Analysis of dependence structure between the Rand/U.S Dollar exchange rate and the gold/platinum prices

by

KAJINGULU MALANDALA

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Abstract

Copulas functions are a flexible tool for modelling the dependence structure between variables. The joint and marginal distributions of Copulas are not constrained by the assumptions of normality. This study examines the dependence structure between the gold, platinum prices and the ZAR/U.S.D exchange rate using Copulas. The study found that marginal distributions of Copulas follows the *ARMA (1, 1)-EGARCH (1, 1)* and *ARMA(1, 1)-APARCH (1, 1)* models under different error terms including the normal, the student-t and the skew student-t error terms. It used the Normal, the Student-t, the Gumbel, the rotated Gumbel, the Clayton, the rotated Clayton, the Plackett, the Joe Clayton and the Normal time varying Copulas to analyse the dependence structure between returns prices of gold, platinum and ZAR/U.S.D exchange rate. The results showed evidence of a positive strong dependence between the returns prices of gold, platinum and returns on the Rand/U.S.D exchange rate for constant and time varying Copulas. The result also showed a co-movement of exchange rates and gold and platinum prices during a rise or declining prices of gold and platinum. The results imply that fluctuations in gold and platinum prices generate Rand/U.S.D exchange rate volatility.

Keys words: Copulas, *ARMA*, *EGARCH*, *APARCH*, dependence structure, exchange rate, commodity prices.

Declaration

I declare that the above dissertation is my own work and that all the sources that I have used and quoted have been indicated and acknowledged by means of complete references.

SIGNATURE ______________________

DATE 19/11/2018

Dedication

I would like to dedicate this dissertation to my loving and supportive wife: Patricia Nyembwe and my children, Emmanuel, Gabriel, Abigael and Dora, for their continuous supports and care.

I would also like to dedicate this dissertation to my mother and father for their prayers and supports.

To God Almighty for the grace and favour, which he renews every morning in my life. Lord Jesus, You are my Rock and my Refuge.

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Contents

List of tables

List of figures

List of abreviations

ARCH: Autoregressive Conditional Heteroskedasticity

ARMA: Autoregressive Moving Average

APARCH: Asymmetric Power *ARCH*

A_{ti}*:*Average of bilateral nominal exchange rate

$Corr$: Correlation

$EGARCH$: Exponential Generalized Autoregressive Conditional Heteroskedasticity

$E(X)$*:* Expected Value of X or mean of X

GARCH: Generalised Autoregressive Conditional Heteroskedasticity

GDP : Gross Domestic Product

IMF: International Monetary Fund

Log: Natural Logarithm

LM: Langrange Multiplier

NEER: Nominal Effective Exchange Rate

U.S.D : United States Dollar

$P_{\boldsymbol{t}}$: Price at time *t*

P_{t-1} : Price at time t-1

PLATI: Platinum

Q-Q: Quantile

RD : Rand/U.S Dollar Exchange Rate

R_t: Return at time t

ZAR: South African Rand

Chapter One: Introduction

1.1 Background of the Study

South Africa produces approximately 4.5 % of gold and 70 % of platinum globally (United States, Geological Survey, 2017). Mineral resources, including gold and platinum are considered as the pillar of the South African economy. Over the past 150 years, the South Africa mining industry has been the main impetus for industrial and economic development. Commodities, including gold and platinum generate economic activities, offer prospects for job creation, stimulate the country's economic growth through investment, and contribute to Gross Domestic Product (GDP) and foreign exchange earnings. According to the Chamber of Mines report, released in 2015, the South Africa mining sector accounted for more than 40 % of merchandises export, traded commodity worth R 391, 4 billion, employed 462000 workers, and contributed R11. 3 billion in taxes and 7.7% in GDP.

In recent years, however, the mining industry has been impacted by deep deviations in commodity prices and lower production. The fluctuation of commodity prices has affected the investment patterns and revenues in this sector. Furthermore, the lower and volatile commodity prices undermine economic growth, due to their devastating effect on employment, and exacerbate the South African fiscus. The South African Mines Annual Report (2015) asserted that "fluctuation of gold and platinum prices might affect cash flow, revenues, assets value and profitability in the mining industry negatively, while volatility in exchange rate of the Rand against foreign currencies may cause accounting volatility". A notable outcome of this instability in commodity prices is job losses in the affected sector. This situation was confirmed by Statistics South Africa (2016) survey released in April-May, which showed that mining sector had shed 55 000 jobs owing to the lower prices of commodities. It has been reported that prices of gold and platinum are on the decline due to the lower demand of commodity driven by the global economic condition and commodity demand in China. In reaction to this, in 2016, the main platinum producer in South Africa reported a decrease in platinum production due to a combination of different factors including impairment to infrastructure, closing of shaft and safety conditions (Johnson & Matthey, 2017).

