Hindawi Publishing Corporation
Advances in Decision Sciences
Volume 2012, Article ID 359082, 14 pages
doi:10.1155/2012/359082

Research Article

Economic Design of Acceptance Sampling Plans in a Two-Stage Supply Chain

Lie-Fern Hsu[1] and Jia-Tzer Hsu[2]

[1] *Department of Management, Zicklin School of Business, Baruch College, The City University of New York, One Bernard Baruch Way, P.O. Box B9-240, New York, NY 10010, USA*
[2] *Department of Computer Science and Information Engineering, Hungkuang University, Shalu, Taichung 43302, Taiwan*

Correspondence should be addressed to Lie-Fern Hsu, lie-fern.hsu@baruch.cuny.edu

Received 2 August 2011; Revised 13 January 2012; Accepted 19 January 2012

Academic Editor: Henry Schellhorn

Supply Chain Management, which is concerned with material and information flows between facilities and the final customers, has been considered the most popular operations strategy for improving organizational competitiveness nowadays. With the advanced development of computer technology, it is getting easier to derive an acceptance sampling plan satisfying both the producer's and consumer's quality and risk requirements. However, all the available *QC tables* and computer software determine the sampling plan on a noneconomic basis. In this paper, we design an economic model to determine the optimal sampling plan in a two-stage supply chain that minimizes the producer's and the consumer's total quality cost while satisfying both the producer's and consumer's quality and risk requirements. Numerical examples show that the optimal sampling plan is quite sensitive to the producer's product quality. The product's inspection, internal failure, and postsale failure costs also have an effect on the optimal sampling plan.

1. Introduction

Supply Chain Management focuses on the material and information flows between facilities and their final customer, and has been considered the most popular operations strategy for improving organizational competitiveness in the 21st century [1]. Recently, due to the pressure to lower manufacturing and service costs and to deliver high-quality products to market quickly, North American companies have become increasingly attracted to outsourcing and off-shoring, which is the usage of overseas workers to produce components, entire products, and services. Many companies have contracted with suppliers in lower-cost countries in order to gain access to a large pool of workers at a mere fraction of the cost of domestic facilities. For this reason, India and China are becoming major players in off-shoring, especially in the areas of manufacturing and service.

Recent product recall scandals have revealed that the benefits of outsourcing and off-shoring also come with its disadvantages—in this case, the threat of quality risks in the supply chain. Some examples of recent product recalls include Toyota's sticking accelerator pedal recall and floor mat recall (e.g., vehicles involved in the sticking accelerator pedal recall include: 2007–2010 Camry, 2009 Camry Hybrid, 2009–2010 Corolla, 2009–2010 RAV4, 2010 Highlander; vehicles involved in the floor mat recall include: 2007–2010 Camry, 2009-2010 Corolla, 2008–2010 Highlander) and China's recent toys, pet food, and melamine milk recalls.

Cao and Zhang [2] showed that firms have been attempting to achieve greater collaborative advantages with their supply chain partners in the past few decades, and that supply chain collaborative advantages have a bottom-line influence on firm performance. In addition, Foster Jr. [3] established that the increasing importance given to supply chain management has resulted in the rethinking of models, constructs, and frameworks for quality management that have been developed for operations management. Although research in quality management has previously focused on an internal versus external view of quality, where the internal view focused on process and the external on the customers, companies must now merge these views as they adopt the systems approach implicitly in supply chain management, in order to internalize upstream and downstream processes with their own. Thus, Foster Jr. [3] defined Supply Chain Quality Management (SCQM) as a system-based approach to performance improvement that leverages opportunities created by upstream and downstream linkages with suppliers and customers.

The purpose of this paper is to design an economic model to determine the optimal sampling plan in a two-stage supply chain that minimizes the producer's and the consumer's total quality cost while satisfying both the producer's and the consumer's quality and risk requirements. The model can be applied to any two-stage supply chain including a vendor and a buyer, where a vendor deliver a batch of product to the buyer, and the buyer decides whether to accept or reject the entire lot based on the quality of the sample selected from the lot. Acceptance sampling is often used to monitor the quality of raw material, purchased parts, and finished products when product testing is destructive, time-consuming, or expensive. An acceptance plan is the overall scheme for either accepting or rejecting a lot based on information gained from samples. The acceptance plan identifies both the size and type of samples and criteria to be used to either accept or reject the lot. Samples may be either single, double, multiple, or sequential.

Single sampling plans are simple to use. However, if the incoming quality level is particularly good or particularly poor, a double, multiple, or sequential sampling plan will reach an acceptance or a rejection decision sooner and, therefore, reduce the average sample number. Moreover, if a single sampling plan is applied, very often the producer is at a "psychological" disadvantage, since no second chance is given for the rejected lots. In such situations, taking a second sample is preferable.

In a single sampling plan, one sample of items is selected at random from a lot and the disposition of the lot is determined from the resulting information. These plans are usually denoted as (n, c) plans for a sample size n, where the lot is rejected if there are more than c defectives. These are the most common (and easiest) plans to use, although not the most efficient in terms of the average number of samples needed.

