, 1994; Leivant, 1994; Vinay, 1994; Matthews, 1995; Blum, 1996; Goldreich, 1996; Papadimitriou, 1996; Razborov, 1996; Soare, 1996a, 1996b, 1999; Wigderson, 1996; Peterson, 1998; Cooper, 1999; Stillwell, 2004; Lakshmikantham, 2005; Marchal, 2005; Peterson, 2006; Chaitin, 2006; Erdi, 2008; Rosenberg, 2009; Velupillai, 2009; Apon, 2010; Elwes, 2010; Rosenberg, 2010; Pavlus, 2012; Smith, 2012; Tourlakis, 2012) *inductive inference:* (Kuo, 1923; Sloctemyer, 1923; Fisher, 1935; Neyman, 1955; Solomonoff, 1957, 1964; Gold, 1965; Blum, 1974, 1975; Buchanan, 1979; Mohr, 1982; Angluin, 1983; Arnold, 1983; Baird, 1983; Ernest, 1984; Daley, 1986a, 1986b; Pitt, 1989; Kounalis, 1990; Ligomenldes, 1990; Paulson, 1990; Parigot, 1992; Crutchfield, 1993, 2011; Ernest, 1994; Jiang, 1995; Carmesin, 1996; Dybjer, 2000, 2003; Mandler, 2000; Shalizi, 2001; Blackwood, 2004; Urso, 2004; Galik, 2006; Kisner, 2008; Zeugmann, 2008; Lange, 2009; McCaskey, 2010; Palla, 2012; Guedon, 2013; Moller, 2013; Zenil, 2013; Zhou, 2013; Ghani, 2015; Scantamburlo, 2015)

A5.4. emerging situated point-of-view in science and mathematics:

situated cognition: (Brown, 1989; Glenberg, 1997; Greeno, 1998; Kirshner, 1998; Smith, 1999; Auslander, 2001; Clark, 2001; Goldin, 2001; Weng, 2001; Presmeg, 2002; Semin, 2002, 2013; Wilson, 2002; Anderson, 2003; Smith, 2004, 2007; Costa, 2005; Yeh, 2006; Barsalou, 2008, 2009; Clancey, 2009; Gallagher, 2009; Robbins, 2009, 2010; Wilson, 2009; McNerney, 2011; Wilson, 2011; Wilson-Mendenhall, 2011; Wilson, 2016)

self-other; self-identification: (Kauffman, 1987; Kao, 1996; Lee, 1998; Posner, 1998; Kitcher, 2002; Lee, 2000; Hauber, 2001; Reiss, 2001; Wolff, 2001; Churchland, 2002; Craig, 2002; Johnson, 2002; Baars, 2003; LeDoux, 2003; Sedikides, 2003; Mateo, 2004; Thom, 2004; Gusnard, 2005; Hobson, 2005; Macinnes, 2005; Posner, 2005; Bongard, 2006; Cohen, 2006; Decety, 2006; Kozma, 2006; Morin, 2006; Northoff, 2006; Perez, 2006; Plotnik, 2006; Synofzik, 2006; Dudley, 2007; Ross, 2007; Zuger, 2007; Grayling, 2009; Karban, 2009; Yovel, 2009; Edgell, 2011; Lander, 2011; Weir, 2011; Falk, 2012; Palombo, 2013)

neural network self-feedback: (Hopfield, 1982, 1987, 1999; Getz, 1991; Perfetti, 1991; Salam, 1991; Dong, 1992; Liu, 1994; Quero, 1994; Douglas, 1995; Chang-song, 1997; Young, 1997; Brandt, 2007; Narayan, 1997; Stauffer, 2003; Ganguli, 2009; Goldman, 2009; Ma, 2010; Wang, 2011)

predictive; adaptive control: (Widrow, 1964; Khol, 1969; Garcia, 1989; Antsaklis, 1991; Levin, 199; Cota, 1994; Botto, 1998; Miall, 1998; Hagan, 2002; Ou, 2002, 2003; Mears, 2003; Hontoria, 2005; Bastain, 2006; Basso, 2006; Raichle, 2006; Marshall, 2007; Szpunar, 2007; Soon, 2008; Grupe, 2013)

2nd order cybernetics: (von Foerster, 1973, 1979, 1984, 2003; Hoffman, 1985; Griffith, 1990; Scott, 1996, 2004; Leydesdorff, 1997; von Glasersfeld, 1996; Heylighen, 2001; Umpleby, 2001; Vanderstraeten, 2001; Brier, 2004; Glanville, 2004; Scott, 2004; Kauffman, 2005; Romesín, 2005; Ziemke, 2005; Garland, 2007; Froese, 2010)

agent-based modeling: (Bainbridge, 1994; Menczer, 1995; Carley, 1996; Doran, 1997; Parunak, 1997, 1998, 2000, 2004, 2005; Bieszczad, 1998; DeCanio, 1998; Iglesias, 1999; Davidsson, 2000; Filipe, 2000; Moss, 2000; Rose, 2001; Sarjoughian, 2001; Uhrmacher, 2001; Wilson, 2001; Brueckner, 2002; Gulyas, 2002; Kennedy, 2002; Mamei, 2003, 2004; 2005; Odell, 2003, 2005; Suematsu, 2003; Turkett, 2003; Zambonelli, 2003; Guerin, 2004; Luck, 2004; White, 2004; Bernon, 2005; Fletcher, 2005; Janssen, 2005; Kadar, 2005; Lees, 2005; Muhammad, 2005, 2006; Weyns, 2005; Kuznar, 2006; Ota, 2006; Zhu, 2006) *choice structure:* (Simon, 1945, 1955, 1956, 1995; Arrow, 1951, 1958, 1959, 1966; Rubin, 1963; Zuckerman, 1967; Bryant, 1972; Orr, 1972; Luce, 1977, 1994; Beeson, 1980; Richardson, 1982; Shamir, 1982; Troelstra, 1983, 1996; Neuringer, 1986; Swait, 1987; Suck, 1992; Horowitz, 1995; Marahrens, 1998; Feferman, 2000; Stillwell, 2002; Roy, 2003; Xue, 2003; Ben-Akiva, 2004; Bonsall, 2004; Dugundji, 2005; Erev, 2005; Bekhor, 2006; Clerc, 2006; Ioannides, 2006; Davidson, 2007; Wutz, 2007; Minks, 2009; Prato, 2009; Durlauf, 2010; Apt, 2011; Fox, 2011; Hackney, 2011; Han, 2011; Cariani, 2012; Mueller, 2012, 2013; McCausland, 2013; Wang, 2013, 2015)

fuzzy; rough structure: (Zadeh, 1965, 1968a, 1968b, 1971, 1976, 1978, 1980a, 1980b, 1984, 2001, 2002, 2003, 2004, 2005, 2008a, 2008b, 2011; Marinos, 1966, 1969; Wee, 1969; Bellman, 1970; Thomason, 1974; Kraft, 1983; Pedrycz, 1990, 1991, 2009; Carpenter, 1991, 1992; Kacprzyk, 1991; Stout, 1991; Xie, 1991; Drossos, 1992, 1993; Pal, 1992; Satyadas, 1992; Wood, 1992; Castro, 1994, 1997; Atonsson, 1995; Burgin, 1994, 1995, 1999, 2009, 2011; Gustin, 1994a, 1994b; Gerstenkorn, 1995; Wong, 1995; Herencia, 1996; Baraldi, 1999; Pham, 1999; Turksen, 1999; Walczak, 1999; Dubois, 2001, 2002; Maji, 2002; Skowron, 2003, 2005; Ferrero, 2004; Oberkampf, 2004; Singpurwalla, 2004; Bazan, 2006; Behounek, 2006; Ekel, 2006; Pawlak, 2007a, 2007b; Vlachos, 2007)

local symmetry; semi-group; groupoid structure: (Pepper, 1948; Ito, 1950, 1976; Dornberger-Schiff, 1956, 1961, 1972; Bagley, 1970; Baronnet, 1978; Fichtner, 1980, 1986, 1988; Hohne, 1981; Mosseri, 1984; Brown, 1987, 2011; Zvyagin, 1988; Masuda, 1993; Makovicky, 1997; Zvyagin, 1997; Paterson, 1998; Howie, 1999; Schein, 1999, 2002; Sun, 1998; Vainshtein, 2000; Nespolo, 2002; Belokoneva, 2005; Mitra, 2006; Hollings, 2007, 2009; Nespolo, 2008a, 2008b)

homogeneous structure: (Gardiner, 1976, 1978; Ronse, 1978; Vince, 1981; Meyers, 1985; Nedela, 1993; Conder, 1996; D'Atri, 2005; Cameron, 2006; Taylor, 2007; Gray, 2010, 2011; Hedman, 2010; Rusinov, 2010; Dolinka, 2011; Macpherson, 2011; Mašulović, 2011; Hamann, 2012, 2013; Leger, 2014; Lockett, 2014; Al-Addasi , 2015)

embodied; non-classic; unconventional; natural computation: (Latto, 1990; Collin, 1998; Henningsen, 2000; Knight, 2001; Stepney, 2003, 2008, 2009, 2014; McLennan, 2004, 2009, 2010, 2011; Doursat, 2005; Cooper, 2006, 2013a, 2013b; Nadin, 2016; Abramsky, 2007a, 2007b; Bentley, 2007; Bhalla, 2007, 2012; Hamann, 2007; Kaiser, 2007; Dodig-Crnkovic, 2008, 2012b; Kari, 2008; Oxman, 2008; Piccinini, 2008; Teuscher, 2008; Ben-Jacob, 2009; Crutchfield, 2009; Sekanina, 2009; Welch, 2009; Gelenbe, 2011; Traub, 2011; Yue, 2011; Burgin, 2013; Burms, 2015)

local mathematics: (Bell, 1986, 2001; Lambek, 1989; Gernert, 1997; Kosub, 2005; Zakharov, 2005; Dodig-Crnkovic, 2012a; Burgin, 2013)

A6. ARCHITECTURAL IDENTITY OF A CONFIGURATION

Is there a local probabilistic structure underlying the hierarchically local interaction of neighborhoods constructed by topological inflation in the Boolean grain model of pattern formation ?

In the fifth knowledge state space of the investigation, the inductive process of a posteriori conceptual clustering hierarchically structured the specific data grains of individual citations into a single set of categorical generalizations that were used to inform situated constructions:

(structures of external point-of-view mathematics)

A6.1. structures of external point-of-view mathematics

metric space: (Bing, 1951; Shore 1993; Ha, 2006 Korner, 2015)

semi-metric space: (Wilson, 1931; Maehara, 1983; Matthews, 1994; Arandelovic, 2009)

topological space: (von Neumann, 1935; Strong, 1966; Duke, 1971; Krishnamurthy, 1977; Morales, 1980; Kong, 1989; Stewart, 1989; Egenhofer, 1992, ????; Kraw, 1995; Saha, 1995; Weinshall, 1999; Arns, 2001, 2003; Hatcher, 2001; Liu, 2002; Darmochwał, 2003; Damiand, 2004; Kopperman, 2005; Ballerini, 2007; Singleton, 2007; Deza, 2009; Davenport, 2013)

point-set topology: (Lynn, 1967; Farrag, 1999; Muscat, 2006; Shick, 2007; Marijuan, 2010; Kolli, 2014)

graph topology: (Bhargava, 1968; Marijuan, 2010; Diestel, 2011; Richter, 2011; Kannan, 2012; Kim, 2012)

APPENDIX B

BIPARTITE PARALLEL PROCESSING

The emergence of commonality across genetic, cognitive, social group, and population scales of adaptive structural transformation lead to construction of a feedforward connectionist model, where pattern develops through a sequence of threshold-cascades reciprocating between information structures and their physical expression: a hierarchically self-similar parallel distributed processing (PDP) model with predictive control, homomorphic to bipartite ensembles of 'black box' artificial neural networks.

Theoretically capable of being embedded in the physical materials of habitat construction, this is an adaptive meta-model of autonomous nonlinear inductive pattern formation where the model itself evolves in response to the co-evolving relationships of internal and external variables.

B1. APPENDIX B CONTENTS

B2. BIPARTITE RECURSION

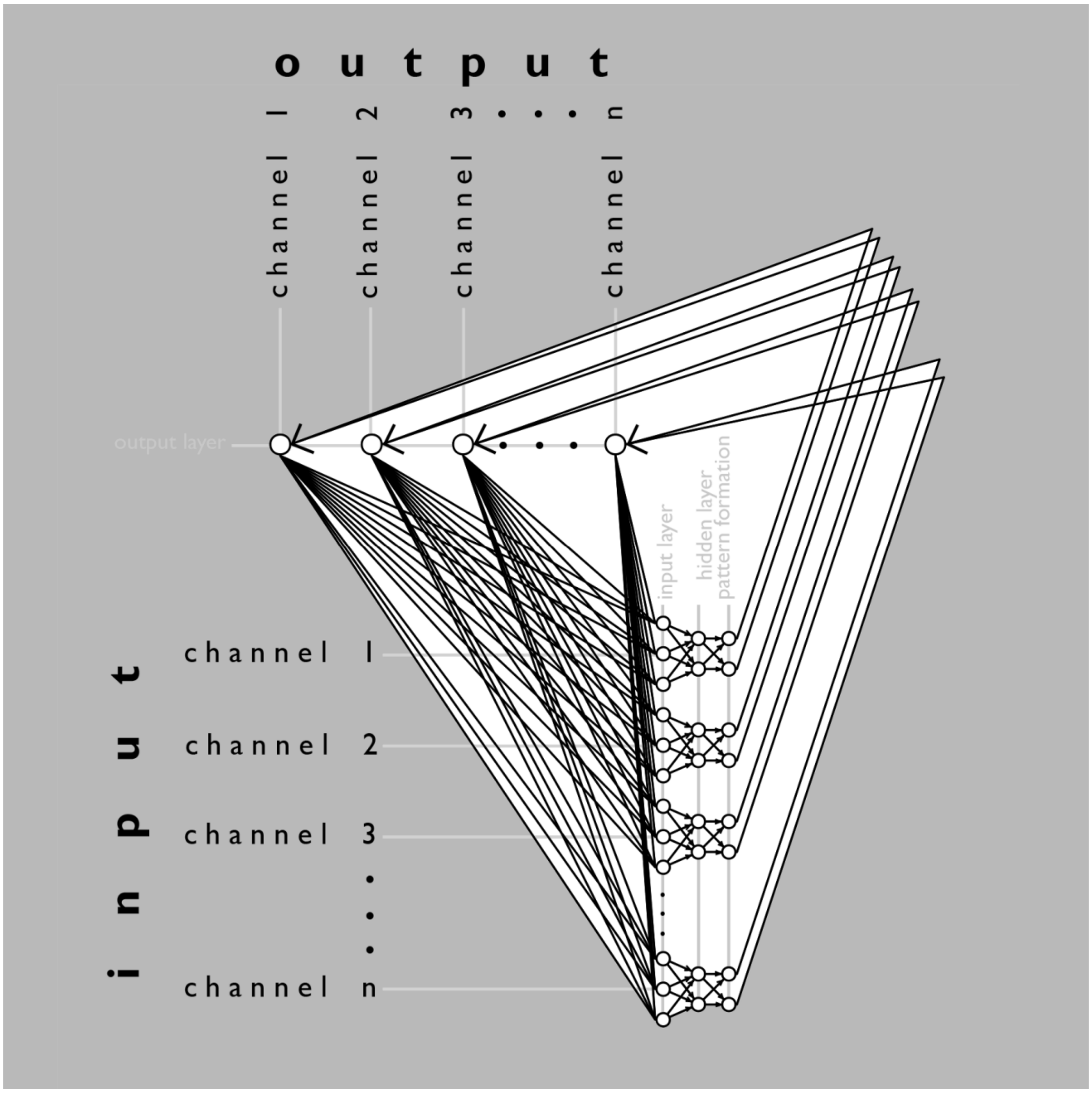

Figure B1. Parallel feedforward inductive pattern formation: internal processing of multi-channel external input with cross-channel feedforward.

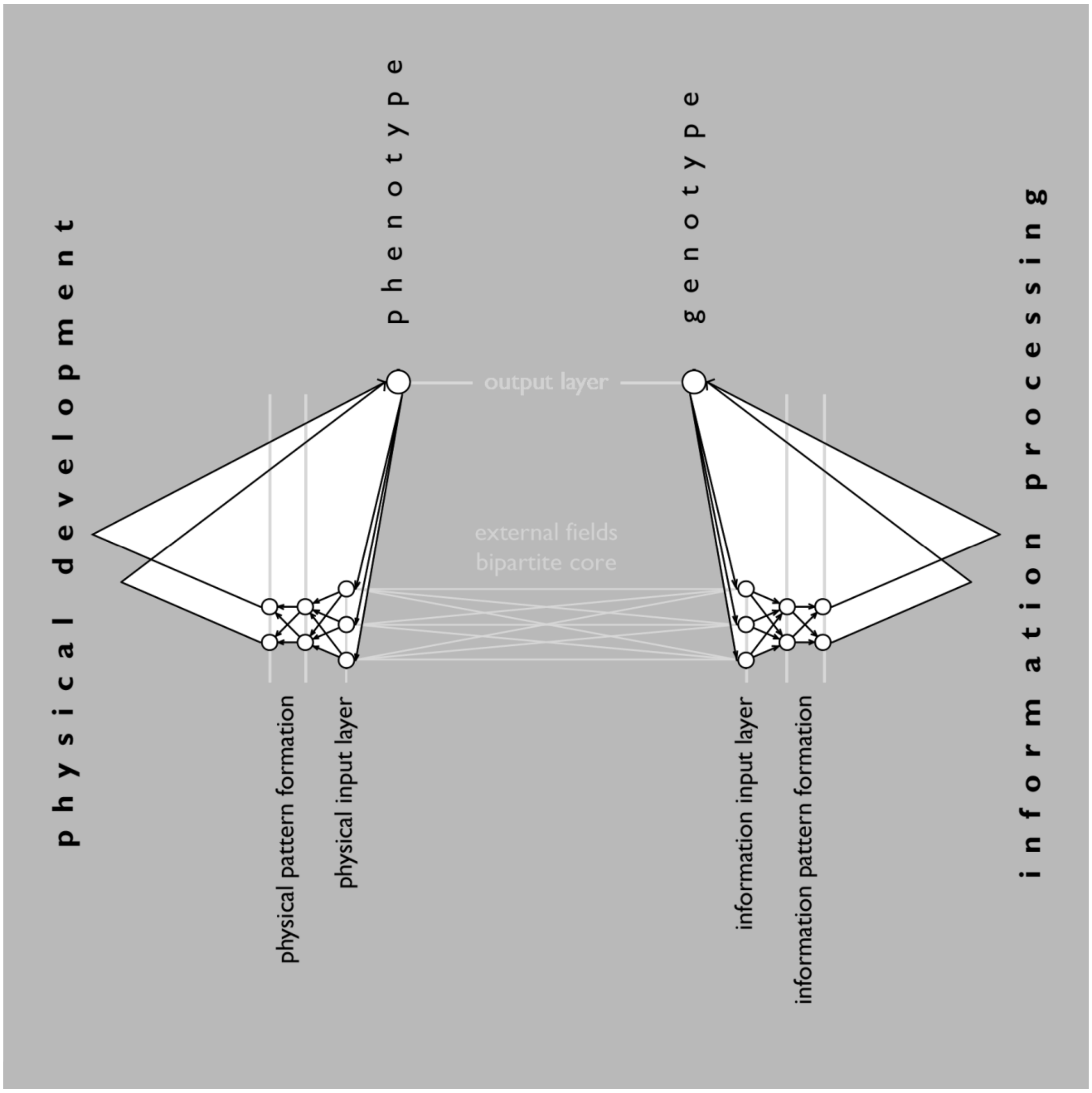

Figure B2. Bipartite feedforward inductive pattern formation.

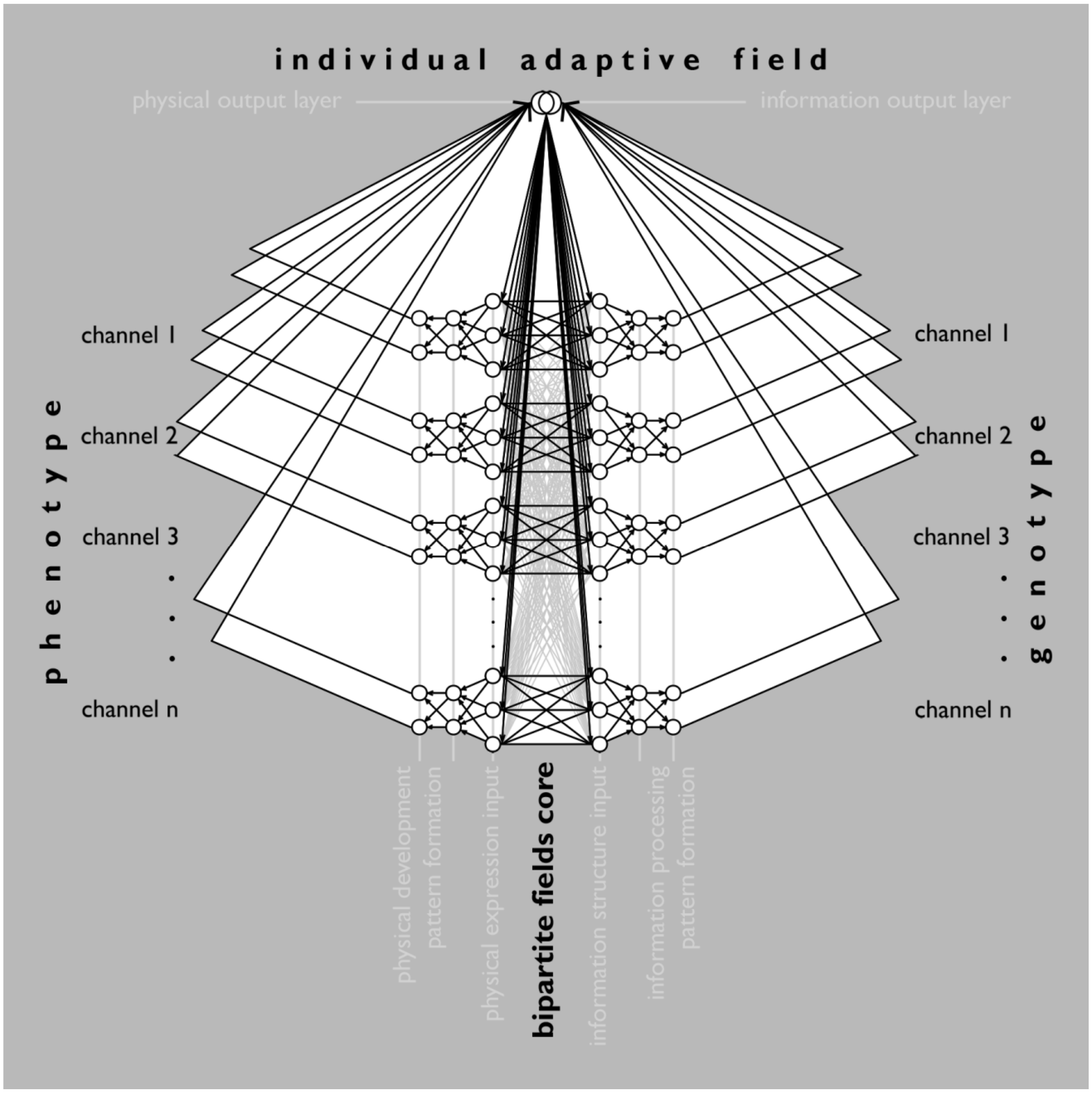

Figure B3. Parallel bipartite feedforward inductive pattern formation:

B3. RECIPROCATING BIPARTITE RECURSION

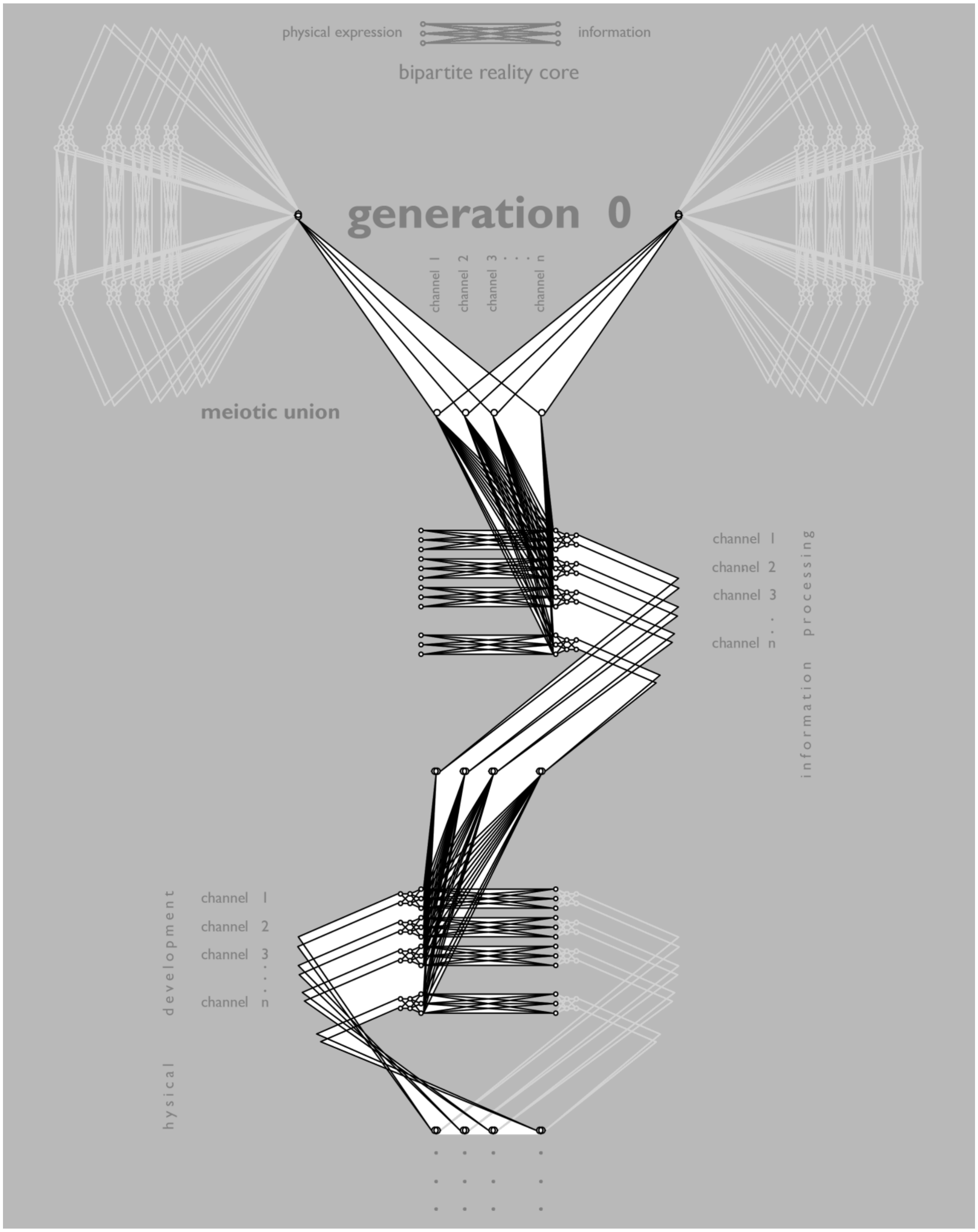

Figure B4a. Reciprocating bipartite recursion: recursive threshold-cascade model of genotype-phenotype development.

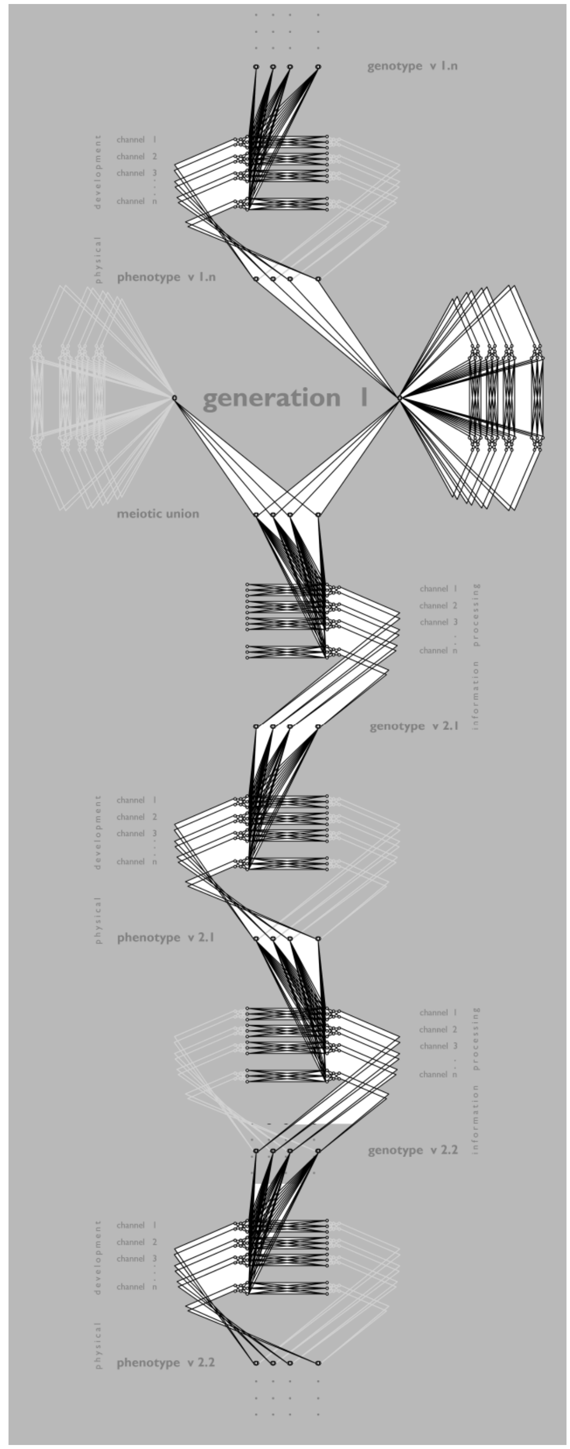

Figure B4b. Reciprocating bipartite recursion (continued).

B4. HIERARCHICAL BIPARTITE RECURSION

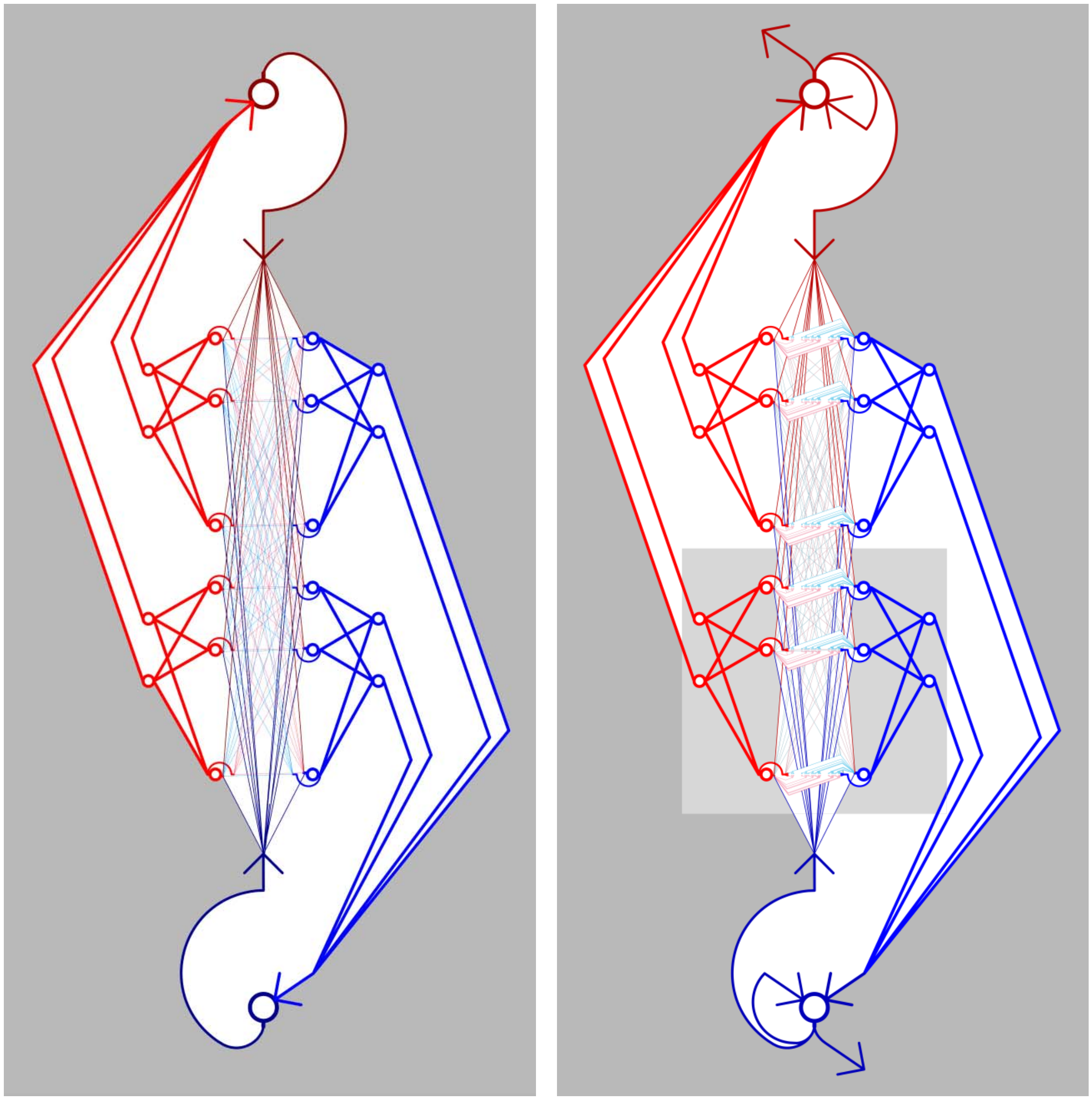

Figure B5a. Hierarchical parallel bipartite feedforward inductive pattern formation.
Left: Non-hierarchical bipartite genotype-phenotype model homomorphic to Figure B2.3.
Right: Self-similar hierarchical model, detail of light grey area follows in Figure B4.2.

