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Research Article

Parabolic Spiral Search Plan for a Randomly Located Target in the Plane

Mohamed Beltagy and Mohamed Abd Allah El-Hadidy

Department of Mathematics, Faculty of Science, Tanta University, Tanta 31527, Egypt

Correspondence should be addressed to Mohamed Abd Allah El-Hadidy; melhadidi@eun.eg

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This paper addresses the problem of searching for a located target in the plane by using one searcher starting its motion from the point (X_0, Y_0). The searcher moves along parabolic spiral curve. The position of the target has a known distribution. We show that the distance between the target position and the searcher starting point depends on the number of revolutions, where the complete revolution is done when $t = 2\pi$. Furthermore, we study this technique in the one-dimensional case (i.e., when the searcher moves with linear search technique). It is desired to get the expected value of the time for detecting the target. Illustrative examples are given to demonstrate the applicability of this technique assuming circular normal distributed estimates of the target position.

1. Introduction

Search problem dates back to World War II, and the works of Koopman [1–3] and Stone [5, 6] offer a classic treatise of this area from an operational research perspective. Many variants and extensions of this problem, in a wide variety of directions, have been presented in both the statistical and operations research literature since Koopman [1] solved this problem for the unidimensional case. This problem has been discussed under some specific hypotheses by Richardson [4]. However, as pointed out by Koopman [2, 3], there is so much complexity in real search and rescue missions that any statistical model can only reflect part of the real-life situation and ours are no exception. On search theory in general, Stone [5] has given a good account of various results presently available, with some informative examples, and also has provided a rigorous mathematical treatment of the subject, for both discrete and continuous cases. On the other hand, Stone [6] provided an overview of different areas in the development of search theory, which could be designated as classical, mathematical, algorithmic, and dynamic. Also, exhaustive surveys of works realized on this topic have been given by Iida [7] and Benkoski et al. [8].

Studying of searching problem in the plane had found considerable interest among researchers due to its wide applications in our life. Feinerman et al. [9] introduced some search trajectories to find the desert Ants Nearby Treasure. They used multiple searchers without communication between them in the plane. Edelsbrunner and Maurer [10] obtained the optimal solutions for the postoffice problem. The certain point location problems in two dimensions are derived via geometric transforms from an optimal solution for the search problem in three dimensions: find the first point hit by a rotating or sweeping plane. Mohamed et al. [11, 12] obtained more interesting search plans that give the minimum expected value of the cost for detecting the lost target. This problem has been discussed when the located target has symmetric and asymmetric distributions and with less information available to the searchers. In both papers, they desired to minimize the expected time for detecting the target. El-Hadidy [13] introduced a new search plan in the plane which is divided into identical cells and the searcher moves along spiral with line segment curve. He found the optimal value of the arcs that the searcher should do and finds the target with minimum cost. Also, Mohamed and El-Hadidy [14] discussed the existence of the search plan that finds the conditionally deterministic target motion. They have shown the existence of the optimal search plan and found the necessary conditions that make the expected value of the first meeting time finite. When the target starts its motion as parabolic spiral from a random point in the plane, Mohamed and El-Hadidy [15] studied this problem such that no time information about the target's position is available to the searcher. The searcher starts its motion from the origin. They formulated a search model and found the conditions that make this search plan finite. Recently, El-Hadidy [16] formulated another search model for a helix target motion in the space by using a team of three searchers. He studied the existence of the optimal search plan and found it.

In this paper, the search for a randomly located target in the plane by a single searcher is studied. The target's position is unknown, but the searcher knows its probability distribution. The searcher moves along parabolic spiral curve as in Figure 1, starting its motion from the point (X_0, Y_0). Furthermore, we study this technique in the one-dimensional case (i.e., when the searcher moves with linear search technique but here the searcher moves with speed depending on time). This problem is more effective and more applicable to real-world search scenarios such as searching for underwater target by firing with 91RTE2 missile system; see website http://www.youtube.com/watch?v=EgQk-FEGV-Y and also the electrons movement in a cyclotron under the influence of a constant magnetic field. The purpose here is to obtain the expected value of the time for detecting the target, assuming circular normal distributed estimates of its position.

This paper is organized as follows. In Section 2 we formulate the problem. The expected value of the time is given when the target position has circular normal distribution in Section 3. One-dimensional case that rides to the generalized linear search problem is discussed in Section 4. Finally, the paper concludes with a discussion of the results and directions for future research.

2. Modeling of Search Problem and Formulation

Here, assumptions of a parabolic spiral search plan for a randomly located target are described and the problem is mathematically formulated as an allocation of searching effort which is the expected value of the time for detecting the target.

