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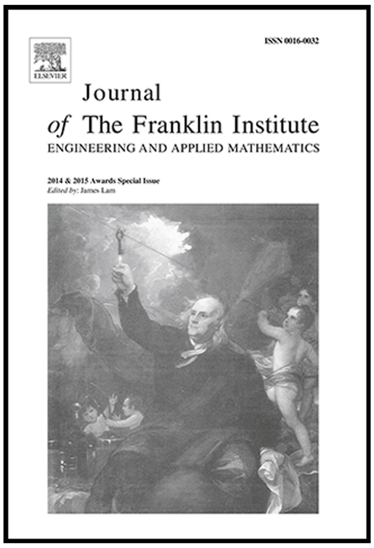

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Highlights

- This paper investigates the stabilization problem for rectangular descriptor systems through delayed dynamic compensator.
- It presents an effective delayed dynamic compensator design which could stabilize the rectangular descriptor systems while delay-free methods fail the purpose.
- A numerical example demonstrates the usefulness and merits of the proposed method.

Stabilization for a Class of Rectangular Descriptor Systems via Time Delayed Dynamic Compensator*

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Abstract - This paper focuses on the stabilization problem for a class of rectangular descriptor systems through dynamic compensation. Such class of systems may not be stabilized by delay-free dynamic compensators, while delayed dynamic compensator could achieve such purpose. We provide a design scheme of time-delayed dynamic compensator which makes the closed-loop system admissible. The design involves solving a quadratic matrix inequality, and consequently, we build a linear matrix inequality (LMI) based algorithm to compute compensator gains. We verify that, under certain circumstances for which delay-free dynamic compensators fail to stabilize, the proposed method works well. An illustrative example demonstrates the usefulness of the present scheme.

Keywords: Rectangular descriptor systems, dynamic compensator, time delay, stabilization, admissibility.

1 Introduction

Descriptor systems, also called singular systems, implicit systems and generalized state-space systems, were originated in 1960's and the study has been undergone thoroughly since 1970's [1].

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Applications of such type of systems cover electrical circuits, economic systems, population statistics, and robotics [2, 3]. Descriptor systems could better describe physical systems and thus the theory receives much research attention. Rectangular descriptor systems, in which the numbers of state variables and equations can be inconsistent, serve as a wider class of descriptor systems and they possess much more complicated behaviours [4]. The generalized regularity and regularizability issues have been studied in [4, 5], and necessary and sufficient conditions have been obtained. Impulse controllability and observability problems are investigated in [6, 7], while filtering, estimation and observer design are presented in [8–10, 31].

Recently, dynamic compensation is suggested in [11, 12] to make the closed-loop rectangular descriptor systems regular. Therefore, the study of rectangular descriptor systems is transformed into a problem of normal systems or descriptor systems which can make things easier. Subsequently, dynamic compensators are designed for achieving purposes such as linear quadratic optimal control, stabilization and H_∞ control [12–14]. Note that the stabilization cannot be fulfilled by using state feedback, static output feedback, etc. However, the design of dynamic compensators is usually difficult as it amounts to solving an NP-hard problem. The LMI-based methods are effective to solve the NP-hard problem [15]. In [12], the path-following method is used, but it has the disadvantage of non-convergence. Quite recently, we develop a PD-type dynamic compensator for a class of T-S fuzzy rectangular descriptor systems, and study the normalization and stabilization problem [16], extending the idea in [17]. The design method therein is related to solving a set of matrix inequalities and an efficient algorithm is built to compute design parameters. In [21], we propose an existence condition for the dynamic compensation, and design a dynamic compensation for rectangular T-S fuzzy discrete-time systems with time delay.

Time delay is a relatively common phenomenon in the real world [22–24]. In practice control, delay-free controllers is the most common in industrial control [28–30, 32–34]. Time-delayed controllers are normally used to compensate the performance of time-delay systems [24, 27]. It is worth noting that time-delayed controller may take active effect on the stabilization task whereas delay-free controllers fail. For instance, for certain oscillatory systems, delayed feedback could make the resultant system stable while delay-free controllers can not stabilize them [18]. So far, the research results for time-delay systems are considerably abundant [25–27]. Nevertheless, there are few theoretical results to discuss the effects of the delay in controllers on system performance. For rectangular descriptor systems, the corresponding problems have not been explored hitherto.

In this paper, we study the stabilization problem for linear rectangular descriptor systems via time-delayed dynamic compensator. We propose a design method together with an iterative

LMI-based algorithm to compute related parameters. The main idea comes from the concept of dynamic compensation [11, 12], the adopting of our design method in [16] [17] and the use of Lyapunov-Krasovskii functional method combined with Wirtinger-based integral inequality [19]. The main characteristics of the proposed method are summarized as follows.

1. We show explicitly that, by using time-delayed dynamic compensator, the resulting criterion becomes solvable for some classes of systems for which the criterion based on delay-free dynamic compensation method fails to produce solutions.