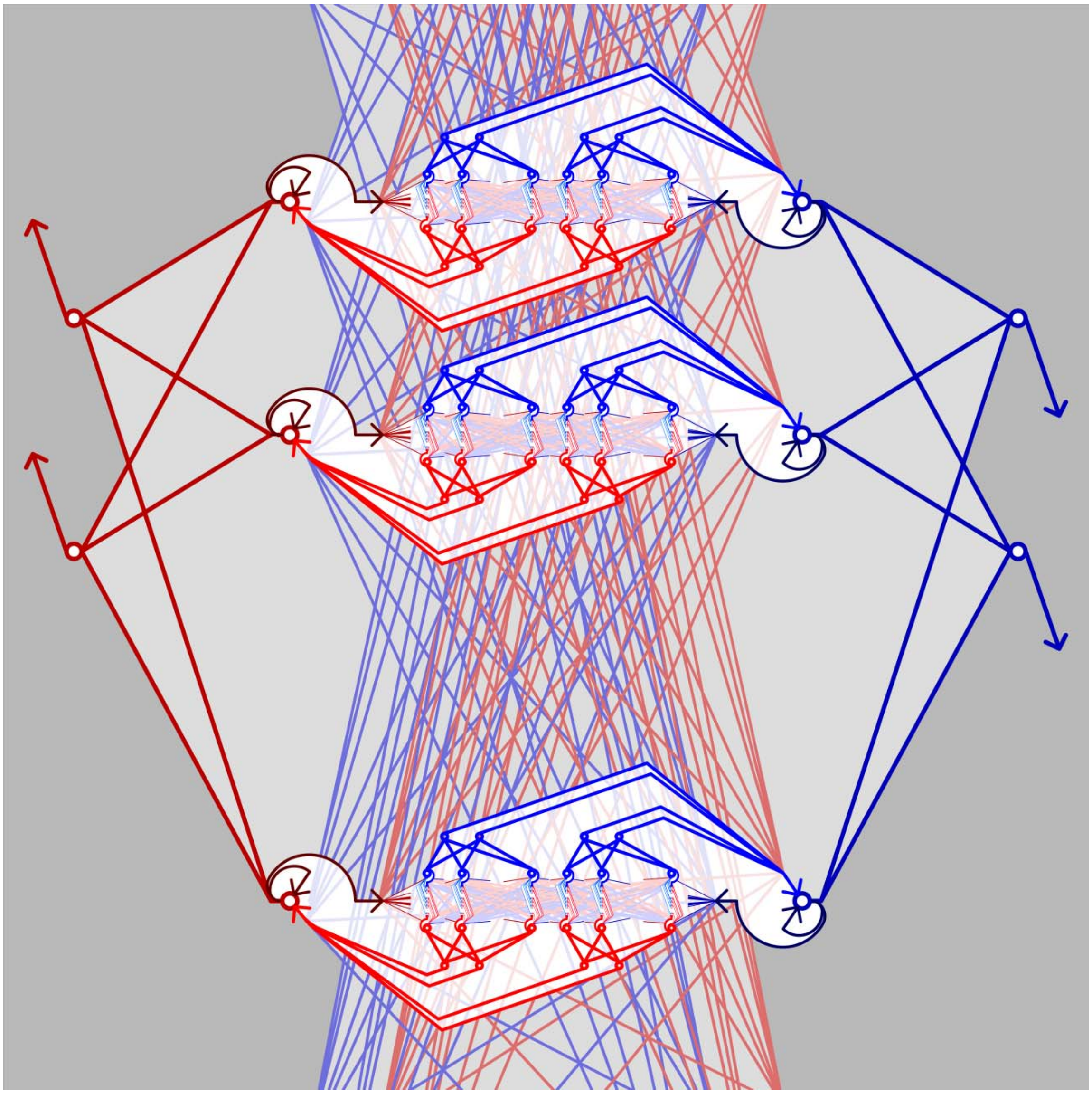

Figure B5b. Detail A. Hierarchical parallel bipartite feedforward inductive pattern formation:

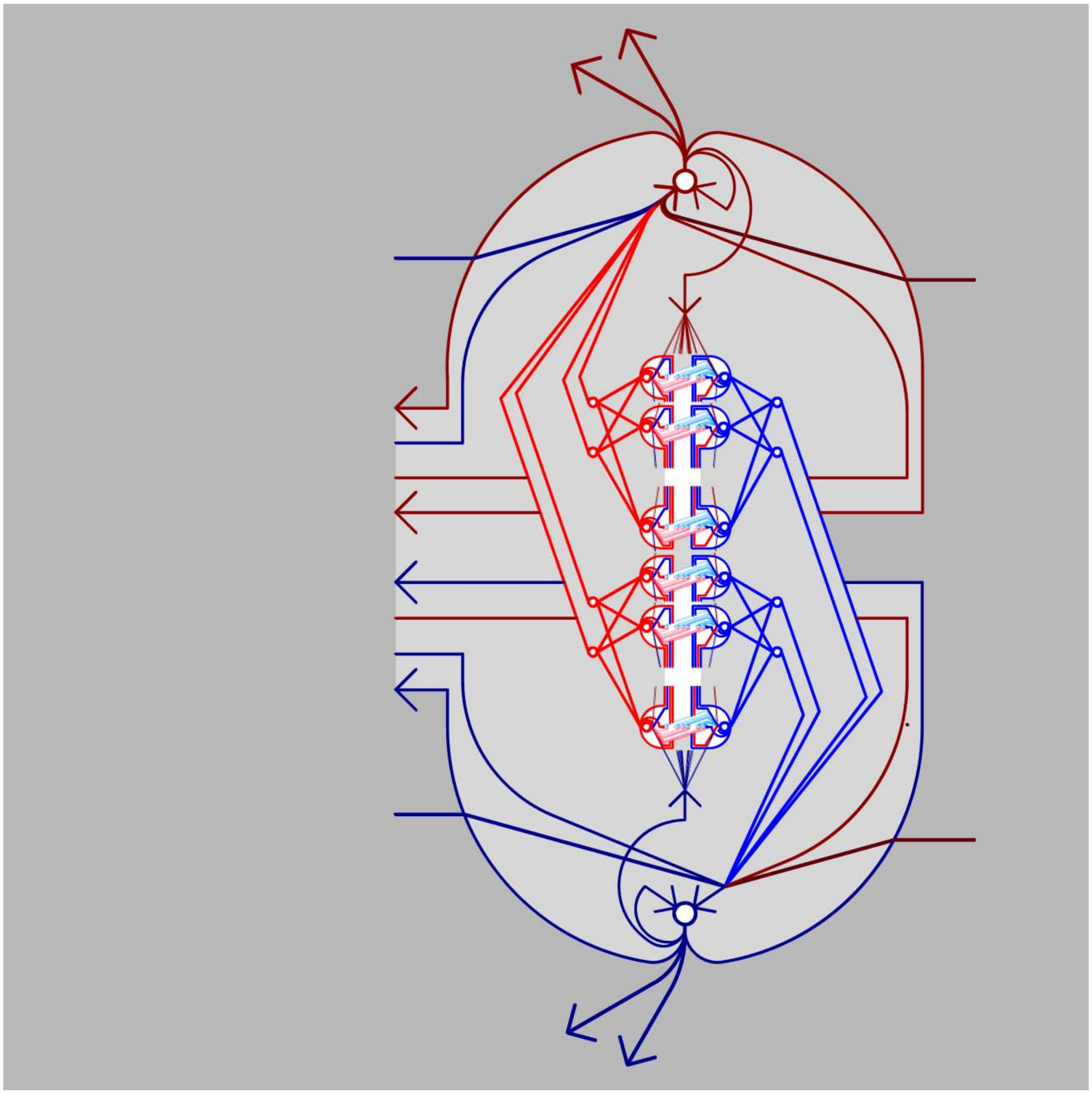

Figure B6. Hierarchical local fundamental group parallel bipartite feedforward inductive pattern formation:

APPENDIX C

SITUATED STRUCTURAL MODELING

Defining boundaries of hierarchical structure has historically presented problems for atomistic models of units of analysis and synthesis. The interpretation of local information is a situated response within a level of a structural hierarchy, presenting the problem of synthesis and analysis units as one of uniquely defining an interacting configuration of elements and its boundary.

The third recursive meta-loop developed the topological operation of Poincare inflation of a point to form its neighborhood, effectively modeling a field surrounding an element. A set of elements undergoing simultaneous Poincare inflation structures a situated Boolean grain model of the set, with continuous inflation constructing hierarchical sequence of topological fundamental groups and their embedded graph structures.

C1. APPENDIX C CONTENTS

Page

C.2. MODELING RANDOM STRUCTURE

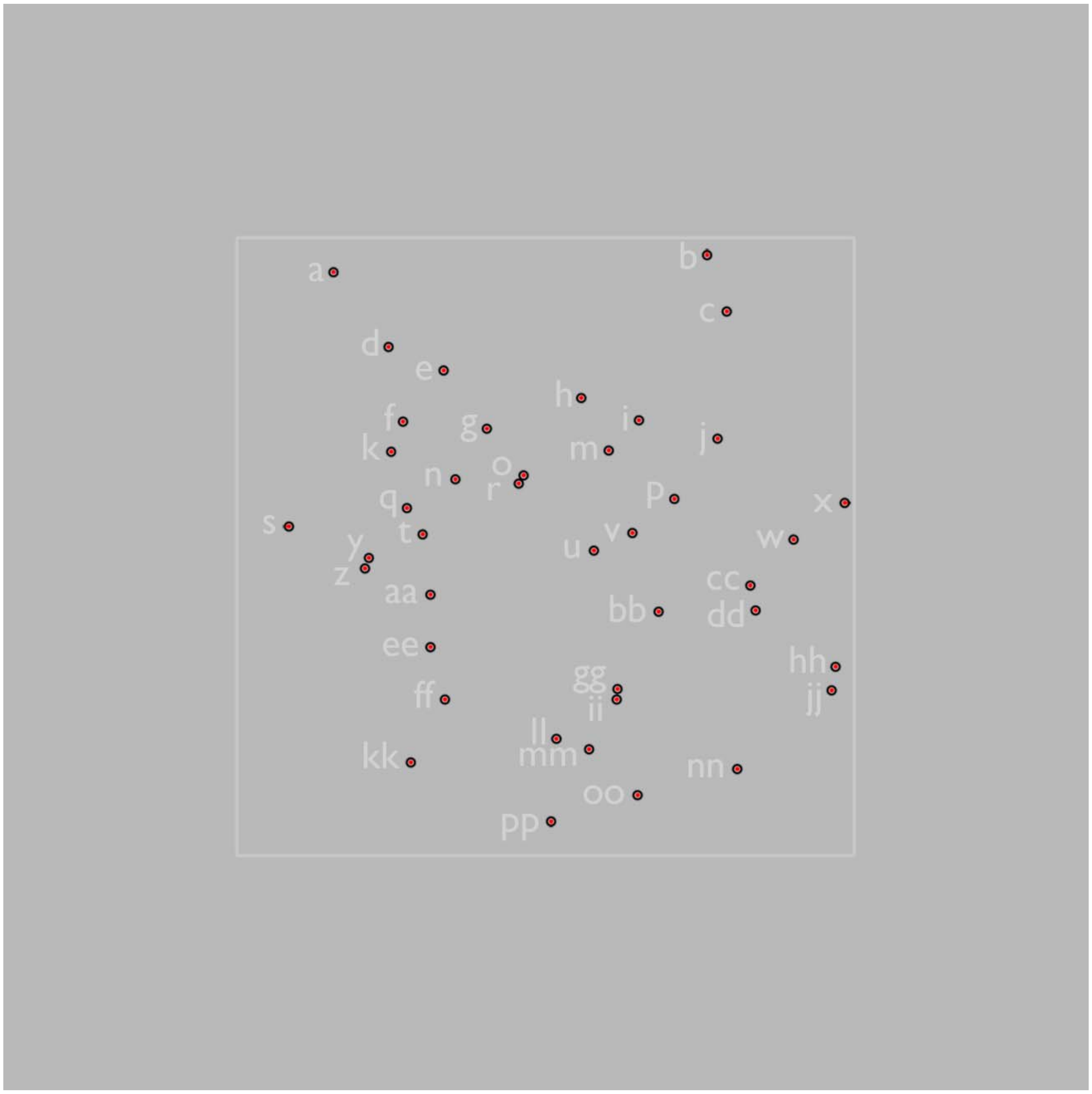

Figure C1. Initial state of a random *situated set*: a static random distribution of 42 points.

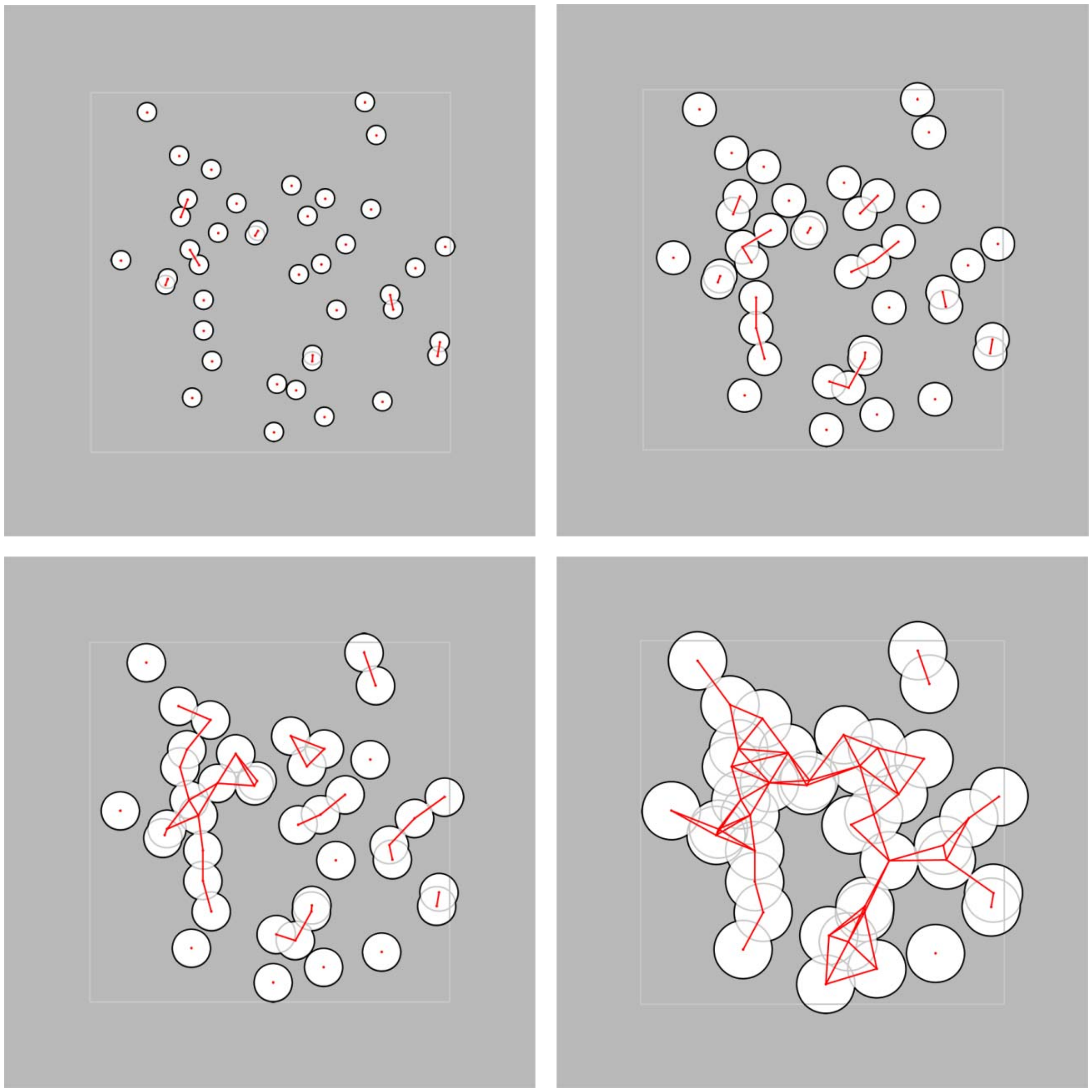

Figure C2.a. Situated development of local space.

Sequence of Boolean topology and graph connectivity states developed by parallel Poincare neighborhood inflation of initial random situated set.

External neighborhood boundary structure (black) defines fundamental group of local finite topological space. Internal neighborhood boundary structure (gray) defines connectivity of the situated Boolean topology of the set. Graph edges (red) define metric distance, semi-metric probabilistic distance, semi-metric topologic distance.

Discrete sequence of local space states: top left, 35 local fundamental groups [1]; top right, 27 local fundamental groups [1]; bottom left, 15 local fundamental groups [1]; bottom right, 3 local fundamental groups [1].

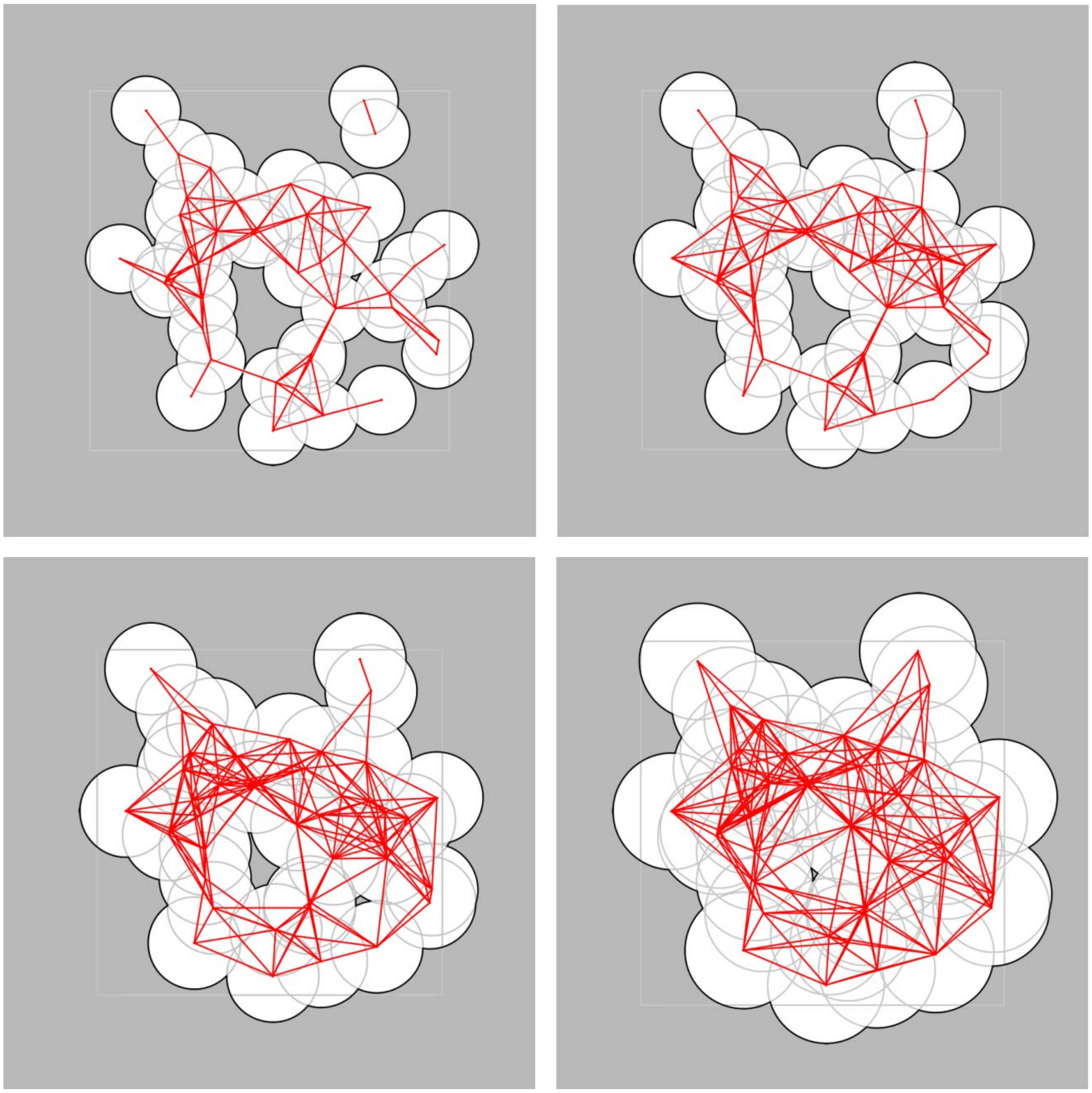

Figure C2.b. Situated development of local space. (continued)

Sequence of discrete states: fundamental group boundary (black); Boolean topology (light gray neighborhoods); graph connectivity (red).

Top left: 1 local fundamental group [1], 1 local fundamental group [3]. Top right: 1 local fundamental group [3]. Bottom left: 1 local fundamental group [3]. Bottom right: 1 local fundamental group [2].

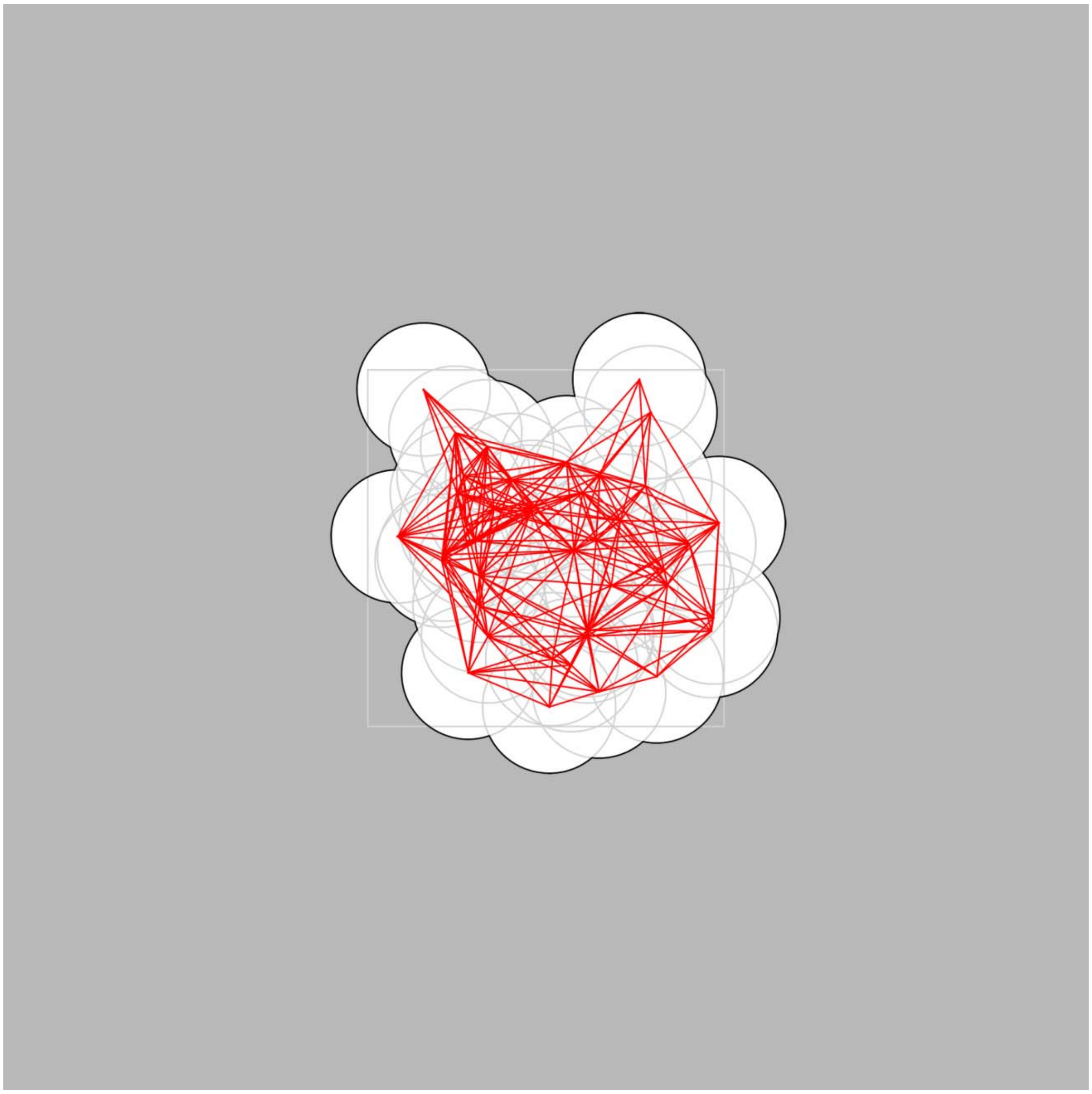

Figure C2.c. Situated development of local space. (continued)

Completely connected finite topological space, local fundamental group [1] boundary (black); Boolean topology (light gray neighborhoods); graph connectivity (red).

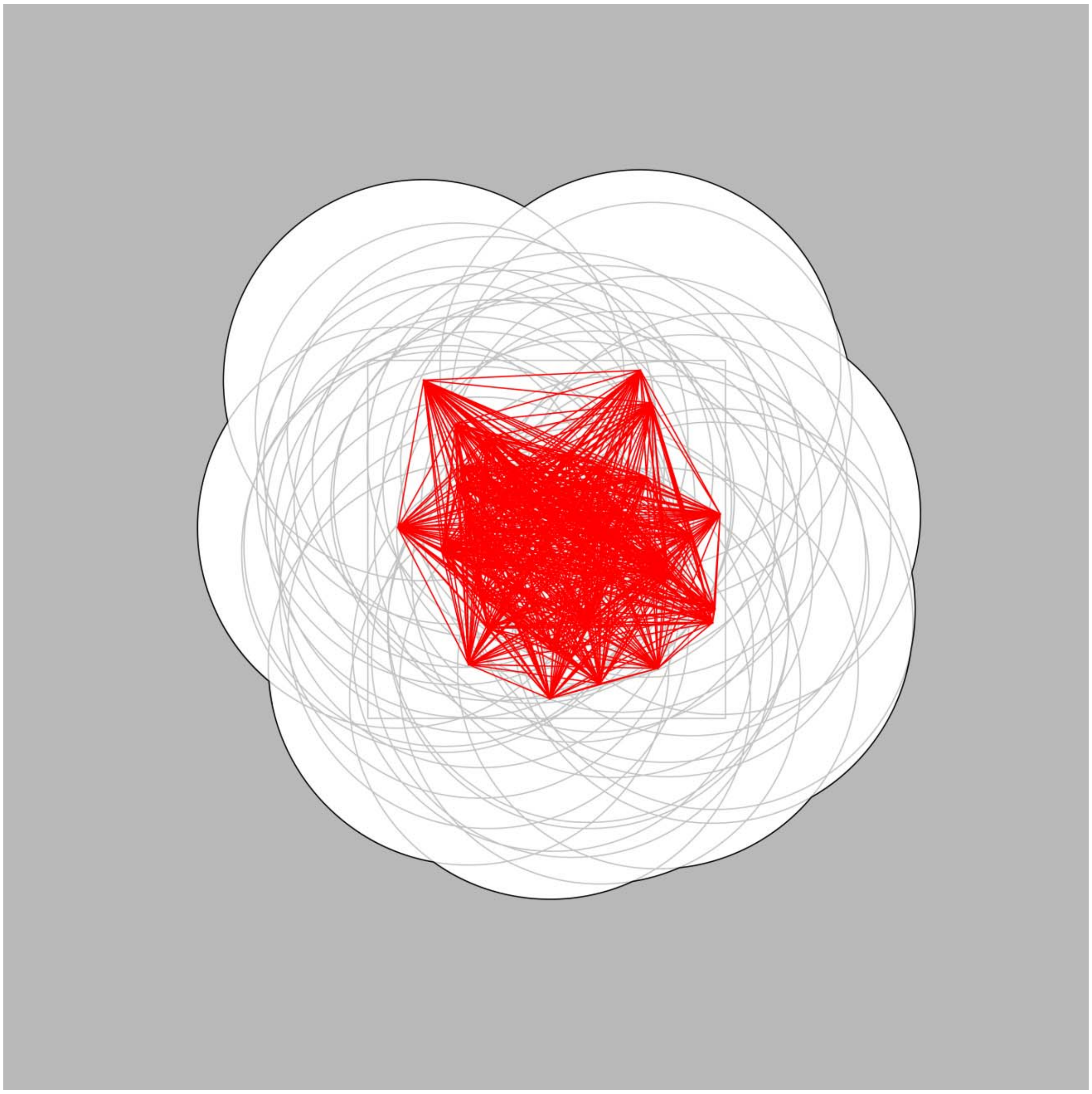

Figure C2.d. Situated development of local space. (continued)

Terminal connectivity and morphological development: local fundamental group [1] boundary (black); complete situated Boolean topology (light gray neighborhoods); complete graph (red).

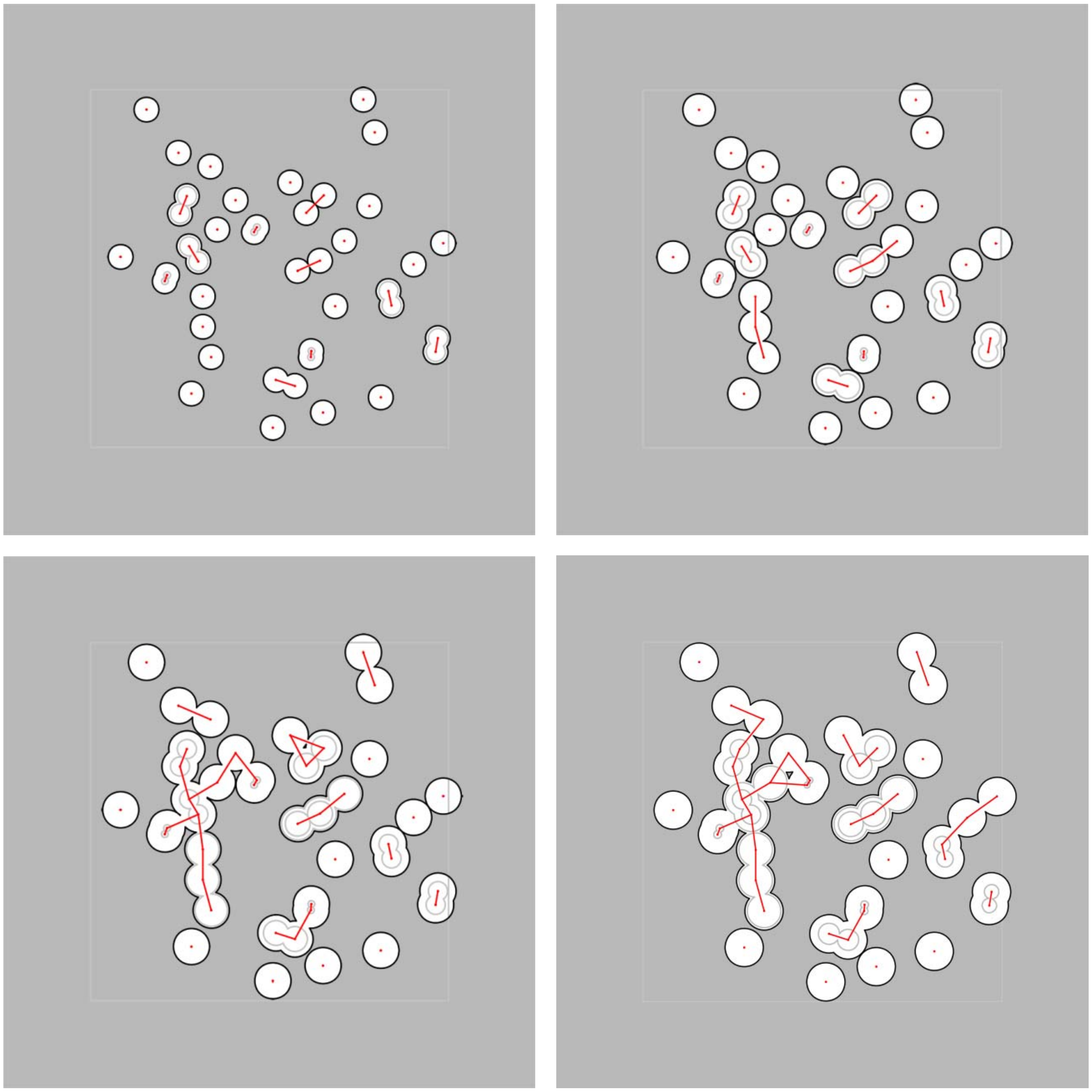

Figure C3.a. Discrete-event hierarchical development of local space.

An event is defined as a change in neighborhood connectivity that results in a boundary change of the local space.

Fundamental group (black) defines boundary of local topological space. Discrete-event chronology (light gray neighborhood) records the hierarchical discrete state sequence of neighborhood intersections. *Minimum hierarchical spanning ring* (red) defines connectivity sequence of hierarchical subset structure in coherence with topological fundamental group of the local space.

