Asuquo, MP, Wang, J, Phylip-Jones, G and Riahi, R

Condition monitoring of marine and offshore machinery using evidential reasoning techniques

Condition Monitoring of Marine and Offshore Machinery using Evidential Reasoning Techniques

(1)Maurice Asuquo, (1), *Jin Wang, (1)Geraint Phylip-Jones, (2)Ramin Riahi

(1)Liverpool Logistics Offshore and Marine (LOOM) Research Institute, Department of Maritime and Mechanical Engineering, Faculty of Engineering and Technology, Liverpool John Moores University, Liverpool, UK

(2) Columbia Shipmanagement, GroÃŸe ElbstraÃŸe 275 - 22767 Hamburg, Germany

Abstract

This paper first assesses the operational uncertainties of a particular piece of equipment in a marine and offshore system based on an oil analysis technique. Trend analysis, family analysis, environmental analysis, human reliability analysis and design analysis for each criterion are aggregated using evidential reasoning (ER) and analytical hierarchy process (AHP) algorithms. Data is collected from available statistics and supplemented by expert judgement from the related industry. The results provided in this study will be beneficial to the marine and offshore industries as indicators for monitoring and diagnosis of faults in machinery and thus assist practitioners in making better decisions in their maintenance management process. Furthermore, by changing the conditions that affect the operation of machinery, and through calculating a value for this operation, a benchmark for condition monitoring is constructed. The operational condition of machinery depends on many variables and their dependencies; thus, alteration of a criterion value will ultimately alter the operational conditions of the machinery. For any deviation to be corrected in a timely manner, the operational condition of the machinery has to be monitored properly and frequently.

Keywords: Condition Monitoring, analytical hierarchy process, evidential reasoning, maintenance analysis, design analysis

1 Introduction

Machine condition monitoring is the practice of assessing a machine's condition by periodically gathering data on key machine-health indicators to determine when to schedule maintenance

* : corresponding author: j.wang@ljmu.ac.uk

(Zhao, 2008). The existence of debris and particles from wearing parts, erosion and contamination provides insights about the issues affecting performance and reliability. The growing failure of marine and offshore machinery, such as main engines, cranes, pumps, *etc*., coupled with operator concern over their reliability, has motivated this research and the development of an efficient condition monitoring methodology and reliability procedures. Furthermore, with the increasing complexity and cost of equipment, accurate diagnosis is important. The fundamental element of machinery condition monitoring on-board ship is watch-keeping (Lloyd's Register, 2010). Watch-keeping involves the ability to recognize changes in performance, as indicated by alarms, alerts, gauges and readings, as well as responding aptly to these changes. However, as the industry becomes more dynamic, there is a need to introduce concepts of flexibility and agility (Bastos *et al.*, 2012), to enable companies to deliver customized condition monitoring (CM) which can react swiftly to machinery operating in highly uncertain environments like sea.

In their normal day-to-day schedules, deck and engineering officers do carry out many condition monitoring activities, such as monitoring the condition of individual components in a piece of equipment. For example, some of the routine condition monitoring activities carried out in marine vessels include the use of electronic equipment for main and auxiliary engine performance measurement, the installation of temperature sensors in cylinder liners to monitor piston rings blow-by, and visual inspection of piston rings and liners through scavenge space (Lloyd's Register, 2010). However, the inability of the deck and engineering officers to evaluate a large number of discrete variables, such as trend analysis, family analysis, environment analysis, human reliability analysis, and design analysis, has prompted questioning of the effectiveness of these routine condition monitoring activities.

This research will demonstrate the framework of monitoring and diagnosing machinery faults in marine and offshore industries will be demonstrated. Evidential reasoning (ER) and analytical hierarchy process (AHP) algorithms will be employed to synthesise the data gathered from all the components, in what is called a *data mining process* (DMP). This will identify the behaviour patterns of each component, thus allowing early and accurate detection of faults in the equipment.

The structure of this paper will be as follows. The second section presents a literature review on monitoring the condition of marine and offshore machinery. The process of building a generic model of a hierarchical structure for monitoring the condition of the machinery is presented in the third section, in which trend, family, environment, human reliability, and design analysis information are processed. The methodology is then explained and applied to the monitoring of the operational conditions of machinery in the fourth section. This proposed methodology, along with a previously accepted condition monitoring methodology, is then tested by a case study, followed by a discussion and conclusion.

2 Literature Review

Monitoring the condition of marine and offshore machinery has become a point of interest, since the environment imposes a high demand for reliability on the installed machinery. Condition monitoring has a unique benefit, in the sense that conditions that would shorten normal lifespan of a piece of machinery can be addressed before they develop into a major failure. Many researchers, such as Courrech *et al.* (2014), Galloway (2014), have conducted research in this area. Given that the input data for determining the condition of the machinery is normally expressed in both quantitative and qualitative terms, decision makers may often carry out judgements based on both quantitative data and experiential subjective assessments of the machinery. Consequently, a proposed methodology for monitoring the condition of marine and offshore machinery should be capable of processing both quantitative and qualitative data.

Condition monitoring (CM) can be defined as the process of monitoring a parameter of condition in machinery (oil, vibration, temperature, *etc.*), in order to identify a significant change that is indicative of a developing fault. In the case of oil, condition monitoring is the assessment of oil failure modes through the monitoring of reliable condition indicators (Toms 1998). Moreover, the operational condition of a machine can be defined as its reliability value if a condition is known to occur. Machinery operational condition depends upon many variables and their dependencies. The most important elements that affect safety and efficiency in machinery performance will be discussed in detail in the methodology section of this paper.

2.1 Analytic Hierarchy Process (AHP)

Analytic hierarchy process is a structured technique commonly used in analysing complex decisions. It is based on mathematics and psychology and was developed by Thomas L. Saaty in the 1970s. Since then, it has been extensively studied and refined by many researchers. AHP provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions. Instead of stipulating a "correct" decision, AHP helps decision makers find solution that best suits their goal and their understanding of the problem. It is used around the world in a wide variety of decision making situations (Saaty, 1983, 2008).

The main aim of AHP is to assist decision makers in organising their thoughts and judgements to make decisions that are more valuable. AHP also provides the objective mathematics to process the inevitable subjective and personal preferences of an individual or group in making decisions. AHP works by developing priorities for alternatives and the criteria are used to judge these alternatives. Firstly, priorities are derived for the criteria in terms of their importance to achieving the goal. Secondly, priorities are derived for the performance of the alternatives on each criterion. These priorities are derived based on pair-wise assessments using judgements, or ratios of measurements from a scale, if one exists. Finally, a weighting and adding process is used to obtain overall priorities for the alternatives regarding how they contribute to the goal (Saaty and Vargas, 2001).

Using the AHP to calculate the relative importance of each attribute requires a careful review of its principles and background (Saaty, 1990). When considering a group of attributes for evaluation, the main objectives of this technique is to provide judgement on the relative importance of these attributes and to ensure that they are quantified to an extent that permits quantitative interpretation of the judgement among these attributes (Pillay *et al.,* 2003). The quantified judgements on pairs of attributes A_i and A_j are represented by an n-by-n matrix E. The entries a_{ij} are defined by the following entry rules.

Rule 1: If a_{ij} = α, then a_{ji} = $1/_{\alpha}$, α ≠ 0

Rule 2: If A_i is judged to be of equal relative importance as A_j, then a_{ij} = a_{ji} = 1.

According to above rules, the matrix E has the form as follows:

$$E = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \cdots & 1 \end{bmatrix}$$

where, i, j=1, 2, 3..., n and each a_{ij} is the relative importance of attribute A_i to attribute A_j. Having recorded the quantified judgements of comparisons on pair ($A_{i,}$ A_j) as the numerical entry a_{ij} in the matrix E, what is left is to assign to the n contingencies A_1, A_2, …, A_n a set of numerical weights w_1, w_2, …, w_n that should reflect the recorded judgements.

In general, the weights w_1, $w_{2...,}$ w_n can be calculated (Pillay *et al.*, 2003) using the following equation:

$$w_k = \frac{1}{n}\sum_{j=1}^{n} \frac{a_{kj}}{\sum_{i=1}^{n} a_{ij}} \; (k = 1, 2, 3, \ldots, n) \tag{1}$$

where, a_{ij} represents the entry of row i and column j in a comparison matrix of order n.

The weight vector of the comparison matrix provides the priority ordering. However, it cannot ensure the consistency of the pairwise judgements. Hence AHP provides a measure of the consistency for the pairwise comparisons by computing a consistency ratio (CR). The CR informs the decision makers how consistent they have been when making the pair-wise comparisons (Kunz, 2010). It is designed in such a way that a CR value greater than 0.10

indicates an inconsistency in the pair-wise judgements and according to Andersen *et al.* (2008), the decision maker should review the pair-wise judgements before proceeding.

Consequently, if the CR is 0.10 or less, the consistency of the pair-wise comparisons is considered reasonable, and the AHP approach can continue with the computations of the weight vectors. A higher number means the decision maker has been less consistent, whereas a lower number means the decision maker has been more consistent (Kunz, 2010). If the CR is > 0.10, the decision maker should seriously consider re-evaluating the pair-wise comparisons. The source(s) of inconsistency must be identified and resolved and the analysis re-done. The CR value is computed according to the equations (Andersen *et al.,* 2008).

$$CR = \frac{CI}{RI} \tag{2}$$

$$CI = \frac{\lambda_{max} - n}{n-1} \tag{3}$$

$$\lambda_{max} = \frac{\sum_{j=1}^{n}[(\sum_{k=1}^{n} w_k a_{jk})/w_j]}{n} \tag{4}$$

where, CI is the consistency index, RI is the average random index, n is the matrix order as shown in Table 1 (Saaty, 1990) and λ_{max} is the maximum weight value of the n-by-n comparison matrix E.

Insert Table 1 here

Insert Table 2 here

Saaty (2004) recommended equivalent scores from 1 to 9, as shown in Table 2. A preference of 1 indicates equality between two attributes, while a preference of 9 indicates that one attribute is nine times larger or more important than the attribute with which it is being compared.

2.2 Evidential Reasoning (ER)

The evidential reasoning approach provides a means for dealing with the aggregation problem. The ER approach was developed particularly for multiple criteria decision making (MCDM) problems with both qualitative and quantitative criteria under uncertainty and utilises individuals' knowledge, expertise, and experience in the forms of belief functions (Riahi, 2010).

There are a number of studies where ER is used. For example, Riahi (2010) used a fuzzy evidential reasoning (FER) to evaluate a seafarer's reliability; Wang and Elhang (2007) used fuzzy group decision making for bridge risk assessment; Zeng *et al.* (2006) applied an aggregative risk assessment model for information technology project development; Yang *et al.* (2005) carried out risk analysis of container supply chains using discrete fuzzy sets and an

ER approach using fuzzy set theories (FST) and ER specifically on risk assessment and decision making; and Liu *et al.* (2003) used the fuzzy rule-based ER approach to analyse the safety of an engineering system with various types of uncertainties.

While MCDM is described using a decision matrix, the ER approach applies an extended decision matrix, in which each attribute of an alternative is described by a distributed assessment using a belief structure (Xu *et al.*, 2001). Each criterion is assigned with belief degrees on several linguistic evaluation grades to assess the subjective uncertainties and ambiguities associated with both quantitative and qualitative criteria. Incompleteness (or ignorance) and vagueness (or fuzziness) are among the most common uncertainties in decision analysis. Subjective judgments may be used to differentiate one alternative from another on qualitative attributes. To evaluate the quality of the operation of equipment, for example, typical judgments may be that "the condition of that equipment is poor, good, or very good to certain degrees." In such judgments, *poor, good,* and *very good* represent distinctive evaluation grades. In equipment evaluation problems, such as ones in a ship propulsion engine, a set of evaluation grades is defined by:

E = $\{poor\ (\beta_1)\ very\ poor\ (\beta_2)\ average\ (\beta_3)\ good\ (\beta_4)\ very\ good\ (\beta_5)\}$
where, $\beta_1, \beta_2, \beta_3, \beta_4,$ and β_5 stand for belief degrees.

The operational condition of the engine is a broad technical approach that is not easy to assess directly. The detailed components of the engine, such as piston, connecting rod, and crankshaft, *etc*. need to be considered separately to simplify the assessment. If a detailed component is still too abstract to assess directly, it may be further broken down to more detailed sub-components. For instance, the piston component (y) may be measured by examining the condition of rings (B_1), pin (B_2), and skirt (B_3), which can be directly assessed and therefore referred to as basic attributes. Assessment attributes often constitute a multilevel hierarchy (Yang and Xu, 2002).

In hierarchical assessment, a high level attribute is assessed through associated lower level attributes. For example, if the *ring, pin,* and *skirt* of the engine piston are all assessed to be exactly *good,* then its piston should also be *good*. According to Yang and Xu (2002), when evaluating qualitative attributes, uncertain judgments can be used. For example, in assessment of the engine piston, assessors may be:

1. 30% sure that its ring is at average condition and 60% sure that it is good.
2. Absolutely sure that its pin is good.
3. 50% sure that its skirt is good and 50% sure that it is very good.

In the above assessments, 30%, 50%, 60%, and 100% (absolutely sure) are referred to as degrees of belief and can be used in decimal format as 0.3, 0.5, 0.6, and 1, respectively.

- Assessment (1) is incomplete as the total degree of belief is 0.9 (0.3 + 0.6).
- Assessments (2) and (3) are complete.
- The missing 0.1 in assessment (1) represents the degree of ignorance or uncertainty.

Difficulty can be encountered as to how an overall assessment about the engine piston is generated by aggregating the above three judgments in a rational manner. The ER approach provides a means for dealing with such an aggregation problem. The basic ER applications and algorithm are discussed in the next two subsections.

2.2.1 ER algorithm

ER is one of the many multiple criteria decision analysis (MCDA) methods. ER is applied to deal with MCDA problems for aggregating multiple criteria based on belief degree matrix (BDM) and D-S theory.

A belief degree represents the strength to which an answer is believed to be true. It must be equal to or less than 100% or it can be described as the degree of expectation that, given an alternative, it will yield an anticipated outcome on a particular criterion. The use of individual belief degrees depends on the decision makers' expertise, knowledge of the subject matter and level of experience regarding the operations of the system. The justification for the use of belief degrees is as a result that human decision making involves ambiguity, uncertainty, imprecision, and where individuals make judgements in probabilistic terms aided by their knowledge.

For instance, let *S* represent a set of five condition monitoring expressions that are synthesized by two subsets, S_1 and S_2 from two different assessors. Then, S, S_1 and S_2 can be expressed independently as follows:

$$S = \{\beta^1 \text{"Very low"}, \beta^2\text{"Low"}, \beta^3\text{"}Medium\text{"}, \ \beta^4 \text{"High"}, \ \beta^5 \text{"}Very\ high\text{"}\}$$

$$S_1 = \left\{{\beta_1}^1 \text{"Very low"}, {\beta_1}^2\text{"Low"}, {\beta_1}^3\text{"}Medium\text{"}, \ {\beta_1}^4 \text{"High"}, \ {\beta_1}^5 \text{"}Very\ high\text{"}\right\}$$

$$S_2 = \left\{{\beta_2}^1 \text{"Very low"}, {\beta_2}^2\text{"Low"}, {\beta_2}^3\text{"}Medium\text{"}, \ {\beta_2}^4 \text{"High"}, \ {\beta_2}^5 \text{"}Very\ high\text{"}\right\}$$

where "Very low", "Low", "Medium", "High", and "Very high" (the condition monitoring expression) are assessed with their respective degrees of belief.

If the normalised relative weights of the two assessors in the evaluation of the condition monitoring process are given by w_1 and w_2 (w_1 + w_2 = 1), then w_1 and w_2 can be estimated by using established methods such as a simple rating method or based on pair-wise comparisons (Yang *et al.,* 2001).

Suppose $M_1{}^m$ and $M_2{}^m$ (m = 1, 2, 3, 4 or 5) are individual degrees to which the subsets S_1 and S_2 support the hypothesis that the condition monitoring evaluation is confirmed to the five evaluation grades. Then, $M_1{}^m$ and $M_2{}^m$ can be derived as follows:

$$M_1{}^m = w_1\beta_1{}^m \; ; \; M_2{}^m = w_2\beta_2{}^m \tag{5}$$

where m = 1, 2, 3, 4, and 5 respectively.

$M_1{}^1=w_1\beta_1{}^1$, $M_2{}^1=w_2\beta_1{}^1$; $M_1{}^2=w_1\beta_1{}^2$, $M_2{}^2=w_2\beta_1{}^2$; $M_1{}^3=w_1\beta_1{}^3$, $M_2{}^3=w_2\beta_1{}^3$;

$M_1{}^4=w_1\beta_1{}^4$, $M_2{}^4=w_2\beta_1{}^4$; $M_1{}^5=w_1\beta_1{}^5$, $M_2{}^5=w_2\beta_1{}^5$

Suppose H_1 and H_2 are the individual remaining belief values unassigned, then H_1 and H_2 can be obtained as follows (Yang and Xu, 2002):

$$H_1 = \overline{H}_1 + \widetilde{H}_1 \; , \; H_2 = \overline{H}_2 + \widetilde{H}_2 \tag{6}$$

where $\overline{H}_n (n = 1 \; or \; 2)$ represents the degree to which the other assessor can play a significant role in the assessment.

$\widetilde{H}_n (n = 1 \; or \; 2)$, causes the likely incompleteness in subsets S_1 and S_2. $\overline{H}_n \; (n = 1 \; or \; 2) \; and \; \widetilde{H}_n (n = 1 \; or \; 2)$ can be described as follows:

$\overline{H}_1$ = 1-w_1 = w_2 , $\overline{H}_2$ = 1-w_2 = w_1

$$\widetilde{H}_1 = w_1\left(1 - \textstyle\sum_{m=1}^{5} \beta_1{}^m\right) = \; w_1\left[1 - \left(\beta_1{}^1 + \beta_1{}^2 + \beta_1{}^3 + \beta_1{}^4 + \beta_1{}^5\right)\right] \tag{7}$$

$$\widetilde{H}_2 = w_2\left(1 - \textstyle\sum_{m=1}^{5} \beta_2{}^m\right) = w_1\left[1 - \left(\beta_2{}^1 + \beta_2{}^2 + \beta^3{}_2 + \beta^4{}_2 + \beta_2{}^5\right)\right] \tag{8}$$

Suppose $\beta^{m'}$ $(m = 1, 2, 3, 4 \; or \; 5)$ represents the non-normalised degree to which the five condition monitoring expressions are confirmed as a result of the synthesis of the judgements obtained by assessors *1* and *2* respectively. Suppose $H_{U'}$ represents the non-normalised remaining belief unassigned after the commitment of belief to the five condition monitoring expressions because of the synthesis of the judgements obtained from assessors *1* and *2*. The ER algorithm can be derived as follows (Yang and Xu, 2002):

$$\beta^{m'} = K(M_1{}^m M_2{}^m + \; M_1{}^m H_2 + \; H_1 M_2{}^m) \tag{9}$$

$$\overline{H}_{U'} = K\left(\overline{H}_1 \overline{H}_2\right) \tag{10}$$

$$\widetilde{H}_{U'} = K\left(\widetilde{H}_1\widetilde{H}_2 + \widetilde{H}_1\overline{H}_2 + \overline{H}_1\widetilde{H}_2\right) \tag{11}$$

$$K = \left[1 - \sum_{T=1}^{5}\sum_{\substack{R=1 \\ R\neq 1}}^{5} {M_1}^{T}\,{M_2}^{R}\right]^{-1} \tag{12}$$

After the above aggregation, the combined degree of belief β^m is generated by assigning $H_{U'}$ back to the five condition monitoring expressions in the normalisation process below (Yang and Xu, 2002):

$$\beta^m = \frac{\beta^{m'}}{1-\overline{H}_{U'}},\ (m = 1, 2, 3, 4, 5) \tag{13}$$

$$H_U = \frac{\widetilde{H}_{U'}}{1-\overline{H}_{U'}} \tag{14}$$

where, H_U is the unassigned degree of belief representing the level of incompleteness in the assessment. The process above highlights the sequence of combining two given sets. The algorithm can also be followed when encountering three or more sets in a hierarchical structure. If three subsets are required to be combined, the result obtained from the combination of any of the two subsets can be further synthesized with the third subset using the above algorithm. Similarly, the judgement of multiple assessors or the evaluations of the condition of the lower-level criteria in the chain systems (components or sub-components) can also be combined.

