Introduction to Isogeometric Analysis (IGA)

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Motivation

The key idea of isogeometric analysis (IGA) (Hughes *et al.* 2005) is to approximate the physical fields with the same basis functions as that used to generate the CAD model.

1. Alleviate meshing burden

2. Direct communication with geometry

- Exact representation of geometry.
- Shape sensitivity analysis.
- Shape optimization.

3. High order continuous field

4. Flexible refinement scheme.

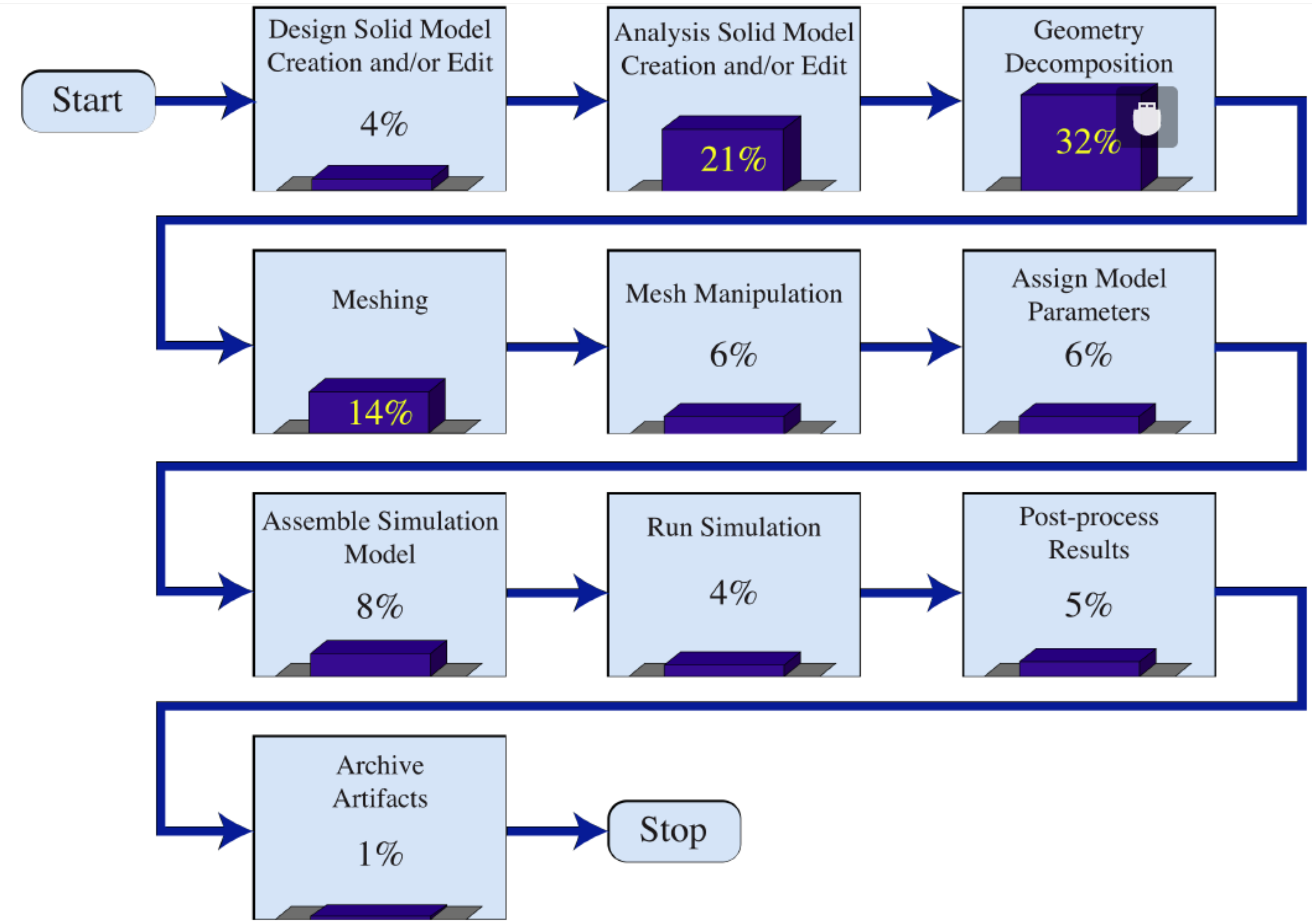
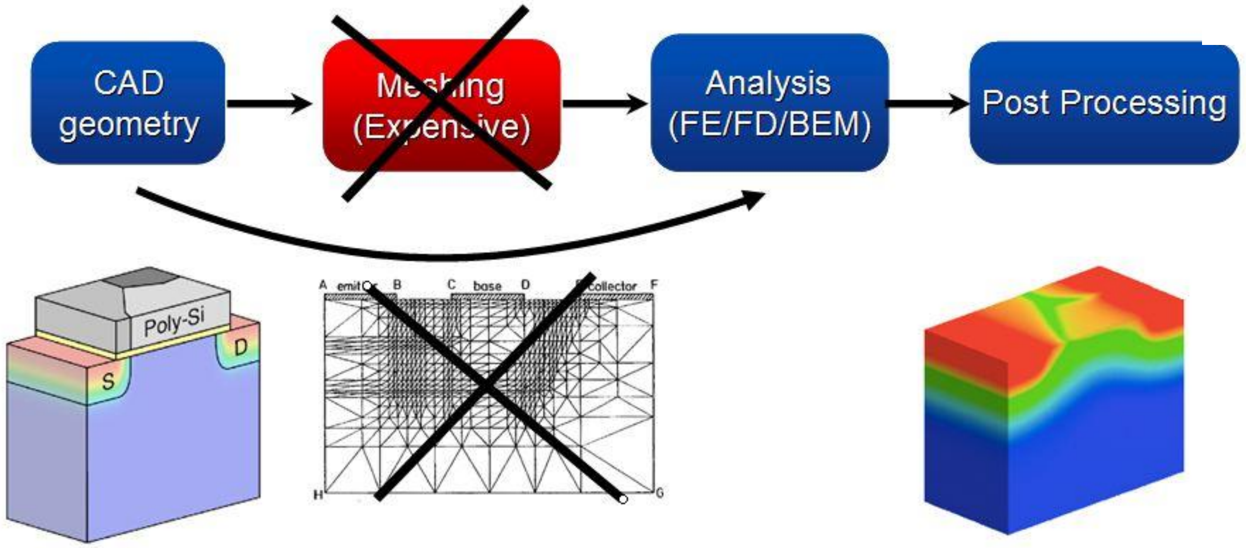


