

Hindawi  
Advances in High Energy Physics  
Volume 2017, Article ID 2356174, 8 pages  
<https://doi.org/10.1155/2017/2356174>



## Research Article

# Holographic van der Waals Phase Transition for a Hairy Black Hole

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Received 2 March 2017; Accepted 29 May 2017; Published 12 July 2017

Academic Editor: Bum-Hoon Lee

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The van der Waals (VdW) phase transition in a hairy black hole is investigated by analogizing its charge, temperature, and entropy as the temperature, pressure, and volume in the fluid, respectively. The two-point correlation function (TCF), which is dual to the geodesic length, is employed to probe this phase transition. We find the phase structure in the temperature-thermal entropy plane besides the scale of the horizontal coordinate (geodesic length plane resembles that in the temperature). In addition, we find the equal area law (EAL) for the first-order phase transition and critical exponent of the heat capacity for the second-order phase transition in the temperature-thermal entropy plane (geodesic length plane is consistent with that in temperature), which implies that the TCF is a good probe to probe the phase structure of the black hole.

## 1. Introduction

Phase transition of the black holes is always a hot topic in theoretical physics for it provides a platform to relate the gravity, thermodynamics, and quantum theory. Phase transition in AdS space is more fascinated owing to the AdS/CFT correspondence. The Hawking-Page phase transition [1], which portrays the transition of thermal gas to the Schwarzschild black hole, can be used to describe the confinement to deconfinement transition of the quark-gluon plasma in Yang-Mills theory [2]. The phase transition of a scalar field condensation around charged AdS black holes can be used to describe the superconductor transition [3–5]. In particular, one often can use the nonlocal observables such as holographic entanglement entropy, Wilson loop, and TCF to probe these phase transitions [6, 7].

VdW phase transition is another important property of the charged AdS black hole. It was observed that a charged black hole will undergo a first-order phase transition and

a second-order phase transition successively as the charge of the black hole increases from small to large, which is analogous to the van der Waals liquid-gas phase transition [8]. This phase transition was perfected recently by regarding the cosmological constant as the pressure for in this case we need not any analogy [9, 10].

Whether the VdW phase transition of the charged black hole can be probed by the nonlocal observables thus is worth exploring. Recently, Chamblin et al. [11] investigated the VdW phase transition of the Reissner-Nordström AdS black hole from the viewpoint of holography and found that the phase structure in the temperature-entanglement entropy plane resembles that in temperature-thermal entropy plane. Thereafter Nguyen [12] investigated exclusively the EAL in the temperature-entanglement entropy plane and found that the EAL holds regardless of the size of the entangling region. Now there have been some extensive studies [13–19] and all the results show that, in the case of thermal entropy, the entanglement entropy exhibits the similar VdW phase transition.

In this paper, we are going to use the equal time TCF to probe the phase structure of the hairy black holes. It has been shown that the TCF has the same effect as that of the holographic entanglement entropy as it was used to probe the thermalization behavior [20–33]; thus, it will be interesting to explore whether this observable can probe the phase structures of the black holes. The hairy AdS black hole is a solution of Einstein-Maxwell- $\Lambda$  theory conformally coupled to a scalar field [34]. This model has at least two advantages. One is that it is an ordinary and tractable model for studying superconducting phase transition with consideration of the back-reaction of the scalar field. Another is that it exhibits more fruitful phase transition behavior, namely, not only the VdW behavior in both the charged and uncharged cases, but also reentrant phase transition in the charged case [35]. In this paper, we mainly concentrate on the VdW phase transition behavior. Beside the thermal entropy-temperature plane, we will also study the EAL and critical exponent of the heat capacity in the geodesic length-temperature plane. We find that the results obtained in both frameworks are consistent.

Our paper is outlined as follows. In Section 1, we present the hairy AdS black hole solution and study the VdW phase transition in the thermal entropy-temperature plane. In Section 2, we employ the TCF to probe the VdW phase transition. In particular, we study the EAL and critical exponent of the heat capacity in the framework of holography and find that the result is similar as that obtained in the thermal entropy-temperature plane. The conclusion and discussion are presented in Section 3. Hereafter in this paper we use natural units ( $G = c = \hbar = 1$ ) for simplicity.

