# Measurement of Branching Fractions and Charge Asymmetries in $B^{ \pm} \rightarrow \rho^{ \pm} \pi^{0}$ and $B^{ \pm} \rightarrow \rho^{0} \boldsymbol{\pi}^{ \pm}$Decays, and Search for $B^{0} \rightarrow \rho^{0} \boldsymbol{\pi}^{\mathbf{0}}$ 

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We present measurements of branching fractions and charge asymmetries in $B$-meson decays to $\rho^{+} \pi^{0}, \rho^{0} \pi^{+}$, and $\rho^{0} \pi^{0}$. The data sample comprises $89 \times 10^{6} \mathrm{Y}(4 S) \rightarrow B \bar{B}$ decays collected with the BABAR detector at the PEP-II asymmetric-energy $B$ Factory at SLAC. We find the charge-averaged
branching fractions $\mathcal{B}\left(B^{+} \rightarrow \rho^{+} \pi^{0}\right)=[10.9 \pm 1.9($ stat $) \pm 1.9($ syst $)] \times 10^{-6}$ and $\mathcal{B}\left(B^{+} \rightarrow \rho^{0} \pi^{+}\right)=$ $(9.5 \pm 1.1 \pm 0.9) \times 10^{-6}$, and we set a $90 \%$ confidence-level upper limit $\mathcal{B}\left(B^{0} \rightarrow \rho^{0} \pi^{0}\right)<2.9 \times$ $10^{-6}$ We measure the charge asymmetries $A_{C P}^{\rho^{+} \pi^{0}}=0.24 \pm 0.16 \pm 0.06 \quad$ and $A_{C P}^{\rho^{0} \pi^{+}}=-0.19 \pm 0.11 \pm 0.02$.

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The study of $B$-meson decays into charmless hadronic final states plays an important role in the understanding of $C P$ violation in the $B$ system. Recently, the $B A B A R$ experiment performed a search for $C P$-violating asymmetries in neutral $B$ decays to $\rho^{ \pm} \pi^{\mp}$ final states [1], where the mixing-induced $C P$ asymmetry is related to the angle $\alpha \equiv \arg \left[-V_{t d} V_{t b}^{*} / V_{u d} V_{u b}^{*}\right]$ of the unitarity triangle [2]. The extraction of $\alpha$ from $\rho^{ \pm} \pi^{\mp}$ is complicated by the interference of decay amplitudes with differing weak and strong phases. One strategy to overcome this problem is to perform an $\mathrm{SU}(2)$ analysis that uses all $\rho \pi$ final states [3]. Assuming isospin symmetry, the angle $\alpha$ can be determined free of hadronic uncertainties from a pentagon relation formed in the complex plane by the five decay amplitudes $B^{0} \rightarrow \rho^{+} \pi^{-}, \quad B^{0} \rightarrow \rho^{-} \pi^{+}, \quad B^{0} \rightarrow$ $\rho^{0} \pi^{0}, B^{+} \rightarrow \rho^{+} \pi^{0}$, and $B^{+} \rightarrow \rho^{0} \pi^{+}$[4]. These amplitudes can be determined from measurements of the corresponding decay rates and $C P$ asymmetries. The branching fractions have been measured for $B^{0} \rightarrow$ $\rho^{+} \pi^{-}$and $B^{+} \rightarrow \rho^{0} \pi^{+}$, and an upper limit has been set for $B^{0} \rightarrow \rho^{0} \pi^{0}[1,5]$.

In this Letter we present measurements of the branching fractions of the decay modes $B^{+} \rightarrow \rho^{+} \pi^{0}$ and $B^{+} \rightarrow$ $\rho^{0} \pi^{+}$, and a search for the decay $B^{0} \rightarrow \rho^{0} \pi^{0}$. All three analyses follow a quasi-two-body approach [1,6]. For the charged modes we also measure the charge asymmetry, defined as

$$
\begin{equation*}
A_{C P} \equiv \frac{\Gamma\left(B^{-} \rightarrow f\right)-\Gamma\left(B^{+} \rightarrow \bar{f}\right)}{\Gamma\left(B^{-} \rightarrow f\right)+\Gamma\left(B^{+} \rightarrow \bar{f}\right)}, \tag{1}
\end{equation*}
$$

where $f$ and $\bar{f}$ are the final state and its charge conjugate, respectively.