2.2.2 Application of ER

Over the years, ER has progressively been applied to diverse multi-attribute problems (Yang, 2001), (Yang and Sen, 1997), (Wang *et al.,* 1996), (Yang and Sen, 1996), (Wang *et al.,* 1995), (Yang and Singh, 1994), and (Yang and Sen, 1994). The unique features of the ER approach have made it necessary for use in tailoring decisions that represent incomplete and subjective judgements for machinery condition monitoring. ER has been initiated for wider application in many real-world decision making issues (Zhou *et al*., 2010). Some areas in which it has been applies include: Strategic research and development projects' assessments (Liu *et al*., 2008) and (Zhou *et al*., 2007); Experts systems (Beynon *et al*., 2001); Knowledge reduction (Wu *et al*., 2005); Oil reserve forecast (Zhang *et al*., 2005); Prequalifying construction contractors (Sonmez *et al*., 2002); Risk analysis (Srivastava and Liu, 2003), (Srivastava and Lu, 2002); Motor-cycle evaluation (Yang and Xu, 2002), (Yang, 2001), (Yang and Singh, 1994), (Yang and Sen, 1994); New product development (Chin *et al*., 2008); Marine system safety analysis and synthesis (Wang *et al.,* 1996), (Wang *et al.,* 1995); and General cargo ship design (Sen and Yang, 1998).

Riahi (2010) believes that in real-world decision making, ER applications have been found to have the following advantages:

- Offers a rational and reproducible methodology to aggregate data in a hierarchical evaluation process.
- Capability to provide users with greater flexibility by allowing them to express their judgement in a subjective and quantitative manner.
- Capability to accept or represent the uncertainty and risk that is inherent in decision-making.
- Great effectiveness in processing and obtaining assessment outputs using mature computing software called Intelligent Decision System (IDS).
- Capability to handle incomplete, uncertain, and vague data as well as complete and precise data.

2.3 Degree of Membership

Items can belong to a fuzzy set to different degrees known as degrees of membership. An item that is completely within a set has a membership degree of 1, while those completely outside a set have a membership degree of 0. All degrees of membership must sum to 1. An item can be both A and not-A to different degrees *e.g.* A to a degree of 0.8, not-A 0.2. Degrees of membership are expressed with membership functions. The range of values a variable can take is called the universe of discourse (Watts, n.d.).

2.3.1 Membership functions

A membership function, normally referred to as 'MF', describes the degree of membership of a value in a fuzzy set. Membership function can be expressed as $\mu(x)$ where x is the value being fuzzified. Depending on the problem being considered, any one of the singleton, rectangular, triangular and Gaussian membership functions can be used to solve that particular problem.

2.3.2 Triangular membership functions (TMF)

In this research, only triangular membership functions will be considered in detail. Amongst the various shapes of fuzzy numbers, the membership function of the triangular fuzzy number (TFN) is the most popular and frequently used. A triangular fuzzy number is a fuzzy number represented with three points, as follows:

$A = (a, b, c)$

This representation is interpreted as membership functions.

$$\mu_A(x) = \begin{cases} (x-a)/(b-a), & a \le x \le b \\ (c-x)/(c-b), & b \le x \le c \end{cases}$$

where, a and b stand for the lower and upper bounds of the TFN respectively, and c for the modal value.

Insert Figure 1 here

2.3.3 Linguistic variables

A linguistic variable is a variable whose values are words or sentences in a natural or artificial language. According to Zadeh (1975), it is very difficult for conventional quantification to express reasonably those situations that are clearly complex or hard to define. Therefore, the concept of a linguistic variable is necessary in such situations. Linguistic variables are currently being used extensively. The linguistic effect values of the best metal element alternatives found in this study are primarily used to assess the linguistic ratings given by the evaluators. Here each membership function (scale of fuzzy number) is defined by three parameters of the symmetric triangular fuzzy number: the left point, middle point, and right point of the range over which the function is defined. Moreover, linguistic variables are used as a way to measure the performance value of the best metal element alternative for each criterion as "very good," "good," "fair," "poor" and "very poor" (Chen *et al.,* 2009). TFN, as shown in Figure 1, is used to indicate the membership functions of the linguistic terms. The horizontal axis indicates the quantitative number and the vertical axis indicates the degree of belief (membership value). If any quantitative number $(e.g.\ h_i)$ is found in the range of $h_{n+1,i}$ (with a grade H_{n+1}) and $h_{n,i}$ (with a grade H_n), its belief degrees can be evaluated as follows:

If $h_{n,i} < h_i < h_{n+1,i}$ $then\ \beta_{n,i} = \frac{h_{n+1,i} - h_i}{h_{n+1,i} - h_{n,i}}$ (15)

$$\beta_{n+1,i} = 1 - \beta_{n,i} \quad (16)$$

where, $\beta_{n,i}$ is the degree of belief of the concerned quantitative number with the grade H_n, and $\beta_{n+1,i}$ is the degree of belief of the concerned quantitative number with the grade H_{n+1}.

3. Methodology

The risk of major failures in marine and offshore machinery is an area that is not thoroughly addressed in academic literature. It is clear that complexity of the machinery stems from the interaction of their dependencies and the high levels of uncertainty in their operations make it extremely difficult to identify the vulnerability of the machinery in order to assess their risks. A survey conducted by Lloyd's Register (2011) indicates that several slew bearings failures have occurred in cranes in recent years, with catastrophic consequences. Moreover, based on an incident report by Aldridge (2012) and a case study by Konecranes (2012), gearbox malfunction is very common in ship cranes, while the crane reliability survey (CRS) shows that gearbox failures can result in catastrophic crane failure. The main obstacle in maintenance methodology that can be utilised by the marine and offshore industry is the difficulty of accommodating the differing types of information and processes of trend analysis, family analysis, design analysis, environmental analysis, and human reliability analysis.

The proposed methodology will provide a framework to optimise the maintenance and inspection activities of the machinery using qualitative and quantitative risk-based techniques to help make the right decisions on how to improve the condition of marine and offshore machinery operating in a highly uncertain environment. The condition of the equipment is evaluated using a combination of different decision making techniques, such as AHP, ER, data mining process, and expected utility. This methodology is chosen because the research work is based on assessing the operational condition of different components in a machine to ascertain which components are prone to failure.

The proposed methodology in stepwise regression is presented in the following sections. The flow diagram for evaluating the condition of machine is shown in Figure 2.

Insert Figure 2 here

3.1 Identification of Risk Criteria (**Step one**)

It is very important for the decision makers to fully understand and have a clear picture of the whole problem before attempting to find a solution, especially when there are many criteria that need to be considered, which may in turn consist of sub-criteria and sometimes even sub-sub-criteria. In such situations, the problem can be displayed in the form of a hierarchical structure. Using hierarchical order, the goal of the problem is indicated at the first level, while in the second level, there are several criteria, each of which contribute to measuring and helping to achieve the overall goal, some of these criteria can further be broken down. This process can continue up to the point where the decision makers are able to make practical evaluation. When constructing a hierarchical structure, it is important to pay attention to only significant criteria, in order to avoid a superfluously large model.

Based on the literature review of the condition monitoring of the marine and offshore machinery, a generic model with a hierarchical structure is constructed. The main criteria, sub-criteria, and sub-sub-criteria that contribute to the condition monitoring of the machinery (goal) are presented in Figure 3. The goal (E) of the condition monitoring is stated in the first level. In the second level, the main criteria (C_1, C_2, C_3 and C_4) contributing to the condition monitoring of the goal (E) are stated. Then in the third level, the sub-criteria {(C_{11}, C_{12}, C_{13},), (C_{21}, C_{22},

C_{23}), (C_{31}, C_{32}, C_{33}), (C_{41}, C_{42}, C_{43})} contributing to the condition monitoring of the main criteria and the goal are stated. Then finally in the fourth level, sub-sub-criteria showing different contributions to measuring and achieving the goal of the problem are stated. However, this can be further broken down into sub-criteria sub-sub-sub-criteria until a point where decision makers can make practical and informed decisions on the lower level criteria.

Insert Figure 3 here

3.2 Application of Analytic Hierarchy Process (**Step two**)

AHP is used to determine the weights of each risk factor by conducting a pair-wise comparison. TFN are used to calculate the preference of one criterion over another because of their computational simplicity in promoting representation of information in an uncertain environment. The comparison is usually based on an estimation scheme which places intensity of importance using qualitative variables. Each of the variables has a corresponding TFN that is employed to transfer experts' judgement into a corresponding matrix.

3.2.1 Experts composition

Table 3 indicates the position, service time and the qualifications of the experts used for the survey.

Insert Table 3 here

3.3 Evaluation of Trend Analysis (TA) (**Step three**)

Trend analysis is an aspect of technical analysis that tries to predict the future performance of machinery based on past data recorded. It is centred on the idea that what has happened in the past gives an idea of what will happen in the future. Trend analysis allows the development of a pattern of behaviour for a particular unit. This pattern of behaviour may develop within a short or long term period. In trend analysis, graphs of a condition-related parameter versus time can be utilized to determine when the parameter is likely to exceed a given limit. This time could be dates or running hours.

The goal of a successful condition monitoring program is to predict the time of an expected breakdown well in advance of its occurrence in order to shut down the machine in ample time and allow for the ordering of spare parts for repairs, thus minimizing the shutdown time. According to Courrech and Eshleman (2014), all condition monitoring criteria indicate that equal changes on a log scale correspond to equal changes in severity; therefore, data for a trend analysis should be plotted on a logarithmic scale in decibels. A linear trend on a logarithmic scale is found occasionally, but the actual trend may follow another path. For example, when the fault feeds back on the rate of deterioration (*e.g.* gear wear), the trend, when plotted on a logarithmic scale, may then be exponential. In some cases, the fault changes suddenly in finite steps, making it very difficult to extrapolate the time of the shutdown. An example is a spall caused by gradual subsurface fatigue.

The following precautions are very vital in ensuring that accurate trend analysis is being obtained (Courrech and Eshleman, 2014):

1. Determining a trend based on measurements of a parameter directly related to a specific type of fault, not on measurements of overall levels.
2. Diagnosing faults before attempting to interpret a trend curve in order to:
    a) Select the appropriate parameter for the type of fault that is being monitored. For example, the parameter may be the level of an individual component, or of a selected frequency range.
    b) Observe critically the results of the trend analysis so as to determine if the linear or exponential interpolation is adequate.
3. Employing a trend of the most recent measurements to obtain the best estimate of the lead time.

Several techniques can be applied in evaluating trend data, such as standard deviations, averages, linear regression, *etc*. All of these techniques are intended to identify a condition that is not normal in relation to the equipment's past behaviour. In this research, trend analysis is evaluated by means of quantitative data transformation (QDT). Each quantitative criterion (*i.e.* grease/oil sample element test result) is transformed to a qualitative criterion (i.e. linguistic variables with the associated belief degrees).

3.4 Evaluation of Family Analysis (**Step four**)

Family Analysis compares the results (*e.g.* wear metal levels) of groups of similar or identical machinery to identify the usual or typical pattern. The extraction of such information provides the data necessary to characterize operating cycles, maintenance schedules, periodic breakdowns, and most importantly, to identify and address abnormal failure rates before critical problems arise. In many cases, systems are grouped together to form a family. A family may consist of identical equipment located in one or many vessels. Equipment can also be grouped together based on: load, size, lubrication type, and operating parameters, such as a group of pumps on-board a vessel. In this way, the wear metal data can then be evaluated as a whole. The data for each component can then be compared to the family to evaluate its wear rate (Clarke 2005).

In family analysis, component patterns are classified to obtain component groups, and machine patterns are also classified to machine groups. The machine component matrix is arranged by placing components within a component group adjacently and repeating the same for the machine. The resulting matrix can then be inspected for bottleneck machines and the number of exceptional cells can be minimized. Comparable to the similarity coefficient in similarity coefficient methods, a degree of similarity between the obtained pattern and the ideal pattern is used. The similarity is measured to ensure whether the obtained pattern is properly classified or not (Dagli *et al*., 1995). However, when determining the family analysis of two similar systems, the similarity is compared with a pre-specified threshold. A different threshold can be specified for the classification of components and machines. From there a different degree of clustering is obtained for each threshold (as in the similarity coefficient method).

Further, if there exists a family of five crane bearings in a vessel, and the average Tin (Sn) reading is 8ppm with 90% of the bearings reading less than 10ppm, it would be safe to assume that it is “normal” for these bearings to have less than 10ppm Tin (Sn) in their oil. However, if one of the bearings has a reading of 35ppm of Tin (Sn), it would be safe to say that its wear rate is “abnormal”. Actions can then be instigated to determine the cause of the higher wear rate and the problem could be corrected. This problem can be detected, identified, and resolved before the damage occurs on the equipment, thus saving a premature bearing failure and replacement costs.

Clarke (2005) opines that family analysis techniques can have a significant impact on both large and small companies’ condition monitoring programmes. A large company can use such a programme to monitor large fleets of similar equipment among their plants, as well as benchmark the performance of individual plants. Conversely, a company with less equipment can use family analysis techniques to compare their equipment wear rates with equipment in many other plants, or taking advantage of the vast laboratory database of equipment data for comparison.

The family analysis is also evaluated using a quantitative data transformation method. Unlike the trend analysis, in which only one deck crane was considered, in family analysis, two deck cranes (Port & Starboard) are being evaluated by calculating the standard deviations of the test results from the laboratory for each of the criterion (element). Each quantitative criterion is then transformed to a qualitative criterion by using TFNs. To move from inaction to action required status, standard deviations are calculated to reveal whether the failure modes under review are very similar and the standard deviation is low and predictable, using the following formula:

$$\text{Standard Deviation} = \sqrt{\frac{\sum(x-\bar{x})^2}{(n-1)}} \tag{17}$$

where x is the sample mean average and n is the sample size.

3.5 Evaluation of Environmental Analysis (**Step five**)

The health and performance of machinery as a whole is vitally important. Rather than focusing on the performance of one part, analysts look at everything together in order to obtain a more complete view of what is achievable and what problems might arise along the way. When machinery operators have comprehensive views of their internal and external environments, they are often better able to plan an effective growth strategy. At the same time, early threat identification allows operators to take timely action in developing a survival plan and setting remediation plans in motion to get the machinery back to good condition.

Environmental analysis evaluates the environmental conditions under which the machinery is currently operating. Environmental conditions will be based on vibration measurement, velocity, and acceleration. However, in the current situation, there is no system to collect the data regarding the environmental conditions of the components involved. Good environmental analysis depends on a constant stream of pertinent information (Camponovo, *et al.,* n.d.). In view of this, the test case will be handled in different types of environment, as suggested.

3.6 Evaluation of Human Reliability (**Step six**)

Human reliability analysis (HRA) will assess the operator's performance during the machinery operations practice. According to MAIB (2010), human error is a factor in the majority of marine machinery failures. Psaraftis *et al.* (2000)'s analysis of maritime accident reports indicated that most of the accidents had a human factor as the prevalent cause.

Researchers have done several studies to evaluate human reliability. Riahi *et al.* (2012) assessed the reliability of a seafarer incorporating subjective judgement; in their assessment, Riahi *et al.* (2012) present a dynamic model capable of coping with changing conditions that affect the performance of a seafarer. Adams (1982) analysed the issues affecting human reliability; Askren (1967) evaluated the reliability of human performance in work; Meister (1964) produced a method of predicting human reliability in man machine systems; and Swain (1963) produced a method for performing a human factors reliability analysis. Given that extensive research works on evaluation of human reliability have been conducted by many researchers and experts, the test case on HRA will rely extensively on the results obtained by Riahi *et al.* (2012) from their assessment and evaluation of a seafarer's reliability.

3.7 Evaluation of Design Analysis (**Step seven**)

Machinery and equipment for shipboard use is designed to operate successfully under severe conditions. Ship machinery systems incorporate all the on-board machinery that is used for propulsion, manoeuvring, cargo handling, fresh water production, space heating, etc. This set of equipment constitutes the ship's energy conversion systems, often referred to as the marine energy system (Kakalis *et al.,* 2012). These marine energy systems are designed to convert the chemical energy of the fuel (lubricants) to the forms required to be used in shipboard, and they tend to be highly complex, having many functions, with variable mission profiles, as well as requirements for flexibility, redundancy, and safety. In addition, the systems have to be cost-effective, energy efficient, and environmental friendly. In order to manage such complexity, it is imperative to adopt a structured and effective approach during the design phase.

Design Analysis will assess the physical behaviour of the machinery and its component as specified by the manufacturer (good or bad). It is based on the prediction of the physical behaviour of just about any part or assembly, under any loading condition.

3.8 Aggregation Operations on Criteria Results Using ER (**Step eight**)

The ER algorithm is used to synthesise the risks in a hierarchical structure. Complex decision making problems are represented hierarchically in a structured and systematic manner, as constructed in the generic model shown in Figure 4. In order to find how well an alternative performs across all criteria, the lowest level criteria evaluation is transformed to the upper level and ultimately to the top level criterion. This complex process requires a robust and systematic decision making tool and ER is a method that can be tailored towards such situations where there is high uncertainty and imprecision in information processing. With the help of ER, the results obtained from AHP and the criteria are aggregated.