In a double sampling plan, after the first sample is tested, there are three possibilities:

(1) accept the lot,

(2) reject the lot,

(3) take a second sample.

If the outcome is (3), a second sample is taken, and the procedure is to combine the results of both samples and make a final decision based on that information.

A multiple sampling plan is an extension of the double sampling plans where more than two samples are needed to reach a conclusion. The advantage of multiple sampling is smaller sample sizes.

A sequential sampling plan is the ultimate extension of multiple sampling where items are selected from a lot one at a time and after inspection of each item a decision is made to accept or reject the lot or select another unit.

One of the most common ways to set the sampling plan parameters is to use what are often referred to as *QC tables*. The two most common sets of these tables are as follows.

(1) ANIS/ASQ Z1.4 and Z1.9-2008 [see [4]]

Using the sample size code letter (which is determined by the shipping lot size and the inspection level), the sampling plan can be read off instantly for a specified acceptable quality level (AQL). The AQL is a percent defective that is the base line requirement for the quality of the producer's product. The producer would like to design a sampling plan such that there is a high probability of accepting a lot that has a defect level less than or equal to the AQL. It provides tightened, normal, and reduced plans to be applied for attributes inspection for percent nonconforming or nonconformities per 100 units [4, 5]. The producer's risk (*Type I* error) is the probability, for a given sampling plan, of rejecting a lot that has a defect level equal to the AQL. The producer suffers when this occurs, because a lot with acceptable quality is rejected. The symbol α is commonly used for the producer's risk and the typical value for α is 0.05.

(2) Dodge-Romig Tables

These attribute acceptance plans set the parameters while assuming the rejected lots are 100 percent inspected and defectives are replaced with nondefectives. Users must specify values for consumer's risk (β), the approximate actual percent defectives, the lot size (N), and the lot tolerance percent defective (LTPD) [6]. The LTPD is a designated high defect level that would be unacceptable to the consumer. The consumer would like the sampling plan to have a *low probability of accepting* a lot with a defect level as high as the LTPD. The consumer's risk (*Type II* error) is the probability, for a given sampling plan, of accepting a lot with a defect level equal to the LTPD. The consumer suffers when this occurs, because a lot with unacceptable quality is accepted. The symbol β is commonly used for the *Type II* error and the typical value for β is 0.10.

Some computer software packages are available now to find the acceptance sampling plans that satisfy the company's quality and risk requirements. For example, Sampling Plan Analyzer 2.0 [7] is a shareware software package for evaluating and selecting acceptance sampling plans. Users can take an existing sampling plan and use the software to evaluate it including calculating and displaying OC (*Operating Characteristic*) curves (the OC curve plots p_a, the probability of accepting the lot (Y-axis) versus p, the lot fraction, or percent defectives (X-axis); the OC curve is a graph of the performance of an acceptance sampling plan, it shows how well an acceptance plan discriminates between good and bad lots). Users can also specify the desired protection and the program will generate a list of sampling plans that might be used.

A plot of the Average Outgoing Quality (AOQ, Y-axis) versus the incoming lot p (X-axis) will start at 0 for $p = 0$ and return to 0 for $p = 1$ (where every lot is 100% inspected and rectified). The AOQ curve shows that as p, the lot fraction or percent defectives, increases, the AOQ initially deteriorates and then improves. The improvement in quality occurs because as the acceptance plan rejects lots, the rejected lots are 100 percent inspected and defectives are either replaced with nondefectives or removed. In between, it will rise to a maximum. This maximum, which is the worst possible long-term AOQ, is called the AOQL.

Acceptance Sampling for Attribute TP105 [8] develops sampling plans for attribute data based on the binomial and the Poisson distributions. The metric used for the OC curve can be either the fraction defective, as in the binomial case of go/no-go data, or counts, as in the Poisson case of defect count.

All the previous *QC tables* or computer software [7, 8] determine the sampling plans on a non-economic basis to satisfy the quality and risk requirements of the producer, the consumer, or both parties. Motivated by the case of a Greek company, which uses the Greek equivalent to the ISO 2859 [9] for the quality control of its incoming raw materials, Nikolaidis and Nenes [10] evaluated the single-sampling plans recommended by the international standard ISO 2859 from an economic point of view. Their evaluation shows that the use of the ISO 2859 rarely leads to satisfactory economic results. Wetherill and Chiu [11] reviewed some major principles of acceptance schemes with emphasis on the economic aspect. Their research indicated that the major approaches for designing an economic acceptance sampling plan include the following.

(1) *The Bayesian approach*. This approach assesses the costs and losses involved in operating a sampling plan and tries to minimize the total costs. The expected cost per batch includes the cost of sampling and the loss due to wrong decisions, which is a function of the process quality p. The optimal single sampling plan (n, c) is obtained by minimizing the expected cost per batch with respect to these two variables.

(2) *The Minimax Approach*. This approach also aims at minimizing costs but without assuming a knowledge of the process quality p. Thus the average cost per batch $C(p)$ is a function of p. For any given sampling plan, $C(p)$ will go through a maximum as p runs from 0 to 1. The minimax principle chooses the plan that minimizes this maximum.

