

## Research Article

# The Design of Robust Controller for Networked Control System with Time Delay

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This paper considers the stability and  $H_\infty$  control problem of networked control systems with time delay. Taking into account the influence of network with delay, unknown input disturbance, and uncertainties of the system modeling, meanwhile we establish a precise, closed-loop model for networked control systems with time delay. By selecting a proper Lyapunov-Krasovskii function and using Lyapunov theorem, a sufficient condition for stability of the system in the form of LMI is demonstrated, corresponding controller parameters are acquired, and the convergence of the control algorithm is proved. The simulation example shows that the construction of the network robust control system with time delay indeed improves the stability performance of the system, which indicates the effectiveness of the design.

## 1. Introduction

With the continuous development of science and technology, the structure of the control system is getting more and more complex, and spatial distribution is becoming wider. In the meanwhile, the requirements of system control performance also increasingly have improved. Networked control system arises at the historic moment. Networked control system is a feedback control system composed of the sharing of communication network, which consists of sensors, controllers, and plants which are often connected over a network medium [1]. Compared with traditional feedback control systems, where these components are usually connected via point-to-point cables, the introduction of communication network media brings great advantages, such as low cost, reduced weight and power requirements, simple installation and maintenance, and higher reliability [2]. Therefore, networked control systems are widely applied to industrial system and have received more and more attention. However, the intervention of the network, because of connection interruption and network congestion, makes the system produce time delay. Considering the characteristics of networked control system, time delay can be divided into input delay, output

delay, state time delay, and uncertain time delay. According to inherent features of delay, it can be divided into inherent delay, stochastic time delay, uncertainty time delay, and so forth. These delays are time varying in nature, and their presence in a system has an adverse impact not only on system performance but also on its stability [3, 4]. Reducing the influence of the system time delay to improve the controlling precision of the system has high practical value. Therefore, the problems of time delay have received more and more attention and have become more and more popular in many practical applications in recent years. At the same time the limitation of network bandwidth and the collision of data transmission cause the phenomenon of packet dropout and cause the data to be out of order [5].

At present, research issues of NCS are concentrated on the influence of the control quality of network induced delay and data packet dropout [6–9]. In the literature [10], the method of setting a cache in the receiving end was raised by Luck and Ray. On the assumption that the maximum transmission delay in the network is known, all the uncertain time delay was defined into the maximum transmission, because of the artificial extending of the transmission time. In 1998, stochastic optimal control method was used to convert

the problem of the random network delay into linear quadratic gauss problem by Nilsson, who considered that the network time delay is less than the sampling period. In the literature [11–13], by using random delay control method and the theory of stochastic control. In the research on the performance of networked control systems, the method can ensure stability of statistical significance. But the precondition is that the network delay must obey a certain distribution. This is difficult to achieve in practical engineering. In the literature [14], the  $H_\infty$  control problem was addressed for a class of networked control systems with induced delays and packet dropouts. The NCS was modeled as a switched system with four subsystems via system states augmentation. By using the notion of time-window packet dropout rate and the average dwell time method, we have achieved sufficient condition of system stability, applied the linear matrix inequality technique and the cone complementarity linearization algorithm, and completed the controller design. At the same time, LMI and interior-point method used for solving the convex optimization problem are proposed, to provide an effective tool for analyzing and solving control problems. Using LMI method, the controller can be designed without adjusting parameters, and can be obtained delay related conclusion with less conservativeness. Compared with the solution of the Riccati equation, LMI toolbox of MATLAB can be used for all variables directly. It brings great convenience for design [15–18]. On the other hand, in the networked control system, consider network induced delay generally and ignore the inherent delay in the system. Time delay system is an infinite dimensional system. Processing method for time delay at this stage includes Smith predictor method. By establishing the model, by implementation of compensating with the aid of the model, transfer the time delay to the outside of the closed loop, to achieve the purpose of treatment delays and improve the performance of system [19]. Proportional integral controller and the first differential control are a traditional control method. There are still many missing, such as slow response speed, low control precision, and cannot fully ensure meet the requirements of high performance of complex system [20]. Internal model control, its main idea is to make the dynamic type of inverse phase approximation between controller and system model. The main research method is to transfer time delay control system to a nondelay system by using the Lyapunov functional analysis.

