Determinations of $|V_{ub}|$ from Inclusive Semileptonic B Decays with Reduced Model Dependence

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We report two novel determinations of $|V_{ub}|$ with reduced model dependence, based on measurements of the mass distribution of the hadronic system in semileptonic B decays. Events are selected by fully reconstructing the decay of one B meson and identifying a charged lepton from the decay of the other B meson from $\Upsilon(4S) \to B\bar{B}$ events. In one approach, we combine the inclusive $\bar{B} \to X_u \ell \bar{\nu}$ rate, integrated up to a maximum hadronic mass $m_X < 1.67\ \mathrm{GeV}/c^2$, with a measurement of the inclusive $B \to X_s \gamma$ photon energy spectrum. We obtain $|V_{ub}| = (4.43 \pm 0.38_{\mathrm{stat}} \pm 0.25_{\mathrm{syst}} \pm 0.29_{\mathrm{theo}}) \times 10^{-3}$. In another approach we measure the total $\bar{B} \to X_u \ell \bar{\nu}$ rate over the full phase space and find $|V_{ub}| = (3.84 \pm 0.70_{\mathrm{stat}} \pm 0.30_{\mathrm{syst}} \pm 0.10_{\mathrm{theo}}) \times 10^{-3}$.

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The measurement of the element V_{ub} of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix [1] plays a critical role in testing the consistency of the standard model description of CP violation. The uncertainties in existing measurements [2,3] are dominantly due to uncertainties in the b-quark mass m_b and the modeling of the Fermi motion of the b quark inside the $\bar{B}$ meson [4]. In this Letter, we present two techniques to extract $|V_{ub}|$ from inclusive $\bar{B} \to X_u \ell \bar{\nu}$ [5] decays where these uncertainties are significantly reduced. Neither method has been previously implemented experimentally.

Leibovich, Low, and Rothstein (LLR) have presented a prescription to extract $|V_{ub}|$ with reduced model dependence from either the lepton energy or the hadronic mass m_X [6]. A technique utilizing weight functions had been proposed previously by Neubert [4]. The calculations of LLR are accurate up to corrections of order α_s^2 and $[\Lambda m_B/(\zeta m_b)]^2$, where ζ is the experimental maximum hadronic mass up to which the $\bar{B} \to X_u \ell \bar{\nu}$ decay rate is determined and $\Lambda \approx \Lambda_{\mathrm{QCD}}$. This method combines the hadronic mass spectrum, integrated below ζ, with the high-energy end of the measured differential $B \to X_s \gamma$ photon energy spectrum via the calculations of LLR.

An alternative method [7] to reduce the model dependence is to measure the $\bar{B} \to X_u \ell \bar{\nu}$ rate over the entire m_X spectrum. Since no extrapolation is necessary to obtain the full rate, systematic uncertainties from m_b and Fermi motion are much reduced. Perturbative corrections are known to order α_s^2. We extract the $\bar{B} \to X_u \ell \bar{\nu}$ rate from the hadronic mass spectrum up to $\zeta = 2.5\ \mathrm{GeV}/c^2$ which corresponds to about 96% of the simulated hadronic mass spectrum.

The measurements presented here are based on a sample of 88.9×10^6 $B\bar{B}$ pairs collected near the $\Upsilon(4S)$ resonance by the *BABAR* detector [8] at the PEP-II asymmetric-energy e^+e^- storage rings operating at SLAC. The analysis uses $\Upsilon(4S) \to B\bar{B}$ events in which one of the B mesons decays hadronically and is fully reconstructed (B_{r}) and the other decays semileptonically ($\bar{B}_{\mathrm{sl}}$). To reconstruct a large sample of B mesons, we follow the procedure described in Ref. [2] in which charged and neutral hadrons are combined with an exclusively reconstructed D meson to obtain combinations with an energy consistent with a B meson. While this approach results in a low overall event selection efficiency, it allows for the precise determination of the momentum, charge, and flavor of the B_{r} candidates.

We use Monte Carlo (MC) simulations of the *BABAR* detector based on GEANT4 [9] to optimize selection criteria and to determine signal efficiencies and background distributions. Charmless semileptonic $\bar{B} \to X_u \ell \bar{\nu}$ decays are simulated as a combination of resonant three-body decays ($X_u = \pi, \rho, \omega, \eta, \eta'$) [10], and decays to nonresonant hadronic final states X_u [11] for which the hadronization is performed by JETSET7.4 [12]. The effect of Fermi motion is implemented in the simulation using an exponential function [11] with the parameters $m_b = 4.79\ \mathrm{GeV}/c^2$ and $\lambda_1 = -0.24\ \mathrm{GeV}^2/c^4$ [13]. The simulation of the $\bar{B} \to X_c \ell \bar{\nu}$ background uses a heavy quark effective theory parameterization of form factors for $\bar{B} \to D^* \ell \bar{\nu}$ [14] and models for $\bar{B} \to D\pi \ell \bar{\nu}$, $D^*\pi \ell \bar{\nu}$ [15], and $\bar{B} \to D \ell \bar{\nu}$, $D^{**} \ell \bar{\nu}$ [10] decays.