Let X, Y be two independent random variables that represent the position of the target. The searcher is assumed to move according to the model [17]

$$\begin{aligned} x(t) &= X_0 + t^2 \cos(t), \\ y(t) &= Y_0 + t^2 \sin(t), \end{aligned} \tag{1}$$

where (X_0, Y_0) is the starting point of the searcher and $t \in \{0\} \cup \mathbb{R}^+$, where $\mathbb{R}$ is the set of real numbers. Equations (1) show that the searcher moves along parabolic spiral curve as in Figure 1.

The searching process is continuous time and continuous space. We consider that the searcher will make a complete revolution when returns the x'-axis; see Figure 1. At time t, the searcher moves along a circle with centre (X_0, Y_0) and radius t^2 given by

$$(x(t) - X_0)^2 + (y(t) - Y_0)^2 = t^4. \tag{2}$$

From (1) we have

$$\begin{aligned} \frac{dx(t)}{dt} &= \dot{x} = 2t\cos(t) - t^2 \sin(t), \\ \frac{dy(t)}{dt} &= \dot{y} = 2t\sin(t) + t^2 \cos(t). \end{aligned} \tag{3}$$

Thus, the speed of the searcher will be given as

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{4t^2 + t^4} = t\sqrt{t^2 + 4}. \tag{4}$$

It is clear that the speed will vanish at $t = 0$ only, and v depends on t. We obtain the time that the searcher will take to detect the target in a complete revolution by considering that the searcher path will intersect the line $y(t) = mx(t) + c$ at time t, where m is the slope of the line and c is the absolute constant. Using (1) we get $c + mX_0 + mt^2\cos(t) = Y_0 + t^2\sin(t)$, but at $t = 0$ we have $Y_0 = mX_0 + c$ then $mt^2\cos(t) = t^2\sin(t)$. Consequently, $t = \tan^{-1} m$, but at $m = 0$ we have $t = 0, 2\pi, 4\pi, \ldots$, (i.e., the complete revolutions will be done at the time values $0, 2\pi, 4\pi, \ldots$).

In addition, the area that the searcher will search through making the complete revolution is given in the following.

After time t, in the first revolution the searcher will arrive to the point $(X_0 + t^2\cos(t), Y_0 + t^2\sin(t))$ and make an angle t with x'-axis; see Figure 2.

With the previous understanding of using polar coordinates, the area that is done by the first revolution is given by $A_1 = \int_0^{2\pi}\int_0^{t^2} g(r,t) r\,dr\,dt$, where $g(r,t)$ is the probability density function of the target position. And the area between the first and second revolution is given by $A_2 = \int_0^{2\pi}\int_{t^2}^{(t+2\pi)^2} g(r,t) r\,dr\,dt$. Similarly, the area between the $(i-1)$th and ith revolution is given by $A_i = \int_0^{2\pi}\int_{(t+2(i-1)\pi)^2}^{(t+2i\pi)^2} g(r,t) r\,dr\,dt$.

In this work, the search space is a standard Euclidean 2-space E with points designated by ordered pairs (x, y). We aim now to evaluate D, that is, the expected value of the time for detecting the target.

Theorem 1. *If the searcher moves along a parabolic spiral curve starting its motion from random point (X_0, Y_0), then the expected value of detecting the target is given by*

$$\begin{aligned} D \le 2\pi \Bigg(& \int_0^{2\pi}\int_0^{t^2} g(r,t) r\,dr\,dt \\ & + \sum_{i=1}^{\infty} i \left[\int_0^{2\pi}\int_{(t+2(i-1)\pi)^2}^{(t+2i\pi)^2} g(r,t) r\,dr\,dt \right] \Bigg). \end{aligned} \tag{5}$$