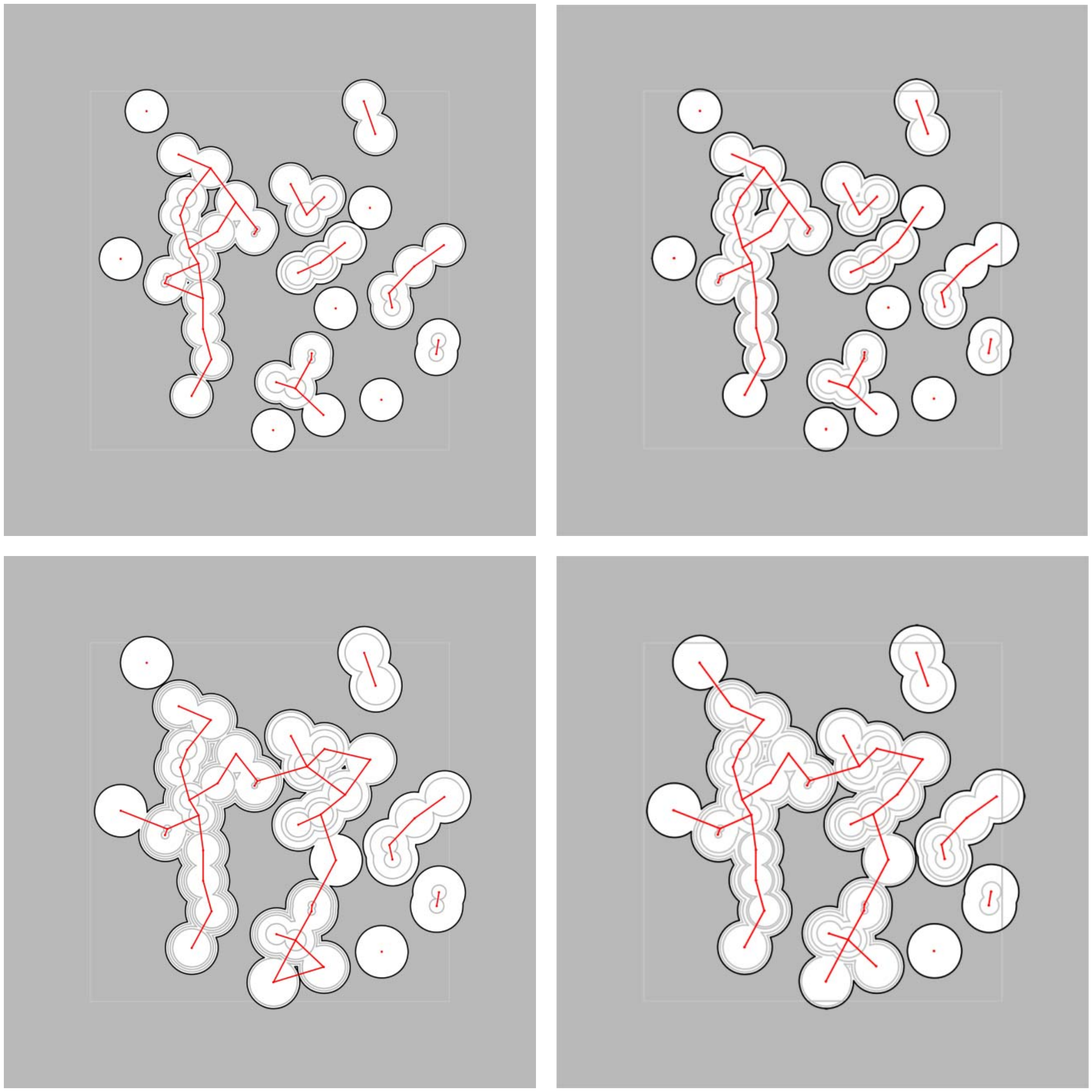

Figure C3.b. Discrete-event hierarchical development of local space. (continued)

Fundamental group boundary (black); discrete-state chronology (light gray neighborhood); minimum hierarchical spanning ring (red).

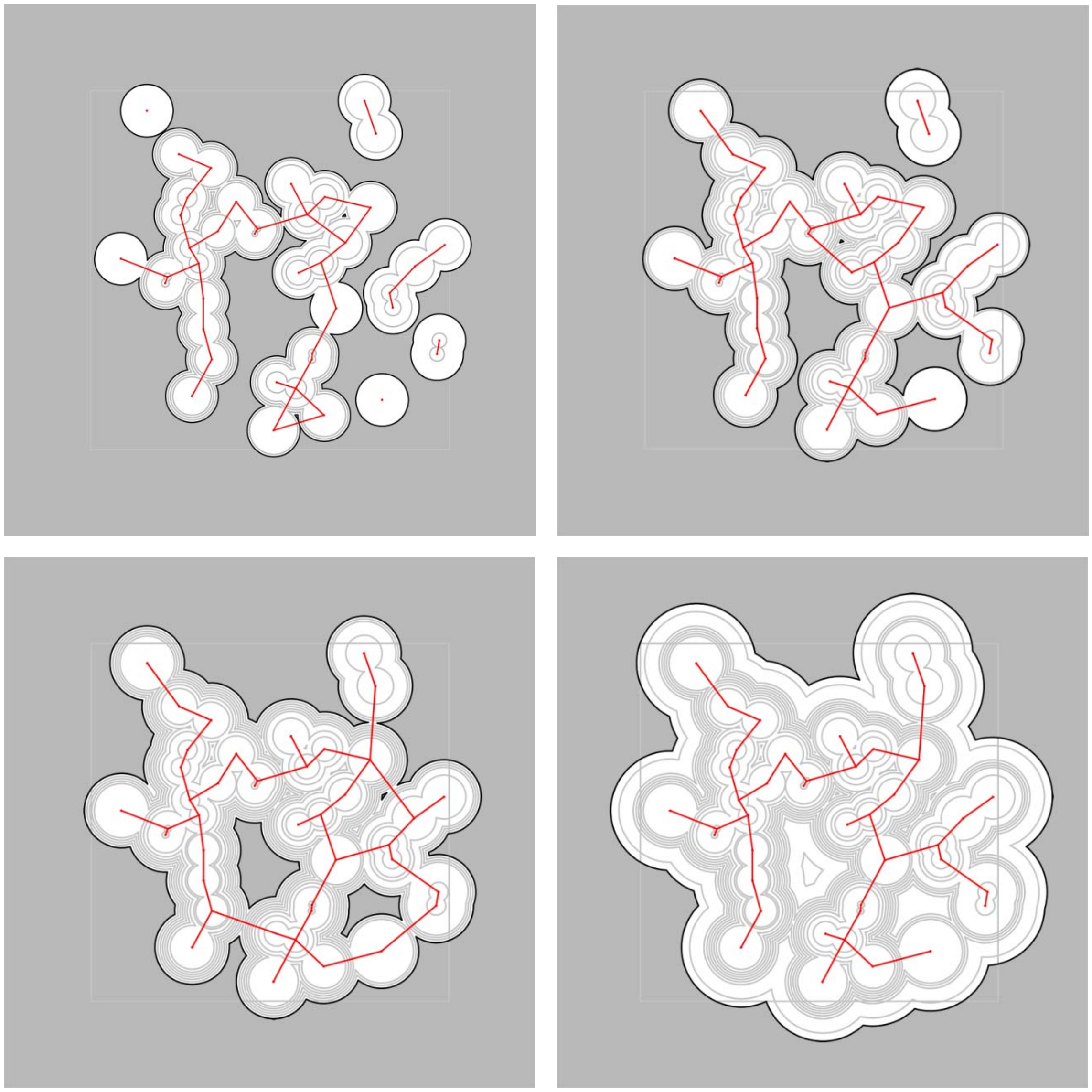

Figure C3.c. Discrete-event hierarchical development of local space. (continued)

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhood); minimum hierarchical spanning ring (red). Lower right: terminal minimum hierarchical spanning ring, fundamental group [1]. With uniform neighborhood inflation, terminal minimum hierarchical spanning ring is equivalent to minimum spanning tree.

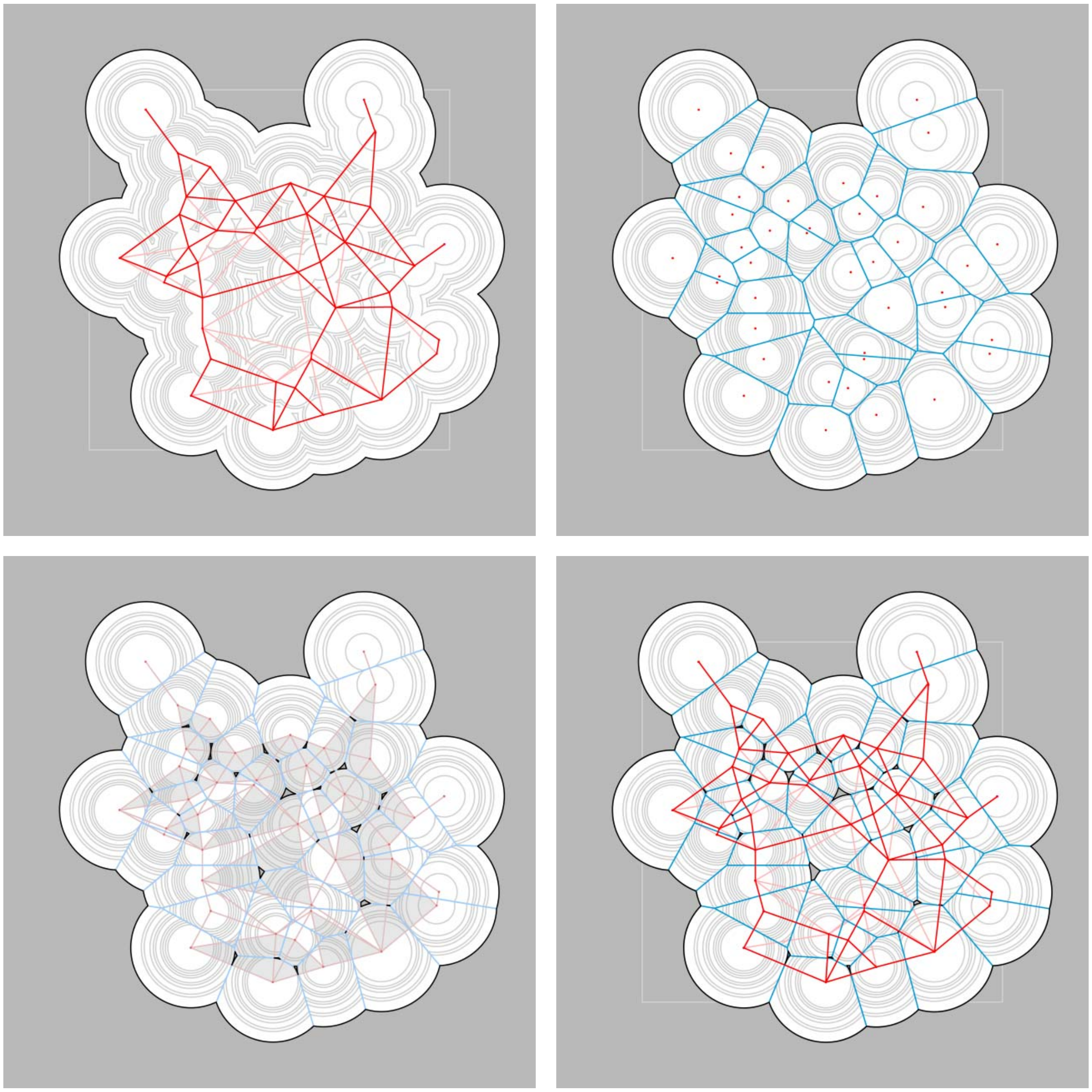

Figure C4. Interstitial void structure in spatial development.

Top left: sum of discrete-state minimum hierarchical spanning rings (red) mapped on Delaunay triangulation (light red). Top right: Voronoi spatial partitioning (blue). Bottom left: sum of discrete-state voids situated in Delaunay triangulation. Bottom right: sum of discrete-state voids situated in sum of discrete-state minimum hierarchical spanning rings.

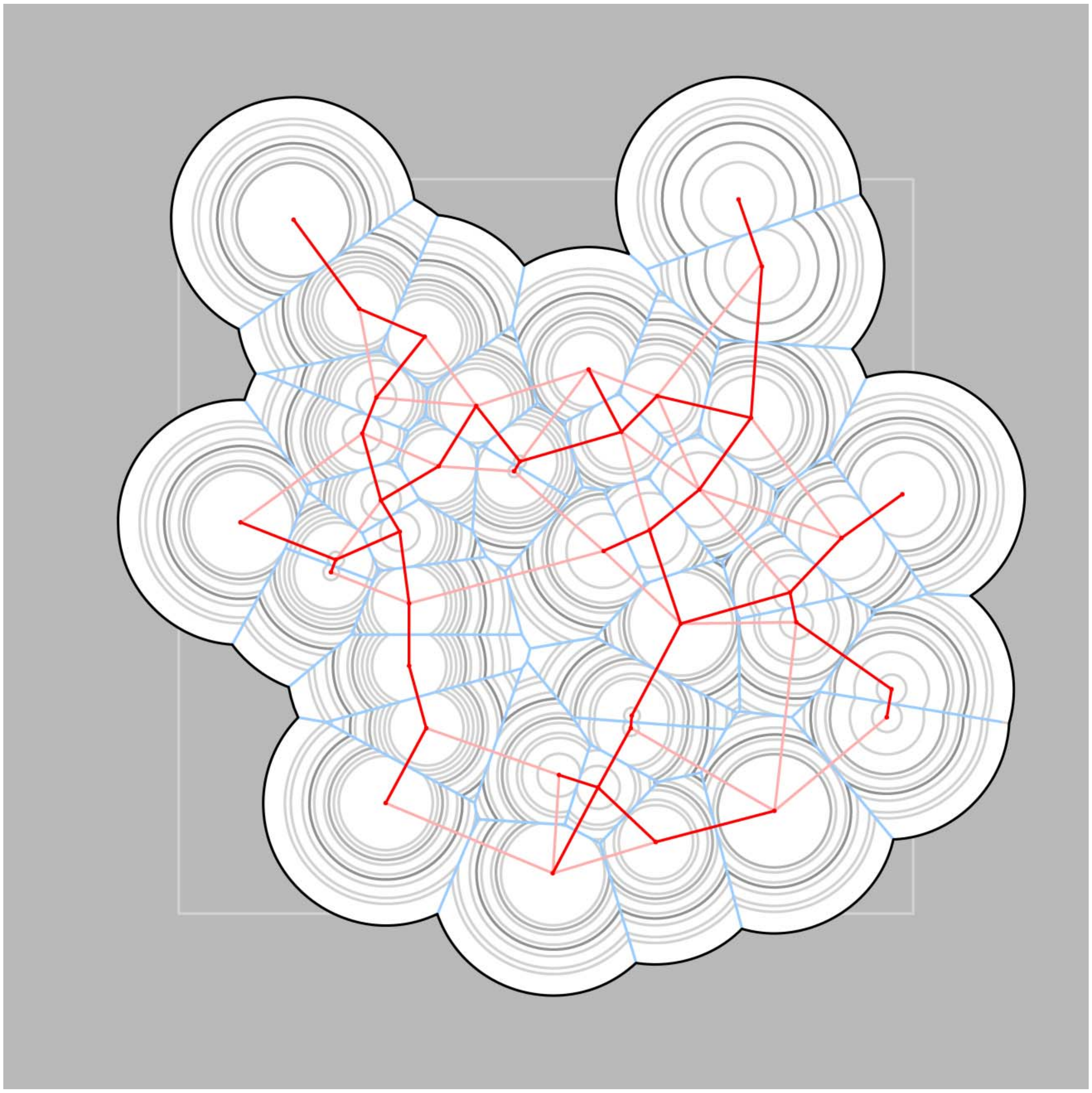

Figure C5. Structures of connected finite topological space.

Topological fundamental group [1] boundary (black). Hierarchical discrete-state chronology (light gray neighborhoods). Giant component boundary at percolation threshold (medium gray neighborhood). Completely connected single finite topological space boundary (dark gray neighborhood). Voronoi spatial partitioning (blue). Terminal minimum hierarchical spanning tree (red) mapped on to summation of discrete-state minimum hierarchical spanning rings (light red).

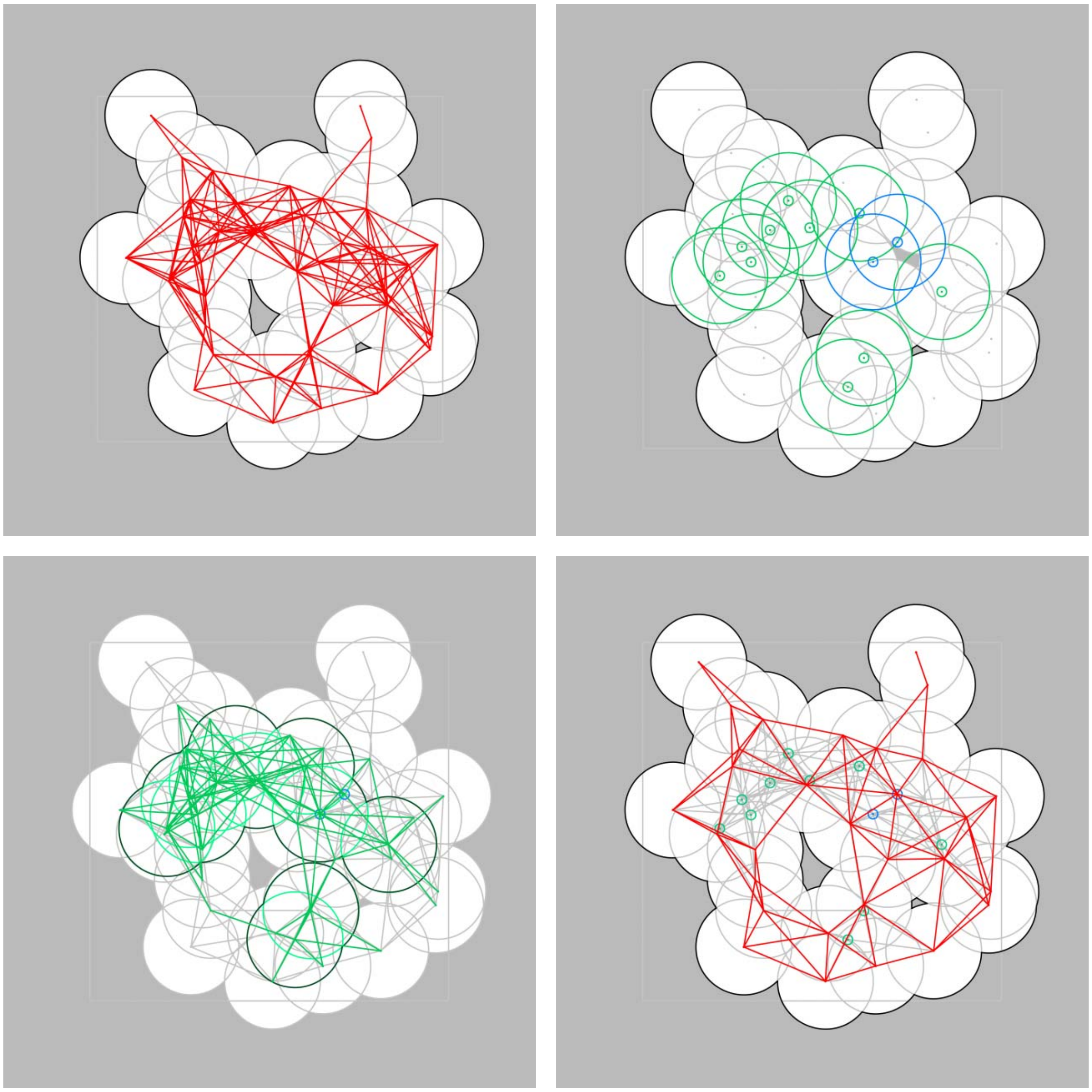

Figure C6. Definition of essential Minkowski set and choice set.

The *essential Minkowski set* is the minimum subset of neighborhoods necessary to generate Minkowski functionals (morphological descriptors)identical to those of the full set of a configuration.

Top left: situated Boolean topology and graph of configuration. Top right: *non-essential Minkowski set* (green), choice set (blue), selection of at least one member is required to complete the essential Minkowski set. Bottom left: non-essential Boolean topology (light green), non-essential graph (green). Bottom right: essential and essential-choice set Boolean topology (gray neighborhoods, essential set graph (red). Non-essential set (green), non-essential choice set (blue), non-essential graph (gray).

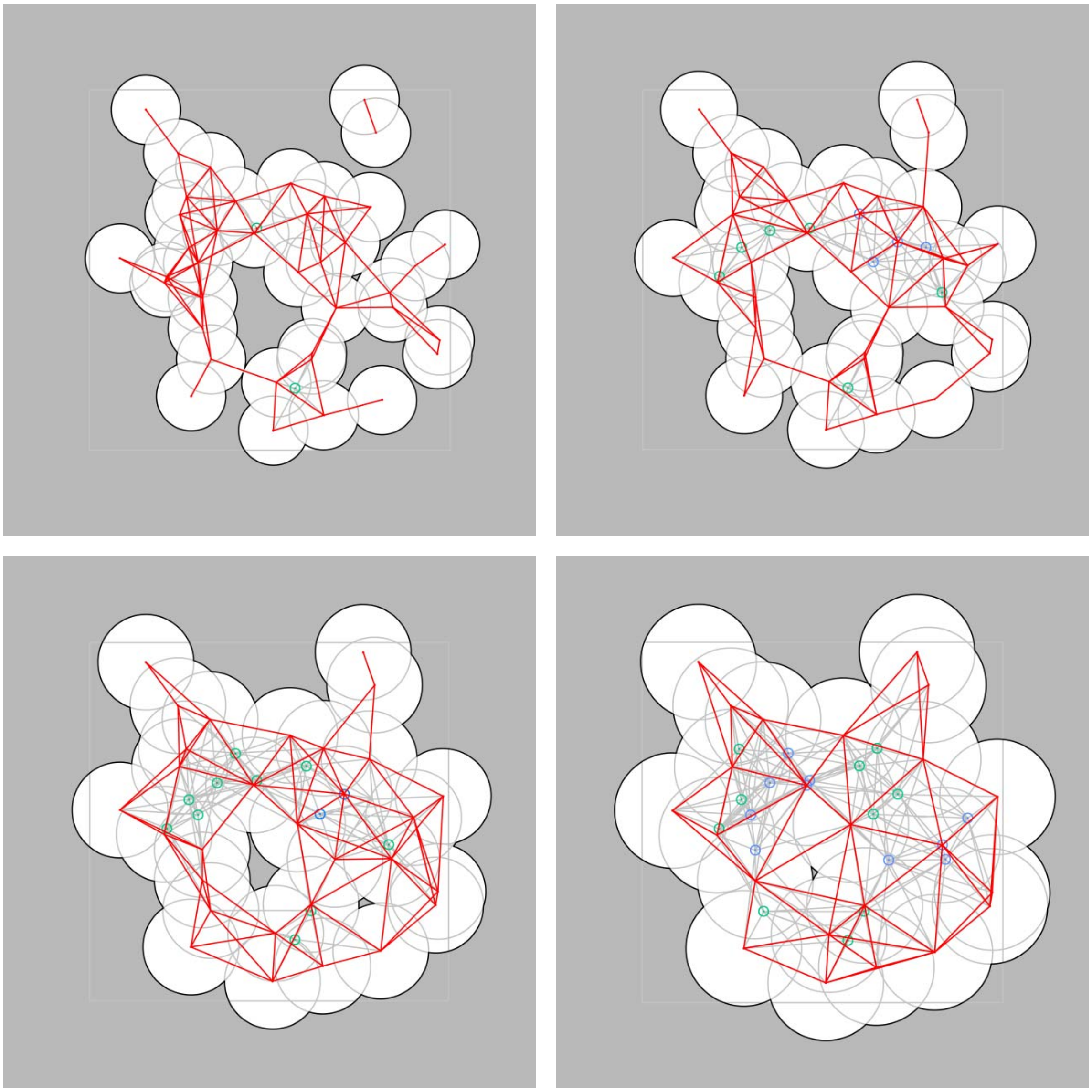

Figure C7.a. Development of local space from essential and choice sets.

Boolean topology (gray neighborhoods) and complete graph (red) of essential Minkowski and essential-choice set. Non-essential set (green), non-essential choice set (blue), non-essential graph (gray).

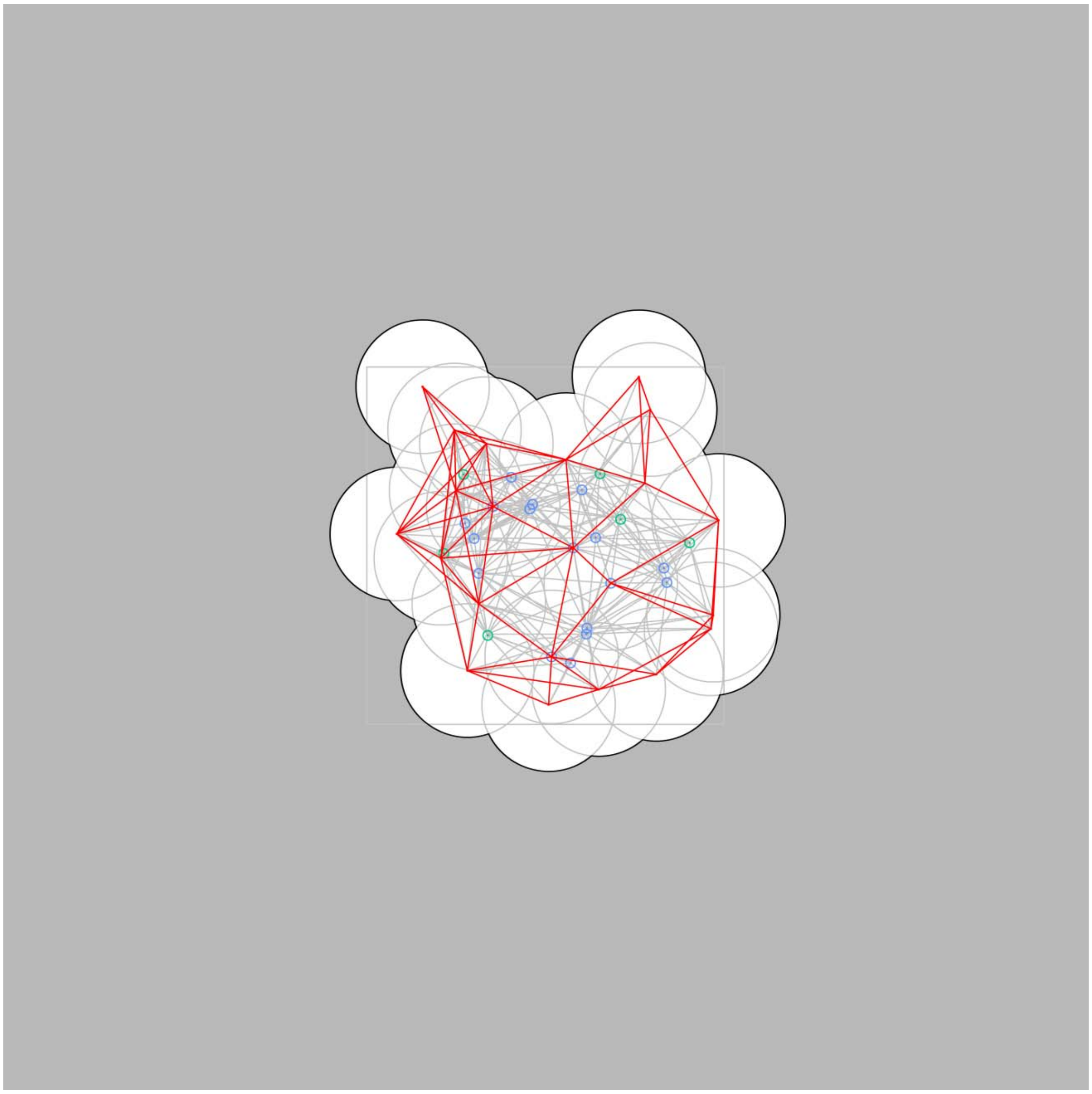

Figure C7.b. Development of local space from essential and choice sets. (continued)

Boolean topology (gray neighborhoods) and complete graph (red) of essential Minkowski and essential-choice set. Non-essential set (green), non-essential choice set (blue), non-essential graph (gray).

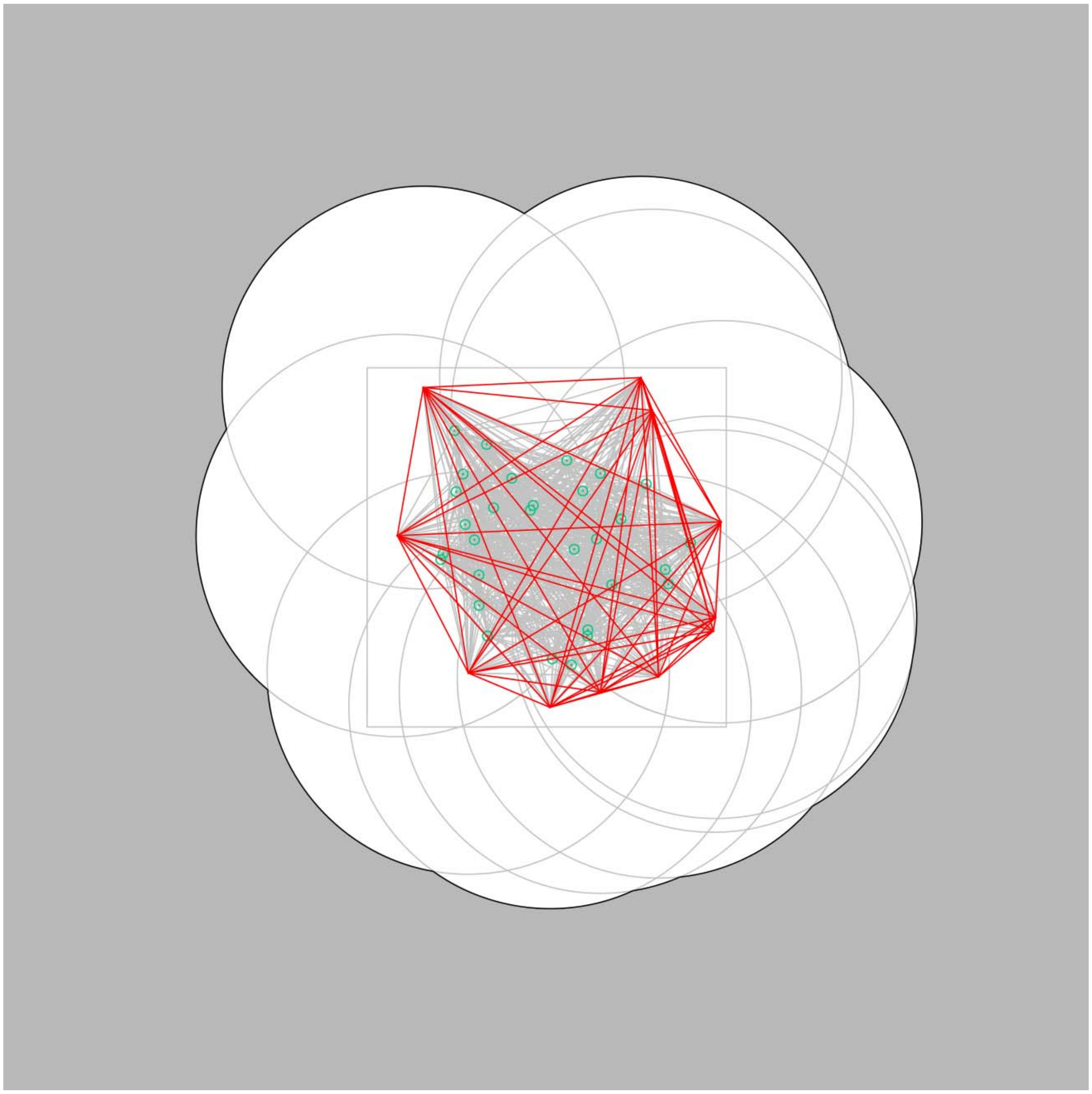

Figure C7.c. Development of local space from essential and choice sets. (continued)

Terminal morphological development and connectivity. Complete Boolean topology (gray neighborhoods) and complete graph (red) of essential Minkowski set. Non-essential set (green), non-essential graph (gray).

