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**Journal of Atmospheric and Oceanic Technology**

DOI:

[10.1175/JTECH-D-17-0059.1](https://doi.org/10.1175/JTECH-D-17-0059.1)

Published: 01/10/2017

Peer reviewed version

[Cyswllt i'r cyhoeddiad / Link to publication](#)*Dyfyniad o'r fersiwn a gyhoeddwyd / Citation for published version (APA):*

Scannell, B., Rippeth, T., Simpson, J., Polton, J., & Hopkins, J. (2017). Correcting surface wave bias in structure function estimates of turbulent kinetic energy dissipation rate. *Journal of Atmospheric and Oceanic Technology*, 34(10), 2257-2273. <https://doi.org/10.1175/JTECH-D-17-0059.1>

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*Journal of Atmospheric and Oceanic Technology*

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The DOI for this manuscript is doi: 10.1175/JTECH-D-17-0059.1

The final published version of this manuscript will replace the preliminary version at the above DOI once it is available.

If you would like to cite this EOR in a separate work, please use the following full citation:

Scannell, B., T. Rippeth, J. Simpson, J. Polton, and J. Hopkins, 2017: Correcting surface wave bias in structure function estimates of turbulent kinetic energy dissipation rate. *J. Atmos. Oceanic Technol.* doi:10.1175/JTECH-D-17-0059.1, in press.



1 **Correcting surface wave bias in structure function estimates of turbulent**  
2 **kinetic energy dissipation rate**

3 Brian D. Scannell\*, Tom P. Rippeth and John H. Simpson

4 *School of Ocean Sciences, Bangor University, Menai Bridge, Isle of Anglesey, United Kingdom*

5 Jeff A. Polton and Joanne E. Hopkins

6 *National Oceanography Centre, Joseph Proudman Building, Liverpool, United Kingdom*

7 \*Corresponding author address: School of Ocean Sciences, Bangor University, Menai Bridge, Isle  
8 of Anglesey, LL59 5AB, United Kingdom.

9 E-mail: brian.scannell@bangor.ac.uk

## ABSTRACT

10 The combination of acoustic Doppler current profilers and the structure  
11 function methodology provide an attractive approach to making extended time  
12 series measurements of oceanic turbulence (the rate of turbulent kinetic en-  
13 ergy dissipation,  $\epsilon$ ) from moorings. However, we show that for deployments  
14 in the upper part of the water column, estimates of  $\epsilon$  will be biased by the  
15 vertical gradient in wave orbital velocities. To remove this bias, we develop a  
16 modified structure function methodology, which exploits the differing length-  
17 scale dependencies of the contributions to the structure function due to turbu-  
18 lent and wave orbital motions. The success of the modified method is demon-  
19 strated through comparison of  $\epsilon$  estimates based on data from instruments at  
20 three depths over a three month period under a wide range of conditions, with  
21 appropriate scalings for wind stress and convective forcing.

## 22 **1. Introduction**

23 Exchanges of heat, freshwater and trace gases between the ocean and the atmosphere are critical  
24 in regulating the climate and depend directly on the properties of the ocean surface boundary layer  
25 (OSBL) (e.g. D'Asaro 2014; Franks 2014; Large et al. 1994). The structure of the OSBL depends  
26 on turbulent processes that cannot be directly simulated in geographical scale numerical models  
27 and which therefore have to be parameterized (Burchard et al. 2008; Belcher et al. 2012; Calvert  
28 and Siddorn 2013).

29 Turbulence in the OSBL is widely recognised as being produced by wind-driven surface shear  
30 stress, destabilising surface buoyancy fluxes and (in shelf seas) tidal current shear at the bottom  
31 boundary (e.g. Brainerd and Gregg 1993; Simpson 1981). Other surface-driven processes include  
32 breaking waves (e.g. Agrawal et al. 1992; Terray et al. 1996), Langmuir circulation (e.g. Thorpe  
33 2004), submesoscale eddies (e.g. Taylor 2016) and swell waves (e.g. Wu et al. 2015). Developing  
34 effective parameterizations for such diverse processes requires robust measurements under a wide  
35 range of environmental conditions, presenting significant observational challenges.

36 The structure function method is an established technique for calculating the turbulent kinetic  
37 energy (TKE) dissipation rate,  $\varepsilon$ , from velocity profiles such as those obtained with an acous-  
38 tic Doppler current profiler (ADCP) (e.g. Wiles et al. 2006; Mohrholz et al. 2008; Lucas et al.  
39 2014; Simpson et al. 2015; McMillan and Hay 2017). The method relates  $\varepsilon$  to the variance of  
40 the along-beam turbulent velocity difference evaluated over a range of separation distances. In-  
41 strument choice and configuration impose constraints on the data collected, but once configured,  
42 ADCP can be deployed to make unattended long-term observations, unlike standard microstruc-  
43 ture techniques.

44 Surface waves induce orbital motions within the water column, the speed of which reduce with  
45 depth. The velocity associated with the orbital motions may be observed by the ADCP, potentially  
46 affecting the structure function and introducing bias in the  $\varepsilon$  estimates. To date, the structure  
47 function technique has typically been applied to observations from sites with small amplitude  
48 surface waves or at depths unlikely to be affected by significant wave orbital velocities (Wiles  
49 et al. 2006; Lucas et al. 2014; Simpson et al. 2015; McMillan and Hay 2017).

50 An exception is the application of the technique by Thomson (2012) to obtain  $\varepsilon$  estimates within  
51 the crests of breaking waves by mounting the ADCP on a surface following Lagrangian float  
52 and by necessity limiting the range of separation distances over which the structure function was  
53 evaluated. Similarly, in order to measure vertical profiles of  $\varepsilon$  in the near-surface under breaking  
54 waves, Sutherland and Melville (2015) adapted the technique by restricting both the range of  
55 separation distances and the time-averaging period over which the statistical properties of the  
56 structure function were evaluated. Restricting the range of separation distances minimises the  
57 difference in the orbital velocity seen by different ADCP bins, whilst adopting a time averaging  
58 period similar to or less than that of the waves will result in the wave orbital velocity being treated  
59 as a background mean flow.

60 Working in a shallow water, wave-dominated environment, Whipple and Luettich (2009) as-  
61 sume that the velocity variance at each depth (calculated over a sampling period much longer than  
62 the wave period) is dominated by the wave orbital velocity at that depth. They fit a theoretical  
63 vertical profile based on linear wave theory to the observations in order to characterise the effec-  
64 tive wave contribution to the structure function over a specified depth range. This is then used  
65 to remove the influence of waves and isolate the much smaller turbulent signal. Whilst this ap-  
66 proach explicitly recognises the contribution of the vertical gradient of the wave orbital velocity

67 to the structure function, it is only applicable in situations where the wave influence dominates the  
68 structure function and does not lend itself to more general application.

69 The aims of this paper are, firstly, to demonstrate that  $\epsilon$  estimates made using the standard struc-  
70 ture function method with ADCP data are inherently susceptible to bias in the presence of surface  
71 waves due to a contribution to the structure function from the vertical gradient in the speed of the  
72 associated wave orbital motion; and secondly, to present a modification to the standard method  
73 that addresses such bias. Section two briefly covers the underlying theory; demonstrates the stan-  
74 dard method's bias using the wave orbital motions under synthetic monochromatic waves; and  
75 describes the proposed modified method based on the application of linear wave equations. Sec-  
76 tion three describes a set of long-term field observations from a shelf sea site that were used to test  
77 the standard and modified methods. Section four uses established similarity scaling approaches to  
78 compare the results under differing surface forcing conditions and section five is a discussion of  
79 the results.

## 80 **2. Theory**

### 81 *a. Structure Function*

82 The theoretical basis of the structure function technique and its derivation from the Kolmogorov  
83 similarity hypotheses is described in detail elsewhere (Sreenivasan 1991; Frisch 1995; Antonia  
84 et al. 1997; Pope 2000; Lucas et al. 2014; McMillan and Hay 2017). In summary, the tech-  
85 nique assumes that for isotropic turbulence in high Reynolds number flows, an inertial sub-  
86 range of length scales exists over which there is a conservative cascade of energy from larger  
87 to smaller motions. The statistical properties of the longitudinal turbulent velocity fluctuation,

88  $\delta u'(x, r) \equiv u'(x+r) - u'(x)$ , where  $u'(x)$  is the along-axis turbulent velocity at location  $x$ , then  
 89 vary as a function of the separation distance  $r$ .

90 Invoking Taylor's frozen field hypothesis to allow sampling of the statistical properties of the  
 91 flow over time, the mean  $\delta u'$  is related to  $\varepsilon$  for  $r$  values within the inertial sub-range as:

$$\langle (\delta u')^n \rangle \propto \langle \varepsilon \rangle^{n/3} r^{n/3} \quad (1)$$

92 where the angle brackets indicate time averaging over a statistically valid sampling period and  $n$   
 93 is the order of the structure function (Kolmogorov 1991a,b; Pope 2000).

94 The second-order structure function,  $D_{LL}(x, r)$ , is then defined as:

$$D_{LL}(x, r) \equiv \langle [u'(x+r) - u'(x)]^2 \rangle \quad (2)$$

95 and for values of  $r$  within the inertial sub-range  $D_{LL}(x, r)$  is related to  $\varepsilon(x)$  as:

$$D_{LL}(x, r) \propto C_2 \varepsilon^{2/3} r^{2/3} \quad (3)$$

96 where  $C_2$  is a universal constant of proportionality, frequently taken to be 2.1 based on atmospheric  
 97 studies (Wiles et al. 2006; Lucas et al. 2014; Simpson et al. 2015), whilst McMillan and Hay (2017)  
 98 use 2.0 based on both theoretical considerations and the comparison of  $\varepsilon$  estimates made using the  
 99 structure function and spectral integral methods.

100 From (3), the second-order structure function exhibits a length-scale dependence on  $r^{2/3}$ , so a  
 101 least-squares linear regression of  $D_{LL}(x, r)$  against  $r^{2/3}$ , at fixed  $x$ , gives:

$$D_{LL}(x, r) = A_0 + A_1 r^{2/3} \quad (4)$$

102 where  $A_0$  is a measure of the Doppler and instrument noise and  $A_1$  is the gradient of the linear  
 103 regression over the range of  $r$  evaluated. From (3),  $A_1 = C_2 \varepsilon^{2/3}$ , which then gives an estimate of  $\varepsilon$   
 104 at  $x$  for the sampling period as:

$$\varepsilon = \left( \frac{A_1}{C_2} \right)^{3/2} \quad (5)$$



105 When applied to ADCP data, a sampling period of several minutes is typically used, during which  
106 multiple individual velocity profiles are collected at a frequency of 1 – 2 Hz. The along-beam  
107 velocity data is processed for each beam separately, with the along-beam turbulent velocity,  $u'$ ,  
108 calculated for each bin by deducting its mean over the sampling period in order to remove the  
109 mean flow and hence any background shear.

110 The structure function,  $D_{LL}$ , is then calculated from the velocity differences at separation dis-  
111 tances,  $r$ , based on multiples of the along-beam bin size. The minimum separation is taken as  
112 two bins due to the lack of independence in the velocities measured in adjacent bins (Teledyne RD  
113 Instruments 2014). The squares of the velocity differences are then averaged over the sampling pe-  
114 riod as in (2). Using a central difference scheme (e.g. Wiles et al. 2006),  $D_{LL}$  is evaluated for each  
115 bin for separation distances centred on the bin, with the  $r$  values that can be resolved dependent  
116 on the bin's position within the range of bins for which the turbulent velocity is available.

117 A maximum separation distance,  $r_{\max}$ , is specified for the regression of  $D_{LL}$  against  $r^{2/3}$ . This  
118 should be chosen to include as much of the inertial sub-range as possible, although in practice  
119 the configuration of the ADCP may restrict the range over which turbulent velocities are resolved.  
120 When this isn't a constraint,  $r_{\max}$  must not exceed the upper length limit of the inertial sub-range,  
121 beyond which  $D_{LL}$  is expected to tend towards a constant. The selection of  $r_{\max}$  therefore depends  
122 on both instrument constraints and the turbulent properties of the observed flow.