The data used in this analysis were collected with the BABAR detector [7] at the PEP-II asymmetric-energy $e^{+} e^{-}$storage ring at SLAC. The sample consists of $(88.9 \pm 1.0) \times 10^{6} B \bar{B}$ pairs collected at the $\mathrm{Y}(4 S)$ resonance ("on-resonance"), and an integrated luminosity of $9.6 \mathrm{fb}^{-1}$ collected about 40 MeV below the $\mathrm{Y}(4 S)$ ("offresonance").

Each signal $B$ candidate is reconstructed from threepion final states that must be $\pi^{+} \pi^{0} \pi^{0}, \pi^{+} \pi^{-} \pi^{+}$, or $\pi^{+} \pi^{-} \pi^{0}$. Charged tracks must have ionization-energy loss and Cherenkov-angle signatures inconsistent with those expected for electrons, kaons, protons, or muons [7]. The $\pi^{0}$ candidate must have a mass that satisfies $0.11<m(\gamma \gamma)<0.16 \mathrm{GeV} / c^{2}$, where each photon is required to have an energy greater than 50 MeV in the laboratory frame and to exhibit a lateral profile of energy deposition in the electromagnetic calorimeter consistent
with an electromagnetic shower [7]. The mass of the reconstructed $\rho$ candidate must satisfy $0.4<m\left(\pi^{+} \pi^{0}\right)<$ $1.3 \mathrm{GeV} / c^{2}$ for $\rho^{+}$and $0.53<m\left(\pi^{+} \pi^{-}\right)<0.9 \mathrm{GeV} / c^{2}$ for $\rho^{0}$. The tight upper $m\left(\pi^{+} \pi^{-}\right)$cut at $0.9 \mathrm{GeV} / c^{2}$ is to remove contributions from the scalar $f_{0}(980)$ resonance, and the tight lower cut is to reduce the contamination from $K_{S}^{0}$ decays. To reduce contributions from $B^{0} \rightarrow$ $\rho^{ \pm} \pi^{\mp}$ decays, a $B^{0} \rightarrow \rho^{0} \pi^{0}$ candidate is rejected if $0.4<$ $m\left(\pi^{ \pm} \pi^{0}\right)<1.3 \mathrm{GeV} / c^{2}$. For the $B^{+} \rightarrow \rho^{+} \pi^{0}$ and $B^{0} \rightarrow$ $\rho^{0} \pi^{0}$ modes, the invariant mass of any charged track in the event and the $\pi^{0}$ must be less than $5.14 \mathrm{GeV} / c^{2}$ to reject the $B^{+} \rightarrow \pi^{+} \pi^{0}$ background. For the $B^{+} \rightarrow \rho^{0} \pi^{+}$ mode, we remove background from charmed decays $B \rightarrow$ $\bar{D}^{0} X, \bar{D}^{0} \rightarrow K^{+} \pi^{-}$, or $\pi^{+} \pi^{-}$, by requiring the masses $m\left(\pi^{+} \pi^{-}\right)$and $m\left(K^{+} \pi^{-}\right)$to be less than $1.844 \mathrm{GeV} / c^{2}$ or greater than $1.884 \mathrm{GeV} / c^{2}$. We take advantage of the helicity structure of $B \rightarrow \rho \pi$ decays by requiring that $\left|\cos \theta_{\rho}\right|>0.25$, where $\theta_{\rho}$ is the angle between the $\pi^{0}$ ( $\pi^{+}$) momentum from the $\rho^{+}\left(\rho^{0}\right)$ decay and the $B$ momentum in the $\rho$ rest frame.

