

Simple hierarchical and general nonlinear growth modeling in sheep

Mostafa GHADERI-ZEFREHEI^{1*}, Farjad RAFEIE², Mohammad Reza BAHREINI BEHZADI¹,
Sajad NAZARI³, Mustafa MUHAGHEGH-DOLATABADY¹, Farhad SAMADIAN¹,
Thomas M R MAXWELL⁴, Hamed AMIRPOUR NAJAFABADI⁴

¹Department of Animal Sciences, Faculty of Agriculture, University of Yasouj, Yasouj, Iran

²Department of Agricultural Biotechnology, Faculty of Agricultural Sciences, University of Guilan, Rasht, Iran

³Cold-water Fishes Genetic and Breeding Research Center, Iranian Fisheries Science Research Institute (IFSRI), Yasouj, Iran

⁴Department of Agricultural Sciences, Faculty of Agriculture and Life Sciences, Lincoln University, Christchurch, New Zealand

Received: 22.11.2017 • Accepted/Published Online: 22.04.2018 • Final Version: 09.08.2018

Abstract: Differential equations and advanced statistical models have been used to predict growth phenomena. In the present study, general nonlinear growth functions such as von Bertalanffy, Gompertz, logistic, and Brody, along with hierarchical modeling were applied to investigate the phenotypic growth pattern of Iranian Lori-Bakhtiari sheep. Growth data from 1410 Lori-Bakhtiari lambs were used in the present study. The results showed that the Brody function outperformed the other three nonlinear growth functions. In addition, including hierarchical growth modeling results allowed the adoption of many random effect structures, suggesting that hierarchical growth modeling has a useful role in growth data modeling. This method provides an estimation of growth parameters based on individual animals, improving individual growth selection. The results suggest this approach for growth modeling. Combining the strength of individual growth modeling with general growth modeling, e.g., von Bertalanffy, Gompertz, logistic, and Brody would be deeply appealing in the future. In this regard, dealing with sheep growth phenomenon using pure mathematical models, i.e. grey system theory models that could be new powerful prediction tools for breeders and experts, has not been done yet. However, running the analysis on large datasets will require significantly higher computational power than is ordinarily available.

Key words: Growth functions, hierarchical modeling, Lori-Bakhtiari sheep, nonlinear models, prediction of growth phenomena

1. Introduction

Growth is described as an increase in the weight and size of a living organism over a certain period of time. Growth curves have different applications such as constructing livestock feeding programs, determining the optimum slaughtering age, and monitoring the effects of selection (1). Sheep growth data have been fitted into random regression models for growth (2). However, the shape of sheep growth curves is influenced by many factors: breed type, management, environment, selection, and nutrition conditions (3). Well-known general growth functions such as Brody, Gompertz, logistic, and von Bertalanffy have been used to monitor growth curves in sheep (4–10). These models are popular in growth trend modeling because of their generality and simplicity of use.

Fixed and mixed models have been used in analyzing growth data. Mixed models provide robust theoretical statistical frameworks to model dependent variables as a function of random and fixed effects. The Bayesian approach utilizes hierarchical models instead of mixed models to estimate model parameters for mixed effects,

containing both fixed and random effects (11). In the present study, a simple hierarchical model was used to estimate individual animal growth parameters. In this case, the hierarchical model shared many features with the growth data random regression model, where fixed effects of regression were fitted to show the overall growth curve. Random effects were used to show deviation of each individual from the overall growth curve (5,12,13). The accuracy of predicting the growth curve of Iranian Lori-Bakhtiari sheep was studied using nonlinear growth functions (von Bertalanffy, Gompertz, logistic, and Brody) to accompany hierarchical modeling. In addition, simple and flexible hierarchical modeling was introduced with general computation requirements that was compatible with different variance-covariance structures when random effects were of interest in repeated data measuring used in animal breeding. Some very advanced pure mathematical models, i.e. the grey system theory models, that could be new powerful growth prediction tools for breeders and experts were pointed out. The mathematical foundation of grey theory was introduced by Deng (14).

* Correspondence: mosmos741@yahoo.com

The grey model has a rigor theoretical basis working quite well in practical scenarios. The results predicted by the grey model are relatively stable and reliable, which gives us fairly accurate results in low time point data. Grey theory could generally be used to predict the development and change of the system behavior value containing both the known and the unknown/uncertain information. In the concept of growth modeling, this idea has been used (15).

