PARAMETRIC STUDY OF ⁹⁹MO PRODUCTION USING A SUB-CRITICAL LOW ENRICHED URANIUM ASSEMBLY DESIGN PROPOSED BY NIOWAVE INC

A Thesis

by

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ABSTRACT

The radioisotope, technetium-99m (t_{1/2}=6 hours) is used in over 80% of diagnostic medical imaging and is the daughter product from the radioactive decay of the isotope, molybdenum-99 (t_{1/2}=66 hours). ⁹⁹Mo is a fission product, with a fission yield of 6.1%, and therefore can be produced by nuclear reactors. While ⁹⁹Mo has been produced using highly enriched uranium (HEU), there is an international interest to produce this isotope using low enriched uranium (LEU) due to the nuclear proliferation concerns of HEU. Niowave Inc. is a facility that has plans to produce ⁹⁹Mo in the United States. The production of ⁹⁹Mo in the US ensures its seamless availability to benefit the people who need ^{99m}Tc based medical diagnostics in the country.

⁹⁹Mo production was studied for an electron beam and sub-critical LEU assembly design proposed by Niowave Inc. by applying Monte Carlo radiation transport and coupled isotope generation-depletion calculations. In addition, the production of ¹³⁵Xe, ¹³⁵I ¹³¹I, ²³⁹Pu, ¹⁰⁵Ru and ¹⁰⁵Rh were also investigated. The Niowave design was studied by varying neutron moderators in the sub-critical system and LEU enrichment to predict optimal production of ⁹⁹Mo and other radioisotopes products of interest. The neutron moderators that were considered for this study are light water, heavy water and beryllium. ⁹⁹Mo production rate

was studied, the predicted value for this study is ~9 kCi per week with a 235 U enrichment of 10% and light water as the neutron moderator. This amount of 99 Mo production could meet 12% of the US demand from one production facility.

Studies found that water is the best neutron moderator for the current design to maximize the production of ⁹⁹Mo. The light-water-moderated system achieves highest criticality level as well as a highest thermal neutron flux and power, when compared to the other two candidates. Heavy water is a better neutron moderator than beryllium for the current design, however, it is not as good as water. Even at 19.9% enriched fuel, heavy water and beryllium do not achieve the neutron flux, power or ⁹⁹Mo production levels when water is used moderator in the system. In conclusion, the studies conducted found that water is the best moderator candidate for this design, maximizing the production of ⁹⁹Mo.

I dedicate this work to my family for their constant support and for believing in me. This work is specially dedicated to my mother for everything that she has done, to serve as an inspiration, role model and instilling in me the ability to dream and shoot for the stars. Mother, I appreciate everything that you do for me and I thank you for always being my rock, you will always be the person who I admire and look up to. Thank you for all that you are to me.

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TABLE OF CONTENTS

| ABSTRACT | ii |
|--|--|
| ACKNOWLEDGEMENTS | v |
| TABLE OF CONTENTS | vi |
| LIST OF FIGURES | viii |
| LIST OF TABLES | ix |
| CHAPTER I INTRODUCTION AND LITERATURE REVIEW | 1 |
| I.A Motivation I.B Production of ⁹⁹ Mo Stages I.C Previous Work I.D Nuclear Safeguards Benefits I.E Legislature I.F Significance of Research | 1 9 .11 .13 .14 |
| CHAPTER II METHODOLOGY | .16 |
| II.A Monte Carlo N-Particle Radiation Transport and Depletion Calculations. II.B Niowave Inc. Design Parameters II.C Niowave Inc. Design Geometry II.D Computational Procedure. | .16 .19 .20 .24 |
| CHAPTER III RESULTS AND DISCUSSION | .31 |
| III.A Neutron Flux and Core Power Calculations III.B Criticality Study III.C Fuel Depletion Study III.D Uranium Fuel Enrichment Parametric Studies III.E Flux and Power Calculations for a 19.9% Enriched Metallic Fuel III.F Depletion Analysis Using 19.9% Enriched Metallic Fuel | .31 .36 .37 .55 .57 .59 |
| CHAPTER IV CONCLUSION AND FUTURE WORK | .65 |

| REFERENCES | 69 |
|----------------|----|
| APPENDIX I. | 73 |
| APPENDIX II. | 77 |
| APPENDIX III. | 81 |
| APPENDIX IV | 85 |
| APPENDIX V | 89 |
| APPENDIX VI | 92 |
| APPENDIX VII. | 95 |
| APPENDIX VIII. | 99 |

LIST OF FIGURES

| P | age |
|--|---------|
| Figure 1 World-Wide Demand for ⁹⁹ Mo [5] | 4 |
| Figure 2 Stages of the ⁹⁹ Mo/ ^{99m} Tc Processes[9] | 7 |
| Figure 3 The Electron LINAC Schematics Used by Niowave Inc [3] | 19 |
| Figure 4 Axial and Cross Sectional Views of the Core Region | 22 |
| Figure 5 Niowave Inc. View of Design[3] | 23 |
| Figure 6 ²³⁵ U Fission Cross Section [20] | 25 |
| Figure 7 ²³⁸ U Fission Cross Section [20] | 25 |
| Figure 8 Six Isotopes Production and Depletion Study for Water Moderator: A) ¹³⁵ Xe B) ¹³¹ I C) ¹⁰⁵ Ru D) ¹⁰⁵ Rh E) ¹³⁵ I and F) ²³⁹ Pu | 39 |
| Figure 9 ⁹⁹ Mo Production and Depletion Studies for Water Moderator for the Entire System. | 44 |
| Figure 10 ⁹⁹ Mo Production and Depletion Studies for Water Moderator for the Entire System using Burnup in the X-Axis | , 45 |
| Figure 11 Total Core ⁹⁹ Mo Production for a Heavy Water Moderator | 50 |
| Figure 12 ⁹⁹ Mo Production for the Beryllium System | 52 |
| Figure 13 ⁹⁹ Mo Radioactivity Values for Heavy Water Moderated System Witl 19.9% Enriched Fuel. | h 60 |
| Figure 14 ⁹⁹ Mo Radioactivity Values for Beryllium Moderated System With 19.9% Enriched Fuel | 63 |

LIST OF TABLES

| Page |
|------|
|------|

| Table 1 Irradiator Facilities Worldwide That Produce ⁹⁹ Mo [5] 2 |
|--|
| Table 2 IAEA List of Significant Quantity of Nuclear Material [2, 4]12 |
| Table 3 Particle Energy Ranges for Monte Carlo Methods[16] 18 |
| Table 4 Details of a LEU Fuel Pin [3] 21 |
| Table 5 Details of the LEU Sub-critical Core 22 |
| Table 6 Moderator Parameters [21] |
| Table 7 Slowing Down Parameters for Moderator Candidates [22-24] |
| Table 8 Half-Lives of Radioisotopes of Interest in This Study 29 |
| Table 9 Estimated Neutron Flux Values Obtained From Each MCNPX Calculation for Three Moderators |
| Table 10 Estimated Sub-Critical Core Thermal Power Values Obtained FromEach MCNPX Calculation for Each Moderator33 |
| Table 11 Power Calculation Results Based on the Reaction Rate |
| Table 12 Effective Neutron Multiplication Factor Values Obtained From MCNPX Criticality Calculations for Each Moderator With 10% 235 U Enriched Metallic Fuel |
| Table 13 ²³⁹ Pu Mass for 10% Enriched Fuel in the Water Moderated System43 |
| Table 14 Results of Fuel Production and Depletion Studies for the Entire Core With Water Moderator 48 |
| Table 15 Results of Fuel Production and Depletion Studies with Heavy Water Moderated Core |
| Table 16 Results of Fuel Production and Depletion Studies with Beryllium Moderator |

| Table 17 99 Mo Production for Each Moderator System | 54 |
|--|----|
| Table 18 Variation of K _{eff} as Function of Increased Uranium Enrichment for Heavy Water Moderated System | 56 |
| Table 19 Variation of K _{eff} as Function of Increased Uranium Enrichment forBeryllium Moderated System | 56 |
| Table 20 Neutron Flux Analysis for the 19.9% Enriched System | 58 |
| Table 21 Estimated Power for the 19.9% Enriched System | 58 |
| Table 22 Depletion Studies with Heavy Water Moderator and 19.9% Fuel Enrichment | 62 |
| Table 23 Depletion Studies with Beryllium Moderator and 19.9% Fuel Enrichment | 64 |

CHAPTER I

INTRODUCTION AND LITERATURE REVIEW

I.A Motivation

Molybdenum-99 (⁹⁹Mo) is a radioisotope of great interest in nuclear medicine. Over 80% of nuclear medical imaging for diagnostics purposes use ^{99m}Tc, which is the radioactive decay product of ⁹⁹Mo. [1, 2] ⁹⁹Mo has a half-life of 66 hours and beta decays into ^{99m}Tc. ^{99m}Tc has 6 hour half-life which makes this isotope a good candidate in nuclear medicine.[3] Not only is the short half-life of this isotope of benefit in use in medical procedures, ^{99m}Tc is also a pure gamma emitter with an energy of 140 keV which makes it ideal because less radiation dose is received by the patient as part of the medical diagnostic procedure.

⁹⁹Mo is produced through nuclear fission and, therefore, can be produced by nuclear reactors or other methods that employs fission. While in the past, ⁹⁹Mo has been manufactured using highly enriched uranium (HEU), there is an international interest to produce the isotope using low enriched uranium (LEU) because of the nuclear proliferations concerns of HEU. The International Atomic Energy Agency (IAEA) defines HEU as containing ²³⁵U in concentrations greater

or equal to 20%. [4] Countries who have produced ⁹⁹Mo are Canada, Netherlands, Belgium, South Africa, France, Poland, Russia among others.[1] Current production facilities are shown in Table 1.

| Deaster/Leastion | Torgoto | Normal | Normal | Detential | Entimoted |
|------------------|---------|-----------|--------------|------------|------------|
| Reactor/Location | Targets | Normal | Normai | Polenilar | Estimated |
| | | Operating | Available | Annual | Stop |
| | | Days | Capacity | Production | Production |
| | | | per week (6- | (6-day Ci) | Rate |
| | | | day Ci) | | |
| BR-2 (Belgium) | HEU | 140 | 7800 | 156000 | 2026 |
| HFR | HEU | 280 | 4680 | 187200 | 2022 |
| (Netherlands) | | | | | |
| LVR-115 (Czech | HEU | 200 | 2800 | 80000 | 2028 |
| Republic) | | | | | |
| MARIA (Poland) | HEU | 165 | 1920 | 42500 | 2030 |
| NRU (Canada) | HEU | 300 | 4680 | 200600 | 2016 |
| OPAL (Australia) | HEU | 290 | 1000 | 41450 | >2030 |
| OSIRIS (France) | HEU | 200 | 1200 | 34300 | 2018 |
| RA-3 (Argentina) | HEU | 336 | 400 | 19200 | 2027 |
| SAFARi-1 | HEU/LEU | 305 | 3000 | 130700 | 2025 |
| (South Africa) | | | | | |

| Table 1 Irradiator Facilities Worldwide That Produce ³³ N | lο | [5] |
|--|----|-----|
|--|----|-----|

Russia has recently become a leading power in the production of ⁹⁹Mo.[5] The country aims to control 20% of the market. However, Russia is also using HEU targets and fuel for irradiator reactors. There are currently four reactors producing ⁹⁹Mo (NIIAR, TPU, NIFKhI, KIR) in Russia. Two of the facilities use HEU for reactor fuel while the other two use HEU targets. However, with the impetus on using LEU instead of HEU, it will be difficult for Russia to sustain the

⁹⁹Mo production using HEU. In addition, two of the four facilities in Russia (KRI and TPU) do not have the capability to ship internationally. [5] The United States has supported Russia in this transition of using LEU instead of HEU for ⁹⁹Mo production. However, for the United States, the majority of these facilities are overseas, making shipment a major disadvantage in the production and use of the isotope.

Canada is currently producing ⁹⁹Mo and selling it to the United States. However, during shutdowns of the facility the United States struggles to maintain the supplies required for treatment. In 2009, there was an isotope supply crisis caused by a shutdown of the ⁹⁹Mo production facility in Canada, National Research Universal, NRU. This reactor is over 52 years old and operates for 300 days in a cycle. [6, 7] Member countries created the OECD's Nuclear Energy Agency after this event occurred, however shutdown of the facility will continue to be an issue. One of the major issues with Canada's ⁹⁹Mo production is the use of HEU fuel. The United States is trying to move away from the use of HEU fuel due to the proliferation concerns. In addition, expansion of ⁹⁹Mo capacity is required in order to sustain the demand in the current industry. [6] Shipment and processing times continue to be an issue when supplying ⁹⁹Mo.

Every year, there are over 17 million people diagnosed or treated using radiopharmaceuticals and radioisotopes.[5] This accounts for about 30 million

3

medical exams conducted annually. The United States is the major purchaser of ⁹⁹Mo, with an estimated annual growth rate of 3 to 5%.[8] Almost half (46%) of the procedures, conducted using ⁹⁹Mo, are conducted in the United States. [5] Figure 1 shows a breakdown of the demand worldwide.



Figure 1 World-Wide Demand for ⁹⁹Mo [5]

There is a loss of 27 to 34% in ⁹⁹Mo radioactivity due to its relatively short halflife and the time needed for production and shipment. This takes into account a 30 to 40-hour time requirement for chemical processing. In addition, only about 90% of ⁹⁹Mo is recovered from chemical extraction. For example, to have 1 Ci remaining after six days of decay a producer must ship 4.54 Ci of material. [5] Due to the loss of time between the chemical processing and shipment, considerable amounts of material is lost. Due to the short half-life, the isotope cannot be stockpiled and must be replaced weekly if not sooner. The current demand of ⁹⁹Mo is 10,000 six-day curies per week. A six-day curie is defined as the amount of material required being present following 6 days after the production process has concluded.[7] The amount needed to be produced by a facility will be greatly affected by the technology and time required to process the fuel following irradiation. While this can be done in as little as 2 days, the process can take up to 8 days depending on the technology available, the location between facilities and any other factor that will involve time.[5, 7, 9, 10]

A total of 160000 Ci of ⁹⁹Mo needs to be produced to meet the six day demand world-wide, assuming a 5 day time period is allocated for the time needed from the end of irradiation to processing.[5, 7] In order to meet the demand of the United States, a total of 73600 Ci of ⁹⁹Mo needs to be produced. Again, this value will be affected by time it takes to process the material and the time allocated to wait post irradiation. The time frames can be as little as 36 to 48 hours, but can also be days depending on the facility. [9] A five-day allocation time was assumed as an average between the 8 and 2 day time frames reported. This urges the production of ⁹⁹Mo to be within the United States.

5

Hence the motivations for this study are two fold; 1) study and analyze a methodology for ⁹⁹Mo production which uses LEU to aid in the reduction of nuclear proliferation concerns and 2) study the efficiency of ⁹⁹Mo production for the case in which it is produced in the United States.

I.B Production of ⁹⁹Mo Stages

⁹⁹Mo production process includes the point from which the material has stopped being irradiated and injected into patients using ^{99m}Tc pharmaceuticals. The raw material from the reactor is separated and purified. Figure 2 illustrates several stages of the general production of ^{99m}Tc using HEU targets from reactors as previously conducted by countries like Canada. The time frames will vary depending on the facility and technology available. For other systems, the process will be similar to that illustrated.