For networked control systems research, this study is more focused on solving the problem of network induced delay and data packet dropout, ignoring the inherent delay of the system. However, the inherent time delay is also a widespread phenomenon of networked control systems. In this paper, being aimed at the networked control systems with time delay, considering the network induced delay, unknown input disturbance, and uncertainties of the system modeling, we establish a precise system model and research on robust control method to design the controller for networked control system.

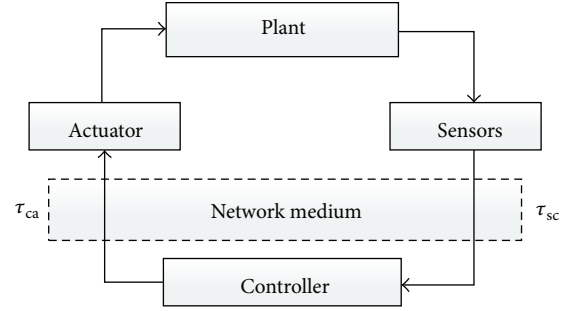


FIGURE 1: Networked control system model.

## 2. System Description and Preliminaries

Consider a typical NCS, as shown in Figure 1, controller, sensor, and actuator; these components are often connected over network media. Define the transmission delays as  $\tau_k$ ;  $\tau_{sc}$  is the delay from the sensor to the controller,  $\tau_{ca}$  is the delay from the controller to the actuator, and  $\tau_c$  is the delay for calculation.

Consider the following networked control system with time delay given by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + A_d x(t-d(t)) + Bu(t), \\ y(t) &= Cx(t),\end{aligned}\quad (1)$$

$$z(t) = Cx(t) + C_d x(t-d(t)) + Du(t),$$

where  $x(t) \in R^n$ ,  $x(t-d(t)) \in R^n$ ,  $u(t) \in R^m$ , and  $\omega(t) \in R^l$  are the system state vector, the system delay state vector, control input vector, and disturbance input vector, respectively; they belong to  $L_2[0, \infty)$ ,  $z(t) \in R^p$  is the output vector control, and  $\varphi(t)$  is continuous-time initial function defined on  $[-\tau, 0]$ .  $A \in R^{n \times n}$ ,  $B \in R^{n \times m}$ ,  $B_\omega \in R^{n \times l}$ ,  $C \in R^{p \times n}$ , and  $D \in R^{p \times m}$  are some constant matrix of appropriate dimensions. The state delay of system is  $d(t)$  that meets  $0 \leq d(t) \leq \tau$ ,  $\dot{d}(t) \leq \rho \leq 1$ .

Consider the influence of uncertainties from modeling inaccuracies and noise disturbance. A model for time delay networked control systems is described by

$$\begin{aligned}\dot{x}(t) &= [A + \Delta A(t)]x(t) + [A_d + \Delta A_d(t)]x(t-d(t)) \\ &\quad + [B + \Delta B(t)]u(t) + [B_\omega + \Delta B_\omega(t)]\omega(t), \\ y(t) &= Cx(t), \\ z(t) &= Cx(t) + C_d x(t-d(t)) + Du(t).\end{aligned}\quad (2)$$

The problem of stability and  $H_\infty$  stabilization of systems with time-varying delay using static state feedback control law have received considerable attention in recent times [21–23]. Suppose the control law for (1) is  $u(t) = Kx(t)$ , and when the network transmission delay is small, the actual value transmitted from sensor to actuator decides that the controller acts on this moment of the current value of the object. When the network transmission delay is large,

the controller acts on the current object value, depending on the latest value of retainer. At the same time, considering the influence of bounded uncertainties of time delay, in Figure 1, the state feedback controller can be described as

$$u(t) = K_1(t - d(t)) + K_2 x(t - \tau), \quad (3)$$

where  $\tau = d(t) + \tau_k$ ; put (3) into (2) as

$$\begin{aligned} \dot{x}(t) &= [A + \Delta A(t)] x(t) + [A_d + \Delta A_d(t) + BK_1 + \Delta BK_1] \\ &\quad \times x(t - d(t)) + (B + \Delta B) K_2 x(t - \tau) \\ &\quad + [B_\omega + \Delta B_\omega(t)] \omega(t), \\ z(t) &= Cx(t) + (C_d + DK_1) x(t - d(t)) + DK_2 x(t - \tau), \\ x(t) &= \phi(t), \quad t \in [-\tau, 0], \end{aligned} \quad (4)$$