2. Materials and methods

2.1. Data collection and management

The growth data of 1410 sheep of the Iranian Lori-Bakhtiari breed (521 males, 889 females) gathered from 1990 to 2011 were used in the present study. The data included birth weight (BW), weaning weight (WW), 6-month-old weight (W6), 9-month-old weight (W9), and 12-month-old weight (W12). These data were from a research flock at the Lori-Bakhtiari sheep breeding station located in Charmahal and Bakhtiari Province, Iran. This research flock was formed in 1989 with about 206 ewes, which had successfully reared at least one lamb, and 28 rams of typical Lori-Bakhtiari breed. The sheep were managed under semimigratory or village production systems. From December to May at the station, the sheep were fed with alfalfa, barley, and wheat stubbles; however, for the rest of the year, the sheep were grazed on range and cereal pasture including the breeding period. By means of natural mating, ewes were assigned to rams for the first time at an average age of 18 months. In this flock, lambing generally started in late January and lambs remained with their dams until weaning. During the preweaning time, lambs had access to both mother's milk and creep feed ad libitum (from 15 days of age). All lambs were weaned at about 90 ± 5 days of age. After weaning, the lambs were separated into female and male flocks and the female lambs were assigned to cultivated alfalfa (in pasture form). Male lambs were treated differently and they received maintenance and growth ration until 6 months of age. The rate of replacement of ewes and rams, after culling the animals with abnormalities, was around

30% and less than 10%, respectively. Voluntary culling for the ewes was at 7 years old but the rams were kept until their offspring were available for replacement. Along with different sort of traits and pedigree data, other information such as age of ewes at mating were recorded routinely. The station was located in Shahrekord at 31°9'N, 32°48'E. The average annual rainfall in the region was about 400 to 600 mm with a mean temperature of 16 °C, and an average elevation of 1734 m above sea level.

2.2. Nonlinear growth functions

Nonlinear growth curve functions of von Bertalanffy, Gompertz, logistic, and Brody (9,10,16,17) were fitted to the data to estimate the parameters of the models and see if the growth curves fit well (Table 1).

Individual estimation of the growth curve parameters and mean square error (MSE) were obtained using the modified Gauss–Newton iterative procedure available in SAS (version 9.2) and the NLMIXED procedure (18). In general, the NLMIXED procedure can be used to fit different forms of linear models, e.g., simple linear regression, multiple linear regression, and analysis of (co) variance when observations are not normally distributed or contain outliers. Here the error part of the model was assumed to be normally distributed and independent of the random effects of the model. Amounts of Akaike's information criteria (AIC) and the logarithm of the likelihood function (-2 LogL) values for each of model were obtained at this step.

2.3. Simple hierarchical modeling

In hierarchical growth modeling, as in repeated growth modeling, multiple observations associated with the same animal were considered. The hierarchical models were used as follows:

$$\text{Model 1: } y_{it} = a_{0i} + b_i(\text{Time})_{it} + \varepsilon_{it} \quad \text{var}(\varepsilon_{it}) \sim a_{0i}^2 1(i=j)$$

$$\text{Model 2: } y_{it} = a_0 + b_{it}(\text{Time})_{it} + \varepsilon_{it} \quad \text{var}(\varepsilon_{it}) \sim a_{0i}^2 1(i=j)$$

$$\text{Model 3: } y_{it} = a_{0i} + b_i(\text{Time})_{it} + \varepsilon_{it} \quad \text{var}(\varepsilon_{it}) \sim a_{0i}^2 1(i=j)$$

In Model 1, it was assumed that the starting point for growth (birth weight) was different across all the sheep in

Table 1. Nonlinear growth functions used in the current study.

Nonlinear functions	Model*	References
von Bertalanffy	$W_t = A(1 - Be^{-kt})^3 + \varepsilon$	16, 10
Gompertz	$W_t = Ae^{(-Be^{-kt})} + \varepsilon$	17, 10
Logistic	$W_t = A(1 - Be^{-kt})^{-1} + \varepsilon$	16, 10
Brody	$W_t = A(1 - Be^{-kt}) + \varepsilon$	9, 10

* W_t = the animal weight at a determined age (t), A = the predicted asymptotic weight at maturity (kg), B = the integration constant to which initial weight is related or animal maturation rate at birth (kg), K = the rate of maturity.

the population but time would imply an identical effect on the growth pattern of all sheep (only in the random intercept model). Model 2 assumed the same starting point for growth for all the sheep in the population, but time would imply different effects on the growth pattern of each animal in the population (only in the random slope model). In Model 3, it was assumed that both the starting point for growth and effects of time on growth were different (in the random intercept and slope models). Models 1 and 2 can be seen as special cases of Model 3. In the above models, y_{it} represents the growth of the i th sheep at time point t , a_i represents specific intercept for i th sheep, b_i represents the specific regression coefficient (slope) for the i th sheep, and ε_{it} represents the error model and it was assumed to be independently identically distributed with parameters $(0, \sigma_e^2)$. Note that the parameters of the model that lack the subscript are assumed to be the same parameters for all sheep (see Models 1 and 2). Model 3 can be expanded as follows:

$$\begin{aligned} a_i &= \alpha_i + a_i^* & a_i^* &\sim N(0, \sigma_a^2) \\ b_i &= \beta + b_i^* & b_i^* &\sim N(0, \sigma_b^2) \end{aligned}$$