(3) *Semieconomic Approach*. Here a point on the OC curve is specified. The fixed point on the OC curve can be the producer's risk point, the consumer's risk point, or the point of indifference quality. The fixed point determines a relationship between c and n. The plan that minimizes the average amount of inspection at the process average quality is chosen.

Tagaras [12] developed an economic model to assist in the selection of minimum cost acceptance sampling plans by variables. The quadratic Taguchi loss function is adopted to model the cost of accepting items with quality characteristics deviating from the target value. Ferrell Jr. and Chhoker [13] presented a sequence of models that addressed 100% inspection and single sampling with and without inspector error when a Taguchi-like loss function is used to describe the cost associated with any deviation between the actual value of a product's quality characteristics and its target value. González and Palomo [14] developed a Bayesian acceptance sampling plan for a lot consisting of N units, when the number of defects in a unit can be described by a Poisson distribution with parameter λ, and the prior distribution of λ takes the form of a gamma or noninformative function. In the acceptance-sampling plan to be constructed, a sample of size n is taken from a lot of size N and all units in the sample are inspected. If the number of defects found in the sample is above a specified value c, the lot is rejected. If the number of defects is at or below c, the lot is accepted and sent to the next stage without further inspection. The sampling plans are obtained following an economic criterion: minimize the expected total cost of quality. Note that none of the research available in the literature focusing on the economic design of acceptance sampling plans has integrated the producer's and the consumer's risk requirements into the design of the model.

In this paper, we consider a two-stage supply chain. For example, one of the major agriculture export products from Taiwan is the orchid. According to the Statistics of the Agriculture and Food Agency of the Taiwanese government, the export value of the seedlings of Phalaenopsis (Butterfly Orchid) to all countries was 13,525,800 US dollars for the year of 2010, among which \$5,497,900′s worth was to the USA and \$1,971,500′s worth was to the Netherlands. In the USA, once the seedlings of Phalaenopsis arrive at the seaports (California or Florida), they are inspected (100 percent inspection for new suppliers and sampling inspections for old suppliers). The defective products are either scrapped at the seaport or returned to Taiwan. In the Netherlands, the defective products are sold at a reduced price. We will develop the optimal sampling plan based on an economic viewpoint. This paper is organized as follows. Section 2 formulates the optimization problem for the economic design of acceptance sampling plan. Section 3 provides numerical examples to illustrate the effects of quality and costs on the optimal sampling plan. Finally, Section 4 concludes this paper with a brief summary of the main results.

2. Economic Design of Acceptance Sampling Plan

Figure 1 illustrates how a *single-sampling plan* for attributes operates.

Let $p_a(p)$ denote the probability of accepting the lot given that the lot quality is p and let ATI denote the average total inspection items. The single sampling plan has the following performance measurements [15]:

$$p_a(p) = \sum_{X=0}^{C} \binom{n}{x} p^X (1-p)^{n-X}, \tag{2.1}$$

$$\text{AOQ} = \frac{p p_a(p)(N-n)}{N} \tag{2.2a}$$

if defective items are replaced with good ones and

$$\text{AOQ} = \frac{p p_a(p)(N-n)}{N - \text{np} - (1-p_a(p))p(N-n)} \tag{2.2b}$$

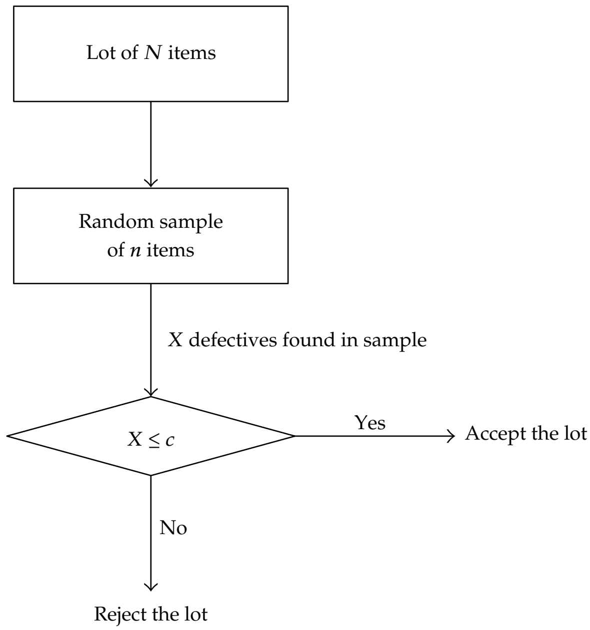

Figure 1: n is the number of items to be sampled; c is the prespecified acceptable number of defectives.

if defective items are removed but not replaced, and

$$\text{ATI} = n + \left(1 - p_a(p)\right)(N - n). \tag{2.3}$$

Let D_d denote the defective items detected; and let D_n denote the defective items not detected, then we have

$$\begin{aligned} D_d &= \text{np} + \left(1 - p_a(p)\right)(N - n)p, \\ D_n &= p_a(p)(N - n)p. \end{aligned} \tag{2.4}$$

Note that if the inspection is 100% reliable, for the sampled n items, the expected defective items np will be detected for sure. If the lot is rejected (with probability $1 - p_a(p)$), it will be 100% inspected and the remaining $(N - n)p$ defective items will be detected. On the other hand, if the lot is accepted (with probability $p_a(p)$), the $(N - n)p$ defective items will not be detected.

