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Research Article

The Effect of Nuclear Elastic Scattering on Temperature Equilibration Rate of Ions in Fusion Plasma

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A plasma with two different particle types and at different temperatures has been considered, so that each type of ion with Maxwell-Boltzmann distribution function is in temperature equilibrium with itself. Using the extracted nuclear elastic scattering differential cross-section from experimental data, solving the Boltzmann equation, and also taking into account the mobility of the background particles, temperature equilibration rate between two different ions in a fusion plasma is calculated. The results show that, at higher temperature differences, effect of nuclear elastic scattering is more important in calculating the temperature equilibration rate. The obtained expressions have general form so that they are applicable to each type of particle for background (*b*) and each type for projectile (*p*). In this paper, for example, an equimolar Deuterium-Hydrogen plasma with density $n = 5 \times 10^{25}$ cm⁻³ is chosen in which the deuteron is the background particle with temperature (also electron temperature $T_p > T_b$. These calculations, particularly, are very important for ion fast ignition in inertial confinement fusion concept.

1. Introduction

In a fusion plasma, because of interactions, collisions, and gain and loss powers, we encounter an unstable plasma [1, 2]. The thermal equilibrium between charged particles is crucial for understanding the overall energy balance in a fusion plasma, where the ignition and burn of the plasma are strongly temperature dependent [3]. Since a plasma is composed of charged particles, they are affected by Coulomb force in the plasma. Coulomb force is a long-range force and, thus, may also be involved in the short-range collisions and the long-range collective effects. The problem of temperature equilibration was studied first by Landau [4, 5] and Spitzer (LS) [6]. The LS model is applicable to dilute, hot, fully ionized plasmas where the collisions are weak and binary. Recently, there have been several theoretical and computational works which aim to study temperature equilibration [7-12]. Among the theoretical studies, Brown, Preston, and Singleton (BPS) by solving the Boltzmann equation for short-range collisions and the Lenard-Balescu equation for long-range collective

effects and also considering the quantum corrections have achieved more accurate results [9]. In a hot and dense fusion plasma, in addition to the Coulomb interactions, the nuclear elastic scattering (NES) is able to role via short-range interactions (hard collisions). The effect of the NES on the stopping power of a charged particle has been investigated in previous studies [13-15]. It has been indicated that the stopping power due to NES is greater at the higher projectile energies. This fact motivated us to study the effect of NES on thermal equilibration rate for a fusion plasma. A fusion plasma is formed from charged particles at high temperatures in which they interact with each other to achieve balance. In this paper, for the simplest case, a plasma with two different ions at different temperatures has been considered, so that each type of ion with Maxwell-Boltzmann distribution function is in temperature equilibrium with itself. Here, for example, proton at higher temperature is considered as projectile and deuteron at lower temperature is assumed as background. By solving the Boltzmann equation and taking into account the thermal motion of plasma ions, the thermal

equilibration rate due to NES is calculated. The results are compared with Coulomb calculations and the conditions in which each contribution is dominated are discussed.

2. Elastic Collision

Consider a projectile with mass m_p and velocity \mathbf{v}_p interacting with a background particle with mass m_b and velocity \mathbf{v}_b in laboratory frame. (Vectors are written in bold type and nonbold letters indicate quantity magnitude.) In an elastic collision, $\mathbf{V}_c = (m_p \mathbf{v}_p + m_b \mathbf{v}_b)/M_{pb}$ is the velocity of the center of mass (C.M.) and relative velocity of the system is $\mathbf{v}_{pb} = \mathbf{v}_p - \mathbf{v}_b$. After the collision, the projectile and target velocities consider \mathbf{v}'_p and \mathbf{v}'_b , respectively. Therefore, $\mathbf{v}'_{pb} =$ $\mathbf{v}'_p - \mathbf{v}'_b$ is the relative velocity of two particles after collision. These quantities are related to each other by