Figure 2 Stages of the ⁹⁹Mo/^{99m}Tc Processes[9]

The final product is tested at the end of each stage for quality control. After quality control tests have been passed ^{99m}Tc solutions are added to vials or kits containing non-radioactive components of the radiopharmaceutical. At the end

of this stage, further quality control tests are conducted. The final product in the form of ^{99m}Tc has a shelf life of 6 to 12 hours before being administrated in the patient specific unit dose and shipped for transportation. Once the facility receives the package, the package is surveyed for contamination, and if approved, each dose is prepared for administration to the patient.[9]

There are several other methods to produce ⁹⁹Mo. These processes are listed below:

- The neutron-capture process: an intense neutron beam generated by a nuclear reactor adds one neutron to a ⁹⁸Mo target to produce ⁹⁹Mo. [9]
- The photo-neutron process: an intense photon beam generated by an electron accelerator removes a neutron from a ¹⁰⁰Mo target to produce ⁹⁹Mo.[9]
- The photo-fission process: a very intense photon beam generated by an electron accelerator causes a uranium target to fission to produce ⁹⁹Mo.[9]
- The neutron-fission process: an intense neutron beam generated from a nuclear reactor strikes uranium, producing ⁹⁹Mo with a 6.1% yield from the fission reactions.[9]

Each method has its own advantages and disadvantages. For example, in neutron capture of ${}^{98}Mo(n, \gamma){}^{99}Mo$ there is nearly no waste stream, however, separation techniques will need to be developed for high volume application. In

8

the photo-neutron process, there is low waste stream, higher predictability of costs, schedule and licensing in comparison to a reactor. In addition, these facilities can be built in small sizes if low power is applicable. The major disadvantages to this process are the higher cost and design of targets along with an expected higher cost for the separation process of ⁹⁹Mo. The third process is photo-fission of ²³⁸U(γ ,f)⁹⁹Mo. About 6.1% of fissions yield ⁹⁹Mo. The major advantage of this process is the lower cost of ²³⁸U. Existing processing techniques can be used to recover the isotope. Disadvantages to this method would be the production of ²³⁹Pu, while it will be low amounts, it will need to be considered. In addition, the facility would have to operate similar to hot-cell which will increase the cost.[9]

I.C Previous Work

There have been several studies conducted to seek better understanding on the production of ⁹⁹Mo and alternatives to produce this isotope. Since 2007, the world supply of ⁹⁹Mo has suffered shortages due to shutdowns and outages in other countries.[11] There have been several designs proposed such as the Aqueous Homogeneous Reactor, AHR. The AHR is a pool type reactor capable of ⁹⁹Mo production (275 Ci/week for six day irradiation).[11] The design only operates for 21 weeks. The reactor has been proposed for two different types of

fuel, UO₂(NO₃)₂ and UO₂SO₄, both operating using LEU fuel.[11] One of the advantages of this design is the minimal loss from ⁹⁹Mo decay by continuous extraction of ⁹⁹Mo. The power can also be varied depending on the demand for ⁹⁹Mo.[11, 12] In addition, Coqui Radio-pharmaceuticals Corporation has also developed a reactor design. The reactor is a pool type reactor operating at 10 MW that uses LEU fuel and light water coolant.[12] Both of these reactors operate at low power and will be solely used for the production of medical isotopes. The use of LEU will decrease the amount of ²³⁵U in a facility providing a safeguards advantage to promote the international non-proliferation objectives. However, while the amount of ²³⁵U is reduced, the amount of ²³⁹Pu production will increase through irradiation. Studies have concluded the amount of plutonium production will be small and will pose less of a safeguard's threat than the HEU targets.[13]

Another method for production being explored is a particle accelerator-based production. These systems rely on converting targets to produce ⁹⁹Mo. In these systems, high intensity electron beams are used to produce photons and then neutrons. The recovery of ⁹⁹Mo is the same of that for HEU making the transition to LEU easier for these systems.[14] These linear accelerators (LINAC) systems are capable of producing ⁹⁹Mo from (γ ,n) reactions using ¹⁰⁰Mo targets. [14] The Canadian design uses 35 MeV electrons to produce photons through bremsstrahlung radiation. The facility is capable of separating and extracting

^{99m}Tc using Na₂MoO₄, making the separation relatively simple when compared to other methods.[14] Most of the LINAC systems use highly pure ¹⁰⁰Mo to produce ⁹⁹Mo.[15] On the other hand, Niowave Inc. (the methodology that was selected for this thesis study) will be using a particle accelerator and converter target to produce a neutron source that will fission LEU targets, which is different from previous work conducted on ⁹⁹Mo production method.

I.D Nuclear Safeguards Benefits

The use of LEU over HEU offers several advantages. The IAEA has set significant quantities for special nuclear materials. A significant quantity is an amount of nuclear material from which the possibility of manufacturing a nuclear explosive cannot be excluded. Significant quantity values take into account losses due to conversion and manufacturing.[4] Table 2 shows the significant quantity values for special nuclear material. For HEU (²³⁵U concentrations equal to or greater than 20%) only 25 kg of ²³⁵U is required to proliferate. However, for LEU fuel (²³⁵U less than 20%) it is required to obtain 75 kg of ²³⁵U for proliferators to have the capability to produce a nuclear weapon. In the process, not only does an LEU facility have less SQ inventory present, the probability of detection of nuclear material diversion by the proliferator is also high as much

larger amount of ²³⁵U is required to weaponize. In addition, the material category changes when using LEU (Indirect Use Nuclear Material) compared to HEU (Direct Use Nuclear Material), which makes the fuel less attractive for an adversary due to the further processing required.[2, 4] The timeliness detection goal also changes from 1 to 12 months, which allows more time to detect and stop a proliferator before they weaponize. For an LEU facility, the proliferator would have to steal numerous pins from the facility in order to have enough material to weaponize.

| Material Category | Material Type | Significant | Timeliness |
|---------------------|---|------------------------|------------------|
| | | Quantities | Goal (months) |
| Direct Use Material | Pu (separated and for Pu | 8 kg Pu | 1 |
| | containing less that 80% Pu) | | |
| | HEU (²³⁵ U enrichment ≥20%) | 25 kg ²³⁵ U | 1 (unirradiated) |
| | | | 3 (irradiated) |
| | Pu in Spent Fuel | 8 kg Pu | 3 |
| | ²³³ U | 8 kg ²³³ U | 1 |
| Indirect Use | LEU (²³⁵ Uenrichment<20%) | 75 kg ²³⁵ U | 12 |
| Material | | | |
| | Th | 20 tons | 12 |

Table 2 IAEA List of Significant Quantity of Nuclear Material [2, 4]

For facilities operating within the United States, Nuclear Regulatory Commission's (NRC) regulations will have to be followed to ensure the facility is proliferation resistant.[4] Overall, an LEU facility such as this one proposed by Niowave Inc. will have less significant quantities present making the facility less attractive to a proliferator.

I.E Legislature

In 2011, the United States Congress passed the American Medical Isotope Act (AMIPA). The act included several goals. One of the primary goals was to "promote the production of ⁹⁹Mo in the United States for medical isotope production, and to condition and phase out the export of highly enriched uranium for the production of medical isotopes".[10] The act allocated the Secretary of Energy 143 million US dollars for the fiscal years of 2011 through 2014. The act also allowed for the Secretary of Energy, to provide assistance to facilities for the development of fuels, targets and processes for the domestic production of ⁹⁹Mo that will not use HEU, unless the reactor already used HEU and relied on that fuel for the production. The lease contracts created also provide that the Secretary of Energy shall have responsibility for the final disposition of the radioactive waste created by any processes that included irradiation, processing, or purification of leased uranium. In addition, the lease also provided

13

compensation in equivalent market amounts for the sale of comparable uranium products.[10] In conclusion, the act aimed to allow the production of ⁹⁹Mo but also reduce proliferation concerns with the use of HEU.

I.F Significance of Research

There is an increased interest to produce ⁹⁹Mo within the United States. In the past years, the production of ⁹⁹Mo has been compromised by plant outages and shutdowns in other countries. Supplying this radionuclide within the United States will ensure that patients are treated in a timely manner. In addition, the production of ⁹⁹Mo within the United States makes the system more economical as losses associated with shipment of the radionuclide are decreased due to lower shipment times.

The production of ⁹⁹Mo is a multibillion-dollar industry, which also helps drive the motivation behind the production of this isotope within the country. There are about 30 million procedures conducted annually in the world. Each procedure averages about 340 dollars, producing 100 billion dollars annually to hospitals and radiopharmaceutical companies. Out of this total 10 billion dollars, only 500 million dollars are spent in the production of ⁹⁹Mo, making the investment of great interest.[3]

Niowave Inc. has recently started producing (small-scale production) ⁹⁹Mo for medical imaging and treatment purposes. The company is one of the first to produce this radionuclide within the United States through the use of a linear accelerator thus helping eliminate the dependency of the United States on foreign production. Niowave Inc., uses an electron accelerator and converter target. The facility uses a Lead Bismuth Eutectic (LBE) Converter to produce photons through bremsstrahlung after electrons hit the LBE.[3] The LBE converter also produces neutrons through (γ, n) reactions from the photons produced, thus providing a neutron source for the system.[3] The neutron source hits the side of the 85 LEU targets (mass of each target is 101.03 g), in a subcritical configuration, causing fissions. In the process, ⁹⁹Mo is produced, as it is one of the fission products. This proposed research is to support Niowave Inc. for a possible optimization of the facility design for ⁹⁹Mo and other radioisotope production. The Niowave facility began small-scale production of ⁹⁹Mo in 2015.

15

CHAPTER II

METHODOLOGY

II.A Monte Carlo N-Particle Radiation Transport and Depletion Calculations

A Monte Carlo Particle Transport code, MCNPX, developed by Los Alamos National Laboratory was used in this study for reactor core modeling and physics simulations of the Niowave subcritical LEU core design. MCNPX is a computer code that solves the linear Boltzmann radiation transport equation and has the capability to represent 3-dimensional complicated geometries as the one needed for this study [16]. Geometry of the problem is modeled using 3-dimensional quadratic equations. The user can use repeated lattice structures and use a variety of sources such as volume (spherical, cylindrical and cartesian), point, K-source and surface source. Importance functions in geometry are given by the user or optimized by the simulation as a function of mean free path thickness of the materials used in the reactor. In addition, MCNPX uses continuous energy interaction cross section. Finally, MCNPX is also capable of calculating several parameters such as particle flux, radiation dose, energy deposition, charge deposition and fission energy deposition among

others. MCNPX is coupled with CINDER90 [16]. Together the codes interact to conduct fuel depletion analysis on materials of choice. The interaction between the two codes is based on flux and reaction rate calculations conducted by MCNPX. When these two parameters are calculated average cross section values are capable of being obtained. The cross sections are transferred to CINDER90 to conduct the depletion at each step. CINDER90 calculates new atom fractions at the end of the burn step which is sent to MCNPX for MCNPX to conduct the new flux and reaction rate calculation. This communication process between the two codes continues for each burn step conducted.

CINDER90 is capable of tracking 3600 fission products. In addition, the code only requires one input deck from the user as opposed to the use of multiple ones as in Monteburns code. The code uses a linear congruential random number generator for its calculations in order to bring in the stochastic nature of solving radiation transport equation. A couple of requirements of the current study were the capability to perform coupled photo-neutron calculation and fuel depletion calculation, both of which are included in MCNPX code. In addition, the code has been benchmarked and is in use for 50 plus years to analyze neutrons, photon and electron particle transport in a variety of applications that includes nuclear reactors, shielding, health physics, nuclear waste, criticality and medical applications among others. Due to these various advantages, MCNPX

17

was selected as the code of choice for these studies. Particle energy ranges that can be simulated in MCNPX are listed in Table 3. [17-19]

| Particle | Energy |
|-----------|-------------------|
| Neutrons | 10e⁻⁵ eV- 150 MeV |
| Photons | 1 keV- 100GeV |
| Electrons | 1keV-1 GeV |

Table 3 Particle Energy Ranges for Monte Carlo Methods[16]

The code simulates the history of a single particle from birth to death (caused by either absorption or loss of the particle) and repeats the simulations for usersupplied number of histories. At the end of the simulation, the code predicts the parameters such as neutron flux, fission product inventories, etc., as an average from the number of histories the code simulated. [17-19]

II.B Niowave Inc. Design Parameters

MCNPX was used to conduct a set of analysis on Niowave Inc.'s ⁹⁹Mo production subcritical reactor core design. As previously mentioned, Niowave Inc. uses a high-energy electron LINAC to produce neutrons and gammas by the impingement of high-energy electrons on a bremsstrahlung production target. Beam details can be found in Figure 3.



Figure 3 The Electron LINAC Schematics Used by Niowave Inc [3]

The beam can produce electrons of energy range 0.5 to 80 MeV and can operate between 1.0 K - 400 kW of power.¹² Niowave Inc. provided specifications regarding the neutron source strength impinging the LEU sub-critical core. Niowave provided a neutron flux of 1.10E15 n/s as the source strength of the

neutron beam. In addition, Niowave Inc. provided the neutron energy hitting the target. The neutron energy was given as 2.0 MeV. [3] Due to the proprietary nature of the work, no other information regarding the electron beam or converter target was provided by the facility.

Niowave Inc. has the ability to operate the core using oxide or metallic LEU fuel. With metallic fuel, a 10% enrichment of 235 U is used. The core operates in a subcritical domain with a maximum neutron multiplication factor, k_{eff} of ~0.95. The neutron beam enters the core on its side.

II.C Niowave Inc. Design Geometry

The LEU core is made up of 85 fuel pins. These pins are surrounded by niobium cladding and submerged in water. The pin details are shown in Table 4. The fuel is metallic uranium. In the original design the ²³⁵U enrichment is 10%, however the enrichment was studied in this design.

| Pin Parameter | Value |
|-----------------------------------|---------|
| Pin Height (cm) | 35.00 |
| Fuel Pin Radius (cm) | 0.2182 |
| Volume per Pin (cm ³) | 5.2351 |
| Mass per pin (g) | 101.037 |
| Fuel Cladding Radius (cm) | 0.3182 |
| Fuel Cladding Thickness (cm) | 0.100 |
| Cladding Height (cm) | 35.00 |

Table 4 Details of a LEU Fuel Pin [3]

As seen in Table 4, fuel pins are relatively small in size and is only 35 cm in height and 0.3182 cm in radius, including the cladding region. The 85 pins are arranged in a hexagonal lattice in the core. The core parameters are shown in Table 5. The core is situated in the center of a pool of light water. A schematic of the core is shown in Figure 4.

| Core Parameters | Value | | |
|--------------------------------|-----------|--|--|
| Core Radius (cm) | 8.80 | | |
| Core Height (cm) | 35.00 | | |
| Mass of Fuel in Total Core (g) | 8588.15 | | |
| Lattice Arrangement | Hexagonal | | |
| Lattice pitch (cm) | 1.76 | | |
| Number of Pins | 85 | | |

Table 5 Details of the LEU Sub-critical Core





Axial View

Cross Sectional View



The core is surrounded by water, including the top and bottom of the fuel pins. The total height of the water in the system is 150.0 cm having a radius of 8.8 cm, same as the core. There is beryllium reflector surrounding the system axially, which is 150 cm height and has a radius of 100 cm. The axial view of the design is shown if Figure 5. The image is not up to scale and should be used only for better understanding of the different regions in the system.



Figure 5 Niowave Inc. View of Design[3]

During the production of ⁹⁹Mo, the neutron source enters the LEU fuel pins on one side of the core in the center axial region of the core. The system does not use pressurized water. Instead, a pool type design is used.

II.D Computational Procedure

The first stage of the research was to better understand the current LEU subcritical core design of Niowave. Criticality calculations were conducted for the system that used non-pressurized water and 10% 235 U LEU metallic fuel. In this study, criticality (or the k_{eff} value) and 235 U enrichment are both limiting parameters based on the licensing regulations in place. The k_{eff} value should be less than 1.0 preferably 0.95 to maintain the sub-critical nature of the core. LEU fuel as noted earlier is defined as fuel with 235 U concentrations less than 20%. The larger the amounts of 235 U enrichment larger will be the production of 99 Mo in thermal neutron energy (0.025 ev) regions, as more fission will occur. Energy dependent neutron cross-section for 235 U and 238 U are shown in Figure 6 and 7. [20]



Figure 6 ²³⁵U Fission Cross Section [20]



Figure 7 ²³⁸U Fission Cross Section [20]

²³⁵U has a higher fission cross section in thermal energy region than ²³⁸U. For this reason, the moderating material plays an important role. The moderator would allow neutrons to slow down from fast energy (when born in fission) to thermal energy and have a higher probability of fission through scattering events within the moderator. [21]

Three different moderators were considered for each study in the Niowave system; light water, heavy water and beryllium. Moderator properties are listed in Table 6 and Table 7.

| | Molecular | Nominal | Microscopic | Microscopic |
|-----------|-----------|---------|-------------------|-------------------|
| Moderator | Weight | Density | absorption cross- | Scattering cross- |
| | (g/mole) | (g/cc) | section (barns) | section (barns) |
| Water | 18.015 | 1.0 | 0.6640 | 103.0 |
| Heavy | 20.027 | 1 10 | 0.0013 | 13.6 |
| Water | 20.021 | 1.10 | 0.0010 | 10.0 |
| Beryllium | 9.012 | 1.85 | 0.0092 | 6.1 |

 Table 6 Moderator Parameters [21]

**Cross-section values were measured in thermal neutron energy 0.025 eV.
| Moderator | £ | Number of collisions from 2 MeV to 1 eV | Σ_{s} (cm ⁻¹) |
|-------------|-------|--|----------------------------------|
| Water | 0.920 | 16 | 1.35 |
| Heavy Water | 0.509 | 29 | 0.176 |
| Beryllium | 0.209 | 69 | 0.158 |

 Table 7 Slowing Down Parameters for Moderator Candidates [22-24]

The average lethargy gain per collision, ξ, corresponds to the average logarithmic energy loss of a neutron in a collision. Water is the better moderator candidate when comparing this property as more energy is lost on average in comparison to the other moderators listed. For water, it takes fewer collisions for a neutron to slow down to thermal energies from fast when compared to the other moderators. Water also has a higher scattering cross section when compared to the other moderating candidates but has a higher absorption cross section when compared to heavy water and beryllium. [22-24]

Criticality calculations were conducted for each moderator. After criticality calculations, the core neutron flux was calculated using a fixed source problem. To do this an F4 tally (cell average neutron flux) was used in MCNPX. The F4 tally calculates the average flux in a cell geometry by averaging the track lengths calculated for each particle modeled and dividing the value by the cell volume, in this case, the pin fuel region. The tally used specific pin locations to calculate the

thermal neutron flux in the fuel per pin and the entire core. The neutron flux tally normalization was done using the Niowave provided neutron source strength.