To derive the total quality cost per lot for a given sampling plan, we define the following cost parameters:

C_i: Inspection cost per item.

C_f: Internal failure cost; that is, the cost of rework, repair, or replacement for a defective item which is not released to the market as a finished product or not released to production as an incoming raw material.

C_o: The cost of an outgoing defective item (i.e., the postsale failure cost, see Hsu and Tapiero [16]). For a finished product, this is the cost of replacement and loss of good will for

Table 1: Single sampling plans satisfying AQL = 0.02, LTPD = 0.07, α = 0.05, β = 0.1, with $n \leq 205$.

TC	n	c	AOQ	ATI	D_d	D_n	$1 - P_a(\text{AQL})$	$P_a(\text{LTPD})$	$P_a(p)$
532.64	131	5	0.0208	306.10	9.18	20.82	0.0487	0.0974	0.7985
518.11	149	6	0.0214	286.98	8.61	21.39	0.0310	0.0970	0.8379
521.23	150	6	0.0213	291.10	8.73	21.27	0.0320	0.0934	0.8340
524.38	151	6	0.0211	295.24	8.86	21.14	0.0330	0.0900	0.8301
527.54	152	6	0.0210	299.40	8.98	21.02	0.0340	0.0867	0.8262
530.73	153	6	0.0209	303.59	9.11	20.89	0.0350	0.0835	0.8222
533.93	154	6	0.0208	307.80	9.23	20.77	0.0361	0.0804	0.8182
537.15	155	6	0.0206	312.04	9.36	20.64	0.0372	0.0775	0.8142
540.39	156	6	0.0205	316.30	9.49	20.51	0.0383	0.0746	0.8101
543.64	157	6	0.0204	320.58	9.62	20.38	0.0394	0.0718	0.8060
546.91	158	6	0.0203	324.89	9.75	20.25	0.0406	0.0690	0.8018
550.20	159	6	0.0201	329.21	9.88	20.12	0.0417	0.0664	0.7976
553.50	160	6	0.0200	333.55	10.01	19.99	0.0429	0.0639	0.7934
556.82	161	6	0.0199	337.92	10.14	19.86	0.0441	0.0615	0.7891
560.15	162	6	0.0197	342.30	10.27	19.73	0.0453	0.0591	0.7848
563.49	163	6	0.0196	346.70	10.40	19.60	0.0466	0.0568	0.7805
566.85	164	6	0.0195	351.11	10.53	19.47	0.0479	0.0546	0.7762
570.22	165	6	0.0193	355.55	10.67	19.33	0.0492	0.0525	0.7718
507.35	166	7	0.0218	272.82	8.18	21.82	0.0192	0.0991	0.8719
509.99	167	7	0.0217	276.31	8.29	21.71	0.0199	0.0957	0.8688
512.66	168	7	0.0216	279.82	8.39	21.61	0.0205	0.0924	0.8656
515.35	169	7	0.0215	283.35	8.50	21.50	0.0212	0.0892	0.8624
518.05	170	7	0.0214	286.91	8.61	21.39	0.0218	0.0861	0.8591
520.78	171	7	0.0213	290.50	8.72	21.28	0.0225	0.0830	0.8559
523.52	172	7	0.0212	294.11	8.82	21.18	0.0232	0.0801	0.8525
526.29	173	7	0.0211	297.75	8.93	21.07	0.0239	0.0773	0.8492
529.07	174	7	0.0210	301.40	9.04	20.96	0.0246	0.0745	0.8458
531.86	175	7	0.0208	305.08	9.15	20.85	0.0254	0.0719	0.8423
534.68	176	7	0.0207	308.79	9.26	20.74	0.0261	0.0693	0.8388
537.51	177	7	0.0206	312.52	9.38	20.62	0.0269	0.0668	0.8353
540.36	178	7	0.0205	316.26	9.49	20.51	0.0277	0.0643	0.8318
543.22	179	7	0.0204	320.03	9.60	20.40	0.0285	0.0620	0.8282
546.10	180	7	0.0203	323.82	9.71	20.29	0.0293	0.0597	0.8246
549.00	181	7	0.0202	327.63	9.83	20.17	0.0302	0.0575	0.8210
551.91	182	7	0.0201	331.46	9.94	20.06	0.0310	0.0554	0.8173
554.83	183	7	0.0199	335.31	10.06	19.94	0.0319	0.0534	0.8136
557.77	184	7	0.0198	339.18	10.18	19.82	0.0328	0.0514	0.8098
504.40	184	8	0.0219	268.95	8.07	21.93	0.0124	0.0971	0.8959
560.73	185	7	0.0197	343.06	10.29	19.71	0.0337	0.0495	0.8061
506.68	185	8	0.0218	271.95	8.16	21.84	0.0128	0.0939	0.8933
563.69	186	7	0.0196	346.96	10.41	19.59	0.0346	0.0476	0.8023
508.98	186	8	0.0218	274.97	8.25	21.75	0.0132	0.0908	0.8907
566.67	187	7	0.0195	350.88	10.53	19.47	0.0356	0.0458	0.7984

Table 1: Continued.