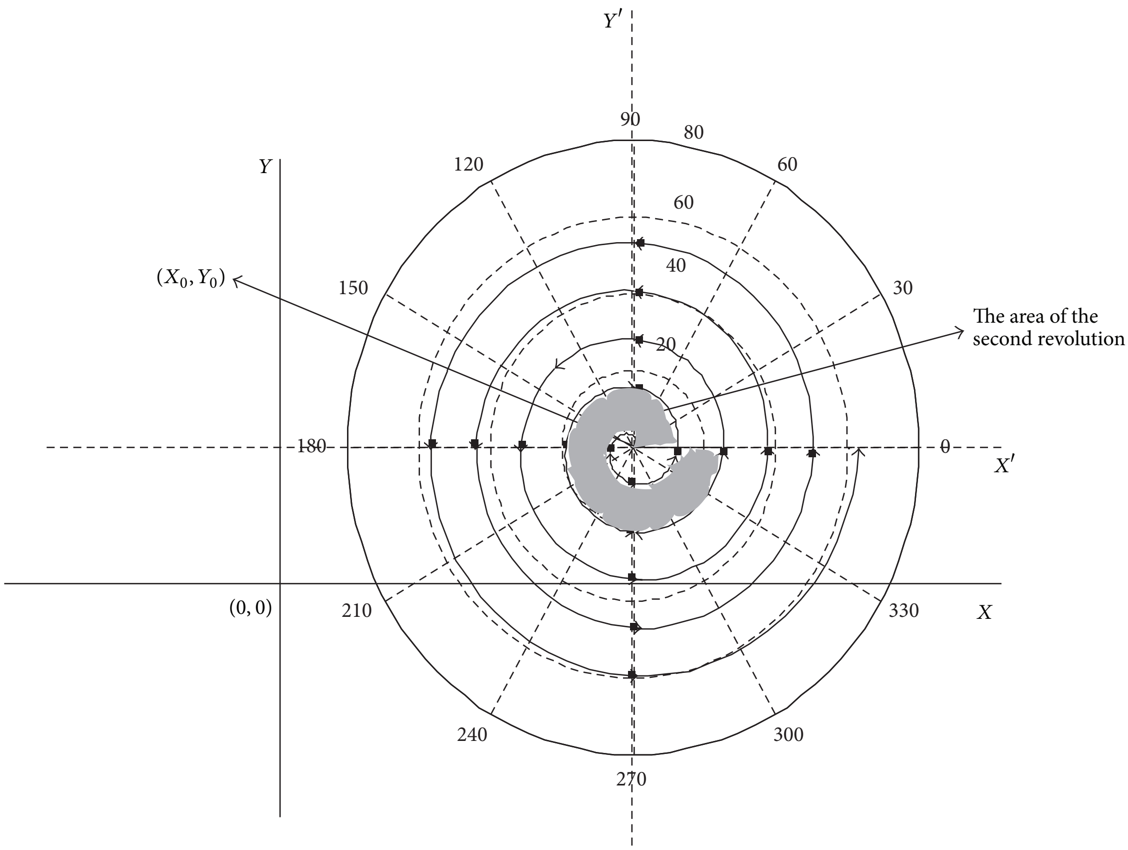

Figure 1: The search path of the searcher.

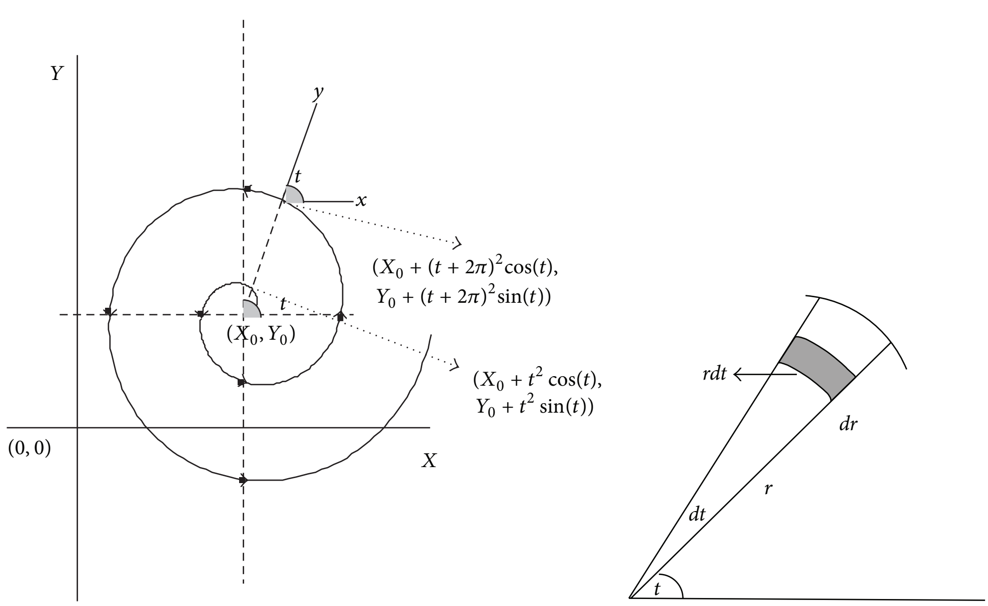

Figure 2: (a) The position of the searcher at time t and $t + 2\pi$. (b) The area at time dt.

Proof. From our hypothesis we can see that

$$\begin{aligned} D &\le 2\pi \int_0^{2\pi} \int_0^{t^2} g(r,t)\, r dr\, dt \\ &\quad + 4\pi \int_0^{2\pi} \int_{t^2}^{(t+2\pi)^2} g(r,t)\, r dr\, dt \\ &\quad + 6\pi \int_0^{2\pi} \int_{(t+2\pi)^2}^{(t+4\pi)^2} g(r,t)\, r dr\, dt + \cdots \\ &= 2\pi \left(\int_0^{2\pi} \int_0^{t^2} g(r,t)\, r dr\, dt \right. \\ &\quad \left. + \sum_{i=1}^{\infty} i \left[\int_0^{2\pi} \int_{(t+2(i-1)\pi)^2}^{(t+2i\pi)^2} g(r,t)\, r dr\, dt \right] \right). \end{aligned} \tag{6}$$

□

If we consider that the target has been detected after n-revolutions, $n > 1$, then we have

$$\begin{aligned} D &\le 2\pi \left(\int_0^{2\pi} \int_0^{t^2} g(r,t)\, r dr\, dt \right. \\ &\quad \left. + \sum_{i=1}^{n} i \left[\int_0^{2\pi} \int_{(t+2(i-1)\pi)^2}^{(t+2i\pi)^2} g(r,t)\, r dr\, dt \right] \right). \end{aligned} \tag{7}$$

It is noticed from (7) that D depends on n. Hence, there exists a relationship between the random distance H (the distance between the target position (X, Y) and the searcher starting point (X_0, Y_0)) and the number of revolutions n.