2. The present dynamic compensator design scheme could achieve the closed-loop system admissibility.

The notation used is fairly standard: $\mathbb{R}^{m\times n}$ is the real matrix space with dimension $m \times n$. $X < 0$ (respectively, $X > 0$) denotes a symmetric negative (respectively, positive) definite matrix. I is identity matrix with compatible dimension. The superscript 'T' represents the transpose, and the symbol $*$ denotes a block matrix inferred by symmetry. $Sym\{M\}$ is defined as $M + M^T$.

2 Problem Formulation and Preliminaries

Consider a rectangular descriptor system

$$\begin{aligned} E\dot{x} &= Ax + Bu, \\ y &= Cx, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^{n_u}$ and $y \in \mathbb{R}^{n_y}$ are the state vector, the control input and the output, respectively; $E \in \mathbb{R}^{m\times n}$ with rank $E = r \leq \min\{m, n\}$; A, B and C are real constant matrices with appropriate dimensions.

One design of the delay-free dynamic compensator form introduced in [11] [12] is

$$\begin{aligned} E_c\dot{x}_c &= A_c x_c + B_c y, \\ u &= C_c x_c + D_c y, \end{aligned} \tag{2}$$

where $x_c \in \mathbb{R}^{n_c}$ is the dynamic compensator state vector; $E_c \in \mathbb{R}^{m_c\times n_c}$ is required to meet $m+m_c = n + n_c$ in order to make the closed-loop system square; A_c, B_c, C_c, and D_c are appropriately dimensional dynamic compensator gains to be designed.

Combing (2) with (1), we obtain the closed-loop system as

$$\begin{bmatrix} E & 0 \\ 0 & E_c \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} A + BD_cC & BC_c \\ B_cC & A_c \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix}. \tag{3}$$

Let $X(t) = [x^T(t),\ x_c^T(t)]^T$, $\bar{E} = \begin{bmatrix} E & 0 \\ 0 & E_c \end{bmatrix}$, $\bar{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix}$, $\bar{C} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}$, $K = \begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix}$. Then, (3) is equivalent to

$$\bar{E}\dot{X}(t) = (\bar{A} + \bar{B}K\bar{C})X(t). \tag{4}$$

System (4) is admissible [3] if and only if there exist matrices $P > 0$, G_1, G_2, K, S, such that

$$\begin{bmatrix} -G_2 - G_2^T & P\bar{E} + VS^T + G_2^T(\bar{A} + \bar{B}K\bar{C}) - G_1 \\ * & (\bar{A} + \bar{B}K\bar{C})^T G_1 + G_1^T(\bar{A} + \bar{B}K\bar{C}) \end{bmatrix} < 0, \tag{5}$$

where V is maximum annihilator of a fixed square matrix which satisfies $\bar{E}^T V = 0$.

Condition (5) is nonlinear, and it can be solved by the path-following methods in [15] [16], which are in fact based on sufficient iterative LMI conditions. That is to say, the delay-free dynamic compensator (2) may be not applicable for producing feasible solutions using existing methods. So we should seek alternative measures to reduce the inherent conservatism of this method.

We propose the following time-delayed dynamic compensator

$$\begin{aligned} E_c\dot{x}_c &= A_c x_c + B_c y + A_{c\tau} x_c(t-\tau) + B_{c\tau} y(t-\tau), \\ u &= C_c x_c + D_c y + C_{c\tau} x_c(t-\tau) + D_{c\tau} y(t-\tau), \\ x &= \phi_1(t), \quad x_c = \phi_2(t), \qquad t \in [-\tau, 0], \end{aligned} \tag{6}$$

where $\tau > 0$ is the introduced time delay; ϕ_1 and ϕ_2 are initial states; $A_{c\tau}$, $B_{c\tau}$, $C_{c\tau}$ and $D_{c\tau}$ are appropriately dimensional delayed dynamic compensator gains to be designed. It is seen that if $A_{c\tau}$, $B_{c\tau}$, $C_{c\tau}$ and $D_{c\tau}$ vanish, equation (6) reduces to the delay-free dynamic compensator (2). In what follows, we will show that the time-delayed dynamic compensator plays an important role in achieving the stabilization purpose.

Under (6), the closed-loop system of (1) is as follows.

$$\begin{bmatrix} E & 0 \\ 0 & E_c \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} A + BD_cC & BC_c \\ B_cC & A_c \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} + \begin{bmatrix} BD_{c\tau}C & BC_{c\tau} \\ B_{c\tau}C & A_{c\tau} \end{bmatrix} \begin{bmatrix} x(t-\tau) \\ x_c(t-\tau) \end{bmatrix}. \tag{7}$$

Then, (7) can be rewritten in a compact form

$$\bar{E}\dot{X}(t) = (\bar{A} + \bar{B}K\bar{C})X(t) + \bar{B}K_\tau\bar{C}X(t-\tau), \tag{8}$$

where $K_\tau = \begin{bmatrix} D_{c\tau} & C_{c\tau} \\ B_{c\tau} & A_{c\tau} \end{bmatrix}$.