In addition, for each pin and the total core, the fission power was calculated using a fixed source calculation. An F7 tally was used for these calculations. This tally calculates the fission power per gram of fuel in the system. A tally multiplier card was used to normalize the flux values, using the neutron source strength supplied by Niowave Inc. The core power is a parameter that needs to be provided in fuel depletion studies. In addition, the power was also calculated using a reaction rate calculation to verify the results. It is expected that the power calculated through the reaction rate are higher than those using an F7 tally as the methodology to calculate it assumes that the entire fission energy is deposited within the pin. Using the power calculated for the system with the F7 tally (due to accuracy in results), a fuel depletion analysis was conducted for each case. The fuel was irradiated for a period of seven days. During this time, ⁹⁹Mo will build up along with other fission products. The system was irradiated at a constant power. After irradiation, the fuel was subjected for a two-week period of decay to estimate the fission product concentrations including the loss of ⁹⁹Mo. The fuel depletion in these studies was conducted using MCNPX and CINDER90, as a coupled code. For every time period the activity of selected nuclides was analyzed. The nuclides that were studied are ⁹⁹Mo, ¹³⁵Xe, ¹³⁵I, ¹³¹I, ¹⁰⁵Ru, ²³⁹Pu and ¹⁰⁵Rh. ¹³⁵Xe and ¹³⁵I are considered neutron poisons as both

28

nuclides have high neutron absorbing cross sections. ²³⁹Pu was studied to analyze the plutonium concentrations in the system. ¹³¹I is a nuclide of interest due to its potential harm if leaked into the environment. ¹³¹I can be absorbed in the soil and consumed by livestock, which can contaminate people that not only live in the region but also consume animals or milk products that were exposed to the radionuclide. In addition, this radionuclide has uses in Lymphoid tissue tumor/hyperthyroidism treatment.[25] ¹⁰⁵Ru is another isotope of interest due to the potential exposure to personnel.[26] ¹⁰⁵Rh is also an isotope of interest due to its potential use in therapeutic applications. For example, this radionuclide can be used to treat cancerous cells in the lungs.[25] The half-lives for each isotope are shown in Table 8. Results for these calculations are discussed in chapter 3.

| Isotope | Half Life |
|---------|-------------|
| Mo-99 | 66 hours |
| Xe-135 | 9.2 hours |
| I-131 | 8.02 days |
| I-135 | 6.57 hours |
| Ru-105 | 4.44 hours |
| Pu-239 | 24110 years |
| Rh-105 | 1.47 days |

 Table 8 Half-Lives of Radioisotopes of Interest in This Study

⁹⁹Mo, ¹³⁵Xe, ¹³⁵I, ¹⁰⁵Ru and ¹⁰⁵Rh have low half-lives which will cause them to decay relatively fast following irradiation. For this reason, the facility will have to consider the processing of the fuel to be able to extract the isotopes of interest in a timely manner. Again, time will be a major factor in the amount of material left for nuclear medicine application, as is the case with ⁹⁹Mo, which has a relatively longer half-life in comparison to some nuclides of interest. ²³⁹Pu also has a longer half-life, which will have to be considered in reprocessing and waste production.

CHAPTER III RESULTS AND DISCUSSION

III.A Neutron Flux and Core Power Calculations

Using a fixed source calculation, the thermal neutron flux and core power of the sub-critical LEU core for each moderating systems were obtained. The neutron source strength provided by Niowave Inc. along with the basic sub-critical core design was used to calculate neutron flux, effective neutron multiplication factor and core power. For the flux calculation, an F4 tally scoring (cell flux average) was used while for a power calculation, an F7 tally scoring (fission energy deposition) was used. Each pin was tallied to calculate each of the parameters as well as for the entire core by using a location delimitation corresponding for each pin in MCNP. Table 9 shows the results obtained for the thermal flux in the fuel region for the entire core. For each run, a total of 1.0E8 particle histories were simulated. An energy range from thermal up to 0.4E-6 eV was used to calculate the thermal flux. Results for each individual pins can be found in appendix I.

| | Thermal Flux | Error | Total Flux | Error |
|-------------|------------------------|-----------|------------|-----------|
| Moderator | (n/cm ² -s) | (n/cm²-s) | (n/cm²-s) | (n/cm²-s) |
| Water | 9.85E12 | ±7.97E9 | 4.81E+13 | ±1.91E+10 |
| Heavy Water | 1.27E12 | ±2.79E10 | 1.36E+13 | ±5.59E+10 |
| Beryllium | 6.44E11 | ±1.63E9 | 9.21E+12 | ±3.46E+09 |

 Table 9 Estimated Neutron Flux Values Obtained From Each MCNPX

 Calculation for Three Moderators

Water provides a higher thermal and total neutron flux when compared to the other two moderators. However, for all cases the thermal flux is relatively low when compared to the total flux. Neutrons are not thermalizing efficiently in the current design for heavy water and beryllium moderators resulting in lower neutron flux levels. Water provides a higher thermal neutron flux for the system. Fast neutrons produced from the external neutron source are capable of being thermalized more efficiently in the water-moderated system than heavy water and beryllium-moderated systems. Heavy-water-moderated system has neutron flux level almost an order of magnitude lower than water, yet better than beryllium. While heavy water has a lower absorption cross sections, neutrons loose more energy per collision in light water than in the two moderators as previously discussed. While heavy water is considered a better moderator due to its moderation ratio, in this system, leakage is a larger issue than the absorption

in the moderator. Since, fewer collisions are required to thermalize a neutron with water, fission is more likely to occur. To implement heavy water as a moderator, a larger moderator to fuel ratio is required in comparison to water to provide adequate slowing down of the fast neutrons. [27]

For the power calculation, individual pin powers were calculated as well as for the entire core for the fuel region. MCNP results were obtained in units of MeV/g-s, the results were then multiplied by the mass of the pin to obtain units of power. Results for each pin can be found in appendix II. Results for the entire core power for each moderator system are shown in Table 10.

| Table 10 Estimated Sub-Critical Core Thermal Power Values Obtained |
|--|
| From Each MCNPX Calculation for Each Moderator |

| Moderator | Power (MW _{th}) | Error (MW _{th}) |
|-------------|---------------------------|---------------------------|
| Water | 0.2720 | ±0.000734 |
| Heavy Water | 0.0433 | ±0.000017 |
| Beryllium | 0.0223 | ±0.000004 |

The design with water moderator has a higher power than heavy water and beryllium. This is consistent with the thermal neutron flux calculations previously discussed. Since neutrons are capable of being thermalized more efficiently with water, they have a higher fission probability than the other systems, as more thermal neutrons are present in the system, which will increase the amount of power produced.

The reaction rate for the system was calculated for each case using an F4 tally for each pin and the entire core. The reaction rate was used along with a tally multiplier within the code to calculate the power for each pin. The atom density was calculated for the system using the fuel density, molecular mass of the fuel and Avogadro's number. The atom density was multiplied by the results obtained through MCNP along with the total volume of fuel in the core and a 200 MeV/fission Q-value to calculate the power for the system. The power was calculated from this calculation for each moderator. The results are shown in Table 11.

| Moderator | Power (MW _{th}) | Power (MW _{th}) |
|-------------|---------------------------|---------------------------|
| Water | 0.3010 | ±0.00126 |
| Heavy Water | 0.0473 | ±0.00002 |
| Beryllium | 0.0354 | ±0.00017 |

Table 11 Power Calculation Results Based on the Reaction Rate

Values for the power calculated by the reaction rate are higher than those calculated using an F7 tally, as described above. MCNPX when calculating the reaction rate assumes all the fission energy is deposited locally, which is not the case as particles leak and move to other regions. The power calculated using the F7 tally was used for the depletion analysis due to the accuracy in the calculation, however, the power calculated with the reaction rate was used as an independent check on the results since the methodology of calculating power using reaction rates is different than the F7 fission energy deposition method.

Niowave reports a neutron flux of ~2.0E13 n/cm²-s and a core power of ~266 kW (compared to the current study predicted value of 272 kW). In the watermoderated system, a thermal flux of 9.9E12 n/cm²-s (error: \pm 7.97E9 n/cm²-s) was obtained for the entire core, while a total flux of 4.81E13 n/cm²-s with an error of \pm 1.91E10 n/cm²-s, was obtained as the total flux for the system. The flux value calculated for the system is in approximation to that reported by Niowave, but it is about two times higher. Details on the calculation procedure used by Niowave Inc. are not available due to the proprietary nature of the design. Hence a better analysis to find out the reasons for the differences between Niowave values and our calculated values of neutron flux and thermal power was not possible.

35

III.B Criticality Study

A criticality calculation study was conducted for each moderator candidate using eighty-five fuel pins and 10% ²³⁵U enriched metallic fuel (90% ²³⁸U) following the power and neutron flux calculations. Effective neutron multiplication factor, k_{eff} was estimated from the calculations for each moderator. In this study, a KCODE feature (criticality source feature) available in MCNPX was used as opposed to a fixed source problem used earlier for neutron flux and core power calculations. For each case 3000 particles were used per cycle for a total of 500 cycles. The first 50 cycles were skipped for each simulation for neutron source convergence in the system. The results are shown in Table 12.

| Table 12 Effective Neutron Multiplication Factor Values Obtained From |
|--|
| MCNPX Criticality Calculations for Each Moderator With 10% ²³⁵ U Enriched |
| Metallic Fuel |

| Moderator | Effective Neutron | Standard Deviation |
|-------------|-----------------------------|--------------------|
| | Multiplication ractor, Reff | |
| Water | 0.95702 | 0.00067 |
| Heavy Water | 0.77661 | 0.00070 |
| Beryllium | 0.7421 | 0.00072 |

Based on the results, water is a better moderator as one can see from Table 12 that the k_{eff} is relatively higher than the other two moderator candidates. More fission occurs in the water-moderated system than in the heavy water and beryllium systems. It is also seen that when using the water-moderated design, Niowave Inc. is not capable of using higher enriched fuel due to the criticality constraint placed on the system by licensing regulations. However, heavy water and beryllium-moderated systems are capable of using higher enrichments of fissile material since the k_{eff} for the system is lower than the criticality constraint in place. Niowave Inc. reports a k_{eff} of ~0.95 (for a water design) which is in agreement to the results obtained in this study for the water moderated system.

III.C Fuel Depletion Study

A fuel depletion study was conducted for each moderator to study the different isotopes of interest previously discussed. The isotopes of interest are the following: ⁹⁹Mo, ¹³⁵Xe, ¹³⁵I, ¹³¹I, ¹⁰⁵Ru, ²³⁹Pu and ¹⁰⁵Rh. The thermal power (272 kW_{th}) calculated previously was used as an input in the code for fuel depletion calculations. MCNPX is coupled with CINDER90 to conduct the analysis. A seven-day irradiation was conducted at constant power using small time steps (0.1, 0.4, 0.7, 1, 2, 3, 4, 5, 6, 7, 14 and 21 days). The time step selection was required to make sure that each time step interval is not too large in comparison

to the half-lives of the isotopes of interest specifically before they reach their equilibrium concentration for a given neutron flux or reactor power level. Following irradiation, a two-step depletion analysis was conducted each accounting for seven days of decay. For this study, a total of 2100 cycles were conducted, skipping the first 100 cycles and each composed of 5000 particles. A total of 10 million histories were simulated for the active cycles. Results obtained for each isotope of interest (except ⁹⁹Mo) are shown in Figures 8A through 8F. Results obtained for ⁹⁹Mo production is shown in Figure 9 separately since it is the prime isotope of interest in this study.











Figure 8 Continued





*A relative error of 0.0005 is given to each depletion value.

Figure 8 Continued

The trends are as expected for each isotope based on their half-lives and yield. ¹³⁵Xe has a yield of approximately 6%, the majority of the production is from the decay of ¹³⁵I. Both nuclides stabilized once a secular equilibrium is reached in the first two days.[24, 28] ¹⁰⁵Ru is also a fission product that decays to ¹⁰⁵Rh by beta decay. Both nuclides have lower fission yields than ⁹⁹Mo.[28] Finally, ¹³¹I is another fission product with fission yields lower than ⁹⁹Mo. Small amounts of ²³⁹Pu are produced for this system based on the radioactivity obtained. After irradiation, isotopes begin to decay at their respective half-lives. With the current design, production of these nuclides does not seem beneficial to Niowave due to the lower amounts produced. As mentioned previously, ⁹⁹Mo, ¹³⁵Xe, ¹³⁵I, ¹⁰⁵Ru and ¹⁰⁵Rh have low half-lives which will mean they will decay relatively fast post irradiation while ²³⁹Pu has a longer half-life which will cause the material to remain, yet the amount of ²³⁹Pu is very small as shown in Table 13.

| Time (Day) | ²³⁹ Pu Mass (g) | Error (g) |
|------------|----------------------------|-----------|
| 0.1 | 2.74E-05 | ±1.37E-08 |
| 0.4 | 5.95E-04 | ±2.98E-07 |
| 0.7 | 1.86E-03 | ±9.30E-07 |
| 1 | 3.78E-03 | ±1.89E-06 |
| 2 | 1.40E-02 | ±7.00E-06 |
| 3 | 2.90E-02 | ±1.45E-05 |
| 4 | 4.76E-02 | ±2.38E-05 |
| 5 | 6.88E-02 | ±3.44E-05 |
| 6 | 9.21E-02 | ±4.61E-05 |
| 7 | 1.17E-01 | ±5.85E-05 |
| 14 | 1.93E-01 | ±9.65E-05 |
| 21 | 2.03E-01 | ±1.02E-04 |

Table 13 ²³⁹Pu Mass for 10% Enriched Fuel in the Water Moderated System

A maximum of 2.03E-1 gram (error ± 0.0001 g) of ²³⁹Pu is produced for the system. Again, using LEU as opposed to HEU is of advantage as the system is more proliferation resistant, as no significant quantities of ²³⁹Pu are present. For this fuel, a total burnup of 2.217E-1 GWd/MTU is reached. There is only 0.0023% of ²³⁹Pu production for the amount of heavy metal fuel in the system. The production of ⁹⁹Mo is shown in Figure 9.



*A relative error of 0.0005 is given to each depletion value.



As expected, ⁹⁹Mo builds up during the irradiation, and decays rapidly during the 14 days of decay following that. A small dip is observed in the data at the 1 day time step. This dip is also seen in Figure 8B and Figure 8D, ¹³¹I and ¹⁰⁵Rh, respectively. The dip in the data is seen in isotopes with relatively longer half lives and is an artifact of plotting the data using time on the x-axis as opposed to potting the data against the burnup in the fuel. It is important to note the scale is not linear as short time steps were used at the beginning to study the effects of each isotope at the beginning of life. Figure 10, shows the data for ⁹⁹Mo plotted

against the burnup as opposed to the time. The dip previously seen in the data is no longer visible in Figure 10.



Figure 10 ⁹⁹Mo Production and Depletion Studies for Water Moderator for the Entire System using Burnup in the X-Axis.

MCNPX does not calculate the error in activity values for any isotope. However, to quantify the error in the results obtained, the random number generator seed was changed using the DBCN card in MCNPX. This card changes the random number sequence in the calculation to compare statistical convergence. [17] The error is mainly statistical as the systematic error depends on the user and interaction cross-section data library, while the statistical error depends on the particle history number and number of neutron generation cycles used in the simulation of the core. Nine different seed calculations were conducted for the 10% enriched water-moderated system. The error obtained was considered for all depletion analysis regarding activity concentrations in the system as well as all moderators. The number of cycles and particles remained the same for all the simulations and depletion analysis conducted, therefore it was decided the error obtained through the seed calculation was a good representation for the other systems. A total of nine different seed calculations were conducted to obtain a good representation of a Gaussian distribution. The depletion values obtained for the 14-day and 21-day time steps were studied. In addition, the reaction rates were also analyzed to study the errors associated with the calculated values.

For each seed calculation, a total of 3000 neutron generation cycles were conducted, each with 2000 particles and skipping the first 500 cycles. From the nine different seeds, the mean value along with the standard deviation was calculated. The relative error was obtained by dividing the standard deviation by the mean value. The relative error for all nine seed calculations was calculated for the 14-day and 21-day time step to be 0.0003 and 0.0002, respectively. The reaction rate relative error was calculated to be 0.0005. A relative error of 0.0005 was used for each depletion calculation reported to be conservative. However, it is expected that the results presented in other calculations will have a lower

46

statistical errors associated with them due to the higher number of particles used for the calculations.

After the seed study concluded, the activity produced for each isotope can be guantified more accurately. The facility produces a total of 1.16E4 Ci (error: ±5.8 Ci) of ⁹⁹Mo for the seven-day irradiation. After seven and fourteen days post irradiations only 1.98E3 Ci (error: ±0.990 Ci) and 3.38E2 Ci (error: ±0.169 Ci) remain, respectively. Again, the relative error for these values is reported to be 0.0005. Based on the results, a large amount of material is lost due to the short half-life of ⁹⁹Mo. Due to the time constraint; the facility needs to process the material rapidly to conserve the majority of the material. Niowave Inc. reports a ~9E3 Ci of ⁹⁹Mo after a week long irradiation, which is in approximation to the results calculated using MCNPX and CINDER90. However, the facility did not specify what they considered a week long irradiation to be, which can vary from 5 to 7 days. Again, due to proprietary nature of the work, the details in the calculation were not provided by Niowave Inc., nor were the uncertainties in the values. These values were given as approximations and therefore we can conclude the results are in agreement to the results obtained in this calculation. Table 14 shows the summary of the results for the entire core. Some values are missing from the table, this is due to the isotope reaching low concentrations, which will cause CINDER90 and MCNPX to stop tracking the material as it does not have a significant contribution.

