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Soteriades, Andreas; Rowland, K.; Roberts, D. J.; Stott, A. W.

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A.D. SOTERIADES[1], K. ROWLAND[2], D.J. ROBERTS[3] and A.W. STOTT[4]

[1] Bangor University, Thoday Building, Deiniol Road, Bangor, Gwynedd, LL57 2UW, UK. a.d.soteriades@bangor.ac.uk

[2] Kingshay Farming & Conservation Ltd, Bridge Farm, West Bradley, Glastonbury, Somerset, BA6 8LU

[3] Scotland's Rural College, Barony Campus, Parkgate, Dumfries, DG1 3NE, UK

[4] Scotland's Rural College, Peter Wilson Building, Kings Buildings, West Mains Road, Edinburgh EH9 3JG, UK

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Competing interests

None

Ethics statement

We hereby state that have paid due regard to ethical considerations relating to the work reported and the work contains no defamatory or unlawful statements.

Abstract

Efficiency benchmarking is a well-established way of measuring and improving farm performance. An increasingly popular efficiency benchmarking tool within agricultural research is Data Envelopment Analysis (DEA). However, the literature currently lacks

sufficient demonstration of how DEA could be tuned to the needs of the farm advisor/extension officer, rather than of the researcher. Also, the literature is flooded with DEA terminology that may discourage the non-academic practitioner from adopting DEA. This paper aims at making DEA more accessible to farm consultants/extension officers by explaining the method step-by-step, visually and with minimal use of specialised terminology and mathematics. Then, DEA's potential for identifying cost-reducing and profit-making opportunities for farmers is demonstrated with a series of examples drawn from commercial UK dairy farm data. Finally, three DEA methods for studying efficiency change and trends over time are also presented. Main challenges are discussed (e.g. data availability), as well as ideas for extending DEA's applicability in the agricultural industry, such as the use of carbon footprints and other farm sustainability indicators in DEA analyses.

1. Introduction

A commonly used measure of efficiency is stated in the ratio of output to input (Cooper *et al*., 2007), and is widely used in benchmarking procedures to identify best-practice management for a given farming system (Fraser and Cordina, 1999). Such procedures, henceforth referred to as 'efficiency benchmarking', are instrumental for guiding farmers on how to reduce costs and resource use, increase profitability and minimize environmental impacts of production (Fraser and Cordina, 1999). This paper demonstrates how an efficiency benchmarking tool that is well-established in agricultural research may be used to solve actual problems facing (dairy) farm managers.

Limitations of conventional efficiency benchmarking

In the farming industry, benchmarking is typically effected by reporting average values (e.g. of input use, production, costs and prices, input-output ratios) from a group of farms with similar characteristics, so that farmers from that group may compare these values to their own performance (AHDB Dairy, 2014; Kingshay, 2017). This type of more 'conventional' benchmarking is myopic and performance indicators such as simple single ratios may mislead when performance and profitability are determined by interrelated multifactorial processes (Cooper *et al.*, 2007). For example, good feed efficiency may be achieved at the expense of inefficient use of labour and nitrogen fertilizer, and at higher replacement rates, resulting in higher costs/lower profits and higher environmental impacts. Moreover, some of these multifactorial processes have public good dimensions, which consumers and society increasingly expect farmers to account for, and they may even reward their delivery if objective metrics can be found that prove contribution while ensuring that the farmer is not left at a disadvantage (Foresight, 2011). Although the agricultural industry is increasingly responding to these demands with novel tools accounting for carbon foot-printing data (Alltech E-CO2, 2017; SAC Consulting, 2017) or other environmental, social and economic indicators (BASF, 2012), developing holistic indicators of farm efficiency performance is mainly confined to academic research, where significant developments have been made with the efficiency benchmarking method Data Envelopment Analysis (DEA; Cooper *et al*., 2007)[1].

Efficiency benchmarking with Data Envelopment Analysis

DEA is becoming extremely popular in agricultural science (Emrouznejad and Yang, 2018), owing to its numerous virtues. DEA gives a more meaningful index of comparative performance that is likely to identify worthwhile opportunities for improvement. Indeed, DEA replaces multiple efficiency ratios by a single weighted sum of outputs over the weighted sum of inputs or by a single 'profit function' (i.e. the weighted sum of outputs *minus* the weighted sum of inputs), with the weights being calculated by the model itself, so that no subjective weighting choices or input and output pricing are necessary (Cooper *et al*., 2007). Therefore, DEA simplifies the analysis by reducing the need to take into account a range of performance indicators (e.g. input-output ratios) and reduces the danger of improving one performance indicator to the detriment of another (which may not even be monitored; Bowlin *et al*., 1984; Fraser and Cordina, 1999).

Another advantage of DEA is that it obviates the need to resort to 'average' values that many of the aforementioned industry tools rely on for benchmarking farm performance. Instead, DEA identifies benchmark farms for *each* farm in the sample and

[1] For an introduction to DEA, see also the excellent textbook by Bogetoft and Otto (2011).

indicates the adjustments that this farm should make to its inputs and outputs to become as efficient as its benchmarks (Cooper *et al*., 2007).

Scope for using DEA as a (dairy) farm management tool

Despite DEA's attractive features and, as shown later, its relative simplicity, it is an ongoing challenge to move the method from the academic to the practitioners' world (Paradi and Sherman, 2014). Paradi and Sherman (2014) identified key reasons why managers are reluctant to adopting DEA, including (i) excessive DEA jargon; (ii) ineffective/insufficient communication/explanation of DEA to managers so that they stop viewing it as a 'black box'; (iii) data availability; and (iv) limited emphasis on managerial applications.

Indeed, the more than 40 peer-reviewed DEA studies of the dairy sector (with which this study is concerned; see Appendix A in Emrouznejad and Yang, 2018; and Appendix I in Soteriades, 2016) mainly explore research questions that *do* inform policy and managerial decision-making, yet *do not* demonstrate how DEA could be tuned to the needs of the farm advisor/extension officer, rather than of the researcher. In our view, two major elements generally missing from DEA dairy studies are the economic (rather than e.g. technical and environmental) insights attached to the DEA models, and the analysis of efficiency over time. Temporal assessments are particularly useful for monitoring performance month-by-month (Kingshay, 2017). Similarly, economic insights are indispensable for decision-making and, unless they are accounted for, a mathematical model (such as DEA) may mean little to a manager (McKinsey & Company, 2017). DEA can help farmers improve economic performance by indicating them how to make best use of their resources, on the one hand, yet, on the other hand, it can be used to guide other priorities such as the improvement of environmental performance (Soteriades *et al*., 2015). This makes DEA a flexible and holistic tool to suit particular objectives for the benefit of both business management and the public good.

Objective

In this study, we demonstrate how DEA can be used to benchmark individual (dairy) farm efficiency performance, as well as indicate the inputs and outputs in which the largest inefficiencies occur. Then, by attaching prices to the inefficiencies, we show how DEA can help guide management actions through a variety of prioritised cost-saving and/or profit-making options for each farm. This deals with point (iv) above. Points (i) and (ii) are addressed by explaining DEA step-by-step and visually, with minimal use of DEA jargon. Formal mathematical formulas describing the DEA model are placed in appendices. Point (iii) is dealt with by using an abundant dairy farm dataset by Kingshay Farming and Conservation Ltd, which also allowed us to demonstrate several temporal DEA approaches of potential interest to farm consultants. We believe that this study provides sufficient insight into how DEA can help identify areas for improvement in (dairy) farm efficiency and so add considerable value to any benchmarking service.

2. Understanding DEA

Numerous DEA models exist with different functions so it is important to choose one that fits the requirements of the problem at hand (Bogetoft and Otto, 2011; Cooper *et al*., 2007). However, most DEA models share two strong advantages: (i) they produce standardized scores between 0 and 1, with unity indicating 100% efficiency and a score less than 1 indicating inefficiency; and (ii) the score is not affected by different measurement units (e.g. milk in L, feed in kg) because DEA uses the data themselves to weight the input and output variables. This study employed a so-called 'additive' model (Cooper *et al*., 2007), which is explained later[2].