Insert Figure 4 here

3.9 Obtaining a Crisp Number for the Goal (**Step nine**)

To obtain a single crisp number for the top-level criterion (goal) of each alternative, a utility approach is used in order to rank them. If the utility of an evaluation grade H_n is denoted by $u(H_n)\ and\ u(H_{n+1}) > u(H_n)$, where H_{n+1} is preferred to H_n, $u(H_n)$ can be estimated using the decision maker's preferences. However, in a situation where no preference information is available, it could be assumed that the utilities of evaluation grades are equidistantly distributed in a normalised utility space. The utilities of evaluation grades that are equidistantly distributed in a normalised utility space are calculated as follows:

$$u(H_n) = \frac{V_n - V_{min}}{V_{max} - V_{min}} \tag{18}$$

where, V_n is the ranking value of the linguistic term H_n that has been considered, V_{max} is the ranking value of the most-preferred linguistic term H_N, and V_{min} is the ranking value of the least-preferred linguistic term H_1.

The utility of the top-level or general criterion $S(E)$ is denoted by $u\big(S(E)\big)$. If $\beta_H \neq 0$ (*i.e.* the assessment is incomplete, $\beta_H = 1 - \sum_{n=1}^{N} \beta_n$) there is a belief interval $\big[\beta_{n,}\ (\beta_n + \beta_H)\big]$, which provides the likelihood that $S(E)$ is assessed to H_n. Without loss of generality, suppose that the least-preferred linguistic term having the lowest utility is denoted by $u(H_1)$ and the most preferred linguistic term having the highest utility is denoted by $u(H_N)$. Then according to Yang (2001), the minimum, maximum, and average utilities of $S(E)$ are defined as:

$$u_{min}\big(S(E)\big) = \sum_{n=2}^{N} \beta_n\, u(H_n) + (\beta_1 + \beta_H) u(H_1)$$

$$u_{min}\big(S(E)\big) = \sum_{n=1}^{N-1} \beta_n\, u(H_n) + (\beta_N + \beta_H) u(H_N)$$

$$u_{average}\big(S(E)\big) = \frac{u_{min}\big(S(E)\big) + u_{max}\big(S(E)\big)}{2} \tag{19}$$

If all the assessments are complete, then $\beta_H = 0$ and the maximum, minimum, and average utilities of $S(E)$ will be the same. Therefore, $u\big(S(E)\big)$ can be calculated as:

$$u\big(S(E)\big) = \sum_{n=1}^{N} \beta_n\, u(H_n) \tag{20}$$

According to Riahi *et al.* (2012), an assessment based on a single value is much easier and more instinctive as a practical tool for a professional decision maker to rank the alternative. Thus, to obtain a single crisp number for the goal, the utility value associated with each linguistic term has to be calculated from Equations (18) to (20).

3.10 Perform Sensitivity Analysis (**Final step**)

It is humanly impossible to define a condition monitoring strategy that has every potential failure covered and it is equally very challenging to have good statistical data which reveals that the failure modes under review are very similar and the standard deviation is low and predictable. As a result, owing to the lack of precise data and the novelty of this model, it has not been possible to find any proven benchmark results for its full validation. Given such difficulties and challenges, a possible method for fully validating the model can be achieved only by using an incremental process and through conducting more industrial case studies. The model that will be developed can then be refined and applied in real-world industrial applications.

In view of the above, sensitivity analysis will be used to help validate the model. Sensitivity analysis refers to analysing how sensitive the model outputs are to a minor change in the inputs. The change may be a variation in the parameters of the model or may be changes in the belief degrees assigned to the linguistic variables used to describe the parameters. Sensitivity analysis is very useful when attempting to determine the impact the actual outcome of a particular variable will have if it differs from what was previously assumed. By forming a given set of scenarios, how changes in one variable will impact the target variable can be determined. If the methodology is sound and its conclusion reasoning is logical, then the sensitivity analysis must follow the following three axioms (Riahi *et al.,* 2012):

Axiom 1: A slight increment or decrement in the degree of belief associated with any linguistic variables of the lowest-level criteria will certainly result in a relative increment or decrement in the degree of belief of the linguistic variable and the preference degrees of the model output.

Axiom 2: If the degree of belief associated with the highest-preference linguistic term of the lowest-level criterion is decreased by m and n, simultaneously the degree of belief associated with its lowest-preference linguistic term is increased by m and n ($1 > n > m$), and the utility values of the model output are evaluated as U_m and U_n respectively, then U_m should be greater than U_n.

Axiom 3: If S and R ($R < S$) criteria from all the lowest-level criteria are selected and the degree of belief associated with the highest-preference linguistic term of each of such S and R criteria is decreased by the same amount (*i.e.* simultaneously the degree of belief associated with the lowest-preference linguistic term of each of such S and R criteria is increased by the same amount) and the utility values of the model output are evaluated as U_R and U_S respectively, then U_R should be greater than U_S.

The implementation of the axioms will help to test the certainty of the delivery of the analysis result. The degrees of belief associated with the highest preference linguistic terms of each sub-criterion are decreased by k and simultaneously, the degrees of belief associated with the lowest preference linguistic terms of the corresponding sub-criterion are increased by k. Thus, the corresponding results are obtained. It is worth noting that when the belief degree of the highest preference linguistic term β_α of a criterion is decreased by k, simultaneously, the belief degree of its lowest preference linguistic term has to be increased by k. However, if β_α is less than k, then the remaining belief degree (*i.e.* $k - \beta_\alpha$) can be taken from the belief degree of the next linguistic term. This process continues until k is consumed (Riahi *et al.*, 2012). The comparative ship crane reliability (SCR) results obtained from this methodology are used to determine which crane's components are susceptible to failure. The component with a low SCR value is identified as the one more prone to failure.

4 Test Case

In order to investigate the possibility of failure throughout the lifespan of a ship crane and during its operations, it is essential to monitor the conditions of its components (given in Figure 4 as main criteria) in terms of their reliability during frequently changing sea and weather conditions, by evaluating the laboratory oil sample test results for these components (*i.e.* bearing, clutch, gearbox and hydraulic pump) based on the given absolute limits for oil. The operating condition of both port and starboard cranes in an FPSO operating within European nautical environments is evaluated based on the information given below. Furthermore, the disparity in their conditions during frequently changing sea and weather conditions is calculated. The characteristics of the cranes, the intended use, type, and size of the vessel, and the environment are listed as follows:

1. Crane type: DONG Nam hydraulic crane on main deck – 10 Ton.
2. Offshore crane used in floating production storage and offloading (FPSO).
3. Crane arrangement: Port and Starboard.
4. Degree of rotation: 350^0.
5. Environmental operating conditions: extremes temperature -20^0C to $+45^0$C.
6. Personnel allowed to be lifted with the crane.
7. The crane has an operator's cabin.
8. Lift Height/Depth: 1200m depth double fall.
9. Overload alarm: set to 100% of Safe Working Load (SWL).
10. The crane has the following main components: double-row ball bearing slewing rings, clutches, gearboxes, and hydraulic pumps. Regular oil/grease sample analysis is carried out for these components, and their laboratory test results are recorded.
11. Using a crane for tasks outside its design intent can significantly increases safety risks, crane failures, and downtime. Consequently, taking into account indication of the design loads, life, and estimated average running time, the overall design of the crane was evaluated as being Good.

Four experts were carefully selected to participate in the analysis. The experts' backgrounds in the industry and their assigned weights are as shown in Table 4.

Insert Table 4 here

4.1 Identification of Risk Criteria (**Step one**)

Considering the generic model for monitoring the condition of the machinery (Figure 3) and the above information, a specific model (Figure 4) for monitoring the condition of a ship's crane can be constructed. Analysis grades are assigned to all the criteria in the hierarchical structure and the qualitative and quantitative criteria are grouped. Four main criteria (bearing, clutch, gearbox, and hydraulic pump) and five sub-criteria (trend analysis, family analysis, environmental analysis, human reliability analysis, and design analysis) are identified for the ship crane.

4.2 Application of Analytic Hierarchy Process Results (**Step two**)

Questionnaires were sent to four experts (listed in Table 4) in the industry. The ratings for expert 1's judgements are used as an example to show how the weights (priority vector) are determined. Following which, the ratings for the four experts' judgement will be aggregated using the AHP software and the results will be shown. There will be one pair-wise comparison matrix for each criterion, within each matrix, the pair-wise comparisons will rate each sub-criterion relative to every other sub-criterion.

4.2.1 Development of the ratings for each decision alternative for each criterion

Based on the five sub-criteria identified, five separate matrices have been developed accordingly: one matrix each for trend analysis (TA), family analysis (FA), environmental analysis (EA), human reliability analysis (HRA), and the design analysis (DA). Within each of the aforementioned five matrices, there will be pair-wise comparisons for each component against every other component relative to that criterion. Since there are five sub-criteria under evaluation, each matrix will be of size 5 x 5. Table 5 shows the pair-wise comparison matrix for the five criteria from Expert 1. From Table 5, using the comparison scale given in Table 2, Expert 1 determines that for the crane bearing:

1. TA is strongly important over FA (5).
2. TA is strongly to very strongly important over EA (6).
3. TA is very strongly important over HRA (7).
4. TA is strongly to very strongly important over DA (6).
5. FA is equally to weakly important over EA (2).
6. FA is strongly important over HRA (5).
7. FA is equally important over DA (1).
8. EA is weakly important over HRA (3).
9. DA is strongly important over EA (5).
10. DA is strongly important over HRA (5).

With the aforementioned pair-wise comparison values, a pair-wise comparison matrix can be constructed. Then the weights for trend analysis, family analysis, environmental analysis, human reliability analysis, and design analysis are computed. The 5 x 5 matrix in Table 5 contains all of the pair-wise comparisons for the criteria. The "equally important" values shown along the upper left to lower right diagonal are comparing each criterion to itself and therefore, by definition, must be equal to one. The remaining values shown in the matrix represent the reciprocal pair-wise comparison of relationships previously mentioned.

Insert Table 5 here

From Table 5, the values in each row are multiplied together and the fifth root of the sub-criteria is calculated and recorded as shown in Table 6. The fifth root of the sub-criteria values (and total) from the previous steps is normalized to obtain the appropriate weights (priority vector) for each criterion. The priority vector (PV) values are the criteria weights. The weights for each criterion must sum to one (*i.e.* the total priority vector), as shown in Table 6.

Insert Table 6 here

The pair-wise comparison values in each column are added together (as the “sum” values) and each sum is then multiplied by the respective weight (from the priority vector column) for those criteria to obtain values for SUM x PV. The aforementioned values (shown in the row labelled “Sum x PV”) are added together to yield a total of 5.459 and this value is called *λ-max*. Note that unlike the weights for the criteria, which must sum to one, λ-max will not necessarily be equal to one.

Using Equation (3), the consistency index (CI) is calculated as:

CI = (λ-max – n) / (n-1); where n = 5

CI = (5.459 – 5) / (5-1) = 0.459 / 4 = **0.115**

The CR is calculated by dividing the consistency index (CI) by a random index (RI), which is determined from a lookup table in Table 1. The RI is a direct function of the number of criteria or components being considered. Using Equation (2), CR is calculated as:

CR = CI / RI

The number of sub-criteria being considered in this test case is 5, thus, from Table 1, RI for 5 is given as 1.12.

CR = 0.115 / 1.12 = **0.10**

If the CR $\leq$ 0.10, the decision maker's pair-wise comparisons are relatively consistent. In this case, the CR is 0.10, which indicates that the pair-wise comparisons are consistent and no correction action is necessary.

Using the same method described above, pair-wise comparison values for Experts 2, 3 and 4 are obtained and their pair-wise comparisons matrices constructed. Their corresponding CR are found to be less than or equal to 0.10, thus depicting that their pair-wise comparisons are also consistent.

Similarly, the ratings and the pair-wise comparisons of the individual four experts for the remaining three components (clutch, gearbox, and hydraulic pump) and their corresponding CR are found to be consistent.

4.2.2 Combining the four experts' judgement to determine the pair-wise comparison matrix for each decision alternative for each criterion

Based on the ratings from the four experts for the crane bearing, by applying Equation (1) and similar techniques used in Section 4.2.1, their value ratings can be combined to determine their pair-wise comparison values for the crane bearing, as shown in Table 7.

Insert Table 7 here

Similarly, the four experts' combined pair-wise comparison values for the crane clutch, gearbox, and hydraulic pump are obtained as shown in Tables 8, 9, and 10 respectively.

Insert Tables 8, 9 and 10 here

4.2.3 Weight assignment

Considering the four experts' pair-wise comparison matrix of the five attributes (sub-criteria) for the main criteria, as shown in Tables 7 to 10, and based on Equations (1) to (4), the CR is calculated as 0.1 As a result, the weights of the five attributes can be accepted for use as shown in Table 11.

Insert Tables 11 here

4.3 Evaluation of Trend Analysis (**Step three**)

Evaluation of trend analysis for the four main criteria (bearing, clutch, gearbox and hydraulic pump) is carried out by transforming the grease sample element test results from the crane bearing and the oil sample element test results from the clutch, gearbox, and hydraulic pump to a linguistic variable with the associated belief degree using TFNs illustrated in subsequent sections. Individual test elements are described utilizing five linguistic terms: *Very Low*, *Low,* *Average*, *High* and *Very Hig*h. The explanation of the linguistic terms describing individual scenario is given in Table 12.

Insert Table 12 here

4.3.1 Evaluation of trend analysis for the crane bearing

Table 12 shows the laboratory test results obtained for grease samples taken from the port crane slewing bearing of a FPSO tanker mentioned in previous Section, while Table 13 shows the absolute limits for a crane bearing used grease sample obtained from a reputable oil company. In order to evaluate the trend analysis for this port crane bearing, each of the grease element test results listed in Table 12, with their corresponding limits in Table 13, is transformed to the linguistic variables with associated belief degrees.

Insert Tables 13 and 14 here

Iron (Fe) element in bearing grease samples:

Based on experts' opinions, the upper limit is found and the rules are written for iron (Fe) element with equal distributions, demonstrated as follows:

1. If a crane bearing grease sample laboratory test has a result of 100ppm iron (Fe) or lower, then it can be categorised as 100% Very Low.
2. If a crane bearing grease sample laboratory test has a result of 200ppm iron (Fe), then it can be categorised as 100% Low.
3. If a crane bearing grease sample laboratory test has a result of 300ppm iron (Fe), then it can be categorised as 100% Average.
4. If a crane bearing grease sample laboratory test has a result of 400ppm iron (Fe), then it can be categorised as 100% High.
5. If a crane bearing grease sample laboratory test has a result of 500ppm iron (Fe) and above, then it can be categorised as 100% Very High.

Based on the above rules, the membership functions of the iron (Fe) can be constructed as shown in Figure 5.

Based on the stated rules and by viewing the iron (Fe) contents for the crane bearing grease test results as an independent criterion, the iron (Fe) contents of 20ppm to 43ppm indicate that the crane bearing is still in good condition. Thus, 20ppm to 43ppm iron (Fe) contents in a grease crane bearing can be categorised as 100% Very Low.

Based on the information in Table 13, the laboratory test result for grease sample 1 indicates iron (Fe) contents of 27ppm. Based on Figure 6 and Equation (15), the belief degrees are calculated as follows:

H_{n+1} is the Very Low grade; $h_{n+1,i} = 100$

h_i = 27, 27 < 100

Thus, based on rule 1, the iron (Fe) contents in grease sample 1 test result set are assessed as:

$\widetilde{Fe}_1$ = {(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}

In the similar way, the iron (Fe) contents in grease samples 2 and 3 test result sets are assessed as:

$\widetilde{Fe}_2$ = {(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}

$\widetilde{Fe}_3$ = {(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}

Using a similar technique, based on expert opinions, the upper limit is found and the rules for other elements are demonstrated (Asuquo, 2018). Based on the given rules, membership functions for the elements are constructed as shown in Figures 6 to 15. Based on the information in Table 13, the laboratory test results set for samples 1, 2, and 3 are assessed and their corresponding belief degrees are calculated and recorded as shown in Table 15. Thus, with the help of the ER algorithm, the trend analysis for the estimates of crane bearing grease samples 1, 2 and 3 can be aggregated as shown in Table 15.

Insert Figures 5 to 15 here

Insert Table 15 here

4.3.2 Evaluation of trend analysis for the crane clutch

Table 16 shows the laboratory test results obtained for the three oil samples taken from the port crane clutch at different intervals, while Table 17 shows the absolute limits for the crane clutch used oil sample. In order to evaluate the trend analysis for this port crane clutch, each of the oil element test results listed in Table 16 with their corresponding limits in Table 17 is transformed to linguistic variables with associated belief degrees (Asuquo, 2018).

Insert Tables 16 and 17 here

In a similar way, the metal elements were modelled based on the information given in Tables 16 and 17. The trend analysis for the crane clutch oil samples 1, 2 and 3 is conducted and the results are shown in Table 18 (Asuquo, 2018).

Insert Table 18 here

4.3.3 Evaluation of trend analysis for the crane gearbox

Table 19 shows the laboratory test results obtained for the three oil samples taken from the port crane gearbox at different intervals, while Table 20 shows the absolute limits for the crane used gearbox oil sample. In order to evaluate the trend analysis for this port crane gearbox, each of the oil element test results listed in Table 19 with their corresponding limits in Table 20 is transformed to linguistic variables with associated belief degrees (Asuquo, 2018).

Insert Tables 19 and 20 here

In a similar way, the trend analysis for the crane gearbox oil samples 1, 2 and 3 is conducted and the results are shown in Table 20 (Asuquo, 2018).

Insert Table 21 here

4.3.4 Evaluation of trend analysis for the crane hydraulic pump

Table 22 shows the laboratory test results obtained for the three oil samples taken from the port crane hydraulic pump at different intervals, while Table 23 shows the absolute limits for the crane used hydraulic pump oil sample. In order to evaluate the trend analysis for this port crane hydraulic pump, each of the oil element test results listed in Table 22 with their corresponding limits in Table 23 is transformed to linguistic variables with associated belief degrees (Asuquo, 2018).

Insert Tables 22 and 23 here

In a similar way, the trend analysis for the crane hydraulic pump oil samples 1, 2 and 3 is conducted and the results are shown in Table 24 (Asuquo, 2018).

Insert Table 24 here

4.4 Evaluation of Family Analysis (**Step four**)

Evaluation of family analysis for the four main criteria (bearing, clutch, gearbox and hydraulic pump) is carried out first by determining the standard deviations of the laboratory test results for each of the elements in the grease/oil samples from both port and starboard cranes, and then, by transforming the grease sample element test results from the two cranes’ bearings and the oil sample element test results from the two cranes’ clutches, gearboxes and hydraulic pumps to a linguistic variables with the associated belief degrees using TFNs, as illustrated in subsequent sections.

4.4.1 Evaluation of family analysis for crane bearing

Table 25 shows the standard deviation of both the port and starboard ship deck crane obtained from their bearing grease samples laboratory test results taken for each element. To evaluate family analysis for each of the crane's bearing, each standard deviation of the element in the crane's bearing grease is transformed into linguistic variables with their associated belief degrees.