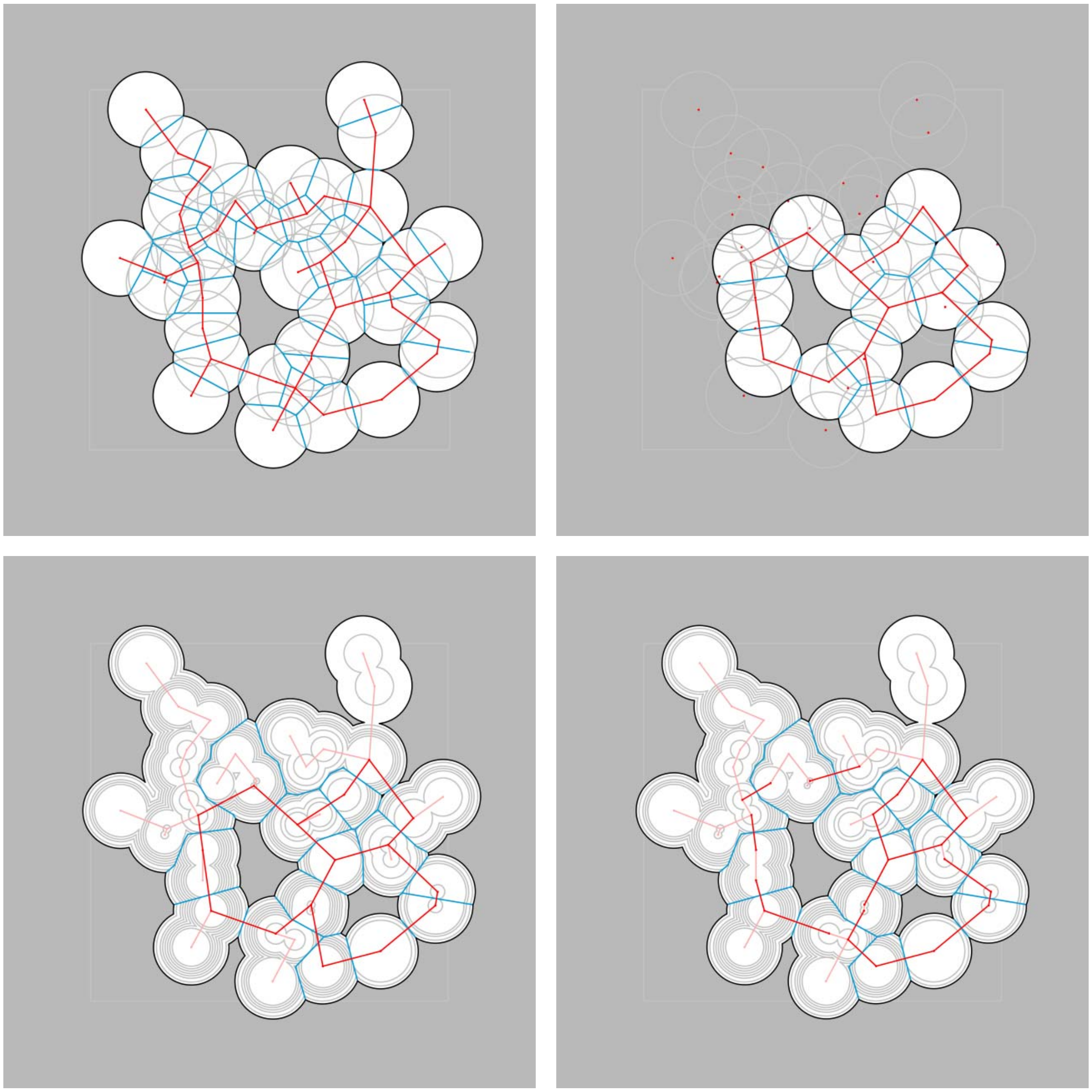

Figure C8. Definition of primitive ring.

A *primitive set* is the minimum subset of intersecting neighborhoods necessary to generate the topological fundamental group of a configuration. A *primitive ring* is the minimum spanning ring of the primitive set.

Top left: minimum spanning ring connectivity (red) and spatial partitioning (blue) of situated Boolean topology (gray neighborhoods). Top right: Primitive ring (red) with spatial partitioning (blue). Bottom left: Primitive ring (red) with spatial partitioning by local minimum spanning tree (light red). Bottom right: primitive ring (red) mapped on to minimum hierarchical spanning ring (light red) with spatial partitioning (blue).

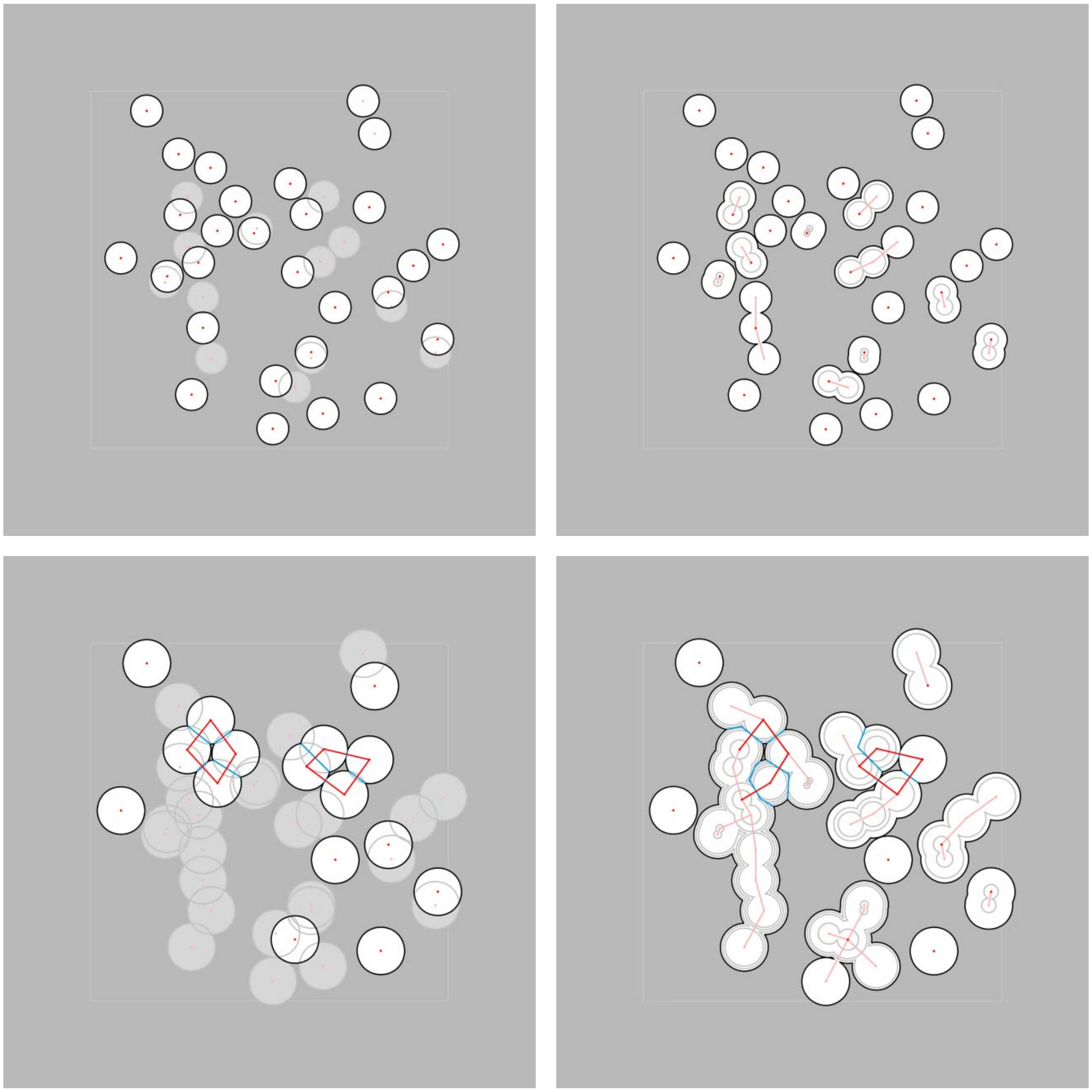

Figure C9.a. Development of spatial partitioning by primitive ring.

Left column: discrete sequence of primitive ring subset spaces equivalent to topological fundamental group of local space. Local topological space (light gray); equivalent *topological primitive* boundary (black); primitive ring (red); spatial partitioning (blue).

Right column: discrete sequence equivalent to left column; primitive ring (red) mapped on to minimum hierarchical spanning ring (light red), spatial partitioning (blue).

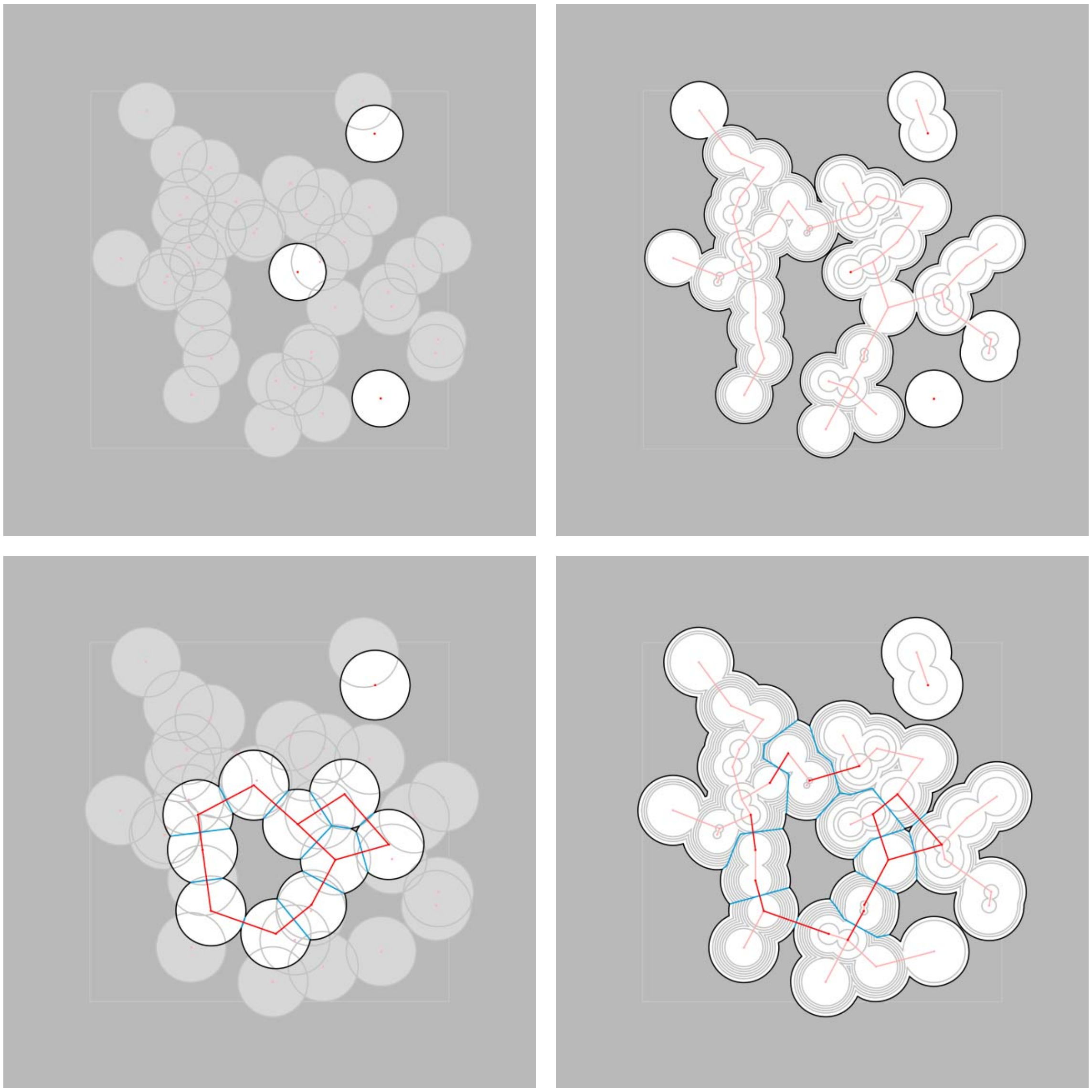

Figure C9.b. Development of spatial partitioning by primitive ring. (continued)

Left column: discrete sequence of primitive ring subset spaces equivalent to topological fundamental group of local space. Local topological space (light gray); equivalent *topological primitive* boundary (black); primitive ring (red); spatial partitioning (blue).

Right column: discrete sequence equivalent to left column; primitive ring (red) mapped on to minimum hierarchical spanning ring (light red), spatial partitioning (blue).

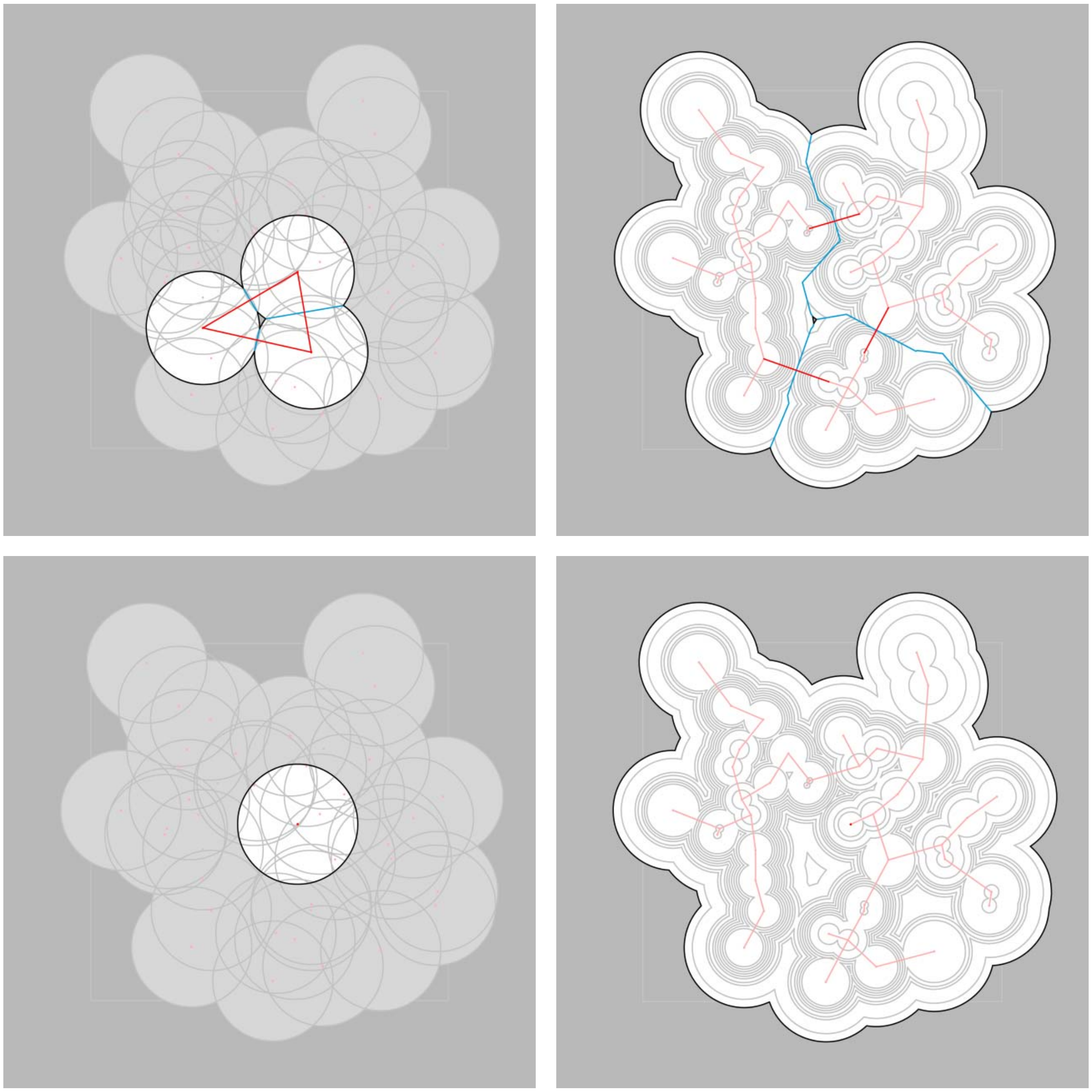

Figure C9.c. Development of spatial partitioning by primitive ring. (continued)

Left column: discrete sequence of primitive ring subset spaces equivalent to topological fundamental group of local space. Local topological space (light gray); equivalent *topological primitive* boundary (black); primitive ring (red); spatial partitioning (blue).

Right column: discrete sequence equivalent to left column; primitive ring (red) mapped on to minimum hierarchical spanning ring (light red), spatial partitioning (blue).

Bottom left, right: connected single finite topological space.

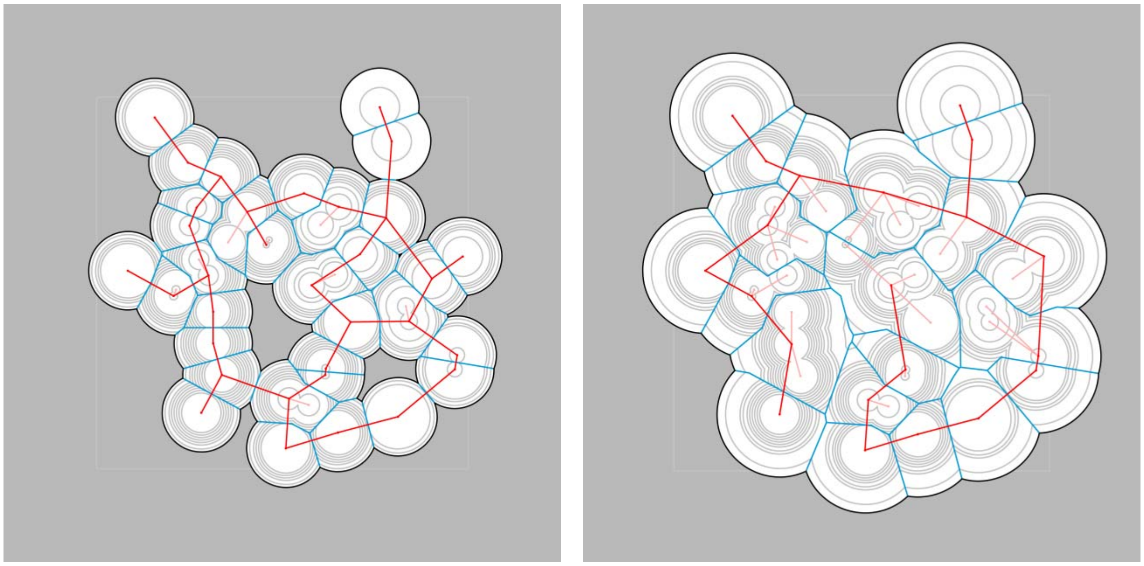

Figure C10.a. Spatial partitioning by primitive ring and nearest neighbor.

Nearest neighbor (light red) connectivity to primitive ring (red). Resulting spatial partitions (blue) lack coherence with hierarchical discrete-event chronology (light grey neighborhoods) in development of local topological space.

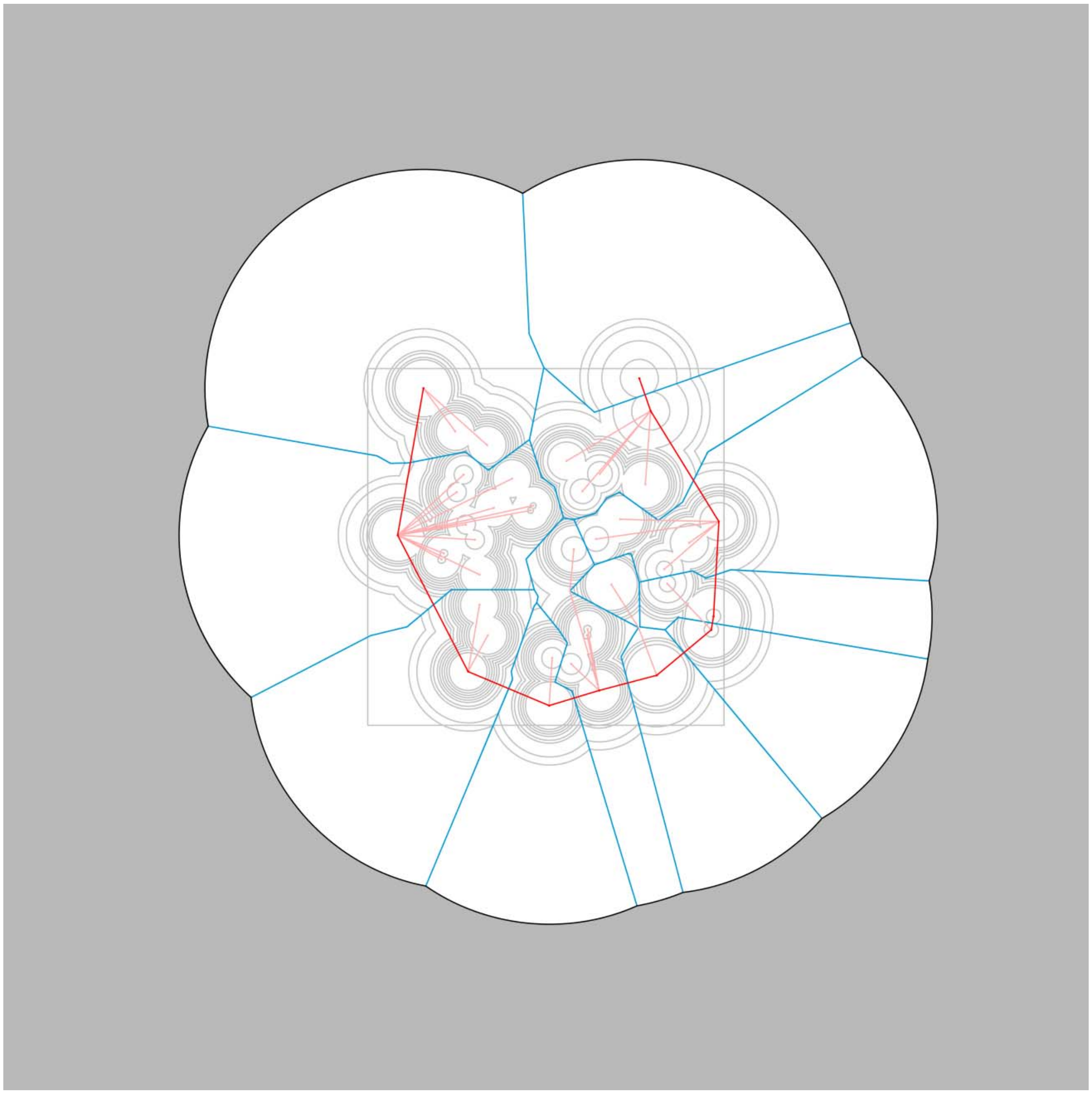

Figure C10.b. Spatial partitioning by primitive ring and nearest neighbor. (continued)

Nearest neighbor graph (light red) connectivity to primitive ring (red). Resulting spatial partitions (blue) lack coherence with hierarchical discrete-event chronology (light grey neighborhoods) in development of local topological space.

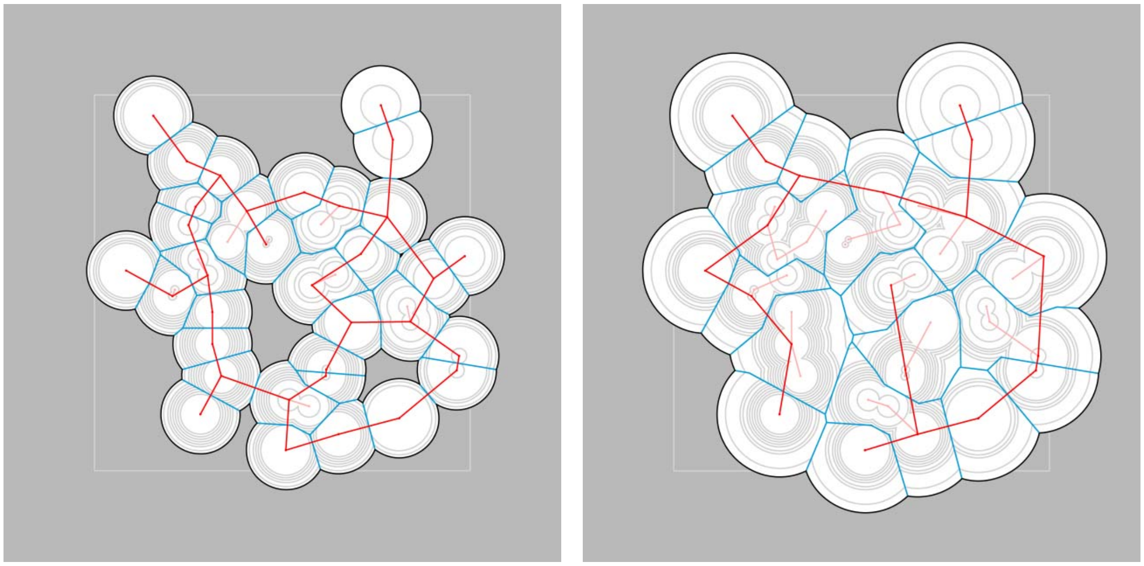

Figure C11.a. Spatial partitioning by primitive ring and minimum local spanning tree.

Minimum local spanning tree (light red) connectivity to primitive ring (red). Resulting spatial partitions (blue) lack coherence with hierarchical discrete-event chronology (light grey neighborhoods) in development of local topological space.

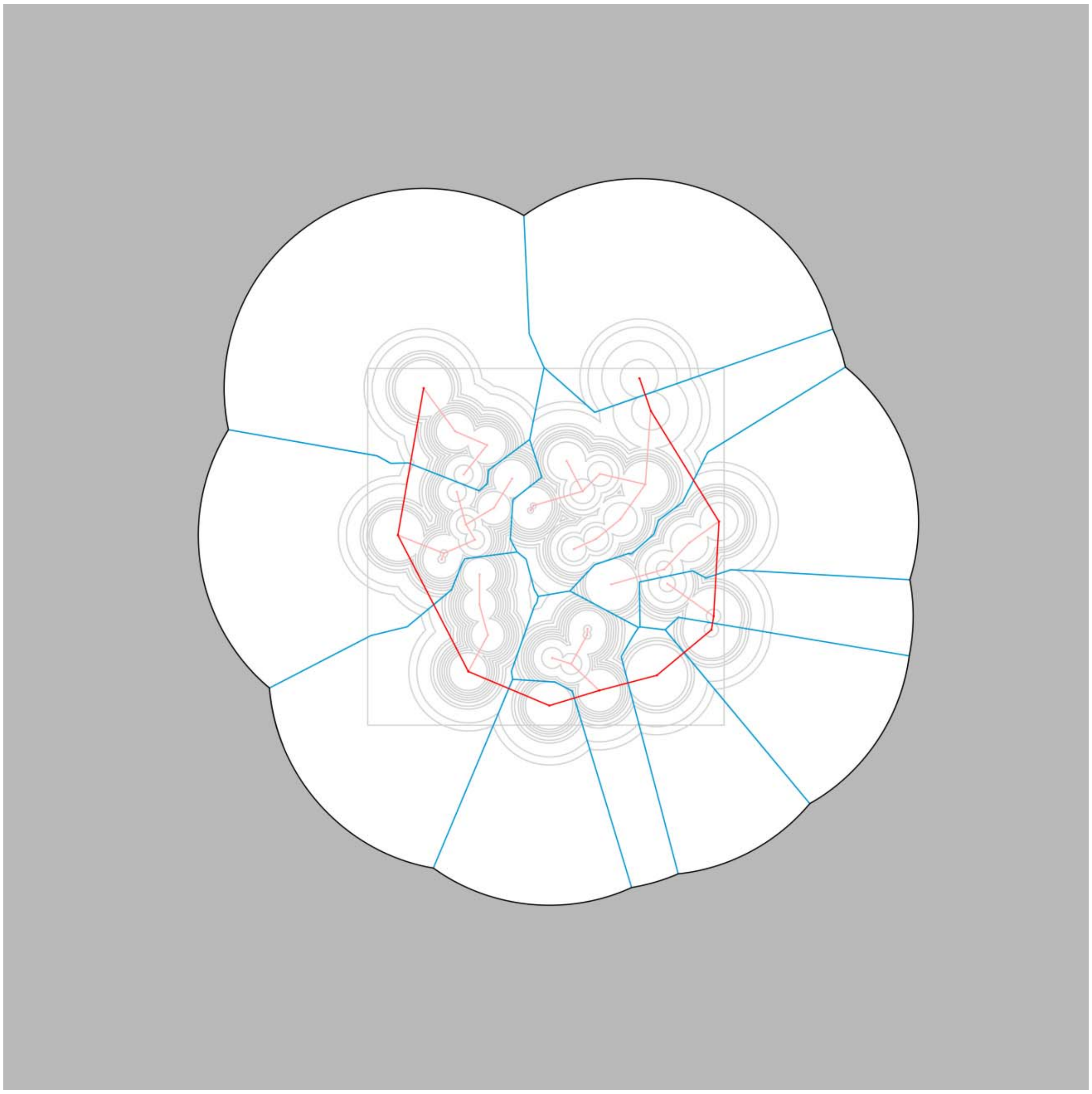

Figure C11.b. Spatial partitioning by primitive ring and minimum local spanning tree. (continued)

Minimum local spanning tree (light red) connectivity to primitive ring (red). Resulting spatial partitions (blue) lack coherence with hierarchical discrete-state chronology (light grey neighborhoods) in development of local topological space.

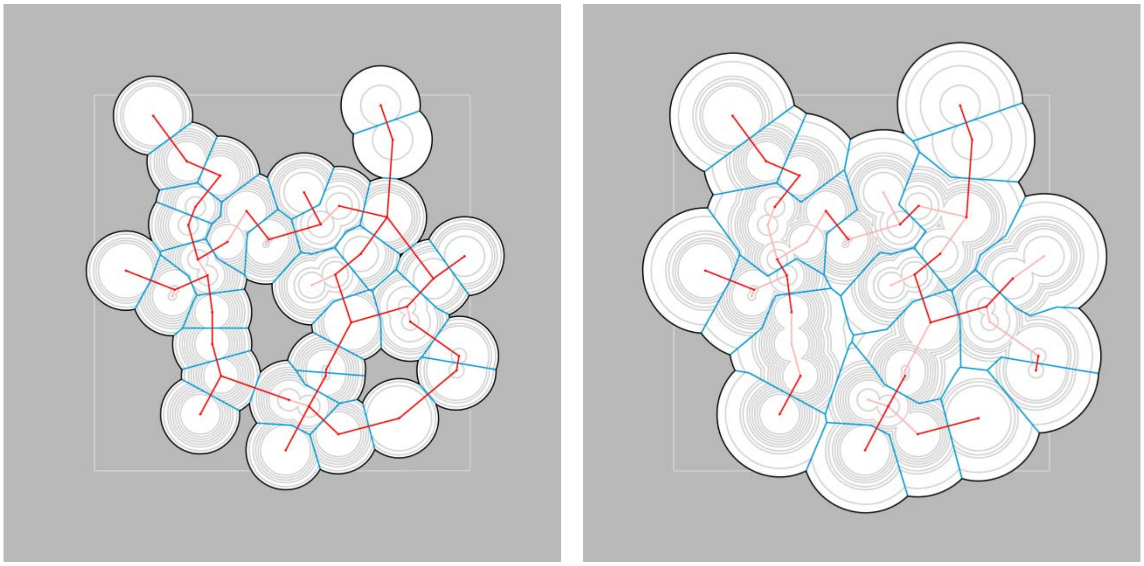

Figure C12.a. Spatial partitioning by embedded primitive ring.

Primitive ring (red) mapped on to minimum hierarchical spanning ring (light red). Resulting spatial partitions (blue) lack coherence with hierarchical discrete-state chronology (light grey neighborhoods) in development of local topological space.

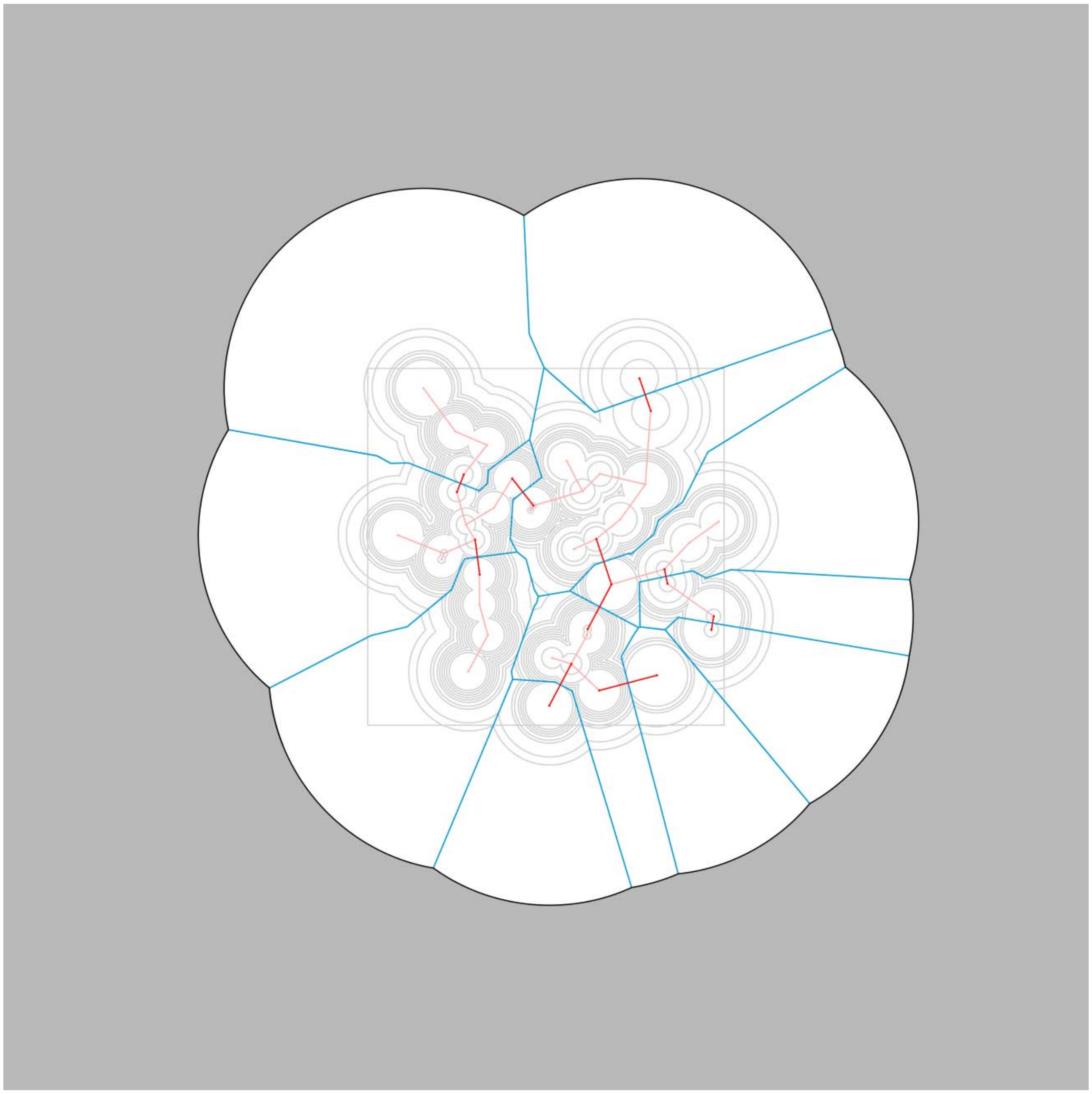

Figure C12.b. Spatial partitioning by embedded primitive ring. (continued)

Primitive ring (red) mapped on to minimum hierarchical spanning ring (light red). Resulting spatial partitions (blue) lack coherence with hierarchical discrete-state chronology (light grey neighborhoods) in development of local topological space.