## 2. Thermodynamic Phase Transition of the Hairy Black Hole

The five-dimensional hairy black hole solution can be written as [34]

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 [d\theta^2 + \sin^2\theta (d\phi^2 + \sin^2\theta d\psi^2)], \quad (1)$$

in which

$$f(r) = \frac{e^2}{r^4} + \frac{r^2}{l^2} - \frac{m}{r^2} - \frac{q}{r^3} + 1, \quad (2)$$

where  $e$  is the electric charge,  $m$  is the mass parameter,  $l$  is the AdS radius that relates to the cosmology constant  $\Lambda$ , and  $q$  is related to the coupling constants of the conformal field  $b_0$ ,  $b_1$ , and  $b_2$ . For the planar solution,  $q = 0$ , while for the spherical symmetric black hole,  $q$  can be expressed as

$$q = \frac{64\pi}{5} \epsilon b_1 \left( \frac{-18b_1}{5b_0} \right)^{3/2}, \quad (3)$$

in which  $\epsilon$  takes the values  $-1$ ,  $0$ , and  $1$ . In addition, to satisfy the field equations, the scalar coupling constants should obey the constraint

$$100b_0b_2 = 9b_1^2. \quad (4)$$

As stressed in [34, 35], the hair parameter  $q$  is not a conserved charge corresponding to some symmetry; it is a variable provided that the scalar field coupling constants are dynamic. In this paper, we will fix  $q$  to investigate the phase structure of this black hole for  $q$  has little effect on the phase structure.

The black hole event horizon  $r_h$  is the largest root of the equation  $f(r_h) = 0$ . At the event horizon, the Hawking temperature can be expressed as

$$T = \frac{-2e^2l^2 + l^2r_h(q + 2r_h^3) + 4r_h^6}{4\pi l^2 r_h^5}, \quad (5)$$

in which we have used the relation

$$m = \frac{e^2l^2 - l^2qr_h + l^2r_h^4 + r_h^6}{l^2r_h^2}. \quad (6)$$

In terms of the AdS/CFT correspondence, the temperature in (5) can be treated as the temperature of the dual conformal field theory. The Maxwell potential in this background is given by

$$A_t = \frac{\sqrt{3}e}{r_h^2}. \quad (7)$$

The entropy of the black hole is

$$S = \frac{\pi^2 r_h^3}{2} - \frac{5\pi^2 q}{4}. \quad (8)$$

Substituting (8) into (5), we can get the following relation:

$$T = \frac{-2\pi^4 e^2 l^2 + 3 \cdot 2^{2/3} \pi^{10/3} l^2 q \sqrt{5\pi^2 q + 4S} + 2 \cdot 2^{2/3} \pi^{4/3} l^2 S \sqrt{5\pi^2 q + 4S} + 25\pi^4 q^2 + 40\pi^2 qS + 16S^2}{\sqrt[3]{2} l^2 (5\pi^3 q + 4\pi S)^{5/3}}. \quad (9)$$

Next, we will employ this equation to study the phase structure of the hairy black hole. The Helmholtz free energy of this system is given by [34]

$$F = M - TS$$

$$= -\frac{5\pi e^2 q}{8r_h^5} + \frac{5\pi e^2}{8r_h^2} + \frac{5\pi q r_h}{4l^2} - \frac{\pi r_h^4}{8l^2} + \frac{5\pi q^2}{16r_h^4} + \frac{\pi q}{8r_h} + \frac{\pi r_h^2}{8},$$

(10)

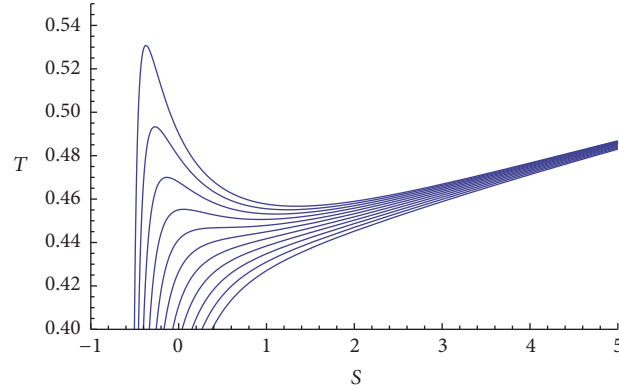


FIGURE 1: Relations between temperature and thermal entropy for different  $e$  with  $q = 0.05$ ; curves from top to down correspond to cases  $e$  varying from 0.1 to 0.2 with step 0.01.