System (8) is in the square form of time-delay descriptor systems. It is defined that such a system is admissible if it is regular, impulse-free and stable. See, *e.g.*, [3] for detailed definitions.

The following lemmas are useful for the development.

Lemma 1. *[20] For matrices T, B, and X with appropriate dimensions, there exists a matrix $W > 0$ such that the following inequalities are equivalent:*

(i) $T < 0, \quad T + BX + X^T B^T < 0;$

(ii) $\begin{bmatrix} T & BW^T + X \\ WB^T + X & -W - W^T \end{bmatrix} < 0 \ .$

Lemma 2. *[19] For positive definite matrix $R \in \mathbb{R}^{n\times n}$, and scalars a and b with $b - a \geq 0$, the following inequality holds:*

$$\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \geq \frac{1}{b-a}\Omega_1^T R\Omega_1 + \frac{3}{b-a}\Omega_2^T R\Omega_2, \tag{9}$$

where

$$\Omega_1 = x(b) - x(a),$$
$$\Omega_2 = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds.$$

3 Main Results

Let matrix V be the maximum annihilator of a fixed square matrix $\bar{E} \in \mathbb{R}^{n+n_c}$, *i.e.*, $\bar{E}^T V = 0$. We define that $e_i = [0_{(n+n_c)\times(i-1)(n+n_c)} \quad I_{(n+n_c)} \quad 0_{(n+n_c)\times(4-i)(n+n_c)}]$, $i = 1, 2, 3, 4$. The following theorem provides a sufficient condition for the admissibility of System (8).

Theorem 1. *For given $\tau > 0$, System* (8) *is admissible, if there exist $P > 0$, $Q > 0$, $R > 0$, K, K_τ, G_1, G_2, G_3 and S, such that*

$$\Xi_1 + \Xi_2 < 0, \tag{10}$$

where

$$\begin{aligned} \Xi_1 &= Sym\{\begin{bmatrix} e_1 \\ \bar{E}e_2 - \bar{E}e_3 \end{bmatrix}^T P \begin{bmatrix} \bar{E}e_2 \\ \tau e_4 \end{bmatrix} + e_2^T S V^T e_1\} + e_2^T Q e_2 - e_3^T Q e_3 + \tau^2 e_1^T R e_1 \\ &\quad -(e_2 - e_3)^T \bar{E}^T R \bar{E}(e_2 - e_3) - 3(\bar{E}e_2 + \bar{E}e_3 - 2e_4)^T R(\bar{E}e_2 + \bar{E}e_3 - 2e_4), \\ \Xi_2 &= Sym\{(G_1 e_2 + G_2 e_1 + G_3 e_3)^T(-e_1 + \bar{A}e_2 + \bar{B}K\bar{C}e_2 + \bar{B}K_\tau \bar{C}e_3)\}. \end{aligned}$$

Proof. Consider a Lyapunov-Krasovskii functional as

$$V(t) = \zeta^T(t)P\zeta(t) + \int_{t-\tau}^t X^T(s)QX(s)ds + \tau\int_{-\tau}^0\int_{t+\theta}^t \dot{X}^T(s)\bar{E}^T R\bar{E}\dot{X}(t)dsd\theta,$$

where

$$\zeta(t) = \begin{bmatrix} \bar{E}X(t) \\ \int_{t-\tau}^t \bar{E}X(s)ds \end{bmatrix}, \quad \dot{\zeta}(t) = \begin{bmatrix} \bar{E}\dot{X}(t) \\ \bar{E}(X(t) - X(t-\tau)) \end{bmatrix}.$$

By Lemma 2, we obtain

$$\begin{aligned}\dot{V}(t) &= Sym\{\dot{\zeta}^T(t)P\zeta(t)\} + X^T(t)QX(t) - X^T(t-\tau)QX(t-\tau) + \tau^2\dot{X}^T(t)\bar{E}^T R\bar{E}\dot{X}(t) \\ &\quad - \tau\int_{t-\tau}^{t}\dot{X}(s)\bar{E}^T R\bar{E}\dot{X}(s)ds \\ &\leq Sym\{\dot{\zeta}^T(t)P\zeta(t)\} + X^T(t)QX(t) - X^T(t-\tau)QX(t-\tau) + \tau^2\dot{X}^T(t)\bar{E}^T R\bar{E}\dot{X}(t) \\ &\quad - (X(t) - X(t-\tau))^T\bar{E}^T R\bar{E}(X(t) - X(t-\tau)) \\ &\quad - 3(X(t) + X(t-\tau) - \frac{2}{\tau}\int_{t-\tau}^{t}X(s)ds)^T\bar{E}^T R\bar{E}(X(t) + X(t-\tau) - \frac{2}{\tau}\int_{t-\tau}^{t}X(s)ds).\end{aligned} \tag{11}$$