TC	n	c	AOQ	ATI	D_d	D_n	$1 - P_a(\text{AQL})$	$P_a(\text{LTPD})$	$P_a(p)$
511.30	187	8	0.0217	278.02	8.34	21.66	0.0136	0.0877	0.8880
569.66	188	7	0.0194	354.82	10.64	19.36	0.0365	0.0441	0.7946
513.63	188	8	0.0216	281.09	8.43	21.57	0.0141	0.0848	0.8854
572.66	189	7	0.0192	358.77	10.76	19.24	0.0375	0.0424	0.7907
515.98	189	8	0.0215	284.19	8.53	21.47	0.0145	0.0819	0.8826
575.68	190	7	0.0191	362.74	10.88	19.12	0.0385	0.0408	0.7867
518.35	190	8	0.0214	287.30	8.62	21.38	0.0150	0.0792	0.8799
578.71	191	7	0.0190	366.72	11.00	19.00	0.0395	0.0392	0.7828
520.74	191	8	0.0213	290.44	8.71	21.29	0.0154	0.0765	0.8771
581.74	192	7	0.0189	370.71	11.12	18.88	0.0405	0.0377	0.7788
523.14	192	8	0.0212	293.61	8.81	21.19	0.0159	0.0739	0.8742
584.79	193	7	0.0188	374.72	11.24	18.76	0.0415	0.0363	0.7748
525.56	193	8	0.0211	296.79	8.90	21.10	0.0164	0.0714	0.8714
587.85	194	7	0.0186	378.74	11.36	18.64	0.0426	0.0349	0.7708
528.00	194	8	0.0210	300.00	9.00	21.00	0.0169	0.0689	0.8685
590.91	195	7	0.0185	382.78	11.48	18.52	0.0437	0.0335	0.7667
530.45	195	8	0.0209	303.23	9.10	20.90	0.0174	0.0665	0.8656
593.99	196	7	0.0184	386.83	11.60	18.40	0.0448	0.0322	0.7627
532.92	196	8	0.0208	306.47	9.19	20.81	0.0180	0.0642	0.8626
597.07	197	7	0.0183	390.88	11.73	18.27	0.0459	0.0309	0.7586
535.41	197	8	0.0207	309.74	9.29	20.71	0.0185	0.0620	0.8596
600.16	198	7	0.0182	394.95	11.85	18.15	0.0470	0.0297	0.7544
537.91	198	8	0.0206	313.04	9.39	20.61	0.0190	0.0598	0.8566
603.26	199	7	0.0180	399.03	11.97	18.03	0.0482	0.0285	0.7503
540.42	199	8	0.0205	316.35	9.49	20.51	0.0196	0.0577	0.8535
606.37	200	7	0.0179	403.12	12.09	17.91	0.0493	0.0274	0.7461
542.95	200	8	0.0204	319.68	9.59	20.41	0.0202	0.0556	0.8504
545.50	201	8	0.0203	323.03	9.69	20.31	0.0208	0.0537	0.8473
503.07	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
548.06	202	8	0.0202	326.40	9.79	20.21	0.0214	0.0518	0.8441
505.03	202	9	0.0219	269.78	8.09	21.91	0.0080	0.0947	0.9151
550.64	203	8	0.0201	329.79	9.89	20.11	0.0220	0.0499	0.8409
507.02	203	9	0.0218	272.39	8.17	21.83	0.0083	0.0917	0.9129
553.23	204	8	0.0200	333.19	10.00	20.00	0.0226	0.0481	0.8377
509.02	204	9	0.0217	275.03	8.25	21.75	0.0085	0.0888	0.9108
555.83	205	8	0.0199	336.62	10.10	19.90	0.0232	0.0464	0.8344
511.04	205	9	0.0217	277.68	8.33	21.67	0.0088	0.0859	0.9086

a defective item which is released to the market. For an incoming raw material, this will be the attendant cost when a defective item is released for production use.