The concept of DEA can be more clearly understood when compared with that of linear regression. The latter measures 'central tendency' (expressed by the regression line) and so we can determine how 'far' observations (dairy farms) are from the 'average' (Cooper *et al*., 2007). Contrariwise, DEA constructs an *efficient frontier* (which we will refer to as the *best-practice frontier*) consisting of the best performers in the sample and all other farms are benchmarked against this frontier. Consider, for instance, seven farms A, B, C, D, E, F and G producing a single output (e.g. grain yield) using a single input (e.g. land; Figure

[2] We have chosen not to present and discuss alternative DEA models here to avoid the danger of making our presentation too 'academic' for the 'intelligent lay' non-academic reader. As with any DEA model, the additive model has pros and cons that we believe are irrelevant to the objective of our study.

1). Farms A, B, C, D, E and F form the frontier, i.e. they do not have to further reduce their input and further increase their output to become relatively efficient- they are the best performers. By contrast, farm G is relatively inefficient as it could be producing more output and using less input relative to one or more efficient farms[3,4]. To become relatively efficient, farm G will have to reduce its input and increase its output until it reaches a point on the frontier. DEA measures the efficiency of farm G by detecting the magnitudes of the inefficiencies that this farm exhibits in its input and output. Consequently, DEA will produce an efficiency score for farm G whose magnitude indicates by 'how much' this farm is inefficient in its input and output. This score is *farm-specific* and thus differs from regression that can only indicate by how much farms deviate from the 'average'. Also, with DEA the single-input single-output case can be easily extended to multiple inputs and outputs, contrary to regression, which, in its simplest and most widely-adopted form, cannot handle more than one dependent variable at a time (Bowlin et al., 1984, p.127).

[3] Note that the input-output frontier lies on the northwest of the dataset, enveloping inefficient farms such as G, hence the term data 'envelopment' analysis. This is by contrast with a regression line, which would be passing *between* the points, leaving some above it and some below it.

[4] Also note that the frontier displayed in Figure 1 is piece-wise linear. This is because we have assumed that farms operate under *variable returns to scale*, under which inefficient farms are only compared to efficient farms of a similar size (Fraser and Cordina, 1999). Alternatively, the frontier can be represented by a single straight line. However, this would imply that an increase in a farm's input would result in a proportional increase in its output (Bogetoft and Otto, 2011; Cooper et al., 2007). This assumption is known as *constant returns to scale* and was considered unreasonable in our case. See also Appendix A.

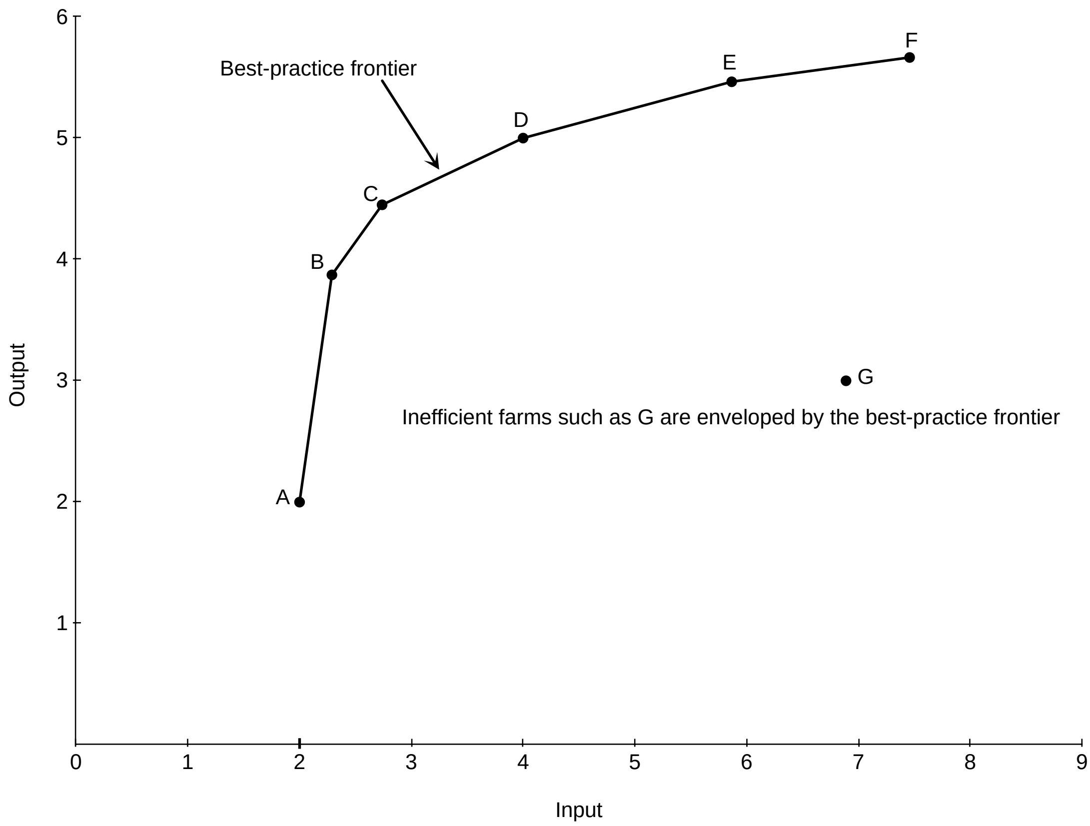

Figure 1: A DEA best-practice frontier ABCDEF and an inefficient farm G in the single-input single-output case

Which efficient farms serve as benchmarks for farm G?

The answer to this question reveals one of DEA's key properties: it can extrapolate from the given dataset by creating 'virtual' or 'synthetic' benchmarks that lie at any point on the frontier ABCDEF (Figure 1; Bogetoft and Otto, 2011). On the one hand, farm G could be benchmarked against, say, efficient farm C or D. On the other hand, it could be benchmarked against a virtual farm represented by a point lying on, say, segment CD. In any case, the benchmark farm's input can be represented by a linear combination of the inputs of farms C and D (see Appendix A).

The above provides an explanation of the idea behind DEA, especially in relation to the construction of the best-practice frontier and the identification of benchmark farms for the farm under evaluation. The additive model is outlined below.

How does the additive model calculate efficiency?

The reason why a farm such as G is inefficient is because it exhibits excess in its input and shortfall in its output relative to its benchmark(s). The excess in inputs and shortfall in outputs represent the *inefficiencies* that G exhibits in its inputs and outputs. These inefficiencies are called *slacks* in the DEA terminology (Cooper *et al*., 2007), but the terms *input inefficiency* and *output inefficiency* will be used in this paper.

The additive model finds the optimal values for the inefficiencies maximizing the total (sum) of input and output inefficiencies and projects farm G onto point C on the frontier. See Figure 2 for a visual representation as well as the Appendices B and C for the mathematical description of the additive model.