On the one hand, the performance of the local currency may have an impact on the exportation of gold and platinum, whereby the appreciation of the Rand against the United States Dollar (U.S.D) makes the former stronger and may encourage imports (Salvatore, 2005). Thus, a weaker Rand may lead to competition, boosting the exportation of gold and platinum while raising the cost of import goods and boosting consumer inflations (Edwards & Garlick, 2007). A strong U.S.D is disadvantageous for resources as it results in rising commodity prices and cost for consumers outside South Africa. Nevertheless, a strong Rand leads to decline in revenues generated by export in terms of Rand; while a softer Rand is also advantageous for dual-listed shares, which constitute the majority of the Johannesburg Stock exchange market value.

Abeysinghe and Yeok (1998) submitted that a weak currency encourages exportation and decreases importation but a strong currency boosts imports and decreases exports. Nevertheless, a flexible exchange rate can help mitigate the impact of external shocks and the transmission mechanism. For South Africa, an exporter of minerals resources including gold and platinum, an increase in the world commodity price may strengthen the exchange rate, leading to a rise in wages, an increase in the prices of goods, an upswing in domestic inflation and demand for the local product (MacDonald & Ricci, 2004).

Economic theories suggest that there are many mechanisms through which the relationship between currencies and commodities can be explained. Some researchers argued that currencies/commodities link is explained by the unsteadiness in macroeconomic expectations, driven by the integration of currency returns into commodity prices movements (Mark, 1995; Sephton, 1995; Engel & West, 2005; Klaasen, 2005).

Others, including, Clements and Fry (2008) argued that the correlation between commodities and currencies can be justified by the trade in good transmission theory, which state that: for a commodity producing country, an increase in the global commodity prices affects local currencies with consequences on salaries and the demand for non-transacted merchandises (Egert et al., 2006). From a practical perspective, it is a challenge and complex task for one to examine the dependence of currencies on commodity prices . This is because of the nature of the exchange rate and the prices of gold and platinum make it challenging and risky to analyse: commodity prices and exchange rates are volatile and unpredictable.

It should be noted that the responsiveness of the exchange rates towards commodity prices may differ from one industry to the other and from one country to the others, as such, it may be challenging to generalise these findings for all sectors of the economy (Fouquin et al., 2001). The South Africa economy, which is an open economy, may be influenced by the deviations in commodity prices and the exchange rate volatility, as one of the largest producers of minerals resources. Therefore, a dependence of the economy on commodity export may result in a robust correlation between inflation, commodity prices and exchange rates. Against this backdrop, it can be argued that there is a possible link between commodity (gold and platinum) prices and Rand/U.S.D exchange rate.

The gold and platinum produced in South Africa are exported and traded in the global stock market using the medium of foreign currencies such as the United States Dollar. Whereas, mining input, costs and taxes are denominated in Rand. Due to the pricing of gold and platinum in U.S.D, it can be argued that commodity prices volatility is associated with currency oscillation. The purpose of this study is to analyse the dependence structure between the Rand/ U.S.D exchange rate and the prices of South African gold and platinum using Copulas-*EGARCH* and Copulas-*APARCH.*

Copulas are continuous distribution functions that connect univariate marginal distributions functions and the dependence structure (Patton, 2006). Nelsen (2006) defines Copulas as "functions that tie multivariate distributions function with marginal distributions functions". According to Jondeau and Rockinger (2006), Copulas functions offer an exciting framework for modelling multivariate distributions once marginal distributions are identified. Bouye et al. (2000) described Copula as a flexible method of dependence structure. Meucci (2011) submitted that:" Copula is the omitted facts from the specific marginal to complete the mutual distribution".

Copulas were introduced by Sklar (1959) and were mainly used in the expansion of principle of probability and metrics spaces. In this study, Copulas are used because of their property to separate the dependence structure and the marginal distribution (Nelsen, 1999). Furthermore, Copulas are more flexible in specifying the dependence structure for different types of distributions including fat tails, skewed distribution and non-linear distributions that are relevant to the objective of this study.

Copulas have been used in many fields including actuarial science, medical studies, hydrology, risk management and finances. In actuarial science, Avanzi et al. (2011) used Levy Copulas to model the dependence between compound Poisson processes. Nikoloulopoulos and Karlis (2008) applied logit Copulas to study caries experience in the mandibular and maxillary left and right molars. In hydrology, Vandenberghe et al. (2010) used Copulas to examine the dependence between storm characteristics. Liu et al. (2017) used Copula and the principle of maximum entropy to model hydro-meteorological incidents. In finance, Nguyen and Nguyen (2014) employed Copulas techniques to investigate the association between money markets, bond and equity.

