

Unreported Leak Location using Pressure and Flow Sensitivity in Water Distribution Networks

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Abstract

Water distribution systems are made up of many interdependent elements that enable water supply to meet a demand that is variable in time and space. One of the main concerns for utility managers is quickly locating and repairing a leak after detection, during regular network water balance.

This paper presents a two-stage methodology for locating a leak that is based on the hydraulic model of the network, and, particularly, on the conservation equations that govern network behaviour.

In the first stage, the sensitivity of each element (nodes and pipes) is obtained for a given demand increase in any node. In the second stage, that sensitivity is combined with additional real data provided by the (possibly) existing pressure sensors and flow meters installed throughout the network. As a final result, the system of equations thus obtained produces the theoretical leak flow at each network node that matches the network conditions. A subsequent analysis of the leak flows obtained highlights the node or nodes in which the leak is occurring.

The presented methodology is applied in a case study and assessed.

Keywords

Water leak detection; sensors; sensitivity analysis; hydraulic modelling

INTRODUCTION

Drinking water distribution systems in cities are large and complex infrastructures. Their operating points vary according to the demands at each moment and their management is complex – and even more so, if the appropriate renewal investments are not made. This complexity and gradual aging of the infrastructure does not help managers address one of the main problems in pressurized water distribution systems: the struggle against unreported leaks (IWA, 2000).

The traditional approach in the management of such leaks is essentially passive: the leaks are repaired when the water becomes visible, and so allowing leaks to run for weeks, months, or even years.

One of the most common policies to avoid wasting water is active leakage control (Charalambous et al, 2014, Berardi et al., 2016, Armon et al., 2011). This approach aims at early detection, location (Mounce et al., 2002), and repair of broken pipes – thereby reducing possible damage to third parties, minimising unplanned work, and reducing the volume of water lost.

Methods for the detection and monitoring of leaks are generally efficient, such as night minimum flows (Boulos & Aboujaoude, 2011; Alkassseh et al., 2013), and sectorisation of networks (Gomes et al., 2012; Tzatchkov et al., 2014; Guistolisi & Ridolfi, 2014). Once the existence of a leak is detected, it is located using acoustic techniques (Li, et al. 2015). For large networks this approach may require many resources and considerable time.

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New methodologies based on the sensorisation of distribution networks are gaining momentum for the rapid localisation of uncontrolled leaks. These methods are based on pressure measurements and sensitivity analysis of the distribution networks, taking advantage of the interdependence of all the operating parameters. These fundamentals were proposed by Pudar & Liggett (1992), who studied the relationship between leaked volume, network pressure, and the leaking section. Most recent studies have focused the location of sensors to facilitate fault detection and maximise leak location performance (Blesa et al., 2016; Casillas et al., 2015; Gamboa-Medina & Reis, 2017; Sarrate et al., 2014). The result depends on the number of sensors installed (Xie et al., 2017) and will be limited by the budget and strategy followed (Fuchs-Hanusch & Steffelbauer, 2017). Based on the measurements of the pressure sensors, Perez et al. (2011) propose analysing the difference between said data and the equivalents provided by a simulation model, and in the event of a discrepancy, using the leak sensitivity matrix to determine the location. Under ideal conditions, these methodologies offer excellent results – but these results are less good if errors exist in the demands and measurements. To minimise this effect, subsequent studies (Casillas et al., 2014) introduce in their calculations an extended-period analysis of the measurements. Other authors propose similar approaches based on sensitivity analysis for locating leaks (Gamboa-medina & Reis, 2017; Möderl et al., 2011; Steffelbauer et al., 2014); but all these approaches demand the posing of multiple scenarios and their corresponding simulations in a hydraulic model to explore the changes obtained. Considering the dynamic operation of distribution networks and the always unpredictable appearance of leaks, the time required for such analyses may limit their practical use.

The present work proposes a new methodology for locating leaks that is based on a simplified calculation of the sensitivity of the elements of the network. This methodology combines knowledge of the specific average behaviour of each element following an increase in demand at any point of the network, and the information provided by the pressure sensors. The aim is to produce a matrix formulation of the equations for the network behaviour in which the leak flows in each node are the unknowns, but which can continue to be resolved using the classical gradient procedure (Todini & Pilati, 1987). Finally, the methodology is applied to a study network and the results are discussed.

Only two additional points need to be highlighted. The first is the versatility of the method: although the work is focused from the perspective of the location of the leak (real losses), the concepts used in the mathematical formulation mean it is equally applicable to the location of clandestine or unauthorised consumption. The second point is that this methodology is notable for its rapid calculation in the resolution of each scenario, which is one of its main advantages over similar studies, due to the vast number of combinations possible when locating leaks.

101 METHODOLOGY

102

103 The methodology is presented in three sections. The first section (basic equations) is a reminder of
104 the conservation equations that govern network behaviour. The contributions of the paper are
105 developed in the second (sensitivity) and the third (location) sections.

106

107 Basic equations

108

109 Given a water network with p pipes, n nodes, and m supply points, the method for modelling its
110 hydraulic behaviour through conservation equations have already been soundly set (Todini &
111 Rossman, 2013; Todini & Pilati, 1987). The first equation is the *mass conservation* equation applied
112 to each network node:

113

$$\sum_{k=1}^{n_i} Q_{ik_0} + q_i = 0 \quad (1)$$

114

115 where n_i is the total number of nodes connected to node i , Q_{ik_0} is the circulating flowrate through
116 the pipe that connects node i to node k , and q_i is the demand flowrate at node i .

117

118 The second equation is the *energy conservation* equation applied to each network pipe:

$$H_{i_0} - H_{j_0} = h_{f_{ij_0}} = r_{ij} Q_{ij_0} |Q_{ij_0}| \quad (2)$$

119 where H_{i_0} and H_{j_0} are the head values at nodes i and j , respectively; $h_{f_{ij_0}}$ are the friction losses in
120 the pipe that connects nodes i and j ; and r_{ij} and Q_{ij_0} are, respectively, hydraulic resistance and
121 flowrate through the pipe between nodes i and j .

122

123 Both sets of conservation equations, *mass* and *energy*, can be structured by means of matricial
124 notation (Todini & Pilati, 1987):

125

$$\begin{bmatrix} A_{11} & \vdots & A_{12} \\ \dots & \dots & \dots \\ A_{21} & \vdots & 0 \end{bmatrix} \cdot \begin{bmatrix} Q \\ \dots \\ H \end{bmatrix} = - \begin{bmatrix} A_{10} \cdot H_0 \\ q \end{bmatrix} \quad (3)$$

126

127 where A_{11} is a $(p \times p)$ diagonal matrix that represents the friction losses in each pipe; A_{12} is a
128 connectivity $(p \times n)$ matrix that relates nodes and pipes; A_{21} is the transposed matrix of A_{12} ; Q is
129 the vector of the (unknown) circulating flows in pipes; H is the vector of the (unknown) piezometric
130 heads in the nodes; A_{10} is the fixed head node incidence $(p \times m)$ matrix; H_0 is the vector with the
131 values of (known) fixed piezometric heads; and q is the vector of (known) nodal demands.

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133

134 Network sensitivity (full equations development in Appendix A)

135

136 To establish the effect of a demand increase throughout the network, this study tackles the
137 sensitivity of the network and analyses the flow and pressure variation caused by small variations in
138 water demand that are significant enough to affect the rest of elements. This study is structured in
139 three steps.

140

141 Step 1: Current situation: The first step is to solve the hydraulic performance of the network,
 142 obtaining the piezometric head in the nodes (vector H), and the flowrate circulating through the
 143 lines (vector Q). This is the base scenario for which the sensitivity is to be calculated.

144

145 Step 2: Definition of a consumption increase and consequences: if in node i , the current demand
 146 (q_i) is increased (φ_{q_i}), it will affect the new demand ($q_i + \varphi_{q_i}$) and head ($H_{i_0} + \varphi_{H_{i_0}}$) in node i , and
 147 the head ($H_{j_0} + \varphi_{H_{j_0}}$) in any other node j , as well as the circulating flow in any pipe ($Q_{ik_0} + \varphi_{Q_{ik_0}}$).

148

149 Therefore, the new formulation of the *mass conservation* equation in any node i will be:

150

$$\sum_{k=1}^{n_i} (Q_{ik_0} + \varphi_{Q_{ik_0}}) + q_i + \varphi_{q_i} = 0 \quad (4)$$

151 where $\varphi_{Q_{ik_0}}$ is the flow variation of the pipe connecting nodes i and k , produced by the demand
 152 variation in node i .

153

154 In parallel, the new *energy conservation* equation for any pipe (connecting nodes i and j) will be:

155

$$(H_{i_0} + \varphi_{H_{i_0}}) - (H_{j_0} + \varphi_{H_{j_0}}) - \vartheta_{ij} r_{ij} \cdot \left(Q_{ij_0} |Q_{ij_0}| + \frac{Q_{ij_0}^2 \cdot \varphi_{Q_{ij_0}}}{|Q_{ij_0}|} + \frac{Q_{ij_0} \cdot \varphi_{Q_{ij_0}}^2}{|Q_{ij_0}|} + \varphi_{Q_{ij_0}} |Q_{ij_0}| \right) = 0 \quad (5)$$

156

157 where ϑ is a term that adopts the value $\frac{Q_{ij_0} \cdot \varphi_{Q_{ij_0}}}{|Q_{ij_0}| \cdot |\varphi_{Q_{ij_0}}|}$ if $|\varphi_{Q_{ij_0}}| > |Q_{ij_0}|$ and the unit value otherwise.

158

159 Step 3: Calculation of network sensitivity: from the equations presented in the previous step,
 160 calculating the sensitivity of all the elements of the network is straightforward following a demand
 161 increase, in particular, in node i . Thus, it is suffice to solve the p Equations 5 and the n Equations 4,
 162 for the total of p unknowns that correspond to the variation of the flow of each pipe $\varphi_{Q_{ij_0}}$ and the n
 163 unknowns corresponding to the variation of the head of each node $\varphi_{H_{j_0}}$.

164

165 Since the network consists of n nodes, the repetition n times of this complete resolution of the
 166 network (one for each node) will give a complete view of the sensitivity of the network. Using this
 167 approach, a complete and quantified range of variation of the properties of each element is
 168 obtained. Although, this is perfectly feasible from the equations presented here, it may not be very
 169 operative in practice, since the calculation time for as many network resolutions as there are nodes,
 170 may be excessive. For this reason, the proposed method includes a simplification that means the
 171 system of network equations needs to be solved just once.

172

173 Instead of working separately with the specific variation in each element caused by the increase of
 174 flow in each node, the single average of all these variations is considered. That is, the general
 175 sensitivity of the head in each node i is shown by:

$$\varphi_{H_i} = \frac{\sum_{m=1}^n \varphi_{H_{i_m}}}{n} \quad (6)$$

176 And the general sensitivity of the flowrate in each pipe ik , is shown by:

$$\varphi_{Q_{ik}} = \frac{\sum_{m=1}^n \varphi_{Q_{ikm}}}{n} \quad (7)$$

177 By introducing these averages in the calculation of the sensitivity of the network elements, and also
 178 averaging all the n equations for each node, then the total of $n \cdot (n+p)$ equations (n Equations 4 plus
 179 p Equations 5, and multiplied by the n nodes of the network) is reduced to n node equations such as:
 180

$$\sum_{k=1}^{n_i} (Q_{ik_0} + \varphi_{Q_{ik}}) + q_i + \frac{\varphi_{q_i}}{n} = 0 \quad (8)$$

181 and p pipe equations such as:

$$(H_{i_0} + \varphi_{H_i}) - (H_{j_0} + \varphi_{H_j}) - \vartheta_{ij} \cdot r_{ij} \cdot \left(Q_{ij_0} \cdot |Q_{ij_0}| + \frac{Q_{ij_0}^2 \cdot \varphi_{Q_{ij}}}{|Q_{ij_0}|} + \frac{Q_{ij_0} \cdot \varphi_{Q_{ij}^2}}{|Q_{ij_0}|} + \varphi_{Q_{ij}} \cdot |Q_{ij_0}| \right) = 0 \quad (9)$$

183 where $\varphi_{Q_{ij}}$ y φ_{H_i} are, respectively, the average variation of the flow in pipe i , and the piezometric
 184 head in node i produced by the consumption variation in any node. The new set of $n+p$ equations,
 185 with $n+p$ unknown factors can be expressed in a matrix format as follows:
 186

$$\begin{bmatrix} A_{11\varphi} & \vdots & A_{12} \\ \dots & \dots & \dots \\ A_{21} & \vdots & 0 \end{bmatrix} \begin{bmatrix} \varphi_Q \\ \dots \\ \varphi_H \end{bmatrix} = - \begin{bmatrix} A_{11\vartheta} & \vdots & A_{12} \\ \dots & \dots & \dots \\ A_{21} & \vdots & 0 \end{bmatrix} \begin{bmatrix} Q \\ \dots \\ H \end{bmatrix} - \begin{bmatrix} A_{10} H_0 \\ \dots \\ q + \frac{\varphi_q}{n} \end{bmatrix} \quad (10)$$

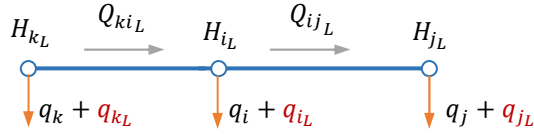
187 where A_{12} , A_{21} , A_{10} , H_0 and q are the same terms as in Equation 3; H and Q are the vectors
 188 explained in Step 1; φ_Q is the vector of the (unknown) sensitivities (average variations) of flow in
 189 network pipes; φ_H is the vector of the (unknown) sensitivities (average variations) of head in
 190 network nodes; $A_{11\varphi}$ is a $(p \times p)$ diagonal matrix with components
 191 $A_{11\varphi}(r, r) = \vartheta_{ij} r_{ij} \left(2 |Q_{ij_0}| + \frac{Q_{ij_0}}{|Q_{ij_0}|} \varphi_{Q_{ij}} \right)$, and $A_{11\vartheta}$ is a $(p \times p)$ diagonal matrix with components
 192 $A_{11\vartheta}(r, r) = \vartheta_{ij} r_{ij} |Q_{ij_0}|$.
 193
 194