Theorem 2. *Assume that the searcher made n complete revolutions; then L (the distance from (X_0, Y_0) to the searcher) is given by $(t + 2n\pi)^2$; n is a natural number.*

Proof. The jump in the first revolution is given by $(t^2 - 0)$ and the jump in the second revolution is given by $(t + 2\pi)^2 - t^2$. In addition, the jump in the last revolution is given by $(t + 2n\pi)^2 - (t + 2(n-1)\pi)^2$. Then, the distance from (X_0, Y_0) to the searcher is given by

$$\begin{aligned} L &= \left[t^2 - 0\right] + \left[(t+2\pi)^2 - t^2\right] + \left[(t+4\pi)^2 - (t+2\pi)^2\right] \\ &\quad + \cdots + \left[(t+2n\pi)^2 - (t+2(n-1)\pi)^2\right] \\ &= t^2 + 4\pi^2 + 4t\pi + 12\pi^2 + 4t\pi + 20\pi^2 + 4t\pi + 28\pi^2 \\ &\quad + 4t\pi + \cdots + (8n-4)\pi^2 + 4t\pi \\ &= t^2 + 4nt\pi + 4\pi^2 \left[1 + 3 + 5 + 7 + \cdots + (2n-1)\right] \\ &= t^2 + 4nt\pi + 4\pi^2 n^2 = (t + 2n\pi)^2. \end{aligned} \tag{8}$$

□

3. The Case of a Target Position Given by Circular Normal Distribution

Stone et al. [18] studied more interesting problem; that is, on June 2009, an Airbus 330–200 with 228 passengers and crew of the Air France Flight 447 is disappeared over the South Atlantic during a night flight from Rio de Janeiro to Paris. An international air and surface search effort recovered the first wreckage on June 6 five and one half days after the accident. More than 1000 pieces of the aircraft and 50 bodies were recovered and their positions logged. A French submarine as well as French and American research teams searched acoustically for the underwater locator beacons on each of the two flight recorder's (black boxes) for 30 days from June 10 to July 10, 2009, with no results. They described this problem and analyzed the results of this analysis. The analysis shows that all impact points are contained within a 20 nautical mile radius circle from the point at which the emergency situation began. The results of this analysis are represented by a second distribution which is circular normal with center at the last known position. One of the important advantages of circular normal distribution is that they are sensitive to shifts in the centering of the searching process.

Thus, we assume that the position of the target has a bivariate normal distribution with parameters σ_1 and σ_2. Let (X, Y) give the initial target's actual position. Then X is normally distributed with mean 0 and standard deviation σ_1. In addition, X is independent of Y, which is normally distributed with mean 0 and standard deviation σ_2. Let

$$f(x,y) = \frac{1}{2\pi\sigma_1\sigma_2} \exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_1^2} + \frac{y^2}{\sigma_2^2}\right)\right], \quad \text{for } (X,Y) \in \mathbb{Q} \tag{9}$$

be the probability density function of the bivariate normal distribution, where $\mathbb{Q}$ denotes a standard Euclidean 2-space. Thus, the distribution of error in the navigation system yields f as given in (10) for the density of the target distribution. If $\sigma_1 = \sigma_2 = \sigma$, then (9) becomes

$$f(x,y) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(x^2 + y^2)}{2\sigma^2}\right], \quad \text{for } (X,Y) \in \mathbb{Q}, \tag{10}$$

and the target distribution is called circular normal. Consequently, (7) becomes

$$\begin{aligned} D &\le 2\pi \left(\left[-\frac{1}{\pi} \int_0^{2\pi} \int_0^{t^2} \exp\left(\frac{-r^2}{2\sigma^2}\right) d\left(\frac{-r^2}{2\sigma^2}\right) dt \right] \right. \\ &\quad \left. + \sum_{i=1}^{n} 2i \left[-\frac{1}{\pi} \int_0^{2\pi} \int_{(t+2(i-1)\pi)^2}^{(t+2i\pi)^2} \exp\left(\frac{-r^2}{2\sigma^2}\right) d\left(\frac{-r^2}{2\sigma^2}\right) dt \right] \right) \end{aligned}$$

Table 1: Values of D given the values of σ and $n = 5, 10, 15, \ldots, 45, 50$.