$$\begin{bmatrix} S & PA_d + PBK_1 & PBK_2 & PB_\omega & C^T & D_x^T & 0 & 0 \\ * & -R + \varepsilon_4^{-1} K_1^T D_u^T & 0 & 0 & C_d^T + K_1^T D^T & 0 & D_d^T & 0 \\ * & * & -Q & 0 & K_2^T D^T & 0 & 0 & K_2^T D_u^T \\ * & * & * & -\gamma I + \varepsilon_5^{-1} D_\omega^T D_\omega & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1^{-1} I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2^{-1} I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_3^{-1} I \end{bmatrix} < 0, \quad (6)$$

where  $S = A^T P + PA + \varepsilon_1 PEE^T P + \varepsilon_2 PEE^T P + \varepsilon_3 PEE^T P + \varepsilon_4 PEE^T P + \varepsilon_5 PEE^T P + R + Q$ . Then the NCS like (4) marches the control law shown in (3) and the system is asymptotically stable with an  $H_\infty$  norm bound  $\gamma$ . Obtained controller  $K_1, K_2$  is state feedback suboptimal  $H_\infty$  controller.

*Proof.* Consider the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(x_t, \omega_t, t) &= x^T(t) Px(t) + \int_{t-d(t)}^t x^T(s) Rx(s) ds \\ &\quad + \int_{t-\tau}^t x^T(s) Qx(s) ds, \end{aligned} \quad (7)$$

where  $P, R$ , and  $Q$  are positive definite matrices. The time derivative of the LK functional along the trajectory of (7) is given by

where  $\Delta A(t), \Delta A_d(t), \Delta B(t)$ , and  $\Delta B_\omega(t)$  are known, real time-varying matrices of appropriate dimensions representing time-varying parametric perturbations; they are assumed to have the following form:

$$\begin{aligned} &[\Delta A(t) \quad \Delta A_d(t) \quad \Delta B(t) \quad \Delta B_\omega(t)] \\ &= EF(t) [D_x \quad D_d \quad D_u \quad D_\omega], \end{aligned} \quad (5)$$

where  $E, D_x, D_d, D_u$ , and  $D_\omega$  are constant matrices of appropriate dimensions and  $F(t)$  is an unknown time-varying matrix, which is Lebesgue measurable in  $t$  and satisfies  $F^T(t)F(t) \leq I, \forall \geq 0$ .

### 3. Performance Analysis and Robust Controller Design

**Theorem 1.** For a given scalar  $\gamma > 0$ , there exist real symmetric matrices  $P, Q, R$ , matrices  $K_1, K_2$ , and scalars  $\varepsilon_1 > 0, \varepsilon_2 > 0, \varepsilon_3 > 0, \varepsilon_4 > 0, \varepsilon_5 > 0$  which satisfy the following inequality:

$$\begin{aligned} \dot{V}(x_t, \omega_t, t) &= \dot{x}^T(t) Px(t) + x^T(t) P\dot{x}(t) + x^T(t) Rx(t) \\ &\quad - x^T(t - d) Rx(t - d) + x^T(t) Qx(t) \\ &\quad - x^T(t - \tau) Qx(t - \tau) \end{aligned} \quad (8)$$

without considering the interference factors  $\omega(t) = 0$ , so

$$\begin{aligned} \dot{V}(x_t, \omega_t, t) &= x^T(t) A^T Px(t) + x^T(t) PAx(t) \\ &\quad + x^T(t) D_x^T F^T x(t) E^T Px(t) + x^T(t) Rx(t) \\ &\quad + x^T(t) Qx(t) \\ &\quad + x^T(t) PEF(t) D_x x(t) + x^T(t - d) A_d^T Px(t) \\ &\quad + x^T(t) PA_d x(t - d) + x^T(t - d) D_d^T F^T(t) E^T Px(t) \\ &\quad + x^T(t) PEF(t) D_d x(t - d) + x^T(t - \tau) K_2^T B^T Px(t) \end{aligned}$$

$$\begin{aligned}
& + x^T(t) PBK_2 x(t-\tau) - x^T(t-d) Rx(t-d) \\
& + x^T(t) PEF(t) D_u K_2 x(t-\tau) \\
& + x^T(t-\tau) K_2^T D_u^T F^T(t) E^T Px(t) - x^T(t-\tau) \\
& \times Qx(t-\tau) \\
& + x^T(t-d) K_1^T B^T Px(t) + x^T(t) PBK_1 x(t-d) \\
& + x^T(t) PEF(t) D_u K_1 x(t-d) \\
& + x^T(t-d) K_1^T D_u^T F^T(t) E^T Px(t).
\end{aligned} \tag{9}$$