Finally, the general model can be written as

$$y_{it} = \alpha + \beta ((Time)_{it} - (\overline{Time})_i) + a_i^* + b_i^* ((Time)_{it} - (\overline{Time})_i) + \varepsilon_{it} \quad \text{var}(\varepsilon_{it}) \sim a_k^2 I(i=j)$$

The above model is a rewritten hierarchical model where $(Time)_{it}$ demonstrates the average time for the i th sheep, α is the fixed effect for average weight of the sheep at start of recording, β is the fixed effects regression coefficient of time, $a_i^* = a_i - \alpha$ is the random sheep specific effect, and $b_i^* = b_i - \beta$ is the sheep specific random effect. It was assumed that the above random effects were jointly normally distributed with zero mean and variance-covariance ψ as follows:

$$\begin{pmatrix} a_i^* \\ b_i^* \end{pmatrix} \sim iidN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \psi = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix} \right]$$

As a result, variance-covariance between observations can be shown as follows:

$$\begin{aligned} \text{var}(y_{it}) &= [1 \quad (Time)_{it}] \psi \begin{bmatrix} 1 \\ (Time)_{it} \end{bmatrix} + \sigma_e^2 = \\ [1 \quad (Time)_{it}] &\begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix} \begin{bmatrix} 1 \\ (Time)_{it} \end{bmatrix} + \sigma_e^2 \\ &= [\sigma_a^2 + \sigma_{ab}(Time)_{it} \quad \sigma_{ab} + \sigma_b^2(Time)_{it}] \begin{bmatrix} 1 \\ (Time)_{it} \end{bmatrix} \\ &+ \sigma_e^2 = \sigma_a^2 + 2\sigma_{ab}(Time)_{it} + \sigma_b^2(Time)_{it}^2 + \sigma_e^2 \end{aligned}$$

In addition, it was assumed that covariance was $\text{cov}(y_{it}, y_{ik}) = \text{cov}(a_i + b_i(Time)_{it} + e_{it}, a_i + b_i(Time)_{ik} + e_{ik}) = \sigma_a^2 + \sigma_{ab}(Time)_{ik} + \sigma_{ab}(Time)_{it} + \sigma_b^2(Time)_{it}(Time)_{ik}$

The above hierarchical model parameters were estimated using REML and implemented in SAS MIXED

procedure (18). In addition, the procedure is featured with many variance-covariance structures (almost 30) to fit to the data. Figure 1 shows this modeling graphically.

2.4. Linear model

To analyze the effect of environmental factors on the observed live weights of sheep and the growth curve parameters A, B, and K, the following linear model was used:

$$y_{ijkl} = \mu + S_i + A_j + T_k + B_l + \varepsilon_{it}$$

where y_{ijkl} = weight, μ = the overall mean, S_i = the effect of i th sex, A_j = the effect of age of i th dam, T_k = the effect of i th type of birth, B_l = the effect of i th year of birth, and ε_{it} = random error of the model. The GLM procedure (SAS 9.2) was used to estimate the parameters of the model.

3. Results and discussion

3.1. Nonlinear growth functions

The estimation of parameters A (predicted live weight at maturity), B (difference between mature live weight and birth weight), and K (growth rate to maturity) and associated R^2 , MSE, AIC, and the $-2\log$ likelihood (-2LogL) for the 4 models are shown in Table 2. Predicted live weight at maturity (A) was largest in the Brody model (59.12 kg) and lowest in the logistic model (54.4 kg). Difference between mature and birth live weight (B) was largest in the logistic model (6.05 kg) and smallest in the von Bertalanffy model (0.85 kg). Predicted growth rate to maturity (K) was largest in the logistic model (0.59 kg) and smallest in the Brody model (0.25 kg), which may indicate an earlier maturity rate in the logistic model compared to those in the other models.

Abegaz et al. (9) reported 37.6, 0.88, and 0.27 kg for predicted live weight at maturity, difference between live weight at maturity and birth, and growth rate to maturity, respectively when applying the Brody growth function to Horro sheep. Similarly, Bathaei and Leroy (4), Topal et al. (6), Gbangboche et al. (7), and Malhado et al. (19) reported different A, B, and K values when they fitted different growth functions to the same growth data, indicating that the applied model sharply affects the estimated model parameters. In the current study, the Brody model based on the lowest MSE and AIC and the highest R^2 was the best fitted model to describe growth in Iranian Lori-Bakhtiari sheep. The results showed high agreement with those obtained by Bahreini et al. (10) in Iranian Balouchi sheep. The results also support Bathaei and Leroy (4), who analyzed and evaluated different growth functions in Iranian Mehraban fat-tailed sheep. They used the Brody function because of its simplicity of interpretation and ease of estimation. Lewis et al. (5) showed that the Gompertz function had desirable properties to describe growth in Suffolk sheep. Topal et al. (6) reported that the Gompertz and the von Bertalanffy models showed the best estimation