The economic sampling plan can be found through the following mathematical model:

$$\text{Minimize} \quad \text{TC} = C_i \cdot \text{ATI} + C_f \cdot D_d + C_o \cdot D_n, \tag{2.5}$$

$$\text{Subject to} \quad 1 - p_a(\text{AQL}) \leq \alpha, \tag{2.6}$$

$$p_a(\text{LTPD}) \leq \beta. \tag{2.7}$$

Table 2: Optimal single sampling plan as a function of the product quality p (other input parameters are given as the base set).

p	TC	n	c	AOQ	ATI	D_d	D_n	$1-P_a(\text{AQL})$	$P_a(\text{LTPD})$	$P_a(p)$
0.01	222.25	131	5	0.0087	132.88	1.33	8.67	0.0487	0.0974	0.9978
0.02	345.61	131	5	0.0165	173.34	3.47	16.53	0.0487	0.0974	0.9513
0.03	503.07	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
0.04	676.49	268	13	0.0237	406.60	16.26	23.74	0.0012	0.0995	0.8106
0.05	862.78	334	17	0.0198	604.63	30.23	19.77	0.0002	0.0997	0.5937
0.06	1020.20	301	15	0.0115	808.08	48.48	11.52	0.0004	0.1000	0.2746
0.07	1102.75	131	5	0.0059	915.35	64.07	5.93	0.0487	0.0974	0.0974
0.08	1146.08	131	5	0.0031	961.33	76.91	3.09	0.0487	0.0974	0.0445
0.09	1175.40	131	5	0.0015	983.57	88.52	1.48	0.0487	0.0974	0.0189
0.10	1198.69	131	5	0.0007	993.44	99.34	0.66	0.0487	0.0974	0.0075
0.11	1219.70	131	5	0.0003	997.52	109.73	0.27	0.0487	0.0974	0.0029
0.12	1239.96	131	5	0.0001	999.11	119.89	0.11	0.0487	0.0974	0.0010
0.13	1260.00	228	8	0.0000	1000.00	130.00	0.00	0.0414	0.0191	0.0000
0.14	1280.00	218	8	0.0000	1000.00	140.00	0.00	0.0326	0.0284	0.0000
0.15	1300.00	198	7	0.0000	1000.00	150.00	0.00	0.0470	0.0297	0.0000
0.16	1320.00	183	7	0.0000	1000.00	160.00	0.00	0.0319	0.0534	0.0000
0.17	1340.00	193	7	0.0000	1000.00	170.00	0.00	0.0415	0.0363	0.0000
0.18	1360.00	187	7	0.0000	1000.00	180.00	0.00	0.0356	0.0458	0.0000
0.19	1380.00	152	6	0.0000	1000.00	190.00	0.00	0.0340	0.0867	0.0000
0.20	1400.00	131	5	0.0000	1000.00	200.00	0.00	0.0487	0.0974	0.0000

Note that for the cases of the export of the seedlings of Phalaenopsis from Taiwan, if the defective products are scrapped at the seaport, the internal failure cost would be the lost profit (i.e., revenue—production cost—transportation cost (from Taiwan to the USA)). If the defective products are returned to Taiwan, the internal failure cost would be the lost profit plus the transportation cost (from the USA to Taiwan) subtract the salvage value when the defective products arrive in Taiwan. If the defective products are sold at a reduced price, the internal failure cost would be calculated as follows: revenue (the original selling price)—the production cost—transportation cost (from Taiwan to The Netherlands)—the reduced selling price.

Note that if the cost of an outgoing defective item C_o is relatively high in comparison to the inspection cost per item C_i and the internal failure cost per item C_f, then the optimal sampling plan is to have a 100% inspection of the entire lot. If C_o is high, then in order to minimize the total cost TC, the defective items not detected D_n should be as small as possible. Since $D_n = p_a(p)(N-n)\ p$, if the sample size n equals the lot size N (100% inspection), then $D_n = 0$. On the contrary, if the inspection cost per item C_i is relatively high in comparison to the internal failure cost per item C_f and the cost of an outgoing defective item C_o, then the optimal sampling plan is to have zero inspection without take into consideration the producer's and the consumer's risk requirements. However, with zero inspection, the consumer's risk would be high and may not be acceptable to the consumer.

Table 3: Optimal single sampling plan as a function of the inspection cost C_i (other input parameters are given as the base set).

C_i	TC	n	c	AOQ	ATI	D_d	D_n	$1 - P_a(\text{AQL})$	$P_a(\text{LTPD})$	$P_a(p)$
0.1	160.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
0.2	260.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
0.3	316.03	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
0.4	342.75	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
0.5	369.47	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
1.0	503.07	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
1.5	636.66	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
2.0	770.26	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
2.5	903.85	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
3.0	1037.45	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
3.5	1171.05	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
4.0	1304.64	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
4.5	1438.24	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
5.0	1571.84	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
5.5	1705.43	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
6.0	1839.03	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
6.5	1972.62	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
7.0	2106.22	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
7.5	2239.82	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
8.0	2373.41	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
8.5	2507.01	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
9.0	2640.60	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
9.5	2774.20	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
10.0	2907.80	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172