195 The process for solving Equation 10 is the same for Equation 3 and, hence, the network sensitivity
 196 of an uncontrolled consumption of a given value at any node, can be calculated.
 197

199 Leak location (full equations development in Appendix B)

200
 201
 202 In the case of a leak occurrence, whose magnitude and location (node) were known, the resulting
 203 pipe flows and node heads could be easily calculated by solving Equation 3. However, in water
 204 distribution networks, while estimating the magnitude of a newly detected leak though a quick
 205 water balance is not difficult, it is not so easy to determine its spatial location. In this leak scenario,
 206 new unknown factors must be introduced (Figure 1): circulating flows in pipes change (Q_{ijL}), node
 207 heads change (H_{iL}), and in addition to normal demand, a leak flow (q_{iL}) should be considered in
 208 each node. It is clear, that most of these added leak flows will be zero, and only those ones (ideally,
 209 just one) in the area close to the leak location will differ from zero, but there is usually no further

210 information to clarify this question. Therefore, the new system would present $2n+p$ unknown
 211 factors (which are the pressures and leak flowrates in the n nodes and the flows through the p pipes)
 212 and only $n+p$ equations.
 213



214
 215 Figure 1. Hypothetical leaks in a distribution network
 216

217 To limit the scope of the problem and find a finite and real solution, two sources of additional
 218 information can be introduced. The first takes advantage of the calculation of node sensitivity
 219 explained above. Since the real magnitude of a leak can be estimated through a quick water balance,
 220 the average head variation for each network node that corresponds to that magnitude of increase in
 221 demand, can also be calculated by Equation 10. Now the following hypothesis is proposed: in a leak
 222 situation the unknown piezometric head of any node can be approximated to the head under that
 223 node's normal working conditions plus its average calculated variation, ($H_{iL} = H_{i_0} + \varphi_{H_i}$).
 224 Therefore, n terms corresponding to the node heads are now considered as known, and the new
 225 scenario can be solved.
 226

227 The second source of additional information are the real measurements of the (likely) existing flow
 228 meters and pressure sensors installed in specific network pipes and nodes. Whatever the number of
 229 devices of each kind, the measurements they provide can be organised in a vector H_D for the known
 230 node heads, and a vector Q_D for the known flowrate pipes.
 231

232 If these two new data sources are introduced into the equations that solve the hydraulic behaviour of
 233 the network, and the resulting set is simplified and adequately reordered, the following matrix
 234 formulation is produced:
 235

$$\begin{bmatrix} A_{11} & \vdots & 0 \\ \dots & \dots & \dots \\ A_{21} & \vdots & 1 \end{bmatrix} \begin{bmatrix} Q_L \\ \dots \\ q_L \end{bmatrix} = - \begin{bmatrix} A_{11} & \vdots & A_{12} \\ \dots & \dots & \dots \\ A_{21} & \vdots & 0 \end{bmatrix} \begin{bmatrix} Q_D \\ \dots \\ H_D \end{bmatrix} - \begin{bmatrix} A_{10} H_0 \\ \dots \\ q \end{bmatrix} \quad (11)$$

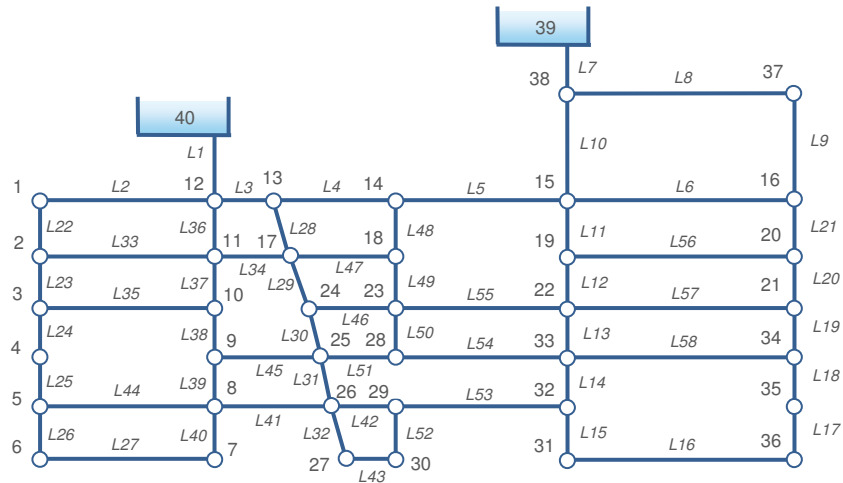
236
 237 Where A_{11} , A_{12} , A_{21} , A_{10} , H_0 and q are the same terms as in Equation 3, Q_L is the vector of the
 238 (unknown) pipe flows (in which values corresponding to flowmetered pipes are set to zero); q_L
 239 is the vector of the (unknown) nodal leak flows; Q_D is the vector of the (known) flowrates as
 240 measured by the meters in the pipes (and zero being the value for the rest of the pipes); H_D is the
 241 vector of the (known) node heads (these values being directly measured by the pressure sensors
 242 installed in some nodes, or the approximation $H_{i_0} + \varphi_{H_i}$ for the rest of the nodes).
 243

244 The two results obtained after solving Equation 11 are the circulating pipe flowrates and, most
 245 importantly, the leak flow in every node. A quick later analysis of the leak flow results will pinpoint
 246 the node (in the best case), or the area (at least) in which the leak is taking place.
 247
 248
 249

250 **CASE STUDY**

251
252 **Network description (full data in Appendix C)**

253
254 What follows is a case study of a synthetic network as shown in Figure 2. The network has a total
255 length of 50 km with 58 pipelines that supply 38 consumption nodes whose elevation is zero. Some
256 6,500 m³ of water is delivered daily (40,000 inhabitants). Both reservoirs are elevated 50 m. The
257 pipe roughness is 0.1 mm, and pipes lengths and diameters are shown in Table C1 (Appendix C).
258



259
260 **Figure 2. Layout of the distribution network**

261
262
263 The base demand is 60 L/minute (0.001 m³/s) for every node between 1 and 16, and nodes 21, 27,
264 and 30; while for the 19 remaining nodes, the base demand is 180 L/minute (0.003 m³/s).
265

266 In its normal operating state, the average pressure is 37.7 mca, with extreme values of 31.7 and
267 47.0 mca (Table C2). Average water velocity and unitary headloss in pipes are 0.41 m/s and
268 2.50 mca/km, respectively (Table C1). The contribution of flow for each reservoir is approximately
269 50%.

270
271 **Sensitivity analysis (full results in Appendix C)**

272
273 For an analysis of the sensitivity of the elements of the network, increments of 15, 30 and 60
274 L/minute have been defined in each node. The reason is that the appearance of leaks from 15 or 30
275 L/minute on, in monitored sectors of a network may be detected using night flow analysis or
276 automatic water balances. In addition, these figures are of the same order of magnitude as those
277 considered by Fuchs-Hanusch & Steffelbauer (2017).
278

279 After solving Equation 10 for all three scenarios, the detailed node results are shown in Table C2.
280 As expected, the greater the increase in demand, the greater the decrease in pressure at the nodes
281 (those that are furthest away from the inlet points being more affected). Thus, the average pressure
282 decrease for network nodes in general is -0.2% for 15 L/minute, and -0.4% and -0.9% for 30 and 60
283 L/minute, respectively.
284

285 Sensitivity pipe results are shown in Table C3. Magnitudes of average flow changes for each

286 demand increase (15, 30 and 60 L/minute) are 0.52%, 1.05%, and 2.13% (respectively). This gives
287 a better idea of the real sensitivity and explains why the two pipes (27 and 44) with variations of
288 more than 20% have two of the lowest network flowrates (about one hundred times less than the
289 average flow per pipe).

290
291 The sensitivity analysis highlights those nodes with larger pressure variations (either in total or
292 percentage terms) – nodes 5, 6 and 25; as well as those whose changes are much less relevant,
293 namely, nodes 12, 15 and 34. In parallel, the most sensitive pipes are 8, 36, and 37 (not taking into
294 account those connecting each reservoir), whereas the least significant are 16, 43 and 46.

295 296 **Leak location (full results in Appendix D)**

297
298 Once the sensitivity analysis is completed, the problem of leak allocation is approached. For
299 demonstrative purposes it is assumed that five pressure sensors and five flowmeters are available
300 for installation in the network. The main criterion for the selection of the elements in which to
301 install the sensors is that of greater sensitivity (as obtained in the previous calculation); while a
302 secondary criterion is that of achieving a reasonable spatial distribution of the sensors. Accordingly,
303 pressure sensors are installed in nodes 5, 18, 27, 30 and 37; and the flowmeters in pipes 1, 9, 20, 36
304 and 45. That is, 13% of the nodes and 9% of the pipes are monitored.

305
306 The application and results of the leak location method are demonstrated as follows: it will be
307 assumed, as each case is different, that a leak of 30 L/minute has occurred in each of the nodes of
308 the network. That is, 38 different cases will be resolved. For each, in addition to knowing in which
309 node the leak is located, the five real pressure values and five flowrate values provided,
310 respectively, by the pressure sensors and flowmeters installed in the network, will be known (by
311 direct simulation of the case). These ten items of data will be transferred to the model in which the
312 method is tested, and whose information is completed for the rest of the nodes with the average
313 head affected by the average sensitivity for an increase in demand of 30 L/minute. Finally, and
314 through the resolution of Equation 11 the leak flow for each node is obtained (as well as the
315 circulating pipe flows). The results are considered successful as the leak flow calculated is about 30
316 L/minute for the leaky node, or for neighbouring nodes, and is negligible for the other network
317 nodes.

318
319 Table D2 (Appendix D) shows the obtained leak flow for each node (rows), measured in L/minute,
320 for each of the simulations (columns). The table shows that the methodology has been successful in
321 63% (24 of the 38 simulations), since in 8 cases the leaking node was exactly identified (in green),
322 and in 16 other cases the area was identified (node adjacent to the node with the leak – in
323 yellow). From the data in Table D2 we can observe that the accurate location of the node, or the
324 area where the uncontrolled leak is taking place, depends to some extent on the existence of a
325 nearby flow or pressure sensor. It is also important to highlight the existence of nodes with a
326 negative consumption (as in the case of node 36 for most simulations). It is apparent that some
327 nodes may need a flow input to counterbalance the deviations introduced when estimating the
328 known piezometric heads.

329
330 As expected, the leak location is less accurately shown if instead of choosing elements with a large
331 variation in pressure and flow, those with a lower average variation are chosen. Thus, if nodes 1, 12,
332 15, 16 and 38, and pipes 6, 16, 43, 46 and 56 were selected, then less than 50% of the cases (16 of
333 the 38) would be successful (Table D3). If sensitivity criteria is left aside and only spatial
334 distribution criteria is considered, then the results show a greater variability. For example, if nodes
335 2, 8, 18, 20 and 32; and pipes 11, 17, 25, 36 and 42 were monitored, the success rate would be

336 exactly 50% (Table D4).

337

338 Logically, the more sensors installed in the network, the better the results. Thus, in Appendix D up
339 to seven cases with different configurations of sensors are resolved, and a success rate of 87% is
340 reached with 15 pressure sensors and 5 flowmeters. Generalised conclusions on the usefulness and
341 versatility of the proposed method follow below.

342

343

344 **CONCLUSION**

345

346 Leakage control is a crucial part of the daily management of water distribution networks, and one of
347 its key pillars is the rapid localisation of leaks. The range of instrumentation currently available for
348 the monitoring of hydraulic variables in the network is very helpful in this task. Thus, pressure
349 sensors and flow meters whose measurements can be transmitted in real time are frequently used,
350 and this is of great value for the methodology proposed in this article. The number and location of
351 these devices is limited by several factors, but mainly the cost of purchase, installation, and
352 maintenance.

353

354 A methodology is proposed in this article that is doubly useful for the control of leaks. Firstly,
355 based on the characteristics of the network and on the direct measurements provided by sensors, it
356 reveals the location of a leak whose magnitude has been previously detected. Secondly, this same
357 methodology can be used to determine the optimal location for the installation of sensors and
358 control flow meters in a network – given that the effectiveness of any possible configuration can be
359 simulated.

360

361 The potential provided by the proposed methodology for both objectives is based on its versatility
362 and speed of calculation. Versatility because only the mathematical model of the network is needed
363 and this is a very common support tool in supply management, and also because model equations
364 are used directly – without need for (nor dependence on) specialised software. Thus, the speed of
365 the resolution of the system of equations proposed here (using any spreadsheet and with a minimum
366 of customised programming ability by the user) is the same, or even faster, than when using
367 hydraulic simulation software.

368

369 The methodology itself is structured in two parts. In the first part, the sensitivity analysis focuses on
370 the variations in the circulating flows and the piezometric heads in the nodes when a slight demand
371 increase occurs. As a result, the average variation of these parameters is obtained and this enables a
372 prioritisation of the hydraulic importance of the elements. The second part resolves mass and
373 energy conservation equations – while considering as variables the unknown and uncontrolled water
374 demands in each circulating flow and node, as well as considering an approximation of the
375 piezometric head in the nodes as known variables. The measurements recorded by the pressure and
376 flow sensors are considered as additional information. The final leak location may vary
377 considerably depending on the number and location of the sensors, while the best locations for
378 installing these sensors are on those elements whose variation is greater than average in the
379 sensitivity analysis. This methodology is finally used in a case study network with a limited number
380 of sensors to detect a water leak of 30 L/minute.

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- 454

455 **Appendix A. Development of equations for network sensitivity**

456 Development of equations to calculate the network **sensitivity**.

457

458 The **mass conservation equation for each node i** , in normal conditions with no leaks is:

459

$$\sum_{k=1}^{n_i} Q_{ik_0} + q_i = 0 \quad (A1)$$

460

461 where n_i is the total number of nodes connected to node i , Q_{ik_0} is the circulating flowrate through
462 the pipe that connects node i to node k , and q_i is the demand flowrate at node i .