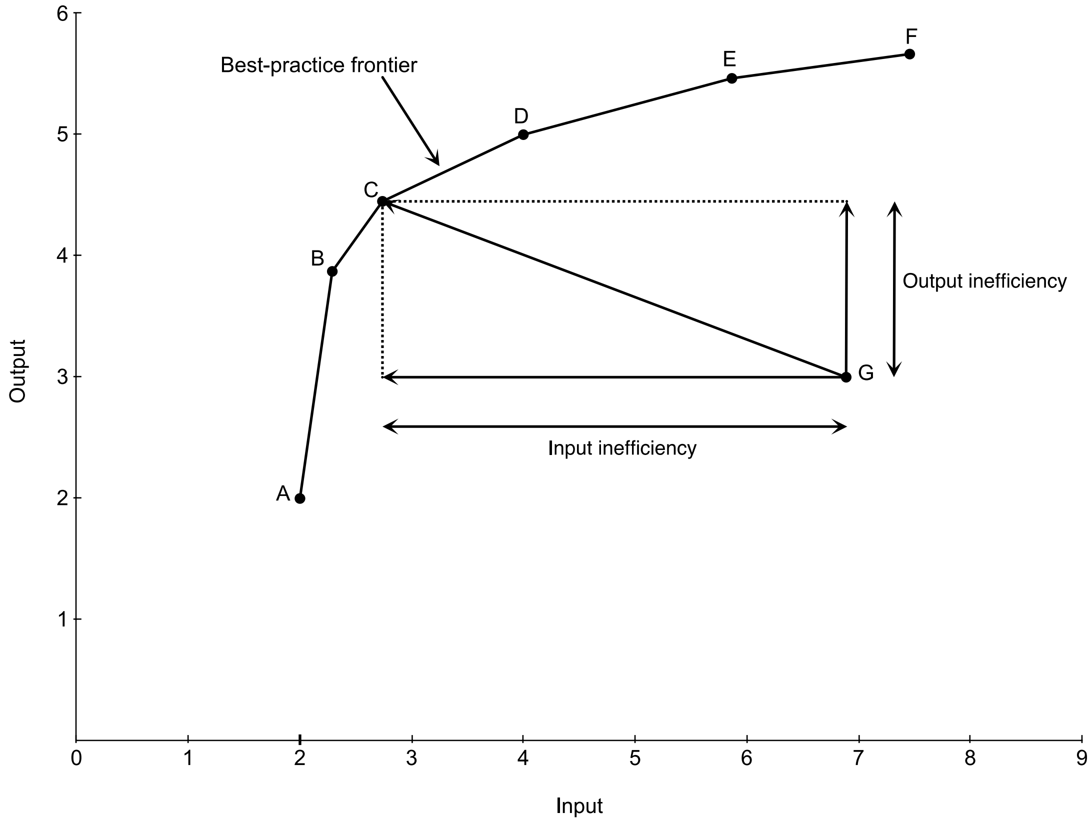

Figure 2: Visual representation of the additive model run for farm G

Before turning to the application with the sample data, it might be more reasonable to consider some of the DEA inputs and outputs as fixed. In this case, the DEA model will not seek to increase/decrease them, yet these inputs and outputs still play a role in shaping the best-practice frontier. This concerns variables that a farmer may not be looking to increase/decrease on the short-term but rather in longer time-horizons. For instance, it might be more appropriate to model cows in herd, forage area and milk yield as fixed, for the following reasons. First, a farmer would for example maintain their herd size fixed and seek to reduce the number of replacements in response to improved output efficiency, rather than reduce the number of cows in the herd. Second, in the short run, it would seem unreasonable to expect that a farmer would reduce their land area. Third, given a low milk price, a farmer would rather increase butterfat and protein rather than milk yield. To illuminate the idea of fixed variables, had the input of farm G (Figure 2) been fixed, this farm would have to move vertically towards the frontier towards a point on segment EF. Similarly, had the output of farm G been fixed, this farm would have to move horizontally towards the frontier towards a point on segment AB. See Appendix D.

3. Application

Data

Data from 675 UK dairy farms were selected, covering the year 2014–2015. Six inputs and three outputs were considered for aggregation into a single DEA efficiency score per farm (Table 1). The six inputs were cows in herd (numbers); forage area (ha); replacements (numbers); purchased feed (kg dry matter [DM]); somatic cell count (SCC; ‘000s/mL); and bacterial count (BC; ‘000s/mL). Cows in herd and forage area were considered as fixed (see previous section). Variables SCC and BC do not represent ‘typical’ physical farm inputs. However, including them in the model allowed us to estimate the inefficiencies that these two inputs exhibited in each farm, thus offering a way of demonstrating the financial benefits (better milk price) that a farm would gain by reducing them to the levels of their benchmarks (i.e. by eliminating these inefficiencies). Other inputs of interest, such as labour and fertiliser, were absent from the dataset and thus were not included in the model.

The three outputs were milk yield (L); butterfat yield (kg); and protein yield (kg). Milk yield was considered as fixed. As with SCC and BC, setting the DEA model to increase butterfat and protein yield allowed us to estimate the milk price benefits of eliminating the inefficiencies in these two outputs.

Table 1: Statistics of the DEA variables

Variables	**Min**	**Mean**	**Max**	**SD**
Inputs				
Cows in herd (numbers)	14	186	1,257	114
Forage area (ha)	17	99	621	58
Replacements (numbers)	2	54	375	42
Purchased feed (kg DM[1])	13,293	558,187	6,253,623	481,680
SCC[2] (‘000s/mL)	64	165	368	48
BC[3] (‘000s/mL)	7	26	144	13
Outputs				
Milk yield (L)	79,628	1,532,009	14,031,479	1,103,397
Butterfat yield (kg)	3,203	60,763	531,894	42,526
Protein yield (kg)	2,692	50,278	448,481	36,034

[1] DM: dry matter. [2]Somatic cell count. [3]bacterial count.

In summary, by setting the DEA model to increase butterfat and protein; and to reduce SCC and BC for the given milk yield, we obtained a ‘new’ milk price for the farm under evaluation. The difference between the actual and ‘new’ prices can be seen as the reward for producing more efficiently.

Finally, we have added a bound to the inefficiencies of butterfat and protein to avoid getting unreasonably large inefficiency values for these two outputs[5]. Specifically, we demanded that the optimal values for butterfat and protein constrain the percentages in

[5] We noted the need for imposing bounds to the inefficiencies of these two outputs after running preliminary exercises without the bounds, where the DEA model unreasonably indicated that some farms had to increase their butterfat content to as much as 12% to reach the best-practice frontier.

butterfat and protein below the maximal percentages in these two outputs observed in the dataset[6]. These bounds can be set extrinsically by the manager. See Appendix E.

Software

We ran the exercise in programming language R (R Core Team, 2017) using the R package 'additiveDEA' (Soteriades, 2017), that is specifically designed to run additive DEA models. Visualizations were also produced with R.

Results

The additive model (formulas (9a)-(9i) and (11a)-(11b) in the Appendices) indicated that the DEA best-practice frontier consisted of 82 farms out of 675, i.e. 12% of the farms in the sample were efficient. The remaining 593 farms were benchmarked against these 82 farms.

In what follows, we provide five examples to demonstrate DEA's potential as a tool that can help guide farm management. In Example 1 we demonstrate that the DEA scores can disagree with widely-used dairy farm efficiency indicators, because the latter are not comprehensive. In the same example, we compare the technical characteristics of DEA's benchmark farms with the top 25% farms in terms of margin over purchased feed (MOPF) per L of milk[7] (from now on referred to as 'Top 25% Farms'). In Examples 2-4 we choose specific farms exhibiting high inefficiencies in their inputs and outputs and show that these farms could be earning/saving substantial amounts of money by producing more efficiently. Example 5 shows how temporal efficiency analysis can be done with DEA.

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[6] Although the bounds can help calculate more reasonable butterfat and protein inefficiencies, it may be argued that they can still be a source of concern because they allow the butterfat and protein inefficiencies of any dairy farming system to become as large as the bounds. This may not be a sensible expectation for e.g. a system based on a by-products diet that may never give high butterfat for biological reasons. This can be dealt with by running DEA within groups of farming systems. We did not do this here, however, for simplicity.

[7] We got the idea from the *Milkbench+ Evidence Report* (AHDB Dairy, 2014). The report uses net margin/L rather than MOPF/L to identify the top 25% farms. However, net margin was not available in the sample dataset, hence our choice of MOPF/L.

Example 1: comparison of DEA efficiency with widely-used dairy farm efficiency indicators

In this example, we compare the DEA efficiency scores with four widely-used indicators of dairy farm efficiency: MOPF per cow (£[8]); feed efficiency (FE) defined as kg of energy-corrected milk per kg DM of purchased feed; milk yield per cow (L); and concentrate use per cow defined as kg DM of purchased feed per cow. The DEA scores are plotted against each of these indicators in Figure 3. It is shown that high DEA efficiency can be achieved at varying- and sometimes low- levels of MOPF per cow, FE, milk yield per cow and concentrate use per cow. This demonstrates that, contrary to DEA, partial efficiency ratios fail to provide a measure of overall farm efficiency.