3. Numerical Examples and Discussion

For the purpose of our illustration, we consider the following set of input parameters: $N = 1{,}000$, AQL $= 0.02$, LTPD $= 0.07$, $\alpha = 0.05$, $\beta = 0.10$, $p = 0.03$, $C_i = 1.0$, $C_f = 2.0$, and $C_o = 10$. We use MATLAB computer software to obtain all the single sampling plans with sample size n less than or equal to 1000 that satisfy both the producer's and consumer's quality and risk requirements. To indicate the performance measurements, Table 1 lists all the single sampling plans for n up to 205. From Table 1, one can see that both the producer's risk $(1-p_a(\text{AQL}))$ and average total inspection (ATI) increase, and the consumer's risk $p_a(\text{LTPD})$ decreases as n increases and c remains unchanged; on the contrary, both the producer's risk and average total inspection decrease, and the consumer's risk increases as c increases and n remains unchanged. Based on the previous input parameters, the optimal sampling plan is $n = 201$ and $c = 9$ with the total cost TC $= 503.07$. Note that without constraints (2.6) and (2.7), the optimal decision for the producer is to have zero inspection ($n = 0$) with the total cost TC $= 300$, the producer's risk $\alpha = 0$, and the consumer's risk $\beta = 1$, which obviously is not acceptable to the consumer. This example indicates that without integrating the producer's and the consumer's risk requirements into the economic design of the acceptance sampling plans, the plan obtained by the model, although minimizing the producer's and the consumer's total quality cost, may not be acceptable to the consumer.

Table 4

(a) Optimal single sampling plan as a function of the internal failure cost C_f (other input parameters are given as the base set).

C_f	TC	n	c	AOQ	ATI	D_d	D_n	$1-P_a(\text{AQL})$	$P_a(\text{LTPD})$	$P_a(p)$
0.0	487.03	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
0.5	491.04	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
1.0	495.05	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
1.5	499.06	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
2.0	503.07	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
2.5	507.07	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
3.0	511.08	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
3.5	515.09	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
4.0	519.10	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
4.5	523.11	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
5.0	527.11	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
5.5	531.12	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
6.0	535.13	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
6.5	539.14	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
7.0	543.14	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
7.5	547.15	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
8.0	551.16	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
8.5	555.17	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
9.0	559.18	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
9.5	563.18	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
10.0	567.19	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172

(b) Optimal single sampling plan as a function of the internal failure cost C_f (with $C_i = 0.2$ and other input parameters are given as the base set).

C_f	TC	n	c	AOQ	ATI	D_d	D_n	$1-P_a(\text{AQL})$	$P_a(\text{LTPD})$	$P_a(p)$
0.0	200.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
0.5	215.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
1.0	230.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
1.5	245.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
2.0	260.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
2.5	275.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
3.0	290.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
3.5	301.34	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
4.0	305.34	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
4.5	309.35	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
5.0	313.36	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
5.5	317.37	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
6.0	321.38	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
6.5	325.38	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
7.0	329.39	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
7.5	333.40	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
8.0	337.41	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
8.5	341.41	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
9.0	345.42	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
9.5	349.43	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
10.0	353.44	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172

Table 5: Optimal single sampling plan as a function of the post sale failure cost C_o (other input parameters are given as the base set).

C_o	TC	n	c	AOQ	ATI	D_d	D_n	$1-P_a$(AQL)	P_a(LTPD)	$P_a(p)$
5	393.14	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
10	503.07	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
15	612.99	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
20	722.91	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
25	832.83	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
30	942.75	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
35	1052.67	201	9	0.0220	267.19	8.02	21.98	0.0077	0.0978	0.9172
40	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
45	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
50	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
55	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
60	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
65	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
70	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
75	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
80	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
85	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
90	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
95	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009
100	1060.00	1000	28	0.0000	1000.00	30.00	0.00	0.0329	0.0000	0.4009

Figure 2 shows the total cost with different sampling plans that satisfy both the producer's and consumer's quality and risk requirements (i.e., AQL $=$ 0.02, LTPD $=$ 0.07, $\alpha = 0.05$, $\beta = 0.10$). For a given c value, as n increases, the total cost increases. However, when n increases or decreases to a certain value, the sampling plan becomes infeasible (i.e., the consumer's or the producer's risk becomes too big).

Table 2 shows the sensitivity analyses of the optimal single sampling with different levels of p. As p increases, the optimal sample size first increases and then decreases. For $p \geq 0.13$, the optimal sampling plan will have a near zero probability of accepting the lot, resulting in a 100% inspection of the entire lot. As a result, all the defective products will be detected and replaced (ATI = 1000 and AOQ = 0).

Table 3 shows the sensitivity analysis of the inspection cost C_i. If $C_i \leq 0.2$, the inspection cost is relatively low compared to the failure costs (C_f and C_o). Therefore, the optimal sampling plan is to have a 100% inspection of the entire lot. For $0.3 \leq C_i \leq 10$, the optimal sampling plans remain at $n = 201$ and $c = 9$.

As shown in Table 4(a), one can see that the internal failure cost C_f is relatively insensitive to the optimal sampling plan. However, when the inspection cost C_i is small, for example, $C_i = 0.2$ (see Table 4(b)), the internal failure cost C_f has an effect on the optimal sampling plan.

Table 5 shows the sensitivity analysis of the postsale failure cost C_o. For $C_o \leq 35$, the optimal sampling plans remain to be $n = 201$ and $c = 9$. However, when $C_o \geq 40$, the optimal sampling plan changes to have a 100% inspection of the entire lot.