C3. MODELING MORPHOLOGICAL PARAMETERS

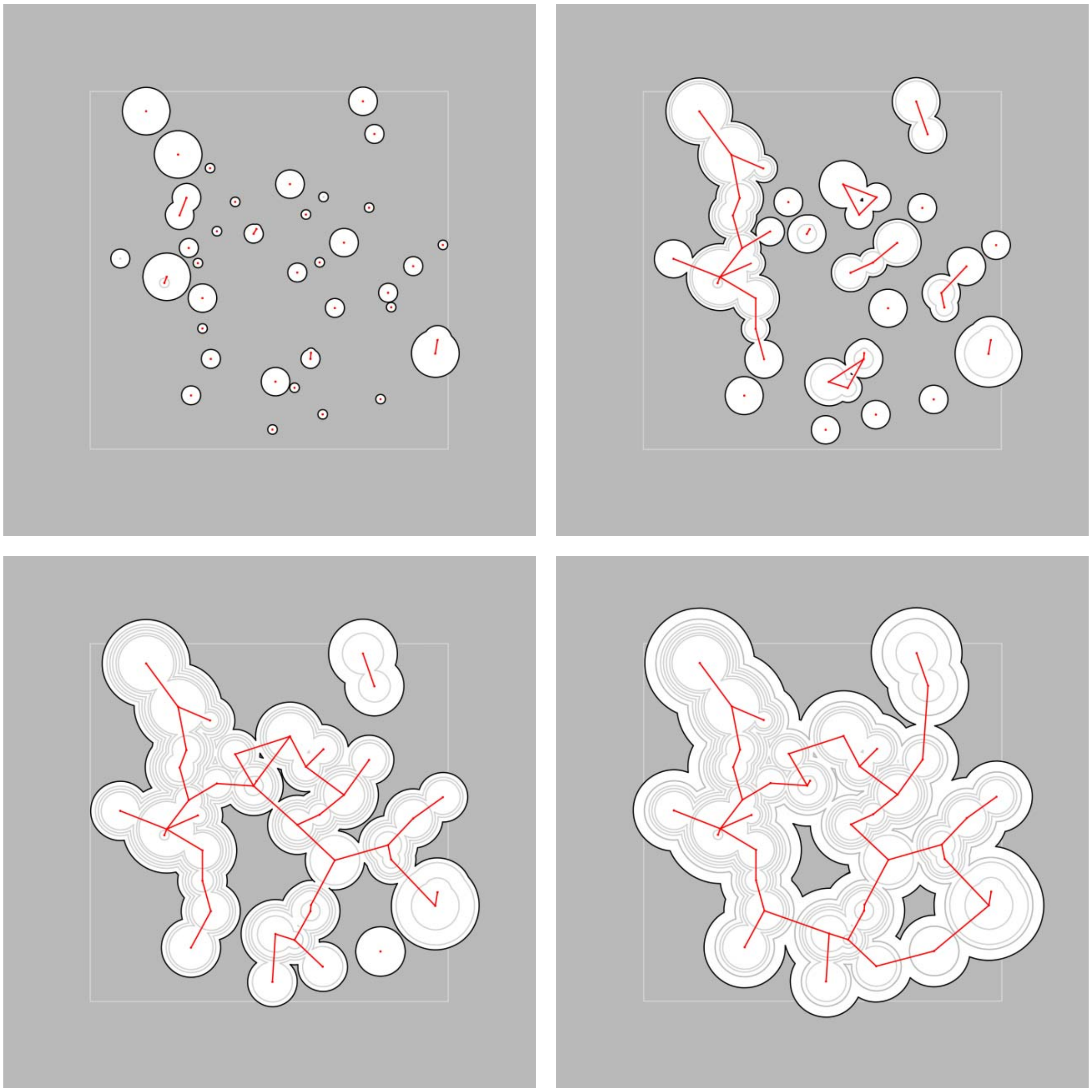

Figure C13. Synchronous arithmetic inflation, variable radius initial state neighborhoods.

Discrete-event hierarchical development of local space from initial points randomly selected for differing initial neighborhood radii undergoing uniform arithmetic inflation.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); minimum hierarchical spanning ring (red).

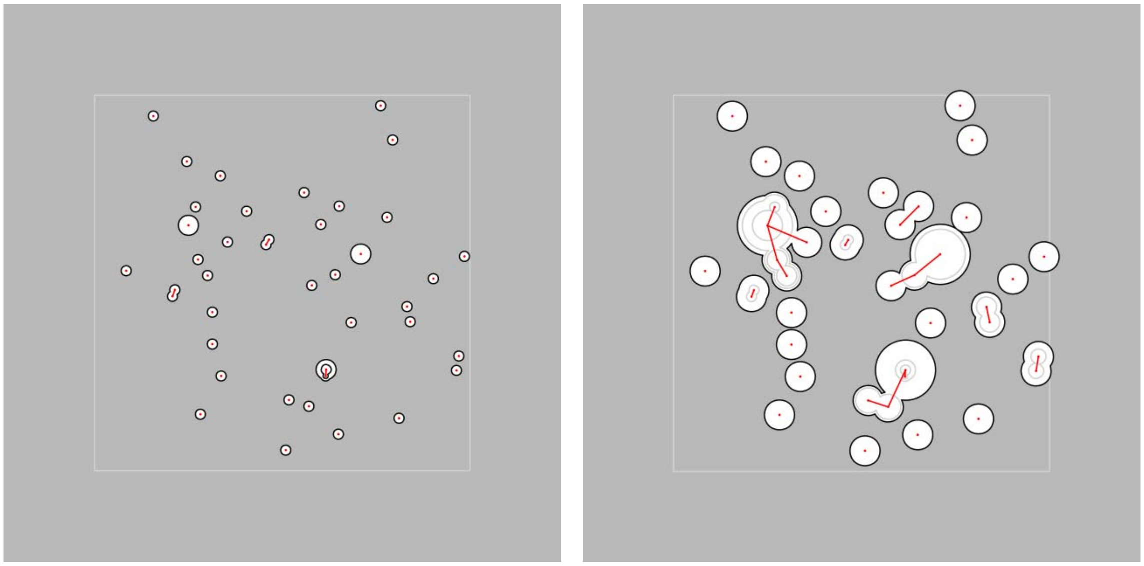

Figure C14.a. Asynchronous arithmetic inflation.

Discrete-event hierarchical development of local space from initial points randomly selected for differing arithmetic inflation rates.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); minimum hierarchical spanning ring (red).

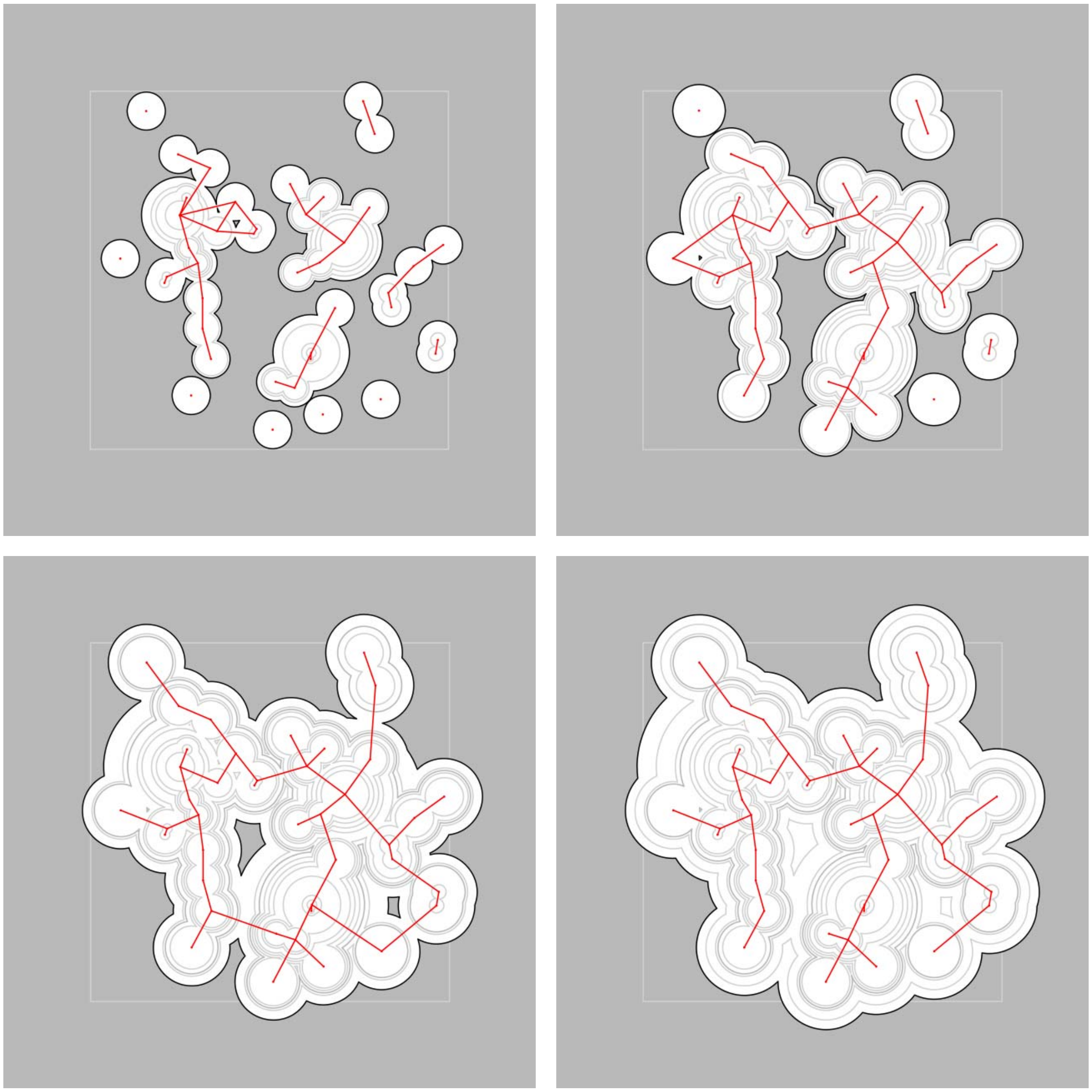

Figure C14.b. Asynchronous arithmetic inflation. (continued)

Discrete-event hierarchical development of local space from initial points randomly selected for differing arithmetic inflation rates.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); minimum hierarchical spanning ring (red).

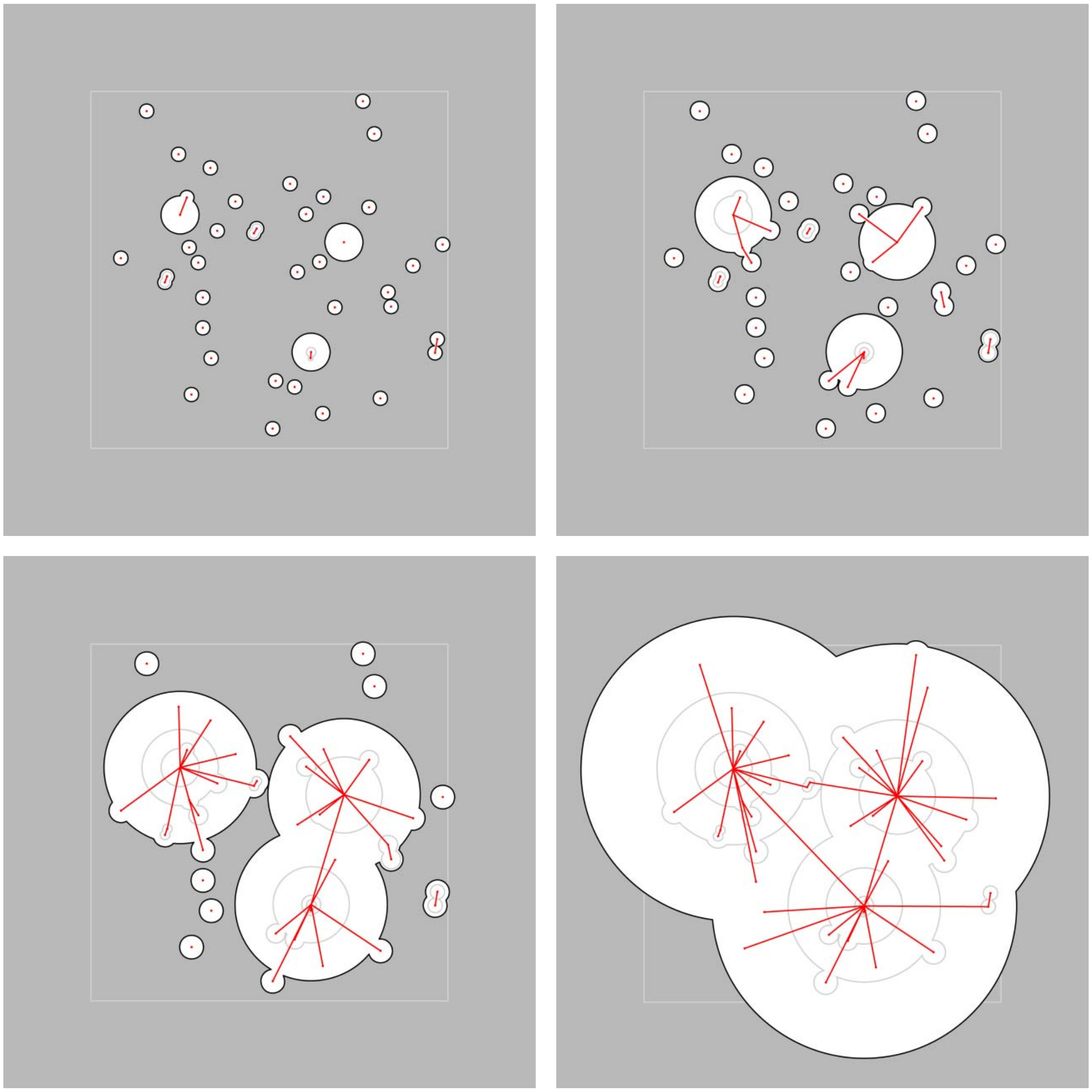

Figure C15. Asynchronous geometric-arithmetic inflation.

Discrete-event hierarchical development of local space from initial points randomly selected for geometric or arithmetic inflation rates.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); minimum hierarchical spanning ring (red).

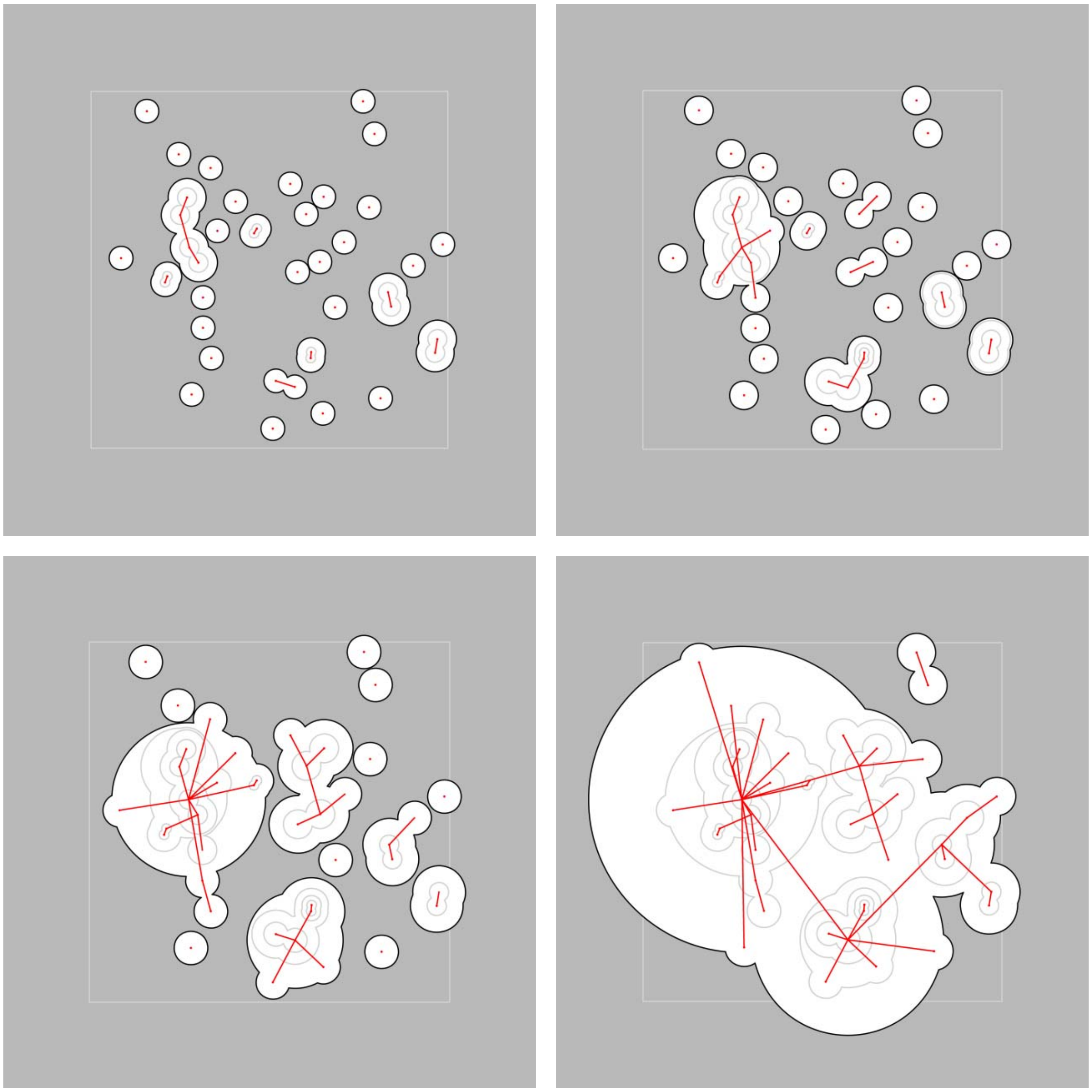

Figure C16.a. Asynchronous excitatory geometric-arithmetic inflation.

Discrete-event hierarchical development of local space from initial points with default arithmetic inflation, interrupted by situated geometric inflation. Neighborhood intersection triggers 1 subsequent excitatory state of geometric inflation, followed by return to default.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); minimum hierarchical spanning ring (red).

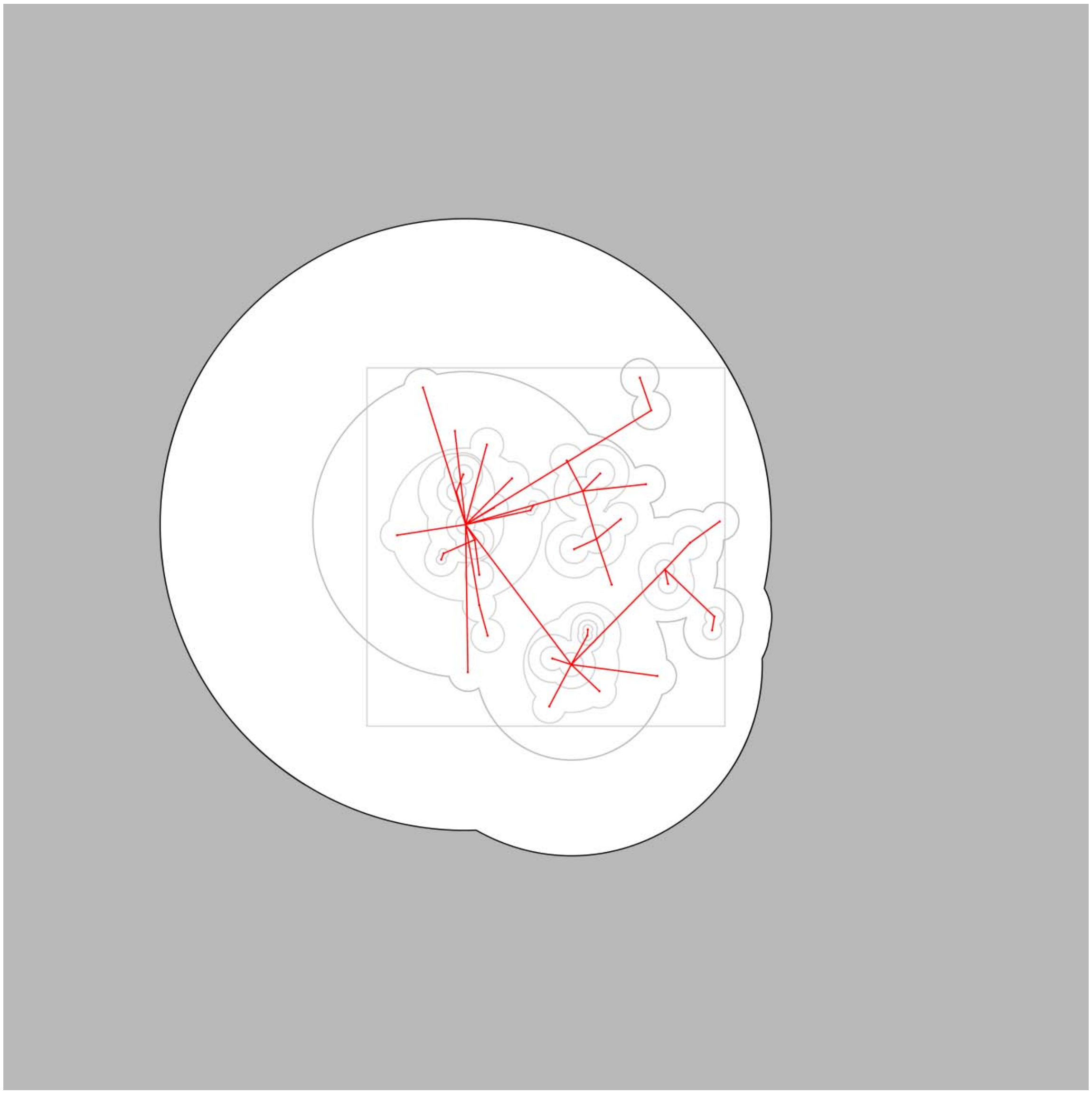

Figure C16.b. Asynchronous excitatory geometric-arithmetic inflation. (continued)

Discrete-event hierarchical development of local space from initial points with default arithmetic inflation, interrupted by situated geometric inflation. Neighborhood intersection triggers 1 subsequent excitatory state of geometric inflation, followed by return to default.

Local fundamental group [1] boundary (black); discrete-state chronology (light gray neighborhoods); terminal minimum hierarchical spanning ring (red).

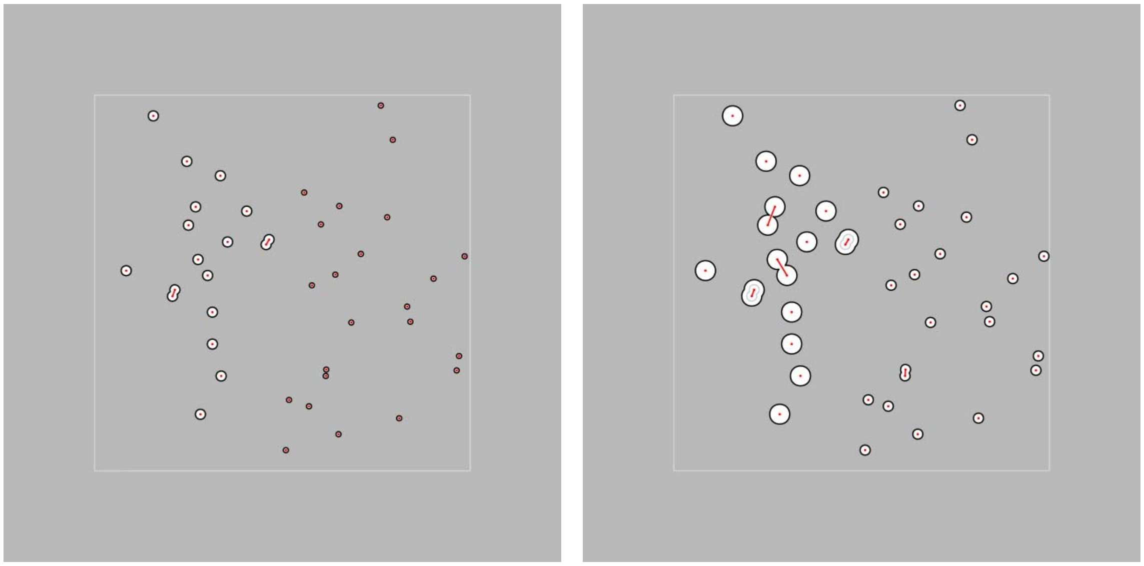

Figure C17.a. Asynchronous bipartite geometric-arithmetic inflation.

Discrete-event hierarchical development of local space from an initial situated set spatially partitioned for geometric or arithmetic inflation.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); minimum hierarchical spanning ring (red).

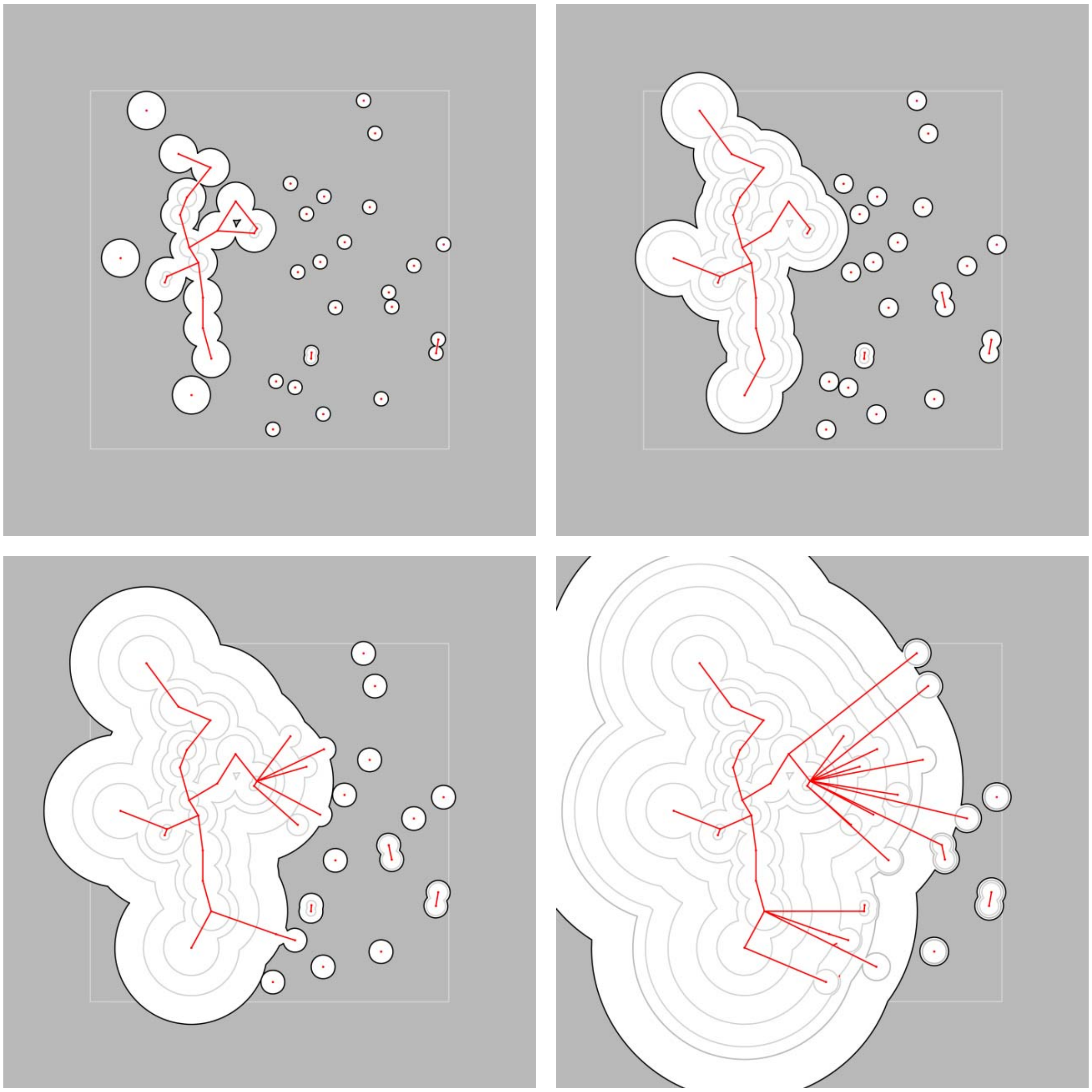

Figure C17.b. Asynchronous bipartite geometric-arithmetic inflation. (continued)

Discrete-event hierarchical development of local space from an initial situated set spatially partitioned for geometric or arithmetic inflation.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); minimum hierarchical spanning ring (red).

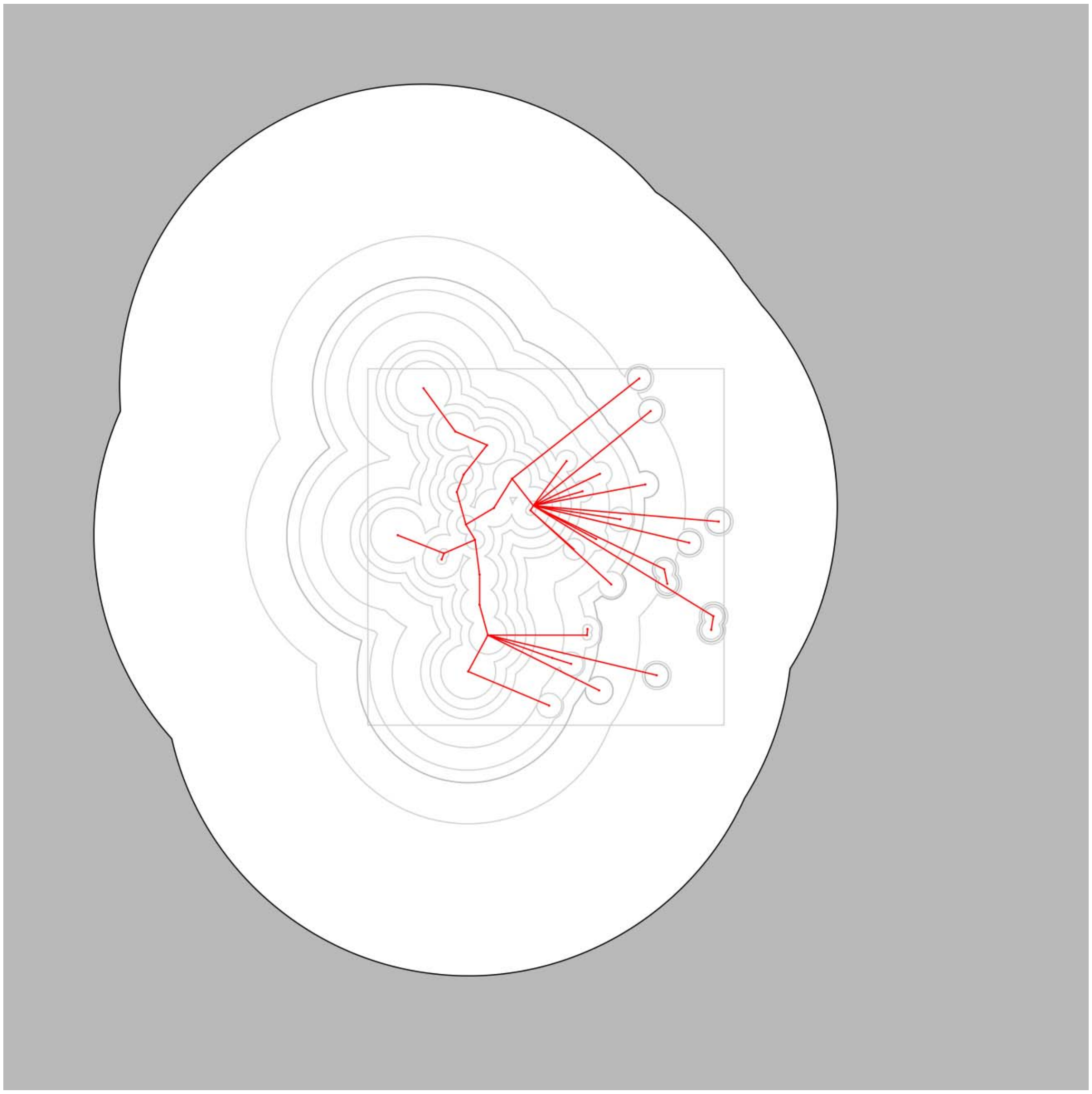

Figure C17.c. Asynchronous bipartite geometric-arithmetic inflation. (continued)

Discrete-event hierarchical development of local space from an initial situated set spatially partitioned for geometric or arithmetic inflation.

Fundamental group [1] boundary (black); discrete-state chronology (light gray neighborhoods); terminal minimum hierarchical spanning ring (red).