As a methodology, Copulas have limitations: some of the Copulas, such as Normal Copula have a zero tail dependence, making it unbearable to measure the asymmetric dependence (Cheung, 2009). Gumbel Copula does not measure the negative tail dependence (Trivedi & Zimmer, 2009). It was expect that Copulas-*APARCH* and Copulas-*EGARCH* would address the drawbacks inherent of normality assumptions and the challenge in modelling the asymmetric and nonlinear dependence structure between mineral resources prices and the Rand/U.S.D exchange rate.

1.2 Gap in Knowledge

The relationship between commodities and currencies has been extensively investigated and documented (Jain & Ghosh, 2013; Schaling et al., 2014; Le Roux & Els 2013; Wu et al., 2012). To date, a limited number of studies have been conducted on the dependence structure between commodity price and the exchange rate, particularly in South Africa and using Copulas as a methodology. There is therefore a need to investigate the currencies/commodities relationship using a more robust method of dependence such as Copulas.

Wu et al. (2012) used Copulas - *GARCH* models to examined the correlation between oil prices and exchange rates. Although *GARCH* presents some useful properties and some advantages, it has some drawbacks including restrictions on its coefficients and *GARCH* is a symmetric model. Furthermore, positive and negative errors have the same effects on volatility when using a *GARCH* model. As a result, these properties make it impossible for *GARCH* model to account for leverage effects and to model asymmetric distributions. This study proposes an asymmetric power Arch (*APARCH*) and an Exponential GARCH (*EGARCH)* models both of which are asymmetric *GARCH* models associated with Copulas. According to Enders (2010) and Ding et al. (1993), *EGARCH* captures asymmetric effects and does not necessitate any restrictions of the non-negativity of the coefficients while *APARCH* describes the leverage effects. In essence, this research presents an enhanced approach for the analysis of the relationship between the exchange rate of the Rand versus the U.S.D and gold and mineral resources prices (gold and platinum) using Copulas *EGARCH* and *APARCH*.

This study contributes to the literature on the time-varying Copulas when analysing the change in dependence structure between the Rand/U.S.D exchange rate and the prices of South Africa main commodities (gold and platinum). The other contribution is the use of different Copulas with *APARCH* proposed by Ding et al. (1993) and *EGARCH* models proposed by Nelson (1991) in studying the dependence structure between the prices of mineral resource (gold and platinum) and exchange rate of the Rand quoted against the U.S.D. Emphasis is placed on whether the fluctuation of exchange rates of the Rand affects the gold and platinum prices or whether the co-movement in gold and platinum prices contributes to the fluctuation of the Rand/U.S.D exchange rate.

1.3 Research questions

This research is guided by the following questions:

- Does the fluctuation in gold and platinum prices affect the Rand/ U.S.D exchange rate? Alternatively, does the exchange rate of Rand/ U.S.D affects the fluctuations in gold and platinum prices?
- What type of dependence structure exists between South Africa's main mineral resources (gold and platinum) prices and the the Rand / U.S.D exchange rate?
- How strong is the relationship between mineral resources prices (gold and platinum) and the exchange rate of the Rand quoted against the U.S.D?

1.4 Aim and Objectives of the Study

In the light of the above questions, the aim of this study therefore, is to analyse the possible relationship between commodity prices (gold and platinum) and the exchange rate of the South African Rand (ZAR) against the U.S.D.

The objectives of this study are to:

- Find an appropriate *ARMA-EGARCH* and *ARMA-APARC*H Copulas models for currencies (Rand/U.S.D exchange rate) and prices of commodities(gold and platinum prices),

- Model the relationship between South African main commodity prices (gold and platinum) and the exchange rate of the Rand against the U.S.D using an *ARMA-EGARCH*, and *ARMA- APARCH* Copulas,

- Explore the nature and the degree of the possible relationship between commodity (platinum and gold) prices, and the exchange rate of the Rand versus the U.S.D.

1.5 Importance of the Study

This study is important because it provides an understanding of the dependence between currencies (Rand/U.S.D) and commodities prices (gold and platinum), which may be of great interest to policymakers and business decision-making process when faced with uncertainty. It also provides information on the level of trade and foreign investment for commodity economy countries, such as South Africa.

1.6 Outline of the Study

This dissertation comprises of five chapters. Chapter one, the introduction to the study presents the background of the study, the gap in knowledge, the research question, aims, objectives and importance of the study. Chapter two defines and reviews some key concepts utilised in the study. The chapter further discusses previous studies on the link between commodities prices and exchange rates. Chapter three introduces the approach used for the investigation of dependence between gold, platinum prices and ZAR/U.S.D exchange rate. Furthermore, the chapter also describes the concept of *APARCH*, *EGARCH* and Copulas as well as the families of Copulas and estimation methods. Chapter four provides an application of Copulas to returns prices of gold, platinum prices and returns on the Rand/U.S.D exchange rate. It also presents the results and discussions of the study. Chapter five offers a summary of the result and conclusion.