Based on expert opinions and by equal distribution of standard deviation, the following rules are demonstrated for all the test elements in Table 25:

1. If both cranes bearing grease sample laboratory test results have a standard deviation of 5 or lower, then it can be categorised as 100% Very Good.
2. If both cranes bearing grease sample laboratory test results have a standard deviation of 10 to 15, then it can be categorised as 100% Good.
3. If both cranes bearing grease sample laboratory test results have a standard deviation of 20 to 25, then it can be categorised as 100% Average.
4. If both cranes bearing grease sample laboratory test results has a standard deviation of 30 to 35, then it can be categorised as 100% Bad.
5. If both cranes bearing grease sample laboratory test results has a standard deviation of 40 and above, then it can be categorised as 100% Very Bad.

Insert Table 25 here

Iron (Fe) element in bearing grease samples:

Based on the stated rules, the membership functions of iron (Fe) element in crane bearing grease samples can be constructed as shown in Figure 16. Then, by viewing the standard deviation in iron (Fe) element as an independent criterion, the 19.07 deviations in the grease samples laboratory test results for the two cranes bearings indicate medium iron (Fe) contents in the grease samples. Thus, 19.07 deviation in iron (Fe) contents can be categorised as partially Average and partially Good.

Based on Figure 16 and Equation (15), the belief degrees are calculated as follows:

H_{n+1} is the Average grade; $h_{n+1,i} = 20$

H_n is the Good grade; $h_{n,i} = 15$

h_i = 19, 15 < 19 < 20

$\beta_{n,i} = \frac{20-19}{20-15} = \frac{1}{5} = 0.2 = $ 20% with the Good grade.

$\beta_{n+1,i} = 1 - 0.2 = 0.8 = \ 80\%$ with the Average grade.

Therefore, the standard deviation in iron (Fe) for the bearing grease samples set are assessed as:

$\widetilde{Fe}$ = {(0, Very Good), (0.2, Good), (0.8, Average), (0, Bad), (0, Very Bad)}

Similarly, the membership functions for other elements in Table 25 for the crane bearing grease samples are constructed as shown in Figures 17 to 26. The standard deviations for the oil samples set are assessed and their corresponding belief degrees are calculated and recorded in Table 26. With the help of the ER algorithm, the family analysis results for the crane bearing grease samples are recorded in Table 26.

Insert Figures 16 to 26 here

Insert Table 26 here

4.4.2 Evaluation of family analysis for crane clutch

Table 27 shows the standard deviation of both the port and starboard ship deck crane obtained from the clutch oil samples laboratory test results taken for each element. By applying the same techniques described in Section 4.4.1, and based on the information in Table 27, the family analysis results for the crane clutch oil samples are obtained and shown in Table 28 (Asuquo, 2018).

Insert Tables 27 and 28 here

4.4.3 Evaluation of family analysis for crane gearbox

Table 29 shows the standard deviation of both the port and starboard ship deck crane obtained from their gearbox oil samples laboratory test results taken for each element. Applying the same techniques described Section 4.4.1, and based on the information in Tables 29, the family analysis results for the crane gearbox oil samples are obtained and shown in Table 30 (Asuquo, 2018).

Insert Tables 29 and 30 here

4.4.4 Evaluation of family analysis for crane hydraulic pump

Table 31 shows the standard deviation of both the port and starboard ship deck crane obtained from their hydraulic oil samples laboratory test results taken for each element. Applying the same techniques described in Section 4.4.1, and based on the information in Table 31, the family analysis results for the crane hydraulic pump oil samples are obtained and shown in Table 32 (Asuquo, 2018).

Insert Tables 31 and 32 here

4.5 Evaluation of Environmental Analysis (**Step five**)

The ship crane operating environmental information is not readily available, making it difficult to know the exact environmental conditions during crane operations. With this lack of environmental data, the environmental conditions for the crane are assessed in different conditions of operation, with weights distributed evenly, when the environment is 100% very good, 100% good, 100% average, 100% bad, and 100% very bad, respectively.

A ship crane operating in a 100% very good environment is assessed as:

$\widetilde{E_1}$ = {(0, Very Bad), (0, Bad), (0, Average), (0, Good), (1, Very Good)}

A ship crane operating in a 100% good environment is assessed as:

$\widetilde{E_2}$ = {(0, Very Bad), (0, Bad), (0, Average), (1, Good), (0, Very Good)}

A ship crane operating in a 100% average environment is assessed as:

$\widetilde{E_3}$ = {(0, Very Bad), (0, Bad), (1, Average), (0, Good), (0, Very Good)}

A ship crane operating in a 100% bad environment is assessed as:

$\widetilde{E_4}$ = {(0, Very Bad), (1, Bad), (0, Average), (0, Good), (0, Very Good)}

A ship crane operating in a 100% very bad environment is assessed as:

$\widetilde{E_5}$ = {(1, Very Bad), (0, Bad), (0, Average), (0, Good), (0, Very Good)}

4.6 Evaluation of Human Reliability Analysis (**Step six**)

Based on the research carried out by Riahi *et al.* (2012), the human reliability belief degrees for the crane bearing, clutch, gearbox and the hydraulic pump are assessed as:

$\widetilde{HRA}$ = {(0.1649, High), (0.1958, Fairly High), (0.4355, Medium), (0.2038, Fairly Low), (0, Low)}

4.7 Evaluation of Design Analysis (**Step seven**)

Considering the four machine components (main criteria) of the crane, according to the crane manufacturer, these components are said to be in good condition. Thus, based on the manufacturer’s recommendation, the design analysis belief degrees for each crane bearing, clutch, gearbox, and the hydraulic pump, can be assessed as:

$\widetilde{DA}$ = {(0, Very Bad), (0, Bad), (0, Average), (1, Good), (0, Very Good)}

4.8 Aggregation Operations on Criteria Results using ER (**Step eight**)

Aggregation operations on the sub-criteria and the main criteria are carried out using the ER algorithm (Equations (5) to (14)), and the weights (Table 10) obtained with the help of AHP, as follows.

4.8.1 Aggregation of sub-criteria

The sub-criteria (TA, FA, EA, HRA and DA) for crane bearing, clutch, gearbox, and hydraulic pump of sample 1 are aggregated and recorded as shown in Tables 33 to 36. Similarly, the sub-criteria for crane bearing, clutch, gearbox, and hydraulic pump of samples 2 and 3 are aggregated and results recorded in Tables 37 and 38.

Insert Tables 33 to 38 here

4.8.2 Aggregation of the main criteria

Based on the expert judgements, the main criteria are equally important. Therefore, the weights for the main criteria are evenly distributed among them. Samples 1, 2 and 3 estimates – (B_1, C_1, G_1, H_1), (B_2, C_2, G_2, H_2) and (B_3, C_3, G_3, H_3) respectively – are aggregated with the help of the ER algorithm and the results are presented in Tables 39, 40 and 41.

Insert Tables 39, 40 and 41 here

4.9 Obtaining a Crisp Number for the Goal (**Step Nine**)

Based on Tables 39, 40, and 41, the estimates for the crane's condition (*i.e*. Goal) of samples 1, 2, and 3 are obtained as S_1, S_2 and S_3 respectively.

To obtain a single crisp value for each of the three samples, the utility value associated with each linguistic term is calculated using Equations (18) to (20) as shown in Table 42. Considering the fact that the estimate for the crane (Goal) is characterised by five linguistic terms, the highest preference is given to the Very Good linguistic term, while the lowest preference is given to the Very Bad linguistic term. Therefore, the ranking value is apportioned from five (i.e. highest preference) to one (*i.e.* lowest preference).

Insert Table 42 here

The crane's assessments, as shown in Table 42, are complete. The utility values of the crane based on sample 1 (S_1), sample 2 (S_2), and sample 3 (S_3), as shown in Table 42, are calculated to be:

$\mathbf{S_1}$ = 0.8548, $\mathbf{S_2}$ = 0.9500, $\mathbf{S_3}$ = 0.9123

From the utility values obtained, it can be noted that sample 2 (S_2) scores the highest utility value of 0.950. From these results it can be deduced that the crane's condition was not very good when oil sample 1 was being taken from the components and sent for testing, then the condition was improved when oil sample 2 was taken, but started deteriorating when oil sample 3 was taken. However, it may be argued that either the oil topping or sampling intervals can influence the results.

Similarly, to assess the condition of the main criteria, the utility values for each main criterion in samples 1, 2, and 3 are calculated and the results shown in Tables 39, 40, and 41 respectively.

4.10 Sensitivity Analysis (**Final step**)

To test the certainty of the delivery of the analysis results, the three axioms mentioned in Section 3.10 are used in the sample 2 input data (details in Tables 5-3E to 8-3E of Appendix 3E (Asuquo, 2018)). The degrees of belief associated with the highest preference linguistic values of all the combined sub-criteria are decreased by 0.2, while simultaneously increasing the degrees of belief associated with the lowest preference linguistic values of each of the combined sub-criteria (see details in Tables 1-3F to 4-3F of Appendix 3F (Asuquo, 2018)). The aggregation results obtained are shown in Table 43. All the results obtained remain in harmony with axioms 1 and 2. Also, by using a similar technique to that described in Section 4.9, the crane's utility value from a 0.2 decrement of sample 2 input data is evaluated to be 0.7774, as shown in Table 43.

To examine the alignment of the model with axiom 3, each original estimate for sample 2 in Table 37 is varied with the 0.2 decrement in Table 43 (see details in Tables 1-3G to 4-3G of Appendix 3G (Asuquo, 2018)). The results obtained are shown in Table 44. The comparative utility values (ship crane reliability) for the crane bearing (B_2), clutch (C_2), gearbox (G_2), and hydraulic pump (H_2) obtained are also listed in Table 44 and shown in Figure 27. The lowest utility value of the ship crane is evaluated as 0.909. In view of the fact that 0.7774 (value of aggregation result in Table 43) is smaller than 0.909, this means that the result is aligned with Axiom 3.

Insert Tables 43 and 44 here

Insert Figure 27 here

From Figure 27, it can be seen that the differences between the ship crane components (bearing, clutch, gearbox and hydraulic pump) are relatively marginal. However, with both the crane bearing (B_2) and the gearbox (G_2) having the lowest values (0.909 and 0.912 respectively), it is obvious that the ship crane is more sensitive to the bearing (B_2) and gearbox (G_2) than to the other two components. Therefore, the ranking orders in Figure 27 are consistent with those given by Lloyd's Register (2011), Aldridge (2012) and Konecranes (2012).

5 Discussions

This paper outlines a novel methodology for evaluating a ship's crane performance by means of its conditional reliability. The methodology for evaluating a ship's crane reliability and the procedure for applying it in a real life scenario has been illustrated in the case study in Section 4. This model is one of the first to concede that a ship's crane reliability value is not fixed and it may change due to factors, such as the trend analysis (*i.e.* pattern of behaviour developed over a period of time), family analysis (*i.e.* typical identical pattern of behaviour), environmental analysis (*i.e.* changes in the sea state), human reliability analysis (*i.e.* operator's well-being), and design analysis (*i.e.* crane's physical behaviour as stated by the manufacturer).

For example, if the grade a ship's crane bearing in design analysis is very bad, and the grade of the environment (sea and weather conditions) is very rough, then owing to the roughness of the sea, hostile weather condition and instability of the ship, the engineer on-board would only be able to carry out limited scheduled maintenance work such as oiling, greasing, *etc.* on the crane's bearing. Thus, the crane grade will decrease from a good grade to an average grade. As a result, the reliability of the crane will alter. Therefore, during the conceptual stage of the ship's crane bearing design, the manufacturer should take into consideration uncertain environmental conditions throughout the life cycle of the crane bearing. Based on the analysis results, it can be deduced that if the grade in a ship's crane bearing is very high (0.874 in Table 40), then the crane's reliability value is about 39% more than that of the same crane with low bearing grade (0.6268 in Table 39).

The gearbox is another component that can significantly influence a ship's crane reliability. Based on the analysis, it can be deduced that if the grade in a ship's crane gearbox is very high (0.9095 in Table 40), then the crane's reliability value is about 20% more than that of the same crane with a very low gearbox grade (0.7590 in Table 39). Furthermore, according to Figure 27, the analyses give emphasis to the importance of design, inspections, and condition monitoring in a ship's crane components.

The evaluation of a ship crane's condition monitoring results can be used to develop a preventive measure against incidents. This can be achieved by correctly measuring the crane's condition and regularly taking oil samples from the crane's components and analysing it as scheduled. The grade of a ship crane's condition monitoring result is significant in identifying and taking preventive measures against incidents at sea, as well as in ports, and for ensuring the appropriate condition of operations on-board.

A ship's crane design is highly dependent on the crane's manufacturer and the ship owner's requirements, whereas, the ship's crane trend analysis, family analysis, and human reliability analysis are highly dependent on the ship owner's strategies. Unfortunately, not much can be done with regards to the environmental analysis, as this is a natural phenomenon that is not dependent on either the ship owner or the ship crane manufacturer. However, with proper ship crane design and the implementation of correct condition monitoring strategies, the environmental impacts can be significantly reduced and well managed, therefore leading to a reduction in the frequency of ship crane incidents. Furthermore, a well-structured maintenance regime, in accordance with the recommendations from original equipment manufacturers (OEMs), Port State Control, classification societies and certifications, can reduce the chances of unexpected defects occurring and can ultimately improve the reliability and operational life of the crane.

3.6 Conclusion

This research proposes a novel approach to monitor the ship's crane risk of failure in a systematic fashion. The usefulness of the approach is demonstrated for condition-based decision-making. The approach outlines how a subjective condition-based decision making process can be achieved during situations of high uncertainties in ship's crane operations. The subjective condition monitoring of the investigated system parameters was first carried out using an AHP approach, then assessment grades were mapped into a common utility space before synthesizing for robust decision-making. This generic approach has highlighted a unique feature associated with the performance and unification of input and output data.

The ER approach employed provides a procedure for aggregation which can preserve the original features of multiple attributes under high and imprecise situations. The inclusion of trend analysis, family analysis, environmental analysis, human reliability, and design analysis to the ship's crane condition monitoring approach will help to ensure that findings are incorporated within the maintenance management process for future reference. If each of the analyses is applied to each wear metal for each crane component tested in a programme, the data evaluation process will become too clumsy. Therefore, realistically, the ideal analysis programme would be a combination of the five analysis techniques discussed in this research work.

It can therefore be reasonably expected that the application of this approach will facilitate that the machinery system is maintained to cope with the uncertain environment it is operating in. As revealed in the final result, the developed approach does provide a level of confidence in monitoring the condition of a ship's crane components. Component manufacturers often define limits for single parameters that have a direct impact on the component's lifetime or performance, for example, a roller bearing manufacturer can state that a bearing can reach the calculated fatigue lifetime only if the contamination level is within a certain range. Although this information is valuable, it is often too general and limited to certain aspects of an oil sample. Therefore, component limits are a good reference point if OEM limits are not provided.

Acknowledgements

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 730888. Shell Marine Products and IPU Group are thanked for the provision of test data for the ship cranes. Sincere thanks are also given to the reviewers for their useful comments to improve the quality of this paper.

References

Aczel, J. and Saaty, T. L. (1983), Procedures for synthesizing ratio judgments. J. Math. Psychol., 27: 93-102.

Adams, J. A. (1982), Issues in human reliability. *Hum Factors*; 24(1); 1–10.

Aldridge, Joseph (2012). *Cranes Malfunction at Bay Port*. Bay of Plenty Times. 7.52am Thursday September 27. New Zealand Herald.

Andersen, D. R.; Sweeney D. J.; Williams T. A and Martin K (2008), An Introduction to Management Science: quantitative approaches to decision making. 12th edition. Mason, OH: *Thomson South Western*, pp. 706-718.

Askren, W. B. E. (1967), Symposium on reliability of human performance in work. *Technical Report AMRL-TR-67-88*, Aerospace Medical Research Laboratories, Wright–Patterson Air Force Base, Dayton, OH, USA.

Asuquo, M. P. (2018), Development of an Efficient Planned Maintenance Framework for Marine and Offshore Machinery Operating under Highly Uncertain Environment. PhD Dissertation, Department of Maritime and Mechanical Engineering, Liverpool John Moores University, Liverpool, UK.

Bastos, P.; Lopes, L. and Pires, L. (2012), A Maintenance Prediction System using Data Mining Techniques. *Proceedings of the World Congress on Engineering 2012 Vol III WCE*, July 4 - 6, London, U.K. ISBN: 978-988-19252-2-0; ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online).

Beynon, M.; Cosker, D. and Marshall, D. (2001), An Expert System for Multi-Criteria Decision Making Using Dempster–Shafer Theory. *Journal of Expert Systems with Applications,* Vol. 20, pp. 357–367.

Camponovo, G.; Ondrus, J. and Pigneur Y. (n.d.), *Environmental Context Significance in Strategic Decision Support Systems*. HEC School of Business, University of Lausanne BFSH1 Dorigny, 1015 Lausanne, Switzerland.

Chen, V. Y. C.; Lien, H. P.; Lui, C. H.; Liou, J. J. H.; Tzeng, G. H. and Yang, L. S. (2009), Fuzzy MCDM Approach for Selecting the Best Environment-watershed Plan. *Applied Soft Computing* 11 (2011) 265-275 Elsevier Journal.

Chin, K. S., Xu, D. L., Yang, J. B. and James Ping-Kit, L. (2008), Group-based ER–AHP System for Product Project Screening, *Expert Systems with Applications*, Vol. 35, No. 4, pp. 1909–1929.

Clarke, R. A. (2005), Liquefied Natural Gas Facilities in Urban Areas. A Security Risk Management Analysis for Attorney General. *Patrick Lynch*, Rhode Island.

Courrech, J. and Eshleman, R. L. (2014). Condition Monitoring of Machinery, Chapter 16.

Dagli, C. and Huggahalli, R. (1995), Machine-part family formation with the adaptive resonance theory paradigm. *INT. J. PROD. RES*., VOL. 33, No.4, 893-913.

Galloway, James (2014). Condition Monitoring in RCCL.

Jackson, S. (2010), Architecting Resilient Systems, Accident Avoidance and Survival and Recovery from Disruptions*. Wiley Series in Systems Engineering and Management*, ISBN: 978-0-470-40503-1.

Kakalis, N. and Dimopoulos, G. (2012), Managing the Complexity of Ship Machinery Systems. DNV COSSMOS: Complex Ship Systems Modelling & Simulation. *DNV Research and Innovation*, Position Paper 11.

Konecranes (2012). Reliable Information about Crane Performance and Reliability. Case Study, Hydro, Karmoy, Norway.

Kunz, J. (2010), The Analytic Hierarchy Process (AHP). *Presentation to Eagle City Hall Location Options Task Force*, February/March 2010.

Lloyd's Register (2013), Ship Right, Design and Construction: Annexes to Machinery Planned Maintenance and Condition Monitoring.

Lloyd's Register (2011), Survey and Examination of Ships' Lifting Appliances. In conjunction with UK P&I Club, Maggregor and Liebherr.