### 123 *b. Wave Orbital Motion*

124 A basic representation of deep-water surface gravity waves is to treat them as sinusoidal, with  
125 amplitude  $A$ , wavelength  $\lambda$  and period  $T$ , giving a radian frequency of  $\omega = 2\pi/T$ , wavenumber  
126  $k = 2\pi/\lambda$  and phase speed  $c$  given by  $c^2 = \omega^2/k^2 = g/k$ , with  $g$  being the acceleration due to  
127 gravity.

128 The simplest model for the motion in the water column below such waves (e.g. Phillips 1977;  
 129 Simpson and Sharples 2012), is of non-rotational circular motion with a speed at depth  $z$  (zero at  
 130 surface, positive up) of:

$$v_{\max} = \omega A e^{kz} \quad (6)$$

131 Over a vertical distance  $\delta z$  around depth  $z_0$ , the difference in the speed of the orbital motion is:

$$\begin{aligned} \delta v_{\max}(z_0) &= \omega A \left[ e^{k(z_0 + \delta z/2)} - e^{k(z_0 - \delta z/2)} \right] \\ &\approx k v_{\max}(z_0) \delta z \end{aligned} \quad (7)$$

132 subject only to the adoption of the small angle approximation that  $\sinh(k\delta z/2) \approx k\delta z/2$ , which is  
 133 valid to within 2% for  $\delta z < \lambda/10$ . Hence, at all depths, the vertical difference in the orbital speed  
 134 varies linearly with the vertical separation distance.

135 As illustrated in figure 1, this vertical variation in the speed of the orbital motion will result in a  
 136 contribution to the structure function even in the absence of turbulence. Under a monochromatic  
 137 wave, the along-beam velocity measured in the ADCP bins will vary sinusoidally in phase in all  
 138 bins, but with an amplitude that depends on the depth of the bin. Since the sampling period used  
 139 to determine the structure function is normally much longer than the surface wave period (several  
 140 minutes versus typically less than 15 seconds), the mean of the along-beam component of the  
 141 wave orbital motion measured by any bin is  $\sim$  zero and will not contribute to the mean velocity  
 142 deducted to calculate the fluctuating turbulent along-beam velocity  $u'$ . Consequently,  $u'$  retains the  
 143 along-beam component of the time-varying wave orbital motion. Any differences in  $u'$  between  
 144 bins will be treated as a turbulent velocity variation when calculating  $D_{LL}$ , potentially resulting in  
 145 a bias in the calculated  $\varepsilon$  estimates.

146 In order to quantify the potential bias,  $\varepsilon$  values were calculated using wave orbital velocities cal-  
 147 culated from linear wave theory for a range of monochromatic waves with amplitudes and periods

148 representative of an exposed shelf-sea environment. These synthetic wave orbital velocities were  
 149 calculated for the bin locations of virtual ADCP at depths of 20, 35 and 50 m with an upward-  
 150 looking orientation, sampling via a beam with a  $20^\circ$  beam angle (inclination from the vertical)  
 151 with 30 bins at a 0.1 m vertical bin spacing and bin one centred at 0.97 m from the transducer.  
 152 The measurement frequency was 1 Hz with a sampling period of 300 s resulting in 300 velocity  
 153 profiles.

154 Assuming waves propagating in the  $x$  direction and the ADCP beam in the  $y = 0$  plane, the  
 155 horizontal ( $u$ ) and vertical ( $w$ ) velocities vary as:

$$\begin{aligned}
 u &= \omega A e^{kz} \sin(kx - \omega t) \\
 w &= -\omega A e^{kz} \cos(kx - \omega t)
 \end{aligned}
 \tag{8}$$

156 with  $t$  being time.

157 The along-beam velocities in each bin were calculated by applying a rotation matrix based on the  
 158 virtual ADCP beam geometry (Teledyne RD Instruments 2010). The structure function,  $D_{LL}$ , was  
 159 calculated using a central difference scheme and  $\varepsilon$  estimates were determined for each bin from  
 160 the regression of  $D_{LL}$  against  $r^{2/3}$  with  $r_{\max}$  equal to 2.0 m. Beam average  $\varepsilon$  values were calculated  
 161 as the geometric mean of the individual values for all bins for which the structure function was  
 162 resolved for all  $r \leq r_{\max}$ .

163 Figure 2 shows the beam average  $\varepsilon$  estimates for each of the three instruments for surface  
 164 waves with amplitudes up to 2 m and periods between 7 and 13 s. The bias in  $\varepsilon$  is more than  
 165  $1 \times 10^{-5} \text{ W kg}^{-1}$  for an ADCP at a depth of 20 m under waves with an amplitude of 1.8 m and a  
 166 period of 8 s. Even for an instrument at 50 m depth, swell waves with a period of 11-12 s and an  
 167 amplitude of 1.6 m could potentially introduce a bias of  $\mathcal{O} 10^{-7} \text{ W kg}^{-1}$ , two orders of magnitude  
 168 above the expected noise floor (Lucas et al. 2014).

169 The bias in  $\varepsilon$  depends on the difference in the speed of the wave orbital motion over distance  
170  $r_{\max}$ , which depends on both the amplitude and the attenuation rate of the speed of the orbital  
171 motion. Since the attenuation rate depends on wave number, the period of the waves contributing  
172 most to any bias will typically increase with ADCP depth.

173 For a spectrum of waves, linear wave theory would suggest that the along-beam velocities ob-  
174 served by the ADCP will be the sum of the wave orbital velocities due to the various component  
175 waves. Whilst the velocity contribution from each component wave will depend on its surface  
176 properties and attenuation rate, each will exhibit the linear variation with vertical separation in  
177 (7). The composite wave orbital velocity can therefore also be expected to demonstrate a linear  
178 length-scale dependency.

179 Though the leading order water motions associated with the surface waves are periodic and do  
180 not affect the time-averaged current profile. Surface waves also produce a second order, depth-  
181 varying Lagrangian transport in their direction of propagation, the Stokes drift (e.g. Phillips 1977;  
182 Ardhuin et al. 2009). Within the structure function calculation, any non-periodic velocity observed  
183 by an ADCP bin is considered as part of the mean flow and removed when the turbulent velocity  
184 is calculated. Asymmetric periodic flows, such as the difference between the upper and lower  
185 portions of a wave orbital motion that leads to Stokes drift, may result in a non-zero contribution  
186 to the mean flow as well as a contribution to the structure function based on the depth dependent  
187 variation in the periodic motion. The Stokes drift speed decays exponentially with depth at twice  
188 the rate of the wave orbital motion (Phillips 1977). It is therefore also expected to exhibit a linear  
189 length-scale dependence over a limited vertical separation distance.

190 Exploiting the differing length-scale dependencies of the turbulent and wave-related components  
191 of the observed velocity offers the possibility of separating these two components of the structure  
192 function.

193 *c. Modified Methodology to Reject Impact of Wave Orbital Motion*

194 From (1), the  $n^{\text{th}}$  order structure function varies as  $r^{n/3}$ , hence  $D_{LL}$  will vary linearly against  
195  $r^{2/3}$ . By contrast, from (7), the difference in the maximum wave orbital velocity magnitude  $\delta v_{\text{max}}$   
196 varies linearly with  $r$ , hence from (2), the contribution to  $D_{LL}$  varies as  $r^2$ . In the regression of  $D_{LL}$   
197 against  $r^{2/3}$ , the contribution to the structure function from the vertical variation in wave orbital  
198 velocity will therefore increase as  $(r^{2/3})^3$ .

199 The differing rates at which the contribution of the turbulent and wave orbital motion compo-  
200 nents of the structure function vary with separation distance provides the basis for the modified  
201 method. Instead of the standard least-squares linear regression of  $D_{LL}$  against  $r^{2/3}$  as in (4), a  
202 least-squares fit is done to determine the coefficients for the linear model:

$$D_{LL}(x) = A_0 + A_1 r^{2/3} + A_3 (r^{2/3})^3 \quad (9)$$

203 The modified method essentially assumes that the wave orbital motion and turbulence do not in-  
204 teract and the associated velocities are simply additive. The contribution to  $D_{LL}$  due to the vertical  
205 gradient in the speed of the wave orbital motion (contained in the  $A_3$  coefficient) can therefore be  
206 extracted without affecting the turbulent contribution. Hence the  $A_0$  coefficient continues to de-  
207 scribe the instrument and Doppler noise and the  $A_1$  coefficient continues to describe the turbulence,  
208 with  $\varepsilon$  still calculated using (5).

209 The effectiveness of the modified method was tested by applying it to the synthesized wave  
210 orbital velocity data described in section 2b. Figure 3 shows the regression of  $D_{LL}$  against  $r^{2/3}$   
211 for both the standard and modified methods for the instrument at depth 35 m with a surface  
212 wave of amplitude 1 m and a period of 10 s. The standard method results in a calculated  $\varepsilon$  of  
213  $1.4 \times 10^{-7} \text{ W kg}^{-1}$  and a physically meaningless negative  $A_0$  value of  $-2.6 \times 10^{-5} \text{ m}^2 \text{ s}^{-2}$ . By

214 contrast, the  $A_0$  and  $A_1$  coefficients for the modified method correctly reflect the fact that there was  
215 no turbulent motion or system noise in the synthesized velocity data.

#### 216 *d. Similarity Scaling*

217 In order to compare the results of the standard and modified methods at different depths and un-  
218 der widely varying environmental conditions, two distinct surface forced regimes with established  
219 similarity scalings are considered. The relevant scaling factors are applied to  $\varepsilon$  estimates calcu-  
220 lated using both the standard and modified methods to illustrate the conformance of the results  
221 from the two methods to the standard scalings.

222 **1 Wind stress forcing.** Following Monin-Obukhov similarity theory, a local balance is as-  
223 sumed between  $\varepsilon$  and TKE production based on a constant stress “law of the wall” relation-  
224 ship (Anis and Moum 1995; Lombardo and Gregg 1989; Brainerd and Gregg 1993; Lozo-  
225 vatsky et al. 2005; Tedford et al. 2014; Bogucki et al. 2015; D’Asaro 2014). This results in a  
226 scaling factor  $\varepsilon_s$  given by:

$$\varepsilon_s = -\frac{u_*^3}{\kappa z} \quad (10)$$

227 where  $u_*$  is the friction velocity in the water, calculated as  $u_* = (\tau_s/\rho_0)^{1/2}$  for surface wind  
228 stress  $\tau_s$  and water density  $\rho_0$ ;  $\kappa$  is the von Kármán constant (0.41); and  $z$  is depth (zero at  
229 surface, positive up). Within the mixed layer, but below the region of direct impact from  
230 breaking waves (Agrawal et al. 1992; Anis and Moum 1995),  $\varepsilon$  estimates would be expected  
231 to scale as  $\varepsilon/\varepsilon_s \approx 1$ , with reported values typically in the range 1 - 2 based on limited duration  
232 observations (Lombardo and Gregg 1989; Lozovatsky et al. 2005; Shay and Gregg 1986;  
233 Thorpe 2005).

234 **2 Convective forcing.** By convention a positive surface buoyancy flux,  $B_0 > 0$ , indicates a loss  
235 of heat from the ocean surface to the atmosphere, increasing the ocean surface density and cre-  
236 ating unstable conditions leading to convection and an increase in  $\varepsilon$ . Within the mixed layer,  
237 but below the Monin-Obukhov length (the depth at which wind stress forcing and convective  
238 forcing match),  $\varepsilon$  is expected to be constant, reducing only at the base of the mixed layer  
239 when it encounters stratification and contributes to mixing by entrainment (Shay and Gregg  
240 1986; Lombardo and Gregg 1989). Hence under low wind conditions,  $\varepsilon$  estimates would be  
241 expected to scale as  $\varepsilon/B_0 \approx 1$ , with reported values based on limited duration observations  
242 typically being in the range 0.5 to 0.8 under conditions of both sustained and diurnal con-  
243 vection, with some indication of a time dependence as convection becomes established (Anis  
244 and Moum 1992; Brainerd and Gregg 1993; Lombardo and Gregg 1989; Shay and Gregg  
245 1984a,b, 1986; Thorpe 2005).

