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Cherenkov-like emission of Z bosons

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Abstract. We study CPT and Lorentz violation in the electroweak gauge sector of the Standard Model in the context of the Standard-Model Extension (SME). In particular, we show that any non-zero value of a certain relevant Lorentz violation parameter that is thus far unbounded by experiment would imply that for sufficiently large energies one of the helicity modes of the Z boson should propagate with spacelike four-momentum and become stable against decay in vacuum. In this scenario, Cherenkov-like radiation of Z bosons by ultra-high-energy cosmic-ray protons becomes possible. We deduce a bound on the Lorentz violation parameter from the observational data on ultra-high energy cosmic rays.

1. Introduction

Lorentz invariance is a fundamental ingredient of both quantum field theory and General Relativity. Nevertheless, in the past two decades there has been a growing interest in the possibility that Lorentz symmetry may not be exact in Nature. On the theoretical side, a number of candidate theories for quantum gravity have been shown to involve Lorentz invariance violation as a possible effect. We may mention in particular string field theory [1] and loop quantum gravity [2]. More simple toy models involving spontaneous breaking of Lorentz symmetry that have been studied are bumblebee models [3] and lattice models where fermion bilinears carrying Lorentz indices acquire nonzero vacuum expectation values [4]. Another scenario involves cosmologically varying scalars [5] in which a scalar acquires a vacuum expectation value that has a (slow) variation as a function of spacetime. In other words, a preferred direction is selected through a non-zero value for the gradient of a scalar. A class of Lorentz-violating models that have received a lot of attention involve noncommutative geometry. Here the coordinates of spacetime are taken to be noncommuting quantities [6], $[x_\alpha, x_\beta] = i\frac{1}{\Lambda_{NC}^2}\theta_{\alpha\beta}$, where $\theta_{\alpha\beta}$ is a tensor-valued set of coefficients of $\mathcal{O}(1)$, while Λ_{NC} denotes the noncommutative energy scale. Noncommutative quantum field theories can be constructed by taking an ordinary quantum field theory and replacing the ordinary multiplication of fields with Moyal products. It is possible to re-express resulting noncommutative field theory in terms of a conventional one, by use of the so-called Seiberg-Witten map [7]. This yields a field theory with Lorentz-violating terms that are at least of mass dimension six. Finally, it is worthwhile to mention Hořava-Lifshitz gravity [8, 9] as a fundamentally Lorentz-noninvariant model, with Lorentz invariance arising at low energy as an emergent symmetry.

However, the most important reason for the recent interest in experimental testing of Lorentz and CPT symmetry has been the development of low-energy effective field theories with Lorentz-invariance violation, in particular de Standard-Model Extension (SME) [10]. The Lagrangian of the matter sector of this framework contains all Lorentz-violating gauge-invariant effective operators that can be build from the conventional Standard-Model fields, coupled to vector and tensor coefficients that parametrize the Lorentz violation. In fact, the SME also contains all CPT-violating operators, since in any local interacting quantum field theory CPT violation implies Lorentz violation [11]. The SME can thus be used to provide a general quantification of the exactness of Lorentz and CPT symmetry in the form of observational contraints on the Lorentz-violation coefficients [12].

A possible observational consequence of Lorentz violation, that can be addressed using astrophysical data, is vacuum Cherenkov radiation [13]. The Lorentz-violation coefficients can in some cases be interpreted as inducing a refractive index for the vacuum. The velocity of charged particles above some energy threshold might then exceed the phase velocity of light. This causes these particles to rapidly lose energy through photon emission. The mere observation of high-energy cosmic particles can then be used to constrain the Lorentz violation coefficients.