Semileptonic $\bar{B}_{\mathrm{sl}}$ candidates are identified by the presence of at least one electron or muon with momentum $p_\ell^* > 1\ \mathrm{GeV}/c$ in the $\bar{B}_{\mathrm{sl}}$ rest frame. For charged B_{r} candidates, we require the charge of the lepton to be consistent with a primary decay of a $\bar{B}_{\mathrm{sl}}$. For neutral B_{r} candidates, both charge-flavor combinations are retained and the average B^0-$\bar{B}^0$ mixing rate [16] is used to determine the primary lepton yield. Electrons (muons) are identified [17] (Ref. [8]), with a 92% (60–75%) average efficiency and a hadron misidentification rate ranging between 0.05% and 0.1% (1–3%).

The hadronic system X in the $\bar{B} \to X \ell \bar{\nu}$ decays is reconstructed from charged tracks and energy depositions in the calorimeter that are not associated with the B_{r} candidate or the identified lepton. The neutrino four-momentum p_ν is estimated from the missing momentum four-vector $p_{\mathrm{miss}} = p_{\Upsilon(4S)} - p_{B_{\mathrm{r}}} - p_X - p_\ell$, where all momenta are measured in the laboratory frame and $p_{\Upsilon(4S)}$ refers to the $\Upsilon(4S)$ momentum.

To select $\bar{B} \to X_u \ell \bar{\nu}$ candidates we require exactly one lepton with $p_\ell^* > 1\ \mathrm{GeV}/c$ in the event, charge conservation ($Q_X + Q_\ell + Q_{B_{\mathrm{r}}} = 0$), and a missing four-momentum consistent with a neutrino hypothesis, i.e., missing mass consistent with zero ($-1.0 < m_{\mathrm{miss}}^2 < 0.5\ \mathrm{GeV}^2/c^4$), $|\mathbf{p}_{\mathrm{miss}}| > 0.3\ \mathrm{GeV}/c$, and $|\cos\theta_{\mathrm{miss}}| < 0.95$, where θ_{miss} is the polar angle of the missing momentum three-vector $\mathbf{p}_{\mathrm{miss}}$. These criteria suppress the majority of $\bar{B} \to X_c \ell \bar{\nu}$ decays that contain additional neutrinos or an undetected K_L^0 meson. Additionally we reject events with charged or neutral kaons (reconstructed as $K_S^0 \to \pi^+\pi^-$ decays) in the decay products of the $\bar{B}_{\mathrm{sl}}$. We suppress $\bar{B} \to D^* \ell \bar{\nu}$

backgrounds by partial reconstruction of charged and neutral D^* mesons via identification of charged and neutral slow pions. The reconstruction of the mass of the hadronic system is improved by a kinematic fit that imposes four-momentum conservation, the equality of the masses of the two B mesons, and $p_\nu^2 = 0$. The resulting m_X resolution is $\sim$250 MeV$/c^2$ on average.

The extraction of $|V_{ub}|/|V_{ts}|$ from the selected events starts from the equation [6]

$$\frac{|V_{ub}|}{|V_{ts}|} = \left\{\frac{6\alpha(1 + H^\gamma_{\rm mix})(C_7^{(0)})^2}{\pi[I_0(\zeta) + I_+(\zeta)]}\delta\mathcal{R}_u(\zeta)\right\}^{1/2}, \quad (1)$$

where $\delta\mathcal{R}_u(\zeta)$ is the partial charmless semileptonic decay rate extracted from the number of $\bar{B} \to X_u \ell \bar{\nu}$ events up to a limit ζ in the m_X spectrum. $H^\gamma_{\rm mix}$ accounts for interferences between electromagnetic penguin operator O_7 with O_2 and O_8 [18], and $C_7^{(0)}$ is the effective Wilson coefficient. The terms $I_0(\zeta)$ and $I_+(\zeta)$ are determined by multiplying the photon energy spectrum $d\Gamma^\gamma/dE_\gamma$ in $B \to X_s\gamma$ decays [13] with weight functions [6] and integrating. The weights are zero below a minimum photon energy $E_\gamma^{\rm min} = m_B/2 - \zeta/4$.

