A MODEL PREDICTIVE CONTROL FRAMEWORK FOR AUV FORMATION CONTROL

Rui Gomes [*,1] **João Borges de Sousa** [*] **Fernando Lobo Pereira** [*]

[*] *Faculdade de Engenharia da Universidade do Porto, Rua Dr. Roberto Frias s/n, 4200-465 Porto, Portugal*

Abstract: This article concerns a Model Predictive Control (MPC) Framework for the control of AUV formations. The strict resource constraints - communications (which are acoustic), computation, and power - make the problem of decentralized control of AUV formations extremely difficult. Here, we present the first developments in the investigation of this problem. It involves the development of a simulation environment implementing a numerically efficient scheme based on a linear quadratic optimization problem. The next stage concerns the reformulation of this framework so that minimal information is exchanged.

Keywords: Model Predictive Control, Autonomous Underwater Vehicles, Formation Control

1. INTRODUCTION

Real world challenges that human kind perceives today, such as climate change, bio-diversity, environment, natural resources management, territory management, security and surveillance, to name just a few, impose a number of increasingly sophisticated requirements for field studies data gathering. Spatial and temporal distribution, persistence, combination of wide area with local area data sampling, etc, are some general requirements calling for a concerted instrumentation of the earth which encompass networked fixed, mobile sensor platforms, and other devices. Moreover, one can easily devise many instances of missions involving, possibly heterogeneous, networked unmanned vehicles, say Autonomous Underwater Vehicles (AUVs), in which there is the need to distribute different sensors by different vehicles that should move in a certain formation defined to fulfill the needed data sampling requirements.

The underwater milieu poses tremendous challenges for the design of advanced data gathering systems. Onboard space and energy (required for the actuation, sensing, computation, and communication) are at a premium and acoustic communications feature

very low data rates, are unreliable and power hungry. This makes the case for systems for which the overall management of onboard resources has to be carefully optimized. Moreover, hydrodynamic effects make precise models too complex from the computational point of view and this implies that operational AUV models have necessarily to be approximated by simpler concentrated parameter models. The price to pay for this is that modeling becomes more difficult and uncertainty increases which is aggravated by the impact of typical pervasive underwater perturbations.

In order to answer these challenges in the context of controlling a formation of AUVs tracking a given trajectory, a decentralized Model Predictive Control (MPC) framework is proposed.

Each vehicle runs an MPC algorithm that, by taking into account its own and its neighbors' navigation data, generates a control strategy that balances a minimal quadratic error to the reference trajectory and deviation from the pre-specified formation pattern with minimal employed control effort over time. Control and state constraints are also considered to reflect control saturations as well as to avoid the collision with obstacles. The obtained control is applied for a short time interval, after which the state is sampled and information is exchanged among the pertinent neighboring vehicles via acoustic communication links. Then, the cycle is restarted with the new optimization carried

out over a shifted time horizon and with the most recent data.

The partiality of the information available to each AUV motivates a decentralized version of this problem. However, this calls for a level of communication and of computation in each vehicle that strongly conflicts with the available onboard resources. Thus, two main issues may arise in the networked MPC scheme.

- acoustic communications may exhibit delays and packets loss.
- computational complexity which, while taking into account the strict limitations of the AUV onboard resources, also has to meet real-time requirements.

A substantial amount of research work has been done on the control of formations of autonomous vehicles, (Franco *et al.*, 2004; Franco *et al.*, 2008; Keviczky *et al.*, 2006; Keviczky *et al.*, 2008; Fax and Murray, 2004; Olfati-Saber and Murray, 2004; Semsar-Kazerooni and Khorasani, 2008; Goodwin *et al.*, 2004; Fontes *et al.*, 2009; Gruene *et al.*, 2009; Allen *et al.*, 2002; Liu *et al.*, 2001). However, to the best of our knowledge, there are no satisfactory developments meeting the requirements of our applications.

This article is organized as follows: In the next section we discuss the various ingredients of the formation control problem and justify the options made. Then, in section 3, we present an overview of the state-of-the-art on MPC. In section 4, we discuss our decentralized MPC framework for the control of formations of AUVs and its implementation in a simulation environment. Finally, some brief conclusions are drawn.