463

464 If the demand at node i is increased by a given amount (φ_{q_i}) because of a leak occurrence:

465

466 • The new demand at node i will be: $q_i + \varphi_{q_i}$

467

468 • A variation in the circulating flowrate through the pipe that connects node i to node k will
469 appear. That variation will be $\varphi_{Q_{ik_i}}$, and the new pipe flowrate will be: $Q_{ik_0} + \varphi_{Q_{ik_i}}$

470

471 • The new mass conservation equation for node i (due to a demand increase at node i) will be:

472

$$\sum_{k=1}^{n_i} (Q_{ik_0} + \varphi_{Q_{ik_i}}) + q_i + \varphi_{q_i} = 0$$

473

474 • The new mass conservation equation for any other node j (due to a demand increase at
475 node i) will be:

476

$$\sum_{k=1}^{n_j} (Q_{jk_0} + \varphi_{Q_{jk_i}}) + q_j = 0$$

477

478

478 If the mass conservation equation is considered for each node i , in each case a demand increase
 479 occurs in each one of the network nodes, the complete set of n equations for node i will be:
 480

Node i	Demand increase in node	Mass conservation equation (for node i)
i	1	$(Q_{i1_0} + \varphi_{Q_{i1_1}}) + \dots + (Q_{ik_0} + \varphi_{Q_{ik_1}}) + \dots + (Q_{in_{i_0}} + \varphi_{Q_{in_{i_1}}}) + (q_i) = 0$
	2	$(Q_{i1_0} + \varphi_{Q_{i1_2}}) + \dots + (Q_{ik_0} + \varphi_{Q_{ik_2}}) + \dots + (Q_{in_{i_0}} + \varphi_{Q_{in_{i_2}}}) + (q_i) = 0$

	m	$(Q_{i1_0} + \varphi_{Q_{i1_m}}) + \dots + (Q_{ik_0} + \varphi_{Q_{ik_m}}) + \dots + (Q_{in_{i_0}} + \varphi_{Q_{in_{i_m}}}) + (q_i) = 0$

	i	$(Q_{i1_0} + \varphi_{Q_{i1_i}}) + \dots + (Q_{ik_0} + \varphi_{Q_{ik_i}}) + \dots + (Q_{in_{i_0}} + \varphi_{Q_{in_{i_i}}}) + (q_i + \varphi_{q_i}) = 0$

	n	$(Q_{i1_0} + \varphi_{Q_{i1_n}}) + \dots + (Q_{ik_0} + \varphi_{Q_{ik_n}}) + \dots + (Q_{in_{i_0}} + \varphi_{Q_{in_{i_n}}}) + (q_i) = 0$

(A2)

481
 482 The result of adding the n equations for node i is:
 483

$$\sum_{k=1}^{n_i} (n Q_{ik_0}) + \sum_{k=1}^{n_i} \left(\sum_{m=1}^n \varphi_{Q_{ik_m}} \right) + n q_i + \varphi_{q_i} = 0 \quad (A3)$$

$$\sum_{k=1}^{n_i} \left(n Q_{ik_0} + \sum_{m=1}^n \varphi_{Q_{ik_m}} \right) + n q_i + \varphi_{q_i} = 0 \quad (A4)$$

484
 485 The average variation for each pipe flowrate, after considering the influence of a leak at each of the
 486 n nodes of the network is:
 487

$$\varphi_{Q_{ik}} = \frac{\sum_{m=1}^n \varphi_{Q_{ik_m}}}{n} \quad (A5)$$

488
 489 If Equation A5 is replaced in Equation A4, and the resulting equation is divided by n , the final mass
 490 conservation equation for node i is:
 491
 492

$$\sum_{k=1}^{n_i} (Q_{ik_0} + \varphi_{Q_{ik}}) + q_i + \frac{\varphi_{q_i}}{n} = 0 \quad (A6)$$

493
 494 The first part of the sensitivity analysis is performed by considering the same demand increase (φ_q
 495 taken as a constant value) for each node, and posing the whole set of the mass conservation
 496 equations for all the network nodes:
 497

Node Id	Mass conservation equation
1	$\sum_{k=1}^{n_1} (Q_{1k_0} + \varphi_{Q_{1k}}) + q_1 + \frac{\varphi_q}{n} = 0$
2	$\sum_{k=1}^{n_2} (Q_{2k_0} + \varphi_{Q_{2k}}) + q_2 + \frac{\varphi_q}{n} = 0$
...	...
m	$\sum_{k=1}^{n_m} (Q_{mk_0} + \varphi_{Q_{mk}}) + q_m + \frac{\varphi_q}{n} = 0$
...	...
n	$\sum_{k=1}^{n_n} (Q_{nk_0} + \varphi_{Q_{nk}}) + q_n + \frac{\varphi_q}{n} = 0$

(A7)

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502

System A7 is compounded by n equations, with p unknowns – the average flowrate sensitivity for each one of the p pipes. According to the matrix formulation used by Todini & Rossman (2013) this system can be expressed as:

$$[A_{21}] [Q + \varphi_Q] + \left[q + \frac{\varphi_q}{n} \right] = 0 \quad (\text{A8})$$

503

where:

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- A_{21} is a $(n \times p)$ matrix that relates n nodes to p pipes, so that each element $A_{21}(q, r)$ takes the value:
 - -1 if pipe r leaves node q
 - 0 if pipe r is not connected to node q
 - $+1$ if pipe r enters node q
- Q is the vector of the pipe flowrates Q_{ij_0}
- φ_Q is the vector of the pipe flowrate average sensitivities $\varphi_{Q_{ij}}$
- q is the vector of the node demands q_i
- $\frac{\varphi_q}{n}$ is the vector of the node demand variations. As explained above, since the sensitivity is calculated for the same demand variation in each node, this vector has a unique value for all the nodes.

Finally, Equation A8 can be operated to the final expression:

$$[A_{21} \varphi_Q] = - [A_{21} Q] - \left[q_i + \frac{\varphi_q}{n} \right] \quad (\text{A9})$$

519

520

521 The **energy conservation equation for each pipe**, which connects node i to node j , in normal
 522 conditions with no leaks is:

$$523 \quad H_{i_0} - H_{j_0} = h_{f_{ij_0}} = r_{ij} Q_{ij_0} |Q_{ij_0}| \quad (A10)$$

524 where H_{i_0} and H_{j_0} are the piezometric heads at nodes i and j , respectively; $h_{f_{ij_0}}$ is the pipe friction
 525 headloss; and r_{ij} and Q_{ij_0} are the pipe hydraulic resistance and pipe flowrate, respectively.

526
 527 If the demand at node i is increased by a given amount (φ_{q_i}), because of a leak occurrence:

- 529 • A variation in the circulating flowrate through the pipe that connects node i to node j will
 530 appear – and in any other pipe of the network as well. That variation will be $\varphi_{Q_{ij_i}}$, and the
 531 new pipe flowrate will be: $Q_{ij_0} + \varphi_{Q_{ij_i}}$.
- 532 • A variation in the piezometric head of nodes i and j will appear – and also in any other node
 533 of the network. Those variations will be $\varphi_{H_{i_i}}$ and $\varphi_{H_{j_i}}$, respectively, and the new
 534 piezometric heads will be: $H_{i_0} + \varphi_{H_{i_i}}$ and $H_{j_0} + \varphi_{H_{j_i}}$.

- 535 • The new energy conservation equation for pipe ij (due to a demand increase at node i) will
 536 be:

$$537 \quad (H_{i_0} + \varphi_{H_{i_i}}) - (H_{j_0} + \varphi_{H_{j_i}}) = r_{ij} (Q_{ij_0} + \varphi_{Q_{ij_i}}) |(Q_{ij_0} + \varphi_{Q_{ij_i}})|$$

- 538 • The new energy conservation equation for pipe mn (due to a demand increase at node i) will
 539 be:

$$540 \quad (H_{m_0} + \varphi_{H_{m_i}}) - (H_{n_0} + \varphi_{H_{n_i}}) = r_{mn} (Q_{mn_0} + \varphi_{Q_{mn_i}}) |(Q_{mn_0} + \varphi_{Q_{mn_i}})|$$

541 If the energy conservation equation is considered for any pipe ij , in each case a demand increase
 542 occurs in each one of the network nodes, the complete set of n equations for pipe ij will be:

Pipe	Demand increase in node	Energy conservation equation (for pipe ij)
ij	1	$(H_{i_0} + \varphi_{H_{i_1}}) - (H_{j_0} + \varphi_{H_{j_1}}) = r_{ij} (Q_{ij_0} + \varphi_{Q_{ij_1}}) (Q_{ij_0} + \varphi_{Q_{ij_1}}) $
	2	$(H_{i_0} + \varphi_{H_{i_2}}) - (H_{j_0} + \varphi_{H_{j_2}}) = r_{ij} (Q_{ij_0} + \varphi_{Q_{ij_2}}) (Q_{ij_0} + \varphi_{Q_{ij_2}}) $

	i	$(H_{i_0} + \varphi_{H_{i_i}}) - (H_{j_0} + \varphi_{H_{j_i}}) = r_{ij} (Q_{ij_0} + \varphi_{Q_{ij_i}}) (Q_{ij_0} + \varphi_{Q_{ij_i}}) $

	j	$(H_{i_0} + \varphi_{H_{i_j}}) - (H_{j_0} + \varphi_{H_{j_j}}) = r_{ij} (Q_{ij_0} + \varphi_{Q_{ij_j}}) (Q_{ij_0} + \varphi_{Q_{ij_j}}) $

	n	$(H_{i_0} + \varphi_{H_{i_n}}) - (H_{j_0} + \varphi_{H_{j_n}}) = r_{ij} (Q_{ij_0} + \varphi_{Q_{ij_n}}) (Q_{ij_0} + \varphi_{Q_{ij_n}}) $

(A11)

548 The equations of System A11 cannot be easily added, so the following operation is considered:

549

$$(Q + \varphi) |(Q + \varphi)| = \vartheta \left(Q |Q| + \frac{Q^2 \varphi}{|Q|} + \frac{Q \varphi^2}{|Q|} + \varphi |Q| \right) \quad (\text{A12})$$

550

551 where:

552 • $\vartheta = \frac{Q \varphi}{|Q| |\varphi|}$, if $|\varphi| > |Q|$

553

554 • $\vartheta = 1$, if $|\varphi| \leq |Q|$

555

556 and the new set of n energy conservation equations for pipe ij is:

557

Pipe	Demand increase in node	Energy conservation equation (for pipe ij)
ij	1	$(H_{i_0} + \varphi_{H_{i_1}}) - (H_{j_0} + \varphi_{H_{j_1}}) - \vartheta_{ij} r_{ij} \cdot \left(Q_{ij_0} Q_{ij_0} + \frac{Q_{ij_0}^2 \cdot \varphi_{Q_{ij_1}}}{ Q_{ij_0} } + \frac{Q_{ij_0} \cdot \varphi_{Q_{ij_1}}^2}{ Q_{ij_0} } + \varphi_{Q_{ij_1}} Q_{ij_0} \right) = 0$
	2	$(H_{i_0} + \varphi_{H_{i_2}}) - (H_{j_0} + \varphi_{H_{j_2}}) - \vartheta_{ij} r_{ij} \cdot \left(Q_{ij_0} Q_{ij_0} + \frac{Q_{ij_0}^2 \cdot \varphi_{Q_{ij_2}}}{ Q_{ij_0} } + \frac{Q_{ij_0} \cdot \varphi_{Q_{ij_2}}^2}{ Q_{ij_0} } + \varphi_{Q_{ij_2}} Q_{ij_0} \right) = 0$

	i	$(H_{i_0} + \varphi_{H_{i_i}}) - (H_{j_0} + \varphi_{H_{j_i}}) - \vartheta_{ij} r_{ij} \cdot \left(Q_{ij_0} Q_{ij_0} + \frac{Q_{ij_0}^2 \cdot \varphi_{Q_{ij_i}}}{ Q_{ij_0} } + \frac{Q_{ij_0} \cdot \varphi_{Q_{ij_i}}^2}{ Q_{ij_0} } + \varphi_{Q_{ij_i}} Q_{ij_0} \right) = 0$

	j	$(H_{i_0} + \varphi_{H_{i_j}}) - (H_{j_0} + \varphi_{H_{j_j}}) - \vartheta_{ij} r_{ij} \cdot \left(Q_{ij_0} Q_{ij_0} + \frac{Q_{ij_0}^2 \cdot \varphi_{Q_{ij_j}}}{ Q_{ij_0} } + \frac{Q_{ij_0} \cdot \varphi_{Q_{ij_j}}^2}{ Q_{ij_0} } + \varphi_{Q_{ij_j}} Q_{ij_0} \right) = 0$

	n	$(H_{i_0} + \varphi_{H_{i_n}}) - (H_{j_0} + \varphi_{H_{j_n}}) - \vartheta_{ij} r_{ij} \cdot \left(Q_{ij_0} Q_{ij_0} + \frac{Q_{ij_0}^2 \cdot \varphi_{Q_{ij_n}}}{ Q_{ij_0} } + \frac{Q_{ij_0} \cdot \varphi_{Q_{ij_n}}^2}{ Q_{ij_0} } + \varphi_{Q_{ij_n}} Q_{ij_0} \right) = 0$

558

559 Before adding the n Equations A13 for pipe ij , it must be noticed that $\frac{\sum_{m=1}^n \varphi_{Q_{ijm}}^2}{n}$ is the mean square

560 of the $\varphi_{Q_{ijm}}$ values. However, being $\varphi_{Q_{ij}}$ the variable to be finally solved, the simplification

561 $\frac{\sum_{m=1}^n \varphi_{Q_{ijm}}^2}{n} \approx \varphi_{Q_{ij}}^2$ is assumed. This is based on the fact that, compared to the other terms within

562 the parenthesis, $\frac{Q_{ij_0} \cdot \varphi_{Q_{ij_n}}^2}{|Q_{ij_0}|}$ is the least relevant because of the magnitude of either $\varphi_{Q_{ij}}^2$ (compared

563 to $\varphi_{Q_{ij}}$), or $\frac{Q_{ij_0}}{|Q_{ij_0}|}$ (compared to $|Q_{ij_0}|$). The final result of the addition of the n equations for pipe ij

564 is:

565

$$n H_{i_0} + \sum_{m=1}^n \varphi_{H_{i_m}} - n H_{j_0} - \sum_{m=1}^n \varphi_{H_{j_m}} - \vartheta_{ij} r_{ij} \cdot \left(n Q_{ij_0} |Q_{ij_0}| + \frac{Q_{ij_0}^2}{|Q_{ij_0}|} \sum_{m=1}^n \varphi_{Q_{ijm}} + \frac{Q_{ij_0}}{|Q_{ij_0}|} \sum_{m=1}^n \varphi_{Q_{ijm}}^2 + |Q_{ij_0}| \sum_{m=1}^n \varphi_{Q_{ijm}} \right) = 0 \quad (\text{A14})$$