In terms of measurable quantities, $\delta\mathcal{R}_u(\zeta)$ is

$$\delta\mathcal{R}_u(\zeta) = \frac{N_u(\zeta)f(\zeta)\mathcal{B}(\bar{B} \to X\ell\bar{\nu})}{N_{\rm sl}\varepsilon_u(\zeta)} \frac{\varepsilon_\ell^{\rm sl}}{\varepsilon_\ell^u} \frac{\varepsilon_{\rm reco}^{\rm sl}}{\varepsilon_{\rm reco}^u}. \quad (2)$$

Here, $N_u(\zeta)$ is the number of reconstructed $\bar{B} \to X_u\ell\bar{\nu}$ events with $m_X < \zeta$, $f(\zeta)$ accounts for migration in and out of the region below ζ due to finite m_X resolution, $\mathcal{B}(\bar{B} \to X\ell\bar{\nu})$ is the total inclusive semileptonic branching fraction, and $\varepsilon_u(\zeta)$ is the efficiency for selecting $\bar{B} \to X_u\ell\bar{\nu}$ decays once a $\bar{B} \to X\ell\bar{\nu}$ decay has been identified with a hadronic mass below ζ. $N_{\rm sl}$ is the number of observed fully reconstructed B meson decays with a charged lepton with momentum above 1 GeV$/c$, $\varepsilon_\ell^{\rm sl}/\varepsilon_\ell^u$ corrects for the difference in the efficiency of the lepton momentum selection for $\bar{B} \to X\ell\bar{\nu}$ and $\bar{B} \to X_u\ell\bar{\nu}$ decays, and $\varepsilon_{\rm reco}^{\rm sl}/\varepsilon_{\rm reco}^u$ accounts for the difference in the efficiency of reconstructing a $B_{\rm r}$ in events with a $\bar{B} \to X\ell\bar{\nu}$ and $\bar{B} \to X_u\ell\bar{\nu}$ decay. By measuring the ratio of $\bar{B} \to X_u\ell\bar{\nu}$ events to all semileptonic B decays many systematic uncertainties cancel out.

We derive $N_u(\zeta)$ from the m_X distribution with a binned χ^2 fit to four components: data, $\bar{B} \to X_u\ell\bar{\nu}$ signal MC simulations, $\bar{B} \to X_c\ell\bar{\nu}$ background MC simulations, and a small MC background from other sources (misidentified leptons, $\bar{B} \to X\tau\bar{\nu}_\tau$, and charm decays), fixed relative to the $\bar{B} \to X_c\ell\bar{\nu}$ component. $N_u(\zeta)$ is determined after the subtraction of the fitted background contributions. For all four contributions, the combinatorial background is determined, separately in each bin of the m_X distribution, with unbinned maximum likelihood fits to distributions of the beam energy-substituted mass $m_{\rm ES} = \sqrt{s/4 - \mathbf{p}_B^2}$ of the $B_{\rm r}$ candidate, where $\sqrt{s}$ is the e^+e^- center-of-mass energy. The $m_{\rm ES}$ fit uses an empirical description of the combinatorial background shape [19] with a signal shape [20] peaking at the B meson mass. The combinatorial background varies from 5% (low m_X bins) to 25% (high m_X bins). The fitted m_X distributions are shown in Fig. 1(a) before and in Fig. 1(b) after subtraction of backgrounds. The m_X bins are 300 MeV$/c^2$ wide except that one bin is widened such that its upper edge is at ζ.

We extract $N_{\rm sl} = (3.253 \pm 0.024) \times 10^4$ from an unbinned maximum likelihood fit to the $m_{\rm ES}$ distribution of all events with $p_\ell^* > 1$ GeV$/c$. The efficiency corrections $\varepsilon_\ell^{\rm sl}/\varepsilon_\ell^u = 0.82 \pm 0.02_{\rm stat}$, as well as $\varepsilon_u(\zeta)$ and $f(\zeta)$ (see Table I) are derived from simulations, where we also find $\varepsilon_{\rm reco}^{\rm sl}/\varepsilon_{\rm reco}^u$ in agreement with one, assigning a 3% uncertainty.

We study three categories of systematic uncertainties in the determination of $|V_{ub}|$: uncertainties in the signal extraction, the simulation of physics processes, and the theoretical description. The quoted uncertainties have been determined for a value of $\zeta = 1.67$ GeV$/c^2$ where the total uncertainty on $|V_{ub}|$ is found to be minimal.