In this talk, we consider a process analogous to vacuum Cherenkov radiation, but with the emitted photon replaced by a Z boson. We assume the latter to obey a Lorentz- and CPT violating dispersion relation, originating from the superficially renormalizable part of the SME, called the minimal SME (mSME). In this case, the Lorentz violation originates from a Chern-Simons-like addition to the Standard-Model Lagrangian [14] and is captured by one four-vector: k^μ_{ZZ}. Such a theory has been shown to be consistently and covariantly quantizable, despite the presence of spacelike momenta, which are necessary for vacuum Cherenkov radiation to occur [15]. If k^μ_{ZZ} is timelike, the nonzero gauge-boson mass is an important ingredient that prevents the theory from containing imaginary energies. However, at the relevant energies the Z-boson mass can be considered small, allowing us to obtain stringent limits on the previously unconstrained Lorentz-violation coefficients k^μ_{ZZ}.

2. Vacuum Cherenkov radiation

One of the first SME terms ever to be considered (even before the formulation of the full SME [14]) is the Lorentz- and CPT-violating photon term

$$\mathcal{L}_{AF} = \frac{1}{2}(k_{AF})_\kappa \epsilon^{\kappa\lambda\mu\nu} A_\lambda F_{\mu\nu}\,. \tag{1}$$

Inclusion of this term leads to birefringence. Therefore, observations of cosmological sources with known polarizations permit searching for energy-dependent polarization changes, either from distant sources or from the cosmological microwave background radiation. As a result, the k_{AF} coefficients have been bound with great precision [14]:

$$|(k_{AF})^\kappa| \leq 10^{-43}\,\mathrm{GeV}\,. \tag{2}$$

Another interesting property of the term (1) is that it can provoke vacuum Cherenkov radiation [13]. This can happen because one of the photon helicities acquires a spacelike momentum, allowing for the possibility for a charged particle to move faster than the phase velocity of the photon.

Let us now consider, instead of the photon, a term analogous to (1) for the Z boson:

$$\mathcal{L}_{AF,Z} = \frac{1}{2}(k_{ZZ})_\kappa \epsilon^{\kappa\lambda\mu\nu} Z_\lambda Z_{\mu\nu}\ , \tag{3}$$

where $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ and Z_μ represents the Z boson. The Lorentz violation four-vector coefficient k^μ_{ZZ} is real and can either be timelike, lightlike, or spacelike.

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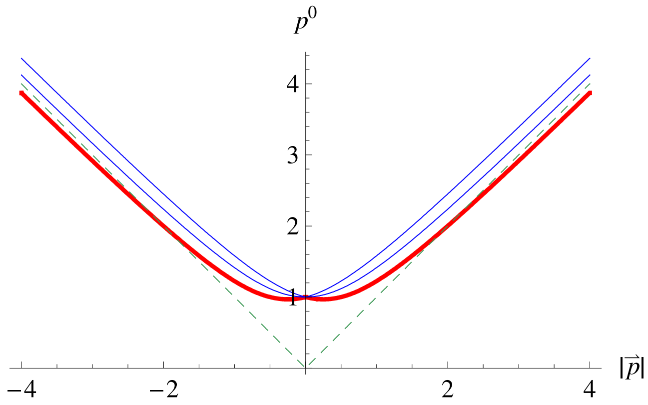

Figure 1. The dispersion relations of the Z boson subject to the inclusion the Lorentz-violating term (3) for purely timelike k^{μ}_{ZZ}, where we took $k^0_{ZZ} = \frac{M}{4}$, in arbitrary units. Indicated in red is the +-mode, which has spacelike four-momenta above an energy threshold.

These Z bosons obey the dispersion relations

$$\Lambda_0(p) = p^2 + M^2 = 0, \qquad \Lambda_\pm(p) \equiv p^2 - M^2 \pm 2\sqrt{(p \cdot k)^2 - p^2 k^2} = 0 \ , \tag{4}$$

where p^μ and M represent the momentum and the mass of the Z boson, while we dropped the subscript on k^{μ}_{ZZ} for simplicity. These dispersion relations are represented in Fig. 1 for the case of purely timelike k^{μ}_{ZZ}. As it turns out, the 0 and − gauge-boson polarization modes are timelike for any momentum. On the other hand, it follows from (4) that the gauge-boson momentum is spacelike for the + mode, if and only if

$$(p \cdot k)^2 > \frac{1}{4} M^4 \ . \tag{5}$$

This relation determines the energy threshold above which the Cherenkov-like process can take place. Since spacelike momenta are a necessary condition for the desired Cherenkov-like process, we will only consider Z bosons in the + polarization mode.