566

567 Again, total average variations for each network element can be considered:

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- For pipe ik : $\varphi_{Q_{ik}} = \frac{\sum_{m=1}^n \varphi_{Q_{ikm}}}{n}$
- For node i : $\varphi_{H_i} = \frac{\sum_{m=1}^n \varphi_{H_{im}}}{n}$

As these two averages are considered in Equation A14, and the resulting equation is divided by n , the final energy conservation equation for each pipe ij will be:

$$(H_{i_0} + \varphi_{H_i}) - (H_{j_0} + \varphi_{H_j}) - \vartheta_{ij} r_{ij} \cdot \left(Q_{ij_0} |Q_{ij_0}| + \frac{Q_{ij_0}^2}{|Q_{ij_0}|} \varphi_{Q_{ij}} + \frac{Q_{ij_0}}{|Q_{ij_0}|} \varphi_{Q_{ij}^2} + |Q_{ij_0}| \varphi_{Q_{ij}} \right) = 0 \quad (A15)$$

$$H_{i_0} + \varphi_{H_i} - H_{j_0} - \varphi_{H_j} - \vartheta_{ij} r_{ij} \cdot \left(Q_{ij_0} |Q_{ij_0}| + \frac{Q_{ij_0}^2}{|Q_{ij_0}|} \varphi_{Q_{ij}} + \frac{Q_{ij_0}}{|Q_{ij_0}|} \varphi_{Q_{ij}^2} + |Q_{ij_0}| \varphi_{Q_{ij}} \right) = 0 \quad (A16)$$

$$\vartheta_{ij} r_{ij} \cdot \left(2 |Q_{ij_0}| + \frac{Q_{ij_0}}{|Q_{ij_0}|} \varphi_{Q_{ij}} \right) \varphi_{Q_{ij}} - \varphi_{H_i} + \varphi_{H_j} + \vartheta_{ij} r_{ij} Q_{ij_0} |Q_{ij_0}| - H_{i_0} + H_{j_0} = 0 \quad (A17)$$

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The second part of the sensitivity analysis is performed by posing the whole set of energy conservation equations for all the network pipes:

Pipe		Energy conservation equation (for pipe $r \equiv ij$)
Id	Nodes connected	
1
2
...
r	ij	$\vartheta_{ij} r_{ij} \cdot \left(2 Q_{ij_0} + \frac{Q_{ij_0}}{ Q_{ij_0} } \varphi_{Q_{ij}} \right) \varphi_{Q_{ij}} - \varphi_{H_i} + \varphi_{H_j} + \vartheta_{ij} r_{ij} Q_{ij_0} Q_{ij_0} - H_{i_0} + H_{j_0} = 0$
...
p

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584

System A18 is compounded by p equations, with $(n + p)$ unknowns – the head average sensitivity for each of the n nodes (φ_{H_i}), and the flowrate average sensitivity for each of the p pipes ($\varphi_{Q_{ij}}$). According to the matrix formulation used by Todini & Rossman (2013), this system can be expressed as:

$$[A_{11\varphi} \quad \vdots \quad A_{12}] \begin{bmatrix} \varphi_Q \\ \vdots \\ \varphi_H \end{bmatrix} + [A_{11\vartheta} \quad \vdots \quad A_{12}] \begin{bmatrix} Q \\ \vdots \\ H \end{bmatrix} = - [A_{10} H_0] - [A_{10} \varphi_{H_0}] \quad (A19)$$

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586
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589

where:

- $A_{11\varphi}$ is a $(p \times p)$ diagonal matrix. Each element refers to one network pipe (r) that connects two nodes (ij):

$$A_{11\varphi}(r, r) = \vartheta_{ij} r_{ij} \left(2 |Q_{ij_0}| + \frac{Q_{ij_0}}{|Q_{ij_0}|} \varphi_{Q_{ij}} \right)$$

590

591

592

- $A_{11\vartheta}$ is a $(p \times p)$ diagonal matrix. Each element refers to one network pipe (r) that connects two nodes (ij):

$$A_{11\vartheta}(r, r) = \vartheta_{ij} r_{ij} |Q_{ij_0}|$$

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- H is the vector of the node heads H_{i_0}
- φ_H is the vector of the node head average sensitivities φ_{H_i}
- A_{10} is a matrix that relates pipes to fixed head nodes.
- H_0 is the vector of the heads for the fixed head nodes.
- φ_{H_0} is the vector of the head average sensitivities for the fixed head nodes, whose elements are, in fact, zeros.
- $\varphi_Q, Q, q, \frac{\varphi_q}{n}$ and A_{21} are the vectors and matrix explained above.
- $A_{12} = A_{21}^T$

Equation A19 can be operated to:

$$[A_{11\varphi} \quad \vdots \quad A_{12}] \begin{bmatrix} \varphi_Q \\ \cdots \\ \varphi_H \end{bmatrix} = - [A_{11\vartheta} \quad \vdots \quad A_{12}] \begin{bmatrix} Q \\ \cdots \\ H \end{bmatrix} - [A_{10} H_0] \quad (\text{A20})$$

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611

The **complete system of conservation equations** can be obtained by merging System A9 and System A20. The new system consists of $(n + p)$ equations, with $(n + p)$ unknowns, and it can take a matrix formulation like:

$$\begin{bmatrix} A_{11\varphi} & \vdots & A_{12} \\ \cdots & \cdots & \cdots \\ A_{21} & \vdots & 0 \end{bmatrix} \begin{bmatrix} \varphi_Q \\ \cdots \\ \varphi_H \end{bmatrix} = - \begin{bmatrix} A_{11\vartheta} & \vdots & A_{12} \\ \cdots & \cdots & \cdots \\ A_{21} & \vdots & 0 \end{bmatrix} \begin{bmatrix} Q \\ \cdots \\ H \end{bmatrix} - \begin{bmatrix} A_{10} H_0 \\ \cdots \\ q + \frac{\varphi_q}{n} \end{bmatrix} \quad (\text{A21})$$

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Being φ_Q and φ_H the vectors of the unknown factors, System A21 can be solved through the Newton-Raphson method to finally obtain the sensitivities for each node and pipe in the network.

620 **Appendix B. Development of equations for leak location**

621 If there is a leak in the network whose location is unknown, then a (also unknown) possible leak
622 flowrate is considered in each node q_{i_L} . The flowrate in each network node will be the addition to
623 the usual demand q_i , plus the potential leak q_{i_L} .

624
625 Because of the disturbance that the leak introduces into the network operating conditions, each pipe
626 flowrate in the (new) leak scenario Q_{ij_L} will likely be different from the usual value in the
627 (reference) no-leak scenario Q_{ij_0} .

628
629 Therefore, the **mass conservation equation for each node i** in the leak scenario is:
630

$$\sum_{k=1}^{n_i} Q_{ik_L} + q_i + q_{i_L} = 0 \quad (\text{B1})$$

631
632 where n_i is the total number of nodes connected to node i , Q_{ik_L} is the new circulating flowrate
633 through the pipe that connects node i to node k , q_i is the demand flowrate at node i , and q_{i_L} is the
634 potential leak flowrate at node i .

635
636 By following the same procedure explained in Appendix A, the system of n mass conservation
637 equations (one equation for each node) can be expressed as:
638

$$[A_{21} \quad \vdots \quad 1] \begin{bmatrix} Q_L \\ \cdots \\ q_L \end{bmatrix} = -[A_{21} \quad Q_D] - [q] \quad (\text{B2})$$

639
640 where:

- 641 • Q_L is the vector of the pipe flowrates Q_{ij_L} that are unknown. In principle, all the pipe
642 flowrates are unknown, but if there are some flowmeters installed in some pipes, the data
643 they provide will be known pipe flowrates, then their corresponding values in Q_L will be set
644 to zero.
- 645 • q_L is the vector of the node potential leak flowrates q_{i_L}
- 646 • Q_D is the vector of the pipe flowrates Q_{ij_L} that are known. Known flow measures provided
647 by possible flowmeters in the network are included in their corresponding elements in vector
648 Q_D , being zero the rest of elements.
- 649 • A_{21} and q are explained in Appendix A.

650
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652
653 System B2 is compounded by n equations, being $(n + p)$ the highest possible number of unknowns.
654 The total number of unknowns would be reduced in the same quantity as flowmeters were installed
655 in the network.

656
657
658 The **energy conservation equation for each pipe**, which connects node i to node j , in the leak
659 scenario is:
660

$$H_{i_L} - H_{j_L} = r_{ij} Q_{ij_L} |Q_{ij_L}| \quad (\text{B3})$$

661 which can be expressed:

$$[A_{11} \quad \vdots \quad A_{12}] \begin{bmatrix} Q_L \\ \dots \\ H_L \end{bmatrix} = -[A_{10} H_0] \quad (\text{B4})$$

662
663 where H_L is the vector of the node heads H_{i_L} , Q_L is explained above, and A_{11} , A_{12} , A_{10} and H_0 are
664 explained in Appendix A

665
666 As explained in the case of the mass conservation equation, if some flowmeters are installed in the
667 network, not all the pipe flowrates will be unknown. Being $[Q_D]$ the vector of known pipe
668 flowrates, System B4 can be written as:

$$[A_{11} \quad \vdots \quad A_{12}] \begin{bmatrix} Q_L \\ \dots \\ H_L \end{bmatrix} = -[A_{11} Q_D] - [A_{10} H_0] \quad (\text{B5})$$

670
671
672 In principle, node heads in the leak scenario H_{i_L} are unknown, but:

- 673 • If there are pressure sensors installed in some nodes, the data they provide are known values
674 for some node pressures.
- 675 • For the rest of the nodes, not being monitored by a pressure sensor, their head could be
676 approximated by taking into account the average sensitivity (as calculated in Appendix A):
677 $H_{i_L} = H_{i_0} + \varphi_{H_i}$
678

680
681
682 Therefore, the node heads in the leak scenario can be considered as known (or approximated),
683 vector H_L is renamed as H_D , and it can be shifted to the right hand side of the equation:

$$[A_{11} Q_L] = -[A_{11} Q_D] - [A_{12} H_D] - [A_{10} H_0] \quad (\text{B6})$$

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688 The **complete system of conservation equations** for the leak scenario can be obtained by merging
689 System B2 and System B6:

$$\begin{bmatrix} A_{11} & \vdots & 0 \\ \dots & \dots & \dots \\ A_{21} & \vdots & 1 \end{bmatrix} \begin{bmatrix} Q_L \\ \dots \\ q_L \end{bmatrix} = - \begin{bmatrix} A_{11} & \vdots & A_{12} \\ \dots & \dots & \dots \\ A_{21} & \vdots & 0 \end{bmatrix} \begin{bmatrix} Q_D \\ \dots \\ H_D \end{bmatrix} - \begin{bmatrix} A_{10} H_0 \\ \dots \\ q \end{bmatrix} \quad (\text{B7})$$

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693
694 The new System B7 is compounded by $(n + p)$ equations, and the maximum number of unknowns
695 would be $(n + p)$ – for the case in which no flowmeters are installed in the network. If some
696 flowmeters are installed, the total number of unknowns would be reduced by the same quantity as
697 flowmeters installed.

698
699 Being Q_L and q_L the vectors of the unknown factors, System B7 can be solved using the Newton-
700 Raphson method to obtain the potential leak flowrates q_L for each node in the network.

701 **Appendix C. Full information about the network of the case study**

702 For reasons of confidentiality, the study case uses a synthetic rather than real network.
 703 Nevertheless, the network is a direct adaptation of a network in the region of Valencia (Spain) that
 704 is managed by the first author of this paper.

705
 706
 707 Table C1. Physical characteristics and normal operation values for pipe variables
 708

Pipe ID	ø (mm)	Length (m)	Velocity (m/s)	Unit. Headloss (m/km)
1	200	977.3	1.22	7.13
2	150	1629.5	0.31	0.79
3	100	559.4	0.73	6.41
4	100	1021.6	0.14	0.33
5	100	1624.3	0.46	2.74
6	50	1862.8	0.22	1.69
7	200	428.1	1.20	6.92
8	200	1838.5	0.71	2.56
9	200	986.9	0.62	1.96
10	150	987.9	0.70	3.53
11	100	481.3	0.92	9.95
12	100	510.8	0.57	3.95
13	100	432.7	0.15	0.35
14	150	461.7	0.50	1.90
15	90	467.7	0.58	4.75
16	90	1830.3	0.11	0.23
17	90	447.1	0.36	1.97
18	90	464.2	0.83	9.32
19	200	471.8	0.45	1.11
20	200	482.7	0.50	1.31
21	200	520.3	0.60	1.85
22	100	486.6	0.57	4.05
23	100	496.0	0.49	2.97
24	90	461.7	0.49	3.49
25	90	473.4	0.34	1.72
26	90	471.8	0.17	0.49
27	50	1625.4	0.03	0.06
28	90	568.8	0.57	4.56
29	90	469.1	0.21	0.72
30	90	498.5	0.28	1.21
31	90	426.1	0.51	3.72
32	50	471.4	0.52	8.04
33	50	1634.9	0.16	0.95
34	50	775.2	0.43	5.78
35	50	1648.6	0.16	1.00
36	200	496.0	0.83	3.42
37	200	476.3	0.76	2.91
38	200	482.7	0.72	2.61
39	90	437.4	0.70	6.73
40	90	512.6	0.15	0.39
41	150	1064.9	0.15	0.20
42	150	497.6	0.10	0.11
43	50	485.2	0.01	0.01
44	50	1652.9	0.04	0.08
45	200	1012.2	0.54	1.55
46	50	699.9	0.04	0.09
47	50	797.1	0.08	0.28
48	90	501.4	0.59	4.94
49	90	456.9	0.15	0.39
50	90	496.0	0.22	0.77
51	200	593.2	0.29	0.48
52	50	490.9	0.50	7.61
53	150	1652.9	0.12	0.15
54	200	1608.8	0.15	0.15
55	90	1606.6	0.10	0.18
56	50	1860.0	0.09	0.36
57	50	1843.8	0.17	1.11
58	150	1841.2	0.34	0.91