Note that

$$0 = Sym\{(G_1X(t)+G_2\bar{E}\dot{X}(t)+G_3X(t-\tau))^T(-\bar{E}\dot{X}(t)+(\bar{A}+\bar{B}K\bar{C})X(t)+\bar{B}K_\tau\bar{C}X(t-\tau))\}, \tag{12}$$

and

$$0 = Sym\{X^T(t)SV^T\bar{E}\dot{X}(t)\}. \tag{13}$$

Combining (11), (12), (13), we obtain

$$\begin{aligned}\dot{V}(t) \quad \leq \quad & \eta^T(t)\{Sym\{\begin{bmatrix} e_1 \\ \bar{E}e_2 - \bar{E}e_3 \end{bmatrix}^T P \begin{bmatrix} \bar{E}e_2 \\ \tau e_4 \end{bmatrix}\} + e_2^T Q e_2 - e_3^T Q e_3 + \tau^2 e_1^T R e_1 \\ & -(e_2 - e_3)^T\bar{E}^T R\bar{E}(e_2 - e_3) - 3(\bar{E}e_2 + \bar{E}e_3 - 2e_4)^T R(\bar{E}e_2 + \bar{E}e_3 - 2e_4) \\ & +Sym\{e_2^T SV^T e_1 + (G_1e_2 + G_2e_1 + G_3e_3)^T(-e_1 + (\bar{A} + \bar{B}K\bar{C})e_2 + \bar{B}K_\tau\bar{C}e_3)\}\}\eta(t) \\ < \quad & 0,\end{aligned}$$

where

$$\eta(t) = \begin{bmatrix} \bar{E}\dot{X}(t) \\ X(t) \\ X(t-\tau) \\ \frac{1}{\tau}\int_{t-\tau}^{t}\bar{E}X(s)ds \end{bmatrix}.$$

This completes the proof. □

Remark 1. *In case of using delay-free dynamic compensator, one could adopt necessary and sufficient conditions like* (5)*. However, the resulting computation methods have to resort to iterative LMI methods which are indeed based on sufficient conditions. See, e.g., [12] [16] and the references therein. In contrast, the inequality* (10) *presents a condition to design delayed dynamic compensator. It is seen that, when the delay terms in dynamic compensator vanish, one could obtain a delay-free method based on condition* (5) *following a similar line. And, inequality* (10) *provides more freedom in the design procedure than that using delay-free conditions. In details, when adopting delayed dynamic compensator, there are additional terms, for instance, the terms* $e_2^T Q e_2 + \tau^2 e_1^T R e_1$ *and* $-(e_2 - e_3)^T\bar{E}^T R\bar{E}(e_2 - e_3) - 3(\bar{E}e_2 + \bar{E}e_3 - 2e_4)^T R(\bar{E}e_2 + \bar{E}e_3 - 2e_4)$ *which could provide freedom in solving matrix inequality. Later, we will show by example that some systems could not* *be stabilized by delay-free dynamic compensators but they can be stabilized by time-delayed dynamic compensators.*

Remark 2. *The delay $\tau > 0$ can be small enough, as long as the integral $\int_{t-\tau}^{t} X^T(s)QX(s)ds + \tau\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{X}^T(s)\bar{E}^T R\bar{E}\dot{X}(t)dsd\theta$ is not equal to zero. It is seen that condition* (10) *is likely to become infeasible with the increase of τ. Thus, a large τ may influence the performance of the system, even make the condition unsolvable.*

The condition (10) is nonlinear which could not be directly solved by the LMI method. We need to deal with the nonlinear term Ξ_2 in condition (10). In the following, Ξ_2 is transformed to $\Xi_2 = Sym\{(G_1e_2+G_2e_1+G_3e_3)^T(-e_1+\bar{A}e_2+\bar{B}K(k)\bar{C}e_2+\bar{B}K_\tau(k)\bar{C}e_3)\}+Sym\{(G_1e_2+G_2e_1+G_3e_3)^T(\bar{B}Y_1^{-1}\Delta K\bar{C}e_2+\bar{B}Y_2^{-1}\Delta K_\tau\bar{C}e_3)$, where $K = K(k)+Y_1^{-1}\Delta K$, $K_\tau = K_\tau(k)+Y_2^{-1}\Delta K_\tau$. The second part is treated by $Sym\{(G_1e_2+G_2e_1+G_3e_3)^T(\bar{B}Y_1^{-1}\Delta K\bar{C}e_2+\bar{B}Y_2^{-1}\Delta K_\tau\bar{C}e_3) = Sym\{(e_1+e_2+e_3)^T(\bar{B}\Delta K\bar{C}e_2+\bar{B}\Delta K_\tau\bar{C}e_3)+((G_1e_2+G_2e_1+G_3e_3)^T(\bar{B}Y_1^{-1}\Delta K\bar{C}e_2+\bar{B}Y_2^{-1}\Delta K_\tau\bar{C}e_3)-(e_1+e_2+e_3)^T(\bar{B}\Delta K\bar{C}e_2+\bar{B}\Delta K_\tau\bar{C}e_3))\}$, and $Sym\{(G_1e_2+G_2e_1+G_3e_3)^T(\bar{B}Y_1^{-1}\Delta K\bar{C}e_2+\bar{B}Y_2^{-1}\Delta K_\tau\bar{C}e_3)-(e_1+e_2+e_3)^T(\bar{B}\Delta K\bar{C}e_2+\bar{B}\Delta K_\tau\bar{C}e_3)\}$ is treated by Lemma 1. Thus, the following main result is obtained for the design of time-delayed dynamic compensator.