3. Cherenkov emission of Z bosons

We will first calculate the rate of Cherenkov emission of Lorentz-violating Z bosons with a spacelike momentum by a elementary Dirac fermion with mass m. Subsequently, we will consider the case of a proton.

First of all, we note that covariant quantization of free Z bosons in the presence of the term (3) can be performed in a fully consistent way, as described in Ref. [15]. Let us label

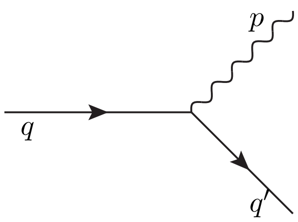

Figure 2. Relevant Feynman diagram for Z boson emission by incoming fermion.

the momentum of the incoming fermion as q, the emitted gauge boson as p, and the outgoing fermion as $q' = q - p$ (see Fig. 2). We will assume that the fermion obeys a conventional Lorentz-symmetric dispersion relation.

The rate at which the fermion loses energy by the Cherenkov-like emission of spacelike gauge bosons is given by the zeroth component of

$$\frac{dP^\mu}{dt} = \int p^\mu d\Gamma \,, \tag{6}$$

where $d\Gamma$ is the differential decay rate, which reads

$$d\Gamma = \frac{1}{2q^0}\frac{d^3p}{(2\pi)^3}\frac{1}{\Lambda'_+(p)}\frac{d^3q'}{(2\pi)^3}\frac{1}{2q'^0}\left(\frac{1}{2}\sum_{\text{spins}}|\mathcal{M}|^2\right)(2\pi)^4\delta^4(q-p-q')\,. \tag{7}$$

Here, the squared matrix element $|\mathcal{M}|^2$ is summed (averaged) over the final (initial) fermion spin. The unconventional factor $\Lambda'_+(p) = \partial\Lambda_+(p)/\partial p^0$ in the denominator defines a normalization in which the phase space and the matrix element are separately observer Lorentz invarian [15], i.e., invariant under simultaneous Lorentz transformations of the momenta and the Lorentz violation four-vector coefficient k^μ_{ZZ}. This is important, as traditional calculations in the literature all have divergent factors in the outgoing boson phase space factors when an observer frame is chosen in which the energy of the radiated boson goes to zero.

The matrix element that follows from the appropriate tree-level Feynman diagram is given by

$$i\mathcal{M} = i\bar{u}(q')\gamma^\mu(g_V + g_A\gamma^5)u(q)e^{(+)*}_\mu(p)\,, \tag{8}$$

where $u(q)$ and $u(q')$ are conventional Dirac spinors corresponding to the ingoing and outgoing particles (the case of antiparticles can be treated in an analogous way). The four-vector $e^{(+)}_\mu(p)$ is the gauge-boson polarization vector that corresponds to the dispersion relation in Eq. (4). (see [15] for explicit expressions). The coupling constants g_V and g_A, which multiply the vector and axial-vector current, respectively, depend on the process under consideration. For example, a charged lepton emitting a Z boson has tree-level values of $g_V = -g(1-4\sin^2\theta_w)/(4\cos\theta_w)$ and $g_A = -g/(4\cos\theta_w)$, with $g \simeq 0.65$ the $SU(2)$ coupling constant.