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Table C2. Node sensitivity analysis

Node id	Initial pressure (mwc)	Pressure variation (mwc)					
		Increase of 15 lpm		Increase of 30 lpm		Increase of 60 lpm	
1	41.75	-0.07	-0.17%	-0.13	-0.31%	-0.26	-0.62%
2	39.78	-0.09	-0.23%	-0.17	-0.43%	-0.34	-0.85%
3	38.30	-0.10	-0.26%	-0.20	-0.52%	-0.40	-1.04%
4	36.69	-0.11	-0.30%	-0.23	-0.63%	-0.45	-1.23%
5	35.88	-0.12	-0.33%	-0.24	-0.67%	-0.47	-1.31%
6	35.65	-0.12	-0.34%	-0.24	-0.67%	-0.48	-1.35%
7	35.55	-0.11	-0.31%	-0.21	-0.59%	-0.42	-1.18%
8	35.75	-0.10	-0.28%	-0.20	-0.56%	-0.41	-1.15%
9	38.69	-0.08	-0.21%	-0.16	-0.41%	-0.32	-0.83%
10	39.95	-0.07	-0.18%	-0.14	-0.35%	-0.29	-0.73%
11	41.33	-0.06	-0.15%	-0.13	-0.31%	-0.25	-0.60%
12	43.03	-0.05	-0.12%	-0.10	-0.23%	-0.21	-0.49%
13	39.44	-0.07	-0.18%	-0.15	-0.38%	-0.30	-0.76%
14	39.11	-0.08	-0.20%	-0.15	-0.38%	-0.31	-0.79%
15	43.55	-0.04	-0.09%	-0.09	-0.21%	-0.17	-0.39%
16	40.40	-0.06	-0.15%	-0.12	-0.30%	-0.25	-0.62%
17	36.85	-0.09	-0.24%	-0.17	-0.46%	-0.35	-0.95%
18	36.63	-0.09	-0.25%	-0.18	-0.49%	-0.35	-0.96%
19	38.76	-0.07	-0.18%	-0.15	-0.39%	-0.29	-0.75%

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Node id	Initial pressure (mwc)	Pressure variation (mwc)					
		Increase of 15 lpm		Increase of 30 lpm		Increase of 60 lpm	
20	39.43	-0.07	-0.18%	-0.14	-0.36%	-0.26	-0.66%
21	38.80	-0.09	-0.23%	-0.15	-0.39%	-0.34	-0.88%
22	36.75	-0.10	-0.27%	-0.17	-0.46%	-0.40	-1.09%
23	36.45	-0.11	-0.30%	-0.18	-0.49%	-0.45	-1.23%
24	36.52	-0.12	-0.33%	-0.18	-0.49%	-0.47	-1.29%
25	37.12	-0.12	-0.32%	-0.17	-0.46%	-0.48	-1.29%
26	35.53	-0.11	-0.31%	-0.20	-0.56%	-0.42	-1.18%
27	31.74	-0.10	-0.32%	-0.31	-0.98%	-0.41	-1.29%
28	36.83	-0.08	-0.22%	-0.18	-0.49%	-0.32	-0.87%
29	35.48	-0.07	-0.20%	-0.20	-0.56%	-0.29	-0.82%
30	31.74	-0.06	-0.19%	-0.31	-0.98%	-0.25	-0.79%
31	33.50	-0.05	-0.15%	-0.21	-0.63%	-0.21	-0.63%
32	35.72	-0.07	-0.20%	-0.19	-0.53%	-0.30	-0.84%
33	36.60	-0.08	-0.22%	-0.18	-0.49%	-0.31	-0.85%
34	38.28	-0.04	-0.10%	-0.15	-0.39%	-0.17	-0.44%
35	33.96	-0.06	-0.18%	-0.20	-0.59%	-0.25	-0.74%
36	33.08	-0.09	-0.27%	-0.21	-0.63%	-0.35	-1.06%
37	42.33	-0.09	-0.21%	-0.10	-0.24%	-0.35	-0.83%
38	47.04	-0.07	-0.15%	-0.04	-0.09%	-0.29	-0.62%

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Table C3. Pipe sensitivity analysis

Line id	Initial flow (L/minute)	Flow variation					
		Increase of 15 (L/minute)		Increase of 30 (L/minute)		Increase of 60 (L/minute)	
1	2298.28	8.01	0.35%	16.02	0.70%	32.04	1.39%
2	330.18	1.64	0.50%	3.27	0.99%	6.53	1.98%
3	345.39	1.05	0.30%	2.1	0.61%	4.2	1.22%
4	68.12	0.15	0.22%	0.31	0.46%	0.61	0.90%
5	218.79	0.77	0.35%	1.53	0.70%	3.06	1.40%
6	25.85	0.08	0.31%	0.15	0.58%	0.31	1.20%
7	2261.72	6.99	0.31%	13.98	0.62%	27.96	1.24%
8	1341.27	4.11	0.31%	8.22	0.61%	16.44	1.23%
9	1161.27	3.71	0.32%	7.43	0.64%	14.86	1.28%
10	740.45	2.49	0.34%	4.97	0.67%	9.94	1.34%
11	435.81	1.25	0.29%	2.5	0.57%	4.99	1.14%
12	266.51	0.89	0.33%	1.78	0.67%	3.56	1.34%
13	70.17	0.44	0.63%	0.87	1.24%	1.75	2.49%
14	531.57	1.91	0.36%	3.82	0.72%	7.65	1.44%
15	221.97	0.4	0.18%	0.8	0.36%	1.59	0.72%
16	41.97	0	0.00%	0.01	0.02%	0.01	0.02%
17	138.03	0.39	0.28%	0.78	0.57%	1.57	1.14%
18	318.03	0.79	0.25%	1.57	0.49%	3.15	0.99%
19	855.99	2.5	0.29%	5	0.58%	10.01	1.17%
20	936.42	2.97	0.32%	5.93	0.63%	11.87	1.27%
21	1127.12	3.4	0.30%	6.79	0.60%	13.59	1.21%
22	270.18	1.24	0.46%	2.48	0.92%	4.95	1.83%
23	228.87	0.97	0.42%	1.94	0.85%	3.87	1.69%
24	188.08	0.73	0.39%	1.45	0.77%	2.87	1.53%
25	128.08	0.33	0.26%	0.66	0.52%	1.29	1.01%
26	63.74	0.18	0.28%	0.36	0.56%	0.72	1.13%
27	3.74	-0.21	-5.61%	-0.43	-11.5%	-0.86	-23.0%
28	217.27	0.5	0.23%	1	0.46%	2	0.92%
29	78.98	0.22	0.28%	0.43	0.54%	0.87	1.10%

Line id	Initial flow (L/minute)	Flow variation					
		Increase of 15 (L/minute)		Increase of 30 (L/minute)		Increase of 60 (L/minute)	
30	105.77	0.18	0.17%	0.36	0.34%	0.72	0.68%
31	194.82	0.77	0.40%	1.55	0.80%	3.1	1.59%
32	60.88	0.4	0.66%	0.79	1.30%	1.59	2.61%
33	18.69	0.13	0.70%	0.25	1.34%	0.5	2.68%
34	50.92	0.13	0.26%	0.27	0.53%	0.53	1.04%
35	19.21	0.15	0.78%	0.29	1.51%	0.58	3.02%
36	1562.71	4.93	0.32%	9.86	0.63%	19.74	1.26%
37	1433.10	4.28	0.30%	8.56	0.60%	17.13	1.20%
38	1353.88	3.74	0.28%	7.48	0.55%	14.97	1.11%
39	267.49	0.94	0.35%	1.87	0.70%	3.76	1.41%
40	56.26	0.61	1.08%	1.22	2.17%	2.44	4.34%
41	155.58	-0.31	-0.20%	-0.63	-0.40%	-1.27	-0.82%
42	109.52	-0.33	-0.30%	-0.66	-0.60%	-1.33	-1.21%
43	0.88	0	0.00%	0	0.00%	0.01	1.14%
44	4.34	-0.24	-5.53%	-0.49	-11.3%	-1.01	-23.3%
45	1026.39	2.4	0.23%	4.81	0.47%	9.63	0.94%
46	4.75	0	0.00%	0.01	0.21%	0.01	0.21%
47	9.21	0.02	0.22%	0.04	0.43%	0.09	0.98%
48	226.91	0.52	0.23%	1.05	0.46%	2.1	0.93%
49	56.12	0.15	0.27%	0.3	0.53%	0.6	1.07%
50	82.35	0.11	0.13%	0.23	0.28%	0.45	0.55%
51	545.79	1.06	0.19%	2.11	0.39%	4.23	0.78%
52	59.12	0.39	0.66%	0.79	1.34%	1.57	2.66%
53	129.60	1.12	0.86%	2.24	1.73%	4.48	3.46%
54	283.45	0.55	0.19%	1.1	0.39%	2.2	0.78%
55	36.78	0.13	0.35%	0.25	0.68%	0.51	1.39%
56	10.70	0.03	0.28%	0.07	0.65%	0.14	1.31%
57	20.43	0.07	0.34%	0.14	0.69%	0.28	1.37%
58	357.96	1.32	0.37%	2.64	0.74%	5.28	1.48%

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722 **Appendix D. Leak location solutions for various installations of pressure sensors and**
723 **flowmeters**

724 Table D1. Summary table for the various sensor configurations tested in the case study
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	Config. 1	Config. 2	Config. 3	Config. 4	Config. 5	Config. 6	Config. 7
# Pressure sensors	5	5	5	10	10	15	15
# Flowmeters	5	5	5	0	5	0	5
Total cases	38	38	38	38	38	38	38
Detection of node	8	8	9	10	10	15	16
Detection of area	16	8	10	10	13	15	17
No detection	14	22	19	18	15	8	5
Successful cases	24	16	19	20	23	30	33
Success rate	63%	42%	50%	53%	61%	79%	87%

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727

728 This table shows both the number and type of sensors. Configurations 1 to 3 show the influence of
729 the position of the sensors according to the criteria of node sensitivity and spatial distribution. The
730 results show that the first criterion has a greater weight on the success rate than the second.
731 Configurations 4 to 7 explore the influence of the number of sensors used. It is clear that with a
732 maximum of 20 sensors (out of a total of 96 elements) the success rate nearly reaches 90%. The
733 detailed results for each configuration are shown in the following tables.

