PAPER • OPEN ACCESS

Tests of Lorentz and CPT symmetry with hadrons and nuclei

View the article online for updates and enhancements.

Tests of Lorentz and CPT symmetry with hadrons and nuclei

J P Noordmans[1]

[1] CENTRA, Departamento de Física, Universidade do Algarve, 8005-139 Faro, Portugal

E-mail: `jnoordmans@ualg.pt`

Abstract. We apply chiral-perturbation-theory techniques to the QCD sector of the Lorentz and CPT violating standard-model extension. We derive the effective Lagrangian in terms of pions and nucleons for a selected set of dimension-five operators involving quarks and gluons. This derivation is based on chiral-symmetry properties of the operators, as well as on their behaviour under C,P, and T transformations. We consider the power counting rules and apply the heavy-baryon approach to account for the large nucleon mass. Having obtained the relevant Lorentz-violating contributions to the pion-nucleon Lagrangian, we proceed to derive the particle and anti-particle Hamiltonian, from which we obtain the Lorentz-violating contribution to comagnetometer experiments. This allows us to place stringent limits on some of the parameters. For some other parameters we find that the best bounds will come from nucleon-nucleon interactions, and we derive the relevant nucleon-nucleon potential. These considerations imply possible new opportunities for spin-precession experiments involving for example the deuteron.

1. Introduction

The unification of general relativity with quantum mechanics is one of the main outstanding problems in theoretical physics today. Many different solutions to this problem have been proposed. However, none of these have proven to be the definitive answer yet. One of the main difficulties is the energy scale at which the relevant theories become distinguishable from what should be their low-energy limits: general relativity and the standard model of particle physics. The generally accepted paradigm is that these theories should become unified around an energy scale of about the Planck mass: $m_P \simeq 10^{19}$ GeV. The relevant deviations from the conventional physics should thus become apparent at this energy scale, which is far out of reach of any present-day experiments.

One possible solution to this problem can be found in the study of violations of spacetime symmetries. Many theories that attempt to unify the standard model of particle physics with general relativity modify the notion of spacetime at Planck-scale energies. Consequently, the fate of spacetime symmetries such as Lorentz and CPT symmetry becomes an open issue. Tests of these symmetries might therefore provide valuable insights about a possible theory of quantum gravity. Moreover, it turns out that very sensitive experimental tests are possible for signals that are characteristic for the breaking of Lorentz and/or CPT symmetry. Therefore, tiny remnants of such high-energy symmetry breakings might be detectable at presently attainable energies. This high sensitivity, combined with the fact that Lorentz- and CPT-symmetry-violating signals

(or their absence) might guide us in the quest for a theory of quantum gravity, has motivated a large amount of research in this area over the last two decades.

This research is facilitated by an effective-field-theory approach to Lorentz and CPT violation, called the standard-model extension (SME) [1], which we will discuss in more detail in the next section. One of the merits of the SME is its generality. It includes all Lorentz-violating effects that can be incorporated into a local quantum field theory. Additionally, it allows for a classification of the different Lorentz-violating operators and for quantitative bounds on the corresponding parameters for Lorentz violation. Therefore, one can compare different (potential) experiments and their sensitivity and identify possible sectors in the parameter space for Lorentz violation that might have been tested relatively poorly.

One of the sectors where limits are relatively weak, is the QCD sector of the SME [2]. The reason for this is obvious: the nonperturbative nature of QCD at energies below $1\,\mathrm{GeV}$. Although very precise bounds on effective Lorentz-violating coefficients for protons, neutrons, and other hadrons exist, the corresponding limits on the underlying quark and gluon parameters are hard to obtain. To remedy this situation, we recently started to apply the machinery of chiral perturbation theory to the QCD sector of the SME [3]. Further work along the same lines can be found in Refs. [4, 5]. Using the effective operators in terms of the relevant degrees of freedom at low energies – the pion and nucleon fields – it becomes possible to put stringent constraints on quark and gluon parameters for Lorentz violation.

2. A Lorentz noninvariant effective field theory

It is not hard to argue that the best way to approach the problem of investigating Lorentz violation in theoretical particle physics today, is by using an effective-field-theory approach. A general construction of such an effective field theory was described in Ref. [1]. Similar, albeit more restricted approaches, were taken in Refs. [6, 7]. The resulting framework is called the standard-model extension (SME) and it has been succesfully used many times to describe and parametrize Lorentz- and CPT-violating effects, as well as to obtain quantitative bounds on many of its coefficients for Lorentz- and CPT violation. In Ref. [2] a complete and up-to-date overview of the experimental limits can be found.

Although it is possible to conceive of other methods to describe Lorentz- and CPT-symmetry breaking, the SME has several important advantages [8]. First of all, it is general, in the sense that all possible Lorentz-violating effects are included in the theory, as long as these effects can be described in the context of a local quantum field theory. Secondly, we can describe the SME as being realistic, since it is a perturbation of the conventional standard model of particle physics and thus includes all the known and well-established particle physics. Finally, the SME is coordinate invariant, as it should be, since we do not want the physics to depend on the coordinates we use.

The operators in the Lagrangian of the SME are build from the conventional standard-model quantum fields that represent all the known elementary particles and transform under the Lorentz group in the conventional way. The fields continue to be representations of the standard-model gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ and the Lagrangian is invariant under the corresponding gauge transformations. The only restrictions on the form of the operators that are relaxed, is those of Lorentz and CPT symmetry. Operationally this means that Lorentz indices are no longer contracted to each other, but instead can be contracted to coefficients that parametrize the Lorentz and CPT violation. The value of these coefficients presumably originates from some unknown high-energy theory that incorporates (spontaneous) Lorentz-symmetry breaking.