2. AUV FORMATION CONTROL PROBLEM

In this section, we formulate the AUV formation control problem to control a set of vehicles that have to track a trajectory and, at the same time, keep a given formation pattern while satisfying state (safety requirements) and control (saturations) constraints. Typically, modeling the AUV motion is difficult. Because of the hydrodynamic effects, AUVs are distributed parameter systems and, thus, represented by extremely complex models. For this reason, we use in our developments the following model with coefficients based on results in (Prestero, 2001) and on our own field experiments.

$$
\dot{\eta} = \begin{bmatrix} u\cos(\psi) - v\sin(\psi) \\ u\sin(\psi) + v\cos(\psi) \\ r \end{bmatrix}, \qquad (1)
$$

$$
\dot{\nu} = \begin{bmatrix} \dfrac{\tau_u - (m - Y_{\dot{v}})vr - X_{u|u|}u|u|}{m - X_{\dot{u}}} \\ \dfrac{(m - X_{\dot{u}})ur - Y_{v|v|}v|v|}{m - Y_{\dot{v}}} \\ \dfrac{\tau_r - (Y_{\dot{v}} - X_{\dot{u}})uv - N_{r|r|}r|r|}{I_{zz} - N_{\dot{r}}} \end{bmatrix}, \qquad (2)
$$

where $\eta = [x, y, \psi]^T$ (from here onwards, a "T" in upper script will denote transposed), $\nu = [u, v, r]^T$, $\tau = [\tau_u, \tau_r]$, the coefficients $X_{\dot{u}}$, $Y_{\dot{v}}$, $N_{\dot{r}}$ represents hydrodynamic added mass, $X_{u|u|}$, $Y_{v|v|}$, $N_{r|r|}$ the hydrodynamic drag and m the vehicle mass.

From the above, we are interested in control strategies which, for each AUV i, $i = 1, \ldots, n_v$, minimize, over a given time interval, a cost functional with two terms, one that penalizes the trajectory tracking error forcing vehicles to follow the desired path, η_{ref}^i, and another that penalizes the control effort, thus saving the limited energy on board of vehicles, i.e.,

$$
\int_t^{t+T} (\eta^i(s) - \eta_{ref}^i(s))^T Q(\eta^i(s) - \eta_{ref}^i(s))
$$
$$
+ \tau^{iT}(s) R \tau^i(s) ds, \quad (3)
$$

and, at the same time, satisfies the following:

(i) Kinematic and dynamic equations constraints (vehicle dynamics) given by (1) and (2);
(ii) Endpoint state constraints, $\eta^i(t + T) \in C_{t+T}$;
(iii) Control constraints, $\tau^i(s) \in \mathbb{U}^i$;
(iv) State constraints, $(\eta^i(s), \nu^i(s)) \in \mathbb{S}^i$;
(v) Communication constraints
 $g_{i,j}^c(\eta^i(s), \eta^j(s)) \in C_{i,j}^c$, $\forall j \in \mathcal{G}^c(i)$; and
(vi) Formation constraints
 $g_{i,j}^f(\eta^i(s), \eta^j(s)) \in C_{i,j}^f$, $\forall j \in \mathcal{G}^f(i)$.

While the control constraints (iii) include, for example, saturations, the state constraints (iv) are specified to keep each vehicle in a specified set in order to satisfy safety or some other requirement. For example, to avoid collision with obstacles - known a priori or detected on the fly - or to prevent some variables to take on values that may damage components.

The satisfaction of the acoustic communication constraints (v) ensure that the motion of the vehicles is such that the required connectivity is preserved. The fact that closer the vehicles are, the lower the power consumption and packets loss, makes a strong case for each AUV to communicate with its neighbors and, hence, for decentralized control structure. The communications structure may be described by the triple $(g^c, C^c, \mathcal{G}^c)$, where $g^c : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}^M$, $C^c \in \mathbf{R}^M$ (here, $M \leq n(n_v - 1)n_v$, being n the dimension of the state space component of interest of each vehicle), and $\mathcal{G}^c$ a graph whose i^{th} component defines the vehicles with which the ith vehicle communicates. We point out that the communications graph is, in general, quite different from the formation or control graphs that we will introduce next.