Lloyd's Register (2010), Condition Monitoring of Marine Machinery: Guidelines for Shipowners and Managers. 190083927X, 9781900839273

Liu, X. B.; Zhou, M.; Yang, J. B. and Yang, S. L. (2008), Assessment of Strategic R&D Projects for Car Manufacturers Based on the Evidential Reasoning Approach, *Journal of Computer Intelligent System*, Vol. 1, pp. 24–49.

Liu, J.; Yang, J. B.; Wang, J. and Sii, H. S. (2003), Review of Uncertainty Reasoning Approaches as Guidance for Maritime and Offshore Safety-Based Assessment. *Journal of United Kingdom (UK) Safety and Reliability Society*, Vol. 23, No. 1, pp.63 – 80.

Marine Accident Investigation Branch (MAIB) (2010), Annual Report. Extract from the Merchant Shipping (Accident Reporting and Investigation) Regulations 2005.

Meister, D. (1964), Methods of predicting human reliability in man–machine systems. *Hum Factors*; 6(1): 621–646.

Moore, J. H. and Weatherford, L. R. (2001), Decision Modelling with Microsoft Excel. 6th edition. Prentice-Hall, Inc.: Upper Saddle River, NJ.

Pillay, A. and Wang J. (2003), A risk ranking approach incorporating fuzzy set theory and grey theory. *Reliability Engineering System Safety*; 79(1): 61-67.

Psaraftis, H. N.; Panagakos, G.; Desypris, N. and Ventikos, N. (2000). An Analysis of Maritime Transportation Risk Factors. Paper ISOPE-98-HKP-02. [Online] [Viewed: 3rd May 2013]. Available at: http://www.martrans.org/documents/2000/safeco/Isopepaperfinal.doc

Ramanathan, R. and Ganesh, L. S. (1994), Group preference aggregation methods employed in AHP: An evaluation and an intrinsic process for deriving members' weightages. *European Journal of operational Research* 79 (1994) 249 – 265.

Riahi, R.; Bonsall, S.; Jenkinson, I. and Wang, J. (2012), A seafarer's reliability assessment incorporating subjective judgements. *Proc IMechE Part M*: J Engineering for the Maritime Environment 226(4) 313-334.

Riahi, R. (2010), Enabling security and risk-based operation of container line supply chains under high uncertainties. *PhD Dissertation*, Engineering Department, Liverpool John Moores University, Liverpool, UK.

Saaty, T. L. (2008), Relative Measurement and its Generalization in Decision Making: Why Pairwise Comparisons are Central in Mathematics for the Measurement of Intangible Factors – The Analytic Hierarchy/Network Process. *Review of the Royal Academy of Exact*, Physical and Natural Sciences, Series A: Mathematics (RACSAM) 102 (2): 251–318. Retrieved 2008-12-22.

Saaty, T. L. (2004), Decision Making, the Analytical Hierarchy and Network Processes (AHP/ANP). J Systems Sci System Engineering; 13(1): 1-35.

Saaty, T. L. and Vargas, L. G. (2001), Models, Methods, Concepts & Applications of the Analytic Hierarchy Process. *Kluwer Academic Publishers*: Norwell, Mass, pages 12 & 27.

Saaty, T. L. (1990), How to make decisions: The Analytical Hierarchy Process. Eur J Opl Res 1990; 48(1): 9-26.

Saaty, T. L. (1983), Priority Setting in Completing Problems. *IEEE Transactions on Engineering Management*. Volume 30. Number 3. August 1983. Pages 140-155.

Saaty, T. L. (1989), Group decision making and the AHP. in: B. L. Golden, E.A. Wasil and P. T. Harker (eds.), The Analytic Hierarchy Process: Applications and Studies, Springer-Verlag, New York, 59-67.

Sen, P. and Yang, J. B. (1998), Multiple Criteria Decision Support in Engineering Design. *Springer, New York*.

Sonmez, M.; Holt, G. D.; Yang, J. B. and Graham, G. (2002), Applying Evidential Reasoning to Pre-qualifying Construction Contractors. *Journal of Management Engineering*, Vol. 18, No. 3, pp. 111–119.

Srivastava, R. P. and Liu, L. (2003), Applications of Belief Functions in Business Decisions, a Review. *Journal of Information Systems Frontiers*, Vol. 5, No.4, pp. 359–378.

Srivastava, R. P. and Lu, H. (2002), Structural analysis of audit evidence using belief functions. *Journal of Fuzzy Sets Systems,* Vol. 131, pp. 107–120.

Swain, A. D. (1963), A method for performing a human-factors reliability analysis. *Sandia Corporation Monograph SCR-685*. Albuquerque, NM: Sandia Corporation.

Toms, L. A. (1998), Machinery Oil Analysis, Methods, Automation & Benefits: A Guide for Maintenance Managers, Supervisors & Technicians. Second Edition March 25.

Wang, J. and Trbojevic, V. M. (2007), Design for Safety of Large Marine and Offshore Engineering Products. *Institute of Marine Engineering, Science and Technology (IMarEST)*, London, UK, ISBN: 1-902536-58-4.

Wang, Y. M. and Elhag, T. M. S. (2007), A fuzzy group decision making approach for bridge risk assessment. *Computers & Industrial Engineering* 53, 137–148. doi:10.1016/j.cie.2007.04.009.

Wang, J.; Yang, J. B. and Sen, P. (1996), Multi-Person and Multi-Attribute Design Evaluations Using Evidential Reasoning Based on Subjective Safety and Cost Analyses. *Reliability Engineering & System Safety*, Vol. 52, No. 2, pp. 113-128.

Wang, J.; Yang, J. B. and Sen, P. (1995), Safety Analysis and Synthesis using Fuzzy Sets and Evidential Reasoning based on Subjective Safety and Cost Analysis. *Reliability Engineering and System Safety*, Vol. 47, No. 2, pp. 103-118.

Watts, M. J. (n.d.), Fuzzy Sets and Fuzzification.
http://mike.watts.net.nz/Teaching/IIS/Lecture4.pdf [Accessed on 13.06.2014].

Wu, W. Z.; Zhang, M.; Li, H. Z. and Mi, J. S. (2005), Knowledge Reduction in Random Information Systems via Dempster–Shafer Theory of Evidence. *Journal of Information Science,* Vol. 174, No. 34, pp. 143–164.

Xu, D. L.; Yang, J. B. and Wang, Y. M. (2001) in press, The ER approach for multi-attribute decision analysis under interval uncertainties. *European Journal of Operational Research*.

Yang, Z. L., Wang, J., Bonsall, S. and Fang, Q. G. (2005), Reliable Container Supply Chain, a New Risk Assessment Framework for Improving Safety Performance. *Journal of World Marine University*, Vol. 4, No. 1, pp. 105-120.

Yang, J. B. and Xu, D. L. (2002), On the Evidential Reasoning Algorithm for Multiple Attribute Decision Analysis under Uncertainty. *IEEE Transactions on Systems, Man and Cybernetics Part A:* Systems and Human, Vol. 32, No. 3, pp. 289–304.

Yang, J. B. (2001), Rule and Utility Based Evidential Reasoning Approach for Multi-Attribute Decision Analysis under Uncertainties. *European Journal of Operational Research*; 131(1): 31-61.

Yang, J. B., Deng, M. and Xu, D. L. (2001), Nonlinear Regression to Estimate Both Weights and Utilities via Evidential Reasoning for MADM, Processing. *51^th^ International Conference*, *Optimization: Techniques and Applications*, Dec. 15-17, Hong Kong.

Yang, J. B. and Sen, P. (1997), Multiple attribute design evaluation of large engineering products using the evidential reasoning approach. *Journal of engineering design*, Vol. 8, No. 3, pp. 211-230.

Yang, J. B. and Sen, P. (1996), Preference Modelling by Estimating Local Utility Functions for Multi objective Optimisation. *European Journal of Operational Research*, Vol. 95, pp. 115-138.

Yang, J. B. and Sen, P. (1994), A General Multi-Level Evaluation Process for Hybrid MADM with Uncertainty. *IEEE Transaction on Systems, Man and Cybernetics*, Vol. 34, No. 10, pp. 1458-1473.

Yang, J. B. and Singh, M. G. (1994), An Evidential Reasoning Approach for Multiple Attribute Decision Making with Uncertainty. *IEEE Transactions on Systems, Man and Cybernetics,* Vol. 24, No. 1, pp. 1–18.

Zadeh, L. A. (1975), The Concept of a Linguistic Variable and its Application to Approximate Reasoning. *Information Sciences*, 8 (2), pp. 199–249.

Zeng, Y.; Wang, L. and Zheng, J. (2006), Aggregative Risk Assessment Model for International Technology and Project Development. *Fifth Wuhan International Conference on E-Business*, Integration on Innovation through Measurement and Management, Vol. 1, No. 3, pp. 1139-1144.

Zhang, X. D.; Zhao, H. and Wei, S. Z. (2005), Research on Subjective and Objective Evidence Fusion Method in Oil Reserve Forecast. *Journal of System Simulation*, No.17, Vol. 10, pp. 2537–2540.

Zhao, Y. (2014), The Importance of Lubricant and fluid Analysis in Predictive Maintenance. SPECTRO, INC. OIL_WPv2_2014-03-26.

Zhou, M., Liu, X.B. and Yang, J.B. (2010), “Evidential Reasoning Based Nonlinear Programming Model for MCDA under Fuzzy Weights and Utilities”, *Journal of Intelligent Systems,* Vol. 25, pp. 31-58.

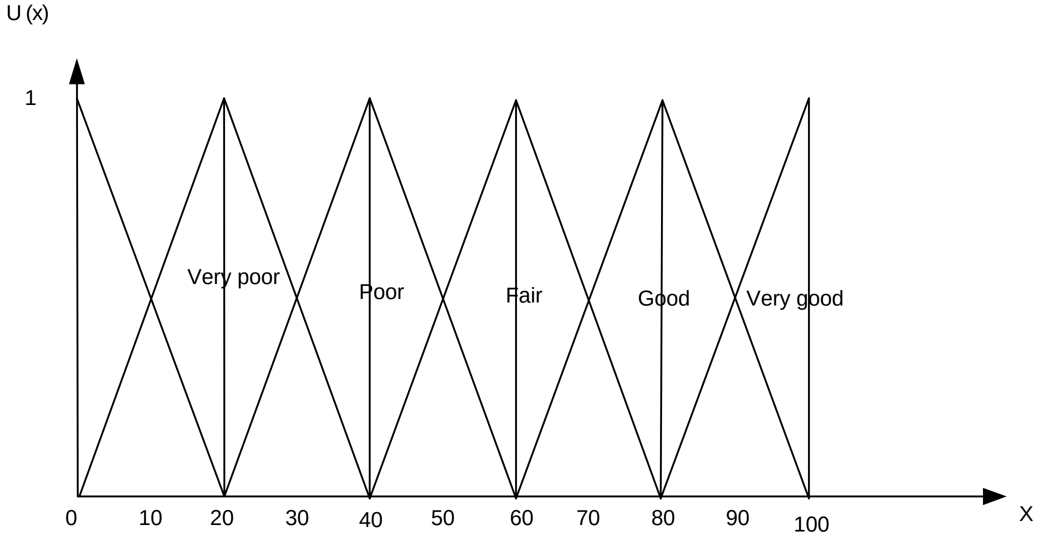

Figure 1: Membership Functions of Linguistics Variables for Measuring the Performance Value of Alternatives