A typical example of an SME operator is

$$\mathcal{L}^{\mathrm{LV}} \supset \langle T \rangle_{\mu\nu}\, \bar{\psi}\gamma^\mu\partial^\nu\psi \,, \tag{1}$$

where $\langle T \rangle_{\mu\nu}$ is the Lorentz-violating coefficient. The Lorentz-violating coefficients are generally taken to have fixed background values and do not transform under so-called 'particle Lorentz transformations'. These are transformations that effect only the quantities associated to the particles, like their momentum and spin, i.e. they are transformations of the particle fields only. Particle Lorentz transformations can be viewed as corresponding to physical rearrangements within the system one is trying to describe. They should be contrasted with 'observer Lorentz transformations', which affect both the particle fields and the Lorentz-violating coefficients. These transformations are just redefinitions of the coordinate system and as such should not influence the physics. Therefore, all SME operators are invariant under observer Lorentz transformations, as clearly exemplified by Eq. (1).

The Lagrangian of the SME in principle contains all local and gauge-invariant Lorentz-violating operators. This is an infinite set of operators, starting at a mass dimensionality three. On phenomenological grounds the coefficients of the operators are assumed to be suppressed by one or more powers of a large mass scale that corresponds to the scale of Lorentz-symmetry breaking (in many cases this scale is taken to be the Planck mass). This leads to a slight conundrum for the minimal standard-model extension (mSME), which contains only operators with a mass dimension of three or four. Naturalness arguments suggest that the coefficients of these operators are proportional the large mass scale or to unity, respectively. This obviously is in sharp contrast with existing experimental limits. One way out of this is to conjecture the existence of some additional symmetry that forbids the offensive operators. This symmetry might be broken at some intermediate energy scale M_{IR}, making low-energy values of the coefficients proportional to powers of M_{IR}/M_P. For example, it seems that in a minimal supersymmetric model the lowest dimension for Lorentz-violating operators is five [9], such that at low energy the symmetry-breaking operators are suppressed by at least one power of the high-energy scale.

Although the SME in principle contains all possible operators, the explicit identification and classification of all the operators has only been performed up to mass dimension five. The power-counting renormalizable operators of dimension three and four are described in Ref. [1], while the authors of Ref. [10] identify all mass-dimension five operators that are subject to certain 'UV-safety constraints', which prevent these operators from mixing with the lower-dimensional operators. For QED, all operators have been identified up to dimension six [11], while for kinetic operators the program of classifying all operators of arbitrary dimensions has been finished for photons, Dirac fermions, and neutrinos [12].

As mentioned in the introduction, in particular the QCD sector of the SME is poorly constrained, due to the nonperturbative nature of QCD. In Ref. [3] we started to remedy this situation, by applying chiral perturbation theory techniques to the Lorentz-violating operators (see Refs. [4] and [5] for more work in this direction). In Ref. [3] we focused on a set of dimension five quark-gluon operators, which are relevant at an energy scale of about 1 GeV. They were first written down in Ref. [10] and are given by

$$\mathcal{L}^{\text{LV}} = \sum_{q=u,d} \left[C^q_{\mu\nu\rho} \bar{q}\gamma^\mu G^{\rho\nu} q + D^q_{\mu\nu\rho} \bar{q}\gamma^\mu\gamma^5 G^{\rho\nu} q \right] + H_{\mu\nu\rho} \text{Tr}\left(G^{\mu\lambda} D^\nu \tilde{G}^\rho{}_\lambda \right) . \qquad (2)$$

The sum in this equation runs over the lightest two quark flavors, the fields q are the quark fields and $G^{\mu\nu} = t^a G^{a,\mu\nu}$ ($t^a = \frac{1}{2}\lambda^a$, $a = 1,\ldots 8$, where λ^a are the Gell-Mann matrices, are the generators of the $SU(3)$ color group). All three operators violate Lorentz symmetry as well as CPT symmetry. This symmetry breaking is parametrized by the coefficients $C^q_{\mu\nu\rho}$, $D^q_{\mu\nu\rho}$, and $H_{\mu\nu\rho}$. In addition to being antisymmetric in their last two Lorentz indices, the $C^{\mu\nu\rho}$ and $D^{\mu\nu\rho}$ coefficients are symmetric in their first and last Lorentz indices, while $H^{\mu\nu\rho}$ is fully symmetric,

i.e.

$$X^q_{\mu\nu\rho} = X^q_{\rho\nu\mu} = -X^q_{\mu\rho\nu} \; , \tag{3}$$

$$H_{\mu\nu\rho} = H_{\mu\rho\nu} = H_{\nu\mu\rho} \; , \tag{4}$$

with $X \in \{C, D\}$. These symmetry constraints follow from the UV-safety considerations, which are discussed in Ref. [10]. The properties of the coefficients under the action of C, P, and T operations are summarized in table 1.

There are many more operators that can be considered in the context of chiral perturbation theory, however these are left for future considerations (see for example Refs. [4] and [5]). Also, we consider only the lightest two quark flavors. In principle the strange quark can be included and our work can be extended to $SU(3)$ chiral perturbation theory (e.g. see Ref. [5]).

3. The Lorentz-violating chiral-perturbation-theory Lagrangian

At energies below $1\,\mathrm{GeV}$, the operators in Eq. (2) induce interactions between the relevant degrees of freedom, i.e. the pions, nucleons, and photons. Chiral perturbation theory [13, 14] (see e.g. Refs. [15, 16] for reviews) can be used to derive the form of the operators that describe these interactions. In Ref. [3] we constructed the low-energy effective Lagrangian that corresponds to the Lorentz-violating operators in Eq. (2) in the $SO(4)$ approach to chiral perturbation theory. This approach is based on the observation that the massless (and Lorentz-symmetric) two-flavor QCD Lagrangian possesses an $SU(2)_L \times SU(2)_R \sim SO(4)$ symmetry under chiral transformations of the quark fields. Moreover, the structure of the baryon and light-meson spectrum implies that the axial part of this symmetry is spontaneously broken, down to its $SO(3)$ isospin subgroup. This happens at the chiral breaking scale: $\Lambda_\chi \simeq 1$ GeV. The pseudoscalar Goldstone bosons associated with this spontaneous symmetry breaking are identified with the pions. The small nonzero mass of the pions originates from the fact that chiral symmetry is not an exact symmetry of the QCD Lagrangian, due to the (small) quark masses.