$$\begin{aligned}
& \times x(t) + x^T(t) [PA_d + PBK_1] x(t-d) + x^T(t-d) \\
& \times [\varepsilon_2^{-1} D_d^T D_d + \varepsilon_5^{-1} K_1^T D_u^T D_u K_1 - R] x(t-d) \\
& + x^T(t) PBK_2 x(t-\tau) + x^T(t-d) [A_d^T P + K_1^T B^T P] \\
& \times x(t) + x^T(t-\tau) K_2^T B^T Px(t) + x^T(t-\tau) \\
& \times [\varepsilon_3^{-1} K_2^T D_u^T D_u K_2 - Q] x(t-\tau).
\end{aligned} \tag{10}$$

Change (9) to get (10) as follows:

$$\dot{V}(x_t, \omega_t, t)$$

$$\begin{aligned}
& \leq x^T(t) [A^T P + PA + \varepsilon_1^{-1} D_x^T D_x + \varepsilon_1 PEE^T P \\
& + \varepsilon_2 PEE^T P + \varepsilon_2^{-1} K_1^T D_u^T K_1 + \varepsilon_3 PEE^T P \\
& + \varepsilon_4 PEE^T P + \varepsilon_5 PEE^T P + R + Q]
\end{aligned}$$

Consider reduction for the following form:

$$\dot{V}(x_t, \omega_t, t) = \xi^T(t) \Sigma \xi(t). \tag{11}$$

Define

$$\xi(t) = \begin{bmatrix} x(t) \\ x(t-d(t)) \\ x(t-\tau) \end{bmatrix}, \quad \Sigma = \begin{bmatrix} S_0 & PA_d + PBK_1 & PBK_2 \\ A_d^T P + K_1^T B^T P & \varepsilon_2^{-1} D_d^T D_d + \varepsilon_4^{-1} K_1^T D_u^T D_u K_1 - R & 0 \\ K_2^T B^T P & 0 & \varepsilon_3^{-1} K_2^T D_u^T D_u K_2 - Q \end{bmatrix}, \tag{12}$$

where  $S_0 = A^T P + PA + \varepsilon_1 PEE^T P + \varepsilon_2 PEE^T P + \varepsilon_3 PEE^T P + \varepsilon_5 PEE^T P + R + Q$ .  $\square$

$$\begin{aligned}
& \leq \omega^T(t) B_\omega^T Px(t) + x^T(t) PB_\omega \omega(t) + x^T(t) \varepsilon_5 PEE^T Px(t) \\
& + \omega^T(t) \varepsilon_5^{-1} D_\omega^T D_\omega \omega(t).
\end{aligned} \tag{13}$$

When  $\Sigma < 0$ , we have  $\dot{V}(x_t, \omega_t, t) < 0$ . The system is robust quadratic stability at this time. Considering the interference factors  $\omega(t)$ , we will have

Now, for a prescribed scalar  $\gamma > 0$ , we define a performance index  $J$  as follows:

$$\omega^T(t) \bar{B}P(x) + x^T(t) P\bar{B}\omega(t)$$

$$\dot{V}(x_t, \omega_t, t) + z^T(t) z(t) - \gamma \omega^T(t) \omega(t) < 0. \tag{14}$$

$$\begin{aligned}
& = \omega^T(t) B_\omega^T Px(t) + \omega^T(t) D_\omega^T F^T E Px(t) + x^T(t) PB_\omega \omega(t) \\
& + x^T(t) PEF(t) D_\omega \omega(t)
\end{aligned}$$