Finally, the formation constraints (vi) specify the relations between data (typically, relative positions) of AUVs which have to be maintained with the help of appropriate control activity. These relative positions are specified in order to ensure the desired requirements of the activity (e.g., data gathering) undertaken by the AUVs. The formation structure may be de-

scribed by triple $(g^f, C^f, \mathcal{G}^f)$ where $g^f : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}^M$, $C^f \in \mathbf{R}^M$ (here, $M \leq n(n_v - 1)n_v$, being n the dimension of the state space component of interest of each vehicle), and $\mathcal{G}^f$ a graph whose i^{th} component defines the vehicles with which the ith vehicle has a formation relation.

3. BRIEF STATE-OF-THE-ART ON MODEL PREDICTIVE CONTROL

There is an extremely vast body of literature on MPC - also designated by Receding Horizon Control (RHC) - that we cannot hope to include in this overview. See, for example, (Mayne *et al.*, 2000). We will focus on the key results that are pertinent to our approach and focus on the class of systems addressed in this report - coordinated control of formations of vehicles.

MPC is a control scheme in which the control action for the current time subinterval - control horizon - is obtained, at each sampling time, by solving online an optimal control problem over a certain large time horizon - the prediction horizon - with the state variable initialized at the current best estimate updated with the latest sampled value. Once the optimization yields an optimal control sequence, this is applied to the plant during the control horizon. Then, once this time interval elapses, the process is re-iterated. The MPC scheme involves the following steps:

1. Initialization. Let t_0 be the current time, and set up the initial parameters or conditions specifying x_0, T, Δ, initial filter parameters (in case the sampled data requires filtering, initial control for the recursive control optimization procedure, etc.
2. Sample the state variable at time t_0.
3. Compute the optimal control strategy, u^*, in the prediction optimal, i.e., $[t_0, t_0 + T]$, by solving the optimal control problem (P).
4. Apply the obtained optimal control during the current control horizon, $[t_0, t_0 + \Delta]$.
5. Slide time by Δ, i.e., $t_0 = t_0 + \Delta$, and adapt parameters and models as needed.
6. Go to step 2.

where x_0 is the initial state, T is the prediction horizon for control optimization, and Δ is the control horizon. A number of variants to this scheme have been considered by enriching some of steps with additional processing:

- For the networked systems implementation, the data obtained in step 4. might be a composition of locally sampled data and data communicated from other vehicles or subsystems. For this class of systems, it might be of interest to replace data that failed to be transmitted by simulated data.
- Filtering the sampled state variable is usually required, being the Kalman filter widely used.
- For situations in which models are significantly uncertain or may vary over time, it might be of interest to use the sampled data to identify or refine the value of model parameters.
- Likewise, if external perturbations act on the vehicles/systems are sensed or estimated, they can be used to improve the models entering in the optimization procedure, and to change the MPC parameters.
- Communication may introduce delays and data packets might fail to arrive with serious consequences to the controller performance. To address this, true data may be replaced by simulated data or MPC parameters may be adjusted.

A typical general formulation of the optimal control problem (P) may be as follows:

$$(P) \text{ Minimize } g(x(t_0 + T)) + \int_{t_0}^{t_0+T} f_0(t, x(t), u(t))dt$$

$$\text{subject to } \dot{x}(t) = f(t, x(t), u(t)) \quad \mathcal{L} - a.e.$$
$$u(t) \in \Omega \quad \mathcal{L} - a.e.$$
$$h(t, x(t)) \leq 0$$
$$g(t, x(t), u(t)) \leq 0$$
$$x(t_0 + T) \in C_f$$

where g is the endpoint cost functional, f_0 is the running cost integrand, f, h, and g represent, respectively, the vehicle dynamics, the state constraints, and the mixed constraints, C is a target that may also be specified in order to ensure stability. If one wants to take into account the uncertainty with respect to the initial state, then one may consider an initial state constraint, i.e., $x(t_0) \in C_i$ where C_i is an estimate of the uncertainty set, being the minimization taken over the worst case of the initial state.