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Table D2. Leak flows obtained with 5 pressure sensors and 5 flowmeters installed by combining criteria on highest sensitivity and spatial distribution (nodes 5, 18, 27, 30, 37 and pipes 1, 9, 20, 36, 45)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
3	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
4	6.1	-3	-9	-22	-32	-28	-5	-1	7.5	8.6	10.0	11.8	10.6	10.9	16.3	13.8	8.7	8.6	11.4	12.8	12.0	8.1	8.1	8.1	7.1	1.7	2.0	7.2	2.3	2.0	6.6	5.0	7.2	11.3	9.1	8.2	15.7	19.9	
5	-14.2	12.6	38.2	125.4	167.6	153.8	22.3	6.7	-18.0	-20.8	-24.5	-29.3	-26.3	-27.0	-40.5	-34.2	-21.2	-21.1	-28.2	-31.7	-29.8	-19.5	-19.6	-19.5	-16.8	-1.4	-2.2	-17.1	-3.3	-2.4	-15.6	-10.9	-17.1	-28.1	-22.3	-19.8	-39.0	-49.0	
6	9.9	-5.8	-20.8	-91.3	-121	-112	-11.6	-2.2	12.0	13.6	15.6	18.2	16.6	17.0	24.2	20.9	13.8	13.7	17.7	19.5	18.5	12.9	12.9	12.9	11.3	2.5	3.0	11.5	3.7	3.1	10.7	8.0	11.5	17.6	14.4	13.0	23.4	28.5	
7	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	
8	1.9	-0.5	-4.5	-8.5	-10.6	-9.9	-1.6	0.1	2.1	2.4	2.6	3.0	2.7	2.8	3.7	3.3	2.4	2.4	2.9	3.1	3.0	2.3	2.3	2.3	2.1	0.8	0.9	2.1	1.0	0.9	2.0	1.6	2.1	2.9	2.5	2.3	3.6	4.2	
9	18.0	16.0	13.4	9.2	6.6	5.9	1.9	1.2	31.9	27.6	23.3	18.9	4.2	1.4	4.3	0.1	-5.1	-6.6	-3.2	-1.2	-2.3	-8.3	-8.8	-9.0	-15.0	-2.9	-3.2	-12.6	-3.6	-3.2	-5.7	-6.6	-9.6	-3.2	-4.3	-4.9	2.7	8.3	
10	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
11	-24.2	-14.3	-8.2	-3.4	-0.4	0.3	4.8	5.6	13.3	17.7	22.6	-29.5	-11.9	-8.8	-12.3	-7.3	-0.1	0.0	-3.6	-5.7	-4.4	2.3	2.2	2.7	7.2	5.3	5.2	5.7	5.1	5.2	2.3	4.3	3.9	-3.3	-0.7	0.4	-10.4	-16.9	
12	35.8	23.7	15.6	8.5	4.1	2.9	-3.6	-4.9	-8.9	-10.7	-13.0	42.1	14.3	7.2	-7.2	-8.6	1.0	-1.0	-6.5	-8.2	-7.9	-5.7	-3.3	-2.3	-6.5	-5.3	-5.4	-6.1	-5.4	-5.4	-6.3	-5.8	-6.1	-7.7	-7.1	-6.8	-9.4	-11.0	
13	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
14	4.6	3.9	3.3	2.6	2.2	2.1	1.5	1.4	2.1	2.9	3.8	4.9	-0.2	-1.7	5.1	4.6	-2.2	-8.7	2.3	4.0	3.6	0.3	-4.0	-1.6	0.8	1.2	1.2	0.5	1.2	1.2	1.6	1.1	0.8	3.3	2.5	2.2	5.7	8.0	
15	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
16	-7.4	-6.0	-4.6	-3.0	-2.0	-1.8	-0.3	0.0	-2.7	-4.4	-6.1	-8.1	-5.3	-5.2	-11.5	20.5	-2.3	-1.6	-2.5	17.6	15.6	1.0	-0.6	-1.3	-0.3	0.5	0.6	0.5	0.7	0.6	4.9	1.7	2.0	13.8	9.7	7.9	-31.5	-18.3	
17	2.8	2.4	2.1	1.7	1.5	1.5	1.1	1.1	1.4	1.9	2.4	3.0	0.3	-0.4	3.1	2.8	-0.6	-3.3	1.6	2.5	2.3	0.6	-1.4	-0.3	0.8	1.0	1.0	0.6	1.0	1.0	1.2	0.9	0.8	2.1	1.7	1.5	3.5	5.0	
18	-16.6	-13.6	-10.7	-7.5	-5.5	-5.0	-1.9	-1.3	-4.7	-8.9	-13.1	-18.1	6.8	15.5	-19.3	-16.8	18.2	87.7	-6.1	-14.1	-12.2	4.2	29.9	14.7	1.8	-0.5	-0.4	3.3	-0.3	-0.4	-2.6	0.4	1.6	-10.5	-6.8	-5.2	-21.8	-32.2	
19	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
20	6.1	4.6	3.2	1.6	0.6	0.3	-1.2	-1.4	1.3	3.0	4.8	6.8	4.4	4.6	12.9	32.8	1.2	0.6	6.2	35.6	-20.0	-1.6	-0.4	0.1	-1.0	-2.0	-2.1	-1.9	-2.2	-2.1	-7.1	-3.2	-3.5	-17.8	-12.8	-10.6	27.7	16.8	
21	-4.4	-2.9	-1.5	0.1	1.1	1.3	2.8	3.1	0.3	-1.4	-3.1	-5.1	-2.8	-3.0	-11.2	-31.1	0.4	1.0	-4.6	-33.9	21.6	3.2	2.1	1.5	2.7	3.6	3.7	3.6	3.9	3.8	8.7	4.8	5.1	19.5	14.5	12.3	-26.0	-15.1	
22	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
23	12.9	10.9	9.0	6.8	5.4	5.1	2.9	2.5	4.9	7.8	10.6	13.9	-3.2	-9.7	14.7	13.1	-11.7	-72.1	5.8	11.3	10.0	-1.4	-20.8	-9.1	0.3	1.9	1.9	-0.7	1.8	1.9	3.4	1.3	0.5	8.8	6.4	5.3	16.3	22.9	
24	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
25	-16.5	-14.5	-11.9	-7.7	-5.0	-4.3	-0.4	0.4	-30.4	-26.1	-21.8	-17.4	-2.6	0.2	-2.7	1.4	6.6	8.2	4.7	2.8	3.8	9.9	10.3	10.6	16.6	4.4	4.7	14.2	5.2	4.8	7.2	8.2	11.1	4.7	5.9	6.4	-1.2	-6.7	
26	2.6	2.3	2.1	1.7	1.5	1.5	1.2	1.1	2.1	2.3	2.5	2.7	2.4	2.4	2.9	2.6	2.1	2.1	2.3	2.5	2.4	1.9	2.0	2.0	1.9	1.0	-14.9	1.9	1.0	-12.9	1.7	1.4	1.8	2.3	2.0	1.9	2.8	3.3	
27	-0.8	-0.5	-0.2	0.1	0.3	0.4	0.7	0.8	-0.2	-0.4	-0.6	-0.9	-0.6	-0.6	-1.1	-0.8	-0.3	-0.2	-0.5	-0.7	-0.6	-0.1	-0.2	-0.2	-0.1	0.9	30.7	0.0	0.8	0.7	0.1	0.4	0.0	-0.5	-0.2	-0.1	-1.0	-1.5	
28	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
29	2.7	2.4	2.2	1.8	1.6	1.6	1.3	1.2	2.2	2.4	2.6	2.8	2.5	2.5	3.0	2.7	2.2	2.2	2.4	2.6	2.5	2.0	2.1	2.0	1.1	-12.8	2.0	1.1	-14.7	1.8	1.5	1.9	2.4	2.1	2.0	2.9	3.4		
30	-0.7	-0.5	-0.2	0.1	0.3	0.3	0.6	0.7	-0.2	-0.4	-0.6	-0.9	-0.6	-0.6	-1.1	-0.7	-0.3	-0.2	-0.5	-0.6	-0.5	-0.1	-0.1	-0.2	0.0	0.8	0.7	0.0	0.9	30.7	0.2	0.4	0.0	-0.5	-0.1	0.0	-1.0	-1.4	
31	8.0	6.8	5.7	4.5	3.7	3.5	2.3	2.1	4.8	5.9	7.1	8.6	6.3	6.1	8.7	3.9	4.5	4.1	4.9	2.7	1.8	2.8	3.5	3.9	3.3	1.7	1.5	2.9	1.4	1.5	-14.2	0.4	2.1	1.0	-18.3	-25.6	6.1	11.3	
32	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
33	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
34	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
35	11.2	9.5	8.0	6.1	5.0	4.7	3.0	2.7	6.6	8.2	10.0	12.0	8.8	8.4	12.2	5.3	6.2	5.6	6.8	3.6	2.2	3.7	4.8	5.3	4.5	2.0	1.8	3.8	1.6	1.8	-23.4	0.1	2.7	1.0	-30.3	-43.1	8.5	15.7	
36	-16.5	-13.7	-11.0	-7.9	-6.0	-5.5	-2.6	-2.1	-8.7	-11.5	-14.5	-18.0	-12.5	-11.8	-18.2	-6.5	-8.1	-7.0	-9.0	-3.6	-1.3	-3.8	-5.7	-6.6	-5.1	-1.0	-0.7	-4.1	-0.3	-0.6	40.3	2.2	-2.1	0.7	51.3	71.3	-12.0	-24.4	
37	9.1	7.7	6.3	4.7	3.7	3.5	2.0	1.7	4.4	6.1	7.8	9.8	7.0	6.9	13.3	-18.8	4.0	3.3	4.3	-15.9	-13.8	0.7	2.3	3.0	2.0	1.2	1.1	1.2	1.0	1.1	-3.2	0							

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Table D3. Leak flows obtained with 5 pressure sensors and 5 flowmeters, installed at the least sensitive elements of the network (nodes 1, 12, 15, 16, 38 and pipes 6, 16, 43, 46, 56)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38			
1	37,7	48,0	18,8	11,8	7,4	6,3	-0,2	-1,4	-2,8	-2,5	-2,6	-3,1	-4,2	-4,8	-8,7	-7,6	-3,8	-4,0	-6,1	-7,0	-6,6	-4,2	-3,9	-3,7	-3,3	-2,2	-2,3	-3,5	-2,4	-2,3	-4,0	-3,2	-3,8	-6,2	-5,2	-4,7	-8,6	-10,8			
2	-13,7	-17,8	-6,2	-3,4	-1,7	-1,2	1,3	1,8	1,7	1,3	0,9	0,6	2,5	3,3	7,1	6,2	2,6	2,9	4,9	5,8	5,4	3,3	2,9	2,6	2,4	2,2	2,2	2,7	2,3	2,2	3,4	2,7	3,0	5,1	4,3	3,9	7,1	9,0			
3	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9		
4	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	
5	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	
6	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1
7	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9
8	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
9	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8
10	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8
11	-14,5	-17,5	-8,5	-5,1	-3,0	-2,5	0,7	1,3	-4,0	-7,7	-11,6	-16,1	-1,2	4,5	30,0	25,0	1,0	3,7	16,7	22,1	20,0	7,2	3,9	1,7	1,3	2,3	2,5	3,0	2,7	2,5	8,1	4,4	5,5	18,1	13,4	11,4	30,4	41,9			
12	0,5	-1,4	3,1	2,4	2,0	1,9	1,2	1,1	10,2	16	22	29,2	6,7	-1,8	-39,0	-31,9	3,0	-1,0	-20,0	-27,8	-24,7	-6,3	-1,3	1,9	2,3	0,0	-0,3	-0,2	-0,6	-0,3	-7,8	-2,7	-3,8	-22,0	-15,4	-12,5	-39,7	-56,3			
13	-0,8	-1,1	-0,1	0,2	0,4	0,5	0,8	0,9	0,3	-0,1	-0,5	-0,9	0,6	1,2	3,9	3,4	0,9	1,1	2,5	3,1	2,9	1,5	1,2	0,9	0,9	1,0	1,0	1,1	1,0	1,0	1,6	1,2	1,3	2,7	2,2	2,0	4,0	5,2			
14	2,1	2,2	1,8	1,6	1,5	1,5	1,3	1,3	1,6	1,7	1,9	2,2	0,9	0,3	-3,3	1,3	1,1	0,9	-0,7	1,2	1,2	0,8	1,0	1,1	1,3	1,3	1,3	1,2	1,3	1,3	1,2	1,2	1,1	1,2	1,2	1,2	1,4	1,7			
15	-4,8	-5,5	-3,3	-2,5	-2,0	-1,8	-1,1	-0,9	-2,0	-3,0	-4,0	-5,2	2,2	5,9	29,1	-3,5	1,0	2,1	11,8	-2,7	-2,4	2,1	1,3	0,5	-0,6	-0,7	-0,7	-0,2	-0,6	-0,7	-0,8	-0,3	0,1	-2,1	-1,5	-1,2	-4,9	-7,9			
16	-62,4	-71,0	-44,1	-33,9	-27,5	-25,9	-16,2	-14,4	-31,5	-42,5	-53,9	-67,0	-41,7	-36,5	-45,4	106	-25,5	-19,8	-13,8	89	76,7	-4,6	-15,0	-20,2	-15,9	-11,1	-10,5	-10,5	-9,7	-10,4	14,8	-3,9	-1,4	66,5	42,3	31,8	50,1	-66,0			
17	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	
18	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	
19	2,9	3,2	2,4	2,1	2,0	1,9	1,6	1,6	2,0	2,4	2,7	3,1	1,1	0,2	-5,8	-0,7	1,3	1,1	2,2	-1,2	-0,8	1,4	1,3	1,4	1,6	1,5	1,5	1,4	1,5	1,5	0,8	1,3	1,3	-0,5	0,1	0,4	0,2	2,0			
20	42,1	47,8	30,3	23,6	19,4	18,3	12,0	10,8	22,0	29,2	36,6	45,2	28,3	24,8	29,5	-67,0	17,8	14,0	6,0	-55,2	-47,5	3,5	10,8	14,3	11,7	8,6	8,2	8,2	7,7	8,1	-7,9	3,9	2,2	-40,9	-25,4	-18,8	-30,2	44,7			
21	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	
22	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8
23	0,6	0,7	0,8	0,9	1,0	1,0	1,1	1,1	0,5	0,5	0,5	0,5	-0,5	1,5	1,5	1,5	-12,0	5,2	2,2	1,5	1,6	2,8	7,0	-12,6	0,5	1,2	1,2	1,5	1,3	1,2	1,5	1,5	1,8	1,6	1,6	1,6	1,6	1,4	1,1		
24	1,4	1,3	1,2	1,1	1,0	1,0	0,8	0,8	1,4	1,4	1,5	1,4	2,5	0,5	0,5	0,5	14,0	-3,3	-0,3	0,4	0,4	-0,9	-5,0	14,5	1,4	0,7	0,7	0,5	0,7	0,7	0,4	0,4	0,2	0,4	0,4	0,4	0,4	0,4	0,6	0,8	
25	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8
26	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9
27	0,9	0,9	1,0	1,0	1,0	1,0	1,0	1,0	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	1,0	21,6	0,9	0,8	-20,5	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	
28	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	
29	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	
30	0,9	0,9	0,9	0,9	0,8	0,8	0,8	0,8	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,8	-19,7	0,9	1,0	22,3	1,0	1,0	0,9	0,9	0,9	0,9	0,9	0,9	0,9	
31	1,8	2,0	2,3	2,6	2,8	2,9	3,2	3,3	2,2	2,0	1,8	1,6	1,8	1,7	0,9	-1,0	2,1	2,1	1,4	-1,2	-1,5	2,1	2,2	2,2	2,3	3,4	3,5	2,3	3,6	3,5	18,1	4,1	2,3	-1,7	-18,5	-25,5	-0,5	0,6			
32	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	
33	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	
34	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9
35	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8
36	0,1	-0,2	-0,5	-0,8	-1,0	-1,1	-1,4	-1,4	-0,4	-0,2	0,0	0,2	0,0	0,1	0,9	2,8	-0,2	-0,2	0,4	3,1	3,3	-0,3	-0,3	-0,3	-0,5	-1,6	-1,7	-0,5	-1,7	-1,7	-16,2	-2,2	-0,5	3,5	20,3	27,3	2,3	1,2			