Fig 1. Conventional FEA procedure (Sandia National Laboratories)

Isogeometric Analysis Process



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NURBS (Non-Uniform Rational B-splines)

NURBS is a mapping from parametric space to physical space.

$$\mathbf{C}(\xi) = \sum_{i=1}^{n} R_{i,p}(\xi)\mathbf{B}_i .$$

1. Knot vector ($\mathcal{X}$)

$$\Xi = \{\xi_1, \ \xi_2, \ \cdots, \ \xi_{n+p+1}\}$$

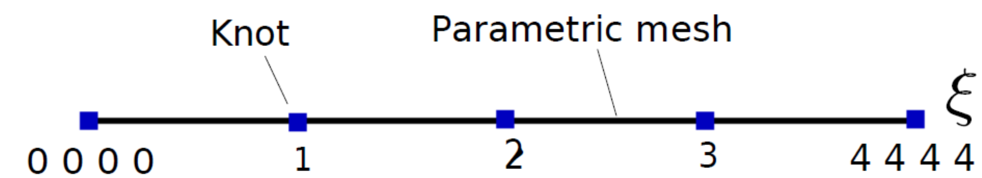

Fig 1: Knot vector

2. Control points ($\mathbf{B}$)

3. NURBS basis function (R)

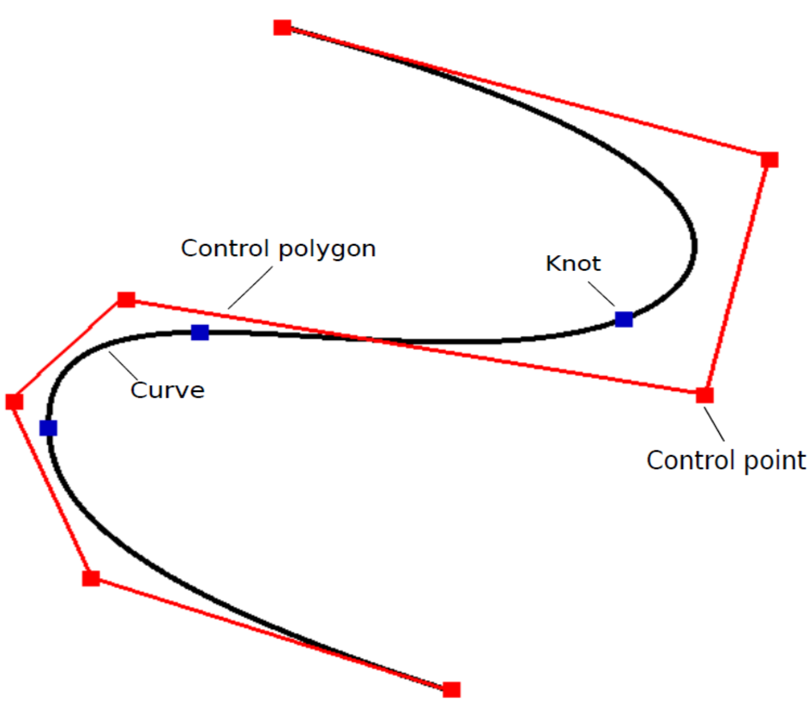

Fig 2: NURBS curve

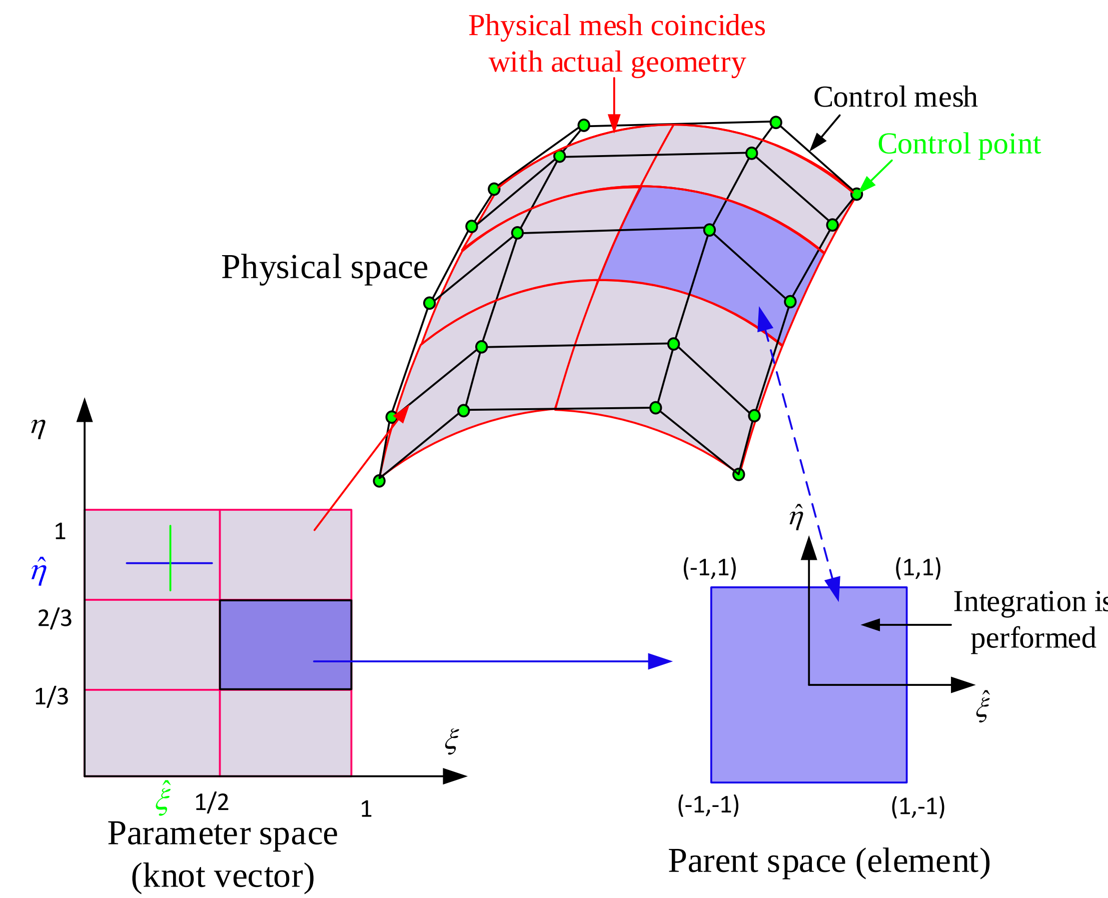

Fig 3: NURBS surface

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B-spline basis functions

B-spline basis function:

$$N_{A,0}(\xi) = \begin{cases} 1 & \text{if } \xi_A \leqslant \xi < \xi_{A+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{A,p}(\xi) = \frac{\xi - \xi_A}{\xi_{A+p} - \xi_A} N_{A,p-1}(\xi) + \frac{\xi_{A+p+1} - \xi}{\xi_{A+p+1} - \xi_{A+1}} N_{A+1,p-1}(\xi)$$