Two kinematic variables, $\Delta E$ and $m_{E S}$, allow the discrimination of signal $B$ decays from random combinations of tracks and $\pi^{0}$ candidates. The energy difference, $\Delta E$, is the difference between the $e^{+} e^{-}$center-of-mass (c.m.) energy of the $B$ candidate and $\sqrt{s} / 2$, where $\sqrt{s}$ is the total c.m. energy. The beam-energy-substituted mass, $m_{E S}$, is defined by $\sqrt{\left(s / 2+\mathbf{p}_{i} \cdot \mathbf{p}_{B}\right)^{2} / E_{i}^{2}-\mathbf{p}_{B}^{2}}$, where the $B$ momentum, $\mathbf{p}_{B}$, and the four-momentum of the initial $e^{+} e^{-}$state $\left(E_{i}, \mathbf{p}_{i}\right)$ are measured in the laboratory frame. For $B^{+} \rightarrow \rho^{0} \pi^{+}$we require that $-0.05<\Delta E<$ 0.05 GeV , while for both modes containing a $\pi^{0}$ we relax this requirement to $-0.15<\Delta E<0.10 \mathrm{GeV}$. For both $B^{+} \rightarrow \rho^{0} \pi^{+}$and $B^{0} \rightarrow \rho^{0} \pi^{0}$ we require that $5.23<$ $m_{E S}<5.29 \mathrm{GeV} / c^{2}$, while for $B^{+} \rightarrow \rho^{+} \pi^{0}$ it is relaxed to $5.20<m_{E S}<5.29 \mathrm{GeV} / c^{2}$.

Continuum $e^{+} e^{-} \rightarrow q \bar{q}(q=u, d, s, c)$ events are the dominant background. To enhance discrimination between signal and continuum, we cut on neural networks (NNs), which combine six discriminating variables: the reconstructed $\rho$ mass, $\cos \theta_{\rho}$, the cosine of the angle between the $B$ momentum and the beam direction in the c.m. frame, the cosine of the angle between the $B$ thrust axis and the beam direction in the c.m. frame, and the two event-shape variables that are used in the Fisher discriminant of Ref. [8]. The event shape variables are sums over all particles $i$ of $p_{i} \times\left|\cos \theta_{i}\right|^{n}$, where $n=0$ or 2 and $\theta_{i}$ is the angle between momentum $i$ and the $B$ thrust axis. The NN for each analysis weighs the discriminating variables differently, according to training on off-resonance data
and the relevant Monte Carlo (MC) simulated signal events.

As further enhancement, we use, for the $B^{0} \rightarrow \rho^{0} \pi^{0}$ mode, the separation between the vertex of the reconstructed $B$ and the vertex reconstructed for the remaining tracks. This separation is related to $\Delta t$, the difference between the two decay times, by $\Delta z=c \beta \gamma \Delta t$, where for PEP-II the boost is $\beta \gamma=0.56$.

Approximately $33 \%, 7 \%$, and $8 \%$ of the events have more than one candidate satisfying the selection in the $B^{+} \rightarrow \rho^{+} \pi^{0}, B^{+} \rightarrow \rho^{0} \pi^{+}$, and $B^{0} \rightarrow \rho^{0} \pi^{0}$ decay mode, respectively. In such cases we choose the candidate with the reconstructed $\rho$ mass closest to the nominal value of $0.77 \mathrm{GeV} / c^{2}$. An event is classified as a misreconstructed signal if the event contains a $B$ that decays to the signal mode, but one or more reconstructed pions are not actually from the decay of that $B$. This misreconstruction is due primarily to the presence of low momentum pions in the $B \rightarrow \rho \pi$ decays. For the charged $B$ modes we distinguish misreconstructed signal events with correct charge assignment from those with incorrect charge assignment. (See Table I.)

We use MC-simulated events to study the background from other $B$ decays ( $B$ background) that include both charmed $(b \rightarrow c)$ and charmless decays. In the selected $\rho^{+} \pi^{0}\left(\rho^{0} \pi^{+}, \rho^{0} \pi^{0}\right)$ sample we expect $205 \pm 46(73 \pm$ $19,59 \pm 18) b \rightarrow c$ and $228 \pm 77(92 \pm 11,74 \pm 22)$ charmless background events. All three analyses share the major $B$-background modes: $B^{0} \rightarrow \rho^{+} \pi^{-}$, longitudinally polarized $B^{0} \rightarrow \rho^{+} \rho^{-}$, and $B^{+} \rightarrow \rho^{+} \rho^{0}$. Other important modes include $B^{+} \rightarrow \rho^{+} \pi^{0}$ (for $B^{0} \rightarrow \rho^{0} \pi^{0}$ ), $B^{+} \rightarrow\left(\mathrm{a}_{1} \pi\right)^{+}$(for $B^{+} \rightarrow \rho^{+} \pi^{0}$ ), $B^{+} \rightarrow K^{*}(892)^{0} \pi^{+}$(for $B^{+} \rightarrow \rho^{0} \pi^{+}$), and background modes containing higher kaon resonances.