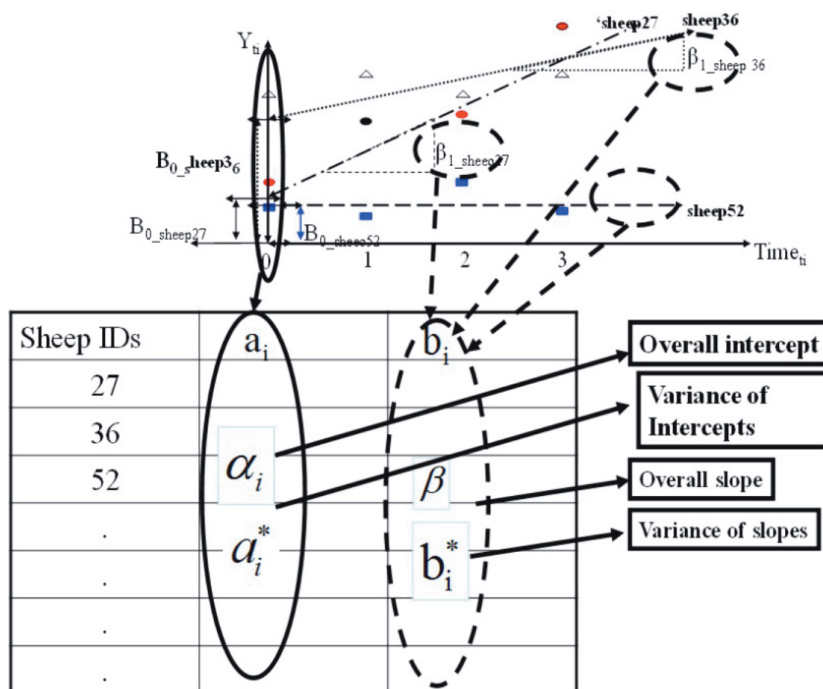


Figure 1. Schematic representation of hierarchical modeling. A simple linear regression, an only random intercept model, and an only random slope model (hierarchical models) of sheep growth over time are shown in this Figure. Here, for the simple linear regression, it is assumed that a hypothetical sheep 36 at three points had repeated records. The vertical axis shows growth and the horizontal axis is the time at which the records were collected. This model can be used for predicting the growth of a whole population, provided the average records for three time points for all the animals are available. However, as stated above, this model cannot show individual sheep growth trend since it is a linear model. The only random slope model shows the individual growth with relevant parameters. This model shows the growth of an individual sheep and can be started with identical intercept and each sheep could have a different growth rate (regression coefficient) over time. For example, the growth of sheep 27 increased sharply with progressing time but this growth was not the case for sheep 52. In addition, it seems that the increase in growth of the sheep does not change, i.e. it is time invariant (only random intercept model). This model can identify an individual animal at the starting point and is useful in selecting the best animals for breeding. It also reveals which animals show better growth over time.

Table 2. Parameter estimations \pm standard error, coefficient of determination (R^2), mean square error (MSE), Akaike’s Information Criterion (AIC), and $-2 \log$ Likelihood (-2 LogL) for 4 models describing the growth curves in Lori-Bakhtiari Sheep breed

Models	Estimated least squares of parameters \pm SE*						
	A	B	K	R^2	Root MSE	AIC	-2 LogL
Brody	59.12 \pm 0.35	5.83 \pm 0.05	0.25 \pm 0.004	0.94	6.1	53,028	53,074
Gompertz	55.23 \pm 0.23	2.93 \pm 0.04	0.42 \pm 0.006	0.93	6.7	53,318	53,310
Logistic	54.4 \pm 0.21	6.05 \pm 0.23	0.59 \pm 0.008	0.92	6.4	54,141	53,152
von Bertalanffy	55.71 \pm 0.25	0.85 \pm 0.009	0.364 \pm 0.006	0.90	6.9	53,028	54,133

*A = the predicted asymptotic weight at maturity (kg), B = the integration constant to which initial weight is related or animal maturation rate at birth (kg), K = the rate of maturity

of growth in Morkaraman and Awassi sheep breeds, respectively. Certain properties such as goodness of fit to data, lower computational cost, biological interpretability of parameters, and managing missing data support the Brody model as the best model to monitor sheep growth

patterns. However, in the current study, there were no missing data values.

The effects of the linear model factors on sheep live weights measured at different ages are shown in Table 3 and they were mostly significant ($P < 0.01$). The male lambs

Table 3. Effects of linear model factors on body weights at different ages of Lori-Bakhtiari sheep.