47

| Time | | | | | | |
|--------|----------|----------|----------|----------|----------|----------|
| (Days) | Xe135 | 1131 | Ru105 | Rh105 | I135 | Pu239 |
| 0.1 | 3.40E+02 | 2.70E+01 | 6.74E+02 | 1.52E+01 | 3.21E+03 | 1.70E-06 |
| 0.4 | 2.65E+03 | 1.73E+02 | 1.78E+03 | 1.89E+02 | 9.14E+03 | 3.69E-05 |
| 0.7 | 4.75E+03 | 3.20E+02 | 2.15E+03 | 4.26E+02 | 1.19E+04 | 1.16E-04 |
| 1 | 6.08E+03 | 4.66E+02 | 2.26E+03 | 6.59E+02 | 1.32E+04 | 2.34E-04 |
| 2 | 7.44E+03 | 9.18E+02 | 2.32E+03 | 1.26E+03 | 1.43E+04 | 8.67E-04 |
| 3 | 7.58E+03 | 1.35E+03 | 2.32E+03 | 1.64E+03 | 1.44E+04 | 1.80E-03 |
| 4 | 7.59E+03 | 1.75E+03 | 2.32E+03 | 1.87E+03 | 1.44E+04 | 2.95E-03 |
| 5 | 7.59E+03 | 2.12E+03 | 2.32E+03 | 2.01E+03 | 1.44E+04 | 4.27E-03 |
| 6 | 7.59E+03 | 2.47E+03 | 2.32E+03 | 2.10E+03 | 1.44E+04 | 5.71E-03 |
| 7 | 7.59E+03 | 2.79E+03 | 2.32E+03 | 2.16E+03 | 1.44E+04 | 7.24E-03 |
| 14 | - | 1.61E+03 | - | 9.28E+01 | - | 1.20E-02 |
| 21 | - | 8.84E+02 | - | 3.45E+00 | - | 1.26E-02 |

Table 14 Results of Fuel Production and Depletion Studies for the EntireCore With Water Moderator

***Quantities for each isotope are represented in curies. A relative error of 0.0005 is given to each depletion value. Results correspond to the entire core.

Similar studies were conducted for heavy water and beryllium moderators in the core. Figures for each isotope production trends are shown in appendices III

and IV. Fewer amounts of isotopes of interest are produced in the beryllium and heavy water systems. It is also seen that the short half-lives of many of these isotopes makes it extremely difficult to process and in delivering the material in a timely manner that will conserve the majority of the activity for each. Some of the isotopes have half-lives less than ⁹⁹Mo which would have to be considered in reprocessing of the fuel. If the facility wants to produce amounts of these isotopes to use in nuclear medicine application, longer irradiation times for the fuel must be conducted, which will have to be studied.

The most important isotope in this study was ⁹⁹Mo. Results obtained for each moderated system are presented next for this nuclide, while the results for the other isotopes of interest can be found in the appendix section. Result for ⁹⁹Mo production in the heavy water moderated system is shown in Figure 11. The results correspond to the entire core region. A total of 2100 cycles were conducted, skipping the first 100 cycles and each composed of 5000 particles. A total of 10 million histories were conducted for the active cycles.

49



*A relative error of 0.0005 is given to each depletion value.



Only 1.84E3 Ci (error: ± 0.92 Ci) of material is produced for a seven-day irradiation system. At the end of the 7 and 14 day decay time only 3.15E2 Ci (error: ± 0.1560 Ci) and 5.38E1 Ci (error: ± 0.0269 Ci) of material remain, respectively. For these values reported a value of 0.0005 as the relative error is given based on the seed calculation. Again, ⁹⁹Mo's half-life is the limiting component and will affect the overall production of the material that can be used for nuclear medicine treatments. Results for the other isotopes are shown in Table 15. The results correspond to the entire core.

| Time | | | | | | |
|--------|----------|----------|----------|----------|----------|----------|
| (Days) | Xe135 | I131 | Ru105 | Rh105 | I135 | Pu239 |
| 0.1 | 5.70E+01 | 4.30E+00 | 1.09E+02 | 2.45E+00 | 5.11E+02 | 5.24E-07 |
| 0.4 | 5.09E+02 | 2.76E+01 | 2.88E+02 | 3.06E+01 | 1.46E+03 | 1.13E-05 |
| 0.7 | 1.01E+03 | 5.10E+01 | 3.46E+02 | 6.90E+01 | 1.90E+03 | 3.55E-05 |
| 1 | 1.41E+03 | 7.42E+01 | 3.65E+02 | 1.07E+02 | 2.11E+03 | 7.19E-05 |
| 2 | 1.98E+03 | 1.46E+02 | 3.74E+02 | 2.05E+02 | 2.28E+03 | 2.66E-04 |
| 3 | 2.08E+03 | 2.15E+02 | 3.74E+02 | 2.67E+02 | 2.29E+03 | 5.51E-04 |
| 4 | 2.10E+03 | 2.79E+02 | 3.74E+02 | 3.06E+02 | 2.29E+03 | 9.04E-04 |
| 5 | 2.10E+03 | 3.38E+02 | 3.75E+02 | 3.30E+02 | 2.29E+03 | 1.31E-03 |
| 6 | 2.10E+03 | 3.93E+02 | 3.74E+02 | 3.45E+02 | 2.29E+03 | 1.75E-03 |
| 7 | 2.10E+03 | 4.44E+02 | 3.75E+02 | 3.55E+02 | 2.29E+03 | 2.22E-03 |
| 14 | - | 2.57E+02 | - | 1.52E+01 | - | 3.66E-03 |
| 21 | - | 1.41E+02 | - | 5.66E-01 | - | 3.84E-03 |

Table 15 Results of Fuel Production and Depletion Studies with HeavyWater Moderated Core

***Quantities for each isotope are represented in curies. A relative error of 0.0005 is given to each depletion value. Results are for the entire core.

Less amounts of ⁹⁹Mo are produced with the heavy water system than the water moderator, as expected. Almost a magnitude less of material is produced between the two systems. The same trend is followed for the other isotopes.

The same study was also conducted for the beryllium system. A total of 2100 cycles were conducted, skipping the first 100 cycles and each composed of 5000 particles, for each moderator. A total of 10 million histories were conducted for the active cycles in each case. Results for the entire core production of ⁹⁹Mo with a beryllium moderator is shown in Figure 12.



*A relative error of 0.0005 is given to each depletion value.

Figure 12 ⁹⁹Mo Production for the Beryllium System

Only 9.47E2 Ci (error: ±0.4735 Ci) of material is produced for this moderating system. After seven and fourteen days following irradiation only 1.62E2 Ci (error:

 ± 0.81 Ci) and 2.77E1 Ci (error: ± 0.0138 Ci) of material remain, respectively. In these calculations a relative error of 0.0005 is assumed. This moderator is less efficient than water and heavy water. Results for the other nuclides of interest are shown in Table 16.

| Time | | | | | | |
|--------|----------|----------|----------|----------|----------|----------|
| (Days) | Xe135 | 1131 | Ru105 | Rh105 | 1135 | Pu239 |
| 0.1 | 2.95E+01 | 2.22E+00 | 5.65E+01 | 1.27E+00 | 2.63E+02 | 2.98E-07 |
| 0.4 | 2.66E+02 | 1.42E+01 | 1.49E+02 | 1.59E+01 | 7.51E+02 | 6.45E-06 |
| 0.7 | 5.35E+02 | 2.63E+01 | 1.80E+02 | 3.58E+01 | 9.78E+02 | 2.02E-05 |
| 1 | 7.50E+02 | 3.83E+01 | 1.89E+02 | 5.54E+01 | 1.09E+03 | 4.09E-05 |
| 2 | 1.07E+03 | 7.54E+01 | 1.94E+02 | 1.07E+02 | 1.17E+03 | 1.51E-04 |
| 3 | 1.14E+03 | 1.11E+02 | 1.94E+02 | 1.39E+02 | 1.18E+03 | 3.13E-04 |
| 4 | 1.15E+03 | 1.44E+02 | 1.94E+02 | 1.59E+02 | 1.18E+03 | 5.13E-04 |
| 5 | 1.15E+03 | 1.74E+02 | 1.94E+02 | 1.71E+02 | 1.18E+03 | 7.42E-04 |
| 6 | 1.15E+03 | 2.03E+02 | 1.94E+02 | 1.79E+02 | 1.18E+03 | 9.92E-04 |
| 7 | 1.15E+03 | 2.29E+02 | 1.94E+02 | 1.84E+02 | 1.18E+03 | 1.26E-03 |
| 14 | - | 1.32E+02 | _ | 7.91E+00 | - | 2.08E-03 |
| 21 | - | 7.25E+01 | - | - | - | 2.18E-03 |

Table 16 Results of Fuel Production and Depletion Studies with BerylliumModerator

***Quantities for each isotope are represented in curies. A relative error of 0.0005 is given to each depletion value. Results are for the entire core.

Beryllium is a less efficient moderator than heavy water and water in the current design. Due to the lower amounts of thermal neutron flux, less fission events are

produced which affects the production of ⁹⁹Mo. Table 17 shows the results for ⁹⁹Mo production for each moderator.

| Time (Days) | Water ⁹⁹ Mo (Ci) | Heavy Water ⁹⁹ Mo (Ci) | Beryllium ⁹⁹ Mo (Ci) | |
|-------------|--------------------------------|--------------------------------------|------------------------------------|--|
| 0.1 | | | | |
| 0.1 | 3.43E+02 | 5.47E+01 | 2.82E+01 | |
| 0.4 | 1.33E+03 | 2.12E+02 | 1.09E+02 | |
| 0.7 | 2.25E+03 | 3.58E+02 | 1.84E+02 | |
| 1 | 3.10E+03 | 4.93E+02 | 2.54E+02 | |
| 2 | 5.51E+03 | 8.78E+02 | 4.52E+02 | |
| 3 | 7.39E+03 | 1.18E+03 | 6.06E+02 | |
| 4 | 8.85E+03 | 1.41E+03 | 7.26E+02 | |
| 5 | 9.99E+03 | 1.59E+03 | 8.19E+02 | |
| 6 | 1.09E+04 | 1.73E+03 | 8.91E+02 | |
| 7 | 1.16E+04 | 1.84E+03 | 9.47E+02 | |
| 14 | 1.98E+03 | 3.15E+02 | 1.62E+02 | |
| 21 | 3.38E+02 | 5.38E+01 | 2.77E+01 | |

 Table 17
 99
 Mo Production for Each Moderator System

***Quantities for each isotope are represented in curies. A relative error of 0.0005 is given to each depletion value. Results are for the entire core.

In comparison, water is a better moderator for the Niowave design. Heavy water is good in relation to beryllium, but produces almost a magnitude less ⁹⁹Mo than

water due to the lack of moderation in the system, as previously discussed. Beryllium is the worst moderator option for the design as it produces the lowest amount of ⁹⁹Mo in comparison to the two other systems.

III.D Uranium Fuel Enrichment Parametric Studies

Based on the depletion studies conducted for the three moderating systems and the criticality analysis it is seen that the enrichment of ²³⁵U can be increased to enhance the production of ⁹⁹Mo as more fissile material will be present, for beryllium and heavy water-moderated systems. Due to the criticality constraint, the enrichment for the water moderated system cannot be increased further. The enrichment of ²³⁵U was studied for heavy water and beryllium. All other parameters were maintained the same. Results obtained for the enrichment variation studies for heavy water moderated system are presented in Table 18. For each case 3000 particles were used per cycle for a total of 500 cycles.

55

Table 18 Variation of K_{eff} as Function of Increased UraniumEnrichment forHeavy Water Moderated System

| Enrichment (%) | K _{eff} | Standard Deviation |
|----------------|------------------|--------------------|
| 10 | 0.77661 | 0.00070 |
| 13 | 0.81026 | 0.00075 |
| 15 | 0.82611 | 0.00073 |
| 17 | 0.84206 | 0.00070 |
| 19 | 0.85420 | 0.00070 |
| 19.9 | 0.85959 | 0.00076 |

A similar study was conducted for the beryllium-moderated system. The results are presented in Table 19.

Table 19 Variation of K_{eff} as Function of Increased Uranium Enrichment forBeryllium Moderated System

| Enrichment (%) | K _{eff} | Standard Deviation | | |
|----------------|------------------|--------------------|--|--|
| 10 | 0.74210 | 0.00072 | | |
| 13 | 0.77514 | 0.00072 | | |
| 15 | 0.79159 | 0.00072 | | |
| 17 | 0.80514 | 0.00072 | | |
| 19 | 0.81906 | 0.00072 | | |
| 19.9 | 0.82318 | 0.00072 | | |

As expected, the increase in fissile content in the system causes the k_{eff} to increase. Heavy water has a higher k_{eff} than beryllium; this factor remained consistent as the 10% enriched fuel studies previously discussed. However, neither moderating candidate reaches a k_{eff} close to 0.95 which is the constraint placed on the design. This means that the fissile content could be increased further (if the system did not use LEU) to increase the fission probability and production rate. For these studies, enrichments of less than 20% for ²³⁵U were the limiting component for the system. Both these moderators will require an increase in moderator to fuel ratio to slow down the neutrons to thermal energy and be able to take advantage of the higher fissile content (if required). The facility will have to consider this balance between moderation (the size of the core) and the enrichment limitation if heavy water and beryllium are considered as moderators.

III.E Flux and Power Calculations for a 19.9% Enriched Metallic Fuel

Using 19.9% ²³⁵U enrichment for both systems, neutron flux and power calculations were conducted for heavy water and beryllium. A fixed source calculation was conducted for each analysis for a total of 1.0E8 particles. The study was conducted for each individual pin as well as the entire core. Results

obtained for the individual pins can be found in appendix V. Results for the entire core, for each moderator, are shown in Table 20.

| | Thermal Flux | Error | Total Flux | Error | |
|-------------|--------------|------------------------|------------|-----------|--|
| Moderator | (n/cm²-s) | (n/cm ² -s) | (n/cm²-s) | (n/cm²-s) | |
| Heavy Water | 2.23E12 | ±3.724E9 | 4.19E+13 | ±1.32E+10 | |
| | | | | | |
| Beryllium | 1.45E12 | ±3.074E9 | 3.53E+13 | ±1.10E+10 | |
| | | | | | |

Table 20 Neutron Flux Analysis for the 19.9% Enriched System

Heavy water achieves a higher thermal and total neutron flux in the fuel than beryllium. However, both systems do not achieve a flux close to the watermoderated system, which used only 10% enriched fuel. Again, there is a lack of thermalization in the system, as neutrons are not slowing down from fast regions efficiently enough to thermal energies. Power calculations were conducted for individual pins as well as for the entire core system. Results for the entire core are listed in Table 21. Individual pin calculation results can be seen in appendix VI.

| Moderator | Power (MWth) | Error (MWth) |
|-------------|--------------|--------------|
| Heavy Water | 0.0748 | ±0.00005 |
| Beryllium | 0.0519 | ±0.00002 |

Table 21 Estimated Power for the 19.9% Enriched System

As expected, the heavy-water-moderated core achieves a higher power in relation to the beryllium-moderated system. However, both systems achieve a power less than the water-moderated system, which only uses 10%, enriched fuel. Fewer neutrons are slowed down to thermal energies in the heavy water and beryllium moderated system design. With fewer thermal neutrons in the systems, less fissions are produced which cause the lower power generation for each case of heavy water and beryllium moderated systems.

III.F Depletion Analysis Using 19.9% Enriched Metallic Fuel

Tracking the same isotopes discussed previously, isotope production and depletion analyses were conducted for a 19.9% enriched fuel for heavy water and beryllium-moderated systems using the respective thermal power calculated and reported above. A total of 2100 cycles were conducted, skipping the first 100 cycles and each composed of 5000 particles. A total of 10 million histories were used for the active cycles. Results obtained for ⁹⁹Mo in the heavy-water

moderated system is shown in Figure 13. Other isotope results can be seen in appendix VII.



*A relative error of 0.0005 is given to each depletion value.

Figure 13 ⁹⁹Mo Radioactivity Values for Heavy Water Moderated System With 19.9% Enriched Fuel.