246 Combined scalings incorporating both wind stress and convective forcing have been developed  
247 as linear combinations of the scalings for the individual forcing regimes (e.g. Lombardo and Gregg  
248 1989; Tedford et al. 2014). However, the variation in the reported weighting coefficients suggests  
249 that the combined scaling may be less robust than the scaling for the individual regimes. The  
250 objective of the current study is not to revisit these scalings, but to use them as the basis for com-  
251 paring the susceptibility of the standard and modified structure function methods to wave-induced  
252 bias. The scalings were therefore applied separately to  $\varepsilon$  estimates based on field observations  
253 made under the relevant forcing conditions and the results compared to a default depth-constant  
254 unity reference value.

### 255 **3. Observations**

#### 256 *a. Dataset*

257 The present analysis is based on observations made during the period January to March 2015  
258 from a site in the Celtic Sea. The site has a water depth of  $\sim 150$  m; is more than 200 km from any  
259 coast, removing it from the direct coastal influences; and is over 125 km from the shelf edge, min-  
260 imising the impact of any shelf break processes. The wave climate included both locally generated  
261 waves and remotely generated swell, unaffected by significant shoaling or coastal reflections.

262 Three Teledyne RD Instruments 600 kHz Workhorse ADCP were deployed on a buoyancy ten-  
263 sioned mooring attached to a seabed anchor weight. The instruments were all configured in pulse-  
264 to-pulse coherent mode (mode 5) (Teledyne RD Instruments 2014) with a sampling frequency of  
265 1 Hz and one ping per ensemble (no ensemble averaging), with a vertical bin size of 0.1 m and  
266 bin one centred 0.97 m from the transducer. The instruments operated for a five-minute sampling  
267 period, followed by a 15-minute rest interval, resulting in three sampling periods per hour, each  
268 comprising 300 velocity profiles for each of the four beams. The uppermost instrument had a  $20^\circ$   
269 beam angle and was deployed upward-looking; the middle instrument also had a beam angle of  
270  $20^\circ$ , but was deployed downward-looking; whilst the lowest instrument had a beam angle of  $30^\circ$   
271 and was upward-looking.

272 The mooring rotated with the tide, the depth-averaged current having spring tide maxima of  
273  $\sim 0.5 \text{ m s}^{-1}$  with a pronounced spring-neap cycle. The instruments' measurement volumes were  
274 centred at mean depths of  $\sim 24.0$ ,  $42.5$  and  $52.5$  m. Reliable velocity measurements were typically  
275 returned for bins 1 to 30 for the  $20^\circ$  beam angle instruments and bins 1 to 28 for the  $30^\circ$  beam  
276 angle instrument, equating to bin centres at along-beam distances of  $\sim 1$  to  $\sim 4.2$  m from the  
277 transducer.



278 Three additional moorings provided supplementary information used in this analysis. All moor-  
279 ings were located within 1 km of each other throughout the observation period. One of the moor-  
280 ings provided full water column temperature, salinity and density (Wihsgott et al. 2016). Another  
281 was a UK Met Office ODAS buoy, which provided meteorological and wave data including hourly  
282 measurements of average wind speeds and direction plus maximum gust speeds at 3 m above  
283 the sea surface based on sampling over a 10 minute period; air and sea surface temperature; at-  
284 mospheric pressure and relative humidity; plus significant wave height and average wave period  
285 based on 17.5 minutes of observations. The third was a UK Centre for Environment, Fisheries and  
286 Aquaculture Science (Cefas) SmartBuoy, which provided half hourly sea surface temperature and  
287 salinity, plus photosynthetically active radiation (used as a proxy for solar irradiance).

## 288 *b. Data Analysis*

289 Surface stress and buoyancy flux were calculated using the TOGA COARE 3 bulk flux algo-  
290 rithm, taking account of the heights of the instruments on the ODAS buoy (Fairall et al. 2003).

291 The ADCP beam coordinate turbulent velocities,  $u'$ , were calculated independently for each bin  
292 in each beam by deducting the mean for that bin over the sampling period. Outlier values were  
293 identified by comparison with the rms value of all turbulent velocities for all bins and beams in  
294 the current sampling period and rejected. Outliers were almost exclusively in the furthest bin for  
295 which the velocity was resolved.

296 The second-order structure function,  $D_{LL}$ , was calculated using a central difference scheme over  
297 all resolvable separation distances,  $r = r_j \Delta r$ , where  $r_j$  is the separation in number of bins and  $\Delta r$  is  
298 the along-beam bin size determined by the vertical bin size and the beam angle. For even number

299 bin separations,  $r_j = 2, 4, 6 \dots$  around bin  $i$ :

$$D_{LL}(x_i, r_j \Delta r) = \left\langle \left[ u' \left( x_i + \frac{r_j}{2} \Delta r \right) - u' \left( x_i - \frac{r_j}{2} \Delta r \right) \right]^2 \right\rangle \quad (11)$$

300 where  $u'(x_i)$  is the turbulent velocity in the bin centred at distance  $x_i$  from the transducer. For odd  
 301 number bin separations,  $r_j = 3, 5, 7 \dots$ , the average of the two possible combinations was used, so  
 302 that:

$$D_{LL}(x_i, r_j \Delta r) = \left\langle \frac{1}{2} \times \left[ u' \left( x_i + \text{floor} \left( \frac{r_j}{2} \right) \Delta r \right) - u' \left( x_i - \text{ceil} \left( \frac{r_j}{2} \right) \Delta r \right) \right]^2 \right. \\ \left. + \frac{1}{2} \times \left[ u' \left( x_i + \text{ceil} \left( \frac{r_j}{2} \right) \Delta r \right) - u' \left( x_i - \text{floor} \left( \frac{r_j}{2} \right) \Delta r \right) \right]^2 \right\rangle \quad (12)$$

303 where floor( ) (ceil( )) means round down (up) to the integer.

304 The  $D_{LL}$  values for all bins were used in least-squares fit regressions against  $r^{2/3}$ , to give a beam  
 305 aggregate  $\varepsilon$  value for the sampling period for both the standard (4) and modified (9) methods. The  
 306 regressions were repeated for a range of  $r_{\max}$  values between 0.8 and 3.0 m (the maximum possible  
 307 given the instrument configurations). Basic result screening rejected regressions if the coefficients  
 308 did not produce a strictly increasing result for  $r > 0$ . Equation (5) was used to calculate  $\varepsilon$  with  
 309  $C_2$  as 2.0. The geometric mean of the individual beam values provided a single representative  $\varepsilon$   
 310 data point per sampling period for each instrument, method and  $r_{\max}$  value over the three months  
 311 of observations, resulting in approximately 6,500 data points for each combination of instrument,  
 312 method and  $r_{\max}$ .

313 The adjusted coefficient of determination,  $R_{\text{adj}}^2 = 1 - (1 - R^2) \left[ \frac{m-1}{m-(p+1)} \right]$ , where  $R^2$  is the un-  
 314 adjusted coefficient of determination;  $m$  is the sample size; and  $p$  is the number of independent  
 315 variables in the regression, was calculated for each regression. Using  $R_{\text{adj}}^2$  rather than  $R^2$  allows  
 316 the quality of the fit from both the standard and modified methods to be compared directly, taking  
 317 account of the additional term in the modified method.

## 318 4. Results

319 The three months of observations included in this analysis cover a wide range of winter condi-  
320 tions. Throughout the period, the water column was negligibly stratified. The surface buoyancy flux,  
321  $B_0$ , was characterised by a destabilising heat flux to the atmosphere ( $B_0$  positive) approximately  
322 70% of the time, when the mean flux was  $6 \times 10^{-8} \text{ W kg}^{-1}$  and the maximum  $1.9 \times 10^{-7} \text{ W kg}^{-1}$ .  
323 Solar irradiance resulted in intermittent diurnal stabilising ( $B_0$  negative) buoyancy fluxes, centred  
324 around midday and increasing in duration and maximum intensity over the period of the obser-  
325 vations. It is anticipated that this warming may have resulted in short periods of diurnal surface  
326 stratification under low wind stress conditions, therefore observations under these conditions were  
327 excluded from the analysis.

328 Wind speeds (at 3 m) had a range from 1 to  $19 \text{ m s}^{-1}$  with a rms of  $9.2 \text{ m s}^{-1}$  and maximum  
329 gusts of  $28 \text{ m s}^{-1}$ . Significant wave height varied between 1.2 and 14.1 m with a rms value of  
330 5.3 m, whilst the average wave period varied between 4.4 and 14.4 s, with a rms of 8.0 s. The  
331 resulting surface wind stress,  $\tau_s$ , varied between  $2 \times 10^{-4}$  and 1.2 Pa, with a rms of 0.27 Pa.

332 The  $\epsilon$  estimates were sorted according to the forcing conditions at the time of the observation,  
333 without any reference to adjustment time scales, resulting in the following datasets:

- 334 • Wind stress forcing:  $\tau_s > 0.05 \text{ Pa}$  giving  $\sim 5,300$  data points per instrument for each model  
335 and  $r_{\max}$  evaluated (81.9% of observations)
- 336 • Convective forcing:  $\tau_s \leq 0.05 \text{ Pa}$  and  $B_0 > 0$  giving  $\sim 870$  data points per instrument for each  
337 model and  $r_{\max}$  evaluated (13.4% of observations)

338 The number of observations varied slightly between instruments and between methods, with the  
339 modified method having the same or fewer  $\epsilon$  estimates for each instrument. Observations made  
340 under conditions when  $\tau_s \leq 0.05 \text{ Pa}$  and  $B_0 \leq 0$  (i.e. low wind and surface heating) comprised

341 4.7% of observations and were excluded from the current analysis. The  $\tau_s$  threshold was chosen  
342 based on the overall distributions of  $\tau_s$  and  $B_0$ , without any structured attempt at optimisation.

### 343 *a. Observation of Wave Orbital Motion*

344 Periodic variations were clearly apparent in much of the along-beam velocity data from each of  
345 the ADCP and were coherent across all bins in a beam. Fourier analysis typically showed a peak at  
346 or around the average surface wave period. In order to test whether the observations demonstrated  
347 the vertical gradient expected of wave orbital motion, the ADCP data was transformed from beam  
348 to earth coordinates and the rms of the earth coordinate vertical velocity,  $w_{\text{rms}}$ , and the difference,  
349  $\delta w_{\text{rms}}$ , over a vertical separation distance,  $\delta z$ , of 2.0 m, was calculated for each instrument and  
350 for each five-minute sampling period. The theoretical variation in the wave orbital speed,  $\delta v_{\text{max}}$ ,  
351 was calculated over  $\delta z$  at each instrument's observation depth using (7), assuming monochromatic  
352 waves of amplitude equal to half of the concurrent significant wave height and with the observed  
353 average period.

354 Figure 4 plots  $\delta w_{\text{rms}}$  versus  $\delta v_{\text{max}}$  together with the linear regression for each instrument. De-  
355 spite the simplistic assumption of monochromatic waves in the calculation of  $\delta v_{\text{max}}$ , all three  
356 instruments demonstrate a linear relationship with nearly identical coefficients over the full range  
357 of conditions. The robust correlation between  $\delta w_{\text{rms}}$  and  $\delta v_{\text{max}}$ , which are derived from indepen-  
358 dent datasets, indicates that wave orbital motions are producing a vertical gradient in the velocity  
359 profiles measured by the ADCP in a manner consistent with the simple theoretical model assumed.

### 360 *b. Comparison of the Standard and Modified Methods*

361 Figure 5 summarises the results for the standard and modified methods for all three instruments  
362 and under both surface wind stress and convective forcing. All regressions are based on  $r_{\text{max}} \sim$

363 2.0 m, the exact value depending on the separation distances evaluated given the ADCP geometry.  
364 The results for the two forcing processes are considered separately:

365 1. **Wind stress forcing.** The median wind stress scaled  $\epsilon$  estimates for each instrument and  
366 for both the standard and modified methods are shown in panel (a) of figure 5 and the data is  
367 summarised in table 1. For the standard method, the median scaled  $\epsilon$  estimates vary from 9.15  
368 for the uppermost instrument to 1.78 for the lowest instrument, with a clear depth dependence.  
369 Over 45% of standard method  $\epsilon$  estimates at 24 m have a bias of an order of magnitude or  
370 greater compared with the default unity scaling, with  $> 97\%$  of observations exhibiting a  
371 bias of two or more. The bias decreases with depth, although over 45% of the observations  
372 at 52.5 m remain subject to a bias of two or more. In contrast, for the modified method,  
373 the median scaled  $\epsilon$  estimates vary between 1.11 and 0.69 for the three instruments, with no  
374 apparent depth dependence, suggesting no significant departure from the “law of the wall”  
375 unity scaling.