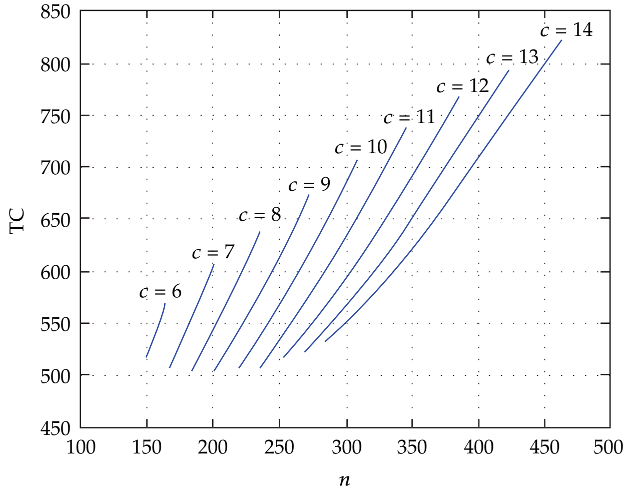

Figure 2: Total cost (TC) versus sample size (n) at different c when $C_i = 1$, $C_f = 2$, and $C_o = 10$.

4. Conclusions

There are many ways to determine an acceptance sampling plan. However, all of them are either settled on a noneconomic basis or did not take into consideration the producer's and consumer's quality and risk requirements. In this paper, we developed a mathematical model for a two-stage supply chain that can help the producer and the consumer to find a single sampling plan that minimizes the producer's and the consumer's total quality cost (inspection and failure costs) and satisfies both the producer's and consumer's quality and risk requirements. From the numerical analyses, we see that the optimal sampling plan is very sensitive to the producer's product quality. The product inspection, internal failure, and postsale failure costs also have an effect on the choice of the economic sampling plan. The results presented in this paper can be further extended to develop models for double or multiple sampling plans. The mathematical model and computer program for determining an optimal double or multiple sampling plans are more complicated. The research work is now being undertaken.

References

[1] O. Kabak and F. Ülengin, "Possibilistic linear-programming approach for supply chain networking decisions," *European Journal of Operational Research*, vol. 209, no. 3, pp. 253–264, 2011.

[2] M. Cao and Q. Zhang, "Supply chain collaborative advantage: a firm's perspective," *International Journal of Production Economics*, vol. 128, no. 1, pp. 358–367, 2010.

[3] S. T. Foster Jr., "Towards an understanding of supply chain quality management," *Journal of Operations Management*, vol. 26, no. 4, pp. 461–467, 2008.

[4] ANSI/ASQ Z1.4 and Z1.9-2008, *Sampling Procedures and Package*, American National Standard Institute, 2008.

[5] Military Standard (MIL-STD-105E), *Sampling Procedures and Tables for Inspection by Attributes*, U.S. Department of Defense, Washington, DC, USA, 1989.

[6] H. F. Dodge and H. G. Romig, *Sampling Inspection Tables*, John Wiley & Sons, New York, NY, USA, 2nd edition, 1998.

[7] *Sampling Plan Analyzer 2.0*, Taylor Enterprises, Libertyville, Ill, USA, 2010.

[8] *Acceptance Sampling for Attributesm TP105*, H & H Servicco, North St. Paul, Minn, USA, 2010.

[9] International Organization for Standardization. Sampling Procedures and Tables for Inspection by Attributes, ISO 2859, Geneva, Switzerland, 1974.
[10] Y. Nikolaidis and G. Nenes, "Economic evaluation of ISO 2859 acceptance sampling plans used with rectifying inspection of rejected lots," *Quality Engineering*, vol. 21, no. 1, pp. 10–23, 2009.
[11] G. B. Wetherill and W. K. Chiu, "A review of acceptance sampling schemes with emphasis on the economic aspect," *International Statistical Review*, vol. 43, no. 2, pp. 191–210, 1975.
[12] G. Tagaras, "Economic acceptance sampling plans by variables with quadratic quality cost," *IIE Transactions*, vol. 26, no. 6, pp. 29–36, 1994.
[13] W. G. Ferrell Jr. and A. Chhoker, "Design of economically optimal acceptance sampling plans with inspection error," *Computers and Operations Research*, vol. 29, no. 10, pp. 1283–1300, 2002.
[14] C. González and G. Palomo, "Bayesian acceptance sampling plans following economic criteria: an application to paper pulp manufacturing," *Journal of Applied Statistics*, vol. 30, no. 3, pp. 319–333, 2003.
[15] D. C. Montgomery, *Introduction to Statistical Quality Control*, John Wiley & Sons, New York, NY, USA, 4th edition, 2000.
[16] L. F. Hsu and C. S. Tapiero, "An economic model for determining the optimal quality and process control policy in a queue-like production system," *International Journal of Production Research*, vol. 28, no. 8, pp. 1447–1457, 1990.


Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
Algebra
Journal of Probability and Statistics
The Scientific World Journal
International Journal of Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
Journal of Optimization