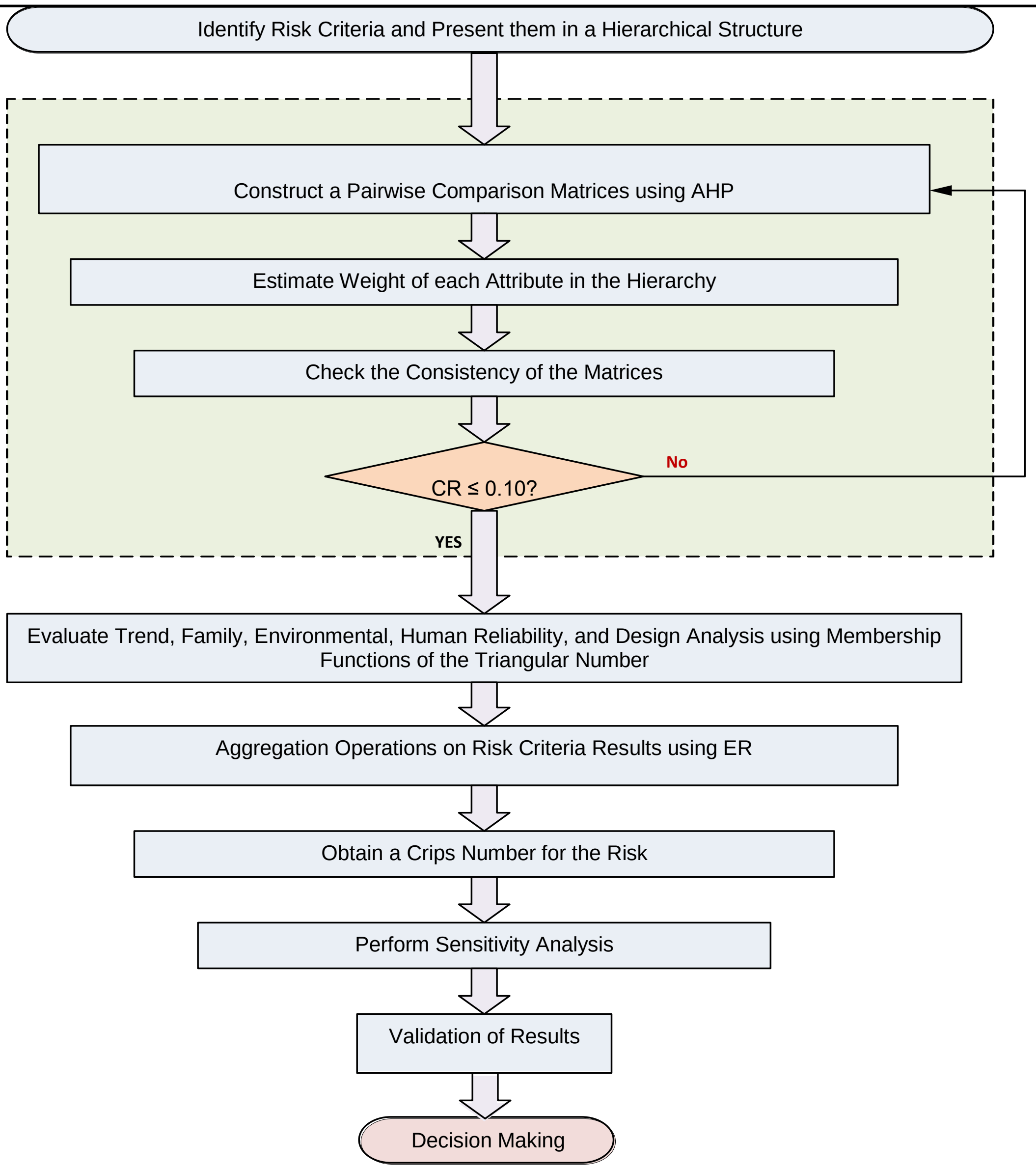

Figure 2: Flow diagram for evaluating the condition of equipment

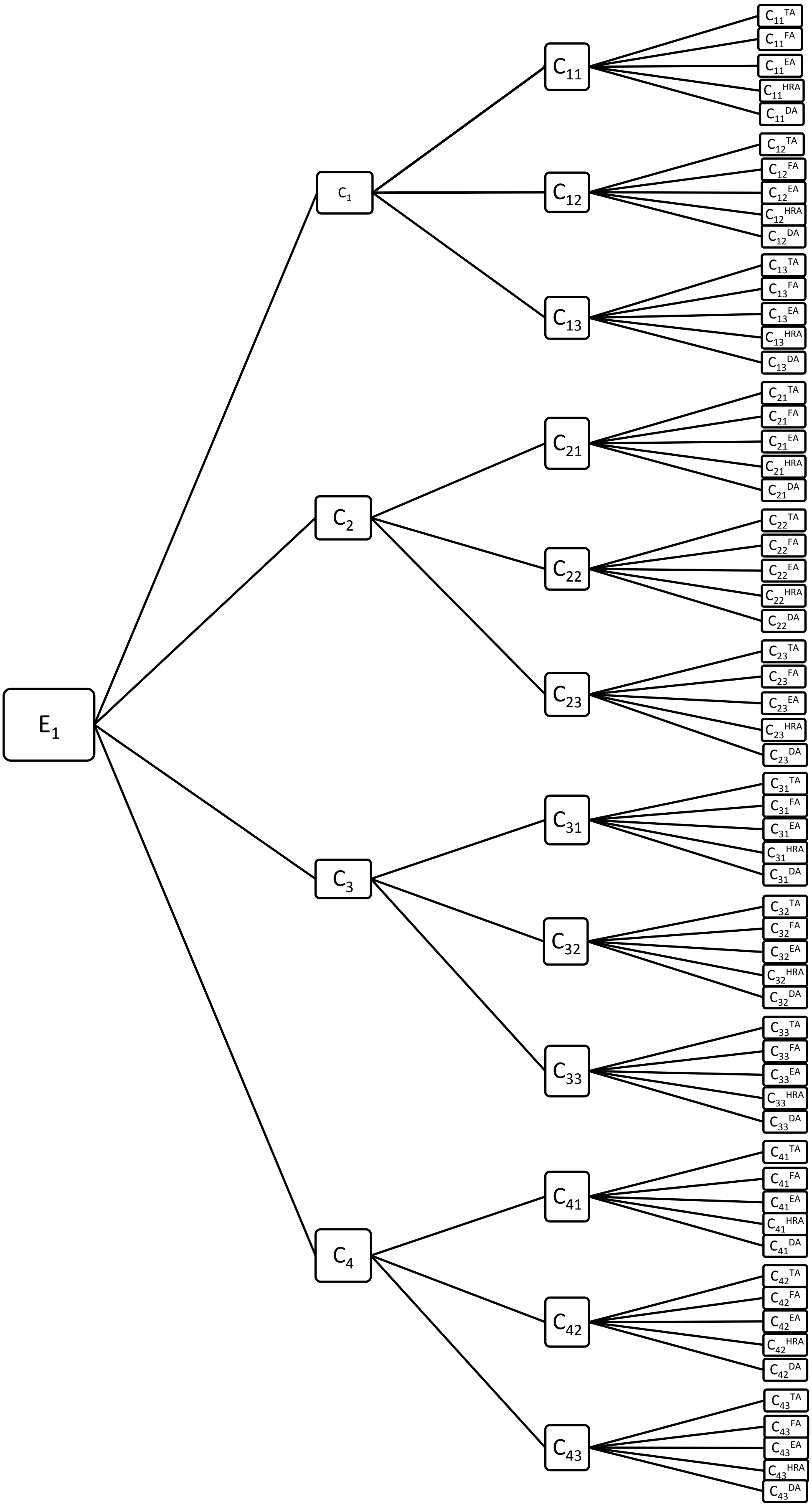

Figure 3: A Generic Model for Condition Monitoring of Machinery

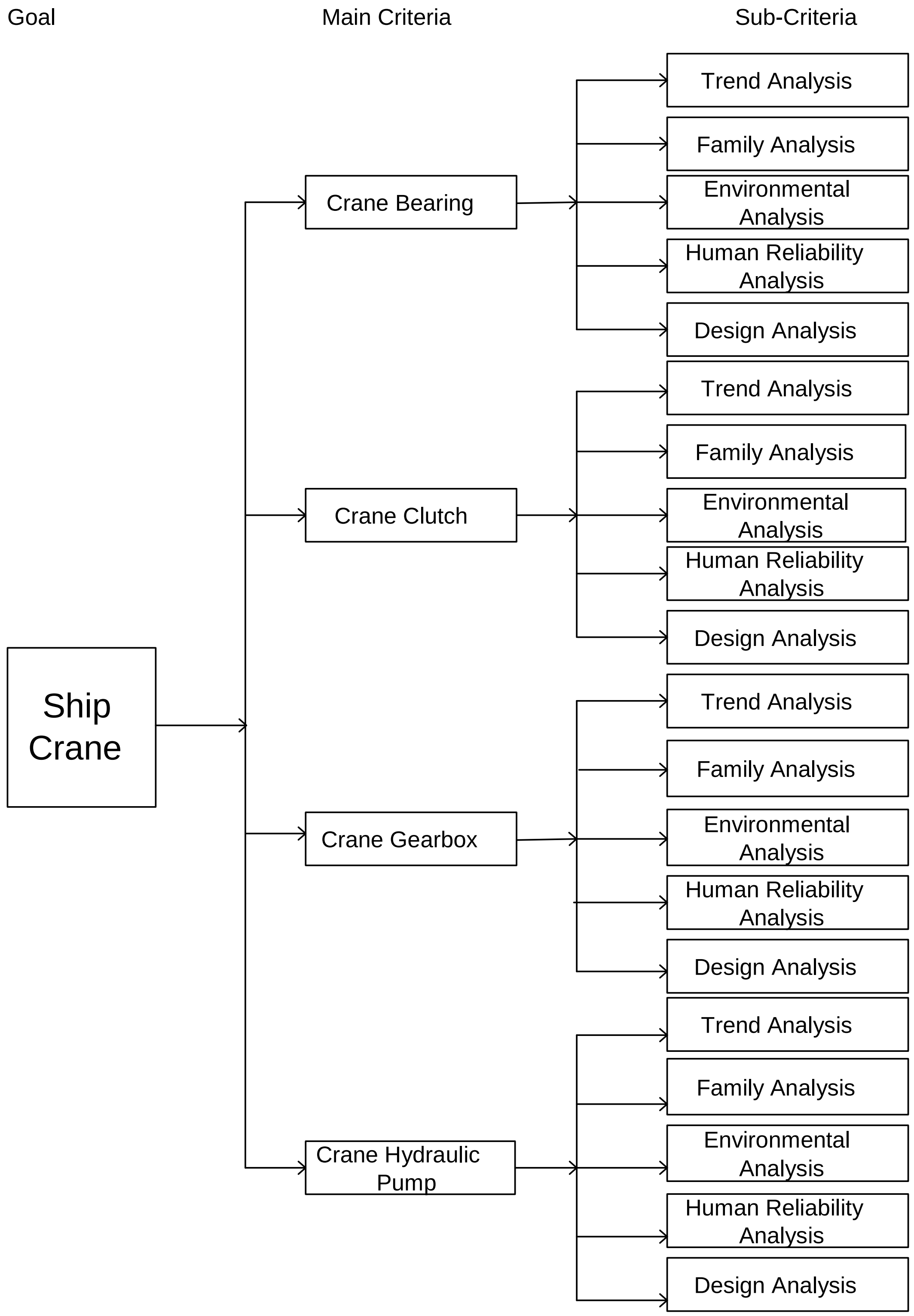

Figure 4: Specific Model for Condition Monitoring of a Ship Crane

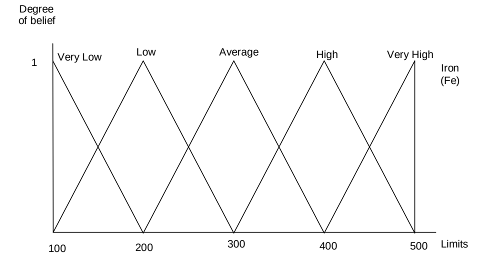

Figure 5: Membership Function of the Iron (Fe) Element

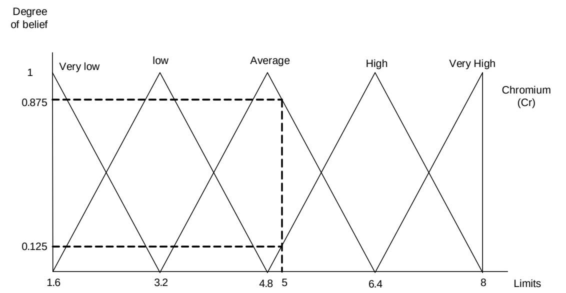

Figure 6: Membership Function of the Chromium (Cr) Element

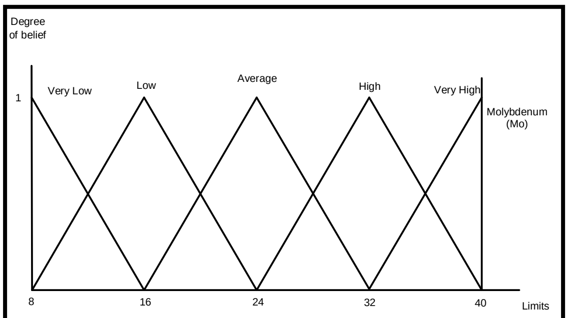

Figure 7: Membership Function of the Molybdenum (Mo) Element

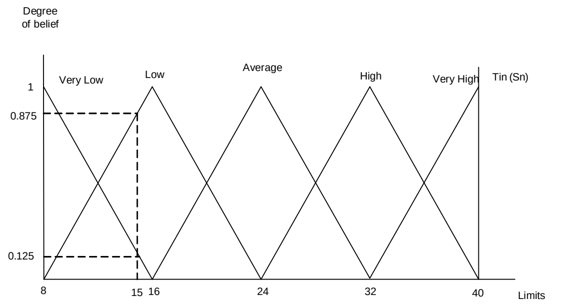

Figure 8: Membership Function of the Tin (Sn) Element

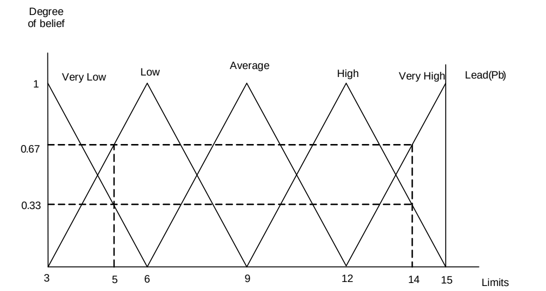

Figure 9: Membership Function of the Lead (Pb) Element

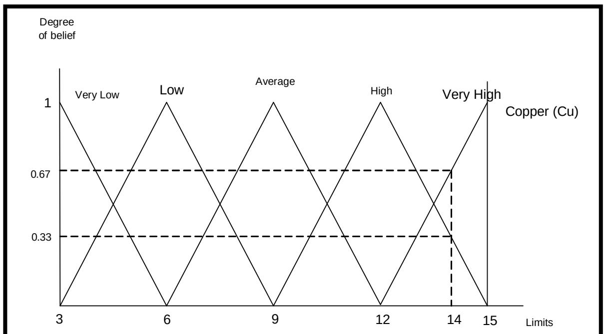

Figure 10: Membership Function of the Copper (Cu) Element

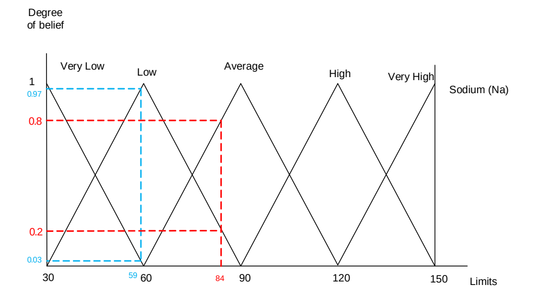

Figure 11: Membership Function of the Sodium (Na) Element

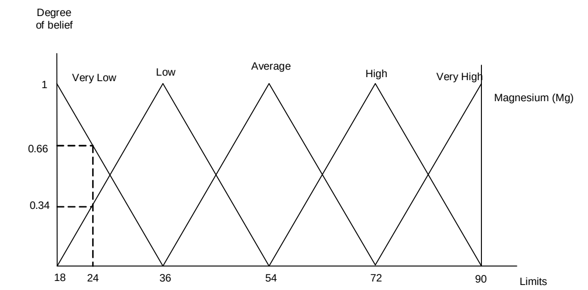

Figure 12: Membership Function of the Magnesium (Mg) Element

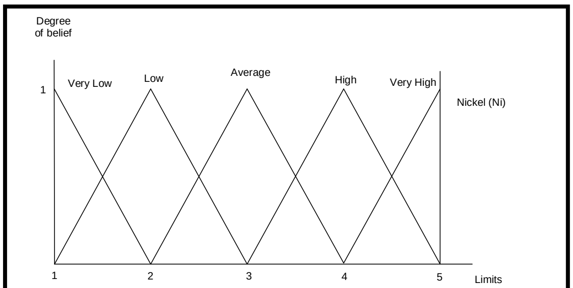

Figure 13: Membership Function of the Nickel (Ni) Element

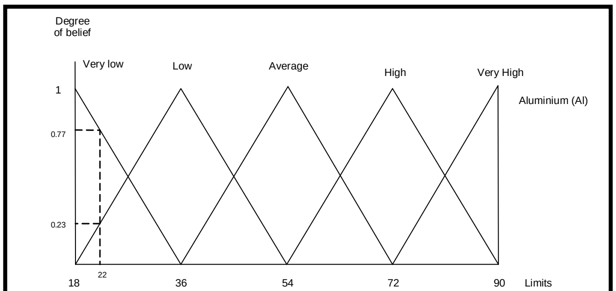

Figure 14: Membership Function of the Aluminium (Al) Element

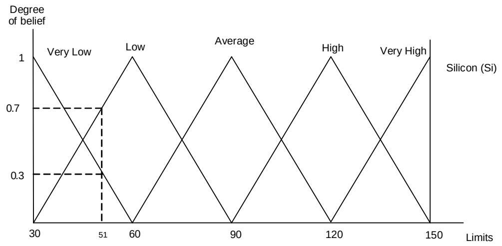

Figure 15: Membership Function of the Silicon (Si) Element

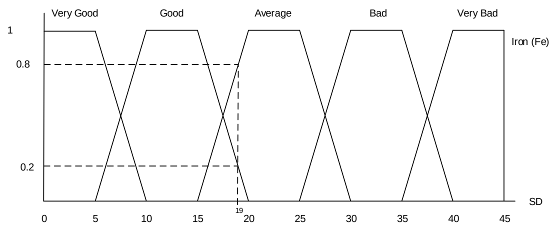

Figure 16: Membership Function of the Iron (Fe) Element

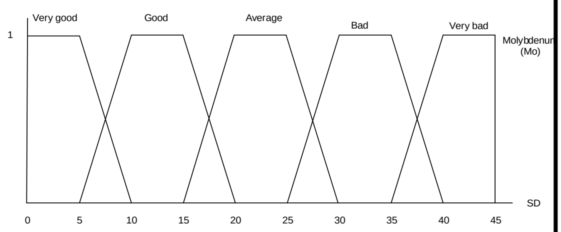

Figure 17: Membership Function of the Molybdenum (Mo) Element

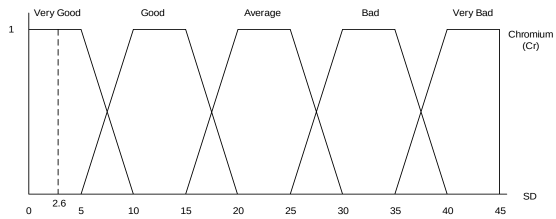

Figure 18: Membership Function of the Chromium (Cr) Element

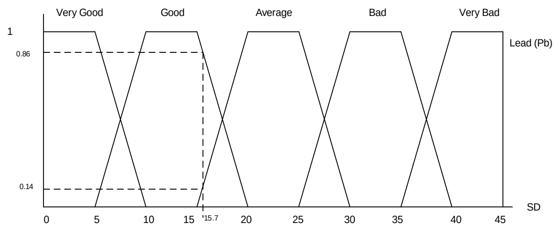

Figure 19: Membership Function of the Lead (Pb) Element

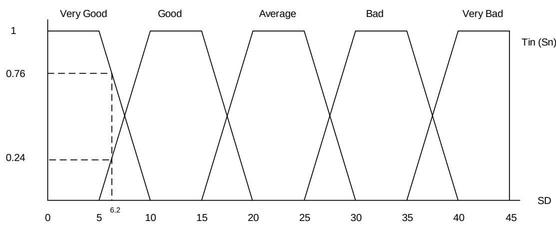

Figure 20: Membership Function of the Tin (Sn) Element

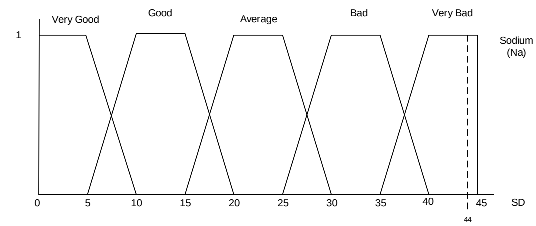

Figure 21: Membership Function of the Sodium (Na) Element

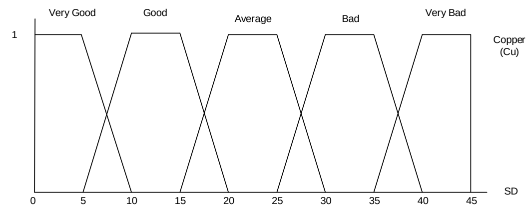

Figure 22: Membership Function of the Copper (Cu) Element

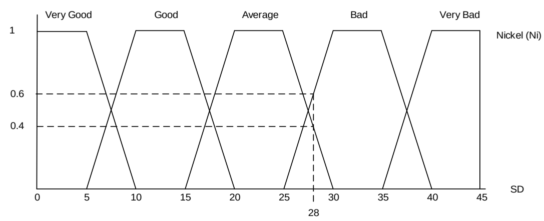

Figure 23: Membership Function of the Nickel (Ni) Element

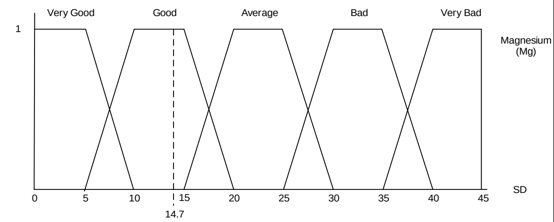

Figure 24: Membership Function of the Magnesium (Mg) Element

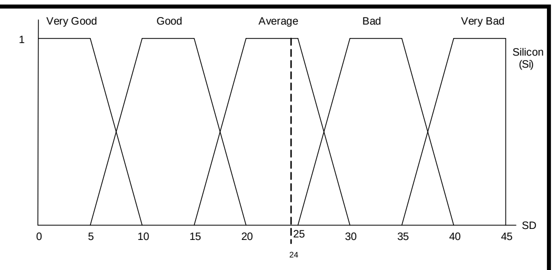

Figure 25: Membership Function of the Silicon (Si) Element

Figure 26: Membership Function of the Aluminium (Al) Element

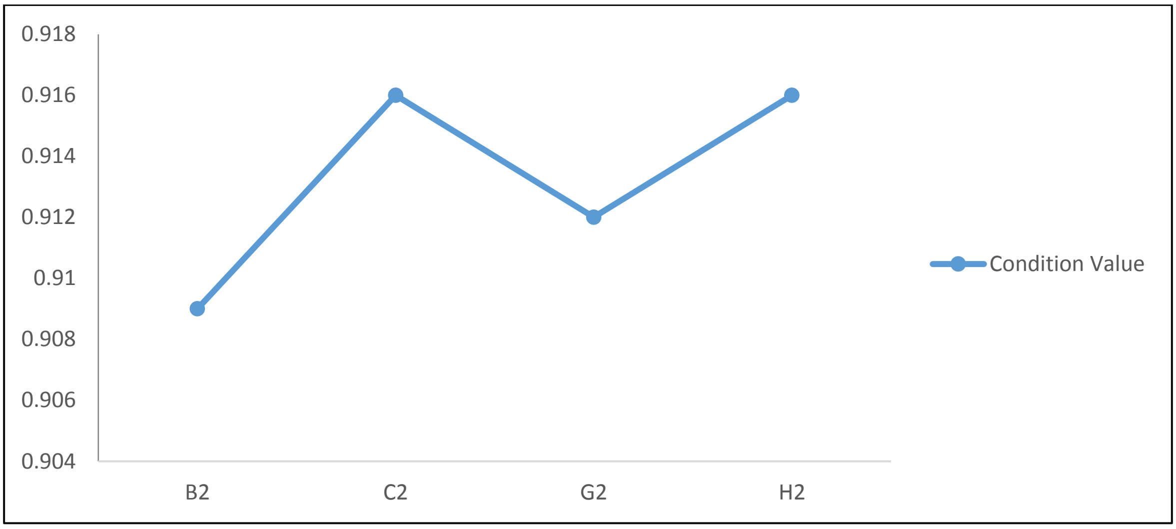

Figure 27: Sensitivity of the Model Output to the Variation of the Alteration with Original in each Main Criterion

Table 1: Value of RI versus Matrix Order

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Source: Hypothetical data [Saaty, (1990)]

Table 2: Comparison Scale

Relative Importance of Attribute (Scale)	Definition
1	Equal importance (EQI)
3	Moderate importance of one over another (MI)
5	Essential or strong importance (SI)
7	Very strong importance (VSI)
9	Extreme importance (EI)
2, 4, 6, 8	Intermediate values between the two adjacent judgements (Int2, Int4, Int6, Int8)

Source: Hypothetical data [Saaty, (1990)]

Table 3: Composition of Experts

Composition	Classification
Industry Position	Production Manager, Cargo Officer, Maintenance Engineer, and Captain
Service Time	> 30 years
Academic Qualification	▪ Master degree ▪ Bachelor degree ▪ HND ▪ Class 1 Certificate of Competency

Table 4: Weighting of Expert Judgments

Number of Decision Makers	Industrial Position	Service Period	Academic Qualification	Experts' Weights
DM1	Production Manager	> 30 years	MSc	0.25
DM2	Senior Cargo Officer	> 30 years	HND	0.25
DM3	Senior Maintenance Engineer	> 30 years	1^{st} Degree	0.25
DM4	Ship Captain	> 30 years	Class 1 Certificate of Competency	0.25
				Total = 1