Then

$$\xi_1^T(t) \Sigma_1 \xi_1(t) + z^T(t) z(t) < 0. \tag{15}$$

Define

$$\xi_1(t) = \begin{bmatrix} x(t) \\ x(t-d(t)) \\ x(t-\tau) \\ \omega(t) \end{bmatrix}, \tag{16}$$

$$\Sigma_1 = \begin{bmatrix} S_1 & PA_d + PBK_1 & PBK_2 & PB_\omega \\ A_d^T P + K_1^T B^T P & \varepsilon_2^{-1} D_d^T D_d + \varepsilon_4^{-1} K_1^T D_u^T D_u K_1 - R & 0 & 0 \\ K_2^T B^T P & 0 & \varepsilon_3^{-1} K_2^T D_u^T D_u K_2 - Q & 0 \\ B_\omega^T P & 0 & 0 & -\gamma I + \varepsilon_5^{-1} D_\omega^T D_\omega \end{bmatrix},$$

where  $S_1 = A^T P + PA + \varepsilon_1 PEE^T P + \varepsilon_2 PEE^T P + \varepsilon_3 PEE^T P + \varepsilon_4 PEE^T P + \varepsilon_5 PEE^T P + R + Q$  as

$$\begin{aligned} & \xi_1^T(t) \Sigma_1 \xi_1(t) \\ & + \begin{bmatrix} C^T \\ C_d^T + K_1^T D^T \\ K_2^T D^T \\ 0 \end{bmatrix} [C \ C_d + DK_1 \ DK_2 \ 0] < 0. \end{aligned} \quad (17)$$

Now, for any  $x(t)$ ,  $x(t-d)$ ,  $x(t-\tau)$ , and  $\omega(t)$  the following holds good:

$$\begin{aligned} & \begin{bmatrix} x(t) \\ x(t-d(t)) \\ x(t-\tau) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} S_1 & PA_d + PBK_1 & PBK_2 & PB_\omega \\ A_d^T P + K_1^T B^T P & \varepsilon_2^{-1} D_d^T D_d + \varepsilon_4^{-1} K_1^T D_u^T D_u K_1 - R & 0 & 0 \\ K_2^T B^T P & 0 & \varepsilon_3^{-1} K_2^T D_u^T D_u K_2 - Q & 0 \\ B_\omega^T P & 0 & 0 & -\gamma I + \varepsilon_5^{-1} D_\omega^T D_\omega \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d(t)) \\ x(t-\tau) \\ \omega(t) \end{bmatrix} \\ & + \begin{bmatrix} x(t) \\ x(t-d(t)) \\ x(t-\tau) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} C^T \\ C_d^T + K_1^T D^T \\ K_2^T D^T \\ 0 \end{bmatrix} [C \ C_d + DK_1 \ DK_2 \ 0] \begin{bmatrix} x(t) \\ x(t-d(t)) \\ x(t-\tau) \\ \omega(t) \end{bmatrix} < 0. \end{aligned} \quad (18)$$

According to the Schur complement lemma, we have

$$\Sigma_2 = \begin{bmatrix} S_1 & PA_d + PBK_1 & PBK_2 & PB_\omega & C^T \\ A_d^T P + K_1^T B^T P & \varepsilon_2^{-1} D_d^T D_d + \varepsilon_4^{-1} K_1^T D_u^T D_u K_1 - R & 0 & 0 & C_d^T + K_1^T D^T \\ K_2^T B^T P & 0 & \varepsilon_3^{-1} K_2^T D_u^T D_u K_2 - Q & 0 & K_2^T D^T \\ B_\omega^T P & 0 & 0 & -\gamma I + \varepsilon_5^{-1} D_\omega^T D_\omega & 0 \\ C & C_d + DK_1 & DK_2 & 0 & -I \end{bmatrix} < 0. \quad (19)$$

Get (19) linear transformation into (6).

The system like (4) is robust asymptotically stable for any  $x(t) \in L_2[0, \infty)$ ,  $\omega(t) \in L_2[0, \infty)$ ,  $x(t-d(t)) \in L_2[0, \infty)$ , and the system state variables march  $\lim_{t \rightarrow \infty} x(t) = 0$ ,  $\lim_{t \rightarrow \infty} x(t-d) = 0$ , and  $V(x_0, \omega_0, 0) = 0$ , and to make  $J = \int_0^\infty [z^T(t)z(t) + \gamma \omega^T(t)\omega(t)] dt$  for any  $\omega(t) \in L_2[0, \infty)$ , we will have

$$J \leq \int_0^\infty [z^T(t)z(t) - \gamma \omega^T(t)\omega(t) + \dot{V}(x_t, \omega_t, t)] dt < 0. \quad (20)$$

From the formula,  $\|z(t)\|_2 \leq \gamma \|\omega(t)\|_2$  can be obtained. So, the system shown in (4) is asymptotically stable with an  $H_\infty$  norm bound  $\gamma$ . By applying successively Schur complement to (19), we deduce the LMIs stated in Theorem 1.