To be able to construct a Lagrangian which has the correct symmetry properties, one constructs chiral- and gauge-covariant derivatives of the pion and nucleon fields. The covariant derivative for the pion field is given by

$$(D_\mu \boldsymbol{\pi})_a = D^{-1}(\partial_\mu \delta_{ab} + eA_\mu \epsilon_{3ab})\pi_b \; , \tag{5}$$

where $\boldsymbol{\pi}$ is the pion triplet, A_μ is the photon field, $e > 0$ is the proton charge, $D = 1 + \boldsymbol{\pi}^2/F_\pi^2$, $F_\pi \simeq \Lambda_\chi/(2\pi) \simeq 185$ GeV is the pion decay constant, and a, b are isospin indices. For the nucleon field the covariant derivative is

$$\mathcal{D}_\mu N = \left(\partial_\mu + \frac{i}{F_\pi^2} \boldsymbol{\tau} \cdot \boldsymbol{\pi} \times D_\mu \boldsymbol{\pi} + \frac{ie}{2} A_\mu (1 + \tau_3) \right) N \; , \tag{6}$$

where $N = (p, n)^T$ is the nucleon doublet and $\boldsymbol{\tau}$ are the Pauli isospin matrices. In the chiral invariant Lagrangian that can now be constructed, pion fields can only appear with one or more derivatives acting on them. Consequently all such operators can be ordered in terms of powers of the small quantity p/Λ_χ, where $p \ll \Lambda_\chi$ is a momentum scale of the order of the pion mass. For the nucleon field on the other hand, the derivative does not give rise to a small quantity. In fact, a time derivative acting on the nucleon field is of order $m_N/\Lambda_\chi = \mathcal{O}(1)$. A way to deal with this is described in Section 4. For now we just consider only the operators with the smallest number of derivatives on either pion or nucleon fields and order all interactions by their chiral index, defined by

$$\Delta = d + f/2 - 2 \; , \tag{7}$$

where d is the number of (covariant) derivatives and f is the number of nucleon fields.

Table 1. The sign that follows from the action of C, P, or T on the Lorentz-violating operators in Eq. (2). As usual $(-1)^\mu = 1$ if $\mu = 0$ and $(-1)^\mu = -1$ otherwise.

	$C_{\mu\nu\rho}$	$D_{\mu\nu\rho}$	$H_{\mu\nu\rho}$
P	$(-1)^\mu(-1)^\nu(-1)^\rho$	$-(-1)^\mu(-1)^\nu(-1)^\rho$	$-(-1)^\mu(-1)^\nu(-1)^\rho$
T	$-(-1)^\mu(-1)^\nu(-1)^\rho$	$-(-1)^\mu(-1)^\nu(-1)^\rho$	$(-1)^\mu(-1)^\nu(-1)^\rho$
C	$+1$	-1	$+1$

Operators that break chiral symmetry can be incorporated in the formalism as well. The way the operators, relevant above 1 GeV, transform under chiral transformations, determines the form of the chiral-breaking operators in the effective Lagrangian below 1 GeV. These operators will generally contain pion fields without derivatives, but are proportional to some small symmetry-breaking parameter. The most important example of this is the breaking of chiral symmetry by the quark masses, which induces the pion mass at low energy.

In a similar fashion we constructed the chiral effective operators corresponding to the Lorentz-violating dimension five quark-gluon operators, given in Eq. (2). The first two operators in Eq. (2) can be split into a chiral invariant part, proportional to $X^+_{\mu\nu\rho} = X^u_{\mu\nu\rho} + X^d_{\mu\nu\rho}$, with $X \in \{C, D\}$, and a chiral noninvariant part, proportional to $X^-_{\mu\nu\rho} = X^u_{\mu\nu\rho} - X^d_{\mu\nu\rho}$. The final operator in Eq. (2) is trivially invariant under chiral transformations, since it does not contain any quark fields.

Using the chiral properties of the operators in Eq. (2), together with their C, P, and T characteristics, given in table 1, it is possible to construct the low-energy effective operators in terms of the relevant degrees of freedom, i.e. the pions and the nucleons (see Ref. [3] for details). The set of lowest-order relativistic operators that is non-redundant under the equations of motion, is given by

$$
\begin{aligned}
\mathcal{L}^{\text{LV}}_\chi &= \frac{i}{m_N} \bar{N} \left(\tilde{C}^+_{\mu\nu\rho} + \tilde{C}^-_{\mu\nu\rho} \left[\tau_3 - \frac{2}{F_\pi^2 D} \left(\boldsymbol{\pi}^2 \tau_3 - \pi_3 \boldsymbol{\tau} \cdot \boldsymbol{\pi} \right) \right] \right) \sigma^{\nu\rho} \mathcal{D}^\mu N \\
&+ \frac{1}{m_N^2} \tilde{H}_{\mu\nu\rho} \bar{N} \gamma^\mu \gamma^5 \mathcal{D}^\nu \mathcal{D}^\rho N + \frac{i}{m_N F_\pi D} \tilde{D}^-_{\mu\nu\rho} \bar{N} (\boldsymbol{\tau} \times \boldsymbol{\pi})_3 \sigma^{\nu\rho} \mathcal{D}^\mu N + \\
&+ \frac{1}{m_N^2 F_\pi} \breve{D}^+_{\mu\nu\rho\alpha\beta} \bar{N} (\boldsymbol{\tau} \cdot D^\mu \boldsymbol{\pi}) \sigma^{\nu\rho} \mathcal{D}^\alpha \mathcal{D}^\beta N + \text{H.c.} ,
\end{aligned}
\tag{8}
$$

where we included a factor of $1/m_N$ for each covariant nucleon derivative, to keep the time derivatives from spuriously lowering the chiral index of the operators. The parameter $\breve{D}^+$ is given by