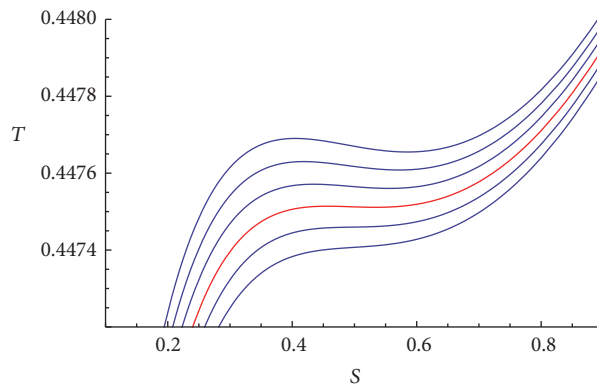


FIGURE 2: Relations between temperature and thermal entropy for different  $e$  with  $q = 0.05$ ; curves from top to down correspond to cases  $e$  varying from 0.1385 to 0.139 with step 0.0001.

in which  $M = 3\pi m/8G$ , where  $G$  is the gravitational constant.

Now we concentrate on studying the phase structure of the hairy AdS black holes. In fluid, the VdW phase transition is depicted in the  $P$ - $V$  plane, where  $P, V$  correspond to pressure and volume of fluid. In black holes, there are two schemes to produce the VdW phase transition. One is presented in [9, 10] where the cosmology constant and curvature are treated as pressure and volume. The other is presented in [8] in which one should adopt analogy shown in Table 1.

In this paper, we will follow the later scheme. In this case, the phase structure depends on the charge of the black hole, and we know that there is a critical charge, for which the temperature satisfies the following relation:

$$\left(\frac{\partial T}{\partial S}\right)_e = \left(\frac{\partial^2 T}{\partial S^2}\right)_e = 0. \quad (11)$$

In this paper, we will get the critical charge numerically. We will set  $l = 1$ . We first plot a bunch of curves by taking different values of  $e$  in the  $T$ - $S$  plane in Figure 1. From this figure, we can read off the rough critical value of the charge which satisfies the condition  $(\partial T/\partial S)_e = 0$ . Having obtained this rough value, we plot several curves in the  $T$ - $S$  plane

TABLE 1

Analogy	
Fluid	AdS black hole
Temperature, $T$	Charge, $e$
Pressure, $P$	Temperature, $T(S, e)$
Volume, $V$	Entropy, $S$

further with smaller step so that we can get the probable critical value of  $e$ . From Figure 2, we find that the probable critical value is 0.1388, which is labeled by the red solid line. Finally, we adjust the value of  $e$  by hand to find the exact value of  $e$  that satisfies  $(\partial T/\partial S)_e = 0$ , which produces  $e_c = 0.13888$ . With this critical value and (11), we can get the critical entropy  $S_c = 0.533926$ . Having obtained the critical charge and critical entropy, the critical temperature and critical free energy also can be produced by relations (9) and (10) directly.

As the critical charge  $e$  is given, we can set different charges to show the VdW phase transition in the  $T$ - $S$  plane, which is shown in Figure 3. From this figure, we know that as the value of the charge is smaller than the critical charge, a small black hole, large black hole, and an intermediate black hole coexist. There is a critical temperature, labeled as  $T_1$ . For

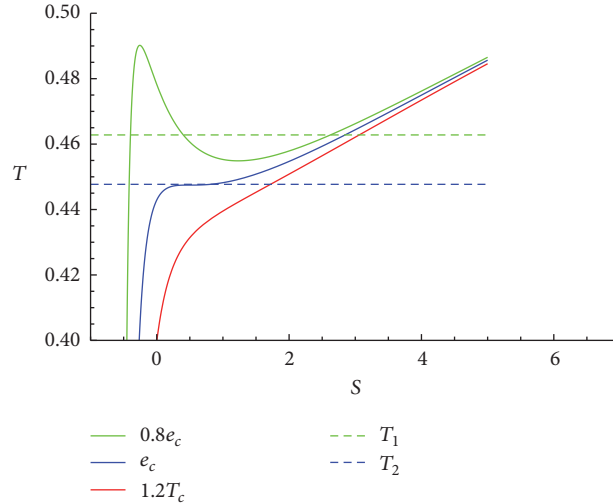


FIGURE 3: Relations between temperature and thermal entropy for different  $e$  with  $q = 0.05$ .