σ	D	σ	D	σ	D	σ	D
	59.18550725		58.97643164		58.81600228		58.68075407
	184.8492134		184.6401378		184.4797084		184.3444602
	389.0527359		388.8436603		388.6832309		388.5479827
	671.7960748		671.5869992		671.4265698		671.2913216
0.5	1033.079230	1	1032.870154	1.5	1032.709725	2	1032.574477
	1472.902202		1472.693126		1472.532697		1472.397449
	1991.264990		1991.055914		1990.895485		1990.760237
	2588.167594		2587.958518		2587.798089		2587.662841
	3263.610015		3263.400939		3263.240510		3263.105262
	4017.592251		4017.383175		4017.222746		4017.087498

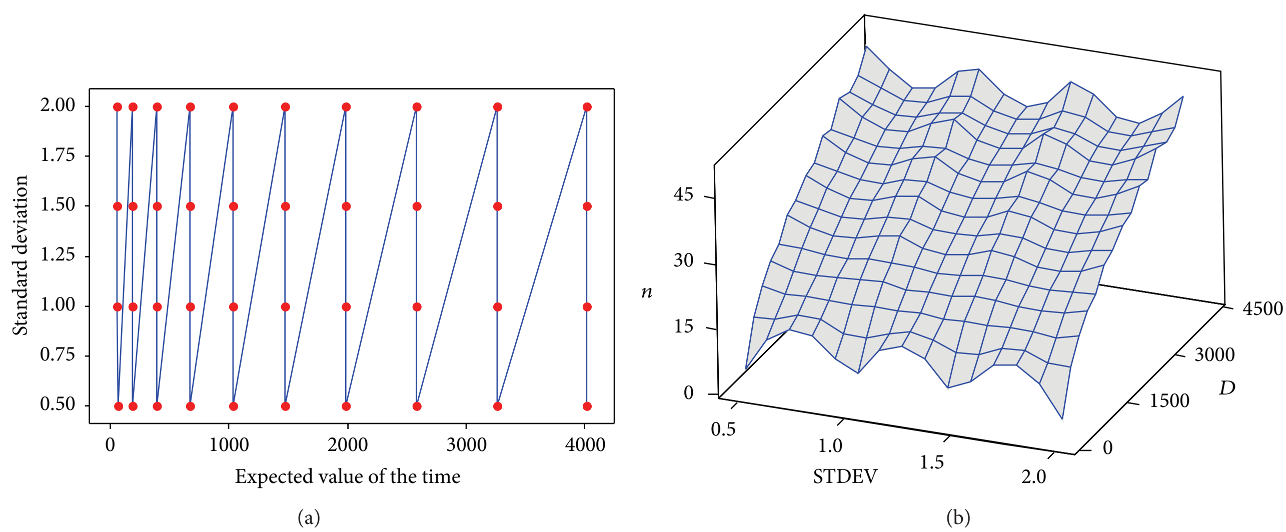

Figure 3: The curves represent the relation between D, σ, and n. In the part (a) Scatter plot of σ versus D and in the part (b) Surface Plot of n versus D, σ.

$$\begin{aligned}
&= -2\Bigg(\left[\int_0^{2\pi}\left[\exp\left(\frac{-r^2}{2\sigma^2}\right)\right]_0^{t^2} dt\right] \\
&\qquad + \sum_{i=1}^{n} i\left[\int_0^{2\pi}\left[\exp\left(\frac{-r^2}{2\sigma^2}\right)\right]_{(t+2(i-1)\pi)^2}^{(t+2i\pi)^2} dt\right]\Bigg) \\
&= 2\Bigg(\left[\int_0^{2\pi}\left[1-\exp\left(\frac{-t^4}{2\sigma^2}\right)\right] dt\right] \\
&\qquad + \sum_{i=1}^{n} i\left[\int_0^{2\pi}\left[\exp\left(\frac{-(t+2(i-1)\pi)^4}{2\sigma^2}\right)\right.\right. \\
&\qquad\qquad \left.\left. - \exp\left(\frac{-(t+2i\pi)^4}{2\sigma^2}\right)\right] dt\right]\Bigg).
\end{aligned} \tag{11}$$

Using Mathematica 7 programme, we get

$$\begin{aligned}
D \le 2\Bigg(& 2\pi + \frac{\Gamma\left(1/4, 8\pi^4/\sigma^2\right) - \Gamma(1/4)}{22^{3/4}\left(1/\sigma^2\right)^{1/4}} \\
&+ \frac{\pi}{2}\sum_{i=1}^{n} i\left[(i-1)\,\mathrm{Ei}\left(\frac{3}{4}, \frac{8((i-1)\pi)^4}{\sigma^2}\right)\right. \\
&\qquad - 2i\mathrm{Ei}\left(\frac{3}{4}, \frac{8(i\pi)^4}{\sigma^2}\right) \\
&\qquad \left. + (1+i)\,\mathrm{Ei}\left(\frac{3}{4}, \frac{8((i+1)\pi)^4}{\sigma^2}\right)\right]\Bigg),
\end{aligned} \tag{12}$$

where Ei gives the exponential integral function.