376 2. **Convective forcing.** The median surface buoyancy flux scaled  $\epsilon$  estimates for each instru-  
377 ment and for both the standard and modified methods are shown in panel (b) of figure 5 and  
378 the data is summarised in table 2. The standard method median bias is higher for all instru-  
379 ments than the equivalent bias for the surface shear stress scaled observations, varying from  
380 21.15 for the uppermost instrument to 2.21 for the lowest instrument and again demonstrat-  
381 ing a clear depth dependence. In contrast, for the modified method, the median scaled  $\epsilon$   
382 estimates vary between 1.36 and 0.79 for the three instruments and again exhibit no apparent  
383 depth dependency, suggesting no significant departure from the unity scaling with  $B_0$ .

384 *c. Method Sensitivity to Selection of  $r_{\max}$*

385 In principle, it is desirable to evaluate the structure function regression over as much of the  
386 inertial sub-range as possible in order to better determine  $\varepsilon$ , subject to the constraint on  $r_{\max}$  being  
387 less than the upper limit of the inertial sub-range.

388 The sensitivity of the standard and modified methods to the choice of  $r_{\max}$  is illustrated in figure 6  
389 for both wind stress and convective forcing with  $r_{\max}$  as close as possible to 1, 2 and 3 m. All of  
390 these  $r_{\max}$  values are expected to be within the inertial sub-range given the water column density  
391 structure and turbulence levels. For  $r_{\max} \sim 1$  m, the regression of  $D_{LL}$  against  $r^{2/3}$  uses data for  
392 just eight separation distances (from two bins to nine bins). The number of separation distances  
393 increases approximately linearly with  $r_{\max}$ , subject to the dependence of the along-beam bin centre  
394 spacing on beam angle. For  $r_{\max} \sim 2$  m (3 m), the regression uses data for 18/16 (27/25) separation  
395 distances for the  $20^\circ/30^\circ$  instrument beam angles.

396 For the standard method, reducing  $r_{\max}$  reduces the bias but does not eliminate it. Even with  
397  $r_{\max}$  reduced to 1 m, the median bias for observations at 24 m remains 4.2 for wind stress forcing  
398 and 8.2 for convective forcing. However, reducing  $r_{\max}$  to 1 m does reduce the median bias to less  
399 than two for the observations at 42.5 m and 52.5 m for both forcing regimes.

400 The impact of reducing  $r_{\max}$  on the quality of the fit for the regression of  $D_{LL}$  against  $r^{2/3}$  and  
401 therefore on the confidence in the calculated  $\varepsilon$  estimate is shown in table 3 for wind stress forcing  
402 and table 4 for convective forcing. Reducing  $r_{\max}$  from  $\sim 2$  m to  $\sim 1$  m dramatically reduces the  
403 mean  $R_{\text{adj}}^2$  values.

404 For the modified method, varying  $r_{\max}$  has only minimal impact on the median scaled  $\varepsilon$  estimates  
405 for all three depths and both forcing regimes. The difference in the median scaled  $\varepsilon$  values is  
406 negligible for  $r_{\max} \sim 1$  m and 2 m, with the values for  $r_{\max} \sim 3$  m being fractionally lower. The

407  $R_{\text{adj}}^2$  values for the modified method consistently indicate a better fit than the standard method,  
408 although the difference is negligible for  $r_{\text{max}} \sim 1$  m, only becoming significant with increasing  
409  $r_{\text{max}}$ .

#### 410 *d. Wave Information from the Modified Method*

411 The additional regression coefficient produced by the modified method ( $A_3$ ) is expected to be  
412 dependent on the vertical difference in the speed of the wave orbital motion over the distance  $r_{\text{max}}$   
413 at the observation depth of the ADCP. Figure 7 plots the  $A_3$  coefficient for each regression for  
414 each instrument against the square of the difference in the theoretical wave orbital speed based  
415 on the concurrent surface wave observations ( $\delta v_{\text{max}}$ ), as described in section 4a, as well as linear  
416 regressions for each instrument.

417 The scatter in figure 7 is considered to result from the assumption of monochromatic waves, with  
418 the average period of the surface waves not being fully representative of the spectrum of waves  
419 contributing to the vertical gradient in the wave orbital speed at the ADCP depths. However,  
420 despite this simplification, the clear linear relationship between the  $A_3$  coefficient and  $(\delta v_{\text{max}})^2$ .  
421 suggests that the modified method is extracting the contribution to the structure function due to the  
422 vertical variation in the wave orbital velocity speed as expected.

423 A specific  $\delta v_{\text{max}}$  cannot be attributed to a unique surface wave condition, even under the as-  
424 sumption of monochromatic waves, since waves with different amplitudes and wavelengths could  
425 produce the same vertical velocity difference. In principle it may be possible to use the variation  
426 of  $A_3$  with depth to determine an "effective" surface monochromatic wave, but this is beyond the  
427 scope for the current study.

## 428 5. Discussion

429 Whilst three decades of ocean turbulence measurements using ship based microstructure profil-  
430 ers have provided strong quantitative links between the dissipation of turbulence kinetic energy  
431 and its forcing, the full geographic and temporal variability of turbulence, and hence mixing, re-  
432 mains a first order problem in oceanographic research (Ivey et al. 2008; Moum and Rippeth 2009;  
433 Mead Silvester et al. 2014). Part of the solution to this problem has been the development of new  
434 techniques for measuring longer time series of turbulence parameters. Amongst the more success-  
435 ful has been the application of moored off-the-shelf acoustic Doppler current profilers (ADCP),  
436 initially through the development of the variance method (Stacey et al. 1999; Lu and Lueck 1999;  
437 Rippeth et al. 2002), but more recently through a structure function approach (Wiles et al. 2006;  
438 Lucas et al. 2014).

439 In particular the structure function technique is an attractive option as the turbulence estimates  
440 are not sensitive to instrument motion, and can therefore be made mid-water column from moored  
441 platforms (Lucas et al. 2014), avoiding the specific processing to remove platform motion required  
442 for spectral techniques (Bluteau et al. 2016). Furthermore the development of pulse-to-pulse co-  
443 herent operating modes has enabled reliable estimates of  $\epsilon$  down to a noise floor estimated as  
444  $\sim 3 \times 10^{-10} \text{ W kg}^{-1}$  (Lucas et al. 2014). However, the averaging period implicit in the structure  
445 function technique is long relative to the period of surface waves, potentially leading to a bias in  $\epsilon$   
446 estimates due to the variation of the speed of the wave orbital motion with depth.

447 Here we have demonstrated the degree to which  $\epsilon$  is biased by the presence of surface waves  
448 using synthetic wave data. We have then developed a modified second-order structure function  
449 method which exploits the differing length-scale dependencies of the contributions due to turbulent  
450 and wave orbital motions in order to remove the surface wave influence. The standard and modified



451 methods were then tested using data collected over a three-month winter period by three ADCP  
452 operating in pulse-to-pulse coherent mode and mounted on a mooring at different depths. The  
453 observational period provided a wide range of wind, wave and surface buoyancy flux conditions.

454 Estimates of  $\varepsilon$  made using both the standard and modified structure function methods were then  
455 scaled using established scaling for either wind stress or convective forcing. The results using  
456 the standard method show a significant departure from the expected value under both forcing  
457 conditions. The bias is greatest for the uppermost instrument and declines significantly with depth.  
458 This accords with the hypothesis that the bias results from the vertical gradient in the speed of the  
459 wave orbital motions, which decay exponentially with depth. The median bias for convective  
460 forcing scaled  $\varepsilon$  estimates were higher than those scaled for wind stress forcing at all depths,  
461 indicating that the bias due to surface waves is more significant under relatively lower turbulence  
462 conditions. In contrast, the scaled  $\varepsilon$  estimates obtained using the modified method collapse to  
463  $\sim$  unity for the observations under both wind stress and convective forcing, indicating that the  $\varepsilon$   
464 profiles are in approximate accordance with the nominal scaling.

465 Analysis of the length-scale dependence of the speed of wave orbital motions for intermediate  
466 depth waves (see Appendix) suggests that the modified method should also be effective in remov-  
467 ing bias in  $\varepsilon$  estimates from observations affected by surface waves in shallower water, providing  
468 the orbital motions match standard wave theory. However, pending evaluation against actual ob-  
469 servations, care is needed in applying the modified method in shallow water conditions.

470 These results lead to the conclusions that:

- 471 • There is significant potential for bias in second-order structure function estimates of  $\varepsilon$  as a  
472 result of the depth variation of surface wave orbital velocities.

473 • A modified method, which exploits the differing length-scale dependencies of the contribu-  
474 tions to the structure function from turbulent and wave orbital motions, is effective in re-  
475 moving the surface wave bias in the  $\varepsilon$  estimates made under both wind stress and convective  
476 forced conditions.

477 *Acknowledgments.* This work was funded by NERC and Defra as a part of the Shelf Sea Biogeo-  
478 chemistry strategic research programme, through grants NE/K001760/1 and NE/K001701/1 (WP1  
479 Candyfloss). Brian Scannell is supported by NERC award 1500369 through the Envision Doctoral  
480 Training Programme. In addition J. Polton was supported by NERC standard grant Pycnocline  
481 Mixing in Shelf Seas (NE/L003325/1). We thank the crew of the RRS Discovery and National  
482 Marine Facilities for their assistance in collecting the data sets. Thanks also to Jon Turton at the  
483 UK Met Office for supplying the ODAS buoy data, to Juliane Wihsgott for the CTD data from  
484 the moored temperature chain and to Tom Hull for the Cefas SmartBuoy data. We also thank two  
485 anonymous reviewers for their comments, which helped to improve the manuscript.

## 486 APPENDIX

### 487 **Application with generalised wave equations**

488 The generalised equations for the motion under surface waves describe elliptical orbits with an  
489 eccentricity that depends on the wave's wavelength, the water depth and the depth of the observa-  
490 tion point. The horizontal and vertical velocity components under an infinitesimal monochromatic  
491 sinusoidal wave travelling in the  $x$  direction are given by:

$$\begin{aligned} u &= \frac{gk}{\omega} A \frac{\cosh(k(z+h))}{\cosh(kh)} \sin(kx - \omega t) \\ w &= -\frac{gk}{\omega} A \frac{\sinh(k(z+h))}{\cosh(kh)} \cos(kx - \omega t) \end{aligned} \quad (\text{A1})$$

492 where  $g$  is acceleration due to gravity;  $k$  is wavenumber given by  $k = 2\pi/\lambda$  and  $\lambda$  is wavelength;  
 493  $\omega$  is radian frequency given by  $\omega = 2\pi/T$  and  $T$  is wave period;  $z$  is depth, with  $z = 0$  at the sea  
 494 surface and positive upwards;  $h$  is water depth so that  $z = -h$  at seabed; and  $t$  is time (Phillips  
 495 1977).