For a seven-day irradiation, a total of 3.18E+03 Ci (error: 1.59 Ci) is produced. Seven days after irradiation only 5.43E+02 Ci (error: 0.2715 Ci) remains. After fourteen days post irradiation only 9.29E+01 Ci (error: 0.0464 Ci) remains. The relative error associated with these reported values is 0.0005. As expected, more ⁹⁹Mo is produced as more fission events are produced in the system compared to the 10% enriched fuel. However, less ⁹⁹Mo is produced in comparison to water. These results are consistent with the power and flux study. Results for the other isotopes are shown in Table 22. The results correspond to the entire core. Again, the lack of moderation in the system is problematic as neutrons leak before being slowed down. With an increase of the moderator to fuel ratio it is expected to see more fissions in this system as neutrons would be able to slow down from fast to thermal energy and take advantage of the ²³⁵U fission cross-section and take advantage of the higher fissile content in the fuel.

| Time | | | | | | | |
|--------|----------|----------|----------|----------|----------|----------|----------|
| (Days) | Xe135 | Mo99 | I131 | Ru105 | Rh105 | I135 | Pu239 |
| 0.1 | 9.87E+01 | 9.45E+01 | 7.43E+00 | 1.86E+02 | 4.18E+00 | 8.83E+02 | 6.27E-07 |
| 0.4 | 8.85E+02 | 3.66E+02 | 4.77E+01 | 4.91E+02 | 5.22E+01 | 2.51E+03 | 1.36E-05 |
| 0.7 | 1.77E+03 | 6.18E+02 | 8.80E+01 | 5.90E+02 | 1.18E+02 | 3.28E+03 | 4.25E-05 |
| 1 | 2.47E+03 | 8.52E+02 | 1.28E+02 | 6.22E+02 | 1.82E+02 | 3.64E+03 | 8.61E-05 |
| 2 | 3.50E+03 | 1.52E+03 | 2.53E+02 | 6.38E+02 | 3.50E+02 | 3.93E+03 | 3.18E-04 |
| 3 | 3.69E+03 | 2.03E+03 | 3.71E+02 | 6.37E+02 | 4.55E+02 | 3.95E+03 | 6.59E-04 |
| 4 | 3.72E+03 | 2.43E+03 | 4.81E+02 | 6.38E+02 | 5.21E+02 | 3.95E+03 | 1.08E-03 |
| 5 | 3.73E+03 | 2746 | 5.84E+02 | 6.38E+02 | 5.63E+02 | 3.95E+03 | 1.56E-03 |
| 6 | 3.73E+03 | 2.99E+03 | 6.79E+02 | 6.38E+02 | 5.88E+02 | 3.95E+03 | 2.09E-03 |
| 7 | 3.73E+03 | 3.18E+03 | 7.66E+02 | 6.38E+02 | 6.04E+02 | 3.95E+03 | 2.65E-03 |
| 14 | - | 5.43E+02 | 4.43E+02 | - | 2.60E+01 | - | 4.38E-03 |
| 21 | - | 9.29E+01 | 2.43E+02 | - | 9.64E-01 | - | 4.60E-03 |

Table 22 Depletion Studies with Heavy Water Moderator and 19.9% FuelEnrichment

***Quantities for each isotope are represented in curies. A relative error of 0.0005 is given to each depletion value. Results are for the entire core.

The same trends are seen as with ⁹⁹Mo production. Less activity amounts of the isotopes are produced for this system than for the water-moderated system as expected.

A similar study was conducted for the beryllium-moderated system. A total of 2100 cycles were conducted, skipping the first 100 cycles and each composed of 5000 particles. A total of 10 million histories were simulated for the active cycles. Results for ⁹⁹Mo in the beryllium-moderated system are described in Figure 14. Other isotope results can be seen in appendix VIII.


*A relative error of 0.0005 is given to each depletion value.

Figure 14 ⁹⁹Mo Radioactivity Values for Beryllium Moderated System With 19.9% Enriched Fuel

For a seven-day irradiation, a total of 2.20E+03 Ci (error: ± 1.1000 Ci) is produced. Seven-days after irradiation only 3.77E+02 Ci (error: ± 0.1885 Ci) of material remains. After fourteen days following irradiation only 6.45E+01Ci (error: ± 0.3225 Ci) remain. A relative error of 0.0005 is associated with each reported value. As expected, more ⁹⁹Mo is produced as more fission events are produced in the system compared to the 10% enriched fuel. However, less ⁹⁹Mo is produced in comparison to water. These results are consistent with the power and neutron flux study. Results for the other isotopes are shown in the Table 23.

The values correspond to the entire core.

| Time | | | | | | | |
|--------|----------|----------|----------|----------|----------|----------|----------|
| (Days) | Xe135 | Mo99 | 1131 | Ru105 | Rh105 | I135 | Pu239 |
| 0.1 | 6.86E+01 | 6.56E+01 | 5.16E+00 | 1.30E+02 | 2.92E+00 | 6.13E+02 | 4.75E-07 |
| 0.4 | 6.19E+02 | 2.54E+02 | 3.31E+01 | 3.43E+02 | 3.64E+01 | 1.75E+03 | 1.03E-05 |
| 0.7 | 1.25E+03 | 4.29E+02 | 6.11E+01 | 4.12E+02 | 8.21E+01 | 2.28E+03 | 3.22E-05 |
| 1 | 1.74E+03 | 5.91E+02 | 8.90E+01 | 4.34E+02 | 1.27E+02 | 2.52E+03 | 6.52E-05 |
| 2 | 2.49E+03 | 1.05E+03 | 1.75E+02 | 4.45E+02 | 2.44E+02 | 2.73E+03 | 2.41E-04 |
| 3 | 2.64E+03 | 1.41E+03 | 2.57E+02 | 4.45E+02 | 3.18E+02 | 2.74E+03 | 4.99E-04 |
| 4 | 2.66E+03 | 1.69E+03 | 3.34E+02 | 4.45E+02 | 3.64E+02 | 2.74E+03 | 8.18E-04 |
| 5 | 2.67E+03 | 1.91E+03 | 4.05E+02 | 4.45E+02 | 3.93E+02 | 2.74E+03 | 1.18E-03 |
| 6 | 2.67E+03 | 2.07E+03 | 4.71E+02 | 4.45E+02 | 4.11E+02 | 2.74E+03 | 1.58E-03 |
| 7 | 2.67E+03 | 2.20E+03 | 5.32E+02 | 4.45E+02 | 4.22E+02 | 2.74E+03 | 2.01E-03 |
| 14 | - | 3.77E+02 | 3.08E+02 | - | 1.81E+01 | - | 3.31E-03 |
| 21 | - | 6.45E+01 | 1.69E+02 | - | 6.73E-01 | - | 3.48E-03 |

Table 23 Depletion Studies with Beryllium Moderator and 19.9% FuelEnrichment

***Quantities for each isotope are represented in curies. A relative error of 0.0005 is given to each depletion value. Amounts correspond to the entire core

The same trends are seen as with ⁹⁹Mo production. Less activity amounts of the isotopes are produced for this system than for the water-moderated system. Beryllium has the worst moderating properties when it comes to water and heavy water with the current design.

CHAPTER IV CONCLUSION AND FUTURE WORK

Three moderated systems were studied for the Niowave Inc. sub-critical LEU core design for estimating various radioisotope productions, the primary one being ⁹⁹Mo. The design is composed of 85 fuel pins with 10% enriched metallic uranium fuel. For each moderator, power, thermal neutron flux, total neutron flux and fuel production/depletion studies were conducted for a variety of isotopes. It was found that the water-moderated system has the highest thermal neutron flux in the fuel region than with the other moderators. In addition, water produces a higher power in the core in comparison to the other moderators of interest. ⁹⁹Mo production was studied for each case; water produced more amount of ⁹⁹Mo as well as the other radionuclides of interest as expected due to the higher thermal neutron flux and power. The higher thermal neutron flux allows more fission to occur for the system in comparison to the other two designs. The results obtained for the water moderated system are reasonably in good agreement with the values presented by Niowave. However, this study made many parametric variation studies.

Enrichment amounts of 235 U, in the metallic fuel, was increased for heavy water and beryllium moderated system up to 20% to determine how far the k_{eff} value

will reach in relation to the design limit of ~0.95. Both, heavy water and beryllium can exceed enrichments greater than 20% to reach the k_{eff} limitation placed on the system. However, due to further constraints the fuel needs to be low enriched fuel. A 19.9% enrichment study was conducted for each fuel. It was found that the heavy water system reached a higher k_{eff} than beryllium along with a higher power and thermal flux. In addition, more quantities of ⁹⁹Mo and radionuclides are produced for the system in comparison to the beryllium-moderated system. Increasing the moderation of both of these candidates will increase the amount of ⁹⁹Mo produced, which might not require higher enrichments of ²³⁵U, this will have to be considered by the facility.

Water is the best moderator for this design. The fuel is better utilized in this system and higher burnup levels can be achieved for the system, as higher power is obtained for the same amount of fuel. ⁹⁹Mo amounts are produced in higher quantities than in the other two cases in the water-moderated system. As previously discussed the amount of ⁹⁹Mo available to use in actual medical treatments will depend on reprocessing time, quality control inspections, transportations among other factors that can prolong the time the radionuclide has to wait to be administered. Time plays a major factor in the efficiency of the production of this nuclide and therefore will have to be considered when deciding the irradiation time. In addition, the system will have to be optimized for the other fission products of interest as not enough material is produced to

account for the small half-life. In the current design, the production of the radioisotopes of interest for medical application is not viable as not enough material is produced and the half-lives are small. Very little ²³⁹Pu is produced for the system. Due to the low burnup, only 0.0023% of ²³⁹Pu is produced in comparison to the mass of fuel in the core in the water moderated system. There are no significant quantities of material present in the current design, making the system more proliferation resistant as it is less attractive to an adversary.

While heavy water is known to be a better moderator than water, due to the low absorption cross section, this design was optimized for a water moderated system. To take advantage of the low absorption cross sections of heavy water, further pin studies will have to be conducted to optimize the design for heavy water moderated system and not light water. Heavy water would be a better candidate for natural uranium fuel, however it does not offer advantages for use in the current design and, if used, it will require an increase the size of the core to offer adequate moderation. The design could also be optimized for the use heavy water. It might be possible to increase the fuel loading in the system and greater enrichments to enhance the ⁹⁹Mo production, however, this could potentially cause a greater proliferation issue depending on the amount of LEU fuel present and the ²³⁹Pu produced in the heavy water optimized design. The current design could also be optimized for a beryllium moderator following the recommendations made for the heavy water moderated system optimization. In

the current design, there is no advantage of using beryllium as water proves to be an efficient system in comparison, as beryllium has worse moderating properties and is more costly than water.

For a seven-day irradiation, Niowave Inc. is capable of producing 1.16E4 Ci (error: ± 5.800 Ci), while the United States' current weekly production need is 94698.5 Ci, assuming a 6-day allocation time for processing of the material. The facility is capable of meeting ~12.25 % of the demand, however, the United States will depend on other facilities, within or outside the country, to supply the remaining amount needed if Niowave does not increase the ⁹⁹Mo production.

Further studies can be conducted to obtain more accurate results. A major parameter affecting these results is the monochromic neutron beam. Niowave Inc. provided the neutron source strength for 2 MeV neutrons, however an energy distribution for the neutron source can be used to obtain more accurate results. This will require the modeling of the converting target and electron LINAC to obtain the neutron source produced in the system. Due to the proprietary nature of the work, this was not conducted. In addition, the core can be optimized for each moderator of interest. Fuel pins can be optimized as well as the amount of moderator in the system for each case.

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APPENDIX I.

FLUX CALCULATIONS PER PIN FOR WATER, HEAVY WATER AND BERYLLIUM FOR A 10.0% ENRICHED FUEL

Thermal flux calculations for individual pins in the fuel region are shown below for each moderator.

| | | | | Heavy | | |
|-----------|------------------------|------------------------|------------|-----------|------------------------|------------------------|
| | Water | Water | Heavy | Water | Beryllium | Beryllium |
| | Flux | ±Error | Water Flux | ±Error | Flux | ±Error |
| Position | (n/cm ² -s) | (n/cm ² -s) | (n/cm2-s) | (n/cm2-s) | (n/cm ² -s) | (n/cm ² -s) |
| [0 0 0] | 9.59E+12 | 7.49E+09 | 1.46E+13 | 1.01E+11 | 4.44E+11 | 1.47E+09 |
| [-1 0 0] | 9.78E+12 | 3.95E+10 | 1.56E+13 | 1.04E+11 | 4.81E+11 | 1.54E+09 |
| [-2 0 0] | 9.91E+12 | 4.00E+10 | 1.12E+13 | 1.09E+11 | 9.72E+10 | 3.71E+08 |
| [-3 0 0] | 1.01E+13 | 7.28E+09 | 1.15E+13 | 1.05E+11 | 1.38E+11 | 4.49E+08 |
| [-4 0 0] | 1.10E+13 | 7.93E+09 | 1.20E+13 | 1.02E+11 | 2.18E+11 | 5.56E+08 |
| [1 0 0] | 9.33E+12 | 7.29E+09 | 9.78E+12 | 9.98E+10 | 7.22E+10 | 3.06E+08 |
| [2 0 0] | 9.03E+12 | 7.05E+09 | 9.31E+12 | 9.31E+10 | 8.42E+10 | 3.33E+08 |
| [3 0 0] | 8.86E+12 | 6.92E+09 | 8.90E+12 | 8.56E+10 | 1.12E+11 | 3.80E+08 |
| [4 0 0] | 9.44E+12 | 7.37E+09 | 8.51E+12 | 7.75E+10 | 1.71E+11 | 4.84E+08 |
| [0 1 0] | 9.42E+12 | 7.36E+09 | 1.00E+13 | 1.02E+11 | 7.30E+10 | 3.10E+08 |
| [-1 1 0] | 9.64E+12 | 7.53E+09 | 1.05E+13 | 1.06E+11 | 7.63E+10 | 3.24E+08 |
| [-2 1 0] | 9.80E+12 | 7.07E+09 | 1.08E+13 | 1.06E+11 | 9.03E+10 | 3.58E+08 |
| [-3 1 0] | 9.95E+12 | 7.17E+09 | 1.13E+13 | 1.07E+11 | 1.21E+11 | 4.10E+08 |
| [-4 1 0] | 1.04E+13 | 7.50E+09 | 1.17E+13 | 1.02E+11 | 1.78E+11 | 5.04E+08 |
| [-5 1 0] | 1.19E+13 | 8.58E+09 | 1.19E+13 | 9.62E+10 | 2.91E+11 | 6.58E+08 |
| [2 1 0] | 8.90E+12 | 6.95E+09 | 9.04E+12 | 8.93E+10 | 1.01E+11 | 3.58E+08 |
| [3 1 0] | 8.99E+12 | 7.02E+09 | 8.63E+12 | 8.08E+10 | 1.42E+11 | 4.22E+08 |
| [4 1 0] | 1.02E+13 | 7.97E+09 | 8.42E+12 | 7.36E+10 | 2.25E+11 | 5.40E+08 |
| [1 1 0] | 9.14E+12 | 7.14E+09 | 9.50E+12 | 9.60E+10 | 8.12E+10 | 3.22E+08 |
| [-1 -1 0] | 9.79E+12 | 7.06E+09 | 1.09E+13 | 1.08E+11 | 9.01E+10 | 3.57E+08 |
| [-2 -1 0] | 9.95E+12 | 7.17E+09 | 1.75E+13 | 1.03E+11 | 6.51E+11 | 1.69E+09 |