$$
\breve{D}^+_{\mu\rho\nu\alpha\beta} = \tilde{D}^{+,1}_{\mu\rho\nu} \eta_{\alpha\beta} + \tilde{D}^{+,2}_{\alpha\nu\rho} \eta_{\mu\beta} + \tilde{D}^{+,3}_{\alpha[\beta\rho]} \eta_{\mu\nu} ,
\tag{9}
$$

where square brackets mean antisymmetrization over the enclosed indices. In Eqs. (8) and (9), the Lorentz-violating coefficients at the hadronic level are denoted by a tilde. This represents the fact that they are proportional to the Lorentz-violating coefficients in Eq. (2), but additionally contain an unknown low-energy constant, i.e. $\tilde{C}^\pm_{\mu\nu\rho} = c^\pm C^\pm_{\mu\nu\rho}$, $\tilde{D}^-_{\mu\nu\rho} = d^- D^-_{\mu\nu\rho}$, $\tilde{D}^{+,i}_{\mu\nu\rho} = d_i^+ D^+_{\mu\nu\rho}$, and $\tilde{H}^{\mu\nu\rho} = h H^{\mu\nu\rho}$, with $c^\pm$, d^-, d_i^+, and h the low-energy constants. The value of these low-energy constants cannot be determined using symmetry arguments, although chiral symmetry does imply that $d^- = 2c^-$. In principle the values of the low-energy constants can be calculated using lattice QCD, however for the Lorentz-violating operators under consideration this has not been done. Therefore we use naive dimensional analysis (NDA) [17] to get an estimate

of the value of the low-energy constants at the order-of-magnitude level. NDA gives that $c^{\pm}, d^{-} = \mathcal{O}(\Lambda_{\chi} F_{\pi})$, $d_i^{+} = \mathcal{O}(F_{\pi})$, and $h = \mathcal{O}(\Lambda_{\chi}^2)$. The operators in Eq. (8) also induce pure-pion operators, however these are all of higher order. Moreover, one can write down additional nucleon-pion operators, that have the correct symmetry properties. However, it can be shown that these are redundant [3].

As expected, we see that the chiral invariant operators (coupled to $C_{\mu\nu\rho}^{+}$ and $H_{\mu\nu\rho}$) only induce nucleon operators without pion fields at lowest order. The chiral noninvariant operators induce kinetic nucleon terms, as well as pion-nucleon interaction terms, which are related by chiral symmetry. Also, in contrast to the $C_{\mu\nu\rho}^{\pm}$ and $H_{\mu\nu\rho}$ parameters, the $D_{\mu\nu\rho}^{\pm}$ coefficient does not introduce any contribution to the nucleon two-point function. This is important, since it turns out that the strongest constraints come from properties of the free nucleon field. Consequently, the bounds on $D_{\mu\nu\rho}^{\pm}$ will be much weaker than those on $C_{\mu\nu\rho}^{\pm}$ and $H_{\mu\nu\rho}$.

4. Heavy baryon chiral perturbation theory

Since an effective field theory in principle contains an infinite number of terms, to be useful there has to be some way to order the effective operators, based on the expected size of their contribution to observables. As mentioned, this ordering is performed based on the chiral index of the operators, defined in Eq. (7). The estimated size of Feynman diagrams is then determined by their chiral order, which, for diagrams with only one baryon in the initial and final state, is given by

$$\nu = 2N_L + 1 + \sum_i \Delta_i \ , \tag{10}$$

where N_L is the number of independent loops and the sum runs over all the interactions that contribute to the relevant diagram. As mentioned, this is based on the assumption that all energy scales in the problem are small compared to the chiral breaking scale, i.e. that we can use the expansion parameter p/Λ_{χ}. However, when doing loop calculations involving nucleons, the nucleon mass constitutes an extra scale in the problem, which is not small, in fact $m_N/\Lambda_{\chi} = \mathcal{O}(1)$. Loop calculations performed with dimensional regularization receive contributions from loop momenta of order m_N. This upsets the assumed power counting. This holds for the Lorentz-violating effective operators in Eq. (8) as well. We therefore apply the framework of heavy-baryon chiral perturbation theory [18]. It consists of introducing heavy-nucleon fields with a fixed velocity v, given by

$$N_v = \frac{1 + \slashed{v}}{2} e^{im_N v^{\mu} x_{\mu}} N \ , \tag{11}$$

where $p^{\mu} = m_N v^{\mu} + k^{\mu}$, with k^{μ} a small residual momentum. Derivatives acting on the heavy-nucleon field will now be proportional to this small residual momentum. Moreover, the propagator of the heavy-nucleon field will no longer contain the nucleon mass, such that the results of loop integrals scale with powers of p/Λ_{χ}. Conventionally, in heavy-baryon chiral perturbation theory, the operators are written in terms of the nucleon velocity v^{μ} and the covariant spin vector S^{μ}, with $S^{\mu} = \frac{i}{2}\gamma^5 \sigma^{\mu\nu} v_{\nu}$. In the heavy-baryon formalism the Lagrangian in Eq. (8) becomes

$$
\begin{aligned}
\mathcal{L}_{\chi}^{\mathrm{HB}} &= 4\left(\epsilon^{\mu\nu\alpha\beta}\tilde{C}_{\rho\alpha\beta}^{+} - \tilde{H}^{\mu\nu}{}_{\rho}\right)v^{\rho}v_{\nu}\bar{N}S_{\mu}N \\
&\quad + 4\epsilon^{\mu\nu\alpha\beta}\tilde{C}_{\rho\alpha\beta}^{-}v_{\nu}v^{\rho}\bar{N}\left[\tau_3 - \frac{2}{F_{\pi}^2 D}\left(\boldsymbol{\pi}^2\tau_3 - \pi_3\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right)\right]S_{\mu}N \\
&\quad + \frac{4}{F_{\pi}D}\epsilon^{\mu\nu\alpha\beta}\tilde{D}_{\rho\alpha\beta}^{-}v^{\rho}v_{\nu}\bar{N}(\boldsymbol{\tau}\times\boldsymbol{\pi})_3 S_{\mu}N + 4\check{D}_{\mu\nu\rho\alpha\beta}^{+}\epsilon^{\nu\rho\lambda\kappa}v_{\lambda}v^{\alpha}v^{\beta}\bar{N}(\boldsymbol{\tau}\cdot D^{\mu}\boldsymbol{\pi})S_{\kappa}N \quad (12)
\end{aligned}
$$

Notice that $\tilde{C}_{\mu\nu\rho}^{+}$ and $\tilde{H}_{\mu\nu\rho}$ are coupled to an identical operator. However, in principle they can still be experimentally distinguished by using the symmetry properties of their Lorentz indices.