Now, we overview some of the typical basic issues and approaches for stability and robustness (Mayne *et al.*, 2000; Langson *et al.*, 2004; Mayne *et al.*, 2009).

Stability. Two major MPC approaches have been considered to stability:

a) Direct method using the fixed horizon value function as a Lyapunov function; and
b) Indirect approach employing the monotonicity property of a sequence of value functions.

Regardless of the approach, a number of formulations involving either a certain terminal state constraint set C, or terminal cost f_0, or both, have been considered. In order to ensure the asymptotic stability of the obtained feedback control law, say $u = k(x)$, the required typical assumptions are:

- $0 \in C$ with C closed;
- $k(x) \in \Omega$ the control constraint set;
- C is positively invariant under $k(\cdot)$; and
- f_0 is locally a Lyapunov function.

Robustness. Robustness concerns the ability of the system in preserving a certain property - e.g., stability or performance - in the presence of uncertainties. For

stability, this can be checked by concluding that the Lyapunov function for the nominal closed-loop system keeps the descent property for sufficiently small disturbances. While this is not very difficult to show for unconstrained problems, the consideration of constraints on states and controls raises substantial challenges as it is required to ensure that the constraints remain satisfied. Inherent robustness, min-max open loop control and feedback control are the general contexts considered to investigate robustness of MPC schemes.

The versatility exhibited by optimal control problems has been exploited in order to formulate and solve problems of controlling formation of vehicles. These typically have a substantially complex structure and may be addressed by using MPC schemes in either a decentralized or a centralized context which may involve two stages: the planning phase - solved off-line to provide the formation reference trajectory -, and the execution phase - solved on-line with the help of locally formulated control problems. Let us overview a selected sample of some of these approaches.

In (Franco *et al.*, 2008), the problem of cooperative control of a team of distributed agents with decoupled nonlinear discrete-time dynamics operating in a common environment and exchanging delayed information is considered. Each agent is assumed to evolve in discrete-time, based on locally computed control laws, which are computed by exchanging delayed state information with a subset of neighboring agents. The cooperative control problem is formulated in a receding-horizon framework, where the control laws depend on the local state variables (feedback action) and on delayed information gathered from cooperating neighboring agents (feedforward action). A rigorous stability analysis exploiting the input-to-state stability properties of the receding-horizon local control laws is carried out. The stability of the team of agents is then proved by utilizing small-gain theorem results.

Building on the work reported in (Keviczky *et al.*, 2006), a decentralized scheme for the coordinated control of formations of autonomous vehicles is presented in (Keviczky *et al.*, 2008). A high level receding horizon control and coordination strategy is obtained for each vehicle by solving a LQ optimization problem featuring control saturation constraints, linear dynamics constraints, and formation constraints with neighboring vehicles defined by a graph. An appropriate graph structure describes the underlying communication topology between the vehicles. On each vehicle, information about neighbors is used to predict their behavior and plan conflict-free trajectories that maintain the coordination and achieve the team objectives. When feasibility of the decentralized control is lost, collision avoidance is ensured by invoking emergency maneuvers that are computed via invariant set theory. A stabilization analysis is also discussed in (Keviczky *et al.*, 2006).

Information exchange strategies that improve the formation stability and performance and, at the same time, are robust to changes in the communication topology are considered in (Fax and Murray, 2004) to address the problem of cooperative control of vehicle formations. The sensed and communicated information flow is modeled by a graph whose topology has implications in the control stability. By exploiting the interplay between communications and control, necessary and sufficient conditions for the stability of an interconnected system of identical vehicles can be derived. Stated in terms of the Popov criterium for networked control systems, these conditions involve the eigenvalues of the graph Laplacian and reveal how to shape the information flow in order to ensure stability and achieve high performance.