**Theorem 2.** For a given scalar  $\gamma > 0$ , there exist real symmetric matrices  $X$ , matrices  $Y_1$ , and scalars  $\varepsilon_1 > 0$ ,  $\varepsilon_2 > 0$ ,  $\varepsilon_3 > 0$ , which satisfy the following inequality:

$$\begin{bmatrix} S & A_d + BY_1 & BK_2 & B_\omega & XC^T & XD_x^T & 0 & 0 \\ * & -R + \varepsilon_4^{-1}Y_1^T D_u^T & 0 & 0 & C_d^T + Y_1^T D^T & 0 & D_d^T & 0 \\ * & * & -Q & 0 & K_2^T D^T & 0 & 0 & K_2^T D_u^T \\ * & * & * & -\gamma I + \varepsilon_5^{-1}D_\omega^T D_\omega & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1^{-1}I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2^{-1}I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_3^{-1}I \end{bmatrix} < 0, \quad (21)$$

where  $S = AX + XA^T + \varepsilon_1 EE^T + \varepsilon_2 EE^T + \varepsilon_3 EE^T + \varepsilon_4 EE^T + \varepsilon_5 EE^T + R + Q$ . The system like (4) under the action of the control law  $u(t) = K_1 x(t - d(t)) + K_2 x(t - \tau)$  is asymptotically stable with an  $H_\infty$  norm bound  $\gamma$ .

*Proof.* Put (6) the linear, at both sides, respectively, by  $\delta = \text{diag}\{P^{-1}, I, I, I, I, I, I, I\}$ , and by  $X = P^{-1}$ ,  $Y_1 = K_1 X$  we can get

$$\begin{bmatrix} S & A_d + BK_1 & BK_2 & B_\omega & XC^T & XD_x^T & 0 & 0 \\ * & -R + \varepsilon_4^{-1}K_1^T D_u^T & 0 & 0 & C_d^T + K_1^T D^T & 0 & D_d^T & 0 \\ * & * & -Q & 0 & K_2^T D^T & 0 & 0 & K_2^T D_u^T \\ * & * & * & -\gamma I + \varepsilon_5^{-1}D_\omega^T D_\omega & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1^{-1}I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2^{-1}I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_3^{-1}I \end{bmatrix} < 0. \quad (22)$$

Based on linear matrix inequality can be obtained  $K_1, K_2$ . Get the parameters of the corresponding memory state feedback controller. If considering state delay does not contain uncertainties, order  $A_d = 0$ .  $\square$

#### 4. Numerical Example

Consider the actual system with the following parameters:

$$\begin{aligned} A &= \begin{bmatrix} -0.9489 & -0.2387 \\ -0.238 & -0.7536 \end{bmatrix}, & A_d &= \begin{bmatrix} -0.01 & 0.02 \\ -0.03 & -0.01 \end{bmatrix}, \\ B &= \begin{bmatrix} -0.3 \\ 0.1 \end{bmatrix}, & B_\omega &= \begin{bmatrix} -0.05 \\ 0.01 \end{bmatrix}, \\ C &= [1.63 \ 0], & Cd &= [0.292 \ 0], & D &= 1, \\ I &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & E &= \begin{bmatrix} 0.004 & -0.003 \\ 0 & -0.004 \end{bmatrix}, & \gamma &= 1, \\ Dx &= \begin{bmatrix} 0.06 & -0.02 \\ 0.05 & -0.06 \end{bmatrix}, & Du &= \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix}, \\ D\omega &= \begin{bmatrix} 0.01 \\ -0.01 \end{bmatrix}, & Dd &= \begin{bmatrix} 0.04 & 0.1 \\ 0.05 & -0.04 \end{bmatrix}. \end{aligned} \quad (23)$$

Using LMI toolbox solving controller, we will have

$$K_1 = [18.1515 \ 14.5412], \quad K_2 = [-1.02653 \ 0.0064]. \quad (24)$$

A general model for continuous system is  $\dot{x}(t) = Ax(t) + Bu(t)$ ,  $z(t) = Cx(t) + Du(t)$ . Taking step signal introducing the system, the output curve of the system is shown in Figure 2.