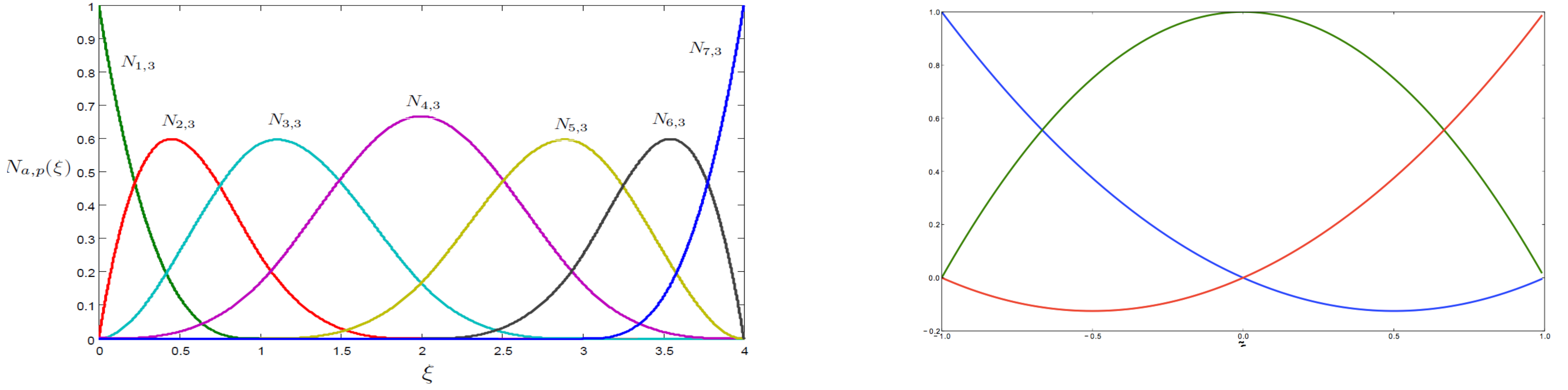

Fig 1: B-spline basis (left) vs quadratic polynomials (right)

- Linear independence**;**
- Partition of unity
- Non-negative*;*
- Locally supported;
- No *Kronecker delta* property

NURBS basis functions

NURBS basis function:

$$R_{A,p}(\xi) = \frac{N_{A,p}(\xi) w_A}{\sum_{A=1}^{n} w_A N_{A,p}(\xi)}$$

where N is the B-spline basis function, and w is the weight associated with the control points.