The problem of unreliable communication channel between the MPC controller output and the actuator input, has been addressed in, among others, (Gruene *et al.*, 2009). Here, a mechanism for compensation of packet dropouts has been incorporated in the MPC scheme for discrete time problems. The basic idea consists in extending the control horizon until the next successful communication event happens and, in the meantime, use the best available control estimate, namely the one that has already been computed for the longer time interval. This article also includes some stability and sub-optimality analysis under an asymptotic controllability assumption. In order to show stability, the authors prove that, under the considered assumptions, the value function associated with the optimal control problem also exhibits properties of a Lyapunov function.

4. A DECENTRALIZED MPC FRAMEWORK FOR AUV FORMATIONS

In this section, we describe the implementation of a decentralized version of a discrete time MPC system to control a formation of AUVs in a simulation environment. The main features are:

- The decentralized character of the overall MPC controller is since each vehicle runs its own MPC scheme (which also encompasses the models of its neighboring AUVs) and communicates only with its neighbors;
- Computational efficiency is achieved by replacing the optimal control problem by a LQ optimization problem (for which an efficient MATLAB solver is used) and, for this, we consider (i) quadratic cost functionals, (ii) approximation of each AUV dynamics by a linear model, and (iii) state and control constraints (saturations) given by inequalities;
- Communication delays and packet dropouts can easily be incorporated; and
- Noise and disturbances can be easily considered in the vehicles simulated motion.

Now, we describe the optimization based control synthesis that will be performed in each AUV as part of the overall decentralized MPC scheme implemented in the simulation environment.

Let N_p, n_v, and T be, respectively, the prediction horizon, the number of vehicles, and the sampling period. Then, according to the previous considerations, the discrete time linear model of vehicle $i = 1, \ldots, n_v$, is, for $k = 0, \ldots, N_p - 1$, given by:

$$x_{k+1}^i = \Phi^i(T)x_k^i + \Psi^i(T)u_k^i, \; y_k^i = C^i x_k^i \quad (4)$$

where $\Phi^i(T) = e^{A^i T}$, $\Psi^i(T) = \int_0^T e^{A^i(T-s)}dsB^i$,

and $x_k^i \in \mathbb{R}^{n_s}$, $u_k \in \mathbb{R}^{n_c}$, and $y_k^i \in \mathbb{R}^{n_o}$ are respectively the system state, input and output variables, and n_s, n_c and n_o are the associated space dimensions.

From the considerations of the formation control problem formulation and assumed simplifications, it follows that the underlying optimal control problem for AUV i, (LQP^i), involves data from all its neighboring vehicles as specified by the formation graph, consisting in minimizing the quadratic cost functional

$$\sum_{k=1}^{N_p} \|y_{t+k}^{ref,i} - y_{t+k}^i\|_{Q^i}^2 + \sum_{k=0}^{N_p-1} \|u_{t+k}^i\|_{R^i}^2$$
$$+ \sum_{k=1}^{N_p} \sum_{j \in \mathcal{G}(i)} \|D^{ij}(y_{t+k}^i - y_{t+k}^j) - d^{ij}\|_{L^{ij}}^2 \quad (5)$$

subject to: $x_{t+k+1}^j = \Phi^j(T)x_{t+k}^j + \Psi^j(T)u_{t+k}^j$ (6)

$$y_{t+k}^j = C^j x_{t+k}^j \quad (7)$$

$$x_{t+k}^j \in [x_{LB,t}^j, x_{UB,t}^j] \quad (8)$$

$$u_{t+k}^j \in [u_{LB}^j, u_{UB}^j] \quad (9)$$

$$x_t^j = x_0^j, \quad (10)$$

where constraints hold for $j \in \{i\} \cup \mathcal{G}(i)$, being, for each time k, $\mathcal{G}(i)$ the set of nodes of the graph specifying the vehicles linked to AUV i. Here, y_{t+k}^i and $y_{t+k}^{ref,i}$ are, respectively, the vector of outputs of vehicle i and its reference, x_0^j is the initial state of vehicle j at the initial time t, D^{ij} is a matrix reflecting the formation relation between vehicles i and j, d^{ij} is a parameter vector specifying distances between vehicles i and j, $x_{LB,t}^j$, $x_{UB,t}^j$, u_{LB}^j, and u_{UB}^j are bounds for state and control at time t, respectively.