$$\mathbf{v}_{p} = \mathbf{V}_{c} + \frac{m_{b}}{M_{pb}} \mathbf{v}_{pb},$$

$$\mathbf{v}_{p}' = \mathbf{V}_{c} + \frac{m_{b}}{M_{pb}} \mathbf{v}_{pb}',$$
(1)

where $M_{pb} = m_p + m_b$ is the total mass. The conservation laws for linear momentum and kinetic energy in the C.M. system indicate that the relative speeds of the particles do not change before and after collision as a result of an elastic collision $|\mathbf{v}_{pb}| = |\mathbf{v}'_{pb}| = v_{pb}$. As a result, the magnitude of the relative velocities remains unchanged and only the angle Θ (scattering angle in C.M. system) between \mathbf{v}_{pb} and \mathbf{v}'_{pb} is changed. Since the magnitude and direction of C.M. velocities (before and after a collision) will not change, \mathbf{V}_c can be selected as the reference. Using the above relations between \mathbf{V}_c and \mathbf{v}_{pb} velocities, the kinetic energy of the projectile is changed as follows:

$$E'_{p} - E_{p} = \frac{1}{2}m_{p}\left(v'^{2}_{p} - v^{2}_{p}\right) = m_{pb}\mathbf{V}_{c}\cdot\left(\mathbf{v}'_{pb} - \mathbf{v}_{pb}\right), \quad (2)$$

where $m_{pb} = m_p m_b / M_{pb}$ is reduced mass of the projectile and target system. If φ is the angle between \mathbf{V}_c and \mathbf{v}_{pb} , according to scattering angle in C.M. system Θ , then angle between \mathbf{V}_c and \mathbf{v}'_{pb} is ($\varphi - \Theta$). The change in kinetic energy of the projectile is equal to

$$E'_{p} - E_{p} = m_{pb}V_{c}v_{pb}\left(\cos\varphi\left(\cos\Theta - 1\right) + \sin\varphi\sin\Theta\right).$$
(3)

Since the scattering in the C.M. frame is axially symmetric about the relative speed \mathbf{v}_{pb} , transverse components average is zero in the scattering process, and so the second term on the right of this equation will be removed.

3. Temperature Equilibration Rate due to Coulomb Interaction

The problem of temperature equilibration was first addressed by Landau and Spitzer, who used the Fokker-Planck equation to derive an equilibration rate for one ion species temperature T_p , given another ion species temperature T_b [4–6]:

$$\frac{d\varepsilon_{pb}^{\rm LS}}{dt} = C_{pb}^{\rm LS} \left(T_p - T_b \right), \tag{4}$$

where C_{pb}^{LS} is defined as

$$C_{pb}^{\rm LS} = -\frac{8}{3}\sqrt{2\pi}\frac{\sqrt{m_p m_b}Z_p^2 Z_b^2 n_p n_b \ln \Lambda_{pb}^{\rm LS}}{\left(m_p T_b + m_b T_p\right)^{3/2}},$$
(5)

where $m_{p,b}$ are the ion species masses, $Z_{p,b}$ are the ion charges, and $n_{p,b}$ are the number densities. The LS Coulomb logarithm ln Λ_{pb}^{LS} is defined as follows:

$$\ln \Lambda_{pb}^{\rm LS} = 23 - \ln \left[\frac{Z_p Z_b \left(m_p + m_b \right)}{m_p T_b + m_b T_p} \left(\frac{n_p Z_p^2}{T_p} + \frac{n_b Z_b^2}{T_b} \right)^{1/2} \right].$$
(6)