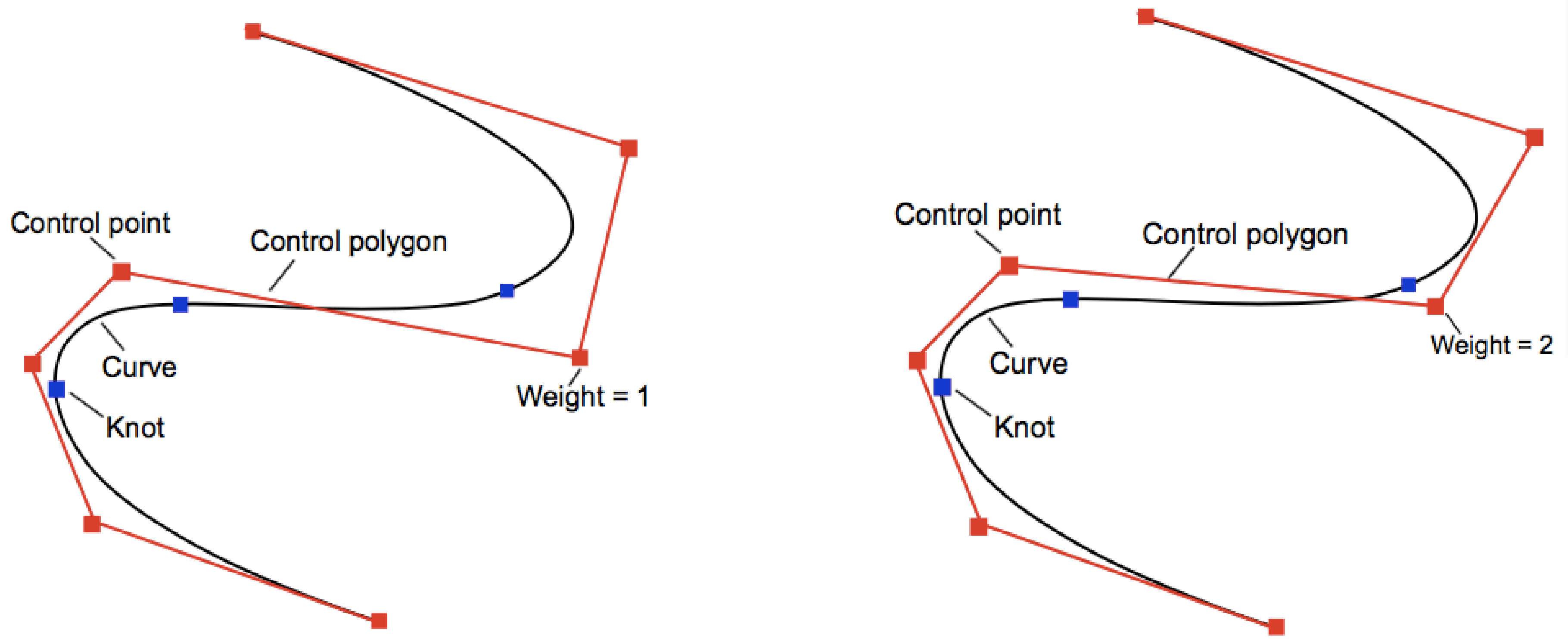

Fig 1: The influence of weights on the geometry

Tensor product property

$$R^{p,q}_{A,B}(\xi,\eta) = \frac{N_{A,p}(\xi) M_{B,q}(\eta) w_{A,B}}{\sum_{\hat{A}=1}^{n} \sum_{\hat{B}=1}^{m} N_{\hat{A},p}(\xi) M_{\hat{B},q}(\eta) w_{\hat{A},\hat{B}}}$$