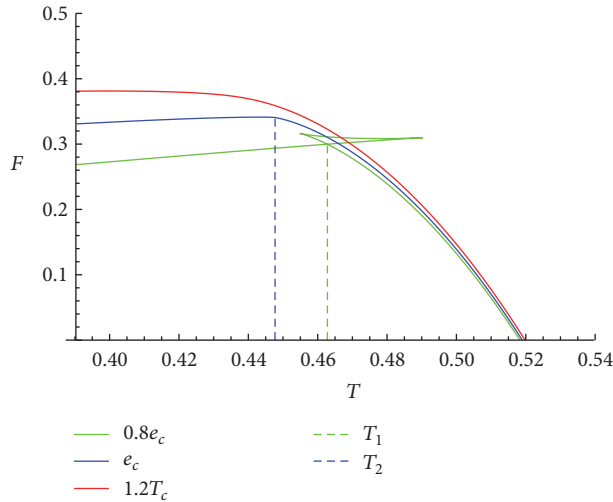


FIGURE 4: The  $F$ - $T$  relation for different  $e$  with  $q = 0.05$ .

the case  $T < T_1$ , the small black hole dominates while for the case  $T > T_1$ , the large black hole dominates. The phase transition for the small black hole to the large black hole is first order. For the case  $e = e_c$ , we find that the unstable region vanishes and an inflection point emerges. The heat capacity in this case is divergent, which implies that the phase transition is second order, and the corresponding phase transition temperature is labeled as  $T_2$ . For the case  $e > e_c$ , the black hole is stable always.

The phase structures can also be observed in the  $F$ - $T$  plane. From the green curve of Figure 4, we know that the swallowtail structure corresponds to the unstable phase in the top curve of Figure 3. The value of the first-order phase transition temperature,  $T_1 = 0.4628$ , can be read off from the horizontal coordinate of the junction. From the blue curve of Figure 4, we also can read off the second-order phase transition temperature  $T_2 = 0.4477$ .

The first-order phase transition temperature  $T_1$  also can be obtained from the EAL:

$$A_L \equiv \int_{S_1}^{S_3} T(S, e) dS = T_1 (S_3 - S_1) \equiv A_R, \quad (12)$$

in which  $T(S, e)$  is the analytical function in (9) and  $S_1$  and  $S_3$  are the smallest and largest roots of the equation  $T(S, e) = T_1$ . On the contrary, as  $T_1$  is given, we can use it to check the EAL numerically. In fact, this relation holds always in thermodynamics. We give the numerical check here in order to compare with the result which will be produced in the framework of holography in the next section.

For the case  $q = 0.05$ , we know that  $T_1 = 0.4628$ ,  $S_1 = 0.40247$ , and  $S_3 = 2.62429$ . Substituting these values into (12), we find  $A_L = 1.01677$ ,  $A_R = 1.02826$ . It is obvious that  $A_L$  and  $A_R$  are equal nearly, which implies that the EAL holds in the  $T$ - $S$  plane within our numerical accuracy.

For the second-order phase transition, we know that near the critical temperature  $T_2$ , there is always a relation [11]

$$\log |T - T_2| = 3 \log |S - S_c| + \text{constant}, \quad (13)$$

in which  $S_c$  is the critical entropy corresponding to the critical temperature  $T_2$ . With the definition of the heat capacity,

$$C_e = T \left. \frac{\partial S}{\partial T} \right|_e. \quad (14)$$

One can get further that  $C_e \sim (T - T_2)^{-2/3}$ ; namely, the critical exponent is  $-2/3$  [11]. Next, we will check whether there is a similar linear relation and critical exponent in the framework of holography.