| | | | | Heavy | | |
|-----------|------------------------|------------------------|------------|-----------|------------------------|------------------------|
| | Water | Water | Heavy | Water | Beryllium | Beryllium |
| | Flux | ±Error | Water Flux | ±Error | Flux | ±Error |
| Position | (n/cm ² -s) | (n/cm ² -s) | (n/cm2-s) | (n/cm2-s) | (n/cm ² -s) | (n/cm ² -s) |
| [-3 -1 0] | 1.04E+13 | 7.50E+09 | 1.96E+13 | 1.18E+11 | 8.59E+11 | 1.88E+09 |
| [-4 -1 0] | 1.19E+13 | 8.58E+09 | 2.21E+13 | 1.28E+11 | 1.22E+12 | 2.14E+09 |
| [0 -1 0] | 9.64E+12 | 7.53E+09 | 1.50E+13 | 1.01E+11 | 4.72E+11 | 1.51E+09 |
| [5 -1 0] | 1.02E+13 | 7.97E+09 | 1.10E+13 | 8.54E+10 | 9.52E+11 | 1.79E+09 |
| [1 -1 0] | 9.42E+12 | 7.36E+09 | 1.80E+13 | 1.09E+11 | 4.56E+11 | 1.46E+09 |
| [2 -1 0] | 9.14E+12 | 7.14E+09 | 1.34E+13 | 9.54E+10 | 4.82E+11 | 1.45E+09 |
| [3 -1 0] | 8.90E+12 | 6.95E+09 | 1.26E+13 | 9.15E+10 | 5.53E+11 | 1.49E+09 |
| [4 -1 0] | 8.98E+12 | 7.01E+09 | 1.18E+13 | 8.81E+10 | 6.94E+11 | 1.58E+09 |
| [1 -2 0] | 9.41E+12 | 7.35E+09 | 1.43E+13 | 9.87E+10 | 5.06E+11 | 1.52E+09 |
| [2 -2 0] | 9.18E+12 | 7.17E+09 | 1.34E+13 | 9.54E+10 | 5.09E+11 | 1.53E+09 |
| [3 -2 0] | 8.96E+12 | 7.00E+09 | 1.28E+13 | 9.29E+10 | 5.62E+11 | 1.51E+09 |
| [0 -2 0] | 9.59E+12 | 6.91E+09 | 1.51E+13 | 1.02E+11 | 5.43E+11 | 1.59E+09 |
| [-1 -2 0] | 9.77E+12 | 7.04E+09 | 1.61E+13 | 1.06E+11 | 6.39E+11 | 1.66E+09 |
| [-2 -2 0] | 1.01E+13 | 7.28E+09 | 1.71E+13 | 1.09E+11 | 8.11E+11 | 1.85E+09 |
| [-3 -2 0] | 1.14E+13 | 8.22E+09 | 1.80E+13 | 1.13E+11 | 1.12E+12 | 2.11E+09 |
| [4 -2 0] | 8.96E+12 | 7.00E+09 | 1.21E+13 | 8.95E+10 | 6.74E+11 | 1.60E+09 |
| [5 -2 0] | 9.87E+12 | 7.71E+09 | 1.13E+13 | 8.68E+10 | 8.89E+11 | 1.75E+09 |
| [-1 2 0] | 9.41E+12 | 7.35E+09 | 1.43E+13 | 9.87E+10 | 5.05E+11 | 1.52E+09 |
| [-2 2 0] | 9.59E+12 | 6.91E+09 | 1.50E+13 | 1.01E+11 | 5.44E+11 | 1.59E+09 |
| [-3 2 0] | 9.77E+12 | 7.04E+09 | 1.60E+13 | 1.06E+11 | 6.39E+11 | 1.66E+09 |
| [-4 2 0] | 1.01E+13 | 7.28E+09 | 1.71E+13 | 1.09E+11 | 8.10E+11 | 1.85E+09 |
| [-5 2 0] | 1.14E+13 | 8.22E+09 | 1.80E+13 | 1.13E+11 | 1.12E+12 | 2.11E+09 |
| [0 2 0] | 9.18E+12 | 7.17E+09 | 1.35E+13 | 9.50E+10 | 5.09E+11 | 1.53E+09 |
| [1 2 0] | 8.96E+12 | 7.00E+09 | 1.28E+13 | 9.29E+10 | 5.61E+11 | 1.51E+09 |
| [2 2 0] | 8.96E+12 | 7.00E+09 | 1.20E+13 | 8.88E+10 | 6.73E+11 | 1.60E+09 |
| [3 2 0] | 9.88E+12 | 7.72E+09 | 1.13E+13 | 8.68E+10 | 8.90E+11 | 1.75E+09 |
| [-1 3 0] | 9.18E+12 | 7.17E+09 | 1.35E+13 | 9.61E+10 | 5.87E+11 | 1.58E+09 |
| [-2 3 0] | 9.37E+12 | 7.32E+09 | 1.41E+13 | 9.84E+10 | 6.07E+11 | 1.63E+09 |
| [-3 3 0] | 9.61E+12 | 6.93E+09 | 1.48E+13 | 1.01E+11 | 6.80E+11 | 1.71E+09 |
| [-4 3 0] | 1.00E+13 | 7.21E+09 | 1.56E+13 | 1.04E+11 | 8.28E+11 | 1.81E+09 |

| | | | | Heavy | | |
|-----------|------------------------|------------------------|------------|-----------|------------------------|------------------------|
| | Water | Water | Heavy | Water | Beryllium | Beryllium |
| | Flux | ±Error | Water Flux | ±Error | Flux | ±Error |
| Position | (n/cm ² -s) | (n/cm ² -s) | (n/cm2-s) | (n/cm2-s) | (n/cm ² -s) | (n/cm ² -s) |
| [-5 3 0] | 1.12E+13 | 8.08E+09 | 1.61E+13 | 1.07E+11 | 1.10E+12 | 2.07E+09 |
| [0 3 0] | 9.04E+12 | 7.06E+09 | 1.28E+13 | 9.29E+10 | 6.18E+11 | 1.61E+09 |
| [1 3 0] | 9.09E+12 | 7.10E+09 | 1.22E+13 | 9.03E+10 | 7.12E+11 | 1.62E+09 |
| [2 3 0] | 9.92E+12 | 7.75E+09 | 1.15E+13 | 8.75E+10 | 9.00E+11 | 1.77E+09 |
| [-1 -3 0] | 1.00E+13 | 7.21E+09 | 1.56E+13 | 1.04E+11 | 8.28E+11 | 1.81E+09 |
| [-2 -3 0] | 1.12E+13 | 8.08E+09 | 1.60E+13 | 1.07E+11 | 1.10E+12 | 2.07E+09 |
| [5 -3 0] | 9.92E+12 | 7.75E+09 | 1.14E+13 | 8.76E+10 | 9.00E+11 | 1.77E+09 |
| [0 -3 0] | 9.60E+12 | 6.92E+09 | 1.48E+13 | 1.01E+11 | 6.79E+11 | 1.70E+09 |
| [1 -3 0] | 9.37E+12 | 7.32E+09 | 1.41E+13 | 9.73E+10 | 6.06E+11 | 1.63E+09 |
| [2 -3 0] | 9.18E+12 | 7.17E+09 | 1.35E+13 | 9.61E+10 | 5.87E+11 | 1.58E+09 |
| [3 -3 0] | 9.04E+12 | 7.06E+09 | 1.28E+13 | 9.29E+10 | 6.18E+11 | 1.61E+09 |
| [4 -3 0] | 9.09E+12 | 7.10E+09 | 1.21E+13 | 8.95E+10 | 7.12E+11 | 1.62E+09 |
| [-1 4 0] | 9.27E+12 | 7.24E+09 | 1.27E+13 | 9.22E+10 | 7.42E+11 | 1.69E+09 |
| [-2 4 0] | 9.36E+12 | 7.31E+09 | 1.34E+13 | 9.62E+10 | 7.33E+11 | 1.67E+09 |
| [-3 4 0] | 9.62E+12 | 6.94E+09 | 1.39E+13 | 9.69E+10 | 7.89E+11 | 1.80E+09 |
| [-4 4 0] | 1.03E+13 | 7.43E+09 | 1.43E+13 | 9.97E+10 | 9.29E+11 | 1.91E+09 |
| [-5 4 0] | 1.14E+13 | 8.22E+09 | 1.48E+13 | 1.03E+11 | 1.16E+12 | 2.08E+09 |
| [0 4 0] | 9.65E+12 | 7.54E+09 | 1.21E+13 | 8.95E+10 | 8.25E+11 | 1.73E+09 |
| [1 4 0] | 1.04E+13 | 8.12E+09 | 1.14E+13 | 8.76E+10 | 9.84E+11 | 1.85E+09 |
| [-1 -4 0] | 1.14E+13 | 8.22E+09 | 1.47E+13 | 1.02E+11 | 1.16E+12 | 2.08E+09 |
| [0 -4 0] | 1.03E+13 | 7.43E+09 | 1.44E+13 | 1.00E+11 | 9.27E+11 | 1.91E+09 |
| [1 -4 0] | 9.62E+12 | 6.94E+09 | 1.38E+13 | 9.72E+10 | 7.89E+11 | 1.80E+09 |
| [2 -4 0] | 9.36E+12 | 7.31E+09 | 1.33E+13 | 9.55E+10 | 7.34E+11 | 1.67E+09 |
| [3 -4 0] | 9.27E+12 | 7.24E+09 | 1.27E+13 | 9.22E+10 | 7.42E+11 | 1.69E+09 |
| [4 -4 0] | 9.65E+12 | 7.54E+09 | 1.20E+13 | 8.96E+10 | 8.25E+11 | 1.73E+09 |
| [5 -4 0] | 1.04E+13 | 8.12E+09 | 1.14E+13 | 8.76E+10 | 9.84E+11 | 1.85E+09 |
| [-1 5 0] | 1.05E+13 | 8.20E+09 | 1.19E+13 | 8.97E+10 | 1.02E+12 | 1.92E+09 |
| [-2 5 0] | 1.03E+13 | 8.04E+09 | 1.24E+13 | 9.18E+10 | 9.71E+11 | 1.91E+09 |
| [-3 5 0] | 1.05E+13 | 7.57E+09 | 1.29E+13 | 9.35E+10 | 9.99E+11 | 1.88E+09 |
| [-4 5 0] | 1.10E+13 | 7.93E+09 | 1.33E+13 | 9.55E+10 | 1.11E+12 | 1.99E+09 |

| | | | | Heavy | | |
|----------|------------------------|------------------------|------------|-----------|------------------------|------------------------|
| | Water | Water | Heavy | Water | Beryllium | Beryllium |
| | Flux | ±Error | Water Flux | ±Error | Flux | ±Error |
| Position | (n/cm ² -s) | (n/cm ² -s) | (n/cm2-s) | (n/cm2-s) | (n/cm ² -s) | (n/cm ² -s) |
| [1 -5 0] | 1.10E+13 | 7.93E+09 | 1.33E+13 | 9.64E+10 | 1.11E+12 | 1.99E+09 |
| [2 -5 0] | 1.05E+13 | 7.57E+09 | 1.29E+13 | 9.44E+10 | 9.99E+11 | 1.88E+09 |
| [3 -5 0] | 1.03E+13 | 8.04E+09 | 1.25E+13 | 9.25E+10 | 9.71E+11 | 1.91E+09 |
| [4 -5 0] | 1.05E+13 | 8.20E+09 | 1.18E+13 | 8.99E+10 | 1.02E+12 | 1.92E+09 |

APPENDIX II.

POWER CALCULATIONS PER PIN FOR WATER, HEAVY WATER AND BERYLLIUM FOR A 10.0% ENRICHED FUEL

Power calculations for individual pins in the fuel region are shown below for each moderator.

| | | | Heavy | Heavy Water | Beryllium | Beryllium |
|-----------|----------|----------|----------|-------------|-----------|-----------|
| | Water | Water | Water | ±Error | (MWth) | ±Error |
| | Power | ±Error | Power | (MWth) | | (MWth) |
| Position | (MWth) | (MWth) | (MWth) | | | |
| [0 0 0] | 3.12E-03 | 3.74E-05 | 3.63E-04 | 3.63E-07 | 1.62E-04 | 1.62E-07 |
| [-1 0 0] | 3.18E-03 | 3.82E-05 | 3.84E-04 | 3.84E-07 | 1.76E-04 | 1.76E-07 |
| [-2 0 0] | 3.23E-03 | 3.88E-05 | 4.30E-04 | 4.30E-07 | 2.05E-04 | 2.05E-07 |
| [-3 0 0] | 3.30E-03 | 3.96E-05 | 4.97E-04 | 4.97E-07 | 2.59E-04 | 2.59E-07 |
| [-4 0 0] | 3.61E-03 | 4.33E-05 | 6.64E-04 | 6.64E-07 | 3.73E-04 | 3.73E-07 |
| [1 0 0] | 3.03E-03 | 3.64E-05 | 3.62E-04 | 3.62E-07 | 1.63E-04 | 1.63E-07 |
| [2 0 0] | 2.94E-03 | 3.53E-05 | 3.78E-04 | 3.78E-07 | 1.75E-04 | 1.75E-07 |
| [3 0 0] | 2.88E-03 | 3.46E-05 | 4.25E-04 | 4.25E-07 | 2.06E-04 | 2.06E-07 |
| [4 0 0] | 3.06E-03 | 3.67E-05 | 5.13E-04 | 5.13E-07 | 2.66E-04 | 2.66E-07 |
| [0 1 0] | 3.06E-03 | 3.67E-05 | 3.67E-04 | 3.67E-07 | 1.66E-04 | 1.66E-07 |
| [-1 1 0] | 3.14E-03 | 3.77E-05 | 3.76E-04 | 3.76E-07 | 1.73E-04 | 1.73E-07 |
| [-2 1 0] | 3.19E-03 | 3.83E-05 | 4.07E-04 | 4.07E-07 | 1.94E-04 | 1.94E-07 |
| [-3 1 0] | 3.24E-03 | 3.89E-05 | 4.66E-04 | 4.66E-07 | 2.35E-04 | 2.35E-07 |
| [-4 1 0] | 3.38E-03 | 4.06E-05 | 5.74E-04 | 5.74E-07 | 3.07E-04 | 3.07E-07 |
| [-5 1 0] | 3.88E-03 | 4.66E-05 | 7.60E-04 | 7.60E-07 | 4.33E-04 | 4.33E-07 |
| [2 1 0] | 2.89E-03 | 3.47E-05 | 4.00E-04 | 4.00E-07 | 1.94E-04 | 1.94E-07 |
| [3 1 0] | 2.92E-03 | 3.50E-05 | 4.68E-04 | 4.68E-07 | 2.38E-04 | 2.38E-07 |
| [4 1 0] | 3.31E-03 | 3.97E-05 | 6.00E-04 | 6.00E-07 | 3.20E-04 | 3.20E-07 |
| [1 1 0] | 2.97E-03 | 3.56E-05 | 3.71E-04 | 3.71E-07 | 1.73E-04 | 1.73E-07 |
| [-1 -1 0] | 3.19E-03 | 3.83E-05 | 4.06E-04 | 4.06E-07 | 1.94E-04 | 1.94E-07 |

| | | | | | Beryllium | |
|-----------|------------------------|------------------------|------------|-------------|----------------------|------------------------|
| | Water | Water | Heavy | Heavy Water | Flux | Beryllium |
| | Flux | ±Error | Water Flux | ±Error | (n/cm ² - | ±Error |
| Position | (n/cm ² -s) | (n/cm ² -s) | (n/cm2-s) | (n/cm2-s) | s) | (n/cm ² -s) |
| [-2 -1 0] | 3.24E-03 | 3.89E-05 | 4.63E-04 | 4.63E-07 | 2.34E-04 | 2.34E-07 |
| [-3 -1 0] | 3.38E-03 | 4.06E-05 | 5.70E-04 | 5.70E-07 | 3.07E-04 | 3.07E-07 |
| [-4 -1 0] | 3.88E-03 | 4.66E-05 | 7.65E-04 | 7.65E-07 | 4.33E-04 | 4.33E-07 |
| [0 -1 0] | 3.14E-03 | 3.77E-05 | 3.79E-04 | 3.79E-07 | 1.73E-04 | 1.73E-07 |
| [5 -1 0] | 3.31E-03 | 3.97E-05 | 6.01E-04 | 6.01E-07 | 3.20E-04 | 3.20E-07 |
| [1 -1 0] | 3.06E-03 | 3.67E-05 | 3.69E-04 | 3.69E-07 | 1.66E-04 | 1.66E-07 |
| [2 -1 0] | 2.97E-03 | 3.56E-05 | 3.75E-04 | 3.75E-07 | 1.73E-04 | 1.73E-07 |
| [3 -1 0] | 2.89E-03 | 3.47E-05 | 4.03E-04 | 4.03E-07 | 1.94E-04 | 1.94E-07 |
| [4 -1 0] | 2.92E-03 | 3.50E-05 | 4.71E-04 | 4.71E-07 | 2.38E-04 | 2.38E-07 |
| [1 -2 0] | 3.06E-03 | 3.67E-05 | 3.89E-04 | 3.89E-07 | 1.82E-04 | 1.82E-07 |
| [2 -2 0] | 2.98E-03 | 3.58E-05 | 3.88E-04 | 3.88E-07 | 1.81E-04 | 1.81E-07 |
| [3 -2 0] | 2.91E-03 | 3.49E-05 | 4.11E-04 | 4.11E-07 | 1.97E-04 | 1.97E-07 |
| [0 -2 0] | 3.12E-03 | 3.74E-05 | 4.09E-04 | 4.09E-07 | 1.95E-04 | 1.95E-07 |
| [-1 -2 0] | 3.18E-03 | 3.82E-05 | 4.57E-04 | 4.57E-07 | 2.27E-04 | 2.27E-07 |
| [-2 -2 0] | 3.29E-03 | 3.95E-05 | 5.44E-04 | 5.44E-07 | 2.84E-04 | 2.84E-07 |
| [-3 -2 0] | 3.70E-03 | 4.44E-05 | 6.96E-04 | 6.96E-07 | 3.86E-04 | 3.86E-07 |
| [4 -2 0] | 2.91E-03 | 3.49E-05 | 4.60E-04 | 4.60E-07 | 2.31E-04 | 2.31E-07 |
| [5 -2 0] | 3.20E-03 | 3.84E-05 | 5.68E-04 | 5.68E-07 | 3.00E-04 | 3.00E-07 |
| [-1 2 0] | 3.06E-03 | 3.67E-05 | 3.88E-04 | 3.88E-07 | 1.82E-04 | 1.82E-07 |
| [-2 2 0] | 3.12E-03 | 3.74E-05 | 4.10E-04 | 4.10E-07 | 1.95E-04 | 1.95E-07 |
| [-3 2 0] | 3.18E-03 | 3.82E-05 | 4.56E-04 | 4.56E-07 | 2.27E-04 | 2.27E-07 |
| [-4 2 0] | 3.29E-03 | 3.95E-05 | 5.46E-04 | 5.46E-07 | 2.84E-04 | 2.84E-07 |
| [-5 2 0] | 3.70E-03 | 4.44E-05 | 6.94E-04 | 6.94E-07 | 3.86E-04 | 3.86E-07 |
| [0 2 0] | 2.98E-03 | 3.58E-05 | 3.92E-04 | 3.92E-07 | 1.81E-04 | 1.81E-07 |
| [1 2 0] | 2.91E-03 | 3.49E-05 | 4.10E-04 | 4.10E-07 | 1.97E-04 | 1.97E-07 |
| [2 2 0] | 2.91E-03 | 3.49E-05 | 4.64E-04 | 4.64E-07 | 2.31E-04 | 2.31E-07 |
| [3 2 0] | 3.20E-03 | 3.84E-05 | 5.60E-04 | 5.60E-07 | 3.00E-04 | 3.00E-07 |
| [-1 3 0] | 2.98E-03 | 3.58E-05 | 4.23E-04 | 4.23E-07 | 2.06E-04 | 2.06E-07 |
| [-2 3 0] | 3.05E-03 | 3.66E-05 | 4.39E-04 | 4.39E-07 | 2.14E-04 | 2.14E-07 |
| [-3 3 0] | 3.12E-03 | 3.74E-05 | 4.68E-04 | 4.68E-07 | 2.38E-04 | 2.38E-07 |