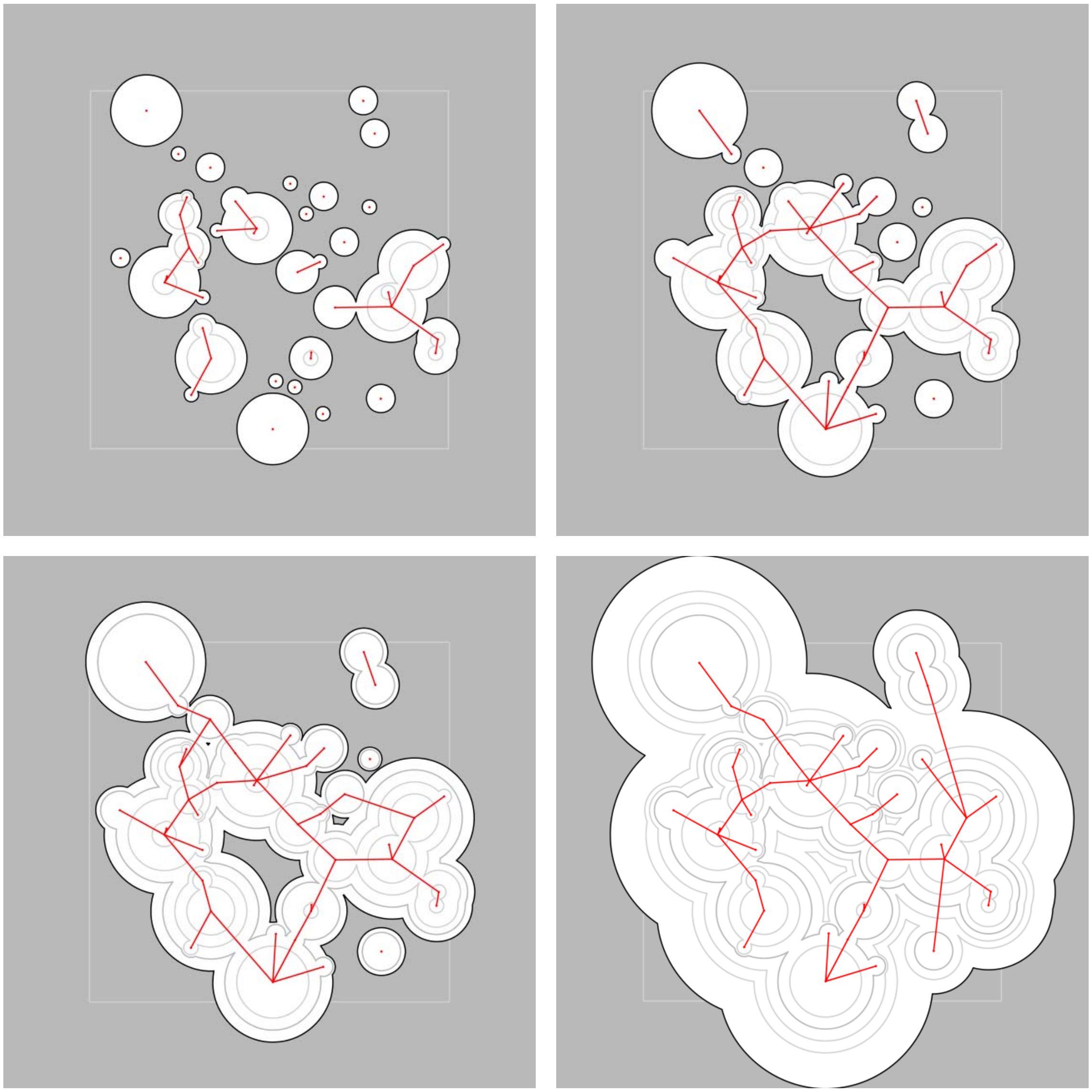

Figure C18. Asynchronous arithmetic inflation, initial state variable radius neighborhoods.

Discrete-event hierarchical development of local space from initial points randomly selected for differing initial neighborhood radii with differing arithmetic inflation rates.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); minimum hierarchical spanning ring (red).

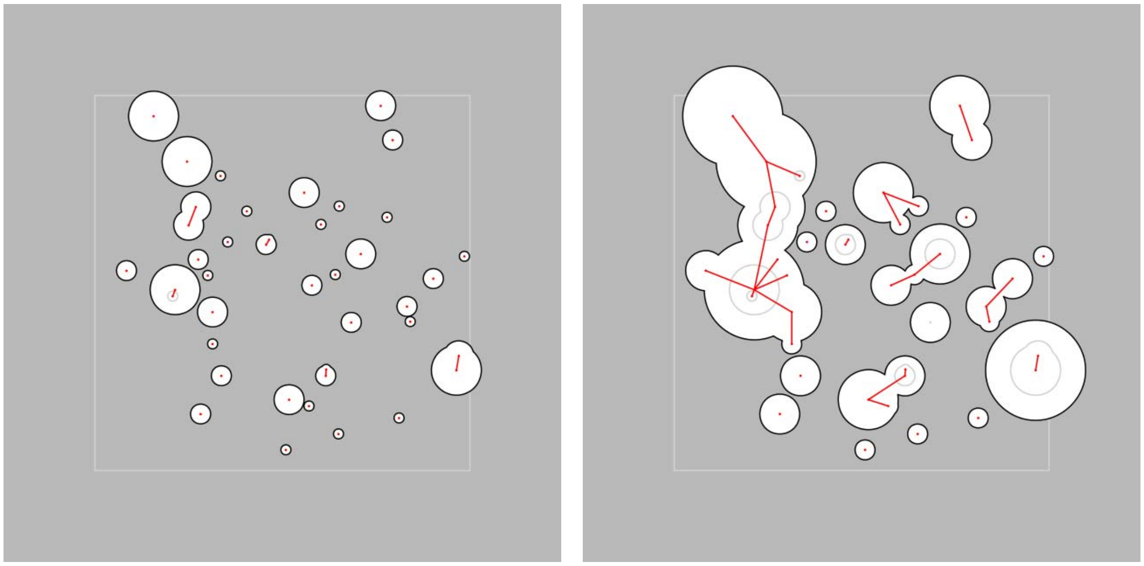

Figure C19.a. Asynchronous geometric-arithmetic inflation, initial state variable radius neighborhoods.

Discrete-event hierarchical development of local space from initial points randomly selected for differing initial neighborhood radii with geometric or arithmetic inflation rates.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); minimum hierarchical spanning ring (red).

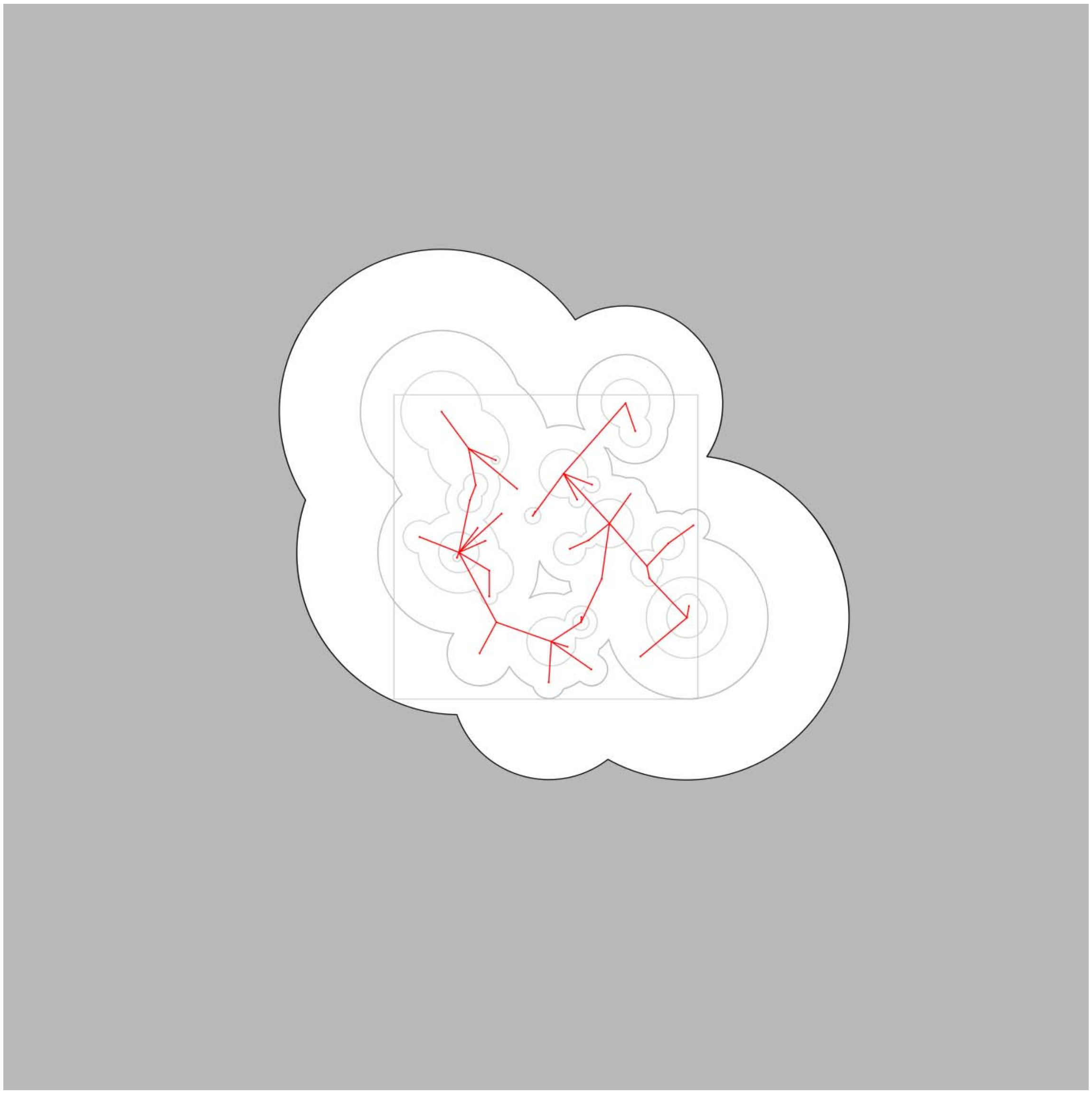

Figure 19.b. Asynchronous geometric-arithmetic inflation, initial state variable radius neighborhoods. (continued)

Discrete-event hierarchical development of local space from initial points randomly selected for differing initial neighborhood radii and geometric or arithmetic inflation.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); terminal minimum hierarchical spanning ring (red), fundamental group [1].

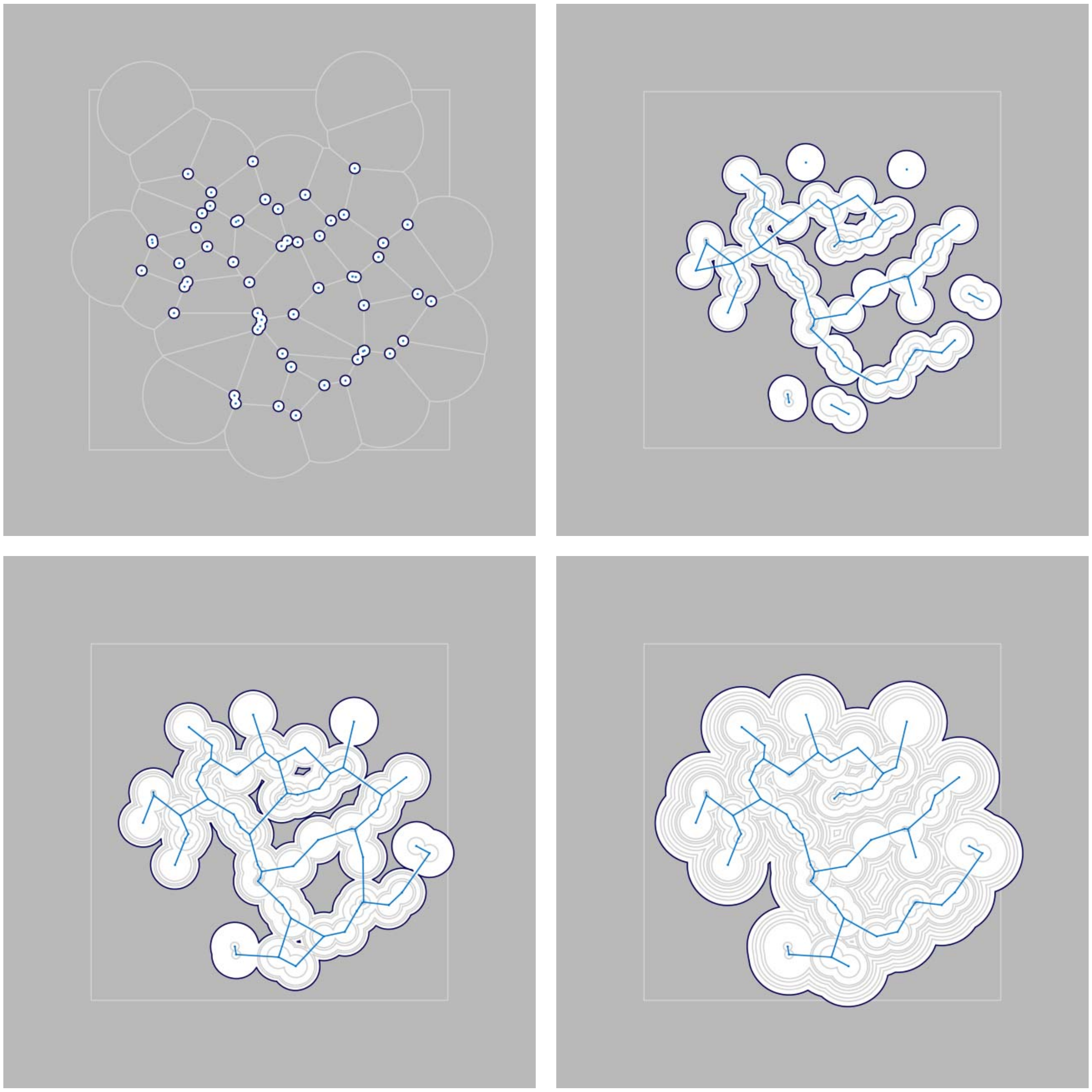

Figure C20. Development of local Voronoi-dual space.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); minimum hierarchical spanning ring (blue).

Top left: initial Voronoi-dual situated set. Bottom right: Connected local topological space, fundamental group [1]

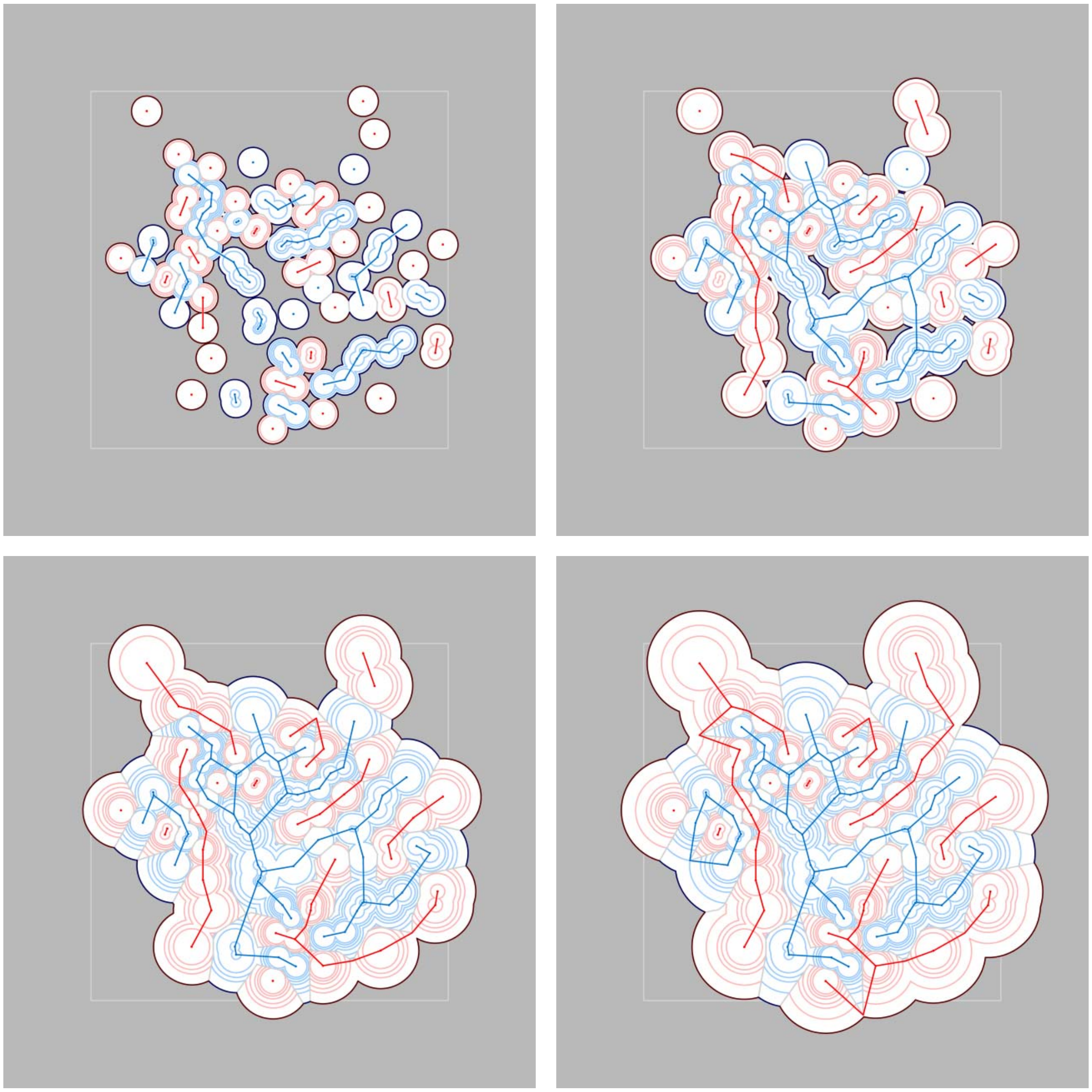

Figure C21.a. Voronoi-dual inhibitory development of local space.

Discrete-event hierarchical development of local space from 2 inhibitory situated sets. Red: local space developed from initial situated set inFigure C1. Blue: local space developed from initial Voronoi-dual situated set in Figure C20.

Top right: blue minimum hierarchical spanning ring achieves percolation.

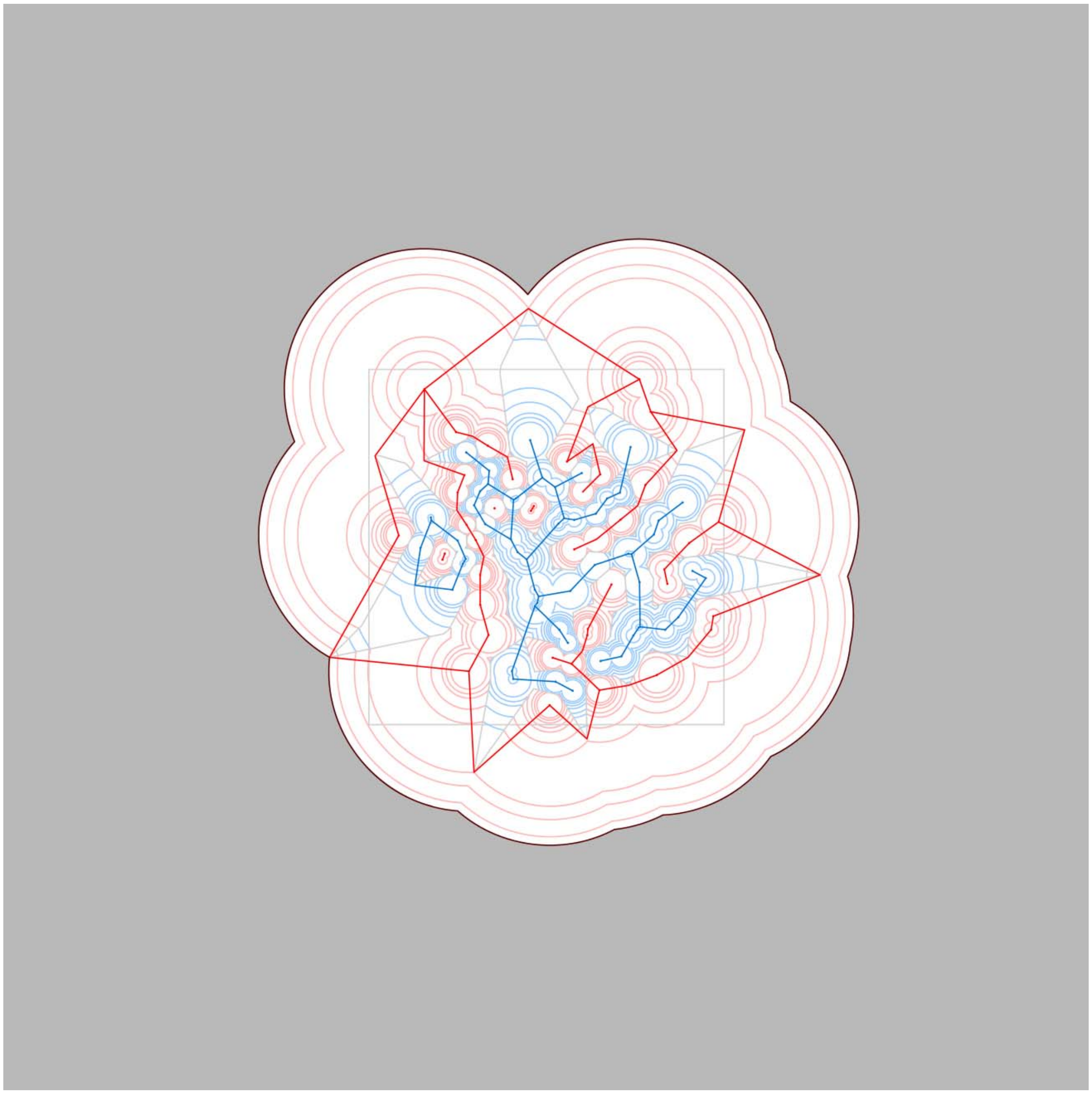

Figure C21.b. Voronoi-dual inhibitory development of local space. (continued)

Discrete-event hierarchical development of local space from 2 inhibitory situated sets. Red: local space developed from initial situated set in Figure C1. Blue: local space developed from initial Voronoi-dual set in Figure C20.

Terminal minimum hierarchical spanning ring, red fundamental group[3] with blue inclusions.

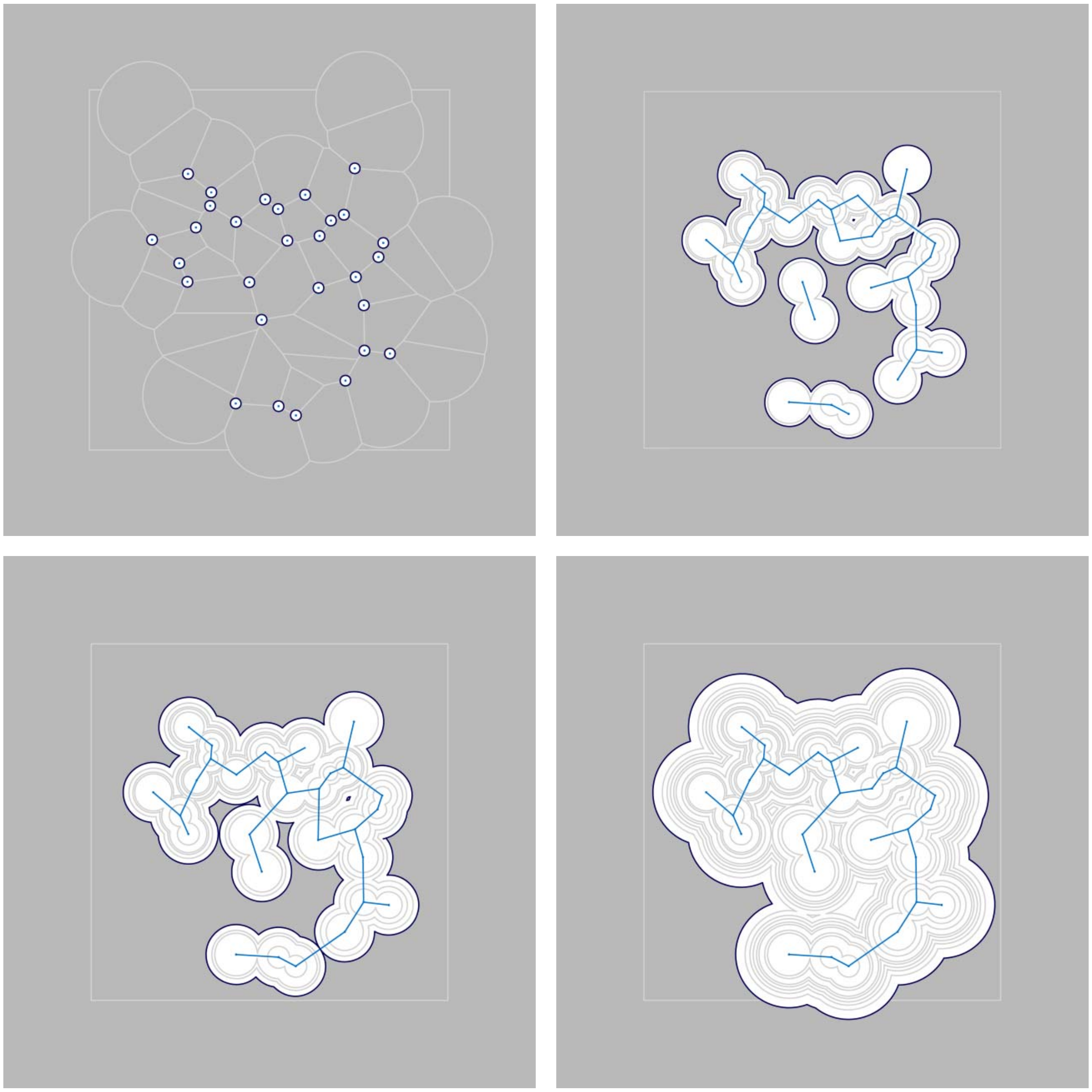

Figure C22. Development of local void-dual space.

Local fundamental group boundary (black); discrete-state chronology (light gray neighborhoods); minimum hierarchical spanning ring (blue).

Top left: initial void-dual situated set. Bottom right: Connected local topological space, fundamental group [1].

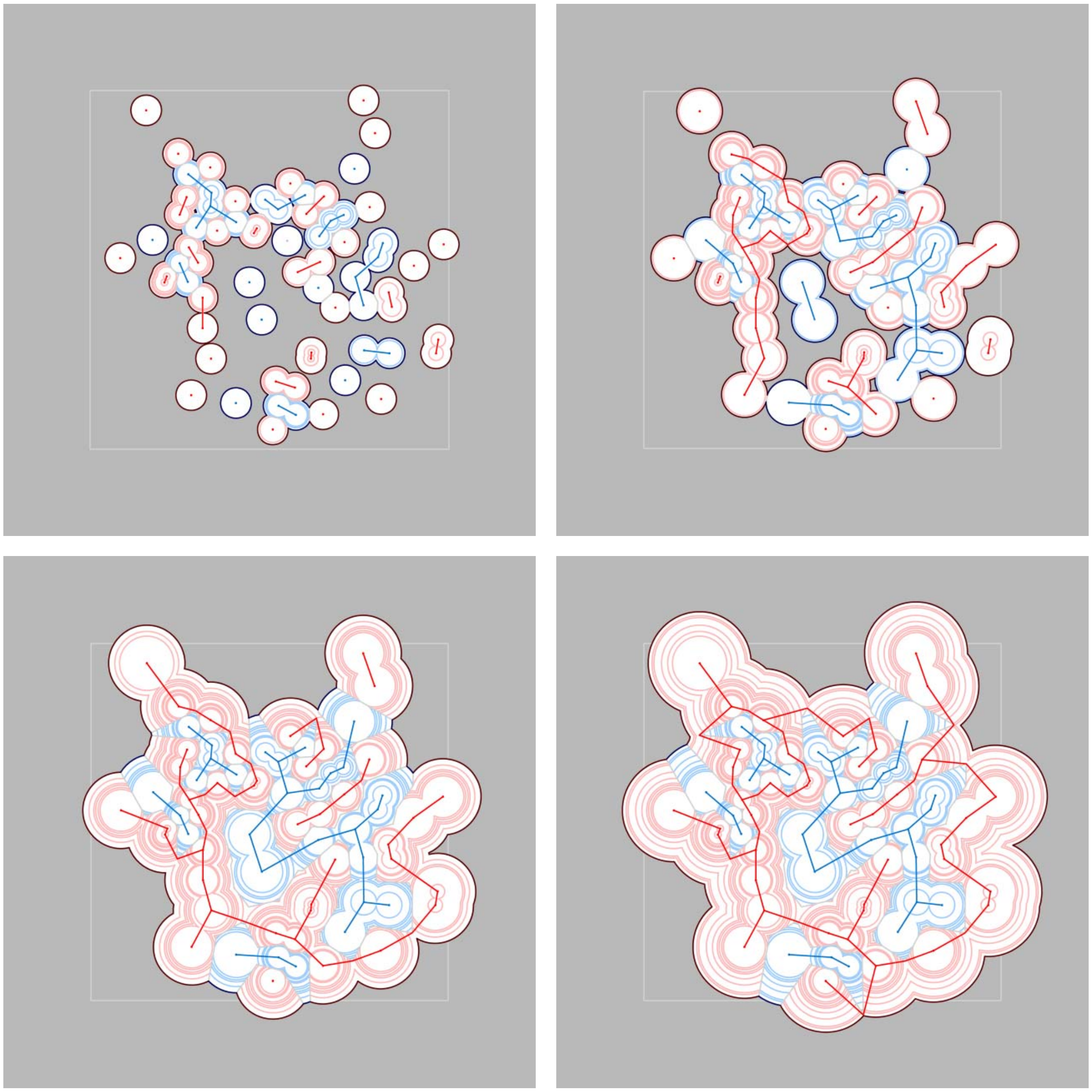

Figure C23.a. Void-dual inhibitory development of local space.

Discrete-event hierarchical development of local space from 2 inhibitory situated sets. Red: local space developed from initial situated set inFigure C1. Blue: local space developed from initial void-dual situated set in Figure C22.

Bottom left: red minimum hierarchical spanning ring achieves percolation. Bottom right: red forms single connected topological space, fundamental group [2].

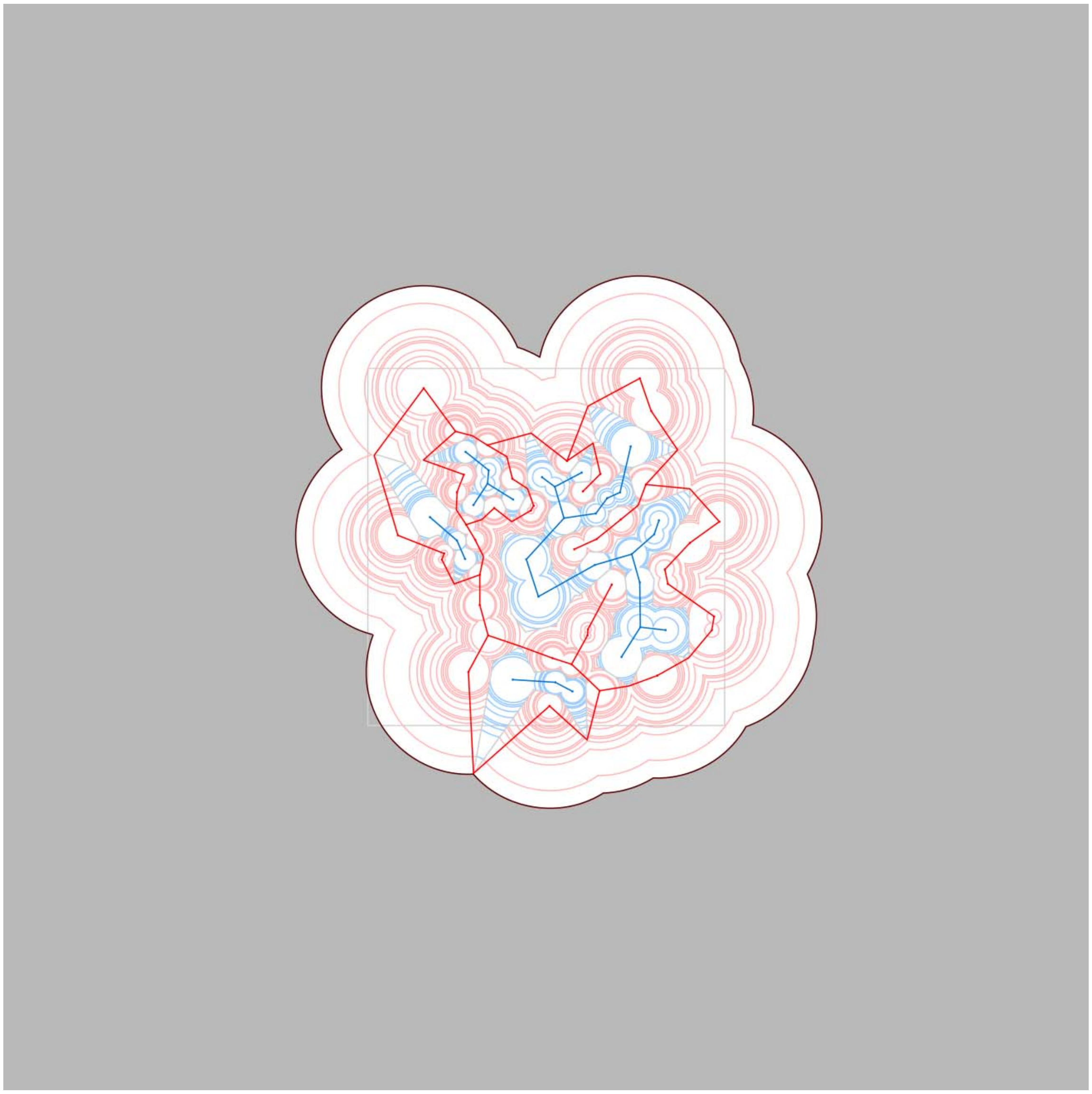

Figure C23.b. Void-dual inhibitory development of local space. (continued)

Discrete-event hierarchical development of local space from 2 inhibitory situated sets. Red: local space developed from initial situated set inFigure C1. Blue: local space developed from initial void-dual situated set in Figure C22.