An unbinned maximum likelihood fit is used for each analysis to determine event yields and charge asymmetries. To enhance discrimination between signal and background events, we use the $B$-flavor-tagging algorithm developed for the $B A B A R$ measurement of the $C P$-violating amplitude $\sin 2 \beta$ [8], where events are separated into categories based on the topology of the event and the probability of misassigning the $B$-meson flavor. The likelihood for the $N_{k}$ candidates tagged in category $k$ is

TABLE I. Numbers of selected events from on-resonance data, signal efficiencies, relative fraction of misreconstructed and wrong charge events from MC simulation. Errors are statistical only.

|  | $B^{+} \rightarrow \rho^{+} \pi^{0}$ | $B^{+} \rightarrow \rho^{0} \pi^{+}$ | $B^{0} \rightarrow \rho^{0} \pi^{0}$ |
| :--- | :---: | ---: | :---: |
| Selected events | 13177 | 8551 | 7048 |
| Signal efficiency | $17.5 \pm 0.1 \%$ | $28.3 \pm 0.1 \%$ | $20.0 \pm 0.1 \%$ |
| Misreconstructed | $38.6 \pm 0.2 \%$ | $7.1 \pm 0.1 \%$ | $9.1 \pm 0.2 \%$ |
| Wrong charge | $8.1 \pm 0.1 \%$ | $1.6 \pm 0.1 \%$ | $\cdots$ |

$$
\begin{equation*}
\mathcal{L}_{k}=e^{-N_{k}^{\prime}} \prod_{i=1}^{N_{k}}\left\{N^{\rho \pi} \epsilon_{k} \mathcal{P}_{i, k}^{\rho \pi}+N_{k}^{q \bar{q}} \mathcal{P}_{i, k}^{q \bar{q}}+\sum_{j=1}^{N_{B}} \mathcal{L}_{i j, k}^{B}\right\} \tag{2}
\end{equation*}
$$

where $N^{\rho \pi}$ is the number of signal events in the entire sample, $\epsilon_{k}$ is the fraction of signal events tagged in category $k, N_{k}^{q \bar{q}}$ is the number of continuum background events that are tagged in category $k$, and $N_{B}$ is the number of $B$-background modes. $N_{k}^{\prime}$ is the sum of the expected event yields for signal $\left(\epsilon_{k} N^{\rho \pi}\right)$, continuum $\left(N_{k}^{q \bar{q}}\right)$, and fixed $B$ background. For the charged modes the asymmetries are introduced by multiplying the signal yields by $\frac{1}{2}\left(1-Q_{i} A_{C P}\right)$, where $Q_{i}$ is the charge of $B$ candidate $i$. The likelihood term $\mathcal{L}_{i j, k}^{B}$ corresponds to the $j_{\text {th }}$ $B$-background contribution of the $N_{B} B$-background classes. The total likelihood is the product of likelihoods for each tagging category.

The probability density functions (PDF) for signal and continuum, $\mathcal{P}_{k}^{\rho \pi}$ and $\mathcal{P}_{k}^{q \bar{q}}$, are the products of the PDFs of the discriminating variables. The signal PDFs are given by $\mathcal{P}_{k}^{(\rho \pi)^{+}} \equiv \mathcal{P}^{(\rho \pi)^{+}}\left(m_{E S}\right) \mathcal{P}^{(\rho \pi)^{+}}(\Delta E) \mathcal{P}_{k}^{(\rho \pi)^{+}}(\mathrm{NN})$ for the charged $B$ decay modes, and by $\mathcal{P}_{k}^{\rho^{0} \pi^{0}} \equiv \mathcal{P}^{\rho^{0} \pi^{0}}\left(m_{E S}\right) \times$ $\mathcal{P}^{\rho^{0} \pi^{0}}(\Delta E) \mathcal{P}_{k}^{\rho^{0} \pi^{0}}(\mathrm{NN}) \mathcal{P}_{k}^{\rho^{0} \pi^{0}}(\Delta t)$ for $B^{0} \rightarrow \rho^{0} \pi^{0}$. Each signal PDF is decomposed into two parts with distinct distributions: signal events that are correctly reconstructed and signal events that are misreconstructed. For the charged $B$ modes, each PDF for the misreconstructed events is further divided into a right-charge and a wrong-charge part. The $m_{E S}, \Delta E$, and NN PDFs for signal and for $B$ background are taken from MC simulation. For continuum, the yields and PDF parameters are determined simultaneously in the fit to on-resonance data.