### 3. Probe the Phase Transition with Two-Point Correlation Function

Having obtained the thermodynamic phase structure of the hairy black hole, we will check whether this property can be probed by the TCF. In terms of the AdS/CFT correspondence, we know that, for the case that the conformal weight  $\Delta$  is large, the equal time two-point function of operators  $\mathcal{O}$  can be computed by the length of space-like geodesics in the bulk geometry; that is [36],

$$\langle \mathcal{O}(t, x_i) \mathcal{O}(t, x_j) \rangle \approx e^{-\Delta L}, \quad (15)$$

in which  $L$  is the length of the bulk geodesic connecting two points  $(t, x_i)$  and  $(t, x_j)$  on the AdS boundary. For the spherically symmetric black hole, we will choose  $(\phi = \pi/2, \theta = 0, \psi = 0)$  and  $(\phi = \pi/2, \theta = \theta_0, \psi = \pi)$  as the two boundary points. Then with  $\theta$  to parameterize the trajectory, the proper length can be written as

$$L = \int_0^{\theta_0} \sqrt{\frac{r'^2}{f(r)} + r^2} d\theta, \quad (16)$$

in which  $r' = dr/d\theta$ . Treating the integral as a Lagrangian, we can get the motion equation of  $r$ ; namely,

$$\begin{aligned} \frac{1}{2} r(\theta) r'(\theta)^2 g'(r(\theta)) - r(\theta) g(r(\theta)) r''(\theta) \\ + 2g(r(\theta)) r'(\theta)^2 + r(\theta)^2 g(r(\theta))^2 = 0. \end{aligned} \quad (17)$$

With the boundary conditions

$$\begin{aligned} r'(0) &= 0, \\ r(0) &= r_0, \end{aligned} \quad (18)$$

we can get the numeric result of  $r(\theta)$  and further get  $L$  by substituting  $r(\theta)$  into (16). As in [11], we are interested in the regularized geodesic length as  $\delta L \equiv L - L_0$ , in which  $L_0$  is the geodesic length in pure AdS. As the value of  $\delta L$  is given, we can get the relation between  $\delta L$  and  $T$  for different charge  $e$ . In this paper, we will also explore whether this relation is  $\theta_0$  independent. Without loss of the generality, we choose  $\theta_0 = 0.1, 0.2$  and set the corresponding UV cutoff in the dual field theory to be  $r(0.099)$ ,  $r(0.199)$ , respectively. The corresponding pictures are shown in Figure 3. It is obvious that both Figures 5 and 6 resemble Figure 3 besides the scale of horizontal coordinate, which implies that the geodesic length owns the same phase structure as that of the thermal entropy. To confirm this conclusion, we will check the EAL for the first-order phase transition and critical exponent for the second-order phase transition in the  $T$ - $\delta L$  plane.

The EAL in the  $T$ - $\delta L$  plane can be defined as

$$\begin{aligned} A_L &\equiv \int_{\delta L_{\min}}^{\delta L_{\max}} T(\delta L, e) d\delta L = T_1 (\delta L_{\max} - \delta L_{\min}) \\ &\equiv A_R, \end{aligned} \quad (19)$$

in which  $(\delta L, e)$  is an interpolating function obtained from the numeric data, and  $\delta L_{\min}, \delta L_{\max}$  are the smallest and largest values of the equation  $T(\delta L, e) = T_1$ . For different  $\theta_0$ , the calculated results are listed in Table 2. From this table, we can see that, for the unstable region of the first-order phase transition in the  $T$ - $\delta L$  plane, the EAL holds within our numeric accuracy. This conclusion is independent of the boundary separation  $\theta_0$ .

We also can investigate the critical exponent of the heat capacity for the second-order phase transition in the  $T$ - $\delta L$  plane by defining an analogous heat capacity

$$C = T \frac{\partial \delta L}{\partial T}. \quad (20)$$

Provided a similar relation as shown in (13) is satisfied, one can get the critical exponent of the analogous heat capacity immediately.