496 A vertically oriented ADCP with a beam in the  $y = 0$  plane will see an along-beam velocity  $b_0$   
 497 in the bin centred at  $x = x_0$  and  $z = z_0$  with contributions from both components depending on the  
 498 beam angle  $\theta$ , which is given by:

$$b_0 = \frac{gkA}{\omega \cosh(kh)} \left[ \begin{aligned} &\sin \theta \cosh(k(z_0 + h)) \sin(kx_0 - \omega t) \\ &- \cos \theta \sinh(k(z_0 + h)) \cos(kx_0 - \omega t) \end{aligned} \right] \quad (\text{A2})$$

499 The velocity difference  $\delta b_0$  over a vertical range  $\delta z$  around depth  $z_0$  will therefore be:

$$\delta b_0 = \frac{gkA}{\omega \cosh(kh)} \left[ \begin{aligned} &\left[ \begin{aligned} &\sin \theta \cosh \left( k \left( z_0 + \frac{\delta z}{2} + h \right) \right) \sin \left( kx_{z_0 + \frac{\delta z}{2}} - \omega t \right) \\ &- \cos \theta \sinh \left( k \left( z_0 + \frac{\delta z}{2} + h \right) \right) \cos \left( kx_{z_0 + \frac{\delta z}{2}} - \omega t \right) \end{aligned} \right] \\ &- \left[ \begin{aligned} &\sin \theta \cosh \left( k \left( z_0 - \frac{\delta z}{2} + h \right) \right) \sin \left( kx_{z_0 - \frac{\delta z}{2}} - \omega t \right) \\ &- \cos \theta \sinh \left( k \left( z_0 - \frac{\delta z}{2} + h \right) \right) \cos \left( kx_{z_0 - \frac{\delta z}{2}} - \omega t \right) \end{aligned} \right] \end{aligned} \right] \quad (\text{A3})$$

500 where  $x_{z_0 - \frac{\delta z}{2}}$  is the  $x$  coordinate of the observation bin centred at  $z = z_0 - \frac{\delta z}{2}$ . For  $\theta$  values of  
 501  $20^\circ$  or  $30^\circ$  and  $\delta z$  appropriate for  $r_{\max}$  values used with the structure function regression, the  
 502 horizontal bin displacement  $x_{z_0 + \frac{\delta z}{2}} - x_{z_0 - \frac{\delta z}{2}}$  will be  $\ll \lambda$ , so that  $kx_{z_0 + \frac{\delta z}{2}} \approx kx_{z_0 - \frac{\delta z}{2}} \approx kx_0$  and the

503 orbital velocity observed in all bins is in phase. Equation (A3) then simplifies as:

$$\delta b_0 = \frac{gkA}{\omega \cosh(kh)} \left[ \sin \theta \sin(kx_0 - \omega t) \left[ \cosh \left( k \left( z_0 + \frac{\delta z}{2} + h \right) \right) - \cosh \left( k \left( z_0 - \frac{\delta z}{2} + h \right) \right) \right] - \cos \theta \cos(kx_0 - \omega t) \left[ \sinh \left( k \left( z_0 + \frac{\delta z}{2} + h \right) \right) - \sinh \left( k \left( z_0 - \frac{\delta z}{2} + h \right) \right) \right] \right] \quad (\text{A4})$$

504 Applying the double angle hyperbolic identities and recognising that  $\cosh$  ( $\sinh$ ) is an even (odd)  
505 function, equation (A4) simplifies as:

$$\delta b_0 = \frac{gkA}{\omega \cosh(kh)} 2 \sinh \left( k \frac{\delta z}{2} \right) \left[ \sin \theta \sin(kx_0 - \omega t) \sinh(k(z_0 + h)) - \cos \theta \cos(kx_0 - \omega t) \cosh(k(z_0 + h)) \right] \quad (\text{A5})$$

506 Grouping all the terms independent of  $\delta z$  into a function,  $F$ :

$$F = \frac{gkA}{\omega \cosh(kh)} \left[ \sin \theta \sin(kx_0 - \omega t) \sinh(k(z_0 + h)) - \cos \theta \cos(kx_0 - \omega t) \cosh(k(z_0 + h)) \right] \quad (\text{A6})$$

507 equation (A5) becomes:

$$\delta b_0 = 2F \sinh \left( k \frac{\delta z}{2} \right) \quad (\text{A7})$$

508 For  $k\delta z \ll 1$ , the approximation  $\sinh(x) \approx x$  can be applied, giving:

$$\delta b_0 \approx kF \delta z \quad (\text{A8})$$

509 For deep water waves,  $\sinh(k(z_0 + h)) \approx \cosh(k(z_0 + h)) \approx \cosh(kh)$ , so that equations (A6) and  
510 (A2) become identical and (A7) becomes  $\delta b_0 \approx kb_0 \delta z$ , recovering equation (7).

511 More generally, equation(A8) suggests that whilst  $F$  may vary with  $z$ ,  $\delta b_0$  will vary linearly  
512 with  $\delta z$  irrespective of the water depth, providing the wave orbital motion is described by the

513 generalised equations (A1), subject only to the constraint of  $\delta z$  being small relative to  $\lambda$ . This  
514 suggests that the modified method has the potential to be effective at removing bias due to wave  
515 orbital motion from  $\epsilon$  estimates over a wider range of water depths.

516 *a. Testing the modified method for non-deep water waves*

517 It is reasonable to anticipate that there will be limits on the effectiveness of the modified method  
518 as the water depth reduces. In order to test this, synthetic velocity data was generated for waves  
519 with a range of wavelengths and amplitudes in different water depths, in the same manner as  
520 described in section 2b, but using the general wave orbital motion equations (A1) rather than the  
521 deep water equations (8).

522 Along-beam velocity data was calculated for a single upward-looking ADCP at a depth of 20 m,  
523 with 30 bins, the first bin centred at 0.97 m from the transducer and with 0.1 m vertical bin centre  
524 spacing. Velocities were calculated at one second intervals for a five minute observation period.  
525 Surface wave wavelengths varied between 50 and 300 m and amplitudes between 0 and 2 m. The  
526 radian frequency was calculated from the dispersion relation  $c^2 = \frac{g}{k} \tanh(kh)$  where  $c$  is the wave  
527 phase speed.

528 The along-beam velocity data was processed to calculate the second-order structure function for  
529 separation distances up to the specified  $r_{\max}$  using a central-difference scheme. A background  $\epsilon$   
530 level was then added to the structure function so that the effectiveness of the modified method  
531 in recovering turbulence levels in the presence of wave orbital motions could be assessed. The  
532 imposed background  $\epsilon$  level varied logarithmically with wave amplitude from  $1 \times 10^{-10}$  to  $1 \times$   
533  $10^{-9}$  W Kg<sup>-1</sup>. The standard and modified methods were then used to calculate  $\epsilon$  estimates for  
534 each bin based on  $r_{\max}$  values between 1.0 and 3.0 m. An average  $\epsilon$  estimate was calculated as the

535 geometric mean of the individual values for all bins for which the structure function was resolved  
536 for all  $r \leq r_{\max}$ .

537 Figure A1 compares the results for the standard (a,c,e,g) and modified (b,d,f,h) methods based  
538 on  $r_{\max} = 2.0$  m for water depths of 150 m (a,b), 75 m (c,d), 50 m (e,f) and 25 m (g,h). Subplots  
539 (a) and (b) represent deep water waves, with subplot (a) being comparable to subplot (a) of figure  
540 2, although the wavelength range 50 to 300 m in figure A1 equates to a wider wave period range  
541 of 5.7 to 13.9 s. The figure shows that for the standard method, the bias introduced by the vertical  
542 gradient in the wave orbital speed overwhelms the imposed background  $\varepsilon$ , with the level of bias  
543 for a given wavelength and amplitude increasing slightly in shallower water depths.

544 The results from the modified method demonstrate that the method is generally effective in  
545 recovering the imposed background  $\varepsilon$  levels, the effectiveness increasing with increasing wave-  
546 length. Reducing the water depth has only a minimal impact, with a slight improvement in effec-  
547 tiveness as the depth is reduced.

548 For the shortest wavelengths and largest wave amplitudes, the modified method exhibits a neg-  
549 ative bias, resulting in calculated  $\varepsilon$  estimates lower than the imposed background values. The  
550 is due to the structure function regression against  $r^{2/3}$  failing to separate the linear term used to  
551 calculate  $\varepsilon$  from the  $(r^{2/3})^3$  term associated with the wave orbital motion. Increasing the imposed  
552 background level or increasing the depth of the observations reduces the effect, whilst increasing  
553  $r_{\max}$  increases the effect. This effectively introduces an observation-depth dependent limit on the  
554 method sensitivity in the presence of high frequency waves.

555 The results from the tests with synthetic data demonstrate that providing the wave-induced or-  
556 bitial motion conforms to the standard equations, reducing the overall water depth does not signif-  
557 icantly compromise the effectiveness of the modified method in removing bias in  $\varepsilon$  estimates due  
558 to the presence of surface waves.

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703 separation ranges up to the specified  $r_{\text{max}}$  for the three observation depths and  
704 for both the standard and modified methods. . . . . 39

705 TABLE 1. Median, 10<sup>th</sup> and 90<sup>th</sup> percentile wind stress scaled  $\varepsilon$  estimates for the three observation depths and  
 706 for both the standard and modified methods.

Depth (m)	Standard Method			Modified Method		
	10%ile	50%ile	90%ile	10%ile	50%ile	90%ile
24.0	3.14	9.15	31.03	0.42	1.11	3.85
42.5	0.82	2.33	7.01	0.18	0.69	1.90
52.5	0.55	1.78	6.27	0.18	0.80	2.71

707 TABLE 2. Median, 10<sup>th</sup> and 90<sup>th</sup> percentile buoyancy flux scaled  $\epsilon$  estimates for the three observation depths  
 708 and for both the standard and modified methods.

Depth (m)	Standard Method			Modified Method		
	10%ile	50%ile	90%ile	10%ile	50%ile	90%ile
24.0	4.13	21.15	90.33	0.29	1.36	7.38
42.5	1.00	3.14	12.94	0.08	0.79	3.40
52.5	0.56	2.21	11.83	0.07	0.85	4.67



709 TABLE 3. Wind stress forcing. Mean  $R_{\text{adj}}^2$  quality of fit for  $D_{LL}$  versus  $r^{2/3}$  regressions for separation ranges  
 710 up to the specified  $r_{\text{max}}$  for the three observation depths and for both the standard and modified methods.

Depth (m)	Standard method			Modified method		
	$r_{\text{max}} = 1$ m	2 m	3 m	1 m	2 m	3 m
24.0	0.58	0.81	0.84	0.59	0.85	0.93
42.5	0.58	0.80	0.85	0.58	0.83	0.91
52.5	0.39	0.57	0.67	0.39	0.58	0.70

711 TABLE 4. convective forcing. Mean  $R_{\text{adj}}^2$  quality of fit for  $D_{LL}$  versus  $r^{2/3}$  regressions for separation ranges up  
 712 to the specified  $r_{\text{max}}$  for the three observation depths and for both the standard and modified methods.

Depth (m)	Standard method			Modified method		
	$r_{\text{max}} = 1$ m	2 m	3 m	1 m	2 m	3 m
24.0	0.50	0.78	0.83	0.51	0.83	0.92
42.5	0.41	0.71	0.80	0.41	0.75	0.85
52.5	0.31	0.51	0.62	0.31	0.52	0.66

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714 **Fig. 1.** Schematic of wave orbital motion contribution to the second-order structure function,  $D_{LL}$ .  
715 Monochromatic, deep water surface waves of amplitude  $A$ , period  $T_p$ , radian frequency  $\omega$ ,  
716 and wavenumber  $k$ , drive irrotational circular motions with speed at depth  $z$  (zero at surface,  
717 positive up) given by  $v_{\max}(z) = A\omega e^{kz}$ . In the absence of any other motion, the ADCP  
718 only measures the along-beam component of the wave orbital motion, such that  $u(z, t) =$   
719  $v_{\max}(z) \sin(\omega t)$ , the velocities being in phase between bins whilst varying in magnitude with  
720 bin depth. The turbulent velocity,  $u' = u - \langle u \rangle$ , retains the wave orbital motion since the bin  
721 mean over a sampling period,  $\langle u \rangle_{T \gg T_p} \approx 0$ . The second-order structure function is the mean  
722 of the turbulent velocity variance,  $\langle (\delta u')^2 \rangle$ , for a range of separation distances, see equation  
723 (2). In the presence of an along-beam gradient in wave orbital motion speed,  $\langle (\delta u')^2 \rangle > 0$   
724 for all separation distances, resulting in an unavoidable non-turbulent contribution to  $D_{LL}$ . . . . . 42

725 **Fig. 2.** Standard second-order structure function method bias in  $\varepsilon$  due to wave orbital motion for  
726 synthetic deep water monochromatic waves observed by virtual ADCP at depths of (a) 20 m;  
727 (b) 35 m and (c) 50 m.  $D_{LL}$  based on a central-difference scheme with regression based on  
728  $r_{\max} \sim 2.0$  m. Beam average  $\varepsilon$  based on geometric mean of bins for which  $D_{LL}$  is resolved  
729 for all  $r \leq r_{\max}$ . ADCP are assumed to have a sampling rate of 1 Hz; a sampling period of  
730 5 minutes; a vertical bin size of 0.1 m with the first bin centred at 0.97 m from the transducer;  
731 and to be upward-looking with a  $20^\circ$  beam angle. . . . . 44