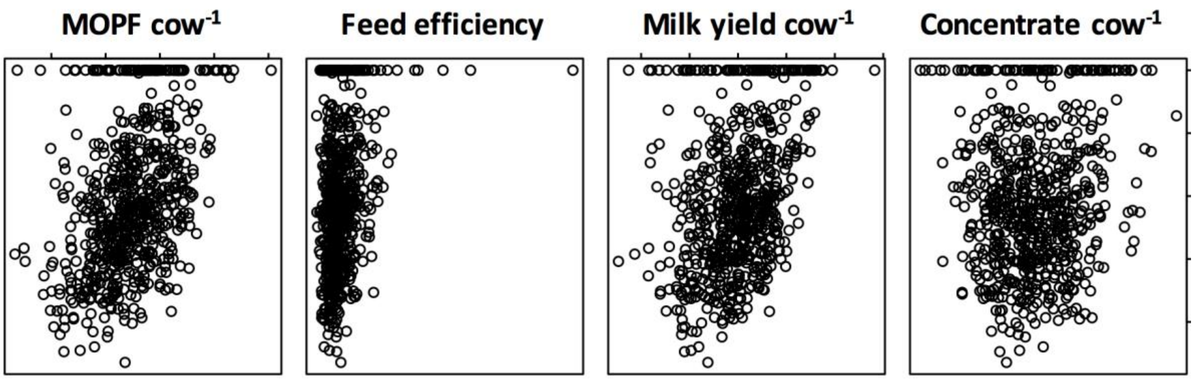

Figure 3: DEA efficiency scores plotted against: margin over purchased feed per cow; feed efficiency; milk yield per cow; and concentrate use per cow

The difference between the way that ratios and DEA measure efficiency can also be seen by comparing the Top 25% Farms (169 farms) with the 82 farms that served as benchmarks in the DEA exercise (Table 2). There are some notable differences between the two groups in milk yield per cow, purchased feed per cow, MOPF per cow and per litre of milk and FE. What is interesting is that DEA benchmark farms are much more inefficient, on average, than the Top 25% Farms for FE and MOPF per cow and per litre of milk. However, this seemingly superior performance of the Top 25% Farms came at the cost of lower yields per cow (Table 2) and per forage hectare (Top 25% Farms: 15,343 L/ha; DEA benchmarks:

[8] In mid-June 2017 £GBP1 was approximately equivalent to €1.15 and $US1.28. £GBP1 equals 100 pence.

18,819 L/ha) and greater numbers, on average, of SCC (Top 25% Farms: 104,688 cells/mL; DEA benchmarks: 100,691 cells/mL) and BC (Top 25% Farms: 24,247 cells/mL; DEA benchmarks: 19,285 cells/mL) than for the DEA benchmarks. This stresses (i) that good performance in some ratios could be achieved at the cost of high inefficiencies in other farm inputs and outputs. For instance, despite the lower MOPF per cow and per litre of milk of DEA benchmarks compared to the Top 25% Farms, the milk price for the latter would be more severely influenced by the higher SCC and BC; and (ii) that DEA offers a more holistic way of measuring efficiency. Finally, it is noteworthy that with DEA the number of 'top farms' is defined by the model itself: 'top farms' are the benchmark farms. This is more subjective than arbitrarily defining the percentage of farms that should be considered as 'top farms' (e.g. 25% as in our example).

Table 2: Comparison of top 25% farms (in terms of MOPF[1]/L) with the 82 DEA[2] benchmark farms in terms of farm characteristics (averaged)

Farm characteristics	Top 25% Farms[8]	DEA benchmarks	Difference
Cows in herd	200	212	-12
Replacement rate (%)	28	25	3
Milk yield/cow (L)	7,590	8,595	-1,005
Purchased feed/cow (kg DM[3])	2,320	2,955	-635
Purchased feed/litre (kg DM[3]/L)	0.30	0.33	-0.03
Butterfat (%)	4.1	4.0	0.1
Protein (%)	3.3	3.3	0
MOPF[1]/cow (£)	1,908	1,878	30
MOPF[1]/litre (ppL[4])	25	22	3
FE[5] (kg ECM[7]/kg DM[3])	3.69	3.54	0.15

[1]MOPF: margin over purchased feed. [2]DEA: data envelopment analysis. [3]DM: dry matter. [4]ppL: pence per L. [5]FE: feed efficiency. [7]ECM: energy-corrected milk. [8]In terms of MOPF/L of milk.

Example 2: increasing MOPF per cow by reducing inefficiency in purchased feed

This example demonstrates how insights from DEA and widely-used partial performance indicators can be coupled to identify profit-making opportunities for farmers. For each farm, we first calculated MOPF per cow:

$$milk\ income - price\ of\ purchased\ feed\ per\ kg\ \times\ purchased\ feed.$$

Then, we calculated the 'optimal' MOPF per cow that each farm would get by reducing its inefficiencies in purchased feed:

$$milk\ income - price\ of\ purchased\ feed\ per\ kg\ \times$$
$$(purchased\ feed - inefficiency\ in\ purchased\ feed).$$

At the final step, we calculated the difference between the actual and 'optimal' MOPF per cow. The largest difference occurred for a farm with actual and 'optimal' MOPF per cow values of £1,595 and £2,319 respectively, i.e. this farm could be improving MOPF per cow by an additional (£2,319 – £1,594) = £725 pounds per year just by using purchased feed more efficiently.

Example 3: increasing milk price by reducing SCC and BC

Another farm exhibited the largest inefficiency in SCC relative to its actual SCC (79%). It also exhibited a high inefficiency in BC relatively to its actual bacterial count (78%). This farm could greatly increase the price it gets for milk by reducing SCC from 339,750 cells/mL to (SCC – inefficiency in SCC) = 71,235 cells/mL and its bacterial count from 66,583 cells/mL to (BC – inefficiency in BC) = 14,619 cells/mL. In more detail, we used AHDB Dairy's Milk Price Calculator (AHDB Dairy, 2017) so as to get milk prices for actual and efficient SCC and bacterial counts[9]. This farm could be earning an additional 9ppL (pence per L) as the price

[9] One referee rightly commented that, in practice, milk price is dependent on SCC and BC *thresholds* rather than *levels*. This, however, does not affect the analysis: reducing SCC and BC to the levels of benchmark farms

for milk would have been improved from 20.43ppL to 29.43ppL[10]. It may achieve this increase in the price of milk by better managing its herd, e.g. by culling cows with the highest SCC and/or improving cow health management. Obviously, there would be costs incurred to improve SCC but the benefits of an extra 9ppL would not be lost on the farmer and would focus the mind on this most important source of inefficiency in this case.

will increase the milk price only if efficient levels of SCC and BC are below the thresholds assumed in the Milk Price Calculator.

[10] Prices are annual prices for Arla Foods-Sainsburys. We used the calculator's standard settings. Monthly milk yields for this farm were available in the sample data.

Example 4: increasing milk price by reducing SCC and BC and by increasing butterfat and protein

The farm studied in Example 3 could be getting an even better price by also eliminating its inefficiencies in butterfat and protein. This farm's butterfat and protein percentages were, respectively, 26,784/638,168 = 4.2% and 21,782/638,168 = 3.4%, while its efficient levels of butterfat and protein were, respectively, (26,784 + 4,995)/638,168 = 5.0% and (21,782 + 4,589)/638,168 = 4.1%. This farm could be earning an additional 9.55ppL as the price for milk would have been improved from 20.43ppL to 29.55ppL. Again, DEA can help focus the mind of the farmer and farm manager on how best to deal with the greatest challenge to efficiency in a given case. The level of efficiency achievable in practice may be less important than the prioritisation of management effort that DEA highlights.

Further applications

Efficiency analysis over time

All previous example applications were based on the rolling data reported in Table 1. Such applications are useful for monitoring farm performance based on annual data. Yet, monitoring efficiency across time is often more appropriate for decision-making, as it can help detect trends that develop slowly, potentially going unnoticed by the manager (Brockett *et al*., 1999).