Example 3. Suppose that the target position (black boxes, etc.) follows the circular normal distribution with probability density function (10). For many values of σ and $n = 5, 10, 15, \ldots, 45, 50$, we get the expected value of the time for detecting the target as in Table 1.

It appears from numerical calculations that the value of D has small increments with decreasing the value of the parameter σ as in Figure 3.

Naturally, σ and D depend on the target position. This means that D does not depend only on σ but also depends on n, where from Theorem 2 we have $n = (\sqrt{L} - t)/2\pi$.

4. One-Dimensional Case

In one-dimensional case, the searcher moves with the model

$$x(t) = X_0 + t^2 \cos(t), \qquad y(t) = Y_0. \tag{13}$$

This model is produced from the projection of model (1) on x-axis; that is, $\Pi_1(x(t), y(t)) = x(t) = X_0 + t^2 \cos(t)$. Consequently, the speed of the searcher will be given as

$$v = \dot{x} = 2t \cos(t) - t^2 \sin(t). \tag{14}$$

At $\dot{x} = 0$, we have $t(2\cos(t) - t\sin(t)) = 0$, which leads to $t = 2\cot(t)$, $t \neq 0$. We choose only positive values of t. From (13), the parabolic spiral search will ride to the linear search problem. The method of linear search is an important methodology to find a lost target on the real line. The position of the target is represented by the random variable X where X has a probability density function $w(x)$. The searcher moves continuously along the real line in both directions of the starting point a_0. The searcher would conduct his search in the following manner (see Figure 4): start at $a_0 = 0$ and go to the left (right) as far as a_1. Then, turn back to explore the right (left) part of $a_0 = 0$ as far as a_2. Retrace the steps again to explore the left (right) part of a_1 as far as a_3, and so forth.

The searcher starts from any point on the real line with uniform velocity and tries to find the target in minimal expected time. It is assumed that the searcher can change the direction of its motion without any loss of time. The target can be detected only if the searcher reaches the target. This problem has been studied extensively in many variations, mostly by A. Beck and M. Beck [19, 20], Reyniers [21, 22], and Balkhi [23, 24]. On the other hand, when the target moves on the real line according to a known random process, the searcher moves continuously along the line in both directions of the starting point until the target is met. In an earlier work, this problem has been studied by Mohamed [25], El-Rayes et al. [26], Washburn [27], and Stone [28]. Recently, Mohamed et al. [29, 30] discussed this problem when the target moves randomly on one of real lines. They formulated a search model and found the conditions under which the expected value of the first meeting time between one of the searchers and the target is finite. Furthermore, they have shown the existence of the optimal search plan that minimized the expected value of the first meeting time and found it.

The problem studied here is more applicable than the linear search problem because the speed of the searcher depends on t as in (14). Equations (11) and (12) show that the searcher moves on the line with the linear search technique as in Figure 5.

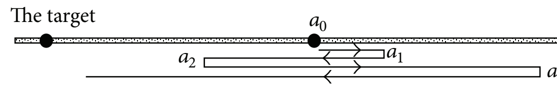

FIGURE 4: The generalized linear search path.

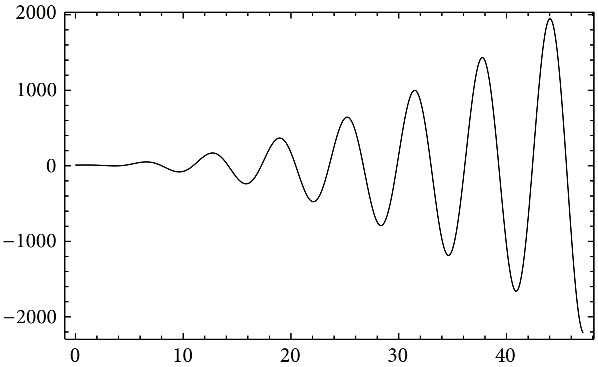

FIGURE 5: The search path of the searcher in one-dimensional case with speed $v = 2t\cos(t) - t^2 \sin(t)$.

In this case, to get the time values of the turning points as in Figure 4, we let $y = t$ and we use the numerical method to solve the system

$$y = t, \qquad y = 2\cot(t). \tag{15}$$

For simplicity, let $X_0 = Y_0 = 0$. Thus, the roots of this system are the intersection points of the line $y = t$ and the equation $y = 2\cot(t)$ as in Figure 6.

Figure 6 gives an illustration of the roots (the time of the turning points) through the time interval $]0, T]$. From (14) the distances h_i that the searcher does through the times t_i, $i = 1, 2, \ldots$ are given by

$$\begin{aligned} |h_i| &= \left| t_i \left(2t_i \cos(t_i) - t_i^2 \sin(t_i) \right) \right| \\ &= \left| t_i^2 \left(2\cos(t_i) - t_i \sin(t_i) \right) \right|. \end{aligned} \tag{16}$$

Now, we can easily obtain the expected value of the time for detecting the target, where the target position has known distribution.