Table 5: Expert 1 Pair-wise Comparison Matrix for the Five Criteria

Crane Bearing	TA	FA	EA	HRA	DA
TA	1	5	6	7	6
FA	0.2	1	2	5	1
EA	0.167	0.5	1	3	0.2
HRA	0.143	0.2	0.333	1	0.2
DA	0.167	1	5	5	1

Table 6: Developing Expert 1 Rating for each Decision Alternative for the Crane Bearing

Crane Bearing	**TA**	**FA**	**EA**	**HRA**	**DA**	**5th Root of Component**	**PV**
TA	1	5	6	7	6	4.169	0.557
FA	0.2	1	2	5	1	1.149	0.154
EA	0.167	0.5	1	3	0.2	0.549	0.073
HRA	0.143	0.2	0.333	1	0.2	0.286	0.038
DA	0.167	1	5	5	1	1.331	0.178
SUM	1.677	7.7	14.333	21	8.4	7.484	**1.000**
SUM * PV	0.934	1.186	1.046	0.798	1.495	**5.459**	
Lambda-max = 5.459							
CI = 0.115							
CR = 0.103							

Table 7: Combined Pair-Wise Comparison Matrix for Crane Bearing

Crane Bearing	**TA**	**FA**	**EA**	**HRA**	**DA**	**5th Root**	**PV**
TA	1	4.4	3.31	4.527	2.783	2.836	0.484
FA	0.226	1	0.574	3.663	1.186	0.562	0.096
EA	0.302	1.732	1	2.711	0.603	0.969	0.165
HRA	0.220	0.272	0.368	1	0.254	0.354	0.060
DA	0.359	0.841	1.655	3.936	1	1.144	0.195
SUM	2.107	8.245	6.907	15.837	5.826	5.865	**1.000**
SUM * PV	1.019	0.791	1.139	0.950	1.136	**5.035**	
Lambda-max = 5.035							
CI = 0.087							
CR = 0.077							

Table 8: Combined Pair-Wise Comparison Matrix for Crane Clutch

Crane Clutch	**TA**	**FA**	**EA**	**HRA**	**DA**	**5th Root**	**PV**
TA	1	3.344	4.606	5.144	4.949	3.301	0.503
FA	0.299	1	1.778	3.499	0.841	1.094	0.167
EA	0.217	0.562	1	1.861	0.379	0.612	0.093
HRA	0.193	0.286	0.537	1	0.293	0.387	0.059
DA	0.203	1.189	2.632	3.409	1	1.167	0.178
SUM	1.912	6.381	10.553	14.913	7.462	6.561	**1.000**
SUM * PV	0.962	1.066	0.981	0.879	1.328	**5.216**	
Lambda-max = 5.216							
CI = 0.054							
CR = 0.05							

Table 9: Combined Pair-Wise Comparison Matrix for Crane Gearbox

Crane Gearbox	**TA**	**FA**	**EA**	**HRA**	**DA**	**5th Root**	**PV**
TA	1	4.729	4.729	6.117	4.162	3.557	0.524
FA	0.212	1	1.861	4.401	1	1.117	0.165
EA	0.212	0.537	1	2.059	0.595	0.674	0.099
HRA	0.163	0.228	0.485	1	0.255	0.341	0.050
DA	0.239	1	1.682	3.936	1	1.096	0.162
SUM	1.826	7.494	9.757	17.513	7.012	6.785	**1.000**
SUM * PV	0.957	1.237	0.966	0.876	1.136	**5.172**	
Lambda-max = 5.172							
CI = 0.043							
CR = 0.04							

Table 10: Combined Pair-Wise Comparison Matrix for Crane Hydraulic Pump

Crane Hydraulic Pump	TA	FA	EA	HRA	DA	5th Root	PV
TA	1	4.472	4.229	3.873	3.761	3.076	0.485
FA	0.224	1	1.732	4.162	1.189	1.139	0.181
EA	0.236	0.577	1	2.449	0.904	0.787	0.124
HRA	0.258	0.239	0.408	1	0.302	0.377	0.059
DA	0.265	0.841	1.107	3.309	1	0.960	0.151
SUM	1.983	7.129	8.476	14.793	7.156	6.339	1.000
SUM * PV	0.962	1.290	1.051	0.873	1.081	5.257	
Lambda-max	= 5.257						
CI	= 0.064						
CR	= 0.057						

Table 11. Weights of the Sub-Criteria

Sub-Criteria	Crane Bearing	Crane Clutch	Crane Gearbox	Crane Hydraulic Pump
Trend Analysis	0.484	0.503	0.524	0.485
Family Analysis	0.096	0.167	0.165	0.181
Environmental Analysis	0.165	0.093	0.099	0.124
Human Reliability Analysis	0.060	0.059	0.050	0.059
Design Analysis	0.195	0.178	0.162	0.151

Table 12: Description for Test Elements and General Interpretation

Linguistic Term for Test Elements	General Interpretation
Very Low	Wear particles present in small quantities. Acceptable amount of normal wear particles.
Low	Wear particles present in small quantities. Acceptable amount of normal wear particles.
Average	Wear particles present in medium quantities. Acceptable amount of normal wear particles.
High	Wear particles present in high quantities. Unacceptable amount of normal wear particles.
Very High	The wear metals content is higher than normal. The crane should be stopped for investigation.

Table 13: Grease Sample Report for Ship Port Crane Bearing

Elements	Sample 3	Sample 2	Sample 1
Iron (Fe) mg/kg	43	20	27
Chromium (Cr) mg	0	0	5
Molybdenum (Mo)	0	0	0
Tin (Sn) mg/kg	15	0	0
Lead (pb) mg/kg	45	5	14
Copper (Cu) mg/k	122	0	14
Sodium (Na) mg/k	84	59	0
Magnesium (Mn) m	0	24	0
Nickel (Ni) mg/k	5	1	72
Aluminium (Al) m	13	22	174
Silicon (Si) mg/k	8	51	30

Data source: From a reputable lubricants manufacturer

Table 14: Absolute Limits for Crane Bearing Used Grease Sample

Test	Upper Attention	Upper Action
Iron (Fe)	500	750
Chromium (Cr)	8	11
Molybdenum (Mo)	40	50
Tin (Sn)	40	60
Lead (Pb)	15	20
Copper (Cu)	15	20
Sodium (Na)	150	200
Magnesium (Mg)	90	100
Nickel (Ni)	5	8
Aluminium (Al)	90	150
Silicon (Si)	150	250

Data source: A reputable lubricants manufacturer

Table 15: Estimates for Crane Bearing Grease Samples – Trend Analysis

Test Elements	Estimates for Sample 1	Estimates for Sample 2	Estimates for Sample 3
Iron (Fe)	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Chromium (Cr)	{(0, Very Low), (0, Low), (0.875, Average), (0.125, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Tin (Sn)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0.125, Very Low), (0.875, Low), (0, Average), (0, High), (0, Very High)}
Molybdenum (Mo)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)},	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)},	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)},
Lead (Pb)	{(0, Very Low), (0, Low), (0, Average), (0.33, High), (0.67, Very High)}	{(0.33, Very Low), (0.67, Low), (0, Average), (0, High), (0, Very High)}	{(0, Very Low), (0, Low), (0, Average), (0, High), (1, Very High)}
Copper (Cu)	{(0, Very Low), (0, Low), (0, Average), (0.33, High), (0.67, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0, Very Low), (0, Low), (0, Average), (0, High), (1, Very High)}
Sodium (Na)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0.03, Very Low), (0.97, Low), (0, Average), (0, High), (0, Very High)}	{(0, Very Low), (0.2, Low), (0.8, Average), (0, High), (0, Very High)}
Magnesium (Mg)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0.66, Very Low), (0.34, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Nickel (Ni)	{(0, Very Low), (0, Low), (0, Average), (0, High), (1, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0, Very Low), (0, Low), (0, Average), (0, High), (1, Very High)}
Aluminium (Al)	{(0, Very Low), (0, Low), (0, Average), (0, High), (1, Very High)}	{(0.77, Very Low), (0.23, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Silicon (Si)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0.3, Very Low), (0.7, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Aggregation Result	**{(0.5858, Very Low), (0, Low), (0.0664, Average), (0.0611, High), (0.2867, Very High)}**	**{(0.7828, Very Low), (0.2172, Low), (0, Average), (0, High), (0, Very High)}**	**{(0.6041, Very Low), (0.0831, Low), (0.0609, Average), (0, High), (0.2519, Very High)}**

Source: Test case data

Table 16: Grease Sample Report for Ship Port Crane Clutch

Elements	Sample 3	Sample 2	Sample 1
Iron (Fe) mg/kg	6	8	8
Chromium (Cr) mg	0	0	0
Molybdenum (Mo)	0	0	0
Tin (Sn) mg/kg	1	0	1
Lead (Pb) mg/kg	1	1	2
Copper (Cu) mg/k	5	6	5
Aluminium (Al) m	1	0	0
Silicon (Si) mg/k	4	5	4
Vanadium (V) mg/k	9	10	8

Data source: From a reputable lubricants manufacturer

Table 17: Absolute Limits for Crane Clutch Oil Tests

Test	Upper Attention	Upper Action
Iron (Fe)	45	68
Chromium (Cr)	5	8
Molybdenum (Mo)	6	8
Tin (Sn)	10	15
Lead (Pb)	5	11
Copper (Cu)	22	32
Aluminium (Al)	10	15
Silicon (Si)	35	55
Vanadium (V)	40	53

Data source: From a reputable lubricants manufacturer

Table 18: Estimates for Crane Clutch Oil Samples – Trend Analysis

Test Elements	Estimates for Sample 1	Estimates for Sample 2	Estimates for Sample 3
Iron (Fe)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}
Chromium (Cr)	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}
Molybdenum (Mo)	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}
Tin (Sn)	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very low), (0, Low), (0, Average), (0, High), (0, Very High)}
Lead (Pb)	{(0, Very Low), (1, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Copper (Cu)	{(0.86, Very Low), (0.14, Low), (0, Average), (0, High), (0, Very High)}	{(0.64, Very Low), (0.36, Low), (0, Average), (0, High), (0, Very High)}	{(0.86, Very Low), (0.14, Low), (0, Average), (0, High), (0, Very High)}
Aluminium (Al)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Silicon (Si)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Vanadium (V)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0.75, Very Low), (0.25, Low), (0, Average), (0, High), (0, Very High)}	{(0.875, Very Low), (0.125, Low), (0, Average), (0, High), (0, Very High)}
Aggregation Result	**{(0.9134, Very Low), (0.0866, Low), (0, Average), (0, High), (0, Very High)}**	**{(0.9562, Very Low), (0.0438, Low), (0, Average), (0, High), (0, Very High)}**	**{(0.9818, Very Low), (0.0182, Low), (0, Average), (0, High), (0, Very High)}**

Source: Test case data

Table 19: Oil Sample Report for Ship Port Crane Gearbox

Test Elements	Sample 3	Sample 2	Sample 1
Water Content %v	0.1	0	0
Total Acid Number (TAN)	0.31	0.42	0.37
Iron (Fe) mg/kg	13	11	15
Chromium (Cr) mg	0	0	0
Molybdenum (Mo)	187	259	513
Tin (Sn) mg/kg	3	0	22
Lead (Pb) mg/kg	0	0	0
Copper (Cu) mg/k	31	29	36
Sodium (Na) mg/k	0	3	0
Aluminium (Al) m	4	3	6
Silicon (Si) mg/	4	4	9
Vanadium (V) mg/	0	0	0

Data source: From a reputable lubricants manufacturer

Table 20: Absolute Limits for Crane Gearbox Oil Tests

Test	Upper Attention	Upper Action
Water Content	0.1	0.21
Total Acid No. (TAN)	1.5	2.5
Iron (Fe)	60	98
Chromium (Cr)	4	6
Molybdenum (Mo)	6	9
Tin (Sn)	7	9
Lead (Pb)	28	47
Copper (Cu)	36	60
Aluminium (Al)	7	10
Silicon (Si)	30	40
Sodium (Na)	30	40
Vanadium (V)	5	10

Data source: From a reputable lubricants manufacturer

Table 21: Estimates for Crane Gearbox Oil Samples – Trend Analysis

Test Elements	Estimates for Sample 1	Estimates for Sample 2	Estimates for Sample 3
Water Content	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0, Very Low), (0, Low), (0, Average), (0, High), (1, Very High)}
TAN	{(0.76, Very Low), (0.24, Low), (0, Average), (0, High), (0, Very High)}	{(0.6, Very Low), (0.4, Low), (0, Average), (0, High), (0, Very High)}	{(0.96, Very Low), (0.04, Low), (0, Average), (0, High), (0, Very High)}
Iron (Fe)	{(0.75, Very Low), (0.25, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0.92, Very Low), (0.08, Low), (0, Average), (0, High), (0, Very High)}
Chromium (Cr)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Molybdenum (Mo)	{(0, Very Low), (0, Low), (0, Average), (0, High), (1, Very High)}	{(0, Very Low), (0, Low), (0, Average), (0, High), (1, Very High)}	{(0, Very Low), (0, Low), (0, Average), (0, High), (1, Very High)}
Tin (Sn)	{(0, Very Low), (0, Low), (0, Average), (0, High), (1, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0, Very Low), (0.85, Low), (0.15, Average), (0, High), (0, Very High)}
Lead (Pb)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Copper (Cu)	{(0, Very Low), (0, Low), (0, Average), (0, High), (1, Very High)}	{(0, Very Low), (0, Low), (0, Average), (0.97, High), (0.03, Very High)}	{(0, Very Low), (0, Low), (0, Average), (0.69, High), (0.31, Very High)}
Aluminium (Al)	{(0, Very Low), (0, Low), (0, Average), (0.71, High), (0.29, Very High)}	{(0, Very Low), (0.85, Low), (0.15, Average), (0, High), (0, Very High)}	{(0, Very Low), (0.14, Low), (0.86, Average), (0, High), (0, Very High)}
Silicon (Si)	{(0.5, Very Low), (0.5, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Sodium (Na)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Vanadium (V)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Aggregation Result	**{(0.6331, Very Low), (0.0694, Low), (0, Average), (0.0484, High), (0.2491, Very High)}**	**{(0.7812, Very Low), (0.0816, Low), (0.0096, Average), (0.0618, High), (0.0658, Very High)}**	**{(0.6298, Very Low), (0.079, Low), (0.0713, Average), (0.0482, High), (0.1717, Very High)}**

Source: Test case data

Table 22: Oil Sample Report for Ship Port Crane Hydraulic Pump

Test Elements	Sample 3	Sample 2	Sample 1
Water Content %v	0	0	0
Iron (Fe) mg/kg	0	0	1
Chromium (Cr) mg	0	0	0
Molybdenum (Mo)	0	0	0
Tin (Sn) mg/kg	0	0	0
Lead (Pb) mg/kg	0	0	0
Copper (Cu) mg/k	0	9	7
Sodium (Na) mg/k	0	9	0
Aluminium (Al) m	0	0	0
Silicon (Si) mg/	0	0	0
Vanadium (V) mg/	0	0	0

Data source: From a reputable lubricants manufacturer

Table 23: Absolute Limits for Crane Hydraulic Pump Oil Tests

Test	Upper Attention	Upper Action
Water Content	0.2	0.5
Iron (Fe)	23	36
Chromium (Cr)	6	10
Molybdenum (Mo)	6	10
Tin (Sn)	6	10
Lead (Pb)	8	13
Copper (Cu)	36	55
Sodium (Na)	30	40
Aluminium (Al)	6	10
Silicon (Si)	30	35
Vanadium (V)	5	10

Data source: From a reputable lubricants manufacturer

Table 24: Estimates for Crane Hydraulic Pump Oil Samples – Trend Analysis

Test Elements	Estimates for Sample 1	Estimates for Sample 2	Estimates for Sample 3
Water Content	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Iron (Fe)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Chromium (Cr)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Molybdenum (Mo)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Tin (Sn)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Lead (Pb)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Copper (Cu)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0.75, Very Low), (0.25, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Aluminium (Al)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Silicon (Si)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Sodium (Na)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(0.5, Very Low), (0.5, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Vanadium (V)	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}	{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}
Aggregation Result	**{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}**	**{(0.9561, Very Low), (0.0439, Low), (0, Average), (0, High), (0, Very High)}**	**{(1, Very Low), (0, Low), (0, Average), (0, High), (0, Very High)}**

Source: Test case data

Table 25: Standard Deviation for Port and Starboard Cranes Bearing Grease Test Results

	PORT CRANE			STARBOARD CRANE				
Test Elements	Sample 3	Sample 2	Sample 1	Sample 3	Sample 2	Sample 1	Average Value	Standard Deviation
Iron (Fe) mg/kg	43	20	27	69	46	20	37.5	19.07
Chromium (Cr) mg	0	0	5	0	0	5	1.667	2.582
Molybdenum (Mo)	0	0	0	0	0	0	0	0
Tin (Sn) mg/kg	15	0	0	7	10	1	5.5	6.221
Lead (Pb) mg/kg	45	5	14	39	14	23	23.33	15.65
Copper (Cu) mg/k	122	0	14	181	0	20	56.17	76.57
Sodium (Na) mg/k	84	59	0	108	56	0	51.17	43.88
Magnesium (Mg) m	0	24	0	0	32	0	9.333	14.68
Nickel (Ni) mg/k	5	1	72	8	3	3	15.33	27.86
Aluminium (Al) m	13	22	174	20	26	15	45	63.37
Silicon (Si) mg	8	51	30	4	66	30	31.5	24.01

Data source: From a reputable lubricants manufacturer

Table 26: Estimates for Crane Bearing Oil Samples – Family Analysis

Test Elements	**Estimates**
Iron (Fe)	{(0, Very Good), (0.2, Good), (0.8, Average), (0, Bad), (0, Very Bad)}
Chromium (Cr)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Molybdenum (Mo)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Tin (Sn)	{(0.76, Very Good), (0.24, Good), (0, Average), (0, Bad), (0, Very Bad)}
Lead (Pb)	{(0, Very Good), (0.86, Good), (0.14, Average), (0, Bad), (0, Very Bad)}
Copper (Cu)	{(0, Very Good), (0, Good), (0, Average), (0, Bad), (1, Very Bad)}
Sodium (Na)	{(0, Very Good), (0, Good), (0, Average), (0, Bad), (1, Very Bad)}
Magnesium (Mg)	{(0, Very Good), (1, Good), (0, Average), (0, Bad), (0, Very Bad)}
Nickel (Ni)	{(0, Very Good), (0, good), (0.4, Average), (0.6, Bad), (0, Very Bad)}
Aluminium (Al)	{(0, Very Good), (0, Good), (0, Average), (0, Bad), (1, Very Bad)}
Silicon (Si)	{(0, Very Good), (0, Good), (1, Average), (0, Bad), (0, Very Bad)}
$\widehat{FA}$ **for Crane Bearings**	**{(0.253, Very Good), (0.2076, Good), (0.2118, Average), (0.0503, Bad), (0.2773, Very Bad)}**