Terminal minimum hierarchical spanning ring, red fundamental group[5], with blue inclusions.

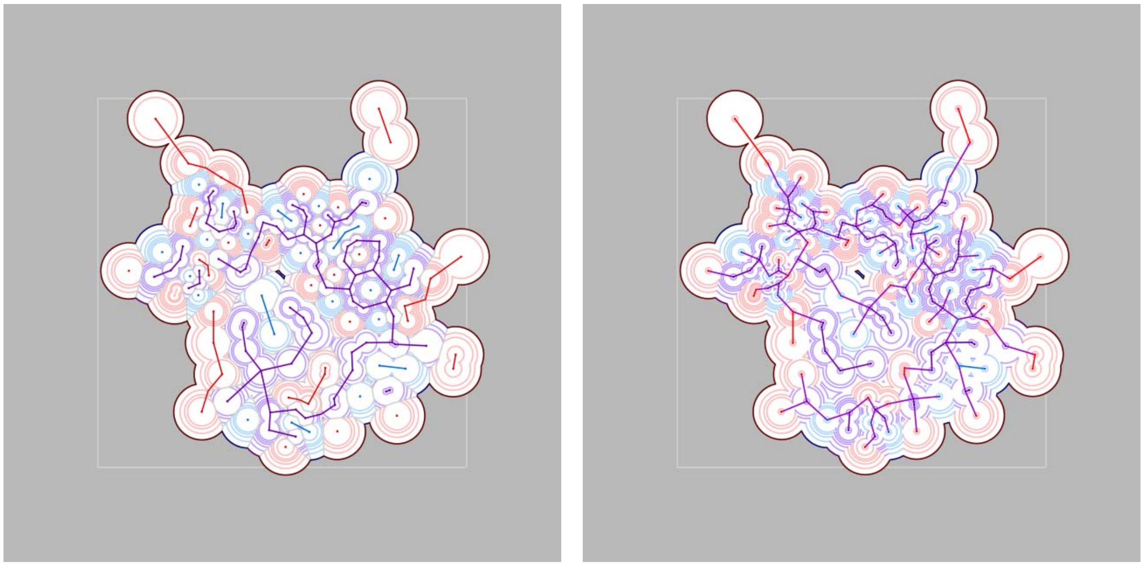

Figure C24. Binding set development of inhibitory local spaces.

Identical discrete-event states in inhibitory (left) and binding (right) development of local topological space.

Left: local space developed from 3 inhibitory initial situated sets. Minimum hierarchical spanning ring, purple fundamental group [3] achieves percolation. Right: Local space developed from 2 inhibitory initial sets (red, blue) and binding initial set (purple). Minimum hierarchical spanning ring, red-purple-blue fundamental group [2], achieves percolation.

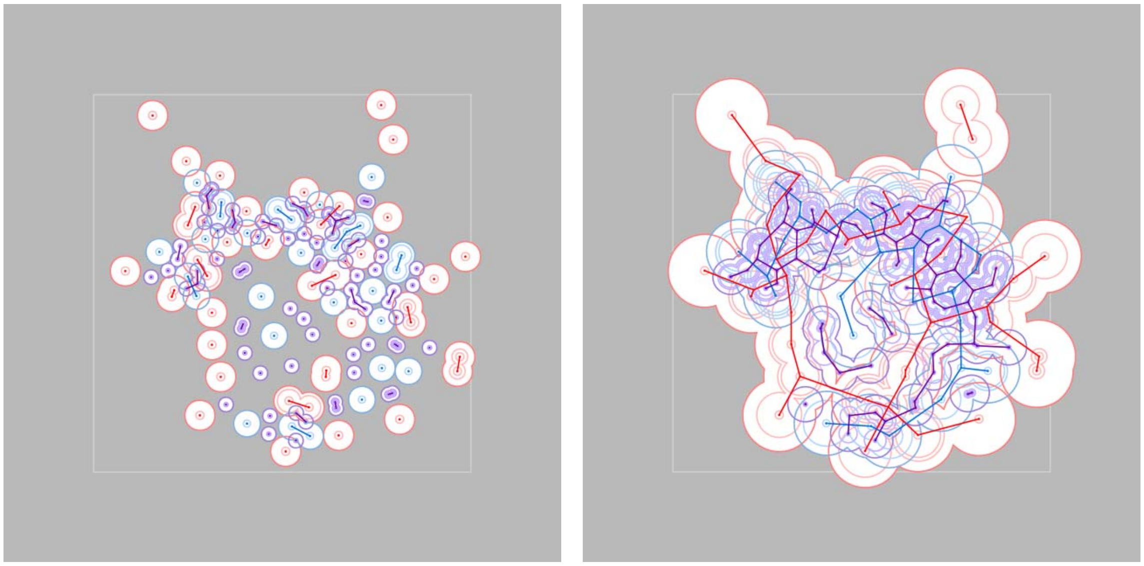

Figure C25. Synchronous development of multi-layer local space.

C4. MODELING POINT FORMATION

Figure C26.a. Deterministic point set automata: 4-neighborhood intersection.

The intersection of 4 neighborhoods in a local space creates a new point. With generative recursion, 4 neighborhood intersections are modeled by intersecting edges of the connectivity graph of a local space, generation G, with intersection points constructing a new situated set, generation G+1.

Local spaces G1 (purple), situated set G2 (red).

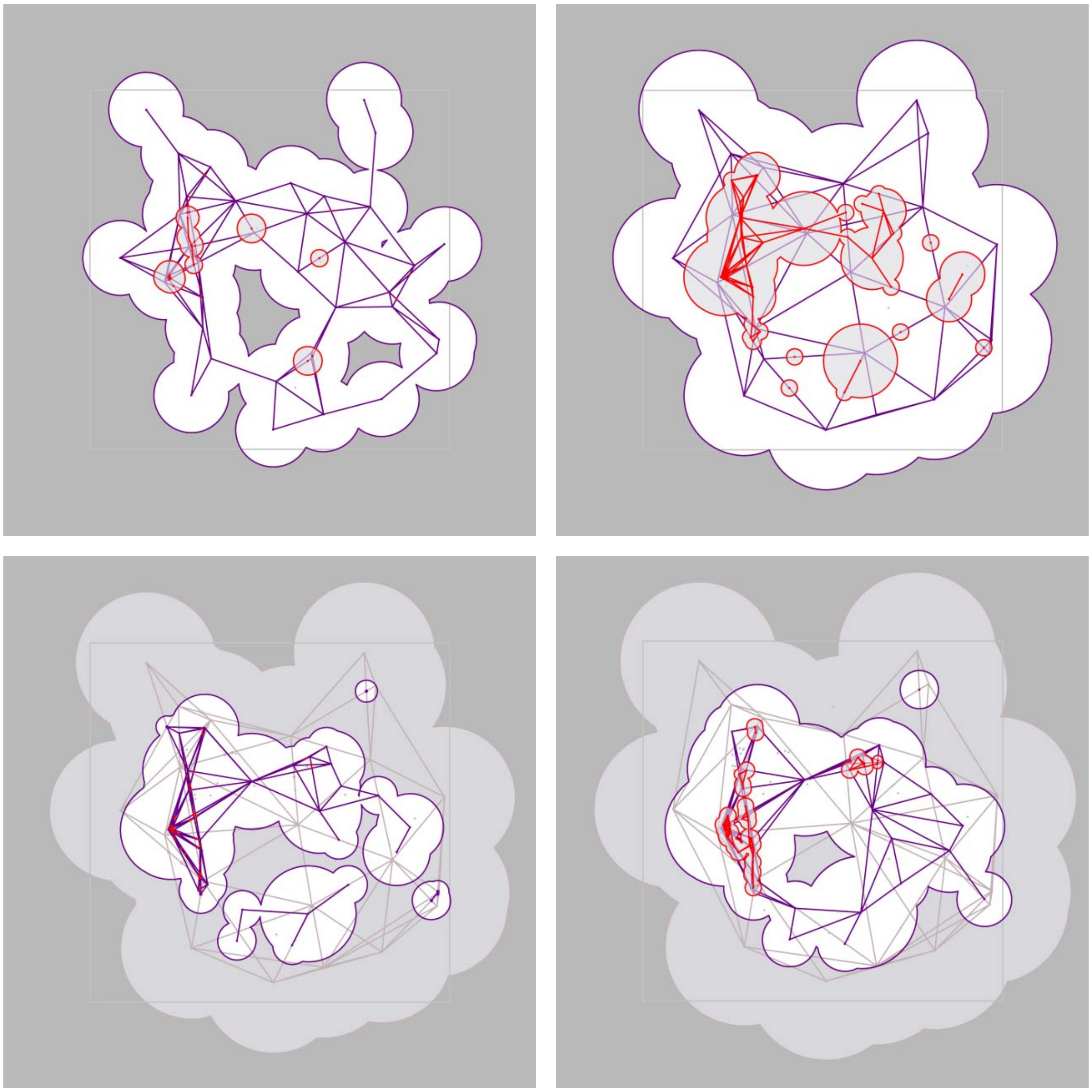

Figure C26.b. Deterministic point set automata: 4-neighborhood intersection. (continued)

Top left: local space G1 (purple), local spaces G2 (red). Top right: local space G1 (purple), local spaces G2 (red). Bottom left: local space G1 (light gray), local spaces G2 (purple), situated set G3 (red). Bottom right: local space G1 (light gray), local spaces G2 (purple), local spaces G3 (red).

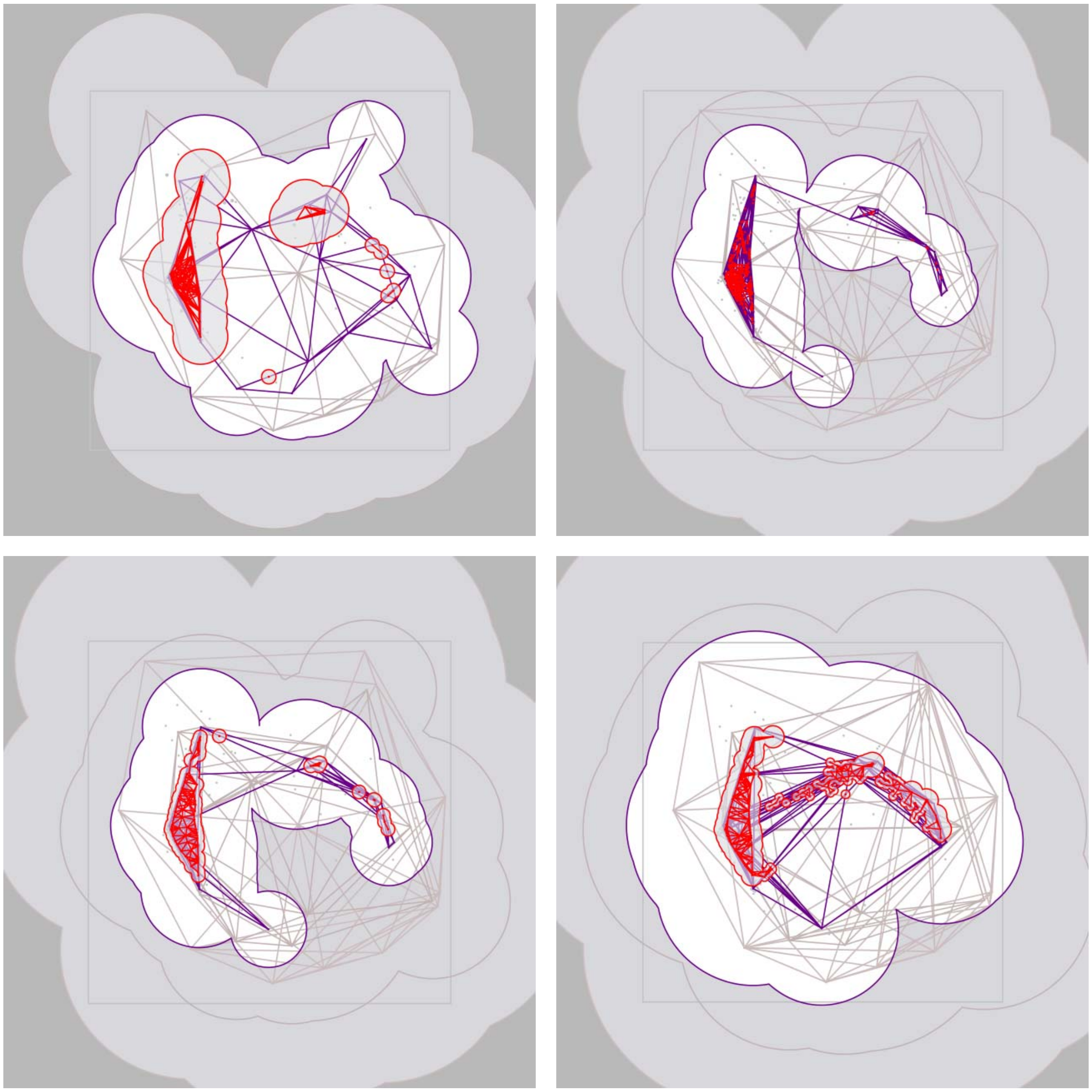

Figure C26.c. Deterministic point set automata: 4-neighborhood intersection. (continued)

Top left: local space G1, (light gray), local space G2 (purple), local spaces G3 (red). Top right: local space G1, G2 (light gray), local space G3 (purple), situated set G4 (red). Bottom left: local space G1, G2 (light gray), local space G3 (purple), local spaces G4 (red). Bottom right: local space G1, G2 (light gray), local space G3 (purple), local spaces G4 (red).

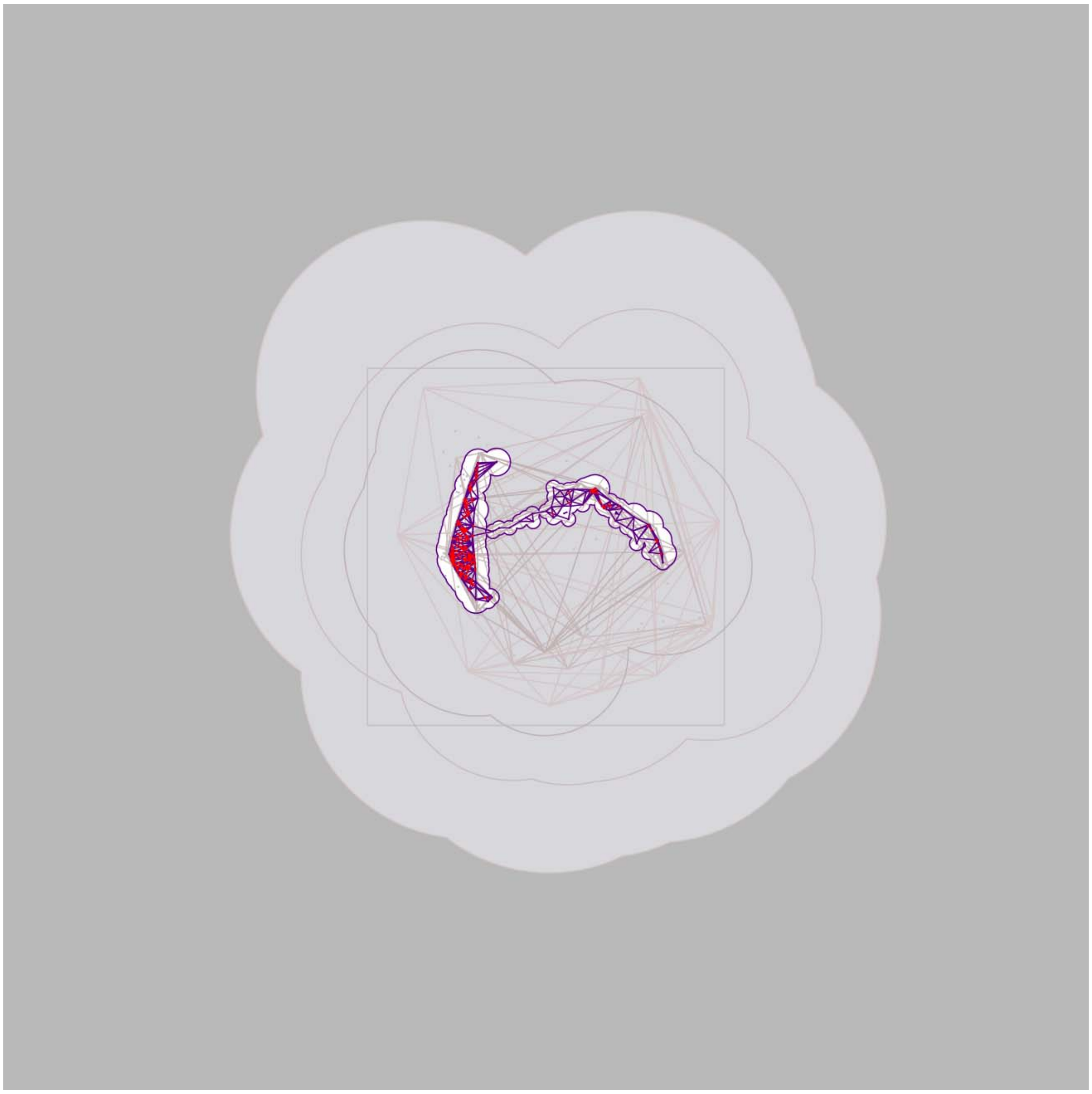

Figure C26.d. Deterministic point set automata: 4-neighborhood intersection. (continued)

Local space G1, G2, G3 (light gray), local space G4 (purple), situated set G5 (red).

Bipartite structure of situated sets constructed on left side of developing spaces suggest possible infinite recursion of point nursery. Inflating boundary of developing local space suggests space-time wavefront.

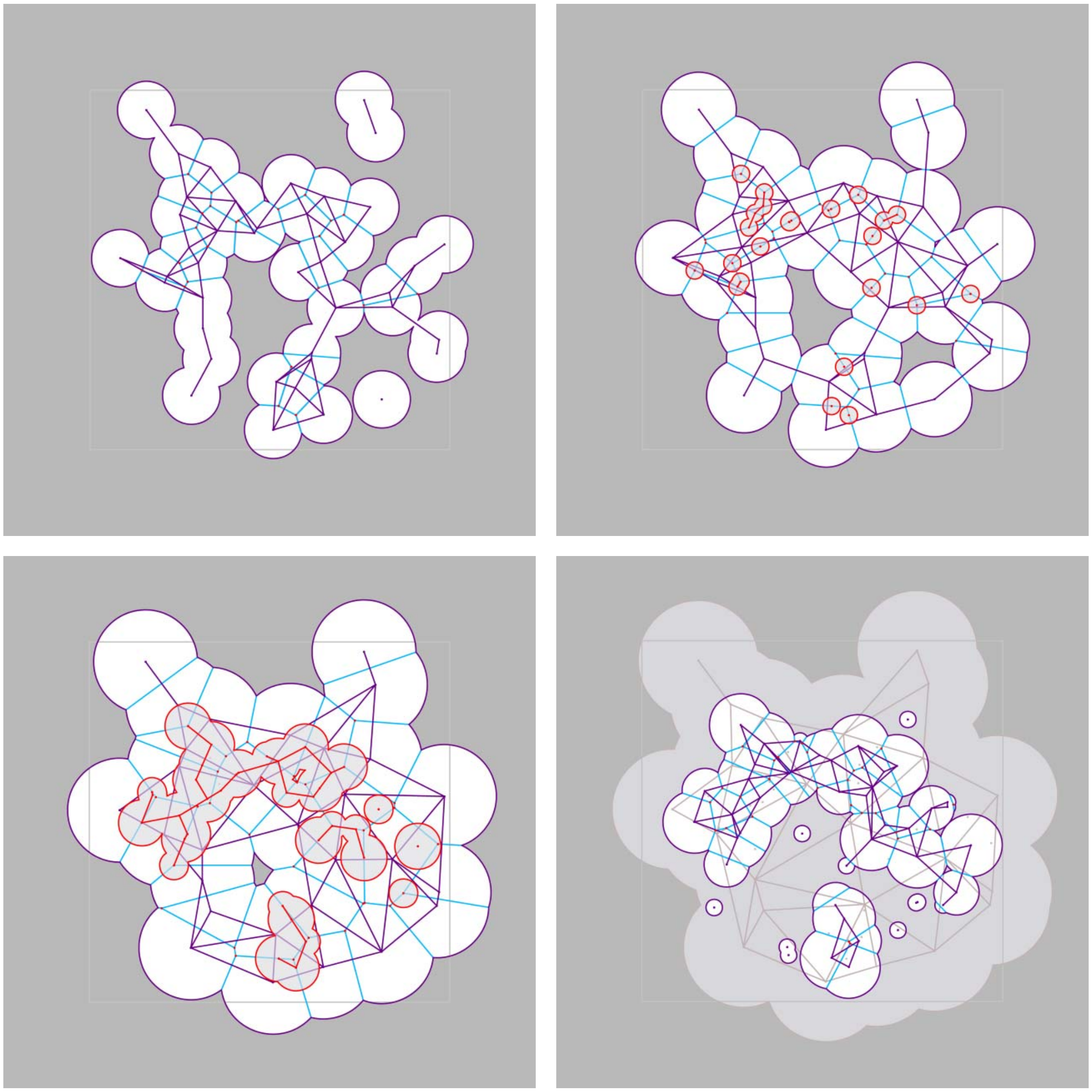

Figure C27.a. Deterministic point set automata: 3-neighborhood intersection.

The intersection of 3 neighborhoods in a local space creates a new point. With generative recursion, 3 neighborhood intersections are modeled by the Voronoi partition of a local space, generation G, with Voronoi vertices structuring the new situated set G+1.

Top left: Local space G1 (purple), Voronoi partition (light blue), situated set G2 (red). Top right: Local space G1 (purple), situated set and local spaces G2 (red). Bottom left: Local space G1 (purple), situated set and local spaces, G2 (red). Bottom right: Local space G1 (light gray), local spaces G2 (purple), situated set G3 (red).

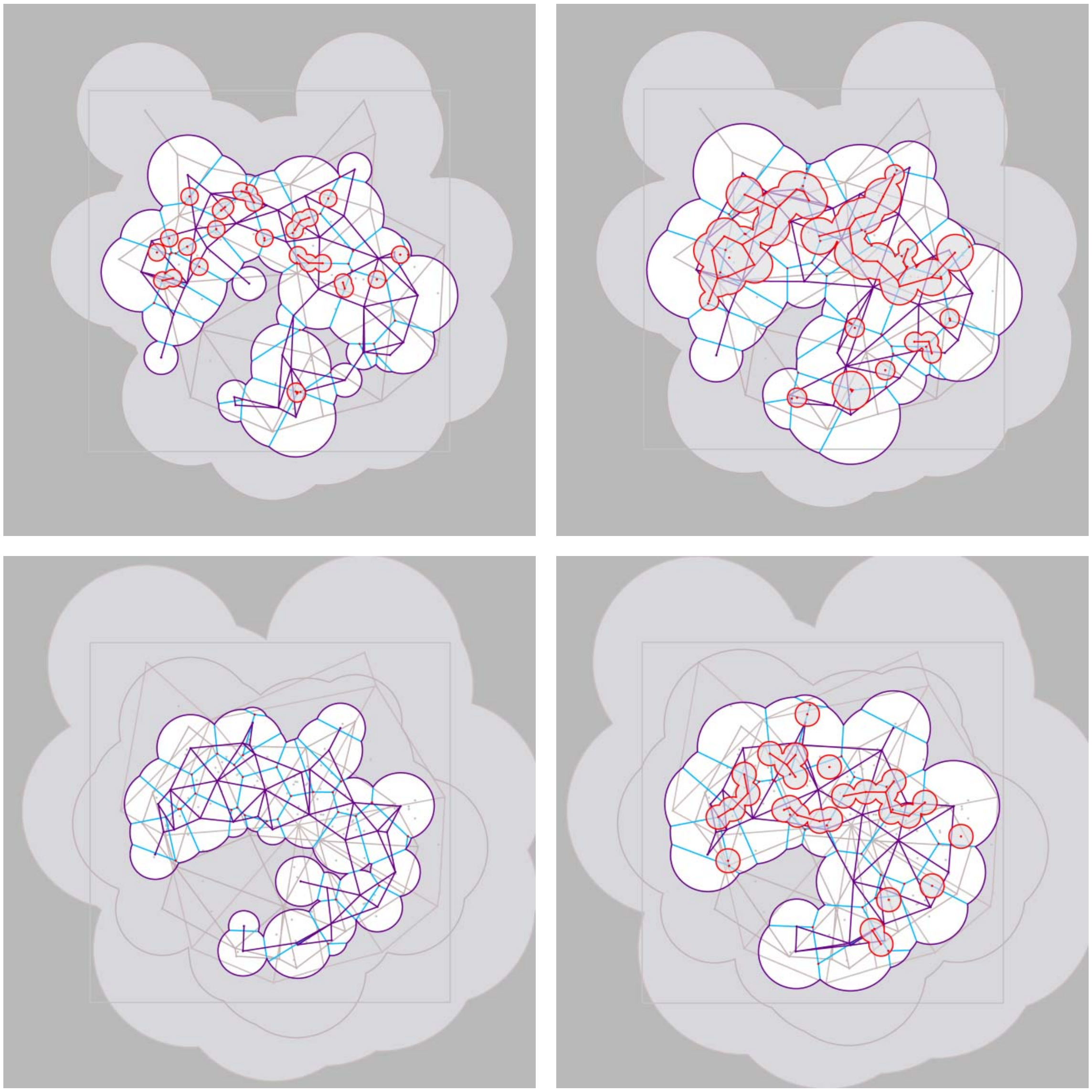

Figure C27.b. Deterministic point set automata: 3-neighborhood intersection. (continued)

Top left: local space G1 (light gray); local spaces G2 (purple); situated set and local spaces G3 (red). Top right: local space G1 (light gray); local spaces G2 (purple); situated set and local spaces G3 (red). Bottom left: local space G1, G2 (light gray); local space G3 (purple); situated set G3 (red). Bottom right: local space G1, G2 (light gray); local spaces G2 (purple); situated set and local spaces G3 (red).

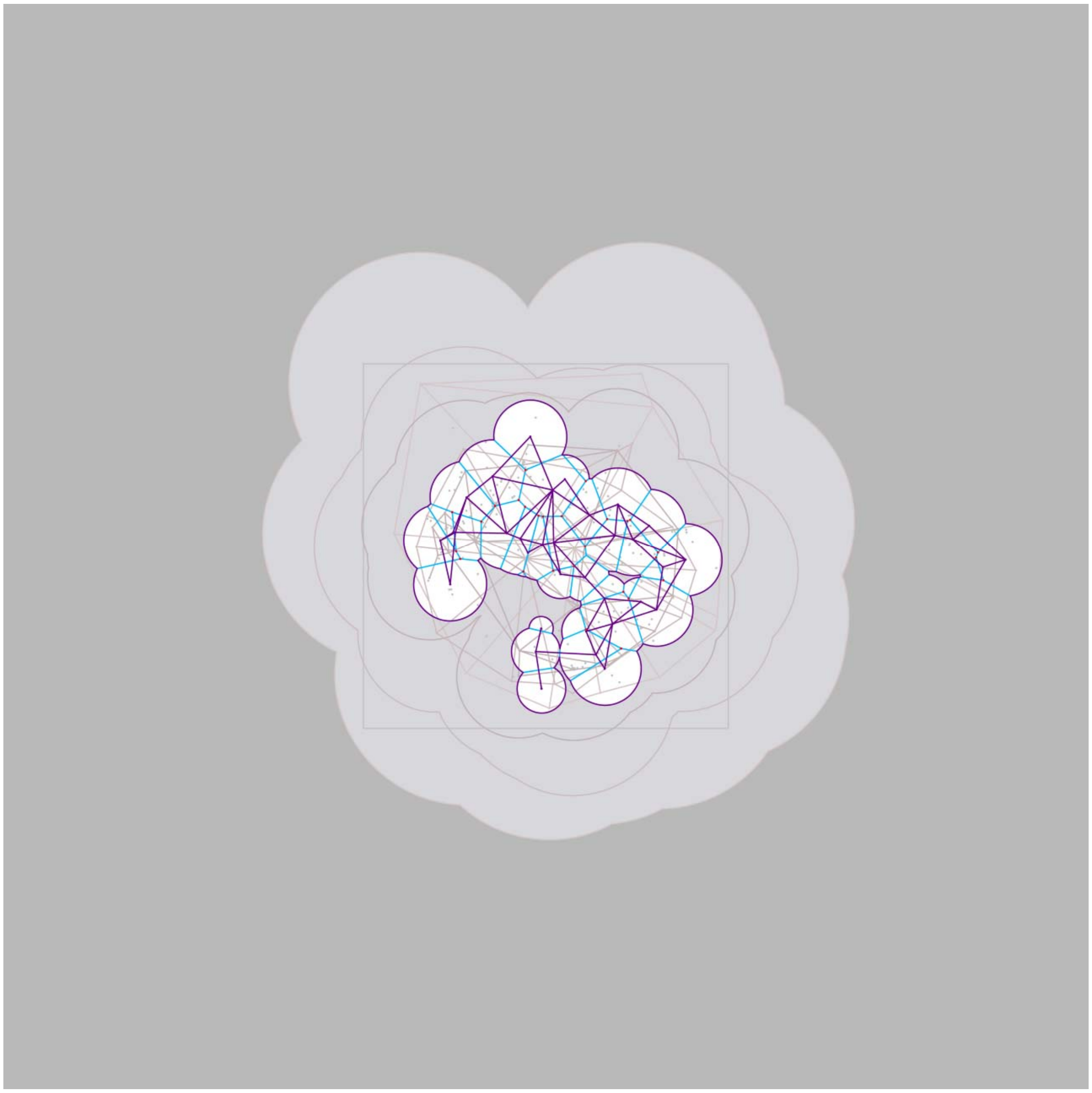

Figure C27.c. Deterministic point set automata: 3-neighborhood intersection. (continued)

Local space G1, G2, G3 (light gray); local space G4 (purple); situated set G5 (red).

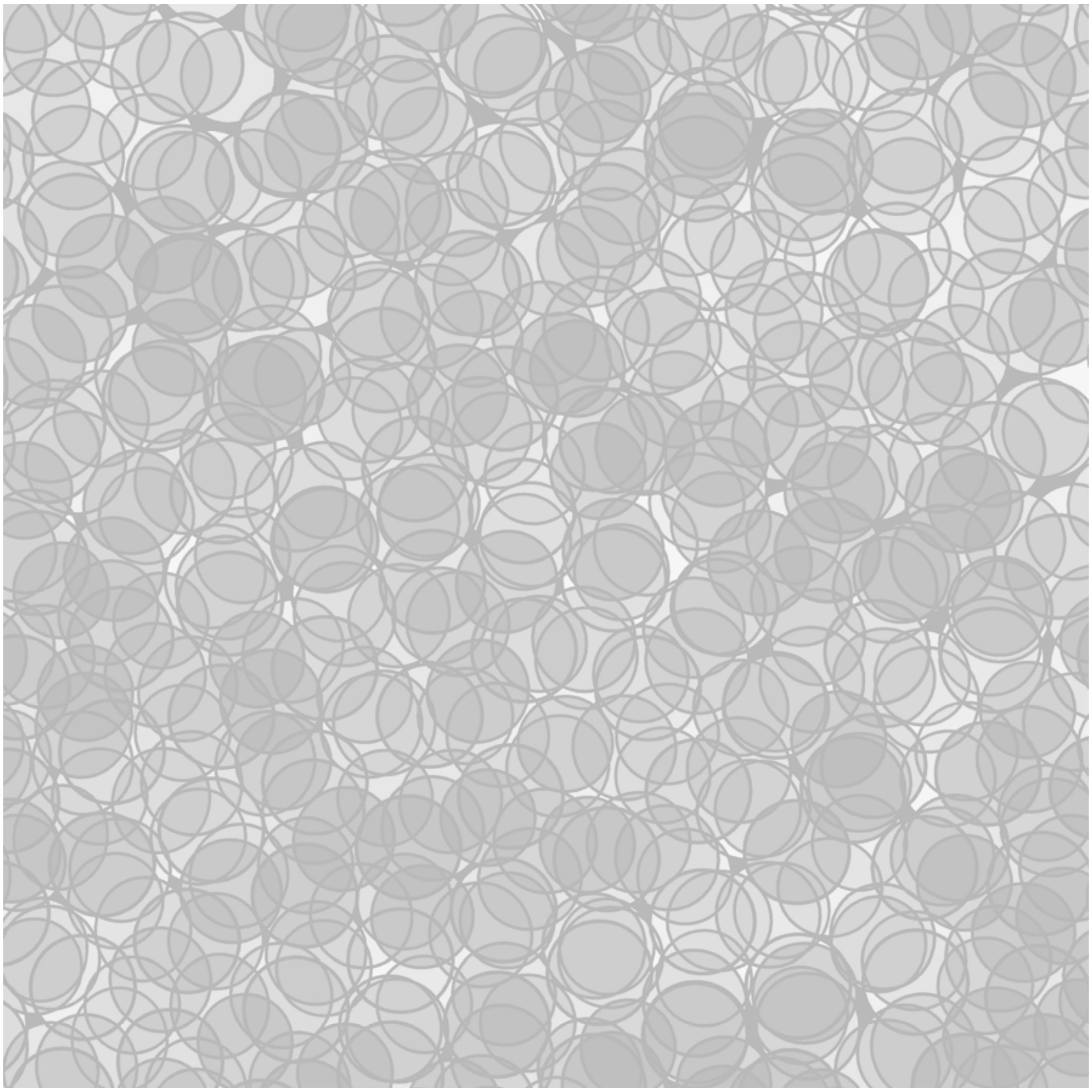

Figure C28.a. Probabilistic empty set automata: local space intersection.