There are several methods for the analysis of efficiency change over time with DEA, each designed to fit particular purposes (interested readers may refer to Asmild *et al*., 2004; Bogetoft and Otto, 2011; Brockett *et al*., 1999; Cooper *et al*., 2007). We discuss three methods that may be of special interest to farm managers: (i) intertemporal analysis (Asmild *et al*., 2004; Brockett *et al*., 1999); (ii) a method by Tsutsui and Goto (2009), which we will refer to as 'cumulative temporal analysis'; and (iii) window analysis (Asmild *et al*., 2004; Cooper *et al*., 2007).

Intertemporal analysis is the simplest form of efficiency analysis over time: all data from different time periods are pooled and evaluated with a single DEA run. Thus, a farm 'FARM A' is considered as a 'different' farm in each period, i.e. FARM A_1, …, FARM A_T, so the single DEA run involves $T \times n$ farms, where T is the number of periods and n is the number of farms. For example, measuring efficiency trends for the period March 2014–March 2015 requires pooling data for all farms from all 13 months and running a single DEA exercise, where all farms are benchmarked against a single best-practice frontier. Doing so allows the farm manager to compare efficiency progress (or deterioration) of individual or groups of farms across all 13 months. Figure 4 illustrates an inter-temporal DEA analysis for the period March 2014–March 2015, with a total of 6,030 'different' farms. The median results are summarized by the six UK regions used in Kingshay's Dairy Manager reports (Kingshay, 2017). In this figure, notable fluctuations in (median) efficiency are observed for Scotland and the Southeast, with the former having the lowest scores for six out of 13 months. By contrast, the Midlands exhibit neither high nor low median efficiency, and these scores are relatively stable throughout the year (between approximately 0.55 and 0.63). Despite the simplicity of intertemporal analysis, its disadvantage is that it may be unreasonable to compare farms over long periods (e.g. years) if large technological changes have occurred meanwhile.

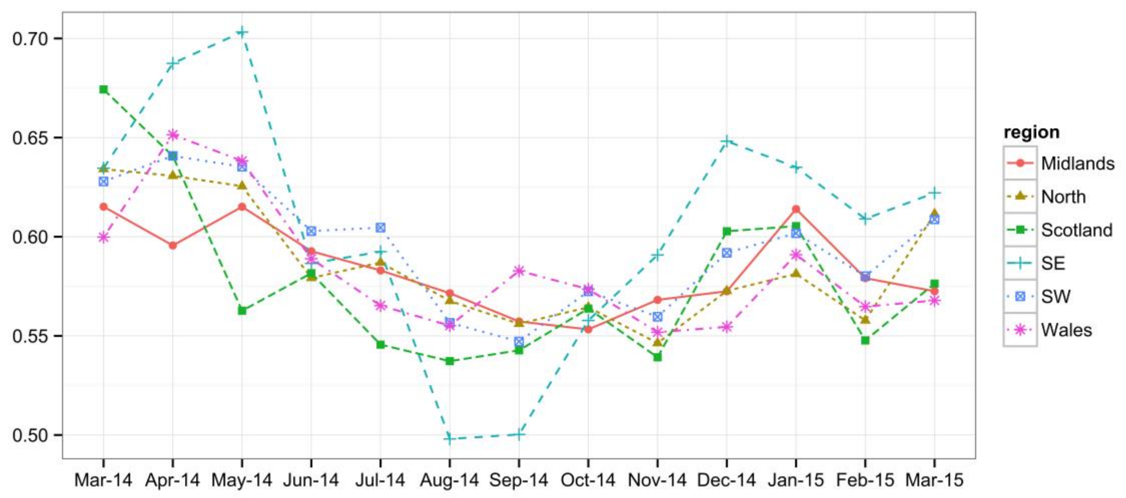

Figure 4: Intertemporal DEA analysis summarized by UK region (median efficiency scores reported). SW: Southwest; SE: Southeast

In cumulative temporal analysis, a farm in a specified period is benchmarked against a best-practice frontier consisting of farms up to that period. For example, a farm in May 2014 is compared to farms in March, April and May 2014. This allows the manager to assess efficiency in each period based on the farms' 'cumulative' performance in inputs and outputs up to that period. As in Figure 4, Figure 5 demonstrates a deep fall in efficiency for Scotland and the Southeast, with Scotland performing at the lowest levels in six out of 13 months. However, all groups have much higher (median) efficiencies than in Figure 4 for up to May 2014. This trend is generally observed for the whole study period, although from June 2014 scores in Figures 4 and 5 tend to get closer for each group. This is intuitive, because in later periods more farms are included in the analysis (note that the DEA run for March 2015 contains all 6,030 farms, hence the resulting scores for this month are identical to those of the inter-temporal analysis).

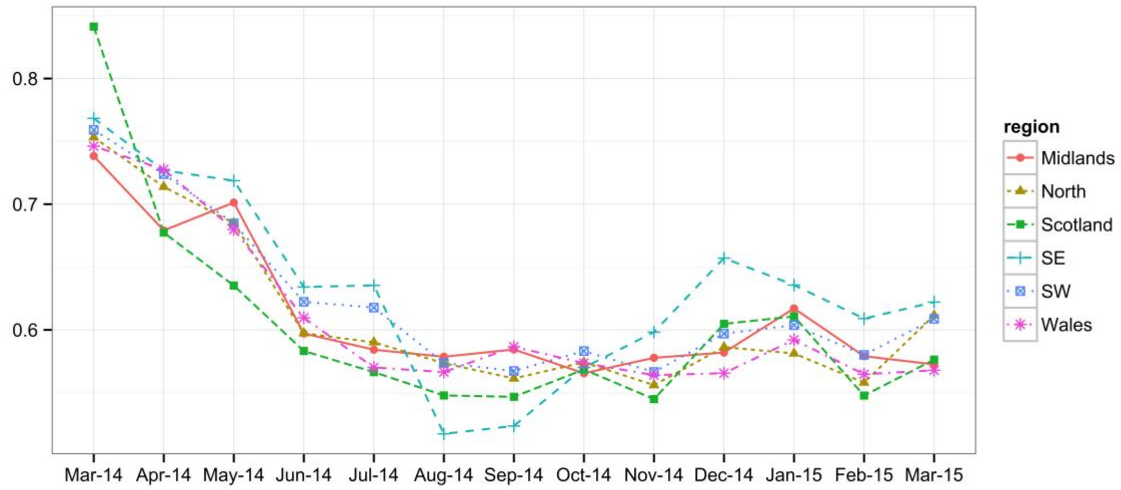

Figure 5: Cumulative temporal DEA analysis summarized by UK region (median efficiency scores reported). SW: Southwest; SE: Southeast

Window analysis resembles the well-known method of 'moving averages' in statistical time-series. Its advantage lies in the fact that it can be used for studying both trends over time as well as the stability of DEA scores within and between time 'windows' specified by the manager. For instance, for a manager interested in evaluating efficiency every four months (four-month 'window') for the period March 2014–March 2015, window analysis first involves a DEA run for all farms in window March 2014–June 2014. Then, March 2014 is dropped and a second DEA run involves all farms in window April 2014–July 2014. The exercise is replicated up to window December 2014–March 2015. The results are reported in such a manner that allows detection of trends and stability. This is illustrated in

Table 3, where results are reported for Scotland (median scores). Looking at the results row-by-row (i.e. window-by-window), we generally observe a decline in efficiency within each row up to window W4. From window W5 efficiency is gradually improving, while results are slightly more mixed within windows W9 and W10. The stability of these findings is confirmed by looking at the scores within each column. In more detail, within each column, scores are generally close, with a few exceptions (e.g. August 2014 where the minimum and maximum scores differ by 0.10), reinforcing the previously mentioned finding that performance deteriorates up to window W4 and then improves (also evident in Figures 4 and 5).