In the $B^{0} \rightarrow \rho^{0} \pi^{0}$ decay the $\Delta t$ distributions for signal and $B$ background are modeled from fully reconstructed $B^{0}$ decays from data control samples [8]. The continuum $\Delta t$ parameters are free in the fit to on-resonance data.

To validate the fit procedure, we perform fits on large MC samples that contain the measured number of signal and continuum events and the expected $B$ background. Biases observed in these tests are largely due to correlations between the discriminating variables, which are not accounted for in the PDFs. For $\rho^{+} \pi^{0}$ and $\rho^{0} \pi^{+}$they are not negligible and are used to correct the fitted signal yields. In addition, the full fit biases are assigned as systematic uncertainties on all three signal yields.

Contributions to the systematic errors are summarized in Table II. Uncertainties in the signal MC simulation, including signal misreconstruction, are obtained from a topologically similar control sample of fully reconstructed $B^{0} \rightarrow D^{-} \rho^{+}$decays. For the $B^{+} \rightarrow \rho^{+} \pi^{0}$ channel we also use $B^{+} \rightarrow K^{+} \pi^{0}$ decays to estimate the uncertainty in the $\Delta E$ model. We vary the signal parameters, which are fixed in the fit, within their estimated errors and assign the effects on the signal yields

TABLE II. Summary of the systematic uncertainties.

|  | $\rho^{+} \pi^{0}$ | $\rho^{0} \pi^{+}$ <br> $($Events $)$ | $\rho^{0} \pi^{0}$ | $A_{C P}^{\rho^{+} \pi^{0}}$ | $A_{C P}^{\rho^{\rho^{0}} \pi^{+}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Error source | 10.7 | 3.8 | 3.3 | 3.4 | 0.3 |
| Signal model | 14.4 | 8.2 | 2.0 | $\cdots$ | $\cdots$ |
| Fit procedure bias | 11.2 | 9.0 | 3.3 | 5.0 | 2.2 |
| $B$ background | $\ldots$ | $\cdots$ | $\cdots$ | 1.0 | 0.9 |
| Detector charge bias | 21.1 | 12.9 | 5.1 | 6.1 | 2.4 |
| Total fit error | $11.6 \%$ | $7.2 \%$ | $7.0 \%$ | $\cdots$ | $\cdots$ |
| Relative efficiency error | 169.0 | 237.9 | 24.9 | $\cdots$ | $\cdots$ |
| Fitted signal yield |  |  |  |  |  |

and charge asymmetries as systematic errors. The expected yields from the $B$-background modes are varied according to the uncertainties in the measured or estimated branching fractions. Since $B$-background modes may exhibit direct $C P$ violation, the corresponding charge asymmetries are varied within their physical ranges. From studies on our data, we find the nonresonant $B^{+} \rightarrow \pi^{+} \pi^{0} \pi^{0}$ contribution to be negligible. For $B^{+} \rightarrow$ $\rho^{0} \pi^{+}$, the systematic uncertainty due to possible interference with $f_{0}(980) \pi^{+}$, a possible $\sigma(400-1200) \pi^{+}$, or nonresonant $\pi^{+} \pi^{-} \pi^{+}$is considered. None of these modes has been measured; their branching fractions are conservatively assumed to be the difference between the inclusive $\pi^{+} \pi^{-} \pi^{+}$branching fraction [9] and the previously measured $\rho^{0} \pi^{+}$branching fraction [5], with uncertainties taken into account. It is found to be 8.7 events. For $B^{0} \rightarrow \rho^{0} \pi^{0}$, the systematic uncertainty due to interference with $B^{0} \rightarrow \rho^{ \pm} \pi^{\mp}$ is found to be 1.5 events. This is obtained by repeating the fit to data, after removing the cut on $m\left(\pi^{ \pm} \pi^{0}\right)$. Systematic error due to possible nonresonant $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decays is also derived from experimental limits [5].