Now, we describe the implemented version of the MPC scheme for the control of a formation of AUVs. This scheme runs in each vehicle and will be the same for all AUVs. Thus, if there is no loss of information in the communication, then, all the vehicles have the same data and the control strategy generated for each vehicle is known to all of them. In the event of packet dropouts or communication delays, the missing sampled data is replaced by simulated data, and there will be some differences between the control strategies computed by the various vehicles for a given vehicle.
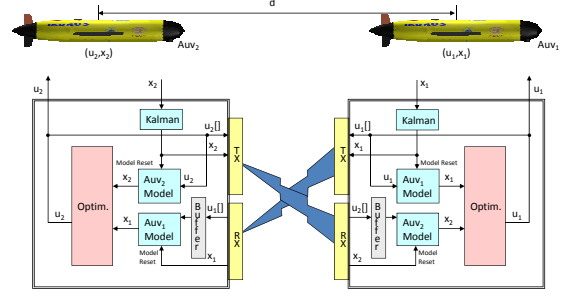

Fig. 1. Formation control scheme for 2 AUVs

The implemented MPC scheme in AUV i is as follows:

1. Initialization: prediction and control horizons, other optimal control problem parameters that depend on specific mission requirements, such as, level of perturbations, existence of obstacles, relative importance of trajectory tracking and formation pattern errors.
2. Sample the state variable, compute its estimate, and communicate it to its neighbors via acoustic modem.
3. Obtain the state variable of its neighbors via acoustic modem.
   (a) If data is available go to step 4.
   (b) Otherwise, generate estimates of the neighbors' state by running their models.
4. Solve the linear quadratic optimization problem (LQP^i) at the current time t, and for the current prediction horizon (of length N_p) and the given reference output trajectory. This yields an optimal control sequence for vehicle i.
5. Apply the control u^{i*} for the current control horizon.
6. Slide time for the optimization problem and adjust parameters if needed.
7. Let time elapse until the end of the current control horizon, and go to step 2.

Simulation results were obtained with the developed simulation environment in which the MATLAB linear quadratic programming solver is used in the context of simple formation of two vehicles that have to travel side by side (see the diagram of figure 1).

This framework exhibits the following features:

- Quadratic cost function weighting reference trajectory tracking error, control effort, and formation pattern error.
- Control systems with linear dynamics and subject to Gaussian noise with "adjustable" mean and variance, added as an additional input in the vehicle dynamics. Once the vehicle state is sampled, a Kalman filter yields a state estimate which is fed in the optimization solver and communicated to neighboring vehicles.
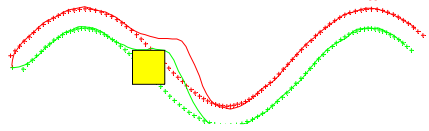- Control constraints enabling saturations.

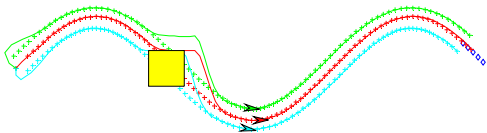Fig. 2. Formation of 2 AUVs with obstacle avoidance

Fig. 3. Formation of 3 AUVs with obstacle avoidance

- Inequality state and output constraints. These enable the incorporation of obstacles and the performance assessment the proposed MPC scheme with obstacle avoidance.
- Communication model. Communicated data is time stamped and may exhibit a time delay proportional to the distance between the vehicles exchanging data. Gaussian random packet dropouts can also be considered. The MPC scheme was assessed in the presence of time delays. Each vehicle has a linear buffer enabling the reception of multiple data samples from other vehicles and whose implementation is described in the previous section.
- Two performance metrics enabling to measure how far the AUVs are from the trajectory to be tracked and how far they are from the defined formation pattern, are used. These measures provide a good assessment of the controller's performance.

Figures 2 and 3 gives the reader an idea of the performance of the MPC controller in the considered cases.

5. CONCLUSIONS

Multiple simulations runs revealed that the proposed framework produced the intended control strategies according to the requirements. Many research challenges remain in order to achieve the computational tractability for problems with more complex formations and larger number of vehicles. This will require new ways of taking into account the decentralization character and are the subject of current research.

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