Experimental uncertainties in the signal extraction arise from imperfect description of data by the detector simulation. We assign 0.5% (0.5%, 0.8%) for the particle identification of electrons (μ, $K^\pm$), 0.7% for the reconstruction efficiency of charged particles, and 0.8% for the resolution and reconstruction efficiency of neutral particles. An additional 0.9% uncertainty is due to imperfect simulation of K_L^0 interactions. By changing the function describing the signal shape in $m_{\rm ES}$ to a Gaussian function and switching from an unbinned to a binned fit method we derive an uncertainty of 2.2%. An uncertainty of 0.8% is determined by letting the contribution from other sources (see above) to the m_X spectrum float freely in the minimum-χ^2 fit. The uncertainties on the inclusive $B \to X_s\gamma$ photon energy

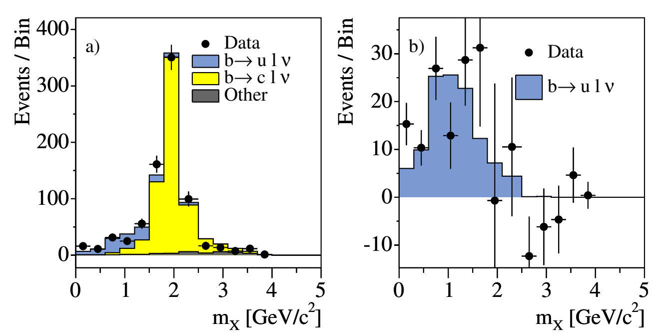

FIG. 1 (color online). The m_X distributions (without combinatorial backgrounds) for $\bar{B} \to X\ell\bar{\nu}$ candidates: (a) data (points) and fit components after the minimum-χ^2 fit, and (b) data and signal MC simulations after subtraction of the $\bar{B} \to X_c\ell\bar{\nu}$ and other backgrounds. The upper edge of the eighth bin is chosen to be at $m_X = 2.5$ GeV$/c^2$. This fit result, with $\chi^2 = 10.2$ for 11 degrees of freedom, is used to extract the number of signal events below 2.5 GeV$/c^2$.

TABLE I. Quantities in Eq. (2) that depend on ζ and their statistical uncertainties. The LLR (full rate) technique is given in the first (second) column.

ζ	1.67 GeV/c^2	2.50 GeV/c^2
f	1.010 ± 0.005	0.998 ± 0.002
N_u	120 ± 17	135 ± 45
ε_u	0.231 ± 0.005	0.231 ± 0.004
$\delta\mathcal{R}_u \times 10^3$	1.43 ± 0.21	1.59 ± 0.53

spectrum are propagated including the full correlation matrix between the individual bins.

The second category of systematic uncertainties arises from imperfections in the composition and dynamics of decays in the simulation, both in signal and background. The uncertainties in the branching fractions of $B \to D^{(*,**)} l\bar{\nu}X$ decays [16] contribute 0.7%. The uncertainties in the form factors in $B \to D^* l\bar{\nu}$ decays [14] introduce a 0.3% uncertainty. Branching fractions of D-meson decay channels [16] contribute 0.2%. The relative contribution of the nonresonant final states has been varied by 20% resulting in an uncertainty of 0.5%. The branching fractions of the resonant final states have been varied by $\pm 30\%$ (π, ρ), $\pm 40\%$ (ω), and $\pm 100\%$ (η and η' simultaneously) resulting in an uncertainties of 1.0%. An uncertainty of 0.7% due to imperfect description of hadronization is determined from the change observed when we saturate the spectrum with the nonresonant component alone. We derive a 1.3% uncertainty due to the imperfect modeling of the $K\bar{K}$ content in the X_u system by varying the fraction of decays to $s\bar{s}$ pairs by 30% for the nonresonant contribution [21]. Even though the extraction of $|V_{ub}|$ does not explicitly depend on a model for Fermi motion, there is still a residual dependency via the simulation of signal events. By varying the Fermi motion parameters m_b and λ_1 within their respective uncertainties, taking correlations into account [13], we derive an uncertainty of 3.5%.

We calculate theoretical uncertainties in the weighting technique by varying the input parameters and repeating the weighting procedure including the calculation of all variables: H^{γ}_{mix}, α_S, and Wilson-coefficients. We vary α between $\alpha(m_b)$ and $\alpha(m_W)$ with a central value of $1/130.3$ and find an uncertainty of less than 1%. For perturbative effects, an uncertainty of 2.9% is derived by varying the renormalization scale μ between $m_b/2$ and $2m_b$. Nonperturbative effects are expected to be of the order $[\Lambda m_B/(\zeta m_b)]^2$, where $\Lambda = 500\ \text{MeV}/c^2$ [22], resulting in an uncertainty of 5.4%. Theoretical uncertainties in the measurement via the full rate are taken from Ref. [23] to be 1.2% (QCD) and 2.2% (HQE). Table II provides a summary of the uncertainties for $\zeta = 1.67\ \text{GeV}/c^2$ and for $\zeta = 2.5\ \text{GeV}/c^2$.