732 **Fig. 3.** Example standard and modified method regression of  $D_{LL}$  against  $r^{2/3}$  for synthetic wave  
733 orbital velocities. Instrument depth: 35 m; wave amplitude: 1.0 m; wave period: 10 s;  $D_{LL}$   
734 based on a central-difference scheme; regression based on  $r_{\max} \sim 2.0$  m. . . . . 45

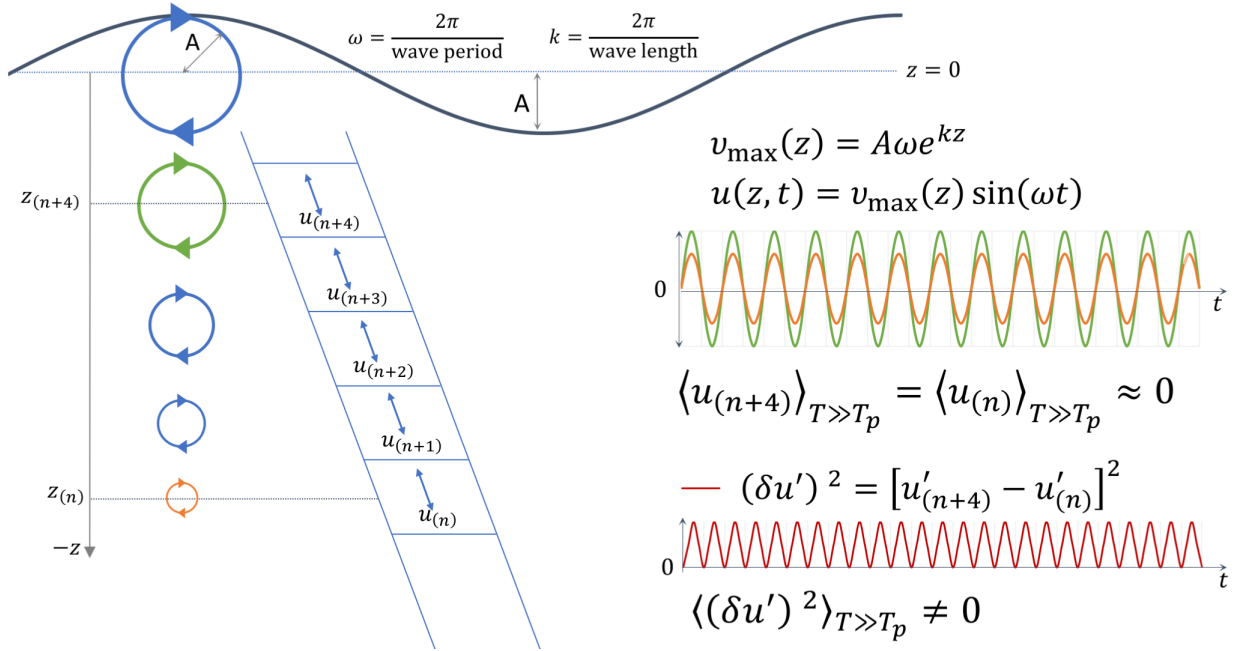
735 **Fig. 4.** Observed difference in rms vertical velocity,  $\delta v_{\text{rms}}$ , versus the difference in theoretical max-  
736 imum wave orbital velocity magnitude,  $\delta v_{\max}$  for the three instruments with observations  
737 centred at depths 24.0 m (red), 42.5 m (orange) and 52.5 m (purple). Differences calcu-  
738 lated over range  $\delta z = 2.0$  m;  $\delta v_{\text{rms}}$  from earth coordinate transformed velocities with rms  
739 over 300 profiles per 5 minute sampling period;  $\delta v_{\max}$  based on monochromatic waves of  
740 amplitude half the observed significant wave height and with the observed average period. . . . . 46

741 **Fig. 5.** Comparison of scaled  $\varepsilon$  estimates using the standard and modified methods. Median scaled  
742  $\varepsilon$  for each instrument with error bars showing 10%ile and 90%ile for standard (blue) and  
743 modified (red) method with (a) surface shear stress scaling ( $\tau > 0.05$  Pa) and (b) buoyancy  
744 flux scaling ( $\tau \leq 0.05$  Pa and  $B_0 > 0$  W kg $^{-1}$ ). Both methods used  $r_{\max} \sim 2.0$  m. Depths are  
745 median values with 10%ile and 90%ile error bars and an offset of 0.5m has been applied to  
746 the standard method data. . . . . 47

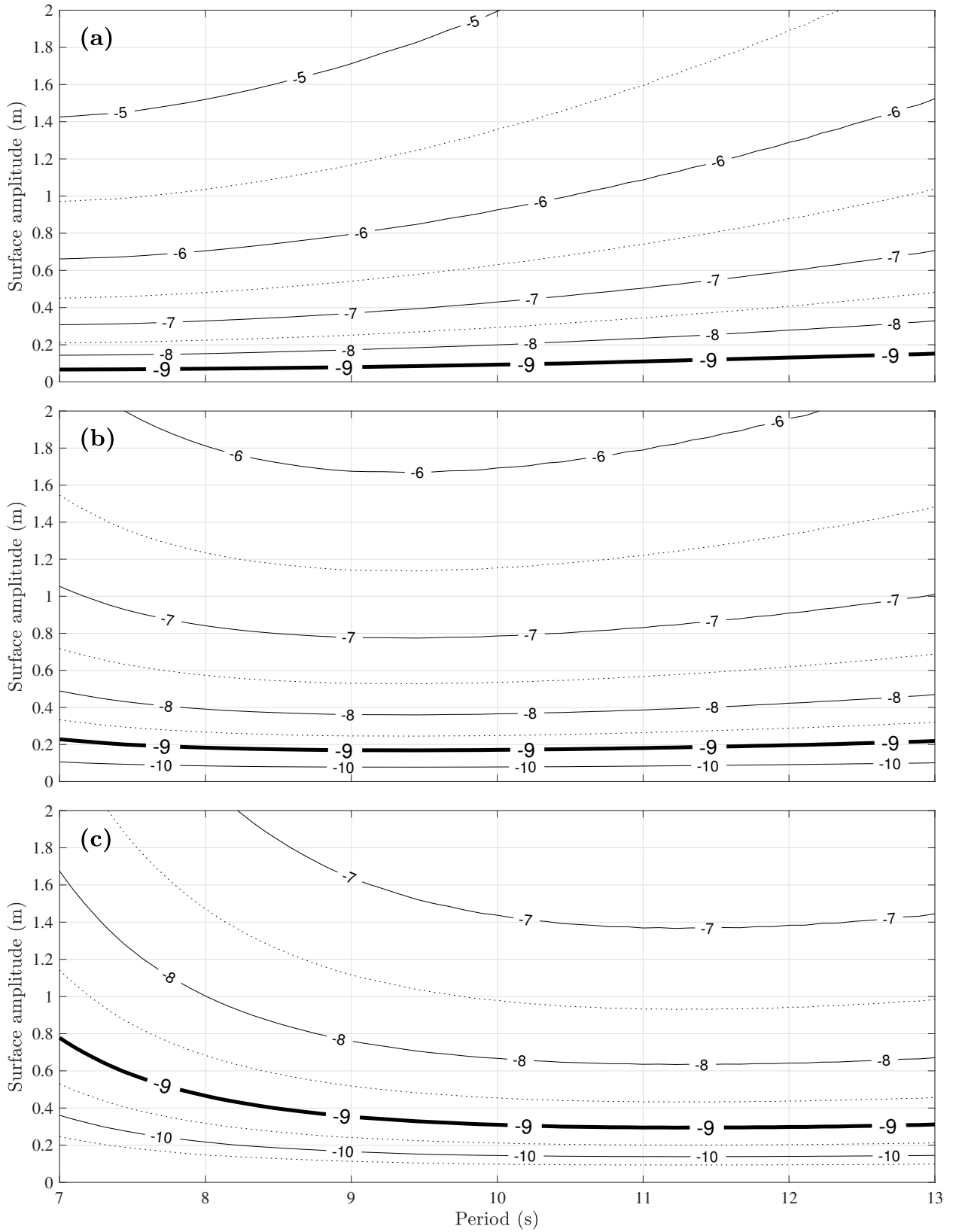
747 **Fig. 6.** Comparison of median scaled  $\varepsilon$  estimates with varying  $r_{\max}$  for the standard and modified  
748 methods. Median scaled  $\varepsilon$  estimates for  $r_{\max} \sim 1, 2$  and 3 m with (a) surface shear stress  
749 scaling ( $\tau > 0.05$  Pa) and (b) buoyancy flux scaling ( $\tau \leq 0.05$  Pa and  $B_0 > 0$  W kg $^{-1}$ ).  
750 Depths are median values. . . . . 48

751 **Fig. 7.** Modified method  $A_3$  regression coefficient versus difference in theoretical maximum wave  
752 orbital velocity magnitude,  $\delta v_{\max}$  for the three instruments with observations centred at  
753 depths 24.0 m (red), 42.5 m (orange) and 52.5 m (purple). Differences calculated over  
754 range  $\delta z = 2.0$  m;  $\delta v_{\text{rms}}$  from earth coordinate transformed velocities with rms over 300  
755 profiles per 5 minute sampling period;  $\delta v_{\max}$  based on monochromatic waves of amplitude  
756 half the observed significant wave height and with the observed average period. . . . . 49

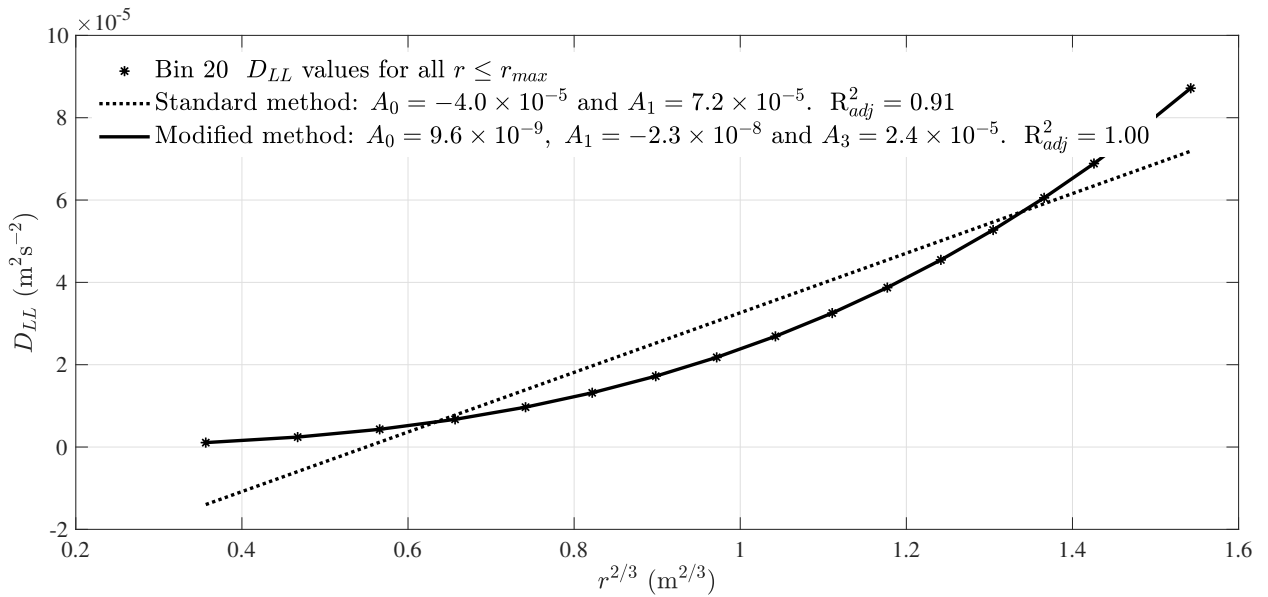
757 **Fig. A1.** Contour plots of  $\log_{10}(\varepsilon)$  estimates from wave orbital velocities synthesized using general  
758 wave velocity equations (A1) for water depths (a,b) 150 m, (c,d) 75 m, (e,f) 50 m and (g,h)  
759 25 m, calculated using (a,c,e,g) standard and (b,d,f,h) modified structure function method.  
760 ADCP at 20 m depth, upward-looking with 30 bins with a vertical bin size of 0.1 m and the  
761 first bin centred at 0.97 m from the ADCP. Wave orbital velocities resolved at 1 s intervals  
762 for 300 s. A background  $\varepsilon$  level is imposed, varying with surface wave amplitude from  
763  $1 \times 10^{-10} \text{ W kg}^{-1}$  for amplitude 0 m to  $1 \times 10^{-9} \text{ W kg}^{-1}$  for amplitude 2 m waves, such  
764 that in the absence of any wave-related bias, contours -9.1, -9.2 ... -9.9 would be equally  
765 spaced horizontal lines. . . . . 51