Considering the influence of bounded inherent time delay in networked control system, the output curve is shown in Figure 3. Further, network system existence of bounded fixed time delay, network with delay, and outside disturbance, obtaining output curve, is shown in Figure 4. In order to improve the system performance index, overcome the influence of system from time delay, external disturbance, the modeling error, and the uncertainty factors. By adding a control law of memory  $u(t) = K_1 x(t - d) + K_2 x(t - \tau)$ , the output response curve is shown in Figure 5.

With reference to Figure 3, when the networked control system with system delay and steady-state error increases, the adjustment time increases. Description that the inherent delay has a great impact on the system. In Figure 4, the system is affected by the inherent time delay, outside disturbance, and



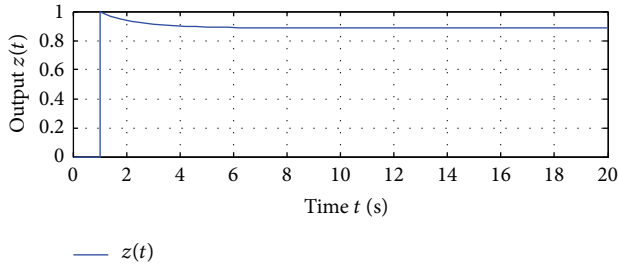


FIGURE 2: The ideal output response curve of the system.

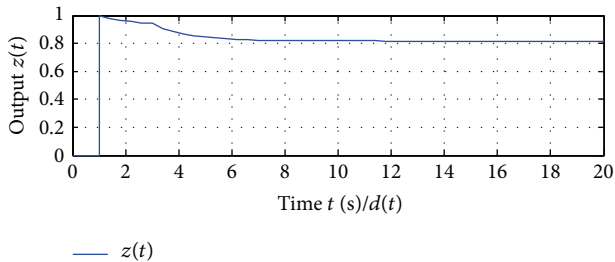


FIGURE 3: Output curve of the system with inherent time delay.

uncertainties. System performance is further deteriorated. From Figure 5, the system output response curve is under the action of the state feedback controller approaching the system ideal output curve. The system can be restored to the stable equilibrium point in a short time. Steady-state error decreases obviously.

## 5. Conclusion

In the networked control system, delay is universal. In this paper, the research object is the networked control systems with bounded time delay. Considering the influence of network induced delay, unknown input disturbance, and uncertainty of the system modeling, we establish the appropriate system model. The convergence of control algorithm has been proved via the selection of Lyapunov-Krasovskii function. Sufficient conditions for the robust stability of the system are given in the form of LMI. The design method of the controller is given. The simulation example shows that the designed networked control system with bounded time delays reduces the system steady-state error and improves the performance of the system. The problem of networked control systems with bounded inherent time delay has been solved. An effective solution has been proposed, which has certain practical value.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

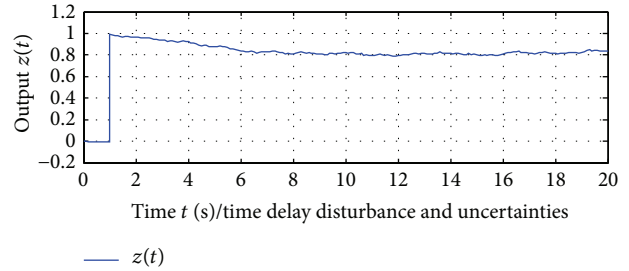


FIGURE 4: The actual output response curve.

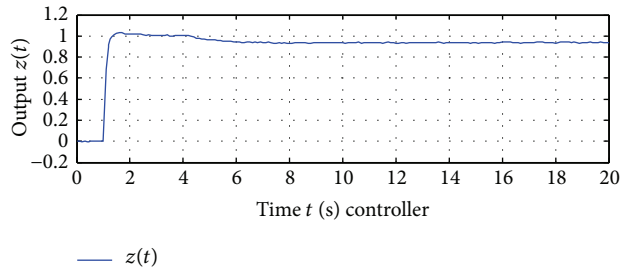


FIGURE 5: The system output response curve under the action of the controller.