5. Particle and antiparticle Hamiltonian

As we mentioned before, the best constraints on the present set of Lorentz-violating coefficients come from kinetic nucleon terms. They are set by clock-comparison experiments. To analyse the effect of nucleon Lorentz violation on such experiments it is convenient to have a block-diagonalized form of the relevant Hamiltonian. Such a form can be obtained by performing a Foldy-Wouthuysen transformation on the nucleon fields. When the Hamiltonian is in the block-diagonalized form, the equations for the particle and the anti-particle become decoupled.

A comparable decoupling is achieved in the heavy-baryon approach, discussed in the previous section. In that approach a nonrelativistic expansion in p/m_N is employed. Using a Foldy-Wouthuysen transformation, in some cases the diagonalization can be achieved exactly, however in many cases the off-diagonal parts are of some higher order in a small quantity, which can also be p/m_N.

For the kinetic nucleon operators in Eq. (8) one can perform a Foldy-Wouthuysen transformation such that the off-diagonal parts of the Hamiltonian are of higher order in the Lorentz-violating coefficients and in the electromagnetic fields. These higher-order terms can safely be ignored and we obtain a relativistic expression for the block-diagonal Hamiltonian. One subtlety involves the higher-order time derivatives in the operators in Eq. (8). To deal with this complication, we use the approach of Ref. [12], i.e. we first diagonalize the Hamiltonian and then substitute $i\partial^0 \rightarrow \sqrt{\vec{p}^2 + m_N^2}$ for the fermion and $i\partial^0 \rightarrow -\sqrt{\vec{p}^2 + m_N^2}$ for the antifermion. Any terms that we miss in this way are of higher order in the Lorentz-violating coefficients.

For a particle of species w, with $w \in \{p, n\}$, the resulting Hamiltonian has a 2×2 upper left block $h_{w,+}$, which we associate with the particle, and a 2×2 lower right block $h_{w,-}$, pertaining to the antiparticle. These particle/antiparticle Hamiltonians can be written as

$$h_{w,\pm} = h_{w,0} \pm \delta h_w \ , \tag{13}$$

where $h_{w,0}$ is the conventional particle/antiparticle Hamiltonian, while the Lorentz-violating contribution is given by

$$\delta h_w = -2\gamma \left[\boldsymbol{\sigma} \cdot \bar{\boldsymbol{\xi}}_w - \gamma \boldsymbol{\sigma} \cdot \boldsymbol{\beta} \left(\bar{\xi}_w^0 - \frac{\gamma}{\gamma+1} \boldsymbol{\beta} \cdot \bar{\boldsymbol{\xi}}_w \right) \right] \ , \tag{14}$$

where $\boldsymbol{\beta} = \boldsymbol{p}/E$ is the (anti)particle velocity, γ is the relativistic boost factor, and

$$\bar{\xi}_w^\mu = \xi_w^{\mu\nu\rho} \beta_\nu \beta_\rho = \left[\tilde{H}^{\mu\nu\rho} - \epsilon^{\mu\nu\alpha\beta} (\tilde{C}^w)^\rho{}_{\alpha\beta} \right] \beta_\nu \beta_\rho \ , \tag{15}$$

with $\beta = (1, \boldsymbol{\beta})$, $\tilde{C}_{\rho\alpha\beta}^p = \tilde{C}_{\rho\alpha\beta}^+ + \tilde{C}_{\rho\alpha\beta}^-$, and $\tilde{C}_{\rho\alpha\beta}^n = \tilde{C}_{\rho\alpha\beta}^+ - \tilde{C}_{\rho\alpha\beta}^-$. Comparing Eq. (15) to the expression in Eq. (12), we see that the same combination of $C_{\mu\nu\rho}^\pm$ and $H_{\mu\nu\rho}$ coefficients appears. This combination, which is symmetric in ν and ρ, is the one to which free nucleon experiments are sensitive. It has 32 independent components. A subset of these can be constrained using clock-comparison experiments, as we will illustrate in the following.

6. Constraints

In Ref. [3] we derived bounds on $C_{\mu\nu\rho}$ and $H_{\mu\nu\rho}$ from comagnetometer experiments. In finding the particular components to which the experiments are sensitive, we followed the approach of Ref. [19]. We will sketch this approach here briefly and refer to Ref. [3] or [19] for more details.

Essentially, one calculates the shift of the rotational energy levels of an atom, ion, or nucleus W, that follows from the Lorentz-violating Hamiltonian described in the previous section. The part of the Hamiltonian for W that is linear in the Lorentz-violating coefficients, is given by

$$h'_W = \sum_w \sum_{N=1}^{N_{w,W}} \delta h_{w,N} \; , \qquad (16)$$

where $\delta h_{w,N}$ is the Hamiltonian for the N-th particle of species w, described in the previous section. The second sum runs over all particles of species w that are present in W.