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Applications

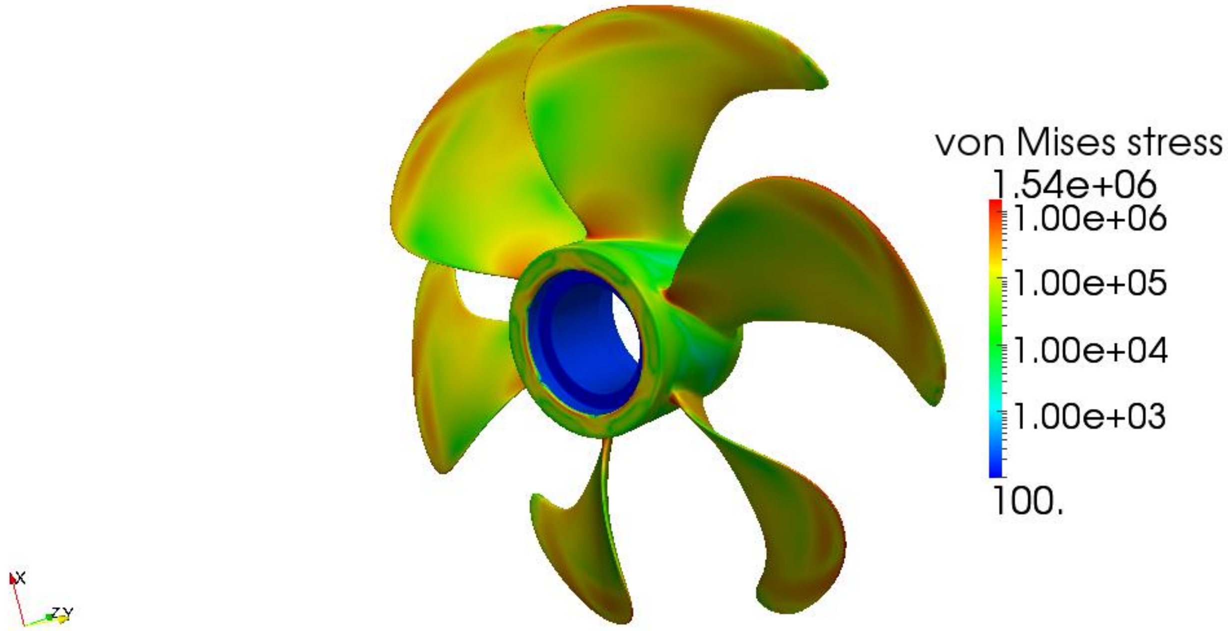

Fig 1. Linear elasticity (propeller)

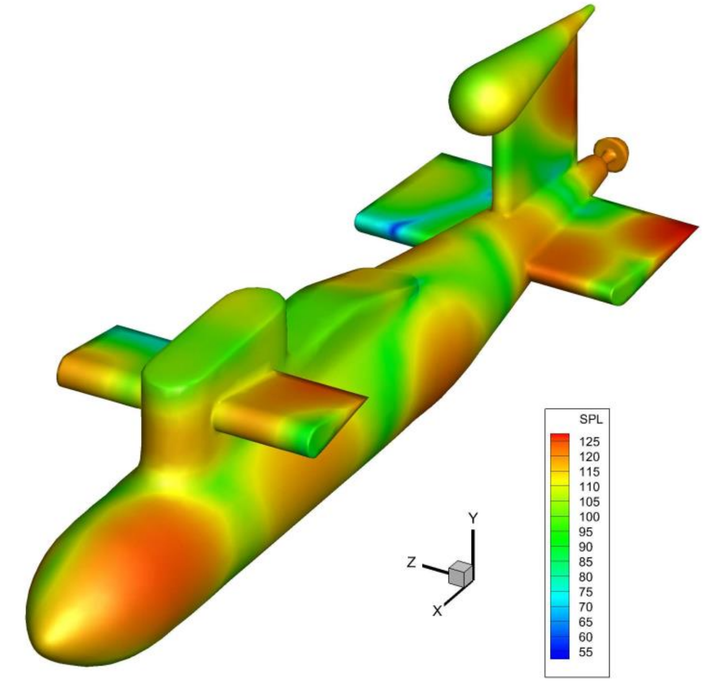

Fig 2. Exterior Acoustics (submarine)

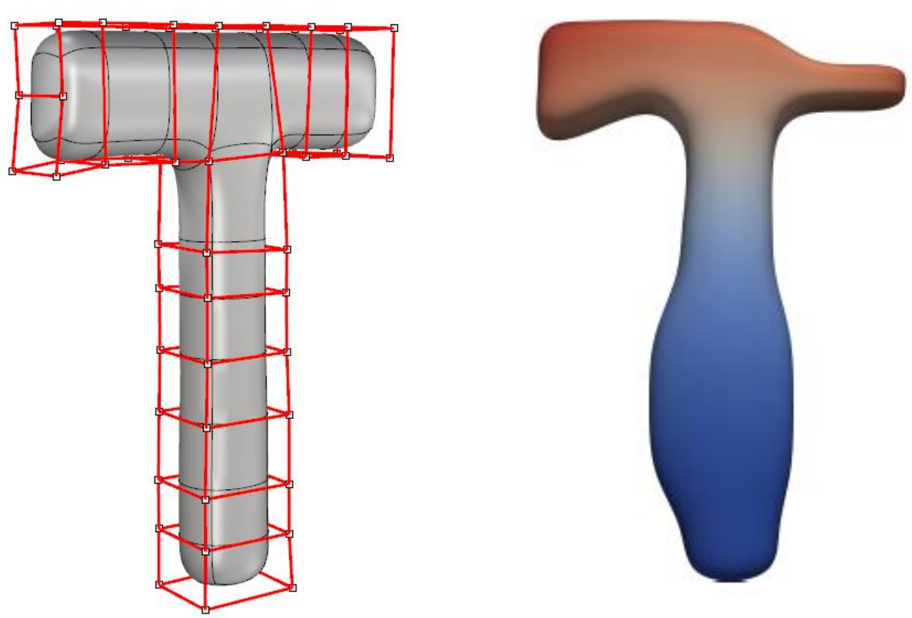

Fig 1. Shape optimization

Fig 4. Crack propagation

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