Chapter Two: Literature review

The following literature review comprises of three main sections. The first section provides common standard definitions of the key operational terms. The second section reviews the history of the South African exchange rate and the exchange rate regime. The last section focuses on the scholarly works with regards to the dependence structure between exchange rate and commodity prices.

2.1 Commodity and Currency

The terms "commodity" is defined as a raw product, an object or an item of value, which creates wealth (Wilmott, 2007: 9). Examples are gold, platinum, coffee, oil, copper, sugar and coffee. There are various types of commodity including agricultural, livestock, industrial, energy commodity and precious metals. Agricultural commodities are plants produce obtained from farming activities, examples in this category are coffee, cane sugar, beans and maize. Livestock commodities are produced poultry and livestock such as goat and cow. Energy commodities include coal, natural gas, and crude oil. While industrial commodities includes aluminium, copper and zink. Precious metals include gold, platinum and silver, which are mined from the ground.

Commodities can be subdivided into two main categories, namely: hard commodities (raw material) and soft commodities. Hard commodities are natural resources or precious metals. One of the characteristics of these commodities is that they are valuable and non-perishable. Soft commodities include organic and perishable products such as agriculture commodities. Commodities are traded in local and foreign currencies. Currency is a form of money used as a medium of exchange and exist in two forms: notes and coins (Volcker & Bernstein, 2008). Countries, which depend greatly on commodity export, are referred to as commodity currencies (Bova, 2009). These countries include developed countries such as U.S.A, New Zealand, Canada and Australia, emerging market countries including Chile, Argentina and South Africa and under-developed countries such as Nigeria, Zambia and the Democratic Republic of Congo.

2.2 Exchange Rate *(ER)*

The amount which a currency is worth when "converted to a certain currency" is called exchange rate. According to Jhingan (2003), an exchange rate is "a rate at which a currency or currencies can be exchanged for other currencies." The exchange rate is "the amount of money paid in Rand in order to get a certain amount in U.S.D or any other foreign currency". The exchange rate is also a system that facilitates trade of goods and service between two countries. It is an important economic variable used to measure the performance of the economy of a nation. It also determines the purchasing power of a local currency against a foreign currency and affects profits of international institutions. There are two types of exchange rate: the nominal exchange rate and the real exchange rate.

2.2.1 Nominal Exchange Rate (*NER)*

The nominal exchange rate is the price of an international currency in relations to the domestic currency. In the context of this study, the nominal exchange rate is the monetary value of the United States Dollar in relation to the Rand. In other words, the nominal exchange rate is an amount of units of international currencies, which can be exchanged with a unit of local currency. Two different approaches are used to determine the nominal exchange rate, namely: the direct and indirect methods. According to Fourie (1997), the direct method is a method in which a local currency can be traded for one unit of an international currency while the indirect approach expresses how a unit of an international currency can be traded for a unit of a local currency.

2.2.2 Nominal Effective Exchange Rate (*NEER)*

The nominal effective exchange rate is given by Opoko-Afari (2004):

$$NEER_{kti} = \sum_{t=1}^{j} W_{ti} \times A_{ti}, \qquad (2.1)$$

where k is the local currency, t the trading partner at time (i) respectevely, A_{ti} is the average of the bilateral nominal exchange rate between the country (t) and its international interchange partner at time i, and W_{ti} is the traded weight of each selected trading partner(t) at time (i) with $t = 1, 2, \dots, j$.

2.2.3 Real Exchange Rate (*RER)*

The real exchange rate is the nominal exchange rate multiply by the inverse of the comparative fee levels. According to the purchasing power theory, the real exchange rate can be expressed as follows:

$$\mathrm{RER}= \mathrm{E}_{\frac{\mathrm{D}}{\mathrm{R}}} \times \frac{\mathrm{S}_{\mathrm{SAR}}}{\mathrm{S}_{\mathrm{USD}}}, \qquad (2.2)$$

where, $E_{\frac{D}{R}}$ is the equivalent of one U.S.D price to 1 Rand, S_{SAR} is the price level in South Africa, and S_{USD} is the price level in the United States.

According to Catão (2007: 46), the real exchange rate is the difference between the costs of a country goods compared to costs of good in another state. It is an estimated cost of goods difference between or among countries. Ganguly and Breuer (2010) used the nominal exchange rate and relative price differential to express the real exchange rate.

2.2.4 Real Effective Exchange Rate (*REER)*

According to the purchasing power parity theory, the real effective exchange rate is described, in the long run as the nominal exchange rate NER, after adjustment to the ratio of the foreign price level S_{ti}^* to the local price level P_{ki}. The real effective exchange rate is given by Ali et al. (2014):

$$\mathrm{REER}_{\mathrm{ki}} = \sum_{\mathrm{i}=1}^{\mathrm{j}} (\mathrm{NER}) \frac{\mathrm{S}_{\mathrm{ti}}^*}{\mathrm{P}_{\mathrm{ki}}}, \qquad (2.3)$$

where k is the local country, t the trading partner at time (i), with S_{ti}^* the total weight price index of the principal interchange partners, and P_{ki} is the price deflator for the country.