Theorem 2. *For given $\tau > 0$, $K(k)$ and $K_\tau(k)$, the closed-loop system* (8) *is admissible, if there exist $P > 0$, $Q > 0$, $R > 0$, ΔK, ΔK_τ, G_1, G_2, G_3, S, Y_1 and Y_2, such that*

$$\begin{aligned}\Psi(k) \;:=\; & \begin{bmatrix} \begin{array}{l}\Xi_1 + Sym\{(G_1e_2+G_2e_1+G_3e_3)^T(-e_1+\bar{A}e_2) \\ \quad +(G_1e_2+G_2e_1+G_3e_3)^T(\bar{B}K(k)\bar{C}e_2+\bar{B}K_\tau(k)\bar{C}e_3) \\ \quad +(e_1+e_2+e_3)^T(\bar{B}\Delta K\bar{C}e_2+\bar{B}\Delta K_\tau\bar{C}e_3)\}\end{array} & * & * \\ \Delta K\bar{C}e_2+\bar{B}^T(G_1e_2+G_2e_1+G_3e_3)-Y_1^T\bar{B}^T(e_1+e_2+e_3) & -Y_1-Y_1^T & * \\ \Delta K_\tau\bar{C}e_3+\bar{B}^T(G_1e_2+G_2e_1+G_3e_3)-Y_2^T\bar{B}^T(e_1+e_2+e_3) & * & -Y_2-Y_2^T \end{bmatrix} \\ < \;& 0, \end{aligned} \tag{14}$$

where Ξ_1 is defined in Theorem 1. If this is the case, the dynamic compensator gains can be solved as

$$K = K(k) + Y_1^{-1}\Delta K, \quad K_\tau = K_\tau(k) + Y_2^{-1}\Delta K_\tau.$$

Proof. By Lemma 1, (14) yields

$$
\begin{aligned}
& \Xi_1 + Sym\{(G_1e_2 + G_2e_1 + G_3e_3)^T(-e_1 + \bar{A}e_2) \\
& +(G_1e_2 + G_2e_1 + G_3e_3)^T(\bar{B}K(k)\bar{C}e_2 + \bar{B}K_\tau(k)\bar{C}e_3) \\
& +(e_1 + e_2 + e_3)^T(\bar{B}\Delta K\bar{C}e_2 + \bar{B}\Delta K_\tau\bar{C}e_3)\} \\
< \; & 0,
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
& \Xi_1 + Sym\{(G_1e_2 + G_2e_1 + G_3e_3)^T(-e_1 + \bar{A}e_2) \\
& +(G_1e_2 + G_2e_1 + G_3e_3)^T(\bar{B}(K(k) + Y_1^{-1}\Delta K)\bar{C}e_2 + \bar{B}(K_\tau(k) + Y_2^{-1}\Delta K_\tau)\bar{C}e_3)\} \\
< \; & 0.
\end{aligned} \tag{16}
$$

Let $K = K(k) + Y_1^{-1}\Delta K$, $K_\tau = K_\tau(k) + Y_2^{-1}\Delta K_\tau$, (16) is equivalent to (10). This completes the proof. □

Remark 3. *Condition* (14) *in Theorem 2 is an LMI for fixed* $K(k)$ *and* $K_\tau(k)$*. The linearization method in Theorem 2 is less conservative even if* $K(k)$ *and* $K_\tau(k)$ *equal to zero matrices. It is worth noting that the choice of* $K(k)$ *and* $K_\tau(k)$ *can affect the solutions of the dynamic compensator parameters. However, it is difficult to find 'good' values of* $K(k)$ *and* $K_\tau(k)$*. In this case, an algorithm is built to find the solution.*

Next, we will give the LMI-based algorithm to solve the dynamic compensator parameters.

Algorithm 1.

Step 1 Set $k = 0$. Let $K(0) = 0$ and $K_\tau(0) = 0$.

Step 2 Solve the following optimization problem with respect to $P > 0$, $Q > 0$, $R > 0$, ΔK, ΔK_τ, G_1, G_2, G_3, S, Y_1, Y_2, and α.

minimize α

s.t. $\Psi(k) < \alpha I$,

where $\Psi(k)$ is defined as in (14) .