Using the fact that [15]

$$e^{(+)}_\mu(p)e^{(+)*}_\nu(p) \;=\; -\frac{1}{2}\eta_{\mu\nu} - \frac{p_\mu p_\nu k^2 + k_\mu k_\nu p^2 - (p_\mu k_\nu + p_\nu k_\mu)(p\cdot k)}{2((p\cdot k)^2 - p^2k^2)} + \frac{i\epsilon_{\mu\nu\alpha\beta}k^\alpha p^\beta}{2\sqrt{(p\cdot k)^2 - p^2k^2}}\,, \tag{9}$$

we find that the spin-averaged and spin-summed squared matrix element is given by

$$\frac{1}{2}\sum_{\text{spins}}|\mathcal{M}|^2 \;=\; \frac{-\xi}{2}\left[p^2\left(1+4X^2-8yX\right) - 4(1-2x)m^2\right]\,, \tag{10}$$

where $\xi = g_V^2 + g_A^2$, $x = g_V^2/\xi$, $y = g_Ag_V/\xi$, and

$$X = \frac{k\cdot(q-\frac{1}{2}p)}{\sqrt{(p\cdot k)^2 - p^2k^2}}\,. \tag{11}$$

The rate of energy-momentum loss becomes

$$\frac{dP^\mu}{dt} = \frac{1}{8\pi^2q^0}\int\frac{d^3p}{\Lambda'_+(p)}\theta(q^0-p^0)\delta((q-p)^2-m^2)p^\mu\left(\frac{1}{2}\sum_{\text{spins}}|\mathcal{M}|^2\right)\,. \tag{12}$$

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To proceed, we note that dP^μ/dt transforms as a four-vector divided by q^0. Therefore, we make an ansatz in terms of all available four-vectors:

$$q^0 \frac{dP^\mu}{dt} = A(q,k)\, q^\mu + B(q,k)\, k^\mu \ , \tag{13}$$

where $A(q,k)$ and $B(q,k)$ are observer-Lorentz-invariant quantities that can depend on q^μ and k^μ. Contracting this expression with q^μ and k^μ gives two observer-Lorentz-invariant equations, which we can solve for $A(q,k)$ and $B(q,k)$ after having performed the integrations over $\vec{p}$, given in Eq. (12).

To achieve this, we make use of the observer Lorentz invariance of $A(q,k)$ and $B(q,k)$ by specializing to an observer frame that simplifies the calculation. For the cases that k^μ is timelike or spacelike, we go the frame where k^μ is purely timelike, i.e., $k = (k^0, \vec{0})$, or purely spacelike, i.e. $k = (0, \vec{k})$, respectively. If k^μ is lightlike, we do not need to specialize to a particular frame.

The calculation of the rate can now be done in any one of these special frames by explicitly performing the integral over $\vec{p}$ in (12). It turns out that, for small Lorentz-violation coefficient, the Z boson is emitted in a narrow forward cone:

$$\cos\theta_{pq} = 1 + \mathcal{O}(\kappa^2/M^2) \tag{14}$$

where θ_{pq} is the angle between the Z boson and the incoming fermion, while

$$\kappa = k^0 - \cos\theta_{pk}|\vec{k}| \tag{15}$$

is a value of the order of the Lorentz-violation coefficient. The θ function in (12) restricts the emitted Z boson momenta to be in a finite range of spacelike momenta. This in turn forces the incoming fermion to have a momentum above a threshold value

$$|\vec{q}|_{\rm th} = \frac{M(M+2m)}{2|\kappa|} \tag{16}$$

in order that there is a nonzero value of the emission rate.

As it turns out, both $A(q,k)$ and $B(q,k)$ in Eq. (13) are of order M^2. Combined with Eq. (16) we thus find that the second term in Eq. (13) is of order κ^2/M^2 compared to the first term. We therefore neglect this second term and find that the rate of energy-momentum loss is given by

$$\frac{dP^\mu}{dt} = \frac{\xi M^2}{96\pi q^0}\theta(a-1) q^\mu F(a, \mathrm{sgn}(\kappa)) \ . \tag{17}$$

where

$$F(a, \frac{m}{M}, \tilde{y}, \kappa) = (a-1)\left[8(1-\tilde{y}) + \left(\frac{5}{a} - \frac{1}{a^2}\right)(1+2\tilde{y})\right] - 12\ln a + \mathcal{O}\left(\frac{m}{M}, \frac{\kappa^2}{M^2}\right) . \tag{18}$$

Here $\tilde{y} = \mathrm{sgn}(\kappa) y$, while the variable a is defined as the ratio of $|\vec{q}|$ to its threshold value, i.e., $a = |\vec{q}|/|\vec{q}|_{\rm th}$. In a general observer frame and upto terms of order κ^2/M^2 and m^2/M^2, we can write this as

$$a = \frac{2|q \cdot k|}{M(M+2m)} \ . \tag{19}$$

The stepfunction demands that $a > 1$ for Eq. (17) to be nonvanishing. This is just the threshold condition for the initial fermion.