Theorem 4. *If the target location has known distribution with probability density function $w(x)$, then the expected value of the time for detecting the target is given by*

$$D \leq \sum_{\substack{i=1 \\ i \text{ is odd}}}^{\infty} t_i \int_{h_{i-1}}^{h_i} w(x)\, dx + \sum_{\substack{i=2 \\ i \text{ is even}}}^{\infty} t_i \int_{h_{i-2}}^{h_i} w(x)\, dx. \tag{17}$$

Proof. If the target lies in $]0, h_1]$, then $D \leq t_1 \int_0^{h_1} w(x)dx$, and if it lies in $]0, h_2]$, then $D \leq t_2 \int_0^{h_2} w(x)dx$. Also, if the target

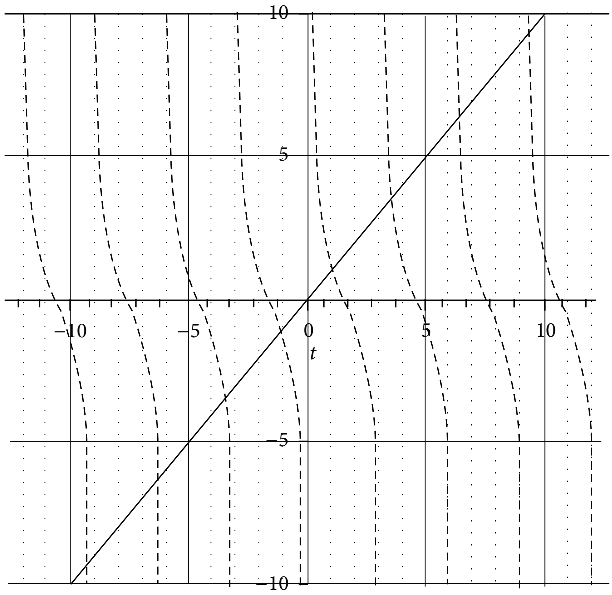

Figure 6: The turning point in one-dimensional case.

Table 2: The time values of the turning points through the time interval $]0, 10]$ and its corresponding distances.

i	t_i	$\|h_i\|$
1	1.07687	0.151 39
2	3.6436	21.522
3	6.57833	80.893
4	9.62956	179.62

Table 3: Values of D given the values of σ.

σ	D
0.5	4.455 6
1	4.779 0
1.5	4.888 8
2	4.944 0
2.5	4.977 1

lies in $]h_1, h_3]$, then $D \le t_3 \int_{h_1}^{h_3} w(x)dx$ and if it lies in $]h_2, h_4]$ then $D \le t_4 \int_{h_2}^{h_4} w(x)dx$, and so forth. Consequently, we have

$$\begin{aligned} D &\le t_1 \int_0^{h_1} w(x)\,dx + t_2 \int_0^{h_2} w(x)\,dx + t_3 \int_{h_1}^{h_3} w(x)\,dx \\ &\quad + t_4 \int_{h_2}^{h_4} w(x)\,dx + \cdots \\ &= \sum_{\substack{i=1 \\ i \text{ is odd}}}^{\infty} t_i \int_{h_{i-1}}^{h_i} w(x)\,dx + \sum_{\substack{i=2 \\ i \text{ is even}}}^{\infty} t_i \int_{h_{i-2}}^{h_i} w(x)\,dx. \end{aligned} \quad (18)$$

□

Example 5. Let the previous illustrative Example 3 and make a projection on x-axis, then (10) becomes

$$f(x, y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{x^2}{2\sigma^2}\right], \quad \text{for } -\infty \le x \le \infty. \quad (19)$$

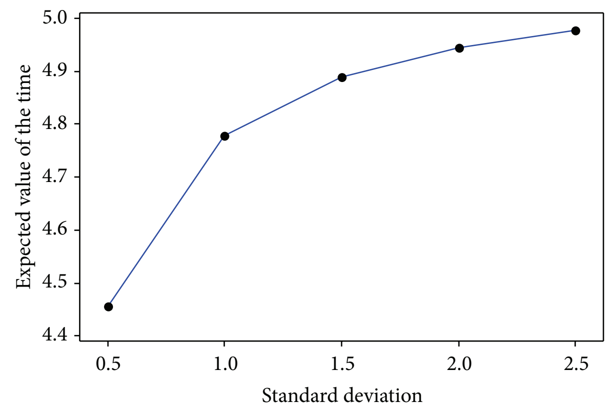

Figure 7: The relation between D and σ in one-dimensional case.

In addition, we let the target be detected through the time interval $]0, 10]$; then by using Figure 6 and (16) we can get the time values of the turning points and its corresponding distances as in Table 2.