Body weights at different ages*						
Factors	Records	BW	WW	W6	W9	W12
Sex:	1410	**	**	**	**	**
Male	521	5.3 ± 0.23	32.27 ± 0.14	47.55 ± 0.17	60.26 ± 0.06	66.44 ± 0.22
Female	889	4.7 ± 0.32	27.38 ± 0.22	36.89 ± 0.32	47.44 ± 0.33	51.22 ± 0.32
Birth type:	1410	**	**	**	**	**
Single	1065	5.33 ± 0.23	30.14 ± 0.42	41.36 ± 0.23	52.17 ± 0.32	57.23 ± 0.33
Twin	345	4.44 ± 0.27	26.57 ± 0.27	39.18 ± 0.46	51.33 ± 0/23	55.16 ± 0.6
Age of dam	1410	**	**	**	ns	ns
Birth year	1410	**	**	**	**	*

*BW = birth weight, WW = weaning weight, W6 = 6-month-old weight, W9 = 9-month-old weight, W12 = 12-month-old weight, ns = not significant ($P > 0.05$), ** = significant ($P < 0.01$).

and the single-born lambs had higher live weights, which may have been due to the hormonal and physiological differences between sexes, maternal conditions and milk production capacity, and the size of the uterus to grow single lambs relative to twin lambs (20). The effect of dam age on 9-month-old weight was significant. Figures 2A and 2B show that the male and the single lambs had higher growth rates than those of the female and the twin lambs. The results were in agreement with those reported in previous papers (5,8). Figure 2C indicates that dams that were 3, 4, and 5 years old delivered lambs with higher growth rates than those delivered by the younger and the older dams. Usually with increasing maternal age, lamb weight also increases. Physical maturity of the dam and reduced need to grow further increases fetal and birth weight as well as allowing better milk production (10). There were significant effects of model factors on all the Brody model parameters (Table 3). Previous studies reported significant effects of type of birth, sex, and birth year on parameter A of the Brody growth model (4,9). In the present study, a significant effect was observed for birth type on parameter B estimated by the Brody growth model. Several studies have reported significant effects of sex on it (4,9,21). The observed significant effect of birth year on the growth model parameter is in agreement with the results reported by Batheai and Leroy (4) and Abegaz et al. (9). However, the effects of birth type, sex, and herd on any Brody and Gompertz growth model parameters were not reported as significant (22). Figures 2D–2H show how data were fitted to the respective models. As the Brody model (Figure 2H) had the best fit, much more attention was given to different aspects of this model, which can be seen in different subsets of Figure 2. Table 4 shows the significant effects of birth type and sex on Brody and

Gompertz growth model parameters. The effect of birth year on maturity weight could be due to management and model factors in the birth year of the sheep because it could affect maternal ability to provide the right motherhood environment for her offspring.

3.2. Simple hierarchical modeling

Table 5 displays partial results of the hierarchical modeling, which is a part of the full hierarchical model in the current study (i.e. model 3 on sheep growth data). In terms of model fitting criteria, for full hierarchical modeling, -2 LogL (53114) was competitive with the Brody model. However, statistical comparisons between the results of these models failed. Comparison of the models should be based on some valid and motivated assumptions and assumptions of these models, in terms of type and number of parameters, were different. The data had no missing values and it took a long time for the model to get converged. This may have been due to the structures of random effects. Two predictions of animal-specific intercept and slope were given for each animal. These specific predicted parameters of animals may help breeders to select the most productive animal. For example, both predicted parameters for sheep 100154 in Table 5 are negative. It might indicate that the growth trend for this sheep is lower than the population mean; therefore, this animal is less desirable for breeding purposes. However, for a predicted specific slope of animal, the null hypothesis cannot be rejected so easily ($P > 0.05$), however, for sheep 100225 this argument is justified. In this type of modeling, parameters for each sheep were randomly selected from a bigger population. Thus, implementing hierarchical modeling utilizes random parameter selection (here the Gaussian distribution was used as it was desirable that weights followed up this distribution asymptotically).