The zeroth component of the decay rate (17) can be viewed as a differential equation $dE(t)/dt = -W(E(t))$ in terms of the energy of the fermion. It can be solved explicitly in

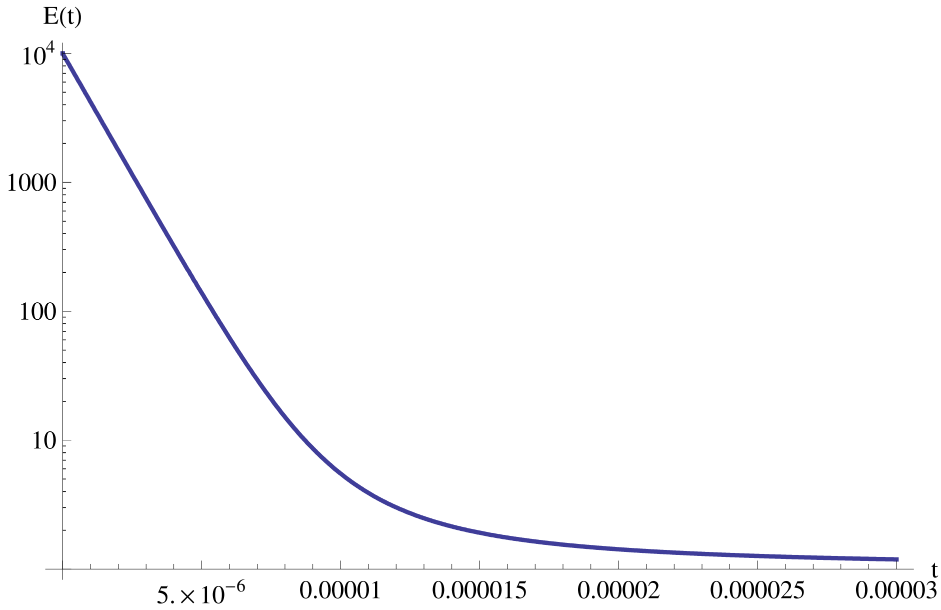

Figure 3. The ratio of the fermion energy to the threshold value as a function of time, for the parameter values: $M = 91\,\text{GeV}$, $m/M = 1/91$, $g_V = -0.0026$, $g_A = 0.0445$, $\kappa = 5\times10^{-15}\,\text{GeV}$.

the regimes $a \gg 1$ and close to threshold, $a-1 \ll 1$. In the former case the decay is exponential, $E(t) = E_{\text{in}}e^{-t/t_\infty}$, with characteristic time $t_\infty = \frac{6\pi\hbar}{\xi|\kappa|(1-\tilde{y})}$. For $a-1 \ll 1$, $E(t)$ decays to the threshold energy E_{th} in finite time, with characteristic time constant $t_{\text{fin}} = \frac{8\pi\hbar}{\xi|\kappa|(1-2\tilde{y})}\left(\frac{M}{m}\right)^{3/2}$. Fig. 3 displays a typical decay curve for the ratio of the fermion energy to the threshold value as a function of time.