This Hamiltonian induces a shift in the energy levels corresponding to

$$\delta E(F, M_F) = \langle F, M_F | h'_W | F, M_F \rangle \; , \qquad (17)$$

where $|F, M_F\rangle$ is the state of W with spin angular momentum F and projection M_F. The Lorentz-violating contribution to a transition frequency $F, M_F \to F', M'_F$ can then be calculated by $\delta\omega = \delta E(F, M_F) - \delta E(F', M'_F)$

What the contribution of different (components of) Lorentz-violating coefficients to Eq. (17) looks like depends on the rotational properties of those coefficients. For the Lorentz-violating coefficients that are presently relevant the Lorentz-violating shift can be written as

$$\delta E(F, M_F) = \widetilde{M}_F^1 E_1^W + \widetilde{M}_F^2 E_2^W + \widetilde{M}_F^3 E_3^W \; , \qquad (18)$$

where $\widetilde{M}_F^n$ ($n = 1, 2, 3$) follow from ratios of Clebsch-Gordan coefficients, while E_n^W represent contributions from spherical tensors of rank n in the original Hamiltonian. Because of the triangle inequality for angular momenta, E_n^W requires W to have an angular momentum of at least $n/2$ to be nonvanishing. In Ref. [3] we calculated E_1^W, E_2^W, and E_3^W for the Lorentz-violating coefficients under consideration, in the nonrelativistic limit. We found that the dominant contribution is part of E_1^W, which constitutes a dipole contribution to the energy. The resulting energy shift is given by

$$\delta E(F, M_F) = -\frac{2M_F}{F} \sum_w \sum_{N=1}^{N_w} \left[H^{300} + (\tilde{C}^w)^{012} - (\tilde{C}^w)^{021} \right] \langle [\sigma_3]_{w,N} \rangle \; , \qquad (19)$$

Here, $\langle [\sigma_3]_{w,N} \rangle$ is an unknown matrix element of the third spin component, defined in the 'stretched' state $|F, F\rangle$ (the 3-axis is the quantization axis). One has to adopt some nuclear-structure model to calculate the values of such matrix elements. Besides Eq. (19), there are other Lorentz-violating contributions to the energy shift, which contain different linear combinations of the Lorentz-violating coefficients. However, these are suppressed by factors of p^2/m_N^2 with respect to the one in Eq. (19).

The best bounds on the Lorentz-violating dipole contribution in Eq. (19) come from a ^{3}He/^{129}Xe comagnetometer [20] for the X and Y direction and from a ^{199}Hg/^{133}Cs comagnetometer [21] for the Z direction. Here the X, Y, and Z direction are defined in the Sun-centered inertial reference frame [2]. These experiments allow us to put bounds on components of $\xi^{\mu\nu\rho}$ in the order of 10^{-33}–10^{-30} GeV^{-1}. Additional bounds can be derived by considering effects due to the nonzero velocity of the Earth. Such effects are ignored in most cases, since they are suppressed by powers of the Earth's velocity with respect to the Sun, i.e. powers of $\beta_\oplus \simeq 10^{-4}$. However, due to the high precision of the comagnetometer experiments, in this case relevant bounds can be derived from such effects. An analysis of such boost-dependent signals was performed in Ref. [22].

Using the experiments and analyses of Refs. [20, 21, 22], we are able to bound 7 linear combinations of components of the Lorentz-violating tensors [3]. These linear combinations

Table 2. Order-of-magnitude bounds on the LV tensor components defined in Eqs. (2).

Tensor component	Limit in GeV^{-1}
$H_{TTX}, H_{TTY}, C^q_{T[XZ]}, C^q_{T[YZ]}$	$< 10^{-33}$
H_{TTZ}	$< 10^{-30}$
$C^q_{T[XY]}$	$< 10^{-29}$
$H_{TXX}, H_{TXY}, H_{TYY}$	$< 10^{-28}$
$H_{TTT}, H_{TXZ}, H_{TYZ}, C^q_{T[TZ]}, C^q_{X[YZ]}, C^q_{Y[XZ]}, C^q_{Y[YZ]}$	$< 10^{-27}$
$C^q_{T[TX]}, C^q_{T[TY]}, C^q_{T[TZ]}, C^q_{X[XZ]}, C^q_{Z[XZ]}, C^q_{Z[YZ]}$	$< 10^{-26}$

involve the low-energy constants described below Eq. (8). Therefore, bounds on the actual coefficients are order-of-magnitude bounds until we can do better than NDA estimates for the low-energy constants. Moreover, we assume that there are no large (unexpected) cancellations between the different coefficients, such that the 7 experimental bounds can be interpreted as bounds on the separate components of the Lorentz-violating tensors. The resulting order-of-magnitude limits are collected in table 2.

7. Spin-precession equation

An alternative way of bounding Lorentz-violating coefficients such as the ones in Eq. (8) is by considering storage-ring experiments that measure spin-precession frequencies. To interpret such experiments, we derived relevant Lorentz-violating spin-precession equation that corresponds to Eq. (13). Using the Heisenberg equation of motion to derive the time evolution of the spin operator, we find that the Lorentz-violating contribution to the spin-precession frequency $\boldsymbol{\omega}_s$ of particle species w is given by

$$\mp \frac{\delta \boldsymbol{\omega}_{s,w}}{4} = -\gamma \bar{\boldsymbol{\xi}}_w + \gamma^2 \boldsymbol{\beta} \left(\bar{\xi}^0_w - \frac{\gamma}{\gamma+1} \boldsymbol{\beta} \cdot \bar{\boldsymbol{\xi}}_w \right), \tag{20}$$

where γ and $\boldsymbol{\beta}$ are the Lorentz boost factor and particle velocity, respectively. The Lorentz-violating coefficient $\bar{\xi}^\mu$ is defined in Eq. (15) and the upper (lower) sign applies to particles (antiparticles).

In Ref. [3], we used Eq. (20) to derive a bound $|\xi^{ZTT}_p| < 2.7 \times 10^{-21}$ GeV by comparing proton and antiproton spin-precession frequencies. These were measured in two double Penning-trap experiments, performed in Mainz [23] and at CERN [24]. Although the sensitivity of these experiments is much lower than the comagnetometer experiments, described in the previous section, they have the added advantage that they do not need to rely on a nuclear model to determine the nuclear matrix elements.