2.3 Classification of Exchange Rate

Exchange rates classification can be subdivided into two groups: De *jure* classification of exchange rate and De *facto* classification of exchange rate. De *jure*, classification of exchange are listed by the government and classified by the international monetary fund (IMF). While, De *facto* classification of exchange rate is an exchange rate classification, which is based on the perceived behaviour of the exchange rate (Cruz-Rodriguez, 2013).

Using the De *jure* classification approach on the exchange rate, exchange rates regimes can be grouped into three main categories (Ishfaq, 2010):

- The fixed exchange rates regimes which include the currency unions, currency board and the fixed exchange rate regime,
- The pegged exchange rates regimes which include the adjustable peg, the crawling peg, basket peg, and the target zones or bands exchange rate regime,
- The floating exchanges rates regimes that include the managed floats and free floats exchange rates regime.

The international monetary fund (IMF), 2014 annual report listed four categories of exchange rates categories namely:

- The hard pegs exchange rates which consist of exchange rate procedures with no separated legal tender and currency board arrangement,
- The soft pegs exchange rates system consist of the conventional peg, pegged exchange rate within horizontal bands, steadied arrangement, crawling peg and crawl-like arrangement,
- The floating exchange rate which includes the floating exchange rate and the free-floating exchange rate,
- The residual exchange rate system, which includes other, types of exchange rates arrangements.

Ghosh et al. (2002) cited ten types of exchanges using the De *jure* and De *facto* classifications of exchange rates:
Monetary union, Dollarization, currency board, single currency peg, basket peg, cooperative regime, crawling peg, target zones, the band managed float and floats.
Using volatility, effective exchange rate, bilateral exchange rate against anchor currency and international reserves, Dubas et al. (2005) classified exchange rates as follows:

- Currency peg,
- Limited flexibility,
- Cooperative arrangement,
- Adjusted according to a set of indicators,
- Managed floating and

- Independently floating.

2.3.1 Exchange Rate Regimes

2.3.1.1 Fixed Exchange Rate Regime

The fixed exchange regime is a financial system in which the local currency is coupled to a foreign currency or a basket of currencies and fixed at a specific level. Under the fixed exchange rate regime, the exchange rate remains constant or varies within constricted boundaries.

2.3.1.2 Floating Exchange Regime

A floating exchange is also called a managed exchange rate. The floating exchange rate is determined by the market's macro-economic variables and by the monetary policy of the central bank. Supply and demand in the economy determine the exchange rate under a flexible exchange rate regime (Samuelson & Nordhaus, 2001). The central bank plays a crucial role as it intervenes in the foreign exchange rate market to control and restrict the volatility of the exchange rate. However, the government allows the foreign exchange market to determine the exchange rate. Trade, government spending, demand for foreign currencies, supply and demand in the market determine the exchange rate (Devereus & Engel, 1998). There are two types of floating exchange rates namely the managed floating exchange rate and the pure floating exchange rate. Under the managed floating exchange rate, the exchange rate is determined by the supply and demand in the foreign exchange rate market. While under a pure float, the exchange rate is determined by the foreign exchange market.

2.4 Factors Affecting the Exchange Rate Volatility

Identifying factors that determine volatility in exchange rate is a controversial topic. Several authors have reported by means of different models and scenarios differing factors. Exchange rates are affected by many factors including inflation, interest rate, economic performance, currents accounts, public debts, terms of trade and political stability. According to Ghalayini (2013), the limitations of exchange rate models including the purchasing power theory, and the interest rate parity motivate the necessity to search for factors affecting the exchange rates volatility. Giannellis and Papadopoulos (2011) investigated the cause of fluctuation of exchange rates for selected Europeans union members and candidate for the Europeans union membership countries. They found evidence of a correlation between stock returns and exchange rates. Furthermore, they argued that economic fundamentals affect the exchange rate. When studying the long run nominal and real exchange rates for 107 countries, Lane (1999) opined that trade openness, country size, central bank independence and government debts are determinant factors of the nominal exchange rate. Furthermore, Jabeen and Khan (2014) identified real out pout, volatility, foreign reserves volatility, inflation, productivity and terms of trade as main determinant of the Pakistan exchange rate.

Others researchers reported that terms of trade such as fluctuations in global commodity prices affect the exchange rates for larger commodity exporters. Chen and Rogoff (2003) claimed that commodity prices determine the real exchange rate of commodity countries such as New Zealand and Australia. Chinn and Alquist (2000) listed money stock and interest rates as main determinant of the Euro/U.S D exchange rate.