Recently, there have been several theoretical and computational works which aim to study temperature equilibration [7–9]. The computational studies have all been done with classical molecular dynamics and have focused on temperature equilibration in a multi-eV hydrogen plasma [10-12]. The results of these studies showed that deviations from LS approach are to be expected, even for moderate values of $\ln \Lambda_{pb}^{LS}$. Among the theoretical works, Brown, Preston, and Singleton (BPS) produced an analytic calculation for Coulomb energy exchange processes for a fusion plasma [9, 16, 17]. These precise calculations are accurate to leading and next-to-leading order in the plasma coupling parameter and to all orders for two-body quantum scattering within the plasma. In general, the energy density exchange rate between the two charged particles *p* and *b* in a plasma can be written as follows:

$$\frac{d\varepsilon_{pb}^{\text{Coul}}}{dt} = C_{pb}^{\text{Coul}} \left(T_b - T_p \right),\tag{7}$$

where C_{pb}^{Coul} is called the rate coefficient and it can be written as a sum of three terms:

$$C_{pb}^{\text{Coul}} = \left(C_{pb,S}^{C} + C_{pb,R}^{C}\right) + C_{pb}^{Q},\tag{8}$$

where the first two arise from classical short and long distance physics and the latter term arises from short distance twobody quantum diffraction. The first term in (8), the shortdistance classical scattering contribution, reads

$$C_{pb,S}^{C} = -k_{p}^{2}k_{b}^{2} \frac{\left(m_{p}\beta_{p}m_{b}\beta_{b}\right)^{1/2}}{\left(m_{p}T_{p} + m_{b}T_{b}\right)^{3/2}} \left(\frac{1}{2\pi}\right)^{3/2} \times \left[\ln\left\{\frac{Z_{p}Z_{b}e^{2}K}{4m_{pb}V_{pb}^{2}}\right\} + 2\gamma\right],$$
(9)

where $\gamma \simeq 0.57721...$ is Euler constant. The second term in (8), the long-distance, dielectric term that accounts for collective effects in the plasma, is given by

$$C_{pb,R}^{C} = \frac{k_{p}^{2}k_{b}^{2}}{2\pi} \left(\frac{m_{p}\beta_{p}}{2\pi}\right)^{1/2} \left(\frac{m_{b}\beta_{b}}{2\pi}\right)^{1/2} \\ \times \int_{-\infty}^{\infty} d\nu\nu^{2} \exp\left\{-\frac{1}{2}\left(m_{p}\beta_{p}+m_{b}\beta_{b}\right)\nu^{2}\right\}$$
(10)
$$\times \frac{i}{2\pi}\frac{F\left(\nu\right)}{\rho_{\text{total}}\left(\nu\right)} \ln\left\{\frac{F\left(\nu\right)}{K^{2}}\right\}.$$

Here, *K* is an arbitrary wave number so that the total result does not depend upon *K*. However, sometimes choosing *K* to be a suitable multiple of the Debye wave number of the plasma simplifies the formula. We write the inverse temperature of the projectile as $\beta_p = T_p^{-1}$ and the plasma species *b* as $\beta_b = T_b^{-1}$, which we measure in energy units. Debye wave number k_b of this species is defined by

$$k_b^2 = 4\pi\beta_b Z_b^2 n_b,\tag{11}$$

where n_b is the number density of species *b*. The total Debye wave number k_D is defined by

$$k_D^2 = \sum_b k_b^2.$$
 (12)

The function F(u) is related to the leading-order plasma dielectric susceptibility in which it may be expressed in the dispersion form:

$$F(u) = -\int_{-\infty}^{\infty} dv \frac{\rho_{\text{total}}(v)}{u - v + i\eta},$$
(13)

where the limit $\eta \to 0^+$ is understood. The spectral weight, $\rho_{\text{total}}(v)$ is defined by

$$\rho_{\text{total}}\left(\nu\right) = \sum_{b} \rho_{b}\left(\nu\right),\tag{14}$$

where

$$\rho_b(v) = k_b^2 v \sqrt{\frac{\beta_b m_b}{2\pi}} \exp\left(-\frac{1}{2}\beta_b m_b v^2\right).$$
(15)