Table D7. Leak flows obtained with 15 pressure sensors (nodes 2, 5, 8, 10, 13, 18, 19, 20, 24, 27, 28, 30, 32, 34, 36)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
1	-6.3	-24.8	-17.4	-10.6	-6.3	-5.2	1.3	2.4	3.8	3.6	3.6	4.1	5.2	5.8	10.0	8.7	4.8	5.1	7.2	8.2	7.7	5.3	5.0	4.7	4.4	3.2	3.3	4.6	3.4	3.3	5.1	4.2	4.9	7.3	6.3	5.8	9.8	12.2
2	17.6	61.7	43.6	27.4	17.5	14.9	0.4	-2.2	-5.2	-4.8	-4.9	-5.9	-8.3	-9.7	-18.7	-16.0	-7.5	-8.1	-12.7	-14.8	-13.9	-8.6	-7.8	-7.2	-6.5	-3.9	-4.1	-7.0	-4.4	-4.2	-8.0	-6.2	-7.6	-13.0	-10.7	-9.7	-18.4	-23.5
3	-7.6	-31.7	-21.6	-12.9	-7.7	-6.4	1.1	2.5	3.8	3.5	3.8	4.5	5.7	6.5	11.3	9.9	5.2	5.5	8.1	9.2	8.7	5.8	5.4	5.1	4.6	3.3	3.5	4.9	3.6	3.5	5.5	4.5	5.3	8.2	7.0	6.4	11.2	13.9
4	6.1	-2.7	-9.2	-22.0	-32.1	-28.4	-5.4	-0.9	7.5	8.6	10.0	11.8	10.6	10.9	16.3	13.8	8.7	8.6	11.4	12.8	12.0	8.1	8.1	8.1	7.1	1.7	2.0	7.2	2.3	2.0	6.6	5.0	7.2	11.3	9.1	8.2	15.7	19.9
5	-12.5	14.3	38.4	125	167	153	19.1	4.1	-17.5	-19.9	-23.2	-27.5	-25.1	-25.7	-38.3	-32.6	-20.5	-20.4	-27.1	-30.3	-28.5	-19.1	-19.1	-19.0	-16.5	-3.0	-3.8	-16.9	-4.7	-3.9	-15.7	-11.5	-16.9	-27.0	-21.7	-19.4	-37.0	-46.2
6	9.9	-5.8	-20.8	-91.3	-121	-112	-11.6	-2.2	12.0	13.6	15.6	18.2	16.6	17.0	24.2	20.9	13.8	13.7	17.7	19.5	18.5	12.9	12.9	12.9	11.3	2.5	3.0	11.5	3.7	3.1	10.7	8.0	11.5	17.6	14.4	13.0	23.4	28.5
7	12.6	7.5	1.8	-5.9	-11.3	-12.8	-22.3	-24.3	4.8	8.1	11.4	15.1	11.0	10.8	18.6	14.2	6.5	5.9	10.3	12.5	11.1	4.0	4.6	4.9	2.6	-14.0	-13.0	2.5	-11.8	-12.9	0.0	-3.9	2.0	10.0	5.5	3.5	17.3	23.7
8	-46.5	-28.2	-8.5	18.9	37.9	44.6	91.1	99.7	-13.9	-27.0	-40.3	-55.2	-38.6	-37.9	-68.8	-51.1	-20.5	-18.3	-35.5	-44.2	-38.9	-10.5	-13.0	-14.1	-4.9	60.3	56.5	-4.7	51.8	56.0	5.3	20.9	-2.7	-34.2	-16.4	-8.4	-63.8	-89.7
9	14.5	6.9	1.0	-3.3	-6.1	-6.8	-10.8	-11.5	-19.5	-28.2	-6.0	18.5	15.4	18.5	54.4	41.4	4.2	6.0	26.4	35.7	31.3	6.5	3.2	0.6	-6.5	-7.8	-7.2	-2.8	-6.5	-7.1	6.3	-1.5	2.0	27.5	17.6	13.3	52.2	75.2
10	-18.5	-7.0	1.0	5.0	7.5	8.2	11.9	12.5	43.0	62.0	21.0	-24.5	-21.5	-27.7	-94.2	-71.1	-2.3	-6.3	-44.0	-60.9	-53.1	-8.5	-1.6	3.7	16.3	9.6	9.0	8.9	8.2	8.9	-10.5	2.5	-0.8	-46.3	-29.7	-22.4	-90.7	-132
11	9.5	2.2	-1.0	-2.3	-3.2	-3.4	-4.6	-4.8	-19.4	-28.5	-8.8	13.5	12.1	15.2	48.6	37.0	2.7	4.6	23.3	31.8	27.9	5.7	2.3	-0.3	-6.4	-3.3	-3.0	-2.8	-2.6	-2.9	6.7	0.3	1.9	24.5	16.2	12.6	46.9	68.2
12	2.3	2.0	1.8	1.6	1.4	1.4	1.2	1.1	1.3	1.6	1.9	2.4	-11.0	-6.6	3.4	3.8	-3.5	-2.1	2.0	3.4	3.1	0.9	-0.8	-1.6	0.7	1.1	1.1	0.7	1.1	1.1	1.6	1.1	1.0	2.8	2.2	2.0	4.6	6.4
13	-7.3	-6.3	-5.5	-4.5	-3.9	-3.8	-2.9	-2.7	-3.2	-4.6	-6.0	-7.8	53.8	31.3	-11.6	-13.4	16.8	10.7	-6.2	-11.8	-10.6	-1.9	5.2	8.5	-1.0	-2.5	-2.5	-1.1	-2.5	-2.5	-4.4	-2.6	-2.3	-9.5	-7.2	-6.1	-16.6	-23.2
14	8.9	7.8	6.7	5.5	4.8	4.6	3.5	3.3	4.2	5.8	7.5	9.5	-31.6	-19.1	11.8	12.2	-11.1	-14.1	6.1	10.8	9.7	1.8	-6.4	-5.8	1.8	3.0	3.0	1.5	3.0	3.0	4.5	2.9	2.5	8.8	6.8	5.9	14.9	20.6
15	4.2	3.6	3.1	2.5	2.1	2.0	1.4	1.3	2.2	2.9	3.7	4.5	2.3	1.7	0.6	2.5	1.4	0.7	-11.5	2.0	1.8	-1.5	0.4	1.1	1.2	1.1	1.1	0.9	1.0	1.1	1.0	0.7	0.4	1.6	1.3	1.2	3.5	5.6
16	47.3	40.1	33.2	25.4	20.5	19.3	12.0	10.7	23.6	32.0	40.7	50.9	31.9	28.4	38.0	-47.8	19.3	15.1	11.7	-64.5	-56.1	3.6	11.3	15.2	11.8	8.2	7.7	7.8	7.1	7.6	-11.2	2.7	0.9	-48.8	-31.4	-23.8	-15.7	55.7
17	14.1	11.5	9.2	6.7	5.2	4.8	2.5	2.1	3.9	7.3	10.9	15.5	-11.2	-7.1	19.5	16.4	-17.6	-10.5	8.3	14.1	12.3	0.9	-7.4	-21.9	-1.2	1.6	1.6	-0.8	1.6	1.6	3.9	1.5	1.0	10.8	7.6	6.2	21.1	31.9
18	-16.6	-13.6	-10.7	-7.5	-5.5	-5.0	-1.9	-1.3	-4.7	-8.9	-13.1	-18.1	6.8	15.5	-19.3	-16.8	18.2	87.7	-6.1	-14.1	-12.2	4.2	29.9	14.7	1.8	-0.5	-0.4	3.3	-0.3	-0.4	-2.6	0.4	1.6	-10.5	-6.8	-5.2	-21.8	-32.2
19	-7.4	-6.0	-4.6	-3.1	-2.1	-1.9	-0.4	-0.2	-2.5	-4.2	-6.0	-8.2	-2.6	-0.9	2.0	-4.4	-0.3	1.4	34.7	-3.3	-2.6	7.2	2.1	0.4	0.1	0.3	0.4	0.9	0.5	0.4	0.3	1.2	2.0	-2.0	-0.9	-0.4	-6.5	-11.0
20	-102	-86.4	-71.4	-54.2	-43.4	-40.6	-24.3	-21.3	-50.2	-68.7	-87.9	-110	-68.8	-61.0	-82.5	116	-40.8	-31.5	-25.9	156.6	135.9	-5.8	-23.0	-31.6	-24.0	-15.7	-14.7	-14.9	-13.3	-14.5	28.7	-3.4	0.7	118	76.3	58.2	39.7	-120
21	129	108	88.7	66.3	52.2	48.6	27.6	23.8	61.2	85.3	110	140	86.9	77.7	113	-101	49.8	38.0	35.0	-145.4	-150.2	5.2	26.8	37.7	27.4	16.6	15.2	15.7	13.4	14.9	-41.1	0.7	-4.4	-155	-102	-79.1	-16.6	164
22	5.7	4.9	4.1	3.2	2.6	2.4	1.6	1.4	2.8	3.8	4.9	6.1	2.9	2.0	0.4	3.2	1.6	0.6	-18.3	2.5	2.2	-2.7	0.2	1.2	1.3	1.1	1.1	0.8	1.0	1.1	1.0	0.6	0.1	1.9	1.5	1.3	4.7	7.7
23	23.5	19.1	14.9	10.2	7.1	6.3	1.6	0.7	5.7	12.1	18.4	25.7	0.5	-6.1	29.1	23.1	-20.9	-75.6	10.9	19.5	16.8	-3.6	-25.7	-22.3	-4.7	-0.5	-0.6	-7.3	-0.8	-0.7	2.8	-1.7	-3.3	14.3	9.0	6.7	29.9	43.5
24	-18.2	-14.3	-10.7	-6.9	-4.5	-3.9	-0.3	0.4	-2.3	-7.7	-13.2	-20.1	3.6	1.9	-25.8	-19.9	33.8	12.1	-9.7	-16.7	-14.2	1.5	11.9	47.4	6.0	1.2	1.2	4.9	1.2	1.2	-2.3	1.3	1.9	-12.1	-7.5	-5.5	-26.3	-40.4
25	84.1	62.6	43.3	22.5	9.9	6.6	-11.9	-15.2	4.2	31.1	59.9	95.1	38.0	34.7	121	77.5	-13.4	-14.6	35.7	61.0	48.7	-18.1	-26.2	-32.2	-34.8	-19.9	-20.3	-48.5	-20.8	-20.4	-8.0	-24.1	-29.8	38.1	16.3	7.0	110	187
26	33.4	19.6	4.5	-16.1	-30.3	-34.2	-59.2	-64.4	12.4	21.2	30.1	40.0	29.0	28.5	49.4	37.5	16.9	15.4	27.1	32.9	29.3	10.2	11.9	12.6	6.4	-37.9	-51.2	6.2	-32.2	-48.8	-0.5	-11.1	4.9	26.1	14.2	8.8	46.0	63.3
27	-0.8	-0.5	-0.2	0.1	0.3	0.4	0.7	0.8	-0.2	-0.4	-0.6	-0.9	-0.6	-0.6	-1.1	-0.8	-0.3	-0.2	-0.5	-0.7	-0.6	-0.1	-0.2	-0.2	-0.1	0.9	30.7	0.0	0.8	0.7	0.1	0.4	0.0	-0.5	-0.2	-0.1	-1.0	-1.5
28	-127	-94.5	-64.7	-32.0	-11.6	-6.4	24.0	29.6	-3.1	-46.0	-90.7	-143	-64.5	-57.8	-180	-115	7.6	21.3	-52.7	-90.5	-71.5	34.0	42.3	32.3	61.9	37.6	38.3	88.1	39.2	38.4	18.5	45.2	55.1	-55.2	-20.8	-5.9	-163	-271
29	26.2	19.7	13.2	5.1	-0.2	-1.6	-9.9	-11.5	9.4	15.7	22.1	29.3	18.9	18.0	33.4	22.6	8.9	6.7	14.4	18.7	15.5	-0.1	3.5	5.4	1.3	-15.3	-30.1	-0.8	-17.6	-32.2	-15.0	-26.5	-5.1	12.8	0.4	-5.3	30.0	44.8
30	-0.7	-0.5	-0.2	0.1	0.3	0.3	0.6	0.7	-0.2	-0.4	-0.6	-0.9	-0.6	-0.6	-1.1	-0.7	-0.3	-0.2	-0.5	-0.6	-0.5	-0.1	-0.1	-0.2	0.0	0.8	0.7	0.0	0.9	30.7	0.2	0.4	0.0	-0.5	-0.1	0.0	-1.0	-1.4
31	12.5	10.1	7.8	5.0	3.3	2.9	0.4	-0.1	6.1	8.4	10.8	13.7	9.4	9.0	14.6	7.6	5.7	4.9	7.1	5.7	4.2	2.4	3.8	4.5	3.2	-1.2	-1.4	2.4	-1.8	-1.5	-17.1	-4.2	0.9	2.9	-18.6	-26.9	11.3	19.6
32	-55.2	-40.0	-24.8	-6.6	4.9	7.9	25.5	28.8	-15.8	-30.4	-45.5	-62.8	-37.8	-35.5	-72.8	-46.4	-14.6	-9.6	-27.0	-37.0	-29.7	5.4	-2.6	-6.7	2.2	36.6	38.7	7.0	41.3	39.0	37.5	60.6	16.3	-23.3	4.6	16.9	-64.3	-102
33	81.4	61.7	42.8	21.0	7.1	3.5	-17.4	-21.3	14.7	39.0	63.5	91.2	49.2	45.2	106	59.4	9.4	1.1	36.3	43.7	31.3	-13.3	-11.1	-4.5	-21.0	-28.4	-29.7	-34.6	-31.3	-29.9	-25.9	-43.0	-27.4	20.3	-1.1	-10.5	89.0	151
34	-83.0	-68.7	-55.3	-40.0	-30.6	-28.1	-14.1	-11.5	-36.7	-53.0	-70.1	-90.2	-54.9	-49.1	-76.3	43.8	-29.5	-21.7	-23.3	68.4	88.5	-0.2	-14.1	-21.2	-14.0	-6.7	-5.8	-6.2	-4.6	-5.7	32.0	3.8	7.1	107	72.5	57.2	-3.9	-112
35	14.2	12.1	10.0	7.6	6.1	5.7	3.5	3.1	8.0	10.2	12.6	15.3	10.9	10.3	15.0	3.6	7.3	6.4	7.7	1.0	-1.2	3.7	5.3	6.2	5.0	2.3	2.1	4.1	1.8	2.0	-24.6	0.0	2.4	-3.2	-33.1	-45.2	8.7	19.8
36	-16.5	-13.7	-11.0	-7.9	-6.0	-5.5	-2.6	-2.1	-8.7	-11.5	-14.5	-18.0	-12.5	-11.8	-18.2	-6.5	-8.1	-7.0	-9.0	-3.6	-1.3	-3.8	-5.7	-6.6	-5.1	-1.0	-0.7	-4.1	-0.3	-0.6								