Also, storage-ring experiments have the potential to bound additional (boost-dependent) components of the Lorentz-violating coefficients. In particular an experiment with deuterons is expected to be benificial, since it will have sensitivity to the $D^\pm_{\mu\nu\rho}$ coefficients that do not contribute to single-nucleon observables. We will discuss this in more detail in the next section.

8. The $D^\pm_{\mu\nu\rho}$ parameter

As can be seen from Eq. (8), the Lorentz-violating coefficient $D^\pm_{\mu\nu\rho}$ does not contribute to any nucleon two-point function, due to its symmetry properties. As such it does not contribute to free nucleon properties and thus cannot be bounded to the same precission as $C^\pm_{\mu\nu\rho}$ and $H_{\mu\nu\rho}$, using comagnetometer experiments.
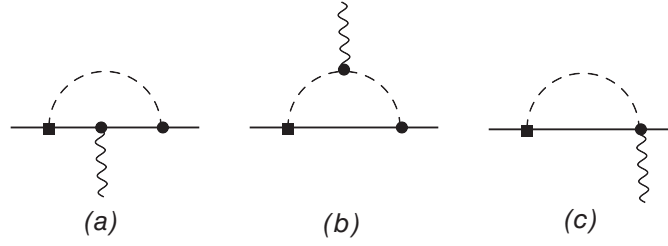
Figure 1. Contribution of $D_{\mu\nu\rho}^{-}$ to the nucleon electromagnetic form factor. The squares denote the pion-nucleon vertex from Eq. (12), while the circles denote leading-order vertices from the usual chiral perturbation Lagrangian.

In Ref. [3], we therefore calculated the pion-loop contribution of $D_{\mu\nu\rho}^{\pm}$ to the nucleon electromagnetic form factor (see Fig. 1). The result for the Lorentz-violating current that follows from the loop calculation is given by

$$I^{\mu}(q) \;=\; \left(F_{\nu\rho\sigma}^{+}(Q^2) + F_{\nu\rho\sigma}^{-}(Q^2)\tau_3\right) v^{\nu} v^{\mu} q^{[\sigma} S^{\rho]} + \cdots , \tag{21}$$

where the dots represent terms that are not relevant for on-shell photons. The function $F_{\nu\rho\sigma}^{+}(Q^2)$ $(F_{\nu\rho\sigma}^{-}(Q^2))$ represents a isoscalar (isovector) form factor, proportional to the Lorentz-violating parameters. For on-shell photons, the loop contribution to the isovector form-factor $F_{\nu\rho\sigma}^{-}$ turns out to vanish, while the loop calculation does give a nonzero isoscalar form factor. After renormalization, the loop contribution to the form factors can be written as

$$F_{\nu\rho\sigma}^{+}(Q^2 = 0) \;=\; \tilde{D}_{\nu\rho\sigma}^{-} \frac{8eg_A}{(2\pi F_\pi)^2} \ln \frac{m_N^2}{m_\pi^2} + \cdots , \tag{22}$$

where the dots represent contributions from other low-energy effective operators. We see that the chiral loop contribution is slightly enhanced by the factor $\ln \frac{m_N^2}{m_\pi^2}$. This result exemplifies how chiral perturbation theory allows for precision calculations for Lorentz-violating hadronic observables.

The problem with using the electromagnetic nucleon form factor to bound Lorentz-violating coefficients is the fact that the presence of physical electromagnetic fields suppresses the effects by many order of magnitudes. The suppression factor with respect to the dominant Lorentz-violating effects due to C and H will be in the order of $eF^{\mu\nu}/\Lambda_\chi^2$. For a magnetic field of 1 tesla this is about 10^{-16}. We thus estimate that the bounds on $D_{\mu\nu\rho}^{-}$ from comagnetometer experiments are at best in the order of $\mathcal{O}(10^{-17})$ GeV.

A more promising option is the contribution of $D_{\mu\nu\rho}^{\pm}$ to nucleon-nucleon interactions. Although it is not possible to write a two-point function for spin-$\frac{1}{2}$ particles involving $D_{\mu\nu\rho}^{\pm}$, this is not necessarily true for higher-spin particles, such as the deuteron. In Ref. [3] we thus calculated the Lorentz-violating nucleon-nucleon potential that follows from one-pion exchange between nucleons in the context of chiral EFT (see Refs. [25, 26, 27] for reviews). In the relevant pion-exchange diagram, one of the two interaction vertices is a Lorentz-violating vertex from Eq. (12). The resulting potential is given by

$$
\begin{aligned}
V_{\mathrm{LV}} \;=\; & -\left(\epsilon^{ijk}\tilde{D}_{0ij}^{-}\right) \frac{2ig_A}{F_\pi^2}(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_3 \frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})\sigma_2^k + (\boldsymbol{\sigma}_2 \cdot \boldsymbol{k})\sigma_1^k}{\boldsymbol{k}^2 + m_\pi^2} \\
& -\left(\epsilon^{jkl}\breve{D}_{ijk00}^{+}\right) \frac{4g_A}{F_\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{(\sigma_1^l \sigma_2^m + \sigma_1^m \sigma_2^l)k^i k^m}{\boldsymbol{k}^2 + m_\pi^2} ,
\end{aligned}
\tag{23}
$$

where $\boldsymbol{\sigma}_{1,2}$ ($\boldsymbol{\tau}_{1,2}$) are the spin (isospin) operators of nucleon 1 and 2 and the momentum transfer $\boldsymbol{k} = \boldsymbol{p} - \boldsymbol{p}'$ flows from nucleon 1 to nucleon 2; $\boldsymbol{p}$ and $\boldsymbol{p}'$ are the relative momenta of the incoming and outgoing nucleon pair in the center-of-mass frame. The Latin indices $i, j, k, ...$ denote spatial directions. There are other contributions from the D-parameter to the potential, which can be found in Ref. [3].

The detailed study of the effects of Eq. (23) are left for future work. However, it seems clear that it will have an effect on for example the spin-precession frequency of the deuteron as well as on other light-nucleon systems or clock-comparison experiments. For such systems a bound on $D_{\mu\nu\rho}^{\pm}$ that is a significant improvement over the current bounds seems feasible.