766 FIG. 1. Schematic of wave orbital motion contribution to the second-order structure function,  $D_{LL}$ . Monochro-  
767 matic, deep water surface waves of amplitude  $A$ , period  $T_p$ , radian frequency  $\omega$ , and wavenumber  $k$ , drive irro-  
768 tational circular motions with speed at depth  $z$  (zero at surface, positive up) given by  $v_{\max}(z) = A\omega e^{kz}$ . In the  
769 absence of any other motion, the ADCP only measures the along-beam component of the wave orbital motion,  
770 such that  $u(z, t) = v_{\max}(z) \sin(\omega t)$ , the velocities being in phase between bins whilst varying in magnitude with  
771 bin depth. The turbulent velocity,  $u' = u - \langle u \rangle$ , retains the wave orbital motion since the bin mean over a sam-  
772 pling period,  $\langle u \rangle_{T \gg T_p} \approx 0$ . The second-order structure function is the mean of the turbulent velocity variance,  
773  $\langle (\delta u')^2 \rangle$ , for a range of separation distances, see equation (2). In the presence of an along-beam gradient in  
774 wave orbital motion speed,  $\langle (\delta u')^2 \rangle > 0$  for all separation distances, resulting in an unavoidable non-turbulent  
775 contribution to  $D_{LL}$ .

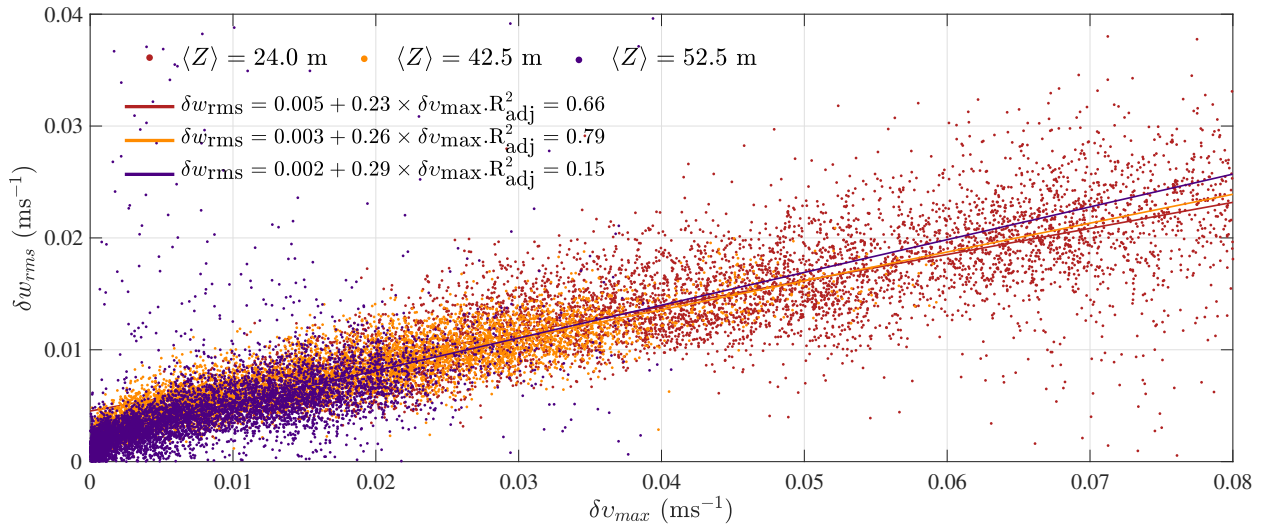


776 FIG. 2. Standard second-order structure function method bias in  $\varepsilon$  due to wave orbital motion for synthetic  
777 deep water monochromatic waves observed by virtual ADCP at depths of (a) 20 m; (b) 35 m and (c) 50 m.  
778  $D_{LL}$  based on a central-difference scheme with regression based on  $r_{\max} \sim 2.0$  m. Beam average  $\varepsilon$  based on  
779 geometric mean of bins for which  $D_{LL}$  is resolved for all  $r \leq r_{\max}$ . ADCP are assumed to have a sampling rate  
780 of 1 Hz; a sampling period of 5 minutes; a vertical bin size of 0.1 m with the first bin centred at 0.97 m from the  
781 transducer; and to be upward-looking with a  $20^\circ$  beam angle.

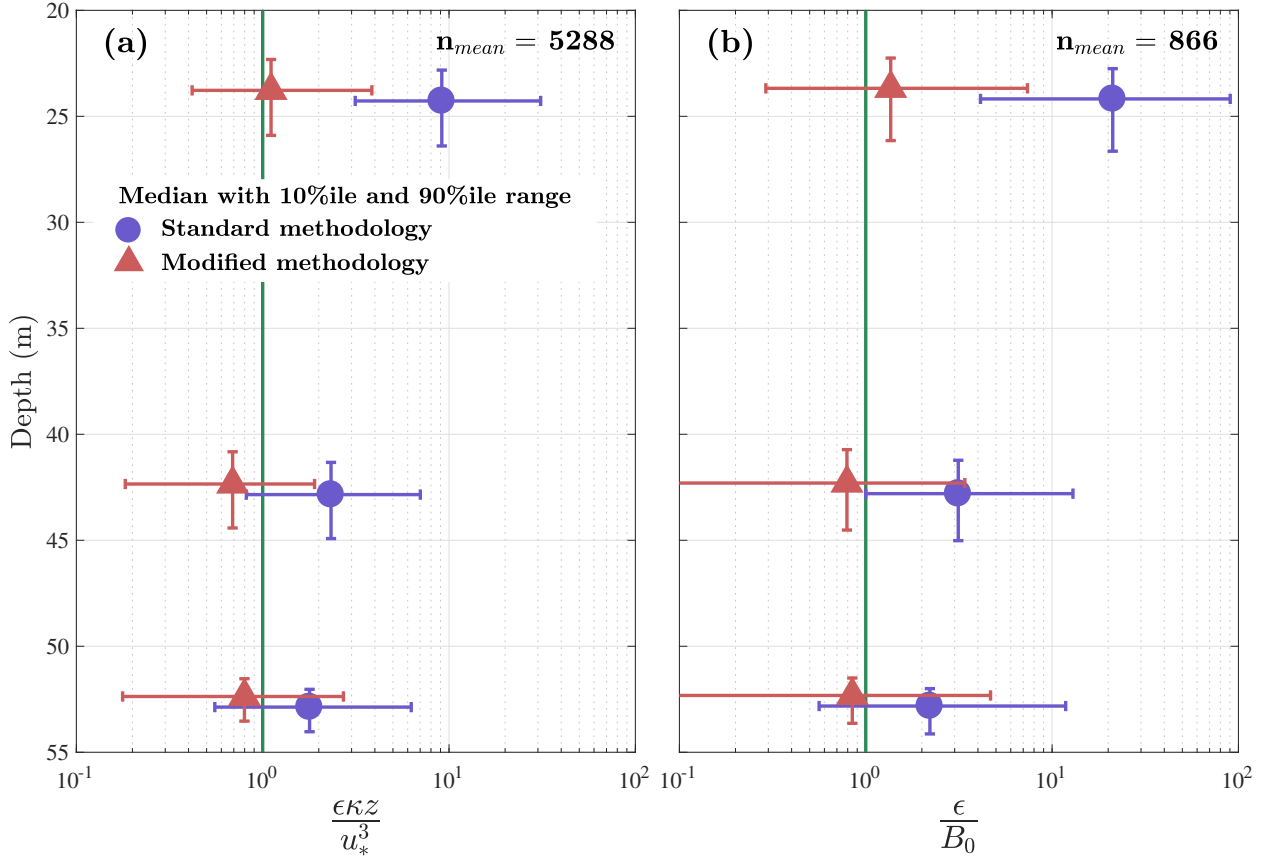


782 FIG. 3. Example standard and modified method regression of  $D_{LL}$  against  $r^{2/3}$  for synthetic wave orbital  
 783 velocities. Instrument depth: 35 m; wave amplitude: 1.0 m; wave period: 10 s;  $D_{LL}$  based on a central-difference  
 784 scheme; regression based on  $r_{max} \sim 2.0$  m.