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759
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Table D8. Leak flows obtained with 15 pressure sensors and 5 flowmeters (nodes 2, 5, 8, 10, 13, 18, 19, 20, 24, 27, 28, 30, 32, 34, 36 and pipes 11, 17, 25, 36, 42)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
1	-6.3	-24.8	-17.4	-10.6	-6.3	-5.2	1.3	2.4	3.8	3.6	3.6	4.1	5.2	5.8	10.0	8.7	4.8	5.1	7.2	8.2	7.7	5.3	5.0	4.7	4.4	3.2	3.3	4.6	3.4	3.3	5.1	4.2	4.9	7.3	6.3	5.8	9.8	12.2
2	17.6	61.7	43.6	27.4	17.5	14.9	0.4	-2.2	-5.2	-4.8	-4.9	-5.9	-8.3	-9.7	-18.7	-16.0	-7.5	-8.1	-12.7	-14.8	-13.9	-8.6	-7.8	-7.2	-6.5	-3.9	-4.1	-7.0	-4.4	-4.2	-8.0	-6.2	-7.6	-13.0	-10.7	-9.7	-18.4	-23.5
3	-7.6	-31.7	-21.6	-12.9	-7.7	-6.4	1.1	2.5	3.8	3.5	3.8	4.5	5.7	6.5	11.3	9.9	5.2	5.5	8.1	9.2	8.7	5.8	5.4	5.1	4.6	3.3	3.5	4.9	3.6	3.5	5.5	4.5	5.3	8.2	7.0	6.4	11.2	13.9
4	2.1	17.9	33.1	59.8	15.0	13.0	1.2	-1.1	-4.6	-4.8	-5.4	-6.4	-6.7	-7.2	-11.6	-10.0	-5.8	-5.9	-8.2	-9.3	-8.8	-5.9	-5.7	-5.5	-4.9	-2.4	-2.6	-5.1	-2.8	-2.6	-5.3	-4.2	-5.3	-8.3	-6.9	-6.3	-11.3	-14.2
5	-8.4	-6.3	-3.8	42.9	120	111	12.4	4.2	-5.4	-6.6	-7.8	-9.2	-7.7	-7.6	-10.4	-8.8	-6.0	-5.9	-7.5	-8.2	-7.8	-5.2	-5.4	-5.5	-4.6	1.1	0.8	-4.6	0.4	0.7	-3.8	-2.3	-4.5	-7.3	-5.7	-5.0	-9.9	-12.0
6	9.9	-5.8	-20.8	-91.3	-121	-112	-11.6	-2.2	12.0	13.6	15.6	18.2	16.6	17.0	24.2	20.9	13.8	13.7	17.7	19.5	18.5	12.9	12.9	12.9	11.3	2.5	3.0	11.5	3.7	3.1	10.7	8.0	11.5	17.6	14.4	13.0	23.4	28.5
7	12.6	7.5	1.8	-5.9	-11.3	-12.8	-22.3	-24.3	4.8	8.1	11.4	15.1	11.0	10.8	18.6	14.2	6.5	5.9	10.3	12.5	11.1	4.0	4.6	4.9	2.6	-14.0	-13.0	2.5	-11.8	-12.9	0.0	-3.9	2.0	10.0	5.5	3.5	17.3	23.7
8	-46.5	-28.2	-8.5	18.9	37.9	44.6	91.1	99.7	-13.9	-27.0	-40.3	-55.2	-38.6	-37.9	-68.8	-51.1	-20.5	-18.3	-35.5	-44.2	-38.9	-10.5	-13.0	-14.1	-4.9	60.3	56.5	-4.7	51.8	56.0	5.3	20.9	-2.7	-34.2	-16.4	-8.4	-63.8	-89.7
9	14.5	6.9	1.0	-3.3	-6.1	-6.8	-10.8	-11.5	-19.5	-28.2	-6.0	18.5	15.4	18.5	54.4	41.4	4.2	6.0	26.4	35.7	31.3	6.5	3.2	0.6	-6.5	-7.8	-7.2	-2.8	-6.5	-7.1	6.3	-1.5	2.0	27.5	17.6	13.3	52.2	75.2
10	-18.5	-7.0	1.0	5.0	7.5	8.2	11.9	12.5	43.0	62.0	21.0	-24.5	-21.5	-27.7	-94.2	-71.1	-2.3	-6.3	-44.0	-60.9	-53.1	-8.5	-1.6	3.7	16.3	9.6	9.0	8.9	8.2	8.9	-10.5	2.5	-0.8	-46.3	-29.7	-22.4	-90.7	-132
11	-15.5	-12.8	-10.0	-6.5	-4.4	-3.8	-0.6	0.0	-6.9	-11.6	13.0	-16.8	-0.6	5.6	35.6	28.9	1.8	3.8	18.9	25.4	22.7	7.3	3.7	1.6	0.0	1.2	1.5	2.1	1.7	1.5	8.2	3.8	5.0	20.4	14.7	12.2	35.7	50.5
12	27.3	17.1	10.8	5.8	2.6	1.8	-2.8	-3.7	-11.2	-15.3	-19.8	32.7	1.7	2.9	16.4	11.9	-2.6	-1.3	6.4	9.9	8.3	-0.6	-2.2	-3.5	-5.7	-3.4	-3.4	-4.1	-3.2	-3.3	0.1	-2.4	-2.1	6.9	3.7	2.3	15.8	24.1
13	-7.3	-6.3	-5.5	-4.5	-3.9	-3.8	-2.9	-2.7	-3.2	-4.6	-6.0	-7.8	53.8	31.3	-11.6	-13.4	16.8	10.7	-6.2	-11.8	-10.6	-1.9	5.2	8.5	-1.0	-2.5	-2.5	-1.1	-2.5	-2.5	-4.4	-2.6	-2.3	-9.5	-7.2	-6.1	-16.6	-23.2
14	8.9	7.8	6.7	5.5	4.8	4.6	3.5	3.3	4.2	5.8	7.5	9.5	-31.6	-19.1	11.8	12.2	-11.1	-14.1	6.1	10.8	9.7	1.8	-6.4	-5.8	1.8	3.0	3.0	1.5	3.0	3.0	4.5	2.9	2.5	8.8	6.8	5.9	14.9	20.6
15	0.2	0.2	0.1	0.1	0.1	0.1	0.0	0.0	0.1	0.2	0.2	0.2	2.4	3.9	15.6	1.5	1.0	1.1	-5.9	1.2	1.0	-1.0	0.4	0.6	0.1	0.0	0.0	0.0	0.0	0.0	0.2	0.0	-0.1	0.8	0.5	0.4	1.9	2.9
16	47.3	40.1	33.2	25.4	20.5	19.3	12.0	10.7	23.6	32.0	40.7	50.9	31.9	28.4	38.0	-47.8	19.3	15.1	11.7	-64.5	-56.1	3.6	11.3	15.2	11.8	8.2	7.7	7.8	7.1	7.6	-11.2	2.7	0.9	-48.8	-31.4	-23.8	-15.7	55.7
17	14.1	11.5	9.2	6.7	5.2	4.8	2.5	2.1	3.9	7.3	10.9	15.5	-11.2	-7.1	19.5	16.4	-17.6	-10.5	8.3	14.1	12.3	0.9	-7.4	-21.9	-1.2	1.6	1.6	-0.8	1.6	1.6	3.9	1.5	1.0	10.8	7.6	6.2	21.1	31.9
18	-16.6	-13.6	-10.7	-7.5	-5.5	-5.0	-1.9	-1.3	-4.7	-8.9	-13.1	-18.1	6.8	15.5	-19.3	-16.8	18.2	87.7	-6.1	-14.1	-12.2	4.2	29.9	14.7	1.8	-0.5	-0.4	3.3	-0.3	-0.4	-2.6	0.4	1.6	-10.5	-6.8	-5.2	-21.8	-32.2
19	-3.4	-2.5	-1.7	-0.7	-0.1	0.0	0.9	1.1	-0.4	-1.4	-2.6	-3.8	-2.7	-3.1	-12.9	-3.3	0.1	1.0	29.1	-2.5	-1.8	6.6	2.2	0.9	1.2	1.4	1.4	1.8	1.5	1.5	1.1	2.0	2.5	-1.2	-0.1	0.3	-4.9	-8.2
20	-102	-86.4	-71.4	-54.2	-43.4	-40.6	-24.3	-21.3	-50.2	-68.7	-87.9	-110	-68.8	-61.0	-82.5	116	-40.8	-31.5	-25.9	156.6	135.9	-5.8	-23.0	-31.6	-24.0	-15.7	-14.7	-14.9	-13.3	-14.5	28.7	-3.4	0.7	118	76.3	58.2	39.7	-120
21	129.2	108	88.7	66.3	52.2	48.6	27.6	23.8	61.2	85.3	110	140	86.9	77.7	113	-101	49.8	38.0	35.0	-145.4	-150.2	5.2	26.8	37.7	27.4	16.6	15.2	15.7	13.4	14.9	-41.1	0.7	-4.4	-155	-102	-79.1	-16.6	164
22	5.7	4.9	4.1	3.2	2.6	2.4	1.6	1.4	2.8	3.8	4.9	6.1	2.9	2.0	0.4	3.2	1.6	0.6	-18.3	2.5	2.2	-2.7	0.2	1.2	1.3	1.1	1.1	0.8	1.0	1.1	1.0	0.6	0.1	1.9	1.5	1.3	4.7	7.7
23	23.5	19.1	14.9	10.2	7.1	6.3	1.6	0.7	5.7	12.1	18.4	25.7	0.5	-6.1	29.1	23.1	-20.9	-75.6	10.9	19.5	16.8	-3.6	-25.7	-22.3	-4.7	-0.5	-0.6	-7.3	-0.8	-0.7	2.8	-1.7	-3.3	14.3	9.0	6.7	29.9	43.5
24	-18.2	-14.3	-10.7	-6.9	-4.5	-3.9	-0.3	0.4	-2.3	-7.7	-13.2	-20.1	3.6	1.9	-25.8	-19.9	33.8	12.1	-9.7	-16.7	-14.2	1.5	11.9	47.4	6.0	1.2	1.2	4.9	1.2	1.2	-2.3	1.3	1.9	-12.1	-7.5	-5.5	-26.3	-40.4
25	84.1	62.6	43.3	22.5	9.9	6.6	-11.9	-15.2	4.2	31.1	59.9	95.1	38.0	34.7	121	77.5	-13.4	-14.6	35.7	61.0	48.7	-18.1	-26.2	-32.2	-34.8	-19.9	-20.3	-48.5	-20.8	-20.4	-8.0	-24.1	-29.8	38.1	16.3	7.0	110	187
26	35.7	25.3	14.0	-1.6	-12.7	-15.8	-36.0	-40.2	14.2	22.6	31.1	40.6	27.7	26.6	46.4	32.6	15.1	12.4	22.2	27.6	23.7	4.0	8.4	10.7	5.6	-10.5	-49.6	2.9	-58.6	-50.8	-14.0	-27.4	-2.2	20.2	4.6	-2.3	42.0	61.0
27	-0.8	-0.5	-0.2	0.1	0.3	0.4	0.7	0.8	-0.2	-0.4	-0.6	-0.9	-0.6	-0.6	-1.1	-0.8	-0.3	-0.2	-0.5	-0.7	-0.6	-0.1	-0.2	-0.2	-0.1	0.9	30.7	0.0	0.8	0.7	0.1	0.4	0.0	-0.5	-0.2	-0.1	-1.0	-1.5
28	-127	-94.5	-64.7	-32.0	-11.6	-6.4	24.0	29.6	-3.1	-46.0	-90.7	-143	-64.5	-57.8	-180	-115	7.6	21.3	-52.7	-90.5	-71.5	34.0	42.3	32.3	61.9	37.6	38.3	88.1	39.2	38.4	18.5	45.2	55.1	-55.2	-20.8	-5.9	-163	-271
29	23.9	14.0	3.8	-9.4	-17.8	-20.0	-33.2	-35.6	7.6	14.3	21.0	28.7	20.2	19.9	36.3	27.5	10.7	9.7	19.2	24.0	21.2	6.1	7.0	7.2	2.1	-42.7	-31.8	2.5	8.9	-30.3	-1.5	-10.2	2.1	18.8	9.9	5.8	34.0	47.0
30	-0.7	-0.5	-0.2	0.1	0.3	0.3	0.6	0.7	-0.2	-0.4	-0.6	-0.9	-0.6	-0.6	-1.1	-0.7	-0.3	-0.2	-0.5	-0.6	-0.5	-0.1	-0.1	-0.2	0.0	0.8	0.7	0.0	0.9	30.7	0.2	0.4	0.0	-0.5	-0.1	0.0	-1.0	-1.4
31	12.5	10.1	7.8	5.0	3.3	2.9	0.4	-0.1	6.1	8.4	10.8	13.7	9.4	9.0	14.6	7.6	5.7	4.9	7.1	5.7	4.2	2.4	3.8	4.5	3.2	-1.2	-1.4	2.4	-1.8	-1.5	-17.1	-4.2	0.9	2.9	-18.6	-26.9	11.3	19.6
32	-55.2	-40.0	-24.8	-6.6	4.9	7.9	25.5	28.8	-15.8	-30.4	-45.5	-62.8	-37.8	-35.5	-72.8	-46.4	-14.6	-9.6	-27.0	-37.0	-29.7	5.4	-2.6	-6.7	2.2	36.6	38.7	7.0	41.3	39.0	37.5	60.6	16.3	-23.3	4.6	16.9	-64.3	-102
33	81.4	61.7	42.8	21.0	7.1	3.5	-17.4	-21.3	14.7	39.0	63.5	91.2	49.2	45.2	106	59.4	9.4	1.1	36.3	43.7	31.3	-13.3	-11.1	-4.5	-21.0	-28.4	-29.7	-34.6	-31.3	-29.9	-25.9	-43.0	-27.4	20.3	-1.1	-10.5	89.0	151
34	-83.0	-68.7	-55.3	-40.0	-30.6	-28.1	-14.1	-11.5	-36.7	-53.0	-70.1	-90.2	-54.9	-49.1	-76.3	43.8	-29.5	-21.7	-23.3	68.4	88.5	-0.2	-14.1	-21.2	-14.0	-6.7	-5.8	-6.2	-4.6	-5.7	32.0	3.8	7.1	107	72.5	57.2	-3.9	-112
35	-4.4	-3.8	-3.1	-2.4	-1.9	-1.8	-1.1	-1.0	-2.5	-3.2	-3.9	-4.8	-3.4	-3.2	-4.7	-1.1	-2.3	-2.0	-2.4	-0.3	0.4	-1.2	-1.7	-1.9	-1.6	-0.7	-0.6	-1.3	-0.6	-0.6	7.6	0.0	-0.8	1.0	54.5	13.8	-2.7	-6.1
36	2.1	2.1	2.1	2.1	2.1	2.1	2.0	2.0	1.8	1.9	2.0	2.1	1.8	1.7	1.4	-1.7	1.6	1.4	1.1	-2.4	-2.9	1.1	1.4	1.5	1.5	2.0	2.0	1.3	2.0	2.0	8.0	2						