| | | | | | Beryllium | |
|-----------|------------------------|------------------------|------------|-------------|----------------------|------------------------|
| | Water | Water | Heavy | Heavy Water | Flux | Beryllium |
| | Flux | ±Error | Water Flux | ±Error | (n/cm ² - | ±Error |
| Position | (n/cm ² -s) | (n/cm ² -s) | (n/cm2-s) | (n/cm2-s) | s) | (n/cm ² -s) |
| [-4 3 0] | 3.25E-03 | 3.90E-05 | 5.43E-04 | 5.43E-07 | 2.87E-04 | 2.87E-07 |
| [-5 3 0] | 3.63E-03 | 4.36E-05 | 6.91E-04 | 6.91E-07 | 3.75E-04 | 3.75E-07 |
| [0 3 0] | 2.94E-03 | 3.53E-05 | 4.36E-04 | 4.36E-07 | 2.15E-04 | 2.15E-07 |
| [1 3 0] | 2.95E-03 | 3.54E-05 | 4.83E-04 | 4.83E-07 | 2.44E-04 | 2.44E-07 |
| [2 3 0] | 3.22E-03 | 3.86E-05 | 5.75E-04 | 5.75E-07 | 3.04E-04 | 3.04E-07 |
| [-1 -3 0] | 3.25E-03 | 3.90E-05 | 5.46E-04 | 5.46E-07 | 2.87E-04 | 2.87E-07 |
| [-2 -3 0] | 3.63E-03 | 4.36E-05 | 6.76E-04 | 6.76E-07 | 3.75E-04 | 3.75E-07 |
| [5 -3 0] | 3.22E-03 | 3.86E-05 | 5.71E-04 | 5.71E-07 | 3.03E-04 | 3.03E-07 |
| [0 -3 0] | 3.12E-03 | 3.74E-05 | 4.73E-04 | 4.73E-07 | 2.38E-04 | 2.38E-07 |
| [1 -3 0] | 3.05E-03 | 3.66E-05 | 4.39E-04 | 4.39E-07 | 2.13E-04 | 2.13E-07 |
| [2 -3 0] | 2.98E-03 | 3.58E-05 | 4.19E-04 | 4.19E-07 | 2.06E-04 | 2.06E-07 |
| [3 -3 0] | 2.94E-03 | 3.53E-05 | 4.33E-04 | 4.33E-07 | 2.15E-04 | 2.15E-07 |
| [4 -3 0] | 2.95E-03 | 3.54E-05 | 4.75E-04 | 4.75E-07 | 2.44E-04 | 2.44E-07 |
| [-1 4 0] | 3.01E-03 | 3.61E-05 | 4.97E-04 | 4.97E-07 | 2.54E-04 | 2.54E-07 |
| [-2 4 0] | 3.04E-03 | 3.65E-05 | 4.94E-04 | 4.94E-07 | 2.52E-04 | 2.52E-07 |
| [-3 4 0] | 3.12E-03 | 3.74E-05 | 5.16E-04 | 5.16E-07 | 2.71E-04 | 2.71E-07 |
| [-4 4 0] | 3.36E-03 | 4.03E-05 | 5.93E-04 | 5.93E-07 | 3.17E-04 | 3.17E-07 |
| [-5 4 0] | 3.70E-03 | 4.44E-05 | 7.06E-04 | 7.06E-07 | 3.93E-04 | 3.93E-07 |
| [0 4 0] | 3.13E-03 | 3.76E-05 | 5.39E-04 | 5.39E-07 | 2.80E-04 | 2.80E-07 |
| [1 4 0] | 3.36E-03 | 4.03E-05 | 6.17E-04 | 6.17E-07 | 3.31E-04 | 3.31E-07 |
| [-1 -4 0] | 3.70E-03 | 4.44E-05 | 7.04E-04 | 7.04E-07 | 3.93E-04 | 3.93E-07 |
| [0 -4 0] | 3.36E-03 | 4.03E-05 | 5.86E-04 | 5.86E-07 | 3.17E-04 | 3.17E-07 |
| [1 -4 0] | 3.12E-03 | 3.74E-05 | 5.21E-04 | 5.21E-07 | 2.71E-04 | 2.71E-07 |
| [2 -4 0] | 3.04E-03 | 3.65E-05 | 4.95E-04 | 4.95E-07 | 2.52E-04 | 2.52E-07 |
| [3 -4 0] | 3.01E-03 | 3.61E-05 | 4.96E-04 | 4.96E-07 | 2.54E-04 | 2.54E-07 |
| [4 -4 0] | 3.13E-03 | 3.76E-05 | 5.29E-04 | 5.29E-07 | 2.80E-04 | 2.80E-07 |
| [5 -4 0] | 3.36E-03 | 4.03E-05 | 6.17E-04 | 6.17E-07 | 3.31E-04 | 3.31E-07 |
| [-1 5 0] | 3.42E-03 | 4.10E-05 | 6.34E-04 | 6.34E-07 | 3.44E-04 | 3.44E-07 |
| [-2 5 0] | 3.34E-03 | 4.01E-05 | 6.08E-04 | 6.08E-07 | 3.28E-04 | 3.28E-07 |
| [-3 5 0] | 3.39E-03 | 4.07E-05 | 6.28E-04 | 6.28E-07 | 3.37E-04 | 3.37E-07 |

| [-4 5 0] | 3.58E-03 | 4.30E-05 | 6.69E-04 | 6.69E-07 | 3.74E-04 | 3.74E-07 |
|----------|----------|----------|----------|----------|----------|----------|
| [1 -5 0] | 3.58E-03 | 4.30E-05 | 6.83E-04 | 6.83E-07 | 3.74E-04 | 3.74E-07 |
| [2 -5 0] | 3.39E-03 | 4.07E-05 | 6.21E-04 | 6.21E-07 | 3.38E-04 | 3.38E-07 |
| [3 -5 0] | 3.34E-03 | 4.01E-05 | 6.04E-04 | 6.04E-07 | 3.28E-04 | 3.28E-07 |
| [4 -5 0] | 3.42E-03 | 4.10E-05 | 6.31E-04 | 6.31E-07 | 3.43E-04 | 3.43E-07 |

APPENDIX III.

HEAVY WATER DEPLETION STUDIES FOR EACH ISOTOPE OF INTEREST FOR A 10.0% ENRICHED FUEL

















| Time (Day) | ²³⁹ Pu Mass (g) | Error (g) |
|------------|----------------------------|-----------|
| 0.1 | 8.44E-06 | ±4.22E-09 |
| 0.4 | 1.83E-04 | ±9.15E-08 |
| 0.7 | 5.72E-04 | ±2.86E-07 |
| 1 | 1.16E-03 | ±5.80E-07 |
| 2 | 4.29E-03 | ±2.15E-06 |
| 3 | 8.88E-03 | ±4.44E-06 |
| 4 | 1.46E-02 | ±7.30E-06 |
| 5 | 2.11E-02 | ±1.06E-05 |
| 6 | 2.82E-02 | ±1.41E-05 |
| 7 | 3.57E-02 | ±1.79E-05 |
| 14 | 5.90E-02 | ±2.95E-05 |
| 21 | 6.20E-02 | ±3.10E-05 |

²³⁹Pu Production for Heavy Water Moderated System

With the 10.0% enriched fuel and a heavy water moderator, it is not viable to produce the isotope of interest in enough quantities compared to water moderated system. Larger volumes of heavy-water moderator should be considered due to the larger number of collisions that need to happen in the heavy water moderator in addition to the amount of leakage in the current design. Again, very little amounts of ²³⁹Pu are present in the system. The design reaches a burnup of 3.53E-2 GWd/MTU for the seven day irradiation.

APPENDIX IV.

BERYLLIUM DEPLETION STUDIES FOR EACH ISOTOPE OF INTEREST FOR A 10.0% ENRICHED FUEL



















| Time (Day) | ²³⁹ Pu Mass (g) | Error (g) |
|------------|----------------------------|-----------|
| 0.1 | 4.80E-06 | ±2.40E-09 |
| 0.4 | 1.04E-04 | ±5.20E-08 |
| 0.7 | 3.26E-04 | ±1.63E-07 |
| 1 | 4.37E-03 | ±2.19E-06 |
| 2 | 2.43E-03 | ±1.22E-06 |
| 3 | 5.04E-03 | ±2.52E-06 |
| 4 | 8.27E-03 | ±4.14E-06 |
| 5 | 1.20E-02 | ±6.00E-06 |
| 6 | 1.60E-02 | ±8.00E-06 |
| 7 | 2.03E-02 | ±1.02E-05 |
| 14 | 3.35E-02 | ±1.68E-05 |
| 21 | 2.30E-06 | ±1.15E-09 |

²³⁹Pu Production for Beryllium Moderated System

Beryllium is a worse moderator candidate when compared to heavy water and water. Again, moderation is low for this system and if considering beryllium as a potential moderator, further studies need to be conducted to increase the amount of fission in the core. Again, very little ²³⁹Pu is produced in the system. The design reaches a burnup of 1.818E-2 GWd/MTU for the seven day irradiation.

APPENDIX V.

FLUX CALCULATIONS PER PIN FOR HEAVY WATER AND BERYLLIUM FOR A 19.9% ENRICHED FUEL

| | Heavy Water | Heavy Water | Beryllium | Beryllium |
|-----------|------------------------|------------------------|------------------------|------------------------|
| | Thermal Flux | Thermal ±Error | Thermal Flux | ±Error |
| Position | (n/cm ² -s) | (n/cm ² -s) | (n/cm ² -s) | (n/cm ² -s) |
| [0 0 0] | 1.03E+14 | 3.72E+10 | 6.89E+13 | 2.49E+10 |
| [-1 0 0] | 7.42E+11 | 1.67E+09 | 3.85E+11 | 1.22E+09 |
| [-2 0 0] | 9.71E+10 | 2.75E+08 | 4.82E+10 | 1.91E+08 |
| [-3 0 0] | 1.19E+11 | 3.03E+08 | 6.44E+10 | 2.28E+08 |
| [-4 0 0] | 1.67E+11 | 3.77E+08 | 1.03E+11 | 2.91E+08 |
| [1 0 0] | 2.74E+11 | 4.66E+08 | 2.01E+11 | 3.98E+08 |
| [2 0 0] | 9.28E+10 | 2.63E+08 | 4.52E+10 | 1.79E+08 |
| [3 0 0] | 1.09E+11 | 2.78E+08 | 5.59E+10 | 1.98E+08 |
| [4 0 0] | 1.47E+11 | 3.32E+08 | 8.50E+10 | 2.52E+08 |
| [0 1 0] | 2.35E+11 | 4.32E+08 | 1.59E+11 | 3.37E+08 |
| [-1 1 0] | 9.39E+10 | 2.66E+08 | 4.56E+10 | 1.81E+08 |
| [-2 1 0] | 9.61E+10 | 2.72E+08 | 4.72E+10 | 1.87E+08 |
| [-3 1 0] | 1.10E+11 | 2.96E+08 | 5.84E+10 | 2.15E+08 |
| [-4 1 0] | 1.45E+11 | 3.48E+08 | 8.52E+10 | 2.65E+08 |
| [-5 1 0] | 2.19E+11 | 4.34E+08 | 1.49E+11 | 3.58E+08 |
| [2 1 0] | 3.73E+11 | 5.82E+08 | 3.01E+11 | 5.12E+08 |
| [3 1 0] | 1.31E+11 | 3.14E+08 | 7.19E+10 | 2.24E+08 |
| [4 1 0] | 1.90E+11 | 3.76E+08 | 1.20E+11 | 2.88E+08 |
| [1 1 0] | 3.18E+11 | 4.96E+08 | 2.36E+11 | 4.34E+08 |
| [-1 -1 0] | 1.04E+11 | 2.80E+08 | 5.23E+10 | 1.92E+08 |
| [-2 -1 0] | 1.01E+12 | 1.86E+09 | 6.04E+11 | 1.44E+09 |
| [-3 -1 0] | 1.31E+12 | 1.99E+09 | 8.90E+11 | 1.64E+09 |
| [-4 -1 0] | 1.87E+12 | 2.26E+09 | 1.51E+12 | 2.02E+09 |
| [0 -1 0] | 7.75E+11 | 1.64E+09 | 4.10E+11 | 1.22E+09 |

| [5 -1 0] | 1.61E+12 | 1.95E+09 | 1.18E+12 | 1.69E+09 |
|-----------|----------|----------|----------|----------|
| [1 -1 0] | 7.60E+11 | 1.61E+09 | 3.98E+11 | 1.18E+09 |
| [2 -1 0] | 8.02E+11 | 1.62E+09 | 4.25E+11 | 1.19E+09 |
| [3 -1 0] | 9.16E+11 | 1.69E+09 | 5.14E+11 | 1.22E+09 |
| [4 -1 0] | 1.15E+12 | 1.75E+09 | 7.19E+11 | 1.35E+09 |
| [1 -2 0] | 8.23E+11 | 1.66E+09 | 4.44E+11 | 1.24E+09 |
| [2 -2 0] | 8.37E+11 | 1.62E+09 | 4.53E+11 | 1.22E+09 |
| [3 -2 0] | 9.24E+11 | 1.70E+09 | 5.20E+11 | 1.24E+09 |
| [0 -2 0] | 8.70E+11 | 1.68E+09 | 4.85E+11 | 1.29E+09 |
| [-1 -2 0] | 9.96E+11 | 1.83E+09 | 5.93E+11 | 1.41E+09 |
| [-2 -2 0] | 1.24E+12 | 1.88E+09 | 8.23E+11 | 1.51E+09 |
| [-3 -2 0] | 1.71E+12 | 2.07E+09 | 1.33E+12 | 1.90E+09 |
| [4 -2 0] | 1.11E+12 | 1.69E+09 | 6.84E+11 | 1.35E+09 |
| [5 -2 0] | 1.49E+12 | 1.86E+09 | 1.06E+12 | 1.52E+09 |
| [-1 2 0] | 8.25E+11 | 1.67E+09 | 4.45E+11 | 1.24E+09 |
| [-2 2 0] | 8.71E+11 | 1.68E+09 | 4.85E+11 | 1.29E+09 |
| [-3 2 0] | 9.98E+11 | 1.84E+09 | 5.91E+11 | 1.41E+09 |
| [-4 2 0] | 1.24E+12 | 1.88E+09 | 8.21E+11 | 1.51E+09 |
| [-5 2 0] | 1.72E+12 | 2.08E+09 | 1.33E+12 | 1.90E+09 |
| [0 2 0] | 8.37E+11 | 1.62E+09 | 4.53E+11 | 1.22E+09 |
| [1 2 0] | 9.24E+11 | 1.70E+09 | 5.21E+11 | 1.24E+09 |
| [2 2 0] | 1.11E+12 | 1.69E+09 | 6.84E+11 | 1.35E+09 |
| [3 2 0] | 1.49E+12 | 1.86E+09 | 1.06E+12 | 1.52E+09 |
| [-1 3 0] | 9.47E+11 | 1.74E+09 | 5.44E+11 | 1.29E+09 |
| [-2 3 0] | 9.65E+11 | 1.78E+09 | 5.62E+11 | 1.34E+09 |
| [-3 3 0] | 1.06E+12 | 1.81E+09 | 6.52E+11 | 1.40E+09 |
| [-4 3 0] | 1.28E+12 | 1.95E+09 | 8.58E+11 | 1.58E+09 |
| [-5 3 0] | 1.70E+12 | 2.06E+09 | 1.31E+12 | 1.87E+09 |
| [0 3 0] | 1.00E+12 | 1.75E+09 | 5.93E+11 | 1.27E+09 |
| [1 3 0] | 1.17E+12 | 1.78E+09 | 7.39E+11 | 1.39E+09 |
| [2 3 0] | 1.50E+12 | 1.88E+09 | 1.07E+12 | 1.53E+09 |
| [-1 -3 0] | 1.28E+12 | 1.95E+09 | 8.58E+11 | 1.58E+09 |
| [-2 -3 0] | 1.70E+12 | 2.06E+09 | 1.31E+12 | 1.87E+09 |
| [5 -3 0] | 1.50E+12 | 1.88E+09 | 1.07E+12 | 1.53E+09 |