For many values of σ we can get the values of D as in Table 3.

It is clear that from numerical calculations the value of D increases with increasing the value of the parameter σ as in Figure 7.

5. Conclusion and Future Work

A parabolic spiral search plan for a randomly located target in the plane has been presented, where the target position is given by independent random variables X, Y. We get the expected value of the time for detecting the target in Theorem 1. In Theorem 2, we show that the distance between the target position and the searcher starting point depends on the number of revolutions, where the complete revolution is done when $t = 2\pi$.

To study this problem in one-dimensional case, we make a projection of the searcher path on x-axis. We find that the case of one-dimension of this problem rides to the generalized linear search problem. In Theorem 4, we can easily get the expected value of the time for detecting the target after knowing the values of the turning points, where the target position is given by a random variable X. The effectiveness of this technique is illustrated using numerical examples assuming circular normal distributed estimates of the target position.

In future research, it seems that the proposed model will be extendible to the multiple searchers case by considering the combinations of movement of multiple targets.

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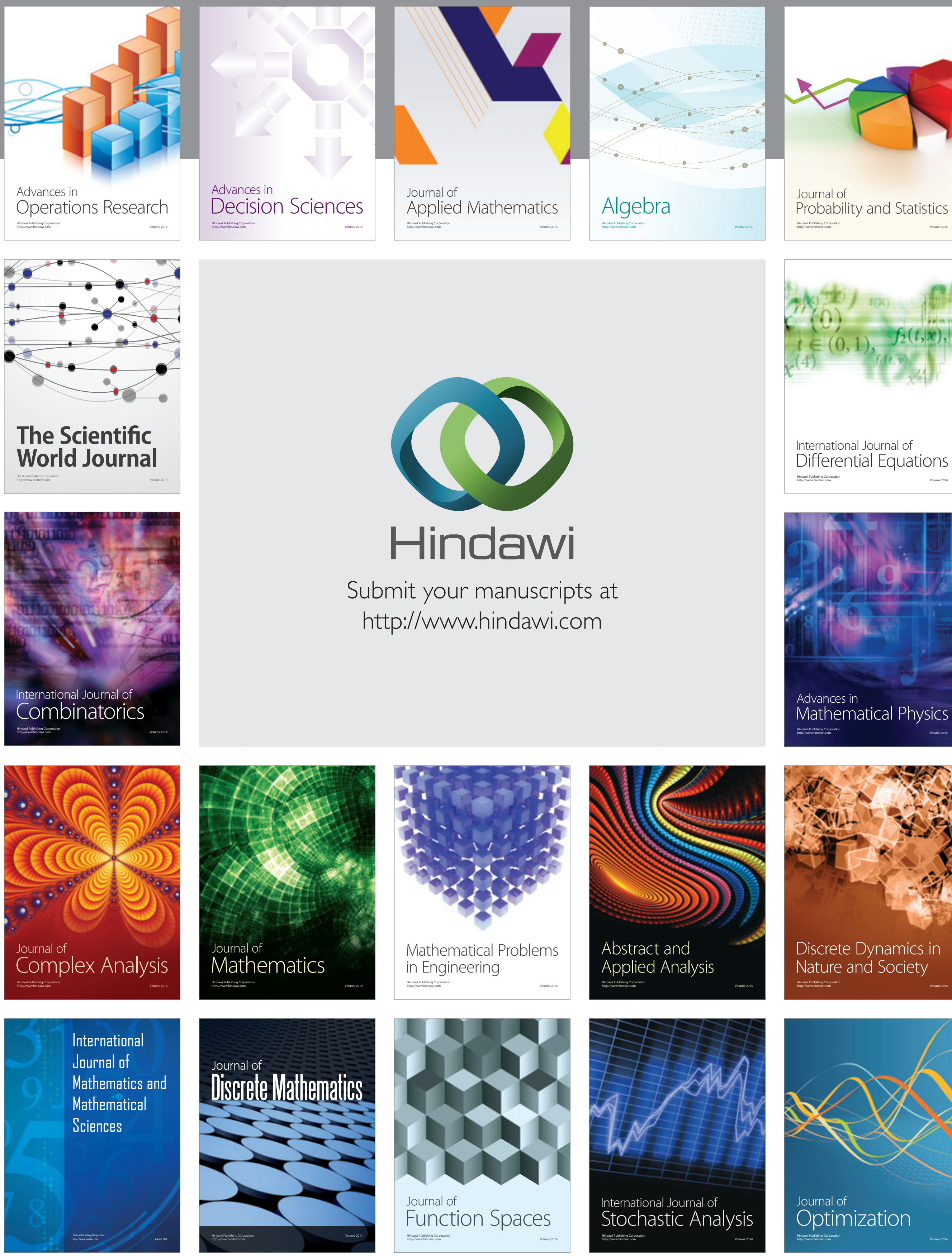
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