Table 3: DEA window analysis for Scotland (median efficiency scores), Mar 14–Mar 15

Window	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**	**Jan**	**Feb**	**Mar**
W1	0.72	0.65	0.56	0.58									
W2		0.65	0.56	0.58	0.57								
W3			0.57	0.58	0.58	0.55							
W4				0.62	0.59	0.56	0.57						
W5					0.62	0.59	0.60	0.63					
W6						0.65	0.65	0.69	0.67				
W7							0.62	0.65	0.65	0.68			
W8								0.65	0.65	0.65	0.69		
W9									0.65	0.65	0.69	0.65	
W10										0.64	0.67	0.64	0.63

Comparing herds managed under different growing conditions

In the DEA runs of the previous examples, an implicit assumption was made that all farms operated under similar growing conditions and thus could be directly compared. The large variation in variables such as growing conditions, regional characteristics, management practices *etc*. may raise concerns about the direct comparison of different types of dairy farms (Soteriades et al., 2016). For instance, Kingshay's Dairy Manager (2017) groups herds by their 'site class', that is, the growing conditions under which these herds are managed (defined by altitude, soil type and rainfall), and compares farms within each group. By contrast, with DEA it also possible to compare farms from different groups with a method by Charnes *et al*. (1981), which is also known as 'corrective methodology' (Soteriades et al., 2016) or the 'meta-frontier' approach (Fogarasi and Latruffe, 2009).

The concept of the 'corrective methodology' or 'meta-frontier' approach is based on the observation that inefficiencies may be attributed to either management or different operating conditions: when both inefficiency sources are amalgamated, there is a risk of granting some 'bad' managers (farmers) good efficiency scores when they are only benefiting from operating under more favourable conditions (Soteriades et al., 2016). Hence, within-group managerial inefficiencies need to be eliminated before comparing groups. This can be done as follows. First, a DEA run is effected within each group to compare 'like with like'. The inefficiencies that inefficient farms exhibit within each group are attributed solely to management. Second, inputs and outputs are adjusted to their efficient levels by eliminating these managerial inefficiencies. For inputs, this means subtracting the inefficiency from the actual input used, for example:

$$'adjusted'\ purchased\ feed = purchased\ feed - inefficiency\ in\ purchased\ feed.$$

For outputs, it means adding the inefficiency to the actual output produced, for example:

$$'adjusted'\ milk\ production = milk\ production + inefficiency\ in\ milk\ production.$$

This is done for all inputs and outputs to eliminate all managerial inefficiencies within each group. Third, farms from all groups are pooled and a single DEA run is effected. Now, all inefficiencies are attributed to differences in operating conditions between groups and so we can determine which groups are more efficient, as well as which of their inputs and outputs exhibit the largest inefficiencies in each group or individual farm.

This methodology (which was not adopted in our study for simplicity and brevity) can be applied to compare any groups of farms that the practitioner feels cannot be directly compared, because of differences in e.g. breed, accumulated T-sums, manure management technology, system (e.g. conventional *versus* organic or pasture-based *versus* housed all year round) etc.

4. Discussion

DEA in agricultural consulting, extension and teaching

As DEA's numerous advantages have made it a well-established method in agricultural and dairy research (see introduction), this article is mainly intended to reach a wider agricultural audience, specifically farm consultants, extension officers, Knowledge Exchange officers and lecturers in farm management. We hope that our examples provide our target audience with sufficient evidence of DEA's potential for farm efficiency assessments, and that they will encourage them to consider using the method. For instance, similar exercises could be used by lecturers to complement teaching based on standard farm management textbooks that focus heavily on partial indicators (Boehlje and Eidman, 1984; Castle and Watkins, 1979; Jack, 2009). Similarly, extension officers and farm consultants could use DEA to get a wider picture of farm performance before discussing with farmers the managerial strategies for improving efficiency. The DEA findings of such exercises could also be presented in online newsletters and reports by farm consultancies and agricultural levy boards (AHDB Dairy, 2014; Kingshay, 2017) to indicate where cost-saving or profit-making opportunities might lie for the farmer (as this study has intended to do). Knowledge Exchange could be achieved through workshops aiming at presenting findings from novel farm management tools and methods to industry stakeholders (SIP Platform, 2017, p.5).

Challenges

A main question is to what extent the indicators that analysts currently use can help them access the insights provided in our examples. However, as demonstrated in our examples, an attractive feature of DEA is that potentially 'already-known' information is summarized into a single score allowing holistic monitoring, while nothing is lost, because the score can be disaggregated into input and output inefficiencies. Moreover, there is great mileage for extending the DEA exercise by linking the scores with other attributes which are not always so well-known, for example casein content and cheese yield. DEA scores may also be linked with data for animal health and welfare, farm management strategies, regional characteristics and other external variables influencing farm efficiency (Barnes et al., 2011; Soteriades et al., 2016), which otherwise tend to be looked at in isolation. Data on the environmental footprints of farms can also be considered as DEA variables to add a sustainability dimension to farm benchmarking (Soteriades et al., 2016).

Missing and incorrect data, as well as unbalanced panel (monthly) data was a challenge that we faced when designing the DEA exercise. We had to remove farms with missing or negative entries in any of the inputs and outputs that we fed to the DEA model. This reduced the size of the available dataset. Similarly, the monthly entries of some farms were not recorded for all months of the 13-month study period, rendering impossible the study of DEA efficiency of individual farms (rather than our regional groups) over all 13 months. Fortunately, developments with precision farming increasingly offer access to precise, well-informed data (Agri-EPI Centre, 2017). Equally important are financial incentives motivating farmers to gather and share their data, such as Scottish Government's Beef Efficiency Scheme (2017). To be sure, Kingshay Farming and Conservation Ltd. and other recording companies provide the means, yet efforts should be made to eliminate variation between farmers in their accuracy of recording- or even their definitions of a record (Jack, 2009). In any case, the analyst can benchmark the farms for which they hold data against farms from the Farm Business Survey data (FBS, 2017), a comprehensive source of information on managerial, socio-economic and physical characteristics of UK farms. The FBS data are used in this manner in a recently developed benchmarking tool for UK farms (Wilson, 2017).

From a methodological viewpoint, this study makes several assumptions and simplifications, so the examples and results should be viewed with the appropriate understanding that they are for illustration purposes. First, we did not correct the data for errors. Second, we ignored outliers. The issue of outliers is debated in the DEA literature, as extreme observations can greatly alter the shape of the best-practice frontier. However, we considered extreme farms as part of what is currently observed in UK dairy farming systems, and it could be argued that '[such farms] reflect the first introduction of new technology into a production process or an innovation in management practice from which [other farms] would want to learn' (Bogetoft and Otto, 2011, p.147). Third, changing the set of DEA variables and/or adding or removing farms from the data will alter the shape of the frontier, consequently changing the set of efficient farms and the efficiency scores. We therefore recommend that DEA results should be seen as a rough proxy of the efficiency gains that may be achieved for the variables of interest in a given dataset. Variable choice is therefore up to the practitioner, and it may expand DEA's usability. This was demonstrated in our examples, with the use of SCC, BC, and butterfat and protein yields to compare current and 'optimal' milk prices.

Towards a DEA-based decision-support tool for farm management

There is currently no DEA-based decision-support tool specifically tailored to the needs of the (dairy) farming industry. Although DEA models can be easily run with standard software that the analyst may be familiar with, such as spreadsheets, all available DEA software (spreadsheet-based or not) we are aware of (Table 4) suffer from excessive use of DEA jargon. As discussed earlier, this is a main factor discouraging analysts from using DEA. Moreover, DEA software tend to be complicated in that they strive to incorporate as many DEA models and techniques as possible. This is a natural consequence, because DEA is founded on the fields of management, economics and operational research, where alternative theories and approaches are continually developed and debated, thus giving birth to alternative DEA models and methodologies to satisfy different needs (Bogetoft and Otto, 2011; Cooper *et al*., 2007). To be sure, this may be of little concern to the farm analyst, who would rather focus their mind on specific objectives that could be dealt with specific DEA models and methods.

That said, it would be bold to assume that the farm analyst would benchmark farms using DEA themselves. As discussed earlier, we are well-aware that our study is a premature and simplified introduction to DEA for farm benchmarking and that many issues were not addressed in our examples. We envisage that this study will evolve to the development of a DEA-based decision-support tool for farm management, following the guidelines in two recent and particularly inspiring papers on the design of decision-support systems for agriculture (Rose et al., 2016, in press).