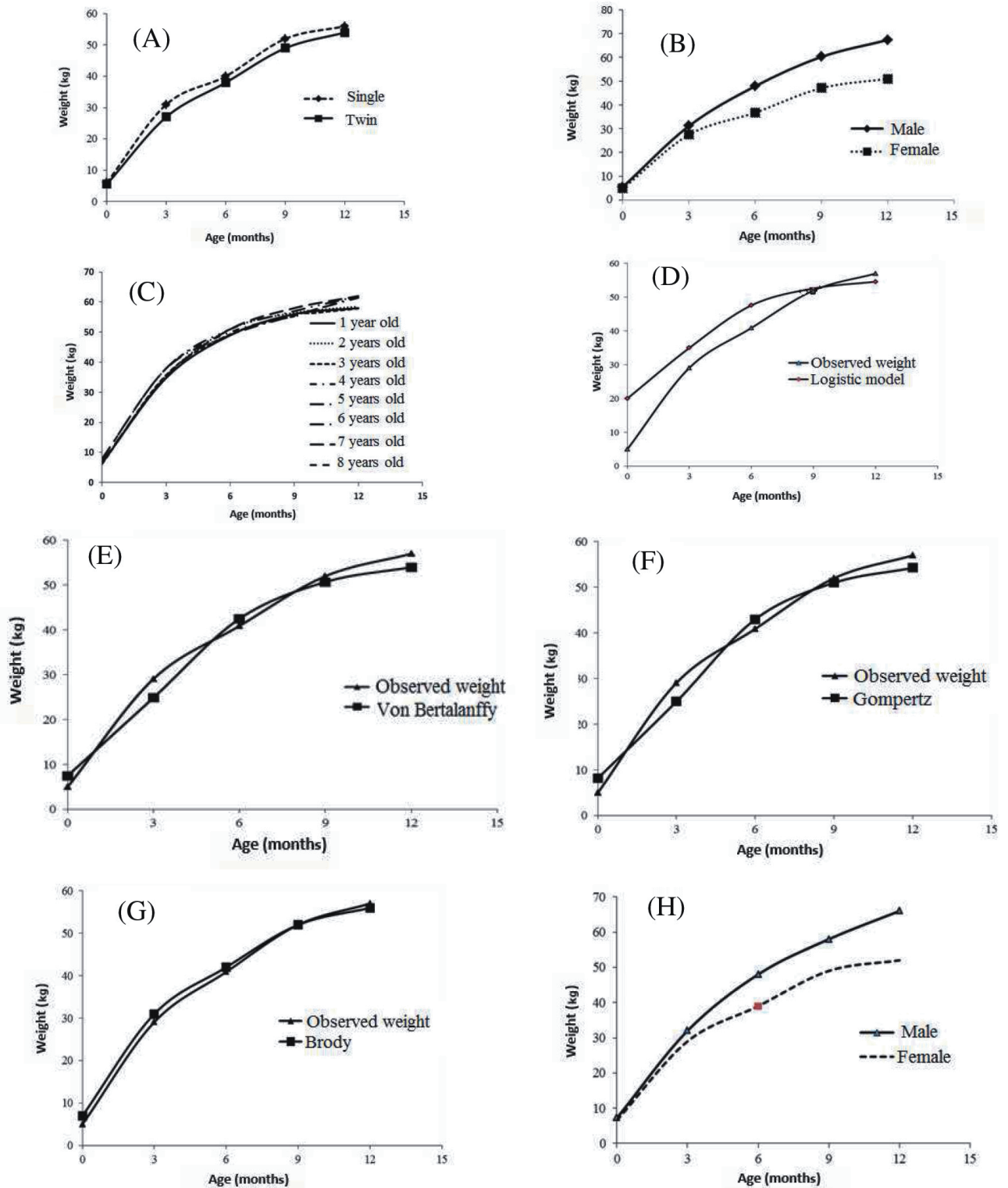


Figure 2. The results of different models for different factors: A) The Brody model for twin and single birth types, B) Trend of growth for the male and female sheep, C) The Brody model at different dam ages, D) Logistic model for average weights at different ages, E) The von Bertalanffy model for average weights at different ages, F) The Gompertz model for average weights at different ages, G) The Brody model for average weights at different ages, H) The Brody model for sex of sheep.

Table 4. Effects of model factors on parameter estimations \pm standard error of the Brody and Gompertz models in Lori-Bakhtiari sheep.

Factors*	Model parameters**					
	Brody			Gompertz		
	A	B	K	A	B	K
Sex:	**	*	**	**	ns	ns
Male (521)	72.12 \pm 0.73	5.44 \pm 0.06	0.275 \pm 0.004	54.15 \pm 0.46	3.19 \pm 0.04	0.443 \pm 0.008
Female (889)	57.54 \pm 0.62	5.85 \pm 0.07	0.228 \pm 0.005	66.43 \pm 0.52	3.59 \pm 0.04	0.445 \pm 0.009
Birth type:	*	**	**	ns	ns	*
Single (1065)	62.24 \pm 0.25	6.14 \pm 0.05	0.249 \pm 0.004	61.32 \pm 0.57	3.40 \pm 0.03	0.482 \pm 0.007
Twin (345)	66.42 \pm 0.77	5.52 \pm 0.05	0.223 \pm 0.005	58.26 \pm 0.42	3.19 \pm 0.05	0.406 \pm 0.001
Age of dam (years)	**	*	*	**	ns	*
Birth year	**	**	**	**	**	**

*The numbers within parentheses show the number of observations, **ns = not significant ($P > 0.05$), * = significant ($P < 0.05$), ** = significant ($P < 0.01$), A = the predicted asymptotic weight at maturity (kg), B = the integration constant to which initial weight is related or animal maturation rate at birth (kg), K = the rate of maturity.