In the case of South Africa, MacDonald and Ricci (2004), Mtonga (2006) and Ricci (2005) claimed that mineral prices influenced the value of the Rand. Coudert et al. (2008) pointed out that financial crises deepened currencies across emerging markets during the 2008 financial crises. However, several authors including Bleany (2008) and Chipili (2012) submitted that trade openness increased Rand volatility.

According to MacDonald and Ricci (2004), economic liberalisation, capital flow and terms of trade are the main determinant factor of the South African exchange rate. A combination of factors including the introduction of the inflation targeting regime in 1990, the termination of the dual financial system in 1995 and the capital account liberation have rendered the local currency (ZAR) to be volatile (Arezki et al., 2014). Mpofu (2016) explored the determination of the exchange rate volatility in South Africa from 1986 to 2003. He reported that the adoption of the floating exchange rate contributed positively to the Rand volatility. Furthermore, he noted that the volatility of the Rand might be generated by the volatility output, commodity prices, money supply and foreign reserves. Ricci (2005) and Arezki et al. (2014) who inferred that that the termination of the dual exchange rate regime and the capital account openness in 1995 contributed to a decline in the volatility of the Rand.

However, Galati and Ho (2003) claimed that current "news" have an impact on the exchange rate; they hypothesed that currencies react differently to announcement, good or bad; good news strengthens currencies while bad news depreciates currencies. Furthermore, Fedderke and Flammand (2005) studied the impact of international macro-economic news on the Rand/ U.S.D exchange rate. They found that good macro-economic information's has more profound impact on the Rand/U.S.D relationship.

Raputsoane and Todani (2008) used the time series approach to study different models of the Rand/U.S.D exchange rate relationship. They reported a long-term correlation between interest, money differentials, income and Rand/ U.S.D exchange rate. Moreover, they claimed that an increase in commodity prices and current account balance leads to a depreciation of the Rand. Mtonga (2011) submitted that short-term interest rate and relative money supply are positively correlated with traded-weighted Rand exchange rate. He also reported an inverse correlation between relative real income and long-terms interest rate differential. Aron et al. (1997) who investigated the short run and long run equilibrium-determining factor of real exchange rate in South Africa claimed that fiscal, monetary, exchange rate policies, terms of trade shocks and shifts in the capital are the main determinants of the Rand exchange rate.

Faulkner and Makrelov (2008) reported that commodity prices are one of the main factors, which influence the South African Rand. It should be noted that the volatility of the Rand is also partly driven by political turmoil. Arezki et al. (2014) claimed that gold price played an important role in the fluctuations of the Rand. Copeland (2005) alleged that export and import, foreign investment and speculation determined the exchange. Kakkar and Yan (2014) submitted that sectoral technology, monetary shocks, price of commodities (oil, gold) and interest rate are the main determinant factors of exchange rate volatility.

2.5 Exchange Rate Regimes and Exchange Rate Fluctuations

There is quite a debate on the relationship between exchange rates regime and exchange rate fluctuations. Literature shows that there is no consensus on how monetary policy affects exchange rates; a shift in exchange rate policies may result in devaluation or appreciation of the local currency. Alaba (2003) purported that the termination of the Bretton exchange regime has contributed significantly to exchange rate fluctuations. Carrera and Vuletin (2002) stated that a fixed exchange rate policy tends to drive volatility compares to a floating exchange rate regime. Contrary to the view of Carrera and Vuletin (2002), Kodongo and Ojah (2014) claimed that floating exchange rate policy caused fluctuation in exchange rate. Kocenda and Valachy

(2006) studied the behaviour of Poland, Hungary, Slovakia and Czech Republic exchange rates and concluded that the exchange rate regime played a major role in the exchange rate volatility. They pointed out that exchange policy is one of the factors that contributed significantly to currency fluctuations.

It has been challenging to find a common ground on factors that determine the exchange rate due to the varying opinions on the association between exchange rates regimes and exchange rate volatility and the resulting research mixed findings. Some authors suggested that time horizon and the economic conditions had a significant role in determining the exchange rate, whereas other claimed that macro-economic fundamentals and time horizon were instrumental.

2.6 History of the South African Rand Exchange Rate

The South African local currency is called the Rand and it is symbolised by ZAR. It is named after a place where the world's largest reserve of gold was discovered: the Witwatersrand basin, an Afrikaans word meaning "white rivers". As a currency, the ZAR was established on 14 February 1961. According to the Bank for International Settlement survey (2016), the Rand is one of the most traded currency in Africa and one of the peak imperative emerging market currency. The Bank for International Settlement ranked the South African currency on the 20th position amongst the most traded currency in the world, contributing to 1% of the global daily currency trading. At the time of its introduction, one Rand was worth $ 1,40 and the country was using a fixed financial system in which the local currency was pegged to U.S D. In 1966, the South African Pound was introduced but it was later abandoned and replaced by the South African Rand, following the termination of South Africa's membership of the Sterling and Commonwealth.