After correcting for the fit biases we find from the maximum likelihood fits the event yields, $N\left(\rho^{+} \pi^{0}\right)=$ $169.0 \pm 28.7, N\left(\rho^{0} \pi^{+}\right)=237.9 \pm 26.5$, and $N\left(\rho^{0} \pi^{0}\right)=$ $24.9 \pm 11.5$, where the errors are statistical only. Figure 1 shows distributions of $m_{E S}$ and $\Delta E$, enhanced in signal content by cuts on the signal-to-continuum likelihood ratios of the other discriminating variables. The statistical significance of the previously unobserved $B^{+} \rightarrow \rho^{+} \pi^{0}$ signal amounts to $7.3 \sigma$, computed as $\sqrt{2 \Delta \log \mathcal{L}}$, where $\Delta \log \mathcal{L}$ is the $\log$-likelihood difference between a signal hypothesis corresponding to the bias-corrected yield and a signal hypothesis corresponding to a yield that equals 1 standard deviation of the systematic error. We find the branching fractions to be

$$
\begin{aligned}
\mathcal{B}\left(B^{+} \rightarrow \rho^{+} \pi^{0}\right) & =(10.9 \pm 1.9 \pm 1.9) \times 10^{-6}, \\
\mathcal{B}\left(B^{+} \rightarrow \rho^{0} \pi^{+}\right) & =(9.5 \pm 1.1 \pm 0.9) \times 10^{-6}, \\
\mathcal{B}\left(B^{0} \rightarrow \rho^{0} \pi^{0}\right) & =(1.4 \pm 0.6 \pm 0.3) \times 10^{-6},
\end{aligned}
$$

where the first errors are statistical and the second system-
atic. The systematic errors include the uncertainties in the efficiencies, which are dominated by the uncertainty in the $\pi^{0}$ reconstruction efficiency and in the case of $\rho^{0} \pi^{+}$, by the uncertainty due to particle identification.

Here we define the $B^{0} \rightarrow \rho^{0} \pi^{0}$ branching fraction by including those events that pass our selection and are fitted as signal but excluding those events that can be interpreted as $B^{0} \rightarrow \rho^{ \pm} \pi^{\mp}$. The signal significance for $\rho^{0} \pi^{0}$, including statistical and systematic errors, is $2.1 \sigma$, and we use a limit setting procedure similar to Ref. [10] to obtain a $90 \%$ confidence level upper limit on its branching fraction. Fits on MC samples are used to find the signal hypothesis for which the ratio of the


FIG. 1. Distributions of $m_{E S}$ and $\Delta E$ for samples enhanced in $\rho^{0} \pi^{+}$signal (a),(b), $\rho^{+} \pi^{0}$ signal (c),(d), and $\rho^{0} \pi^{0}$ signal (e),(f). The solid curve represents a projection of the maximum likelihood fit result. The dashed curve represents the contribution from continuum events, and the dotted line indicates the combined contributions from continuum events and $B$-related backgrounds.


FIG. 2. Distributions of $\rho$ mass and helicity for samples enhanced in $\rho^{0} \pi^{+}$signal (a),(b) with the same line conventions used in Fig. 1.
probability that the fitted signal yield is less than that observed in data and the probability that the fitted yield is less than that in data under the null signal hypothesis is 0.1 . This signal hypothesis is shifted up by one sigma of the systematic error and the efficiency is shifted down also by one sigma. This method gives an upper limit of $\mathcal{B}\left(B^{0} \rightarrow \rho^{0} \pi^{0}\right)<2.9 \times 10^{-6}$.

The good agreement between data and MC simulation shown in Fig. 2 confirms that the effect due to the possible presence of scalar or nonresonant contribution is negligible.

For the charged $B$ decays we find the charge asymmetries, $A_{C P}^{\rho^{+} \pi^{0}}=0.24 \pm 0.16 \pm 0.06, A_{C P}^{\rho^{0} \pi^{+}}=-0.19 \pm 0.11 \pm$ 0.02 , with contributions to the systematic errors listed in Table II.

In summary, we have presented measurements of branching fractions and $C P$-violating charge asymmetries in $B^{+} \rightarrow \rho^{+} \pi^{0}$ and $B^{+} \rightarrow \rho^{0} \pi^{+}$decays, and a search for the decay $B^{0} \rightarrow \rho^{0} \pi^{0}$. We observe the decay $B^{+} \rightarrow \rho^{+} \pi^{0}$ with a statistical significance of $7.3 \sigma$. We also find a branching fraction for $B^{+} \rightarrow \rho^{0} \pi^{+}$that is consistent with previous measurements [5], and set an upper limit for $B^{0} \rightarrow \rho^{0} \pi^{0}$. We do not observe evidence for direct $C P$ violation.

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