Source: Test case data

Table 27: Standard Deviation for Port and Starboard Cranes Clutch Oil Test Results

	PORT CRANE			STARBOARD CRANE				
Test Elements	**Sample 3**	**Sample 2**	**Sample 1**	**Sample 3**	**Sample 2**	**Sample 1**	**Average Value**	**Standard Deviation**
Iron (Fe)	6	8	8	11	11	10	9	2
Chromium (Cr)	0	0	0	0	0	0	0	0
Molybdenum (Mo)	0	0	0	0	0	0	0	0
Tin (Sn)	1	0	1	4	4	3	2.167	1.722
Lead (Pb)	1	1	2	1	1	0	1	0.632
Copper (Cu)	5	6	5	10	10	9	7.5	2.429
Magnesium (Mg)	13	13	10	19	20	17	15.33	3.933
Aluminium (Al)	1	0	0	2	2	0	0.833	0.983
Silicon (Si)	4	5	4	5	5	6	4.833	0.753
Vanadium (V)	9	10	8	15	17	14	12.17	3.656

Source: Test case data

Table 28: Estimates for Crane Clutch Oil Samples – Family Analysis

Test Elements	**Estimates**
Iron (Fe)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Chromium (Cr)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Molybdenum (Mo)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Tin (Sn)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Lead (Pb)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Copper (Cu)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}

Magnesium (Mg)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Aluminium (Al)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Silicon (Si)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Vanadium (V)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
$\widehat{FA}$ for Crane Clutches	**{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}**

Source: Test case data

Table 29: Standard Deviation for Port and Starboard Cranes Gearbox Oil Test Results

	PORT CRANE			**STARBOARD CRANE**				
Test Elements	**Sample 3**	**Sample 2**	**Sample 1**	**Sample 3**	**Sample 2**	**Sample 1**	**Average Value**	**Standard Deviation**
Water Content %v	0.1	0	0	0	0	0	0.017	0.041
Total Acid No. (TAN)	0.31	0.42	0.37	0.36	0.43	0.67	0.427	0.127
Iron (Fe) mg/kg	13	11	15	13	16	20	14.67	3.141
Chromium (Cr) mg	0	0	0	0	0	0	0	0
Molybdenum (Mo)	187	259	513	253	488	598	383	170.2
Tin (Sn) mg/kg	3	0	22	1	0	0	4.333	8.733
Lead (Pb) mg/kg	0	0	0	0	0	4	0.667	1.633
Copper (Cu) mg/k	31	29	36	24	32	38	31.67	5.007
Sodium (Na) mg/k	0	3	0	0	3	4	1.667	1.862
Magnesium (Mg) m	1	0	1	1	0	1	0.667	0.516
Boron (B) mg/kg	3	0	0	0	0	0	0.5	1.225
Aluminium (Al) m	4	3	6	6	7	12	6.333	3.141
Silicon (Si) mg/	4	4	9	9	11	15	8.667	4.227
Vanadium (V) mg/	0	0	0	0	0	0	0	0

Source: Test case data

Table 30: Estimates for Crane Gearbox Oil Samples – Family Analysis

Test Elements	**Estimates**
Water Contents %v	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Total Acid Number (TAN)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Iron (Fe)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Chromium (Cr)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Molybdenum (Mo)	{(0, Very Good), (0, Good), (0, Average), (0, Bad), (1, Very Bad)}
Tin (Sn)	{(0.26, Very Good), (0.74, Good), (0, Average), (0, Bad), (0, Very Bad)}
Lead (Pb)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Copper (Cu)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Sodium (Na)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Magnesium (Mg)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Boron (B)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Aluminium (Al)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Silicon (Si)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Vanadium (V)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
$\widehat{FA}$ for Crane Gearboxes	**{(0.9172, Very Good), (0.0352, Good), (0, Average), (0, Bad), (0.0476, Very Bad)}**

Source: Test case data

Table 31: Standard Deviation for Port & Starboard Cranes Hydraulic Pump Test Results

	PORT CRANE			**STARBOARD CRANE**				
Test Elements	**Sample 3**	**Sample 2**	**Sample 1**	**Sample 3**	**Sample 2**	**Sample 1**	**Average Value**	**Standard Deviation**
Water Content %v	0	0	0	0	0	0	0	0
Total Acid No. (TAN)	0.55	0.5	0.5	0.48	0.34	0.42	0.465	0.074
Iron (Fe)	0	0	1	0	0	0	0.167	0.408
Chromium (Cr)	0	0	0	0	0	0	0	0
Molybdenum (Mo)	0	0	0	0	0	0	0	0
Tin (Sn)	0	0	0	0	0	0	0	0
Lead (Pb)	0	0	0	0	0	0	0	0
Copper (Cu)	0	9	7	0	6	3	4.167	3.764
Sodium (Na)	0	9	0	0	6	0	2.5	3.987
Magnesium (Mg)	0	0	0	0	0	0	0	0

Boron (B)	0	1	0	0	1	0	0.333	0.516
Aluminium (Al)	0	0	0	0	0	0	0	0
Silicon (Si)	0	0	0	0	0	0	0	0
Vanadium (V)	0	0	0	0	0	0	0	0

Source: Test case data

Table 32: Estimates for Crane Hydraulic Pump Oil Samples – Family Analysis

Test Elements	Estimates
Water Contents %v	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Total Acid Number (TAN)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Iron (Fe)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Chromium (Cr)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Molybdenum (Mo)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Tin (Sn)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Lead (Pb)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Copper (Cu)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Sodium (Na)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Magnesium (Mg)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Boron (B)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Aluminium (Al)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Silicon (Si)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
Vanadium (V)	{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}
$\widehat{FA}$ for Crane Hydraulic Pump	**{(1, Very Good), (0, Good), (0, Average), (0, Bad), (0, Very Bad)}**

Source: Test case data

Table 33: Aggregation of Sub-Criteria for Crane Bearing Sample 1

Sub-Criteria	Estimates
E_1 , TA_1, FA_B, HR, DA	{(0.1914, Very Bad), (0.0449, Bad), (0.0761, Average), (0.1693, Good), (0.5183, Very Good)}
E_2 , TA_1, FA_B, HR, DA	{(0.1977, Very Bad), (0.0463, Bad), (0.0787, Average), (0.3266, Good), (0.3507, Very Good)}
E_3 , TA_1, FA_B, HR, DA	{(0.2014, Very Bad), (0.0472, Bad), (0.2162, Average), (0.1781, Good), (0.3572, Very Good)}
E_4 , TA_1, FA_B, HR, DA	{(0.2026, Very Bad), (0.1782, Bad), (0.0806, Average), (0.1792, Good), (0.3594, Very Good)}
E_5 , TA_1, FA_B, HR, DA	{(0.3522, Very Bad), (0.0461, Bad), (0.0783, Average), (0.1742, Good), (0.3492, Very Good)}
Aggregation result (main criteria) B_1	**{(0.2251, Very Bad), (0.0658, Bad), (0.0978, Average), (0.1996, Good), (0.4117, Very Good)}**

Source: Test case data

Table 34: Aggregation of Sub-Criteria for Crane Clutch Sample 1

Sub-Criteria	Estimates
E_1 , TA_1, FA_C, HR, DA	{(0.0053, Very Bad), (0.0063, Bad), (0.0140, Average), (0.1685, Good), (0.8060, Very Good)}
E_2 , TA_1, FA_C, HR, DA	{(0.0056, Very Bad), (0.0066, Bad), (0.0147, Average), (0.2476, Good), (0.7255, Very Good)}
E_3 , TA_1, FA_C, HR, DA	{(0.0057, Very Bad), (0.0067, Bad), (0.0697, Average), (0.1805, Good), (0.7373, Very Good)}
E_4 , TA_1, FA_C, HR, DA	{(0.0057, Very Bad), (0.0607, Bad), (0.0150, Average), (0.1806, Good), (0.7379, Very Good)}
E_5 , TA_1, FA_C, HR, DA	{(0.0596, Very Bad), (0.0067, Bad), (0.0150, Average), (0.1807, Good), (0.7380, Very Good)}
Aggregation result (main criteria) C_1	**{(0.0121, Very Bad), (0.0129, Bad), (0.0191, Average), (0.1552, Good), (0.8006, Very Good)}**

Source: Test case data

Table 35: Aggregation of Sub-Criteria for Crane Gearbox Sample 1

Sub-Criteria	Estimates
E_1 , TA_1, FA_G, HR, DA	{(0.1569, Very Bad), (0.0338, Bad), (0.0118, Average), (0.1582, Good), (0.6392, Very Good)}
E_2 , TA_1, FA_G, HR, DA	{(0.1636, Very Bad), (0.0352, Bad), (0.0123, Average), (0.2430, Good), (0.5458, Very Good)}
E_3 , TA_1, FA_G, HR, DA	{(0.1664, Very Bad), (0.0359, Bad), (0.0748, Average), (0.1677, Good), (0.5552, Very Good)}
E_4 , TA_1, FA_G, HR, DA	{(0.1660, Very Bad), (0.1005, Bad), (0.0125, Average), (0.1673, Good), (0.5538, Very Good)}
E_5 , TA_1, FA_G, HR, DA	{(0.2416, Very Bad), (0.0353, Bad), (0.0123, Average), (0.1649, Good), (0.5459, Very Good)}
Aggregation result (main criteria) G_1	**{(0.1602, Very Bad), (0.0403, Bad), (0.0205, Average), (0.1615, Good), (0.6175, Very Good)}**

Source: Test case data

Table 36: Aggregation of Sub-Criteria for Crane Hydraulic Pump Sample 1

Sub-Criteria	Estimates
E_1 , TA_1, FA_H, HR, DA	{(0.0047, Very Bad), (0.0055, Bad), (0.0123, Average), (0.0058, Good), (0.9718, Very Good)}
E_2 , TA_1, FA_H, HR, DA	{(0.0052, Very Bad), (0.0062, Bad), (0.0138, Average), (0.0750, Good), (0.8998, Very Good)}
E_3 , TA_1, FA_H, HR, DA	{(0.0052, Very Bad), (0.0062, Bad), (0.0833, Average), (0.0064, Good), (0.8989, Very Good)}
E_4 , TA_1, FA_H, HR, DA	{(0.0052, Very Bad), (0.0747, Bad), (0.0138, Average), (0.0065, Good), (0.8998, Very Good)}
E_5 , TA_1, FA_H, HR, DA	{(0.0736, Very Bad), (0.0062, Bad), (0.0138, Average), (0.0065, Good), (0.8999, Very Good)}
Aggregation result (main criteria) H_1	**{(0.0124, Very Bad), (0.0130, Bad), (0.0182, Average), (0.0132, Good), (0.9432, Very Good)}**

Source: Test case data

Table 37: Aggregation Results of Sub-Criteria for Sample 2

Bearing (B_2)	{(0.0460, Very Bad), (0.0275, Bad), (0.0490, Average), (0.1395, Good), (0.7380, Very Good)}
Clutch (C_2)	{(0.0094, Very Bad), (0.0100, Bad), (0.0148, Average), (0.0254, Good), (0.9404, Very Good)}
Gearbox (G_2)	{(0.0412, Very Bad), (0.0366, Bad), (0.0198, Average), (0.0478, Good), (0.8545, Very Good)}
Hydraulic Pump (H_2)	{(0.0127, Very Bad), (0.0134, Bad), (0.0186, Average), (0.0283, Good), (0.9269, Very Good)}

Source: Test case data

Table 38: Aggregation Results of Sub-Criteria for Sample 3

Bearing (B_3)	{(0.1754, Very Bad), (0.0296, Bad), (0.0820, Average), (0.0835, Good), (0.6294, Very Good)}
Clutch (C_3)	{(0.0092, Very Bad), (0.0098, Bad), (0.0146, Average), (0.0162, Good), (0.9501, Very Good)}
Gearbox (G_3)	{(0.0962, Very Bad), (0.0341, Bad), (0.0502, Average), (0.0515, Good), (0.7680, Very Good)}
Hydraulic Pump (H_3)	{(0.0124, Very Bad), (0.0130, Bad), (0.0182, Average), (0.0132, Good), (0.9432, Very Good)}

Source: Test case data

Table 39: Aggregation of Main Criteria from Sample 1

Main Criteria	Estimates	Utility Value
Bearing (B_1)	{(0.2251, Very Bad), (0.0658, Bad), (0.0978, Average), (0.1996, Good), (0.4117, Very Good)}	0.6268
Clutch (C_1)	{(0.0121, Very Bad), (0.0129, Bad), (0.0191, Average), (0.1552, Good), (0.8006, Very Good)}	0.9299
Gearbox (G_1)	{(0.1602, Very Bad), (0.0403, Bad), (0.0205, Average), (0.1615, Good), (0.6175, Very Good)}	0.7590
Hyd. Pump (H_1)	{(0.0124, Very Bad), (0.0130, Bad), (0.0182, Average), (0.0132, Good), (0.9432, Very Good)}	0.9655
Aggregation result (S_1)	**{(0.0829, Very Bad), (0.0261, Bad), (0.0308, Average), (0.1095, Good), (0.7507, Very Good)}**	**0.8548**

Source: Test case data

Table 40: Aggregation of Main Criteria from Sample 2

Main Criteria	Estimates	Utility Value
Bearing (B_2)	{(0.0460, Very Bad), (0.0275, Bad), (0.0490, Average), (0.1395, Good), (0.7380, Very Good)}	0.8740
Clutch (C_2)	{(0.0094, Very Bad), (0.0100, Bad), (0.0148, Average), (0.0254, Good), (0.9404, Very Good)}	0.9694
Gearbox (G_2)	{(0.0412, Very Bad), (0.0366, Bad), (0.0198, Average), (0.0478, Good), (0.8545, Very Good)}	0.9095
Hyd. Pump (H_2)	{(0.0127, Very Bad), (0.0134, Bad), (0.0186, Average), (0.0283, Good), (0.9269, Very Good)}	0.9609
Aggregation result (S_2)	**{(0.0190, Very Bad), (0.0152, Bad), (0.0178, Average), (0.0425, Good), (0.9054, Very Good)}**	**0.9500**

Source: Test case data

Table 41: Aggregation of Main Criteria from Sample 3

Main Criteria	Estimates	Utility Value
Bearing (B_3)	{(0.1754, Very Bad), (0.0296, Bad), (0.0820, Average), (0.0835, Good), (0.6294, Very Good)}	0.7405
Clutch (C_3)	{(0.0092, Very Bad), (0.0098, Bad), (0.0146, Average), (0.0162, Good), (0.9501, Very Good)}	0.9721
Gearbox (G_3)	{(0.0962, Very Bad), (0.0341, Bad), (0.0502, Average), (0.0515, Good), (0.7680, Very Good)}	0.8403
Hyd. Pump (H_3)	{(0.0124, Very Bad), (0.0130, Bad), (0.0182, Average), (0.0132, Good), (0.9432, Very Good)}	0.9655
Aggregation result (S_3)	**{(0.0536, Very Bad), (0.0156, Bad), (0.0299, Average), (0.0298, Good), (0.8711, Very Good)}**	**0.9123**

Source: Test case data

Table 42: Utility Value

H_n	Very Good	Good	Average	Bad	Very Bad
V_n	5	4	3	2	1
$U(H_n)$	$\frac{5-1}{5-1}=1$	$\frac{4-1}{5-1}=0.75$	$\frac{3-1}{5-1}=0.5$	$\frac{2-1}{5-1}=0.25$	$\frac{1-1}{5-1}=0$
$\beta_n(S1)$	0.7507	0.1095	0.0308	0.0261	0.0829
$\sum_{n=1}^{5}\beta_n$	0.7507 + 0.1095 + 0.0308 + 0.0261 + 0.0829 = 1 (complete)				
$\beta_n U(H_n)$	0.7507	0.082125	0.0154	0.006525	0
S_1 Condition value of the crane = $\sum_{n=1}^{5}\beta_n U(H_n) = 0.85475 \approx 0.8548$					
$\beta_n(S2)$	0.9054	0.0425	0.0178	0.0152	0.0190
$\sum_{n=1}^{5}\beta_n$	0.9054 + 0.0425 + 0.0178 + 0.0152 + 0.0190 = 1 (complete)				
$\beta_n U(H_n)$	0.9054	0.031875	0.0089	0.0038	0
S_2 Condition value of the crane = $\sum_{n=1}^{5}\beta_n U(H_n) = 0.949975 \approx 0.9500$					
$\beta_n(S3)$	0.8711	0.0298	0.0299	0.0156	0.0536
$\sum_{n=1}^{5}\beta_n$	0.8711 + 0.0298 + 0.0299 + 0.0156 + 0.0536 = 1 (complete)				
$\beta_n U(H_n)$	0.8711	0.02235	0.01495	0.0039	0
S_3 Condition value of the crane = $\sum_{n=1}^{5}\beta_n U(H_n) = 0.9123$					

Source: Test case data

Table 43: Aggregation Results for Sample 2 due to Decrement by 0.2

Main Criteria	Estimate	UV
Bearing (B_2)	{(0.2483, Very Bad), (0.0301, Bad), (0.0536, Average), (0.1525, Good), (0.5155, Very Good)}	0.6642
Clutch (C_2)	{(0.1797, Very Bad), (0.0115, Bad), (0.0170, Average), (0.0291, Good), (0.7627, Very Good)}	0.7959
Gearbox (G_2)	{(0.2330, Very Bad), (0.0410, Bad), (0.0221, Average), (0.0535, Good), (0.6503, Very Good)}	0.7118
Hydraulic Pump (H_2)	{(0.1863, Very Bad), (0.0153, Bad), (0.0213, Average), (0.0324, Good), (0.7447, Very Good)}	0.7835
Aggregation Result	**{(0.1835, Very Bad), (0.0193, Bad), (0.0225, Average), (0.0536, Good), (0.7211, Very Good)}**	**0.7774**

Source: Test case data

Table 44: Aggregation Results for the Variation of each 0.2 Decrement Values with the Original Estimates in the Main Criteria

Main Criteria	Sample 2 Estimates	Aggregation Results
Bearing (B_2)	{(0.0572, Very Bad), (0.0164, Bad), (0.0195, Average), (0.0470, Good), (0.8599, Very Good)}	0.909
Clutch (C_2)	{(0.0510, Very Bad), (0.0160, Bad), (0.0188, Average), (0.0446, Good), (0.8695, Very Good)}	0.916
Gearbox (G_2)	{(0.0550, Very Bad), (0.0166, Bad), (0.0190, Average), (0.0453, Good), (0.8641, Very Good)}	0.912
Hydraulic Pump (H_2)	{(0.0517, Very Bad), (0.0161, Bad), (0.0189, Average), (0.0448, Good), (0.8686, Very Good)}	0.916

Source: Test case data