Step 3 Let $K(k+1) = K(k) + Y_1^{-1}\Delta K$, $K_\tau(k+1) = K_\tau(k) + Y_2^{-1}\Delta K_\tau$. If $\alpha < 0$, stop, and $K(k+1)$, and $K_\tau(k+1)$ are the solutions.

Else, if the relative improvement of α is inferior to ε (a prescribed tolerance), then stop with the conclusion that the algorithm can not find the solutions.

Step 4 Set $k = k + 1$, and go to Step 2.

It is seen that the above algorithm is convergent as α is decreasing after each iteration.

Remark 4. *Theorem 2 together with **Algorithm 1** presents a delayed dynamic compensator design for rectangular descriptor systems. It provides an effective stabilization scheme in case that the existing methods, e.g., those in [11] [12], fail to achieve the stabilization purpose. This will be seen later through an example.*

4 Numerical Example

Consider System (1) with

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0.2 & 0.8 & -0.2 & -1.5 \end{bmatrix},$$
$$B = [0\ 1\ 0]^T, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

We study the stabilization problem for this example via dynamic compensator. Choose $E_c = [0\ 1]^T$ and $V = [0\ 0\ 0\ 1\ 0]^T$. If K_τ in (8) is set as 0, we can not find a dynamic compensator to achieve the stabilization for this system. Hence, the methods in [11] [12] fail to stabilize the system.

Now, let us adopt our method with the delayed dynamic compensator (6). Let $\tau = 0.2$. By using Algorithm 1, the solutions can be easily found as

$$K = \begin{bmatrix} 0.2455 & 0.0471 & 0 \\ 0.0522 & -1.9708 & 0 \\ 0 & 0 & -1.7003 \end{bmatrix}, \quad K_\tau = \begin{bmatrix} 0.5016 & -0.0248 & 0 \\ 0.0458 & -0.2776 & 0 \\ 0 & 0 & 0.2717 \end{bmatrix},$$

and the iteration number is 2. Under the above gain matrices, Fig. 1 shows the closed-loop responses of the system with initial condition $\phi_1(t) = x(0) = [1.5\ \ 2\ \ 1\ \ -0.0397]^T$, $t \in [-\tau, 0]$. Fig. 2 shows the state responses of the dynamic compensator with initial condition $\phi_2(t) = x_c(0) = 0.5$, $t \in [-\tau, 0]$.

Next, the parameter τ is changed to test the influence of time delay on the system performance. If $\tau = 3$, the dynamic compensator is designable by Algorithm 1. After 4 iterations, the gain matrices are obtained as

$$K = \begin{bmatrix} 0.6630 & 2012.5691 & 0 \\ 3.2896e-05 & -2.7076 & 0 \\ 0 & 0 & -1.5177 \end{bmatrix}, \ K_\tau = \begin{bmatrix} 0.1969 & -241.5260 & 0 \\ 4.9008e-05 & 0.2905 & 0 \\ 0 & 0 & 0.2001 \end{bmatrix}.$$

The closed-loop responses of the system and the dynamic compensator state are shown in Figs. 3-4.

It is easy to see from Figs. 1-4 that the convergence speed of dynamic response of system slows down with the increase of delay τ.