9. Conclusion and outlook

In Ref. [3], we started the program of deriving the hadronic low-energy effective Lagrangian for Lorentz-violation, based on chiral-symmetry considerations and induced by operators of the Lorentz-violating standard-model extension. Here, we limited the discussion to a selected set of dimension-five quark-gluon operators, which obey a set of UV-safety conditions [10]. This work has been extended to several different operators in other work [4, 5].

We applied a Foldy-Wouthuysen transformation to the low-energy effective Lagrangian to obtain a decoupled particle/antiparticle Hamiltonian and derived the resulting shift in the rotational energy levels of atoms. Using these results we derived strict bounds on components of two of the three Lorentz-violating coefficients, from comagnetometer experiments [20, 21]. The resulting limits are collected in table 2. Comparing these to the expected size of the dimension-five operators, i.e. $1/m_P \simeq \mathcal{O}(10^{-19})\,\text{GeV}^{-1}$, we see that many of the components of $C_{\mu\nu\rho}$ and $H_{\mu\nu\rho}$ have bounds that are well beyond the Planck scale.

An important result is that the $D_{\mu\nu\rho}$ coefficients do not contribute to the nucleon two-point function and therefore cannot be limited by the same methods that provided the stringent bounds on $C_{\mu\nu\rho}$ and $H_{\mu\nu\rho}$. In Ref. [5] we found that this also holds for a particular pure-gluon parameter in the mSME and we expect there will be more such parameters in the SME Lagrangian.

We calculated pion-loop contributions to the nucleon electromagnetic form factor in terms of the $D_{\mu\nu\rho}$ coefficient. However, due to the supression that follows from the necessity of the presence of a physical electromagnetic field, the resulting bounds on components of $D_{\mu\nu\rho}$ are in the order of $\mathcal{O}(10^{-17})\,\text{GeV}^{-1}$ at best. This is not yet at the expected level of $1/m_P$ and improvements of this bound are thus desirable.

Such improvement might come from considering effects of the one-pion exchange nucleon-nucleon potential in Eq. (23). The effects of the NN interaction in nuclei could provide much better bounds than the electromagnetic interactions. Especially the spin precession of the deuteron is promising in this respect and for example storage-ring experiments might be able to place stringent constraints [28].

References

[1] Colladay D and Kostelecký V A 1997 *Phys. Rev.* D **55** 6760
    Colladay D and Kostelecký V A 1998 *Phys. Rev.* **58** 116002
[2] Kostelecký V A and Russell N 2016 *Preprint 0801.0287v10 [hep-ph]*
[3] Noordmans J P, de Vries J and Timmermans R G E 2016 *Phys. Rev.* C **94** 025502
[4] Kamand R, Altschul B and Schindler M R 2017 *Phys. Rev.* D **95** 056005
[5] Noordmans J P 2017 *Preprint 1701.04334 [hep-ph]*
[6] Carroll S M, Field G B and Jackiw W 1990 *Phys. Rev.* D **41** 1231
[7] Coleman S R and Glashow S L 1999 *Phys. Rev.* D **59** 116008

[8] Kostelecký V A 2008 *Preprint 0802.0581 [gr-qc]*
[9] Groot Nibbelink S and Pospelov M 2005 *Phys. Rev. Lett.* **94** 081601
    Bolokhov P A, Groot Nibbelink S and Pospelov M 2005 *Phys. Rev. D* **72** 015013
    Mattingly D 2008 *Preprint 0802.1561 [gr-qc]*
[10] Bolokhov P A and Pospelov M 2008 *Phys. Rev. D* **77** 025022
[11] Ding Y and Kostelecký V A 2016 *Phys. Rev. D* **94** 056008
[12] Kostelecký V A and Mewes M 2009 *Phys. Rev. D* **80** 015020
    Kostelecký V A and Mewes M 2012 *Phys. Rev. D* **85** 096005
    Kostelecký V A and Mewes M 2013 *Phys. Rev. D* **88** 096006
[13] Weinberg S 1979 *Physica* A **96** 327
[14] Gasser J and Leutwyler H 1984 *Ann. Phys.* **158** 142
[15] Weinberg S 1996, *The Quantum Theory of Fields* vol 2 (Cambridge: Cambridge University Press)
[16] Bernard V and Meißner U -G 2007 *Ann. Rev. Nucl. Part. Sci.* **57** 33
    Scherer S and Schindler M R 2012 *Lect. Notes Phys.* **830** 1
[17] Manohar A V and Georgi H 1984 *Nucl. Phys.* B **234** 189
    Georgi H and Randall L 1986 *Nucl. Phys.* B **276** 241
[18] Jenkins E E and Manohar A V 1991 *Phys. Lett.* B **255** 558
    Bernard V, Kaiser N, Kambor J and Meißner U -G 1992 *Nucl. Phys.* B **388** 315
[19] Kostelecký V A and Lane C D 1999 *Phys. Rev. D* **60** 116010
[20] Allmendinger F *et al.* 2014 *Phys. Rev. Lett.* **112** 110801
[21] Peck S K *et al.* 2012 *Phys. Rev.* A **86** 012109
[22] Canè F *et al.* 2004 *Phys. Rev. Lett.* **93** 230801
[23] Mooser A *et al.* 2014 *Nature* **509** 596
[24] DiSciacca J *et al.* 2013 *Phys. Rev. Lett.* **110** 130801
[25] Bedaque P F and van Kolck U 2002 *Ann. Rev. Nucl. Part. Sci.* **52** 339
[26] Epelbaum E, Hammer H W and Meißner U -G 2009 *Rev. Mod. Phys.* **81** 1773
[27] de Vries J and Meißner U -G 2016 *Int. J. Mod. Phys.* E **25** 1641008
[28] Eversmann D *et al.* (JEDI Collaboration) 2015 *Phys. Rev. Lett.* **115** 094801