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## References

- [1] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, 2007.
- [2] H. Ishii and B. A. Francis, *Limited Data Rate in Control Systems with Networks*, Lecture Notes in Control and Information Sciences, Springer, 2002.
- [3] Y. Q. Xia, M. Y. Fu, and B. Liu, "Design and performance analysis of networked control systems with random delay," *Journal of Systems Engineering and Electronics*, vol. 20, no. 4, pp. 807–822, 2009.
- [4] A. K. Sharma and G. Ray, "Robust controller with state-parameter estimation for uncertain networked control system," *IET Control Theory & Applications*, vol. 6, no. 18, pp. 2775–2784, 2012.
- [5] Y. L. Wang and G. H. Yang, " $H_\infty$  control of networked control systems with time delay and packet disordering," *IET Control Theory & Applications*, vol. 1, no. 5, pp. 1344–1354, 2007.
- [6] X.-M. Tang and B.-C. Ding, "Design of networked control systems with bounded arbitrary time delays," *International Journal of Automation and Computing*, vol. 9, no. 2, pp. 182–190, 2012.
- [7] M. B. G. Cloosterman, N. van de Wouw, W. P. M. H. Heemels, and H. Nijmeijer, "Stability of networked control systems with uncertain time-varying delays," *IEEE Transactions on Automatic Control*, vol. 54, no. 7, pp. 1575–1580, 2009.

- [8] K. You and L. Xie, "Survey of recent progress in networked control systems," *Acta Automatica Sinica*, vol. 39, no. 2, pp. 101–117, 2013.
- [9] S. Chae, D. Huang, and S. K. Nguang, "Robust partially mode delay dependent  $H_\infty$  control of discrete-time networked control systems," *International Journal of Systems Science: Principles and Applications of Systems and Integration*, vol. 43, no. 9, pp. 1764–1773, 2012.
- [10] R. Luck and A. Ray, "An observer-based compensator for distributed delays," *Automatica*, vol. 26, no. 5, pp. 903–908, 1990.
- [11] F.-L. Lian, J. Moyne, and D. Tilbury, "Network design consideration for distributed control systems," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 2, p. 297, 2002.
- [12] S. Hu and Q. Zhu, "Stochastic optimal control and analysis of stability of networked control systems with long delay," *Automatica*, vol. 39, no. 11, pp. 1877–1884, 2003.
- [13] J. Nilsson, *Real-Time Control Systems with Delays*, Lund Institute of Technology, Lund, Sweden, 1998.
- [14] L. Wei, W. Qing, and D. Chaoyang, "Robust  $H_\infty$  control of networked control systems with short delays and packet dropouts," *Journal of Northeastern University*, vol. 6, no. 35, pp. 774–779, 2014.
- [15] F. Yang, Z. Wang, Y. S. Hung, and M. Gani, " $H_\infty$  control for networked systems with random communication delays," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 511–518, 2006.
- [16] Y. Wang and Z. Sun, " $H_\infty$  control of networked control systems via lmi approach," *International Journal of Innovative Computing, Information and Control*, vol. 3, no. 2, pp. 343–352, 2007.
- [17] S. Chae, D. Huang, and S. K. Nguang, "Robust partially mode delay dependent  $H_\infty$  control of discrete-time networked control systems," *International Journal of Systems Science*, vol. 43, no. 9, pp. 1764–1773, 2012.
- [18] M. Yu, L. Wang, and T. Chu, "An LMI approach to networked control systems with data packet dropout and transmission delays," in *Proceedings of the 43rd IEEE Conference on Decision and Control (CDC '04)*, vol. 4, pp. 3545–3550, Nassau, Bahamas, 2004.
- [19] E. Fridman and U. Shaked, "A new  $H_\infty$  filter design for linear time delay systems," *Browse Journals & Magazines*, vol. 8, no. 7, pp. 2839–2843, 2002.
- [20] G. J. Silva, A. Datta, and S. P. Bhattacharyya, "PI stabilization of first-order systems with time delay," *Automatica*, vol. 37, no. 12, pp. 2025–2031, 2001.
- [21] C. E. de Souza and X. Li, "Delay-dependent robust  $H_\infty$  control of uncertain linear state-delayed systems," *Automatica*, vol. 35, no. 7, pp. 1313–1321, 1999.
- [22] H. Gao and C. Wang, "Comments and further results on a descriptor system approach to  $H_\infty$  control of linear time-delay systems," *Automatic Control*, vol. 48, no. 3, pp. 520–525, 2003.
- [23] E. Fridman and U. Shaked, "Delay-dependent stability and  $H_\infty$  control: constant and time-varying delays," *International Journal of Control*, vol. 76, no. 1, pp. 48–60, 2003.





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