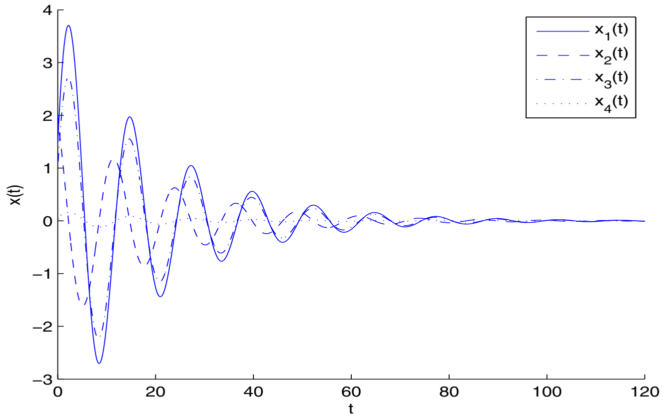

Figure 1: State response curves for the closed-loop system

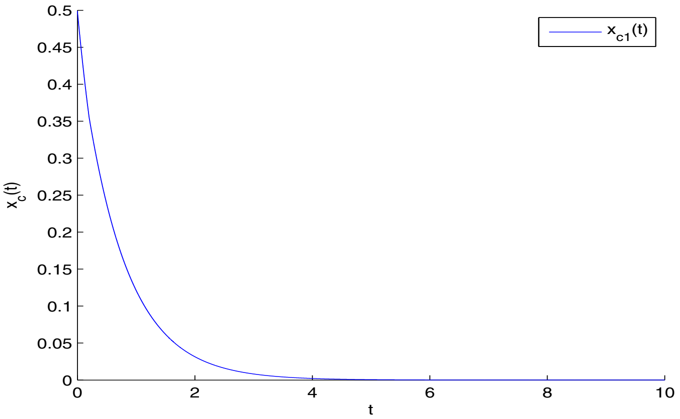

Figure 2: State response curves for the dynamic compensator

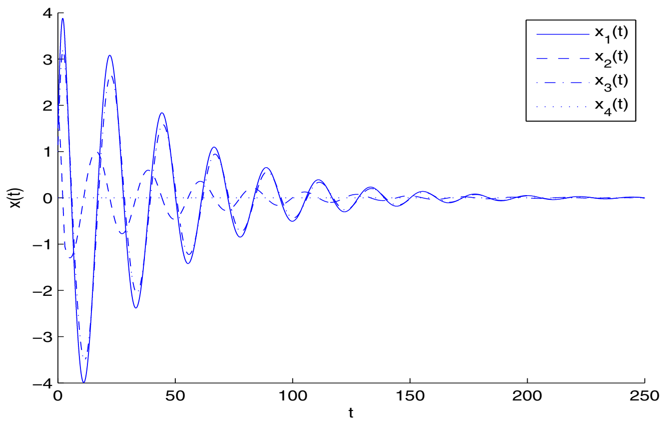

Figure 3: State response curves for the closed-loop system with $\tau = 3$

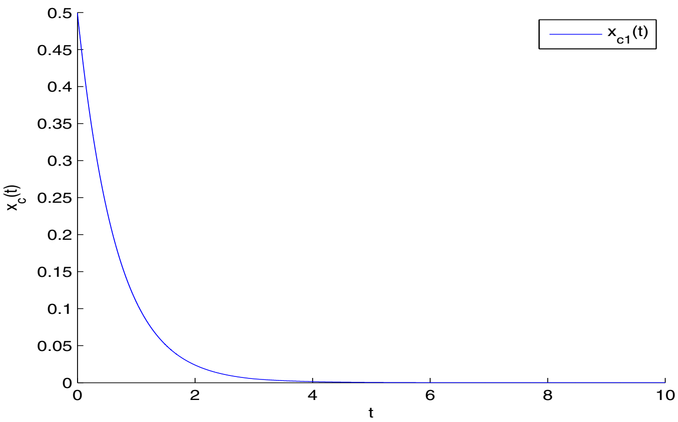

Figure 4: State response curves for the dynamic compensator with $\tau = 3$

If $\tau = 4$, our method is infeasible. This shows numerically that the length of delay affects the performance of system, and even affects the solvability of the method.

5 Conclusion

The stabilization for linear rectangular descriptor systems via time-delayed dynamic compensator has been studied in this paper. A design scheme of delayed dynamic compensator is proposed and an LMI-based algorithm is given to compute related parameters. Verification is provided to show that the present design scheme could make the closed-loop system admissible for rectangular descriptor systems for which the delay-free dynamic compensators fail to achieve the stabilization purpose. A numerical example is given to demonstrate the merits of the present method. The analysis and design method in this paper will lay a foundation for further studies in the synthesis of rectangular descriptor control systems with actuator saturation, model uncertainty, and exogenous signals.

References

[1] H. H. Rosenbrock, Structural properties of linear dynamical systems, International Journal of Control 20(2) (1974) 191-202.

[2] L. Dai, Singular Control Systems, New York: Springer-Verlag, 1989.

[3] S. Xu, J. Lam, Control and Filtering of Singular Systems. Springer, Berlin, 2006.

[4] L. R. Fletcher, Regularizability of descriptor systems, International Journal of Systems Science 17(6) (1986) 843-847.

[5] G. Duan, Y. Chen, Generalized regularity and regularizability of rectangular descriptor systems, Journal of Control Theory and Application 5(2) (2007) 159-163.

[6] J. Y. Ishihara, M.H. Terra, Impulse controllability and observability of rectangular descriptor systems. IEEE Trans. Automatic Control 46(6) (2001) 991-994.

[7] M. Hou, Controllability and elimination of impulsive modes in descriptor systems. IEEE Trans. Automatic Control 49(10) (2004) 1723-1727.

[8] M. Darouach, H_∞ unbiased filtering for linear descriptor systems via LMI, IEEE Trans. Automat. Control 54(8) (2009) 1966-1972.

[9] C.-S. Hsieh, H_∞ kalman estimation for rectangular descriptor systems with unknown inputs, IEEE Trans. Automatic Control 59(3) (2014) 826-832.

[10] M. K. Gupta, N.K.Tomar, S. Bhaumik. Full- and reduced-order observer design for rectangular descriptor systems with unknown inputs, Journal of the Franklin Institute 352(3) (2015) 1250-1264.

[11] G. S. Zhang, Regularizability, controllability and observability of rectangular descriptor systems by dynamic compensation, Proceedings of the American Control Conference, Minneapolis, USA (2006) 4393-4398.

[12] G. S. Zhang, L. Liu, Linear quadratic optimal control based on dynamic compensation for rectangular descriptor systems. Acta Automatica Sinica 36(2) (2010) 1752-1757.