The South African exchange rate system has gone through different stages: Bretton Woods, fixed, transitional and floating exchange system regime. After the Second World War, the Bretton Woods's exchange rate system (1944-1971) was established to regulate the movement of currencies between states (Wang, 2009:29). The Bretton Woods exchange rate was a fixed financial system, under which gold was used as an exchange standard, with the U.S D as the main mode of exchange rate.

Following the collapse of the Bretton Wood system in 1970, the South African government implemented a fixed exchange rate system, in which the Rand was floated against the British Pound in February 1971 and was pegged to the U.S.D (Aron et al., 2000). From December 1971 to September 1972, the Rand was pegged to the British pound and later fixed to the United States Dollar. Additionally, the United States currency was used as a medium of exchange until May 1974. According to Aron et al. (2000), an endeavour to safeguard the country's balance of payment saw eleven adjustments made to the exchange rates system between June 1974 and June 1975. One of the amendments was that the Rand was fixed to the U.S.D. These adjustments lead to the devaluation of the Rand: the Rand depreciated 17. 9% to the U.S.D in September 1975.

The first dual exchange rates were introduced in 1979; namely, the crawling peg commercial Rand and the free-floating financial Rand (Mtonga, 2011), and the Rand was pegged to the U.S.D. Following the recommendation of the de Kock commission in 1983, the Rand was delinked from the U.S.D and determined under free conditions (De Kock commission, 1985) and a managed floating Rand was introduced in 1983. In the same year, the Rand was negatively affected by the declining gold price and the deficiency of financial Rand. From 1984, the Rand lost its ground to the U.S.D and reached a level of R 2 per U.S.D. By 1985, the Rand was trading at R 2.4 against one U.S.D. In 1994, the Rand depreciated further and reached R 3.60 per one U.S.D.

A second dual exchange rate was implemented in 1995; two types of exchange rates were introduced, namely, the commercial Rand and the financial Rand. The commercial Rand was used by residents for current accounts and international trade while the financial Rand was used by non-residents for account transactions, which were primarily used for investments. This dual exchange rate regime was abolished in the same year, making a way for a unified exchange regime (Van der Merve, 1996). With the termination of the double financial regime in 1997, the commercial Rand was endorsed as exchange rate system for current and capital transactions. The government ended the commercial Rand exchange rate system and a freely floating regime was introduced.

In June 2001, The Rand plunged to 50% of its value to the U.S.D (Raddatz, 2008). In December 2001, the Rand extended its losses to the American currency (U.S.D) and was equivalent to R 13.84.

Unprecedented events, including the 2008 global financial crises, affected currencies globally including the Rand. The local currency lost its ground to the U.S.D , trading at R 11. 85 per 1 U.S.D. In May 2009, the Rand showed stability, started regaining previously lost ground to the U.S.D, and reached a level of R 8 per one U.S.D due partially to the recovery from the devaluation caused by the 2008 global financial crisis.

The lower commodity prices, economic growth affected the value of the Rand negatively. For example, in 2013 the Rand reached one of the lowest levels to the U.S.D trading at R 10.28 against the U.S.D. From 2014 up to 2017, the Rand lost further ground to the U.S.D and declined even further due to the tough economic situation and policy uncertainty and one U.S.D traded around R 13 per one U.S.D. In October 2017, the Rand softened against the U.S.D and breached R 14.10 per one U.S.D. Currently, South Africa utilises a floating exchange rate system underpinned by an inflation-based target system.

2.7 Rand-U.S.D Exchange Rate: The Place of the Prices of Gold and Platinum

Gold and platinum are rare and precious metals. Their physical and chemical properties make them unique mineral resources. They are utilised in different applications, including in jewellery and coins design, in medicine and several industrial applications (for instance, in electricity and electronics industries). According to the Gold Survey (2015), gold and platinum are mainly used in jewellery, industry and investment. The Gold Survey (2015) reported that 60 % of tonnes of gold produced worldwide is used in jewellery, 11 % in industry and 29 % in investment. On the one hand, gold is used in finance as investment and a medium of exchange by banks worldwide (Goodman, 1956; Solt & Swanson, 1981). In addition, gold features provide comforts for mining stocks, coins, portfolio diversification and indemnity for financial tragedies (Constable & Wright, 2011). Tiny nanoparticles of gold are used in medicine for the treatment of cancerous cells. Gold is also used in dentistry for restorative and replacement purposes. On the other hand, platinum is a commodity that finds application in the manufacturing of laboratory equipment and electronics. The demand for platinum is driven by tightened legislation on catalyst consumption for diesel engine in the automobile industry while 30 % of platinum produced globally is utilised in jewellery (Johnson & Matthey, 2017).