According to the BPS calculations, the general case is obtained by adding a quantum correction, which is related to the short distance two-body quantum diffraction and to the classical result. Therefore, the third term in (8), C_{pb}^{Q} is a coefficient which is calculated from the quantum correction and is defined as follows:

$$C_{pb}^{Q} = -\frac{1}{2} k_{p}^{2} k_{b}^{2} \frac{\left(m_{p} \beta_{p} m_{b} \beta_{b}\right)^{1/2}}{\left(m_{p} T_{p} + m_{b} T_{b}\right)^{3/2}} \left(\frac{1}{2\pi}\right)^{3/2} \int_{0}^{\infty} d\zeta \exp\left(-\frac{\zeta}{2}\right) \times \left[\operatorname{Re} \psi\left(1 + i\frac{\overline{\eta}_{pb}}{\zeta^{1/2}}\right) - \ln\left\{\frac{\overline{\eta}_{pb}}{\zeta^{1/2}}\right\}\right];$$
(16)

the strength of the quantum effects associated with the scattering of two plasma species p and b is characterized by the dimensionless parameter:

$$\overline{\eta}_{pb} = \frac{Z_p Z_b}{\hbar V_{pb}},\tag{17}$$

where the square of the thermal velocity in this expression is defined by

$$V_{pb}^{2} = \frac{T_{p}}{m_{p}} + \frac{T_{b}}{m_{b}},$$
(18)

and $\psi(z)$ is the logarithmic derivative of the gamma function.

4. Temperature Equilibrium due to Nuclear Elastic Scattering

Since nuclear force is a short-range force, it can only be involved in short-range collisions and, therefore, does not contribute to long-range cumulative effects of plasma. Boltz-mann equations are described generally as short-range encounters. The equation for the phase-space density $f_p(\mathbf{P}_p)$ of species p is [18]

$$\left[\frac{\partial}{\partial t} + \mathbf{v}_{p} \cdot \nabla\right] f_{p}\left(\mathbf{r}, \mathbf{p}_{p}, t\right) = \sum_{b} K_{pb}\left(\mathbf{r}, \mathbf{p}_{p}, t\right).$$
(19)

We suppress the common space and time coordinates \mathbf{r} , t and write the collision term involving species b in the following form:

$$K_{pb}\left(\mathbf{r},\mathbf{p}_{p},t\right) = \int \frac{d^{3}\mathbf{p}_{b}'}{(2\pi\hbar)^{3}} \frac{d^{3}\mathbf{p}_{p}'}{(2\pi\hbar)^{3}} \frac{d^{3}\mathbf{p}_{b}}{(2\pi\hbar)^{3}} \left|T\left(W,q^{2}\right)\right|^{2} (2\pi\hbar)^{3} \delta^{3} \\ \times \left(\mathbf{p}_{b}'+\mathbf{p}_{p}'-\mathbf{p}_{b}-\mathbf{p}_{p}\right) (2\pi\hbar) \delta \qquad (20) \\ \times \left(\frac{1}{2}m_{b}v_{b}'^{2}+\frac{1}{2}m_{a}v_{p}'^{2}-\frac{1}{2}m_{b}v_{b}^{2}-\frac{1}{2}m_{a}v_{p}^{2}\right) \\ \times \left[f_{b}\left(\mathbf{p}_{b}'\right)f_{p}\left(\mathbf{p}_{p}'\right)-f_{b}\left(\mathbf{p}_{b}\right)f_{p}\left(\mathbf{p}_{p}\right)\right].$$

This relation describes the scattering of the particles of masses m_p and m_b and the scattering from the initial momenta $\mathbf{p}_p = m_p \mathbf{v}_p$, $\mathbf{p}_b = m_b \mathbf{v}_b$ to the final momenta $\mathbf{p}'_p = m_p \mathbf{v}'_p$, $\mathbf{p}'_b = m_b \mathbf{v}'_b$ with the scattering amplitude $T(W, q^2)$ depending on the center-of-mass energy, W, and the squared momentum transfer, q^2 . It is convenient to employ this quantum-mechanical notation for several reasons. It explicitly displays the complete kinematical character of a scattering process, including the detailed balance symmetry. Furthermore, it shows that the collision term (19) vanishes when all the particles are in thermal equilibrium with the generic densities $f(\mathbf{p}) \sim \exp[-\beta m v^2/2]$ because of the conservation of energy enforced by the delta function. In case we have a spatial uniformity (homogeneity), include the gradient removed