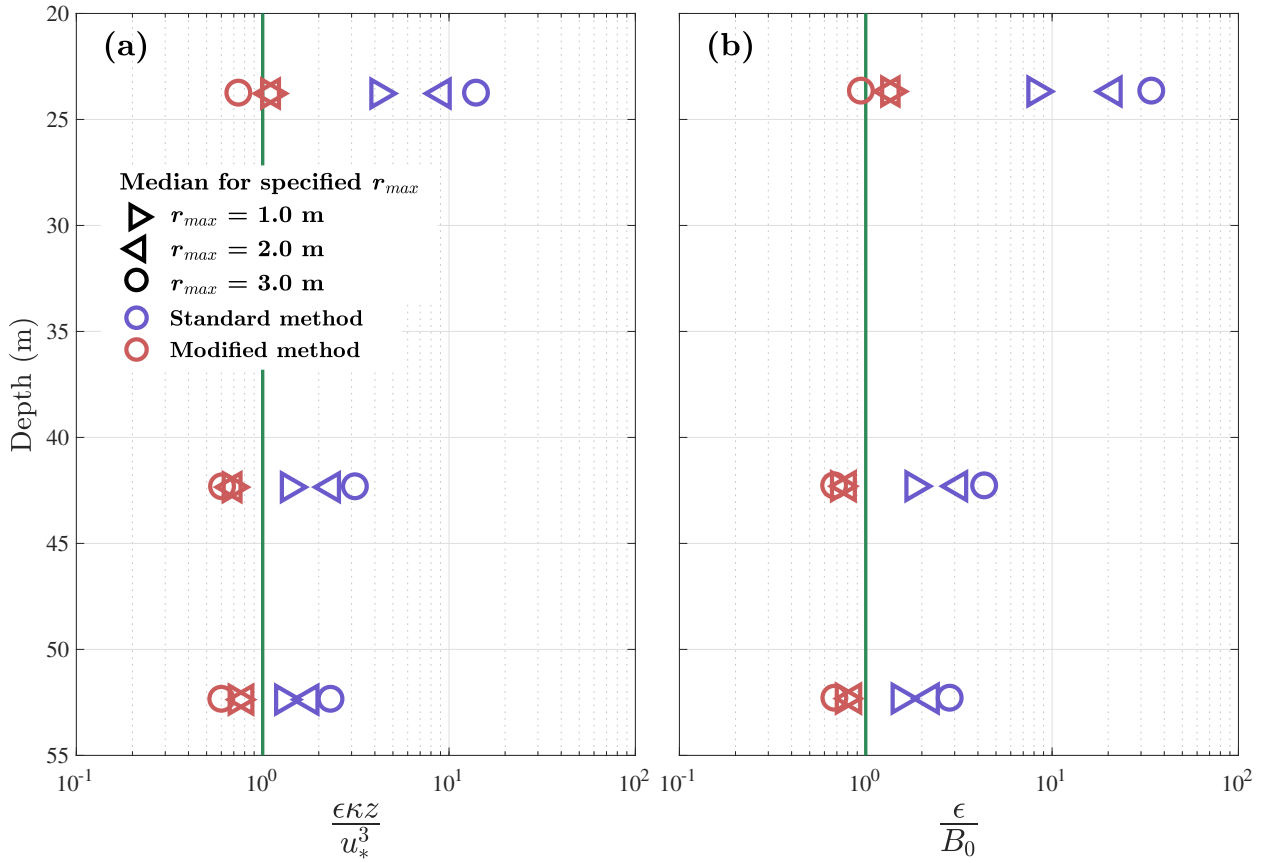




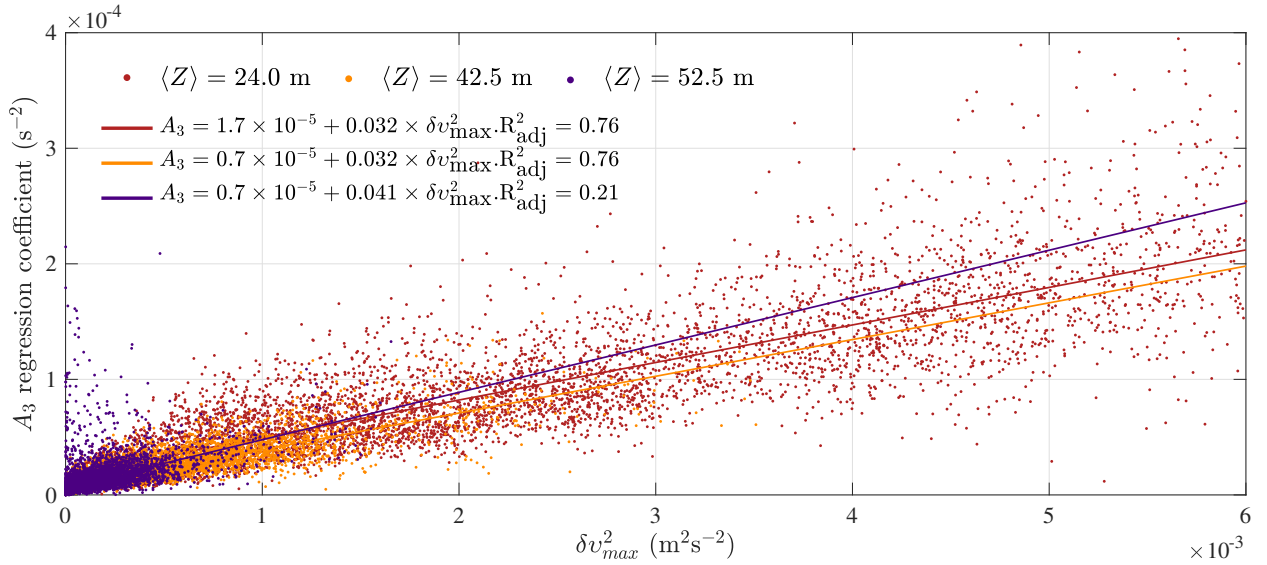
785 FIG. 4. Observed difference in rms vertical velocity,  $\delta w_{rms}$ , versus the difference in theoretical maximum  
 786 wave orbital velocity magnitude,  $\delta v_{max}$  for the three instruments with observations centred at depths 24.0 m  
 787 (red), 42.5 m (orange) and 52.5 m (purple). Differences calculated over range  $\delta z = 2.0$  m;  $\delta w_{rms}$  from earth  
 788 coordinate transformed velocities with rms over 300 profiles per 5 minute sampling period;  $\delta v_{max}$  based on  
 789 monochromatic waves of amplitude half the observed significant wave height and with the observed average  
 790 period.



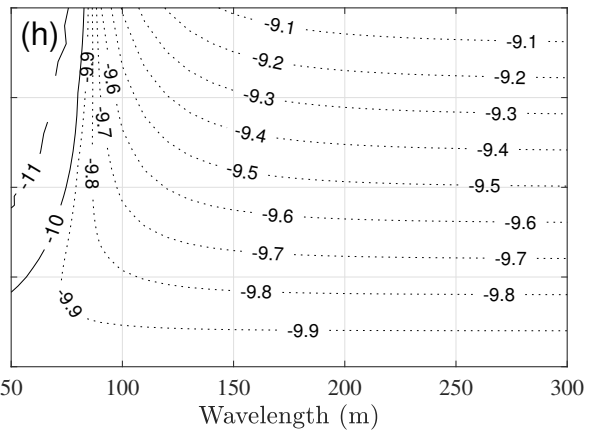
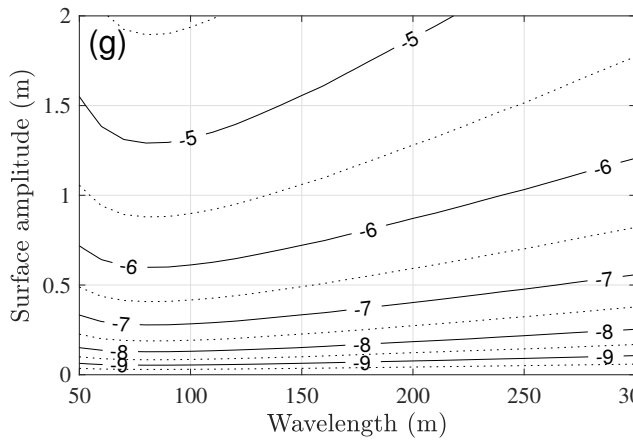
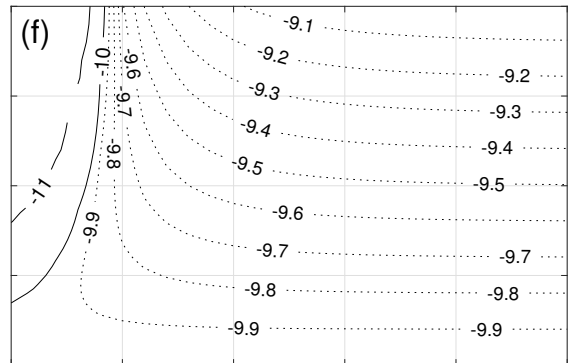
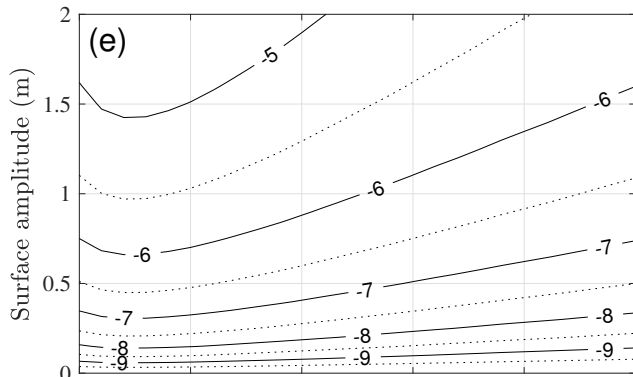
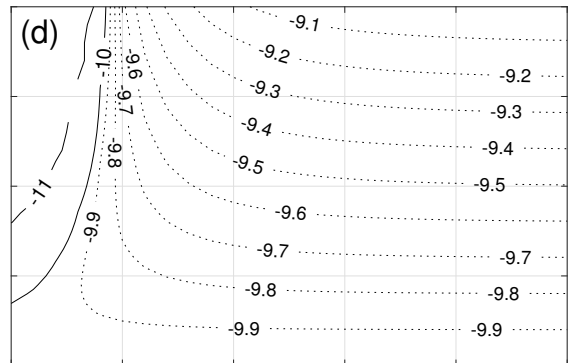
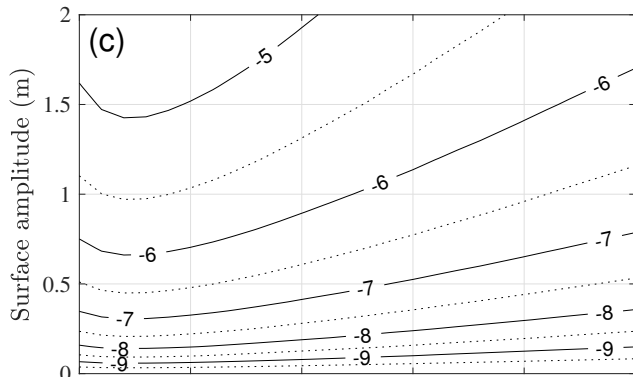
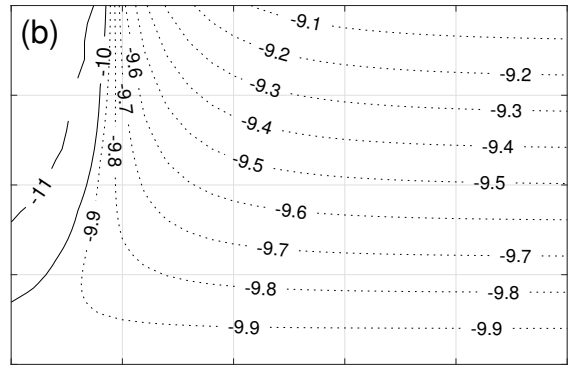
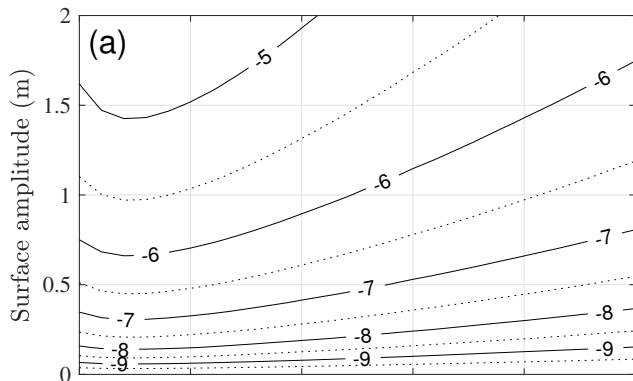
791 FIG. 5. Comparison of scaled  $\epsilon$  estimates using the standard and modified methods. Median scaled  $\epsilon$  for each  
 792 instrument with error bars showing 10%ile and 90%ile for standard (blue) and modified (red) method with (a)  
 793 surface shear stress scaling ( $\tau > 0.05$  Pa) and (b) buoyancy flux scaling ( $\tau \leq 0.05$  Pa and  $B_0 > 0$  W kg $^{-1}$ ). Both  
 794 methods used  $r_{max} \sim 2.0$  m. Depths are median values with 10%ile and 90%ile error bars and an offset of 0.5m  
 795 has been applied to the standard method data.



796 FIG. 6. Comparison of median scaled  $\epsilon$  estimates with varying  $r_{max}$  for the standard and modified methods.  
 797 Median scaled  $\epsilon$  estimates for  $r_{max} \sim 1, 2$  and  $3$  m with (a) surface shear stress scaling ( $\tau > 0.05$  Pa) and (b)  
 798 buoyancy flux scaling ( $\tau \leq 0.05$  Pa and  $B_0 > 0$  W kg $^{-1}$ ). Depths are median values.



799 FIG. 7. Modified method  $A_3$  regression coefficient versus difference in theoretical maximum wave orbital  
800 velocity magnitude,  $\delta v_{\max}$  for the three instruments with observations centred at depths 24.0 m (red), 42.5 m  
801 (orange) and 52.5 m (purple). Differences calculated over range  $\delta z = 2.0$  m;  $\delta w_{\text{rms}}$  from earth coordinate  
802 transformed velocities with rms over 300 profiles per 5 minute sampling period;  $\delta v_{\max}$  based on monochromatic  
803 waves of amplitude half the observed significant wave height and with the observed average period.



804 Fig. A1. Contour plots of  $\log_{10}(\varepsilon)$  estimates from wave orbital velocities synthesized using general wave  
805 velocity equations (A1) for water depths (a,b) 150 m, (c,d) 75 m, (e,f) 50 m and (g,h) 25 m, calculated using  
806 (a,c,e,g) standard and (b,d,f,h) modified structure function method. ADCP at 20 m depth, upward-looking with  
807 30 bins with a vertical bin size of 0.1 m and the first bin centred at 0.97 m from the ADCP. Wave orbital velocities  
808 resolved at 1 s intervals for 300 s. A background  $\varepsilon$  level is imposed, varying with surface wave amplitude from  
809  $1 \times 10^{-10} \text{ W kg}^{-1}$  for amplitude 0 m to  $1 \times 10^{-9} \text{ W kg}^{-1}$  for amplitude 2 m waves, such that in the absence of  
810 any wave-related bias, contours -9.1, -9.2 . . . -9.9 would be equally spaced horizontal lines.