TABLE II. Summary of results and uncertainties on $|V_{ub}|$ for both approaches. The LLR (full rate) technique is given in the first (second) column.

ζ [GeV/c^2]	1.67	2.5
$|V_{ub}| \times 10^3$	4.43	3.84
$\bar{B} \to X_u \ell \bar{\nu}$ stat.	7.7%	18.2%
Experimental syst.	3.3%	3.6%
Background model	1.0%	3.8%
Signal model	3.9%	5.6%
Theoretical	6.2%	2.6%
$B \to X_s\gamma$ (stat., syst.)	3.5%, 2.0%	. . .
$|V_{cb}|$ (exp., theo.)	1.0%, 1.7%	. . .

Finally, we present two different determinations of $|V_{ub}|$. First, using the weighting technique with the photon energy spectrum in $B \to X_s\gamma$ decays from Ref. [13], the hadronic mass spectrum up to a value of $\zeta = 1.67\ \text{GeV}/c^2$, we find $|V_{ub}|/|V_{ts}| = 0.107 \pm 0.009_{\text{stat}} \pm 0.006_{\text{syst}} \pm 0.007_{\text{theo}}$. If we assume the Cabibbo-Kobayashi-Maskawa matrix is unitary then $|V_{ts}| = |V_{cb}| \times [1 \pm \mathcal{O}(1\%)]$ and, taking $|V_{cb}|$ from Ref. [24], we derive

$$|V_{ub}| = (4.43 \pm 0.38 \pm 0.25 \pm 0.29) \times 10^{-3},$$

where the first error is the statistical uncertainty from $\bar{B} \to X_u \ell \bar{\nu}$ and from $B \to X_s\gamma$ added in quadrature, the second (third) is systematic (theoretical). Second, we determine $|V_{ub}|$ from a measurement of the full m_X spectrum, i.e., up to a value of $\zeta = 2.5\ \text{GeV}/c^2$, and find $|V_{ub}| = (3.84 \pm 0.70_{\text{stat}} \pm 0.30_{\text{syst}} \pm 0.10_{\text{theo}}) \times 10^{-3}$, using the average B lifetime of $\tau_B = (1.604 \pm 0.012)$ ps [16,25].

The weighting technique is expected to break down at low values of ζ, since only a small fraction of the phase space is used. Figure 2 illustrates the dependence of the result, and its statistical and theoretical uncertainties, on variations of ζ and also compares it with the value of $|V_{ub}|$ determined from the full rate. The weighting technique

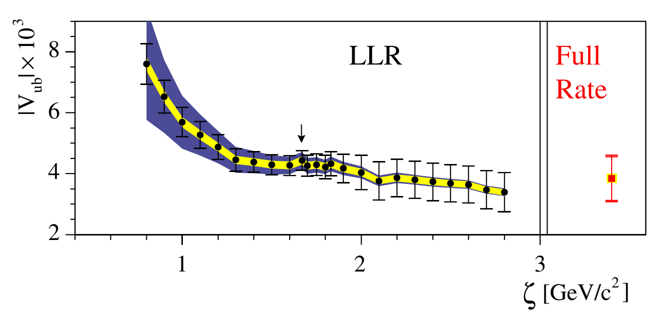

FIG. 2 (color online). $|V_{ub}|$ as a function of ζ with the LLR method (left) and for the determination with the full rate measurement (right). The error bars indicate the statistical uncertainty. They are correlated between the points and get larger for larger ζ due to larger background from $\bar{B} \to X_c \ell \bar{\nu}$. The total shaded area illustrates the theoretical uncertainty; the inner light shaded (yellow online) area indicates the perturbative share of the uncertainty. The arrow indicates $\zeta = 1.67\ \text{GeV}/c^2$.

appears to be stable down to $\zeta \sim 1.4$ GeV$/c^2$. The current uncertainties on the $B \to X_s \gamma$ photon energy spectrum limit the sensitivity with which the behavior at high ζ can be probed.

The above results are consistent with previous measurements [2,3] but have substantially smaller uncertainties from m_b and the modeling of Fermi motion. Both techniques are based on theoretical calculations that are distinct from other calculations normally employed to extract $|V_{ub}|$ and, thus, provide a complementary determination of $|V_{ub}|$.

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