from (19) and, thus, the rate of change of energy is obtained as follows:

$$\frac{d\varepsilon_{pb}}{dt} = \int \frac{d^3 \mathbf{p}_p}{(2\pi\hbar)^3} \frac{p_p^2}{2m_p} \frac{\partial f_p\left(\mathbf{p}_p, t\right)}{\partial t}
= \int \frac{d^3 \mathbf{p}_p}{(2\pi\hbar)^3} \frac{p_p^2}{2m_p} K_{pb}\left(\mathbf{p}_p\right).$$
(21)

Placement of relation (20) in (21) and rate of change of energy density of species p in collisions with plasma species b will be written as follows:

$$\frac{d\varepsilon_{pb}}{dt} = \int \frac{d^{3}\mathbf{p}_{b}'}{(2\pi\hbar)^{3}} \frac{d^{3}\mathbf{p}_{p}'}{(2\pi\hbar)^{3}} \frac{d^{3}\mathbf{p}_{b}}{(2\pi\hbar)^{3}} \frac{d^{3}\mathbf{p}_{p}}{(2\pi\hbar)^{3}} \left(\frac{p_{p}'^{2}}{2m_{p}} - \frac{p_{p}^{2}}{2m_{p}}\right) \times |T|^{2} (2\pi\hbar)^{3} f_{b} (\mathbf{P}_{b}) f_{p} (\mathbf{P}_{p}) \delta^{(3)} \times \left(\mathbf{p}_{b}' + \mathbf{p}_{p}' - \mathbf{p}_{b} - \mathbf{p}_{p}\right) (2\pi\hbar) \delta \times \left(\frac{1}{2}m_{b}v_{b}'^{2} + \frac{1}{2}m_{p}v_{p}'^{2} - \frac{1}{2}m_{b}v_{b}^{2} - \frac{1}{2}m_{p}v_{p}^{2}\right).$$
(22)

Now, in general, the cross section for the scattering of particles p and b into a restricted momentum interval Δ is given by

$$\begin{aligned} v_{pb} \int_{\Delta} d\Omega \left(\frac{d\sigma_{pb}}{d\Omega} \right) \\ &= \int \frac{d^{3} \mathbf{p}_{b}'}{(2\pi\hbar)^{3}} \frac{d^{3} \mathbf{p}_{p}'}{(2\pi\hbar)^{3}} \left| T \left(W, q^{2} \right) \right|^{2} (2\pi\hbar)^{3} \delta^{3} \\ &\times \left(\mathbf{p}_{b}' + \mathbf{p}_{p}' - \mathbf{p}_{b} - \mathbf{p}_{p} \right) (2\pi\hbar) \delta \\ &\times \left(\frac{1}{2} m_{b} v_{b}^{\prime 2} + \frac{1}{2} m_{p} v_{p}^{\prime 2} - \frac{1}{2} m_{b} v_{b}^{2} - \frac{1}{2} m_{p} v_{p}^{2} \right). \end{aligned}$$
(23)

We can write (23) in terms of the speed of the center of mass \mathbf{V}_c and relative speed \mathbf{v}_{pb} . We use the conversion $d\mathbf{v}_p d\mathbf{v}_b = j(\mathbf{v}_p, \mathbf{v}_b; \mathbf{V}_c, \mathbf{v}_{pb}) d\mathbf{V}_c d\mathbf{v}_{pb}$, where $(j(\mathbf{v}_p \mathbf{v}_b, \mathbf{V}_c \mathbf{v}_{pb}) = 1)$ is the Jacobian. Using these definitions, we find that

$$\frac{d\varepsilon_{pb}}{dt} = \int \frac{d^{3}\mathbf{p}_{b}}{(2\pi\hbar)^{3}} \frac{d^{3}\mathbf{p}_{p}}{(2\pi\hbar)^{3}} f_{b}\left(\mathbf{P}_{b}\right) f_{p}\left(\mathbf{P}_{p}\right) v_{pb} \\
\times \int d\Omega \left(\frac{d\sigma_{pb}}{d\Omega}\right) \left[E'_{p} - E_{p}\right].$$
(24)