If the radiating particle is a composite fermion, the latter will disintegrate upon emission of a Z boson. In this case, it makes more sense to consider the decay rate $\Gamma = \int d\Gamma$, rather than the radiation rate calculated above. For a proton the decay rate can be written

$$\Gamma = \frac{1}{2q^0}\int \frac{d^3\vec{p}}{(2\pi)^3}\frac{4\pi}{\Lambda'_+(p)}e^{(+)}_\mu(p)e^{(+)*}_\nu(p)W^{\mu\nu}\ , \tag{20}$$

where $W^{\mu\nu}$ is the hadronic part, which can be calculated in the parton model. The calculation essentially amounts to calculating the decay rate of an elementary quark that caries a fraction x of the longitudonal proton momentum. We can thus use many of the results obtained for the elementary fermion rate. The final result for the decay rate is

$$\Gamma = \frac{|\kappa|}{16\pi}\sum_q \xi_q \int_0^1 (f_q(x) + \bar{f}_q(x))G_q(a,x)\theta(ax-1)\,dx\ . \tag{21}$$

Here $\xi_q = \frac{g^2}{2c_w^2}\big((T_q^3)^2 + 2Q_q s_w^2(Q_q s_w^2 - T_q^3)\big)$, with Q_q the quark charge and T_q^3 the third weak-isospin component. The functions $f_q(x)$ and $\bar{f}_q(x)$ are the parton distributions functions for the quarks and antiquarks of flavor q, respectively. They represent the chance of finding a quark with momentum xq inside the proton. The function $G_q(a,x)$ in (21) is given by

$$G_q(a,\frac{m}{M},y_q,\kappa,x) = (ax-1)\left(\frac{-7+6\tilde{y}_q(x)}{ax} - \frac{1-2\tilde{y}_q(x)}{(ax)^2}\right) - 4\left(1+\frac{1-2\tilde{y}_q(x)}{ax}\right)\ln a + \mathcal{O}\left(\frac{m}{M},\frac{\kappa^2}{M^2}\right) \tag{22}$$

with

$$\tilde{y}_q(x) = \frac{g^2 T_q^3 (2Q_q s_w^2 - T_q^3)}{4\xi_q c_w^2} . \frac{f_q(x) - \bar{f}_q(x)}{f_q(x) + \bar{f}_q(x)} .$$

We see that (21) is essentially the sum over elementary-quark decay rates, weighted by the corresponding parton distribution functions.

4. Limits from ultra-high-energy cosmic rays

We can use the fact that a proton with an energy above threshold will disintegrate to use astrophysical data to limit k^μ. More precisely, such a proton cannot reach Earth if its mean free path L is much smaller than the distance from its source to Earth. Since many ultra-high-energy cosmic ray particles (UHECR) with energies above $57\,\text{EeV} \equiv |\vec{q}|_{\text{obs}}$ have been observed, more or less from all directions [16], it follows that

$$|\kappa| < \frac{M(M+2m)}{|\vec{q}|_{obs}} \approx 1.5 \times 10^{-7}\,\text{GeV} \equiv |\kappa|_0 \tag{23}$$

We see from (21) that the mean lifetime of protons (in the Earth's frame) t_p is proportional to $|\kappa|^{-1}$, but it is enhanced by the minute values of the parton distribution functions for values of x close to one. A conservative (large) estimate gives a mean free path of

$$L \simeq ct_p \sim (\hbar c/|\kappa|_0) \times 10^{15} \sim 10^3\,\text{km} . \tag{24}$$

It is clear that protons with an energy above this threshold will not be able to reach Earth from any viable UHECR source. This allows us to conclude that

$$|k^\mu_{ZZ}| < 1.5 \times 10^{-7}\,\text{GeV} \tag{25}$$

as a bound on the components of k^μ_{ZZ}.

It is tempting to try and relate the coefficient k_{ZZ} to the $SU(2)$ and $U(1)$ gauge boson parameters k_1^μ and k_2^μ parametrizing all possible CPT violation in the $SU(2)\times U(1)$ gauge sector of the mSME [10]. However, it turns out that the latter generate, apart from the terms (3) and (1), a quadratic CPT-violating mixing term between the photon and the Z boson. The terms (3) and (1) by themselves are not invariant under general $SU(2) \times U(1)$ gauge transformations. However, a similar analysis as the one presented can be done in the case of a Lorentz- and CPT violation term for the W bosons, in which case no problematic mixing occurs [17].

Acknowledgments

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