So next, we are interested in the relation between  $\log |T - T_2|$  and  $\log |\delta L - \delta L_c|$ , in which  $\delta L_c$  is the solution that satisfies the relation  $T(\delta L, e) = T_2$ . For different  $\theta_0$ , the numeric results are shown in Figure 7. By data fitting, the straight lines in Figure 7 can be fitted as

$$\log |T - T_2| = \begin{cases} 28.2901 + 3.05999 \log |\delta L - \delta L_c|, & \text{for } q = 0.05, e = e_c, \theta_0 = 0.1, \\ 19.3767 + 3.07505 \log |\delta L - \delta L_c|, & \text{for } q = 0.05, e = e_c, \theta_0 = 0.2. \end{cases} \quad (21)$$

Obviously, for all the lines, the slope is about 3, which resembles that in (13). That is, the critical exponent of

the analogous heat capacity in  $T$ - $\delta L$  plane is the same as that in the  $T$ - $S$  plane, which confirms the conclusion that

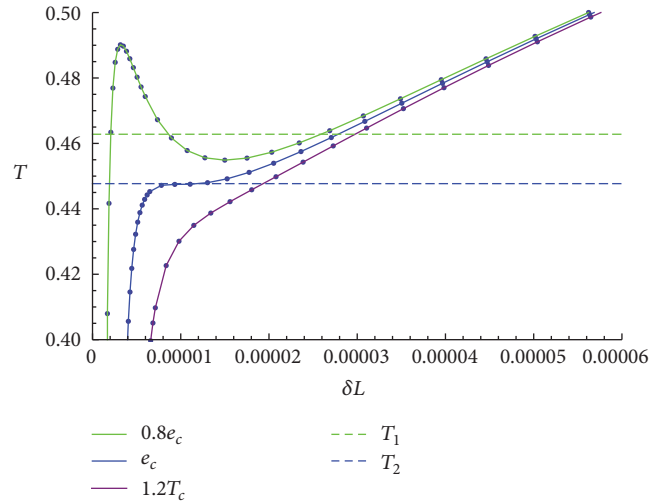


FIGURE 5: Relations between geodesic length and temperature for different  $e$  with  $q = 0.05, \theta_0 = 0.1$ .

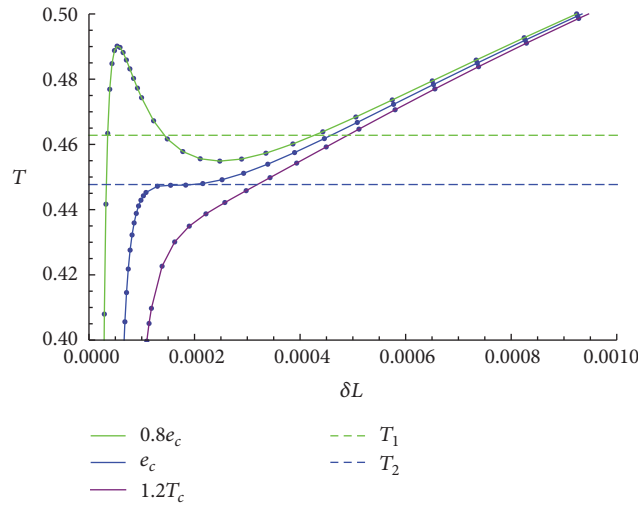


FIGURE 6: Relations between geodesic length and temperature for different  $e$  with  $q = 0.05, \theta_0 = 0.2$ .