Equation (24) represents the rate of change of energy density of projectile particles p in collisions with plasma species b. Now, consider the particles distribution functions p and b, $f_p(\mathbf{p}_p)$, and $f_b(\mathbf{p}_b)$ where

$$f_j\left(\mathbf{p}_j\right) = n_j \left(\frac{2\pi\hbar^2\beta_j}{m_j}\right)^{3/2} \exp\left(-\frac{1}{2}\beta_j m_j v_j^2\right)$$
(25)

is the Maxwellian distribution function. Then temperature Equilibration rate can be calculated as follows;

$$\frac{d\varepsilon_{pb}^{\rm NI}}{dt} = C_{pb}^{\rm NI} \left(T_b - T_p \right), \tag{26}$$

where the coefficient C_{pb}^{NI} is given by

$$C_{pb}^{\rm NI} = -n_p n_b \pi^{1/2} 2^{7/2} \frac{\left(m_p m_b\right)^{1/2} M_{pb}}{\left(m_p T_p + m_b T_b\right)^{5/2}} \\ \times \int_0^\infty E_c^2 dE_c \exp\left(-\frac{M_{pb}}{m_p T_b + m_b T_p} E_c\right)$$
(27)
 $\times I^{\rm NI} \left(E_C\right),$

where the integral

$$I^{\rm NI}(E_C) = \int_0^{\pi} \left(\frac{d\sigma_{pb}^{\rm NI}}{d\Omega}\right) (\cos\Theta - 1)\sin\Theta d\Theta, \qquad (28)$$

and $E_c = 1/2m_{pb}v_{pb}^2$ is the total energy of the projectile and the target is in the center of mass system. Extracting of NES differential cross-section from experimental values, $(d\sigma_{pb}^{\rm NI}/d\Omega)$ rate of temperature equilibration is archived. It is seen that (26) when the two types of ion temperature are equal, $d\varepsilon_{pb}^{\rm NI}/dt = 0$ which indicates the temperature equilibrium.

5. Results and Discussion

Now, we will consider the thermal equilibration rate between the proton and deuteron in an equimolar plasma with density $n = 5 \times 10^{25} \text{ cm}^{-3}$. The deuteron is considered as the background at temperature $T_b = 1 \text{ keV}$ and the proton is the projectile at temperature T_p , greater than the background temperature $(T_p > T_b)$. It is assumed that each type of particle is in equilibrium with itself so that the equilibrium is described by Maxwell-Boltzmann distribution function. The density and temperatures are chosen so that the their values are close to initial conditions of an inertial confinement fusion fuel at ignition moment. The thermal equilibration rate due to Coulomb interactions is obtained from (5) and (7) for LS and BPS relations, respectively (Figure 1). The portions of singular term (see (9)), regular term (see (10)), and quantum term (see (16)) of BPS relation are shown, separately. It is seen that the values obtained from the BPS calculations would give lower values than LS calculations. However, the most important point is that both equilibration rates are further reduction at the higher temperature differences. This event is predictable of course, because the Coulomb differential cross section (Rutherford's differential cross section) is proportional with the total energy in center of mass system (E_C) as $d\sigma^{\text{Coul}}/d\Omega \propto E_C^{-2}$. The total energy in the center of mass system increases with increasing the temperature difference between the two ions, and, thus,



FIGURE 1: The thermal equilibration rate due to Coulomb interactions is obtained from (4) and (7) for LS and BPS relations, respectively.