Table 4: List of available DEA software

Software	**URL**
additiveDEA	https://CRAN.R-project.org/package=additiveDEA
Benchmarking	https://CRAN.R-project.org/package=Benchmarking

DEA-Excel	http://nb.vse.cz/~jablon/dea.htm
DEAFrontier	http://www.deafrontier.net/deasoftware.html
DEAS	https://sourceforge.net/projects/deas/?source=navbar
DEA Solver Pro	http://www.saitech-inc.com/Products/Prod-DSP.asp
DEAP	http://www.uq.edu.au/economics/cepa/deap.php
EMS	http://www.holger-scheel.de/ems/
Frontier Analyst	https://banxia.com/frontier/
InverseDEA	http://maxdea.com/InverseDEA.htm
MaxDEA	http://maxdea.com/MaxDEA.htm
nonparaeff	https://CRAN.R-project.org/package=nonparaeff
Open Source DEA	http://opensourcedea.org/
PIM-DEA	http://deazone.com/en/software

5. Conclusion

DEA can help identify inefficient producers as well as indicate the inputs and outputs in which the largest inefficiencies occur for each farm. That way DEA can help guide management actions through a variety of cost-saving and/or profit-making options for each farm. We showed that detection- and elimination- of input and output inefficiencies can notably increase milk price and reduce the costs of concentrate use for inefficient UK dairy farms. We also demonstrated three simple ways of studying efficiency change over time with DEA to help detect trends in the technical performance of different farms or farm groups. Our DEA exercise could be extended to include other important variables such as labour, fertilizer use, greenhouse gas emissions, nitrogen and phosphorous surpluses *etc*. to account for objectives relevant to both business management and the public good. This flexibility characterizing DEA increases its importance in the context of a post- ‘Brexit’ UK, where a significant challenge will be to improve competitiveness in the world market (BSAS, 2017).

6. Appendices

Appendix A: which efficient farms serve as benchmarks for farm G?

Farm G could be benchmarked against, say, efficient farm C or D (Figure 1). On the other hand, it could be benchmarked against a virtual farm represented by a point lying on, say, segment CD. In any case, the benchmark farm's input can be represented by a linear combination of the inputs of farms C and D. Similarly, the benchmark farm's output can be represented by a linear combination of the outputs of farms C and D. We can express these linear combinations mathematically as follows:

$$x_{Ben} = \lambda_C x_C + \lambda_D x_D \tag{1a}$$

$$y_{Ben} = \lambda_C y_C + \lambda_D y_D, \tag{1b}$$

where x_{Ben}, x_C, x_D are the inputs of the benchmark farm, farm C and farm D respectively; y_{Ben}, y_C, y_D are the outputs of the benchmark farm, farm C and farm D respectively; and λ_C, λ_D are semi-positive variables whose values are calculated by the DEA model. The values of these lambda variables provide information on which farms serve as benchmarks for farm G. For example, if $\lambda_C = 1$ and $\lambda_D = 0$, then farm C is the benchmark of farm G. If $\lambda_C = 0$ and $\lambda_D = 1$, then farm D is the benchmark of farm G. However, if $\lambda_C = 0.1$ and $\lambda_D = 0.9$, then the benchmark of farm G is a virtual farm with input $0.1x_C + 0.9x_D$ and output $0.1y_C + 0.9y_D$.

We note that farm D plays a larger role in the formation of the virtual benchmark because its lambda value is much larger than that of farm C. In other words, farm D contributes to the formation of the virtual benchmark more 'intensively' than farm C. Therefore, the lambdas are referred to as *intensity variables* in the DEA literature. In this study, the term *benchmark variables* will be used instead.

Now note that, as mentioned above, the benchmark variables are calculated by the DEA model, hence the model does not 'know' *a priori* which facet of the frontier farm G is benchmarked against. Therefore, formulas (1a) and (1b) are more appropriately expressed as follows:

$$x_{Ben} = \lambda_A x_A + \lambda_B x_B + \lambda_C x_C + \lambda_D x_D + \lambda_E x_E + \lambda_F x_F + \lambda_G x_G \tag{2a}$$

$$y_{Ben} = \lambda_A y_A + \lambda_B y_B + \lambda_C y_C + \lambda_D y_D + \lambda_E y_E + \lambda_F y_F + \lambda_G y_G, \tag{2b}$$

where $\lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F = 1$. In formulas (2a) and (2b), the benchmark farm is indicated by those benchmark variables that have non-zero values. Efficient farms serve as benchmarks of themselves, e.g. for farm B we have that $\lambda_B = 1$ and $\lambda_A = \lambda_C = \lambda_D = \lambda_E = \lambda_F = \lambda_G = 0$. Note that the condition that the sum of lambdas equals 1 safeguards that the DEA model accounts for economies of scale. This is important when both small and large farms are present in the dataset, as was the case with the sample data. This condition is known as *variable returns to scale* specification. Other returns to scale specifications are available when needed, see Cooper *et al.* (2007).

Based on the above insights, we will demonstrate how the DEA model identifies benchmark farms for each farm in the sample. It is obvious that benchmark farms use at the most the same amount of inputs as the farm under evaluation, say farm G. Similarly, they produce at least the same amount of outputs as farm G. Therefore, we demand that

$$x_{Ben} = \lambda_A x_A + \lambda_B x_B + \lambda_C x_C + \lambda_D x_D + \lambda_E x_E + \lambda_F x_F + \lambda_G x_G \leq x_G \quad (3a)$$

$$y_{Ben} = \lambda_A y_A + \lambda_B y_B + \lambda_C y_C + \lambda_D y_D + \lambda_E y_E + \lambda_F y_F + \lambda_G y_G \geq y_G. \quad (3b)$$

Formulas (3a) and (3b) simply tell us that the benchmark farm cannot be using more input and be producing less output than G. For instance, we could have that $x_{Ben} = 0x_A + 0x_B + 1x_C + 0x_D + 0x_E + 0x_F + 0x_G = x_C \leq x_G$ and similarly $y_{Ben} = y_C \leq y_G$. In this case, the benchmark for farm G is C. Alternatively, we could have that $x_{Ben} = 0.08x_A + 0x_B + 0.67x_C + 0x_D + 0.25x_E + 0x_F + 0x_G \leq x_G$ and $y_{Ben} = 0.08y_A + 0y_B + 0.67y_C + 0y_D + 0.25y_E + 0y_F + 0y_G \leq y_G$. In this case, the benchmarks for farm G are farms A, C and E.

Appendix B: how does the additive model calculate efficiency?

Another way to interpret formulas (3a) and (3b) is that an inefficient farm such as G exhibits excess in its input and shortfall in its output relatively to its benchmark. The excess in inputs and shortfall in outputs represent the *inefficiencies* that G exhibits in its inputs and outputs. We denote input and output inefficiency as s_G^- and s_G^+ respectively, with $s_G^-, s_G^+ \geq 0$. These inefficiencies are central to the way that additive DEA models calculate efficiency. Before expanding on this, first note that $s_G^- = x_G - x_{Ben}$ and $s_G^+ = y_{Ben} - y_G$ so formulas (3a) and (3b) can be re-expressed for farm G as follows:

$$x_G = (\lambda_A x_A + \lambda_B x_B + \lambda_C x_C + \lambda_D x_D + \lambda_E x_E + \lambda_F x_F + \lambda_G x_G) + s_G^- \qquad (4a)$$

$$y_G = (\lambda_A y_A + \lambda_B y_B + \lambda_C y_C + \lambda_D y_D + \lambda_E y_E + \lambda_F y_F + \lambda_G y_G) - s_o^+. \qquad (4b)$$

Using formulas (4a) and (4b) as constraints of a mathematical optimization problem, the additive model seeks the maximal sum of input and output inefficiencies $s_G^- + s_G^+$ that farm G can exhibit (hence the term 'additive'):

$$\text{Maximize}(s_G^- + s_G^+) \qquad (5a)$$

subject to

$$x_G = (\lambda_A x_A + \lambda_B x_B + \lambda_C x_C + \lambda_D x_D + \lambda_E x_E + \lambda_F x_F + \lambda_G x_G) + s_G^- \qquad (5b)$$

$$y_G = (\lambda_A y_A + \lambda_B y_B + \lambda_C y_C + \lambda_D y_D + \lambda_E y_E + \lambda_F y_F + \lambda_G y_G) - s_G^+ \qquad (5c)$$

$$\lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G = 1 \qquad (5d)$$

$$\lambda_A, \lambda_B, \lambda_C, \lambda_D, \lambda_E, \lambda_F, \lambda_G, s_G^-, s_G^+ \geq 0. \qquad (5e)$$

Problem (5a)-(5e) finds the optimal values for the inefficiencies and benchmark variables maximizing $s_G^- + s_G^+$ and projects farm G onto point C on the frontier (i.e. $\lambda_C = 1$ and all other lambdas are zero). See Figure 2 for a visual representation of problem (5a)-(5e) for farm G.