[13] J. Stefanovski, LQ control of rectangular descriptor systems: Numerical algorithm and existence results. Optimal Control Applications and Methods 32(5) (2011) 505-526.

[14] Q. Shao, S. Liu, Robust H_∞ control for uncertain rectangular descriptor system by dynamic compensator, Journal of Computational Information Systems 9(12) (2013) 4639-4646.

[15] J. Chen, C. Lin, B. Chen, An improved path-following method for solving static output feedback control problems, Optimal Control Applications & Methods 37(6) (2016) 1193-1206.

[16] C. Lin, J. Chen, B. Chen, L. Guo, Z. Zhang, Fuzzy normalization and stabilization for a class of nonlinear rectangular descriptor systems, Neurocomputing 219 (2017) 263-268.

[17] C. Lin, Q.-G. Wang, T. H. Lee, Robust normalization and stabilization of uncertain descriptor systems with norm-bounded perturbations, IEEE Transactions on Automatic Control 50(4) (2005) 515-520.

[18] S.-I. Niculescu, C.T. Abdallah, Delay effects on static output feedback stabilization, Proceedings of the 39th IEEE Conference on Decision and Control, Sydney, Australia, Dec. 12-15 3 (2000) 2811-2816.

[19] A. Seuret, F.Gouaisbaut, Wirtinger-based integral inequality: application to time-delay systems, Automatica 49(9) (2013) 2860-2866.

[20] X. H. Chang, J. H. Park, J. Zhou, Robust static output feedback H_∞ control design for linear systems with polytopic uncertainties, Syst. Control Lett. 85 (2015) 23-32.

[21] J. Chen, C. Lin, B. Chen, Q. G. Wang, Regularization and Stabilization for Rectangular T-S Fuzzy Discrete-Time Systems With Time Delay, IEEE Transactions on Systems Man & Cybernetics Systems (2017). DOI: 10.1109/TSMC.2017.2686433

[22] X. Wang, G. Zong, H. Sun, Asynchronous finite-time dynamic output feedback control for switched time-delay systems with nonlinear disturbance, IET Control Theory & Applications 10 (10) (2016) 1142-1150.

[23] R. Guo, Z, Zhang, X. Liu, C. Lin, H. Wang, J. Chen, Exponential input-to-state stability for complex-valued memristor-based BAM neural networks with multiple time-varying delays, Neurocomputing 275 (2018) 2041-2054.

[24] Z. Zhang, X. Liu, D. Zhou, C. Lin, J. Chen, H. Wang, Finite-time stabilizability and instabilizability for complex-valued memristive neural networks with time delays, IEEE Transactions on Systems, Man, and Cybernetics: Systems 48(12) (2018) 2371-2383.

[25] Z. Y. Li, J. Lam, Y. Wang, Stability analysis of linear stochastic neutral-type time-delay systems with two delays, Automatica 91 (2018) 179-189.

[26] X. Xie, J. Lam, P. Li, H_∞ control problem of linear periodic piecewise time-delay systems, International Journal of Systems Science (2018). https://doi.org/10.1080/00207721.2018.1440028

[27] J. Chen, C. Lin, B. Chen, Q.-G. Wang, Fuzzy-model-based admissibility analysis and output feedback control for nonlinear discrete-time systems with time-varying delay, Information Sciences 412-413 (2017) 116-131.

[28] A. Zaafouri, R. C. Ben, A. H. Ben, A. Chaari, DSP-based adaptive backstepping using the tracking errors for high-performance sensorless speed control of induction motor drive, ISA Transactions 60 (2016) 333-347.

[29] R. C. Ben, F. Farhani, A. Zaafouri, A. Chaari, A novel adaptive control method for induction motor based on Backstepping approach using dSpace DS 1104 control board, Mechanical Systems & Signal Processing 100 (2017) 466-481.

[30] C. B. Regaya, F. Farhani, A. Zaafouri, A. Chaari, High-performance control of IM using MRAS-fuzzy logic observer, International Journal of Tomography & Simulation, 30 (2) (2017) 40-52.

[31] C. B. Regaya, F. Farhani, A. Zaafouri, A. Chaari, An adaptive sliding-mode speed observer for induction motor under backstepping control, Innovative Computing, Information and Control (ICIC Express Letters), 11(4) (2017) 763-771.

[32] F. Farhani, R. C. Ben, A. Zaafouri, A. Chaari, Real time PI-backstepping induction machine drive with efficiency optimization, ISA Transactions (2017). DOI: https://doi.org/10.1016/j.isatra.2017.07.003.

[33] J. Dong, G.-H. Yang, H. Zhang, Stability analysis of T-S fuzzy control systems by using set theory, IEEE Transactions on Fuzzy Systems 23(14) (2015) 827-841.

[34] J. Dong, Y. Wu, G.-H. Yang, A new sensor fault isolation method for T-S fuzzy systems, IEEE Transactions on Cybernetics 47(9) (2017) 2437-2447.