| [0 -3 0] | 1.06E+12 | 1.81E+09 | 6.51E+11 | 1.40E+09 |
|-----------|----------|----------|----------|----------|
| [1 -3 0] | 9.65E+11 | 1.78E+09 | 5.61E+11 | 1.34E+09 |
| [2 -3 0] | 9.47E+11 | 1.74E+09 | 5.43E+11 | 1.29E+09 |
| [3 -3 0] | 1.00E+12 | 1.75E+09 | 5.92E+11 | 1.27E+09 |
| [4 -3 0] | 1.16E+12 | 1.76E+09 | 7.39E+11 | 1.39E+09 |
| [-1 4 0] | 1.19E+12 | 1.81E+09 | 7.70E+11 | 1.45E+09 |
| [-2 4 0] | 1.16E+12 | 1.76E+09 | 7.42E+11 | 1.46E+09 |
| [-3 4 0] | 1.24E+12 | 1.88E+09 | 8.16E+11 | 1.50E+09 |
| [-4 4 0] | 1.45E+12 | 1.94E+09 | 1.03E+12 | 1.71E+09 |
| [-5 4 0] | 1.81E+12 | 2.19E+09 | 1.43E+12 | 1.92E+09 |
| [0 4 0] | 1.35E+12 | 1.93E+09 | 9.21E+11 | 1.53E+09 |
| [1 4 0] | 1.63E+12 | 1.97E+09 | 1.22E+12 | 1.63E+09 |
| [-1 -4 0] | 1.81E+12 | 2.19E+09 | 1.43E+12 | 1.92E+09 |
| [0 -4 0] | 1.45E+12 | 1.94E+09 | 1.03E+12 | 1.71E+09 |
| [1 -4 0] | 1.24E+12 | 1.88E+09 | 8.15E+11 | 1.50E+09 |
| [2 -4 0] | 1.16E+12 | 1.76E+09 | 7.43E+11 | 1.46E+09 |
| [3 -4 0] | 1.19E+12 | 1.81E+09 | 7.68E+11 | 1.44E+09 |
| [4 -4 0] | 1.35E+12 | 1.93E+09 | 9.21E+11 | 1.53E+09 |
| [5 -4 0] | 1.64E+12 | 1.98E+09 | 1.22E+12 | 1.63E+09 |
| [-1 5 0] | 1.67E+12 | 2.02E+09 | 1.26E+12 | 1.69E+09 |
| [-2 5 0] | 1.57E+12 | 1.90E+09 | 1.16E+12 | 1.66E+09 |
| [-3 5 0] | 1.60E+12 | 1.94E+09 | 1.19E+12 | 1.70E+09 |
| [-4 5 0] | 1.76E+12 | 2.13E+09 | 1.37E+12 | 1.84E+09 |
| [1 -5 0] | 1.76E+12 | 2.13E+09 | 1.37E+12 | 1.84E+09 |
| [2 -5 0] | 1.60E+12 | 1.94E+09 | 1.19E+12 | 1.70E+09 |
| [3 -5 0] | 1.57E+12 | 1.90E+09 | 1.15E+12 | 1.64E+09 |
| [4 -5 0] | 1.67E+12 | 2.02E+09 | 1.26E+12 | 1.69E+09 |

APPENDIX VI.

POWER CALCULATIONS PER PIN FOR HEAVY WATER AND BERYLLIUM FOR A 19.9% ENRICHED FUEL

| | Heavy Water | Heavy Water | Beryllium | Beryllium |
|-----------|--------------|---------------|--------------|---------------|
| Position | Power (MWth) | ±Error (MWth) | Power (MWth) | ±Error (MWth) |
| [0 0 0] | 6.03E-04 | 6.03E-08 | 3.69E-04 | 3.69E-08 |
| [-1 0 0] | 6.34E-04 | 6.34E-08 | 3.94E-04 | 3.94E-08 |
| [-2 0 0] | 7.00E-04 | 7.00E-08 | 4.47E-04 | 4.47E-08 |
| [-3 0 0] | 8.32E-04 | 8.32E-08 | 5.62E-04 | 5.62E-08 |
| [-4 0 0] | 1.11E-03 | 1.11E-07 | 8.40E-04 | 8.40E-08 |
| [1 0 0] | 6.05E-04 | 6.05E-08 | 3.66E-04 | 3.66E-08 |
| [2 0 0] | 6.39E-04 | 6.39E-08 | 3.87E-04 | 3.87E-08 |
| [3 0 0] | 7.26E-04 | 7.26E-08 | 4.54E-04 | 4.54E-08 |
| [4 0 0] | 9.21E-04 | 9.21E-08 | 6.29E-04 | 6.29E-08 |
| [0 1 0] | 6.11E-04 | 6.11E-08 | 3.72E-04 | 3.72E-08 |
| [-1 1 0] | 6.26E-04 | 6.26E-08 | 3.86E-04 | 3.86E-08 |
| [-2 1 0] | 6.74E-04 | 6.74E-08 | 4.24E-04 | 4.24E-08 |
| [-3 1 0] | 7.72E-04 | 7.72E-08 | 5.06E-04 | 5.06E-08 |
| [-4 1 0] | 9.59E-04 | 9.59E-08 | 6.82E-04 | 6.82E-08 |
| [-5 1 0] | 1.32E-03 | 1.32E-07 | 1.06E-03 | 1.06E-07 |
| [2 1 0] | 6.90E-04 | 6.90E-08 | 4.24E-04 | 4.24E-08 |
| [3 1 0] | 8.23E-04 | 8.23E-08 | 5.38E-04 | 5.38E-08 |
| [4 1 0] | 1.10E-03 | 1.10E-07 | 8.06E-04 | 8.06E-08 |
| [1 1 0] | 6.29E-04 | 6.29E-08 | 3.81E-04 | 3.81E-08 |
| [-1 -1 0] | 6.74E-04 | 6.74E-08 | 4.24E-04 | 4.24E-08 |
| [-2 -1 0] | 7.72E-04 | 7.72E-08 | 5.07E-04 | 5.07E-08 |
| [-3 -1 0] | 9.59E-04 | 9.59E-08 | 6.82E-04 | 6.82E-08 |
| [-4 -1 0] | 1.32E-03 | 1.32E-07 | 1.06E-03 | 1.06E-07 |
| [0 -1 0] | 6.25E-04 | 6.25E-08 | 3.86E-04 | 3.86E-08 |
| [5 -1 0] | 1.10E-03 | 1.10E-07 | 8.06E-04 | 8.06E-08 |

| [1 -1 0] | 6.11E-04 | 6.11E-08 | 3.73E-04 | 3.73E-08 |
|-----------|----------|----------|----------|----------|
| [2 -1 0] | 6.29E-04 | 6.29E-08 | 3.82E-04 | 3.82E-08 |
| [3 -1 0] | 6.90E-04 | 6.90E-08 | 4.25E-04 | 4.25E-08 |
| [4 -1 0] | 8.24E-04 | 8.24E-08 | 5.38E-04 | 5.38E-08 |
| [1 -2 0] | 6.48E-04 | 6.48E-08 | 4.00E-04 | 4.00E-08 |
| [2 -2 0] | 6.50E-04 | 6.50E-08 | 3.99E-04 | 3.99E-08 |
| [3 -2 0] | 6.96E-04 | 6.96E-08 | 4.30E-04 | 4.30E-08 |
| [0 -2 0] | 6.80E-04 | 6.80E-08 | 4.28E-04 | 4.28E-08 |
| [-1 -2 0] | 7.58E-04 | 7.58E-08 | 4.94E-04 | 4.94E-08 |
| [-2 -2 0] | 9.08E-04 | 9.08E-08 | 6.32E-04 | 6.32E-08 |
| [-3 -2 0] | 1.20E-03 | 1.20E-07 | 9.37E-04 | 9.37E-08 |
| [4 -2 0] | 8.01E-04 | 8.01E-08 | 5.19E-04 | 5.19E-08 |
| [5 -2 0] | 1.03E-03 | 1.03E-07 | 7.37E-04 | 7.37E-08 |
| [-1 2 0] | 6.49E-04 | 6.49E-08 | 4.00E-04 | 4.00E-08 |
| [-2 2 0] | 6.80E-04 | 6.80E-08 | 4.28E-04 | 4.28E-08 |
| [-3 2 0] | 7.59E-04 | 7.59E-08 | 4.93E-04 | 4.93E-08 |
| [-4 2 0] | 9.08E-04 | 9.08E-08 | 6.31E-04 | 6.31E-08 |
| [-5 2 0] | 1.20E-03 | 1.20E-07 | 9.36E-04 | 9.36E-08 |
| [0 2 0] | 6.50E-04 | 6.50E-08 | 3.98E-04 | 3.98E-08 |
| [1 2 0] | 6.96E-04 | 6.96E-08 | 4.31E-04 | 4.31E-08 |
| [2 2 0] | 8.01E-04 | 8.01E-08 | 5.19E-04 | 5.19E-08 |
| [3 2 0] | 1.03E-03 | 1.03E-07 | 7.37E-04 | 7.37E-08 |
| [-1 3 0] | 7.15E-04 | 7.15E-08 | 4.50E-04 | 4.50E-08 |
| [-2 3 0] | 7.30E-04 | 7.30E-08 | 4.65E-04 | 4.65E-08 |
| [-3 3 0] | 7.91E-04 | 7.91E-08 | 5.21E-04 | 5.21E-08 |
| [-4 3 0] | 9.24E-04 | 9.24E-08 | 6.45E-04 | 6.45E-08 |
| [-5 3 0] | 1.18E-03 | 1.18E-07 | 9.15E-04 | 9.15E-08 |
| [0 3 0] | 7.44E-04 | 7.44E-08 | 4.73E-04 | 4.73E-08 |
| [1 3 0] | 8.36E-04 | 8.36E-08 | 5.52E-04 | 5.52E-08 |
| [2 3 0] | 1.04E-03 | 1.04E-07 | 7.45E-04 | 7.45E-08 |
| [-1 -3 0] | 9.23E-04 | 9.23E-08 | 6.45E-04 | 6.45E-08 |
| [-2 -3 0] | 1.18E-03 | 1.18E-07 | 9.15E-04 | 9.15E-08 |
| [5 -3 0] | 1.04E-03 | 1.04E-07 | 7.45E-04 | 7.45E-08 |
| [0 -3 0] | 7.91E-04 | 7.91E-08 | 5.21E-04 | 5.21E-08 |
| | | | | |

| [1 -3 0] | 7.30E-04 | 7.30E-08 | 4.65E-04 | 4.65E-08 |
|-----------|----------|----------|----------|----------|
| [2 -3 0] | 7.15E-04 | 7.15E-08 | 4.50E-04 | 4.50E-08 |
| [3 -3 0] | 7.44E-04 | 7.44E-08 | 4.72E-04 | 4.72E-08 |
| [4 -3 0] | 8.35E-04 | 8.35E-08 | 5.52E-04 | 5.52E-08 |
| [-1 4 0] | 8.57E-04 | 8.57E-08 | 5.75E-04 | 5.75E-08 |
| [-2 4 0] | 8.43E-04 | 8.43E-08 | 5.63E-04 | 5.63E-08 |
| [-3 4 0] | 8.91E-04 | 8.91E-08 | 6.11E-04 | 6.11E-08 |
| [-4 4 0] | 1.02E-03 | 1.02E-07 | 7.43E-04 | 7.43E-08 |
| [-5 4 0] | 1.25E-03 | 1.25E-07 | 9.85E-04 | 9.85E-08 |
| [0 4 0] | 9.47E-04 | 9.47E-08 | 6.60E-04 | 6.60E-08 |
| [1 4 0] | 1.12E-03 | 1.12E-07 | 8.33E-04 | 8.33E-08 |
| [-1 -4 0] | 1.25E-03 | 1.25E-07 | 9.85E-04 | 9.85E-08 |
| [0 -4 0] | 1.02E-03 | 1.02E-07 | 7.43E-04 | 7.43E-08 |
| [1 -4 0] | 8.90E-04 | 8.90E-08 | 6.10E-04 | 6.10E-08 |
| [2 -4 0] | 8.42E-04 | 8.42E-08 | 5.64E-04 | 5.64E-08 |
| [3 -4 0] | 8.56E-04 | 8.56E-08 | 5.74E-04 | 5.74E-08 |
| [4 -4 0] | 9.47E-04 | 9.47E-08 | 6.60E-04 | 6.60E-08 |
| [5 -4 0] | 1.12E-03 | 1.12E-07 | 8.34E-04 | 8.34E-08 |
| [-1 5 0] | 1.15E-03 | 1.15E-07 | 8.64E-04 | 8.64E-08 |
| [-2 5 0] | 1.09E-03 | 1.09E-07 | 8.03E-04 | 8.03E-08 |
| [-3 5 0] | 1.10E-03 | 1.10E-07 | 8.25E-04 | 8.25E-08 |
| [-4 5 0] | 1.21E-03 | 1.21E-07 | 9.38E-04 | 9.38E-08 |
| [1 -5 0] | 1.21E-03 | 1.21E-07 | 9.39E-04 | 9.39E-08 |
| [2 -5 0] | 1.10E-03 | 1.10E-07 | 8.26E-04 | 8.26E-08 |
| [3 -5 0] | 1.08E-03 | 1.08E-07 | 8.03E-04 | 8.03E-08 |
| [4 -5 0] | 1.15E-03 | 1.15E-07 | 8.63E-04 | 8.63E-08 |
| | | | | |

APPENDIX VII.

HEAVY WATER DEPLETION STUDIES FOR EACH ISOTOPE OF INTEREST FOR A 19.9% ENRICHED FUEL



















| Time (Day) | ²³⁹ Pu Mass (g) | Error (g) | | | |
|------------|----------------------------|-----------|--|--|--|
| 0.1 | 1.01E-05 | ±5.05E-09 | | | |
| 0.4 | 2.19E-04 | ±1.10E-07 | | | |
| 0.7 | 6.86E-04 | ±3.43E-07 | | | |
| 1 | 1.39E-03 | ±6.95E-07 | | | |
| 2 | 5.13E-03 | ±2.57E-06 | | | |
| 3 | 1.06E-02 | ±5.30E-06 | | | |
| 4 | 1.74E-02 | ±8.70E-06 | | | |
| 5 | 2.52E-02 | ±1.26E-05 | | | |
| 6 | 3.37E-02 | ±1.69E-05 | | | |
| 7 | 4.28E-02 | ±2.14E-05 | | | |
| 14 | 7.06E-02 | ±3.53E-05 | | | |
| 21 | 7.41E-02 | ±3.71E-05 | | | |

²³⁹Pu Production for Heavy Water Moderated System With 19.9% ²³⁵U Enriched Fuel

Heavy water produces less amounts of the isotopes studied as expected due to the lower flux and power produced in the core even with 19.9% enriched fuel. If heavy water is implemented as a moderator, further studies need to be conducted to optimize the amount of moderator in the core. This will also cause the core to be larger as more moderation is required to increase the amount of fission currently being produced in the core. Again, very little ²³⁹Pu is produced in the system. The design reaches a burnup of 6.098E-2 GWd/MTU for the seven day irradiation.
APPENDIX VIII.

BERYLLIUM DEPLETION STUDIES FOR EACH ISOTOPE OF INTEREST FOR A 19.9% ENRICHED FUEL



















| Time (Day) | ²³⁹ Pu Mass (g) | Error (g) |
|------------|----------------------------|-----------|
| 0.1 | 7.66E-06 | ±3.83E-09 |
| 0.4 | 1.66E-04 | ±8.30E-08 |
| 0.7 | 5.19E-04 | ±2.60E-07 |
| 1 | 1.05E-03 | ±5.25E-07 |
| 2 | 3.88E-03 | ±1.94E-06 |
| 3 | 8.04E-03 | ±4.02E-06 |
| 4 | 1.32E-02 | ±6.60E-06 |
| 5 | 1.91E-02 | ±9.55E-06 |
| 6 | 2.55E-02 | ±1.28E-05 |
| 7 | 3.23E-02 | ±1.62E-05 |
| 14 | 5.34E-02 | ±2.67E-05 |
| 21 | 5.61E-02 | ±2.81E-05 |

²³⁹Pu Production for Beryllium Moderated System with 19.9% ²³⁵U Enriched

Fewer amounts of the isotopes of interest are produced with the beryllium moderator in the 19.9% enriched fuel. Further studies could be conducted to help optimize this system if it is of interest. However, leakage is still the major issue as with heavy water and the moderation of the core will have to increase which also suggests that the core will have to be relatively larger than the current design. Again, very little ²³⁹Pu is produced in the system. The design reaches a burnup of 4.231E-2 GWd/MTU for the seven day irradiation.