Now we point out some shortcomings of the additive model and propose adjustments to enhance its applicability in the context of dairy farm efficiency. Note that the optimal sum $s_G^{-*} + s_G^{+*}$ ('*' denotes optimality), i.e. the score of the additive model for farm G, represents the maximal sum of inefficiencies in inputs and outputs that G exhibits. This has three drawbacks: (i) the additive model produces a score of total *inefficiency* rather than *efficiency*; (ii) the inefficiency score is not readily interpretable as it represents a sum of inefficiencies in inputs and outputs potentially measured in different units. For instance, the sum of inefficiency in milk production plus inefficiency in fertilizer use is clearly not intuitive;

consequently, (iii) the optimal solution is affected by the different measurement units in which inputs and outputs are measured.

Problems (ii)-(iii) can be easily overcome by replacing the sum in (5a) with

$$\frac{s_G^-}{x_G} + \frac{s_G^+}{y_G}. \tag{6}$$

In (6) the different measurement units cancel because the inefficiencies are scaled by the actual input and output. In other words, the sum in (6) is *units invariant* and thus deals with problem (iii). The sum in (6) is interpreted as the proportion in input excess in x_G plus the proportion in output shortfall relatively to y_G. In more detail, a ratio of, say $\frac{s_G^-}{x_G} = 0.60$ means that the input of farm G is in excess by 60%, i.e. it could be using $x_G - s_G^- = x_G - 0.60x_G = 0.40x_G = 40\%$ of its input x_G. On the output side, a ratio of $\frac{s_G^+}{y_G} = 0.60$ means that farm G could be producing $y_G + s_G^+ = y_G + 0.60y_G = 1.60y_G = 160\%$ of its output y_G.

However, we are still faced with problem (i), although this can also be easily dealt with. First note from (5b) that s_G^- cannot exceed x_G, i.e. $\frac{s_G^-}{x_G} \leq 1$. However, we note from (5c) that this is not the case with s_G^+, i.e. we may have that $\frac{s_G^+}{y_G} > 1$. Nevertheless, in real life applications it might be unreasonable to have output slacks larger than the actual output because in such a case the farm under evaluation would have to at least double its output to become efficient- an enormous increase. Hence, we may demand that $s_G^+ \leq b_G$, where b_G is an upper bound defined by the user, with $b_G \leq y_G$ (Cooper et al., 2007, ch.13). By safeguarding that $\frac{s_G^-}{x_G} \leq 1$ and $\frac{s_G^+}{y_G} \leq 1$, we have for the optimal solution to (5a)-(5e) that $0 \leq \frac{1}{2}\left(\frac{s_G^{-*}}{x_G} + \frac{s_G^{+*}}{y_G}\right) \leq 1$ and so

$$0 \leq 1 - \frac{1}{2}\left(\frac{s_G^{-*}}{x_G} + \frac{s_G^{+*}}{y_G}\right) \leq 1. \tag{7}$$

Thus, the *inefficiency* score (6) is converted to an *efficiency* score (7) that is bounded by 0 and 1, with 1 indicating full efficiency (zero input and output inefficiencies) and a score less than 1 indicating inefficiency (non-zero input and output inefficiencies). The adjusted additive model for farm G becomes:

$$\text{Minimize}\left[1-\frac{1}{2}\left(\frac{s_G^-}{x_G}+\frac{s_G^+}{y_G}\right)\right] \tag{8a}$$

subject to

$$x_G = (\lambda_A x_A + \lambda_B x_B + \lambda_C x_C + \lambda_D x_D + \lambda_E x_E + \lambda_F x_F + \lambda_G x_G) + s_G^- \tag{8b}$$

$$y_G = (\lambda_A y_A + \lambda_B y_B + \lambda_C y_C + \lambda_D y_D + \lambda_E y_E + \lambda_F y_F + \lambda_G y_G) - s_G^+ \tag{8c}$$

$$\lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G = 1 \tag{8d}$$

$$s_G^+ \leq b_G \tag{8e}$$

$$b_G \leq y_G \tag{8f}$$

$$\lambda_A, \lambda_B, \lambda_C, \lambda_D, \lambda_E, \lambda_F, \lambda_G, s_G^-, s_G^+ \geq 0. \tag{8g}$$

Appendix C: the general case

We consider the general case where there are n dairy farms each using m inputs to produce s outputs, denoted as x_i $(i = 1, \dots, m)$ and y_r $(r = 1, \dots, s)$ respectively. The efficiency score for the farm under evaluation, denoted as FARM_O, is given by the following generalization of problem (8a)-(8g):

$$\rho^* = \underset{\lambda_j, s_{io}, s_{ro}}{\text{Minimize}}\left[1-\frac{1}{m+s}\left(\sum_{i=1}^{m}\frac{s_{io}}{x_{io}}+\sum_{r=1}^{s}\frac{s_{ro}}{y_{ro}}\right)\right] \tag{9a}$$

subject to

$$x_{io} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_{io},\ i = 1, \dots, m \tag{9b}$$

$$y_{ro} = \sum_{j=1}^{n} y_{rj}\lambda_j - s_{ro},\ r = 1, \dots, s \tag{9c}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{9d}$$

$$s_{ro} \leq b_{ro},\ r = 1, \dots, s \tag{9e}$$

$$b_{ro} \leq y_{ro},\ r = 1, \dots, s \tag{9f}$$

$$s_{io}, s_{ro}, \lambda_j \geq 0\ (i = 1, \dots, m, r = 1, \dots, s, j = 1, \dots, n), \tag{9g}$$

where x_{io} and y_{ro} are the inputs and outputs of FARM_O respectively; s_{io} and s_{ro} are the input and output inefficiencies of FARM_O respectively; and b_{ro} is the user-defined upper bound of s_{ro}.

Appendix D: fixed variables

Fixed inputs and outputs can be included in model (9a)-(9g) by adding the following two constraints:

$$x_{ko}^{fixed} \geq \sum_{j=1}^{n} x_{kj}^{fixed} \lambda_j \,, \; k = 1, \ldots, number\ of\ fixed\ inputs \tag{9h}$$

$$y_{lo}^{fixed} \leq \sum_{j=1}^{n} y_{lo}^{fixed} \lambda_j \,, \; l = 1, \ldots, number\ of\ fixed\ outputs. \tag{9i}$$

Appendix E: bounds

The bounds imposed to the slacks of the additive model run in this exercise were the following:

$$\frac{y_o^{butterfat} + s_o^{butterfat}}{y_o^{milk}} \leq \max\left(\frac{y_j^{butterfat}}{y_j^{milk}}\right) \tag{10a}$$

$$\frac{y_o^{protein} + s_o^{protein}}{y_o^{milk}} \leq \max\left(\frac{y_j^{protein}}{y_j^{milk}}\right), \tag{10b}$$

thus

$$b_o^{butterfat} = \max\left(\frac{y_j^{butterfat}}{y_j^{milk}}\right) y_o^{milk} - y_o^{butterfat} \tag{11a}$$

$$b_o^{protein} = \max\left(\frac{y_j^{protein}}{y_j^{milk}}\right) y_o^{milk} - y_o^{protein} \tag{11b}$$

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