Table 5. A small part of full hierarchical modeling results

Effect	Sheep ID	Estimate	Pred	DF	Pr> t
Intercept	100154	-0.1318	0.0	9300	<0.0001
time	100154	-0.1344	0.3205	9300	0.6751
Intercept	100225	-1.6356	0.0	9300	<0.0001
time	100225	-1.1037	0.3205	9300	0.0006
Intercept	100229	-1.3137	0.0	9300	<0.0001
time	100229	-0.9041	0.3205	9300	0.0048
Intercept	100237	-1.2722	0.0	9300	<0.0001
time	100237	-0.9618	0.3205	9300	0.0027
Intercept	100241	-1.3452	0.0	9300	<0.0001
time	100241	-0.9392	0.3205	9300	0.0034
Intercept	100243	-0.6392	0.0	9300	<0.0001
time	100243	-0.5284	0.3205	9300	0.0993

Pertaining to the hierarchal modeling, it should be noted that such a model has its advantages and disadvantages. In this type of modeling, it is possible to obtain the parameters for each animal independently. This model has great ability to fit and explore a wide range of different statistical distributions for random intercept and slope effects.

Variance-covariance structure used in this type of modeling could impose a wide range of communications between random coefficients in the model. As the number of the records increases, it takes too much time to get convergence. Depending on the model structure and

the conditions under which the records are collected, fitting the model could be quite complex. Having a good pedigree, a genetic analysis of two traits (here the first trait is intercept and the second trait is slope) can be carried out and therefore, the amount of heritability and genetic correlation can be grasped. If the covariance is considered to be zero, it can be postulated that early-measured weight will not have any impact on other weight behaviors over time. As mentioned above, most of the models used in the growth pattern modeling have a nonlinear nature (16,23). However, further studies using hierarchical modeling can

find a link between A, B, and K parameters in nonlinear functions, which may help in selecting the right animals for growth purposes.

Hierarchical models are powerful tools that can capture the undefined part of covariance among growth data. However, future growth research on animals or all living organisms should thoroughly be canalized to avoid mistaken inferences. In this regard, apart from the nature of character, adopting new intelligent machine learning algorithms and pure mathematical methods are of great importance. For example, a new algorithm has recently been introduced based on the combination of least square support vector machine and genetic algorithm (24). The algorithm outperformed all general functions of growth phenomenon modeling. However, there has not been any sign of grey system theory models in the context of animal growth modeling. Grey system theory as an interdisciplinary scientific area was introduced by Deng (14,25). This model is inherently suited for modeling positive variables explicitly. Growth phenomena lie in this area as well. Nowadays, growth is recorded over a limited number of times (as in the present study). Superiority of the grey models to the conventional statistical models is that with limited

amount of information, a valid estimation of the behavior of an unknown growth system can be well achieved (26). As a suggestion for groundbreaking further research, the grey systems paradigm can be integrated with BLUP based concept. This integration would pave the way for new mainstream studies.

One of the main advantages regarding the proposed method in comparison with the general growth modeling schemes is that it allows us to derive growth function solely based on data but not on presumptions about growth function (no fixed number of parameters). The proposed method can be integrated with pedigree data and, therefore, a BLUP estimation of growth over any single time can be grasped soundly. Moreover, a large amount information of different (co)variance functions across random affects can be well fitted, which cannot be done with general growth modeling paradigms (like the four models used in the present study).

Acknowledgments

Authors extend their gratitude to Dr Vatankhah (University of Sharekord), and Dr Asadi (University of Sharekord) for providing the data and assisting in the analysis of data.