FIGURE 2: The ratio of the experimental differential cross-section $(d_{pb}^{\text{coul}}\sigma/d\Omega)$ to the Coulomb differential cross-section $(d_{pb}^{\text{Coul}}\sigma/d\Omega)$ is plotted versus the scattering angle in the center of mass system (Θ) for different energies E_C , for proton-deuteron elastic scattering.

the rate of Coulomb interaction decreases due to reduction of its cross-section.

In Figure 2, the ratio of the experimental differential cross-section $(d_{pb}^{exp}\sigma/d\Omega)$ to the Coulomb differential cross-section $(d_{pb}^{Coul}\sigma/d\Omega)$ is plotted versus the scattering angle in the center of mass system (Θ) for different energies, for proton-deuteron elastic scattering. The labeled energies in



FIGURE 3: The integral $I^{\text{NI}}(E_C)$ (see (28)) versus the total energies in the center of mass coordinate E_C .

the figure are the total energy in the center of mass system (E_C) . As can be seen, this ratio is different for different energies and also is associated with fluctuations. However, in general, this ratio is increased more by increasing the energy and this means that the experimental differential cross-section is more deviated from the Coulomb differential cross-section. Deviation increase reflects the growth of the contribution of NES differential cross-section $(d_{pb}^{NI}\sigma/d\Omega)$ at higher energies (the superscript NI denotes the sum of nuclear and interference terms that, for simplicity, are called nuclear elastic scattering (NES)).

By extracting the NES differential cross-section from experimental data $(d_{pb}^{\text{NI}}\sigma/d\Omega = d_{pb}^{\exp}\sigma/d\Omega - d_{pb}^{\text{Coul}}\sigma/d\Omega)$, the integral $I^{\text{NI}}(E_C)$ (see (28)) can be calculated (Figure 3). The integral $I^{\text{NI}}(E_C)$ is submitted in (27) and the thermal equilibration rate due to NES can be obtained. Figure 4 shows the thermal equilibration rate due to NES (NI) and Coulomb interaction (LS and BPS) and also the total term that is obtained from sum of nuclear and Coulomb contributions (NI + BPS). It is seen that the thermal equilibration rate due to Coulomb interaction has the main contribution in the lower temperature differences. With increasing temperature, the Coulomb interaction contribution decreases and increases in the share of NES. Thus, at temperature differences, $\Delta T \ge$ 3.278 MeV; the NES is the dominant contribution to the thermal equilibration rate.

The rapid development of laser-accelerated ion beams is developing the ion fast ignition (IFI) scheme in inertial confinement fusion concept [19]. In the IFI scheme, an intense laser pulse accelerates an ion beam to multi-MeV temperatures. The accelerated ion beam deposits its energy in the ignition region of fuel pellet that is named "hot spot" and provides the required energy for ignition [20, 21]. For this reason, the diver ion beam is named ignitor. The power is deposited through an ion beam energy exchange with the background plasma particles with few-keV temperatures.



FIGURE 4: The thermal equilibration rate due to NES (NI) and Coulomb interaction (LS and BPS) and also the total term that is obtained from the sum of nuclear and Coulomb contributions (NI + BPS).

In the energy exchange process, some of the ion beam energy is delivered to the plasma ions, and another part of energy is delivered to the plasma electrons. The electrons are the source of power loss in a fusion plasma that it is better to be kept at low temperature. At the high temperature difference (multi-MeV) between ion beam and background plasma particles, the rate of energy exchange via Coulomb interaction is reduced and, in turn, the rate of energy exchange via NES is increased. The NES causes the ion temperature to be further increased, resulting in reduced power loss.

The laser-accelerated ion beams can be produced with different distribution functions, such as exponential, Maxwellian, and quasimonoenergy distribution functions [22, 23]. Furthermore, some of ion beams, such as the deuteron beam can be on their way to interact with the background ions [24, 25]. Therefore, research in this area requires more detailed calculations that will be covered in the future.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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