On a continuous surface of fluctuating peaks and valleys of random probability values, a threshold value traces a contour line. Closed contour lines define boundaries of discrete threshold-value neighborhoods, constructing an array of local independent probability spaces.

Four continuous surfaces are superimposed, interacting only above the threshold level, to construct an intersecting neighborhood structure of local spaces. (above) Intersections form a union of local space, with the intersecting sub-space forming the sum of independent neighborhood probabilities.

When a union of 4 neighborhoods constructs a complete situated boolean topology, the complete intersection space creates a sum of independent probabilities > 1. This is a phase transition in probability space: a point is formed.

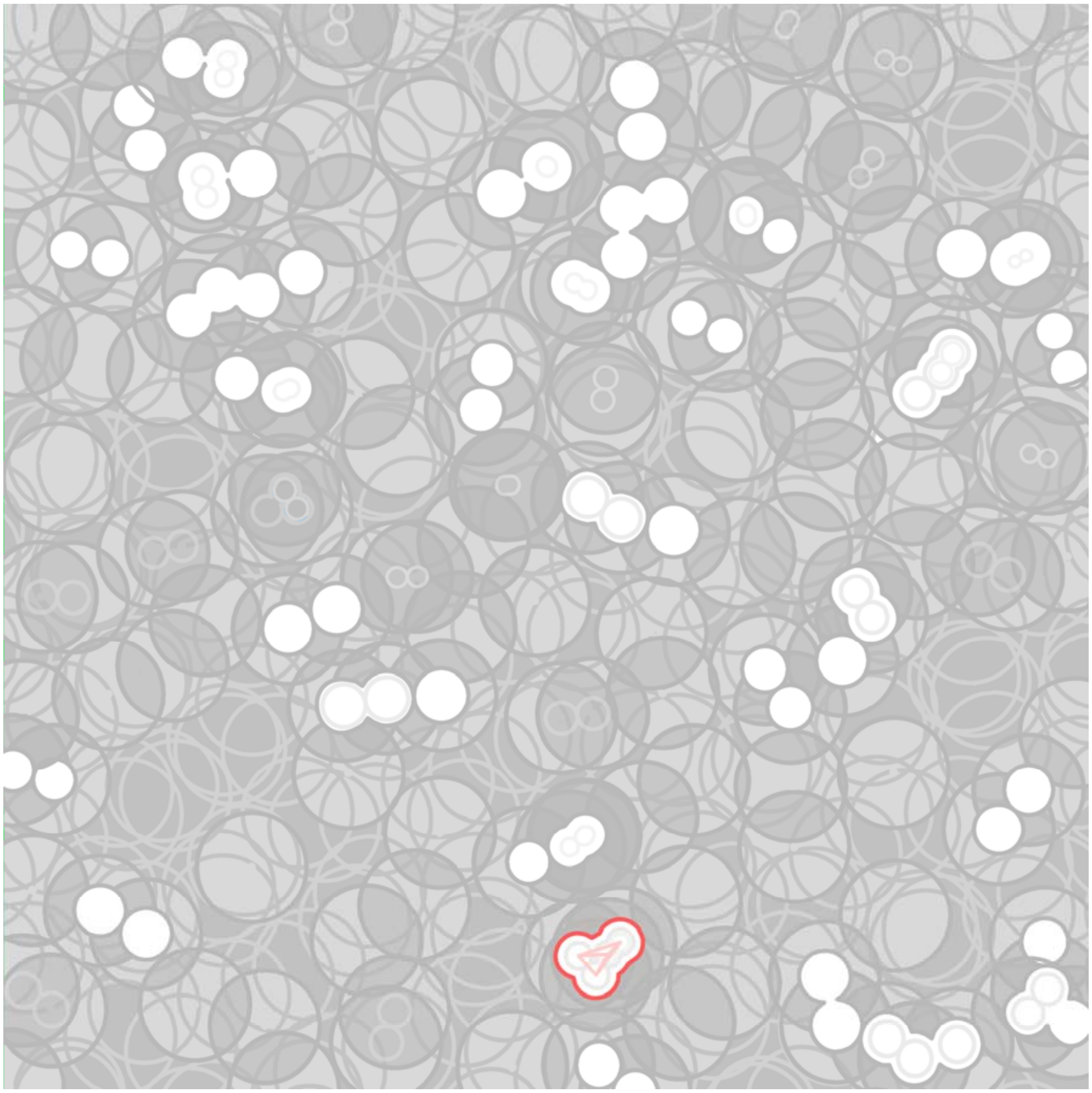

Figure C28.b. Probabilistic empty set automata: local space intersection. (continued)

Critical state of intersecting local probability spaces.

Formation of a complete Boolean topology in a union of local spaces (red) produces a point formation probability >1. Probability threshold neighborhoods (white); discrete-state chronology (light gray neighborhoods).

C5. MODELING PERIODIC STRUCTURE

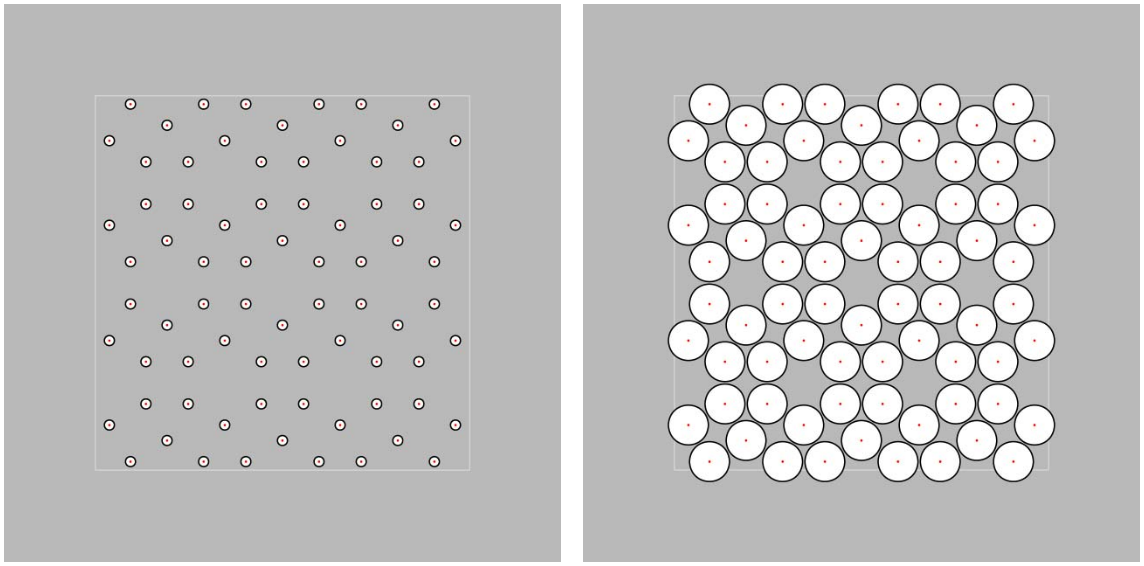

Figure C29.a. Discrete-event development of periodic finite topological space.

Left: initial state of situated set. Right: disjoint local spaces developing from situated set; local fundamental group [1] boundary (black).

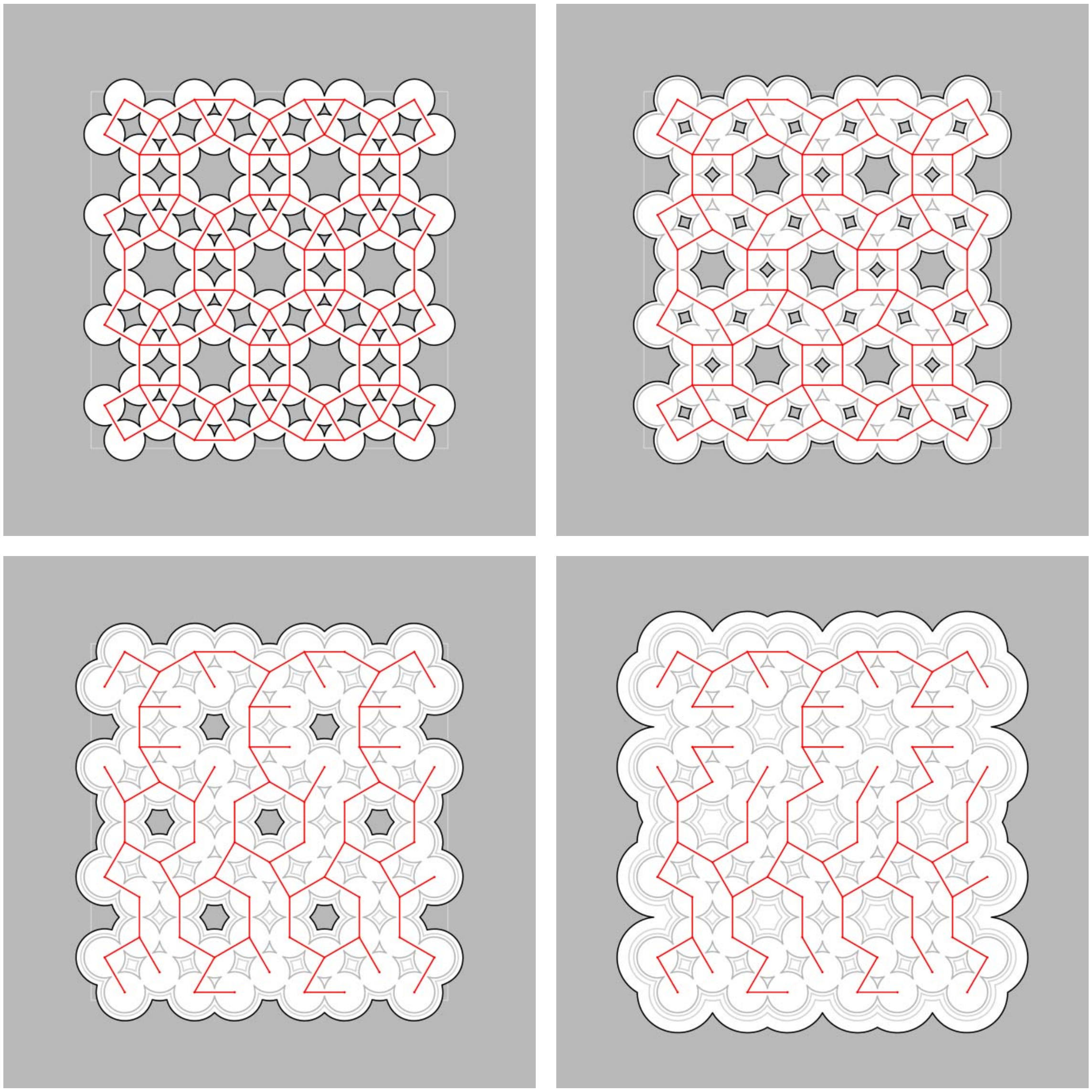

Figure C29.b. Discrete-event development of periodic finite topological space. (continued)

Single finite topological space. Top left: fundamental group boundary (black); hierarchical discrete-event chronology (light gray neighborhoods); minimum spanning ring (red) constructs tesselation 3.4.6.4. Top right, bottom left: fundamental group boundary (black); hierarchical discrete-event chronology (light gray neighborhoods); one member of the set of minimum spanning rings (red). Bottom right: fundamental group [1] boundary (black); discrete-state chronology (light gray neighborhoods); one member of the set of minimum spanning trees (red).

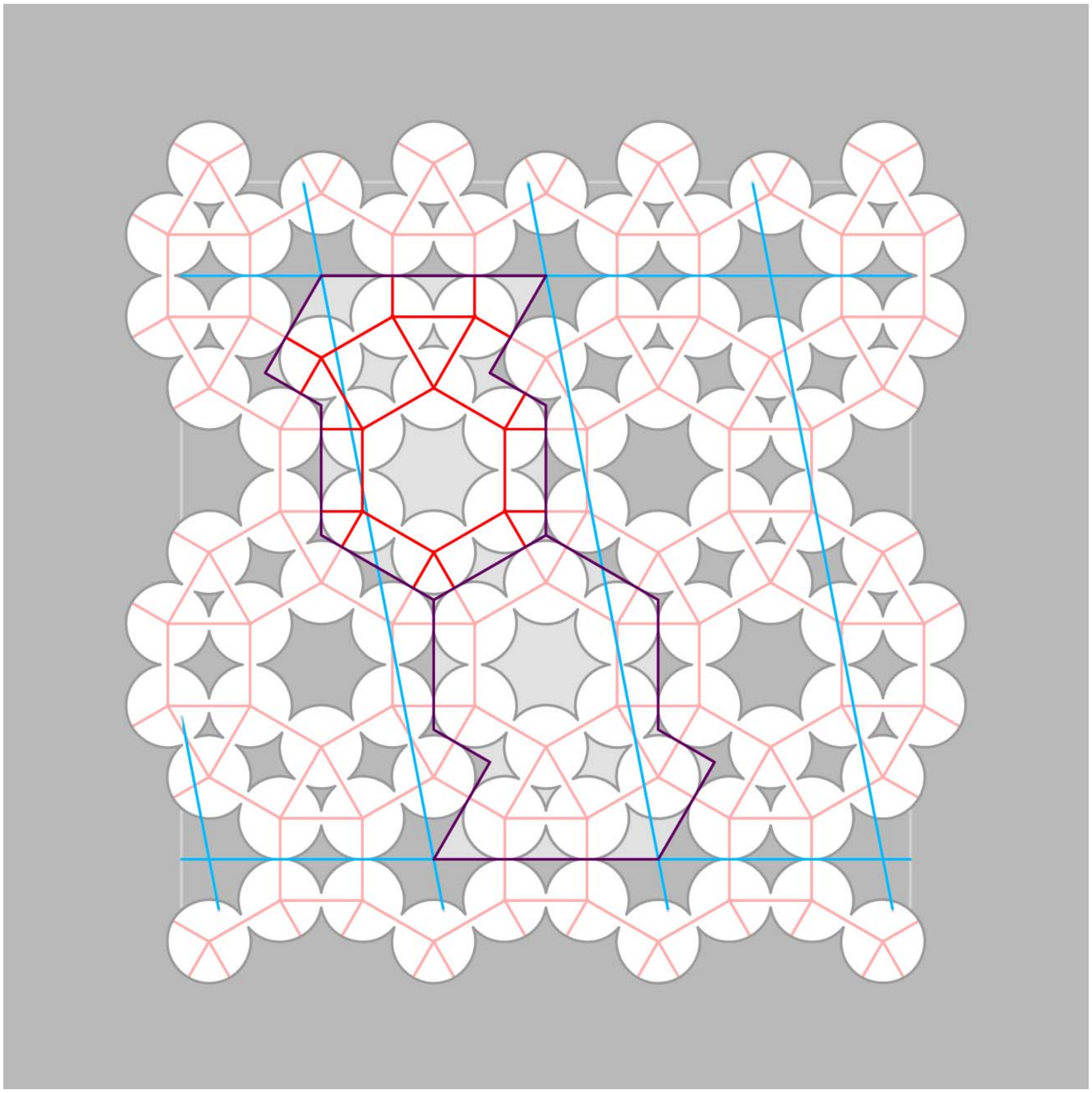

Figure C30. Topological unit cell of infinite periodic local space.

Unit cell boundary (purple); local fundamental group [4] boundary (dark gray neighborhoods); local minimum spanning ring (red).

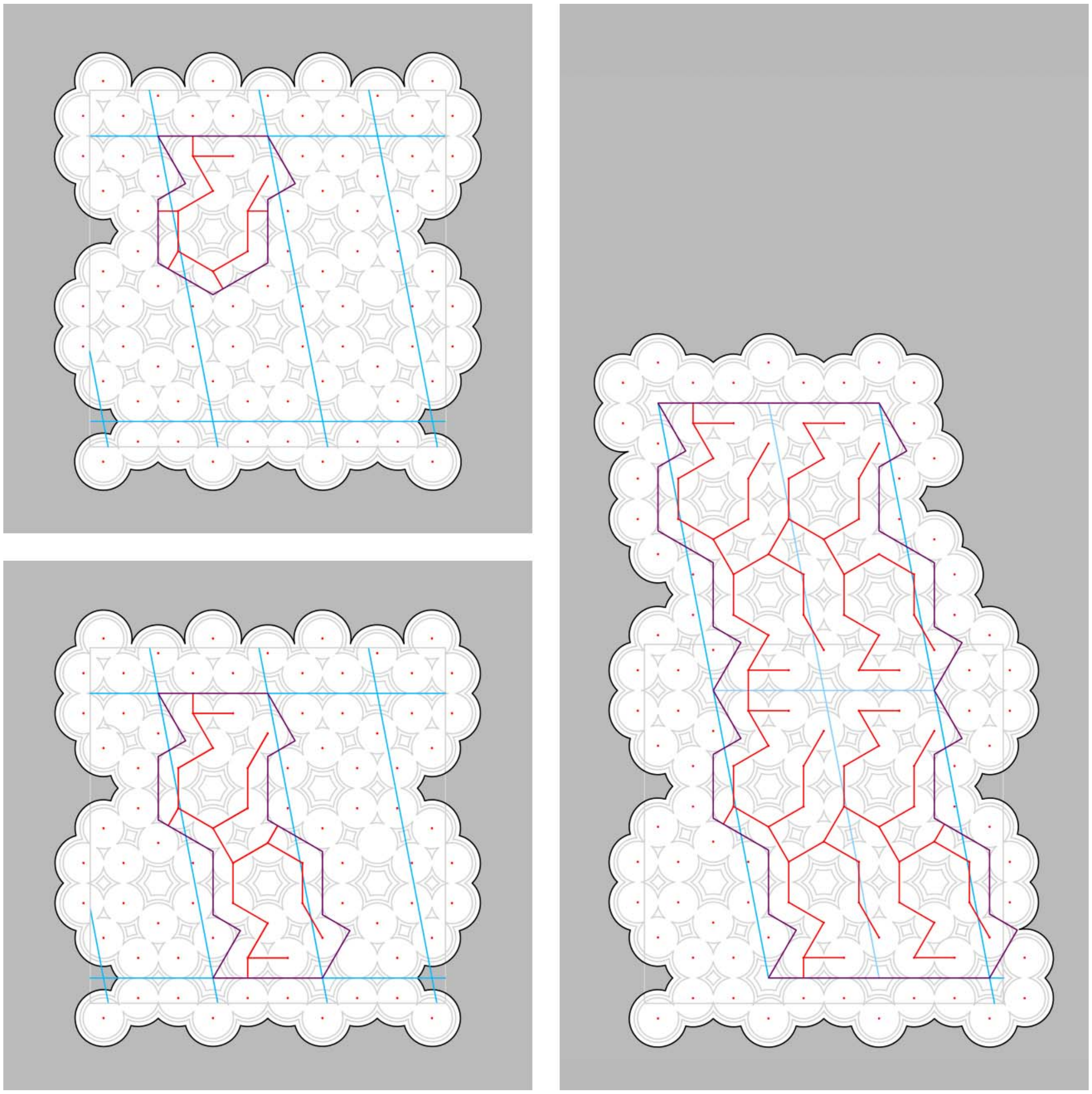

Figure C31.a. Development of Zeno's infinite minimum spanning tree.

Recursive development of one member of a set of Zeno's infinite minimum spanning trees (red); topological unit cell (purple).

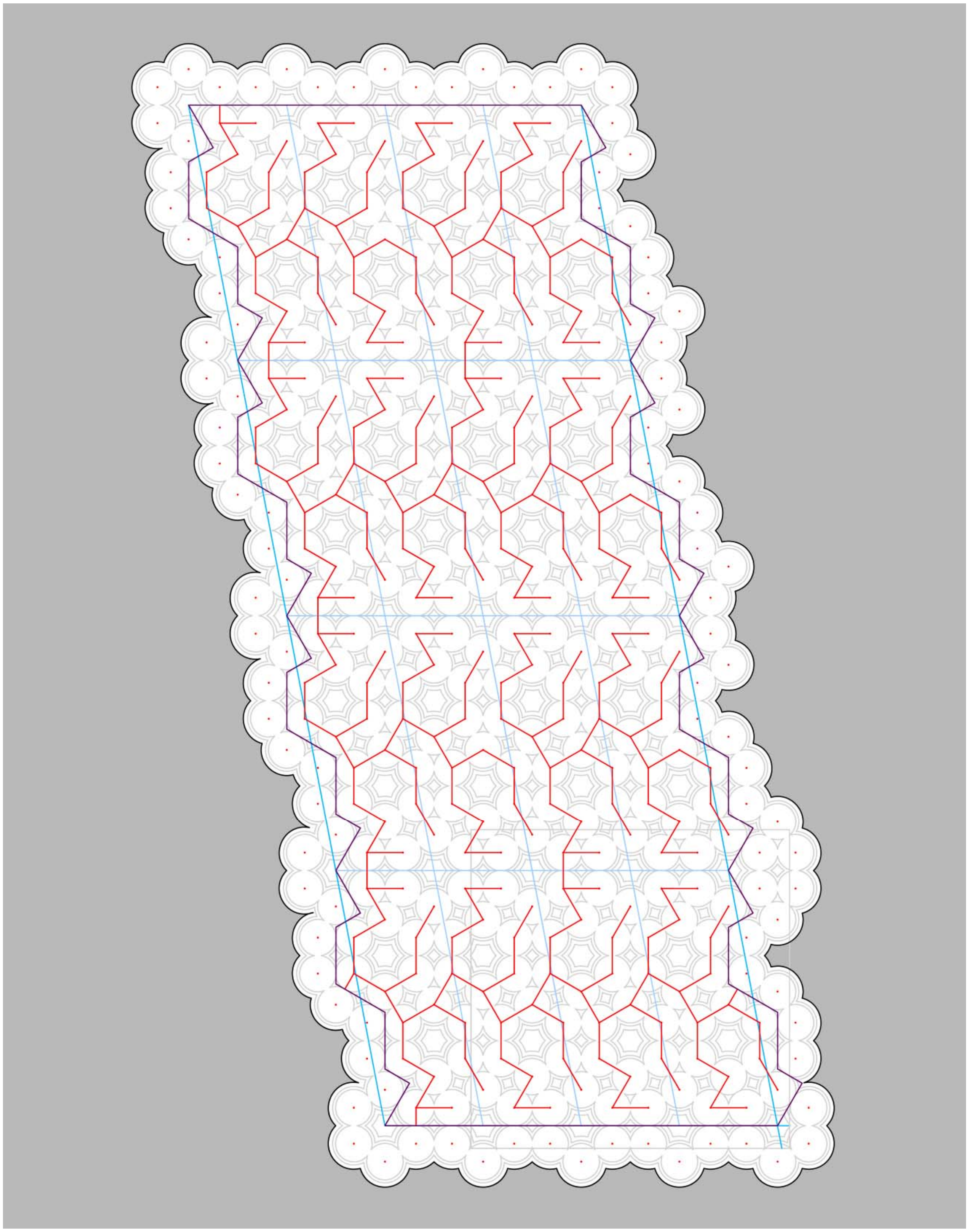

Figure C31.b. Development of Zeno's infinite minimum spanning tree. (continued)

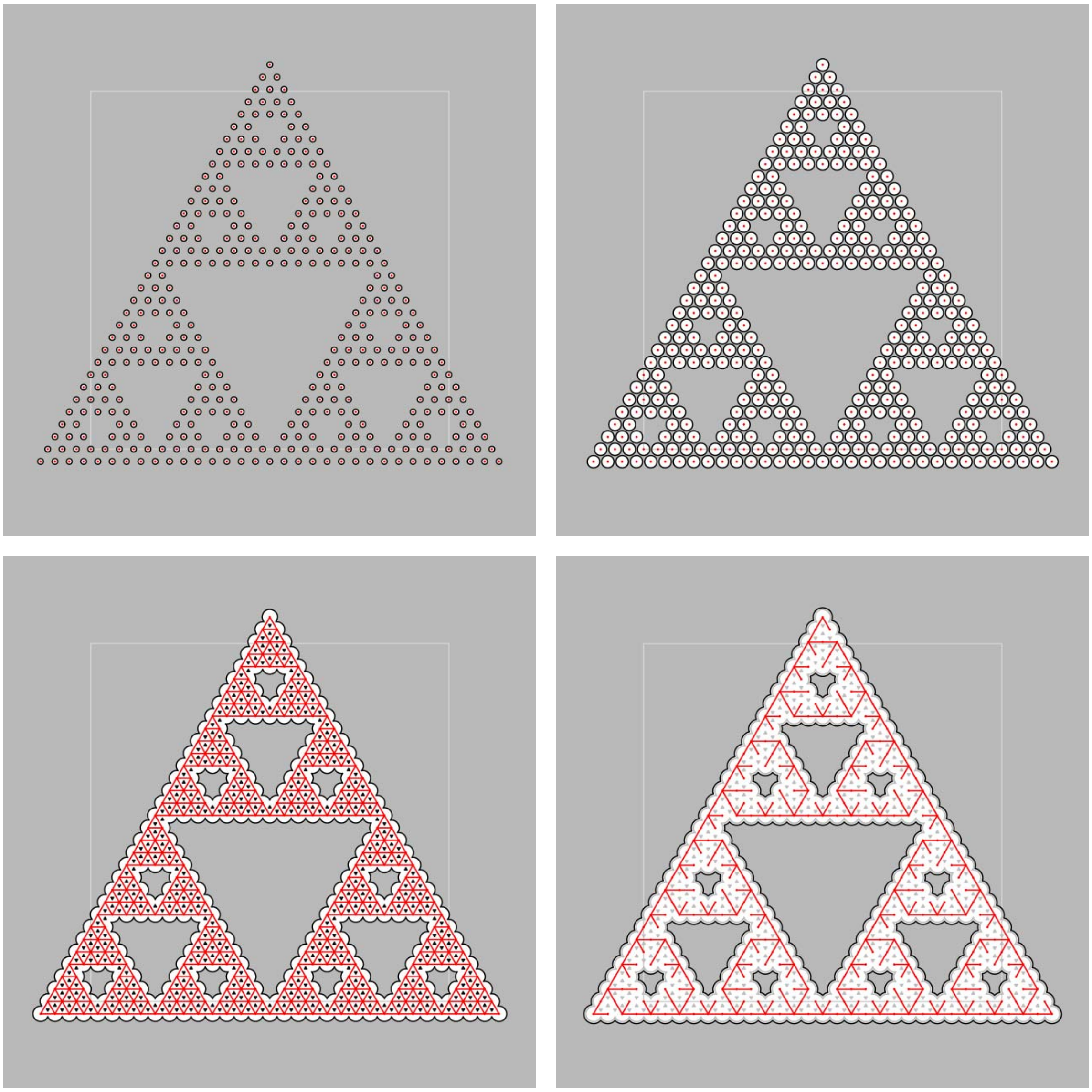

Figure C32.a. Development of hierarchically self-similar finite topological space.

Top left: initial state of situated set. Top right: local spaces developing from situated set; local fundamental group [1] boundary (black). Bottom left: fundamental group [14] boundary (black); minimum spanning ring (red). Bottom right: one member of the set of minimum spanning rings (red); fundamental group [14] boundary (black); discrete-state chronology (light gray neighborhoods).

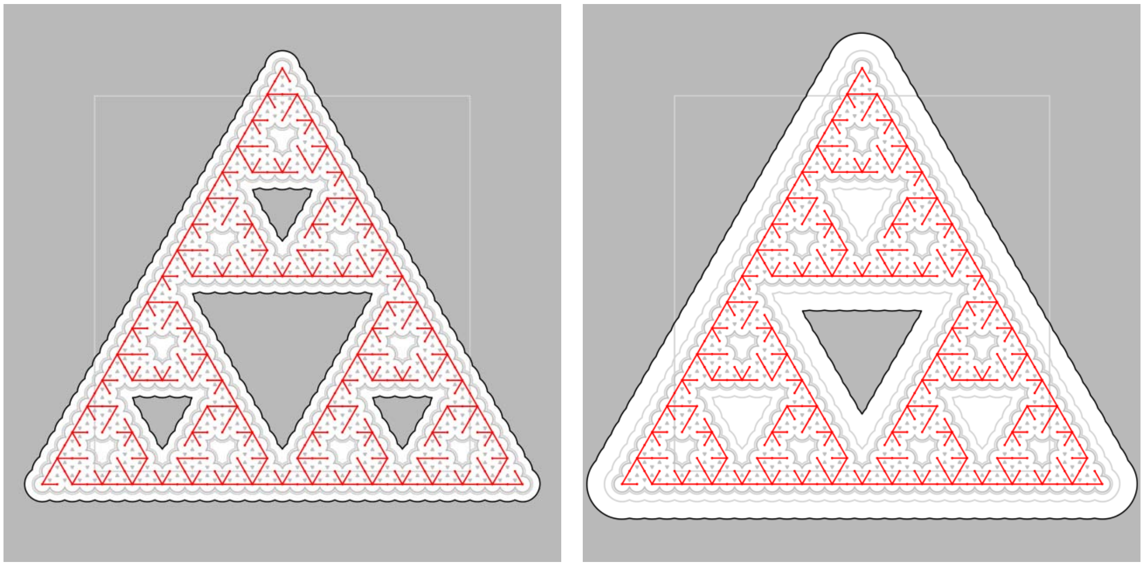

Figure C32.b. Development of hierarchically self-similar finite topological space. (continued)

Left: fundamental group [5] boundary (black); discrete-state chronology (light gray neighborhoods); one member of the set of minimum spanning rings (red). Right: fundamental group [2] boundary (black); discrete-state chronology (light gray neighborhoods); one member of the set of minimum spanning rings (red).

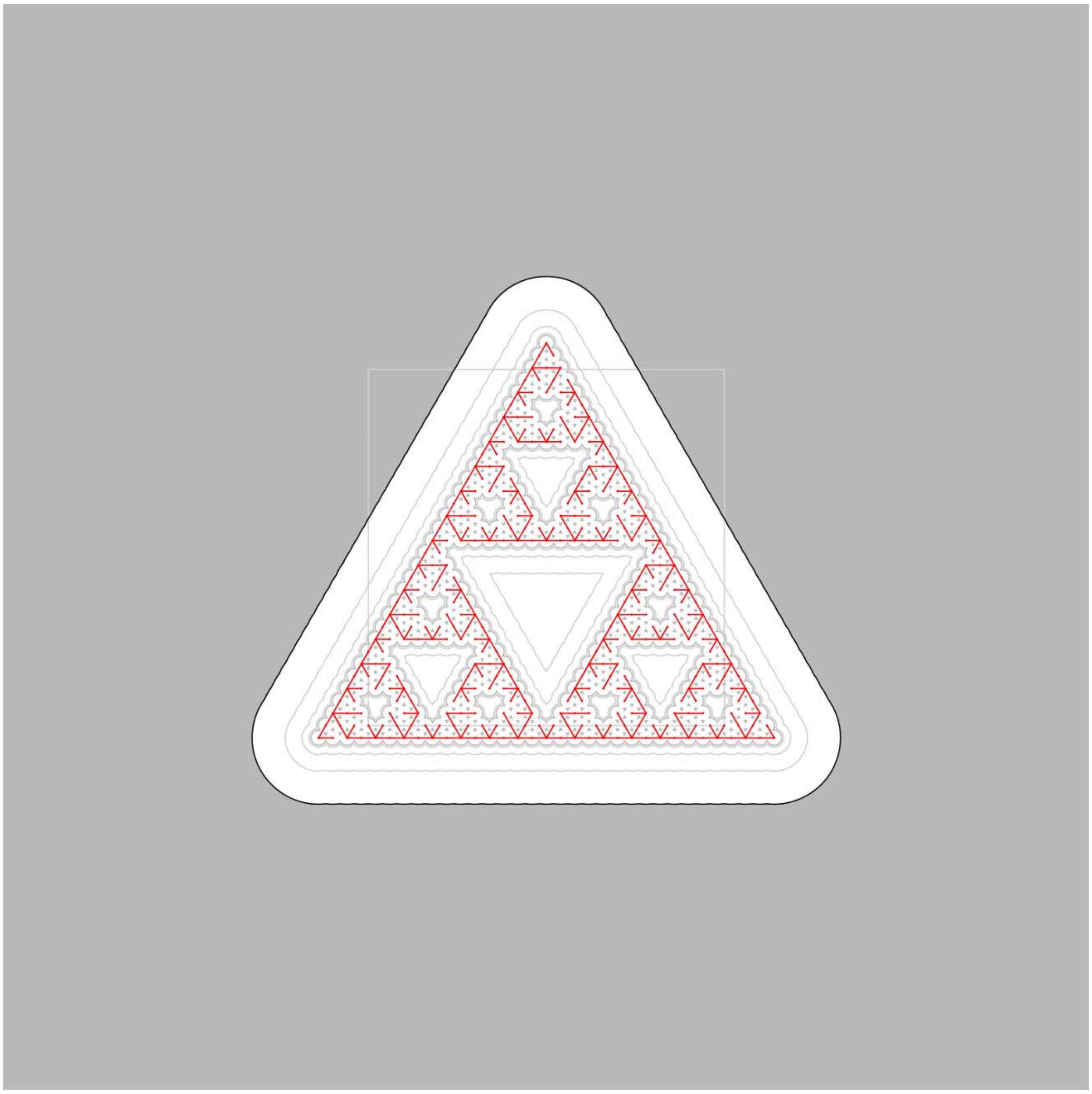

Figure C32.c. Development of hierarchically self-similar finite topological space. (continued)

Terminal hierarchical connectivity and morphological development. Fundamental group [1] boundary (black); discrete-state chronology (light gray neighborhoods); one member of the set of minimum spanning rings (red).