References

1. Blasc A, Gomes E. A note on growth curves of rabbit lines selected on growth rate or litter size. *Anim Prod* 1993; 57: 332-334.
2. Molina A, Menéndez-Buxadera A, Valera M, Serradilla M. Random regression model of growth during the first three months of age in Spanish merino sheep. *J Anim Sci* 2007; 85: 2830-2839.
3. Kume K, Hagno L. Study of growth curve variations for kids 0-6 months old of Alpine goat breed in Albania. *Arch Zootec* 2010; 13: 54-62.
4. Bathaei SS, Leroy PL. Genetic and phenotypic aspects of the growth curve characteristics in Mehraban Iranian fat-tailed sheep. *Small Ruminant Res* 1998; 29: 261-269.
5. Lewis RM, Brotherstone S. A genetic evaluation of growth in sheep using random regression techniques. *Anim Sci* 2002; 74: 63-70.
6. Topal M, Ozdemir M, Aksakal V, Yildiz N, Dogru U. Determination of the best non-linear function in order to estimate growth in Morkaraman and Awassi lambs. *Small Ruminant Res* 2004; 55: 229-232.
7. Gbangboche AB, Adamou-Ndiaye M, Youssao AKI, Farnir F, Dettleux J, Abiola FA, Leroy PL. Non-genetic factors affecting the reproduction performance, lamb growth and productivity indices of Djallonke sheep. *Small Ruminant Res* 2006; 64: 133-142.
8. Gbangboche AB, Youssao AKI, Senou M, Adamou-Ndiaye M, Ahissou F, Farnir F, Michaux FA, Abiola FA, Leroy PL. Examination of non-genetic factors affecting the growth performance of Djallonke sheep in Soudanian zone at the Okpara breeding farm of Benin. *Trop Anim Health Pro* 2006; 38: 55-64.
9. Abegaz S, Vanwyk JB, Olivira JJ. Estimation of genetic and phenotypic parameters of growth curve and their relationship with early growth and productivity in Herro sheep. *Arch Tierzucht* 2010; 53: 85-94.
10. Bahreini Behzadi MR, Aslaminejad AA, Sharifi AR, Simianer H. Comparison of mathematical models for describing the growth of Baluchi sheep. *J Agric Sci Technol* 2006; 14: 57-68.
11. Donnet S, Foulley JL, Samson A. Bayesian analysis of growth curves using mixed models defined by stochastic differential equations. *Biometrics* 2010; 66: 733-741.
12. Safaei M, Fallah-Khair AR, Seyed-Sharifi R, Haghbin H, Nazarpak H, Sobhabi A, Yalchi T. Estimation of covariance functions for growth trait from birth to 180 days of age in Iranian Baluchi sheep. *J Food Agric Environ* 2010; 8: 659-663.
13. Mota R, Luiz F, Aarão M, Paulo S, Lopes L, Pinheiro AM, Hidalgo C, Suguimoto L. Random regression models in the evaluation of the growth curve of Simbrasil beef cattle. *Genet Mol Res* 2013; 12: 528-536.

14. Deng JL. Properties of relational space for Grey Systems. In: Grey System. Beijing, China: China Ocean Press, 1988. pp. 1-13
15. Jing W, Yuesong H, Weilin L, Wenhui C. Application of grey system theory to tree growth prediction. *J Forestry Res* 2000; 11: 34-36.
16. Gbangboche AB, Glele-Kakai R, Salifou S, Albuquerque LG. Comparison of non-linear growth models to describe the growth curve in West African Dwarf sheep. *Animal* 2008; 2: 1003-1012.
17. Lambe NR, Navajas EA, Simm G, Bunger L. A genetic investigation of various growth models to describe of lamb of two contrasting breeds. *J Anim Sci* 2006; 84: 2642-2654.
18. Statistical Analysis System (SAS). SAS/STAT 9.2 User's Guide: The MIXED Procedure. Cary, NC, USA: SAS Institute Inc; 2009.
19. Malhado CHM, Carneiro PLS, Mello P. Growth curves in Dorper sheep crossed with the local Brazilian breeds, Morada Nova, Rabo Largo, and Santa Inês. *Small Ruminant Res* 2009; 84: 16-21.
20. Wilson RT, Peacock C, Sayers AR. Livestock production on Masai Group Ranches. 2. Growth and liveweight in goats and sheep at Elangata Wuas and the factors influencing them. *J Agr Rural Dev Trop* 1982; 21: 191-198.
21. Sarmento JLR, Rezazzi AJ, Souza WH, Torres RA, Breda FC, Menezes GRO. Analysis of the growth curve of Santa Ines sheep. *Rev Bras Zootecn* 2006; 35: 435-442.
22. Waheed A, SajjadKhan M, Safdar A, Sarwar M. Estimation of growth curve parameters in Beetal goats. *Arch Tierzucht* 2011; 54: 287-296.
23. Forni S, Piles M, Blasco A, Varona L, Oliveira HN, Lôbo RB, Albuquerque LG. Comparison of different nonlinear functions to describe Nelore cattle growth. *J Anim Sci* 2009; 87: 496-506.
24. Salehi Broujeni M, Ghaderi-Zefrehei M, Ghane-Golmohammadi F, Ansari-Mahyari S. A novel LSSVM based algorithm to increase accuracy of bacterial growth modeling. *Iran J Biotechnol* 2018; 16: e1542.
25. Sifeng L, Jeffrey F, Yingjie Y. A brief introduction to grey systems theory. *Grey Systems: Theory and Application* 2012; 2: 89-104.
26. Kayacan E, Ulutas B, Kaynak O. Grey system theory-based models in time series prediction. *Expert Syst Appl* 2010; 37: 1784-1789.