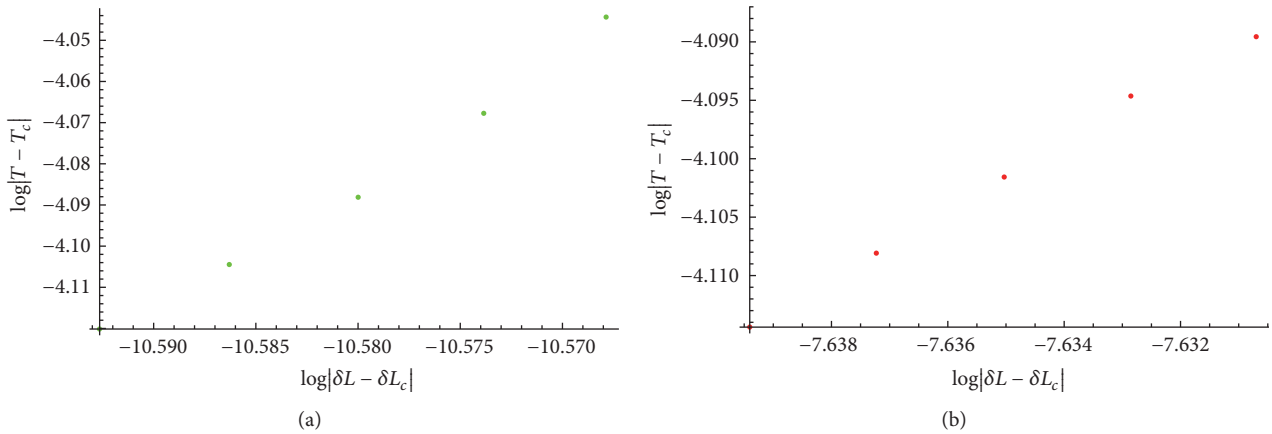


FIGURE 7: Relations between  $\log|T - T_2|$  and  $\log|\delta L - \delta L_c|$  for the case  $q = 0.05, e = e_c$ . (a) corresponds to the case  $\theta_0 = 0.1$  while (b) corresponds to  $\theta_0 = 0.2$ .

TABLE 2: EAL in the  $T$ - $\delta L$  plane; here relative error =  $(A_L - A_R)/A_R$ .

Boundary size	$T_1$	$\delta L_{\min}$	$\delta L_{\max}$	$A_L$	$A_R$	Relative error
$\theta_0 = 0.1$	0.4628	0.0000014	0.0000259	0.0000111	0.0000113	1.8869%
$\theta_0 = 0.2$	0.4628	0.0000354	0.0004267	0.0001812	0.0001811	0.4549%

the TCF can probe the phase structure of the hairy black hole.

#### 4. Conclusion and Discussion

As the usual charged black hole, we find that the hairy black hole also exhibits the VdW phase transition. That is, the phase structure of the hairy black hole depends on the charge of the spacetime. For the case where the charge is smaller than the critical charge, the small black hole, immediate black hole, and large black hole coexist. The small black hole will transit to the large black hole as the temperature reaches the critical temperature  $T_1$ . The order of this phase transition is first for the nonsmoothness of the free energy in the  $F$ - $T$  plane. As the charge equals the critical charge, the immediate black hole vanishes and the order for the small black hole transit to the large black hole is second for the heat capacity is divergent in this case. We also check the EAL numerically for the first-order phase transition and get the critical exponent of the heat capacity for the second-order phase transition.

With the TCF, we also probe the phase structure of the black hole. For the TCF in the field theory is dual to the geodesic length in the bulk, we thus employ the geodesic length to probe the phase structure of the hairy black hole. We find that the phase structure in the  $T$ - $\delta L$  plane resembles that in the  $T$ - $S$  plane, regardless of the size of the boundary separation. In addition, we find in the framework of holography that the EAL still holds and the critical exponent of the analogous heat capacity is the same as that in the usual thermodynamic system.

In this paper, we fix the parameter  $q = 0.05$  to investigate the phase structure of the black hole. For other values of  $q$ , we find that the phase structure is similar as the case  $q = 0.05$ . To avoid encumbrance, we will not list these values. In addition, we employ the analogy in Table 1 to study the VdW phase transition. In fact, as the cosmological constant is treated as a thermodynamic pressure,  $P$ , and its conjugate quantity as a thermodynamic volume,  $V$ , the VdW phase transition also can be constructed in the  $P$ - $V$  plane [13]. In [35], the holography entanglement entropy has been used to probe the phase structure in this case. For the hairy black hole, the TCF also can be used to probe its phase structure in the  $P$ - $V$  plane.

#### Conflicts of Interest

The authors declare that they have no conflicts of interest.

#### Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant no. 11405016), China Postdoctoral Science Foundation (Grant no. 2016M590138), Natural Science Foundation of Education Committee of Chongqing

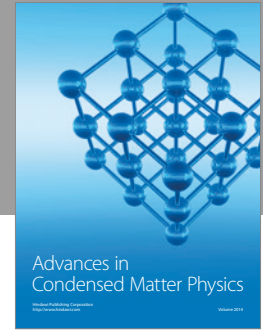
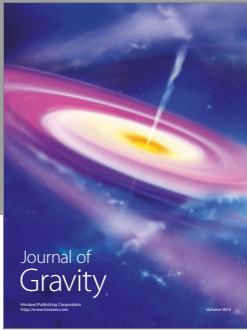
(Grant no. KJ1500530), and Basic Research Project of Science and Technology Committee of Chongqing (Grant no. cstc2016jcyj0364).

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