Gold is traded on the international market, including the London stock exchange, New York stock exchange and Frankfurt stock exchange, while Platinum is mainly traded in the U.S platinum market, Australian platinum market, United Kingdom platinum market and Canadian platinum market. Although gold and platinum are sold worldwide, the main international markets for gold are the American, London, Zurich and Hong Kong gold markets.

South Africa has the largest reserves of gold and platinum. The platinum reserve is found in the Bushwell complex, while the gold reserve is located in the Witwatersrand basin. South Africa produced 5.8% of gold globally and 70 % of platinum (United States Geological Survey, 2017). Mineral resources including gold and platinum are priced in international currency such as the United States Dollar. Generally, the U.S.D is associated with the gold price. An increase in the global gold prices may affect the U.S.D negatively, leading to the depreciation of the green back. Hedge funds, banks and corporations trade the pair United States Dollar and South African Rand (U.S.D/ZAR). According to the 2016 Bank for International Settlement report, the ZAR/U.S.D trade contributed 0.8 % of the global currency trade and accounted for 81 % of the whole Rand market. According to Kissi (2013), the Rand is the most used currency in sub-Saharan Africa.

2.8 Behaviour of Commodity Prices

Commodity market generates large volumes of sales and foreign currencies. Commodity prices (agricultural, metals, industrial, energy and non-energy commodity prices) changes over time. Plourde and Watkins (1998) stated that some commodity prices including the prices of gold, silver, and wheat are less volatile than the prices of oil, which tend to be more volatile. As the supply and demand of commodities affect commodity prices, lower or higher demand of commodities lead to commodity prices fluctuations, also referred to as" cycle". Generally, commodity prices exhibit periods of booms, busts and super cycle, which are characterised by long term trend, medium term cycle and short term trend.

Heap (2005) and Rogers (2004) stated that commodity prices exhibits a medium terms cycle. While Harvey et al. (2010) argued that commodity prices distributions show descending and long term trend. Roberts (2009) pointed out that commodity prices distributions exhibit joint trends and strident price peaks in the short term. However, Jerrett and Cuddington (2008) noted that co-movement in commodity prices is possibly premised on condition that commodity prices are determined by demand instead of metal supply shocks. Radetzki (2006) suggested that commodity prices, which tend to be volatile in the short term are characterised by movement of higher and lower prices. However, Cashin et al. (1999) reported that commodity prices distributions are skewed with lengthy prices slump trends and a smaller price boom trend.

According to Frankel (2007), decreasing interest rates lead to an increase in commodity prices. Cashin and McDermott (2002) stated that real commodity prices exhibited a descending trend with a 1.3 % per year over the last 140 years. Lombardi et al. (2012) pointed out that the behaviour of U.S.D describes an equally steady amount of variations of commodity prices in the short term. Given that, there are diverse market economics variables in the commodity market, including traders, speculations or hedge fund managers, Cashin and McDermott (2002) argued that commodity prices exhibits an asymmetric trend. It was reported that commodity prices are driven by macroeconomics variables, rapid growth of emerging markets countries, growth of the China economy, and the demand and the use of commodity in various industries including the electrical, construction and the automobile industry (Byrne et al., 2013; Lombardi et al., 2012).

2.9 Independence

The event A and event B are independent if the existence of the event A does not affect the likelihood of the event B or if the occurrence of the events B does do not influence the probability of the event A. Mathematically, the two events are independent if the following condition hold:

- $P(A \cap B) = P(A) \times P(B), or\ m_{XY}(x, y) = m_X\ (x) \times\ m_Y\ (Y),$ $\quad (2.4)$

Using the equation (2. 4), it can be stated that two events are dependent if they are not independent.

2.10 Dependence

Dependence can be defined as a state of being dependent on something or somebody else. Dependence also refers to as correlation, a relationship or an association. Key decisions in business and planning can be made through the dependence between two or more variables. According to Scarsini (1984), dependence describes the relationship between X and Y on any measurable function, "as X and Y move further to cluster around the graph of a function, either $y = g(x)$ or$x = f(y)$".

Two random variables X and Y are dependent if the value of one random variable influence the other variable. The random variables X and Y are independent if the events $\{X = x\}$ and $\{Y = y\}$ are independent for all x and y. It follows that the joint probability of two independent events is the product of the probabilities of the two events. Different statistical measures of association are used to analyse the dependence including, linear correlation, Kendall's Tau and Spearman.

2.10.1 Measures of Dependence

Measures of dependence are statistical measures of association, which analyse the strength (weakness), or the level of association between two or more random variables. A measure of dependence $\mathcal{M}_C$ = $\mathcal{M}_{X,Y}$ is called as a measure of association if:

- $\mathcal{M}_{X,Y}$ is given for every random variable (X, Y),
- $0 \leq \mathcal{M}_{X,Y} \leq 1$,
- $\mathcal{M}_{X,Y}$ = $\mathcal{M}_{Y,X}$,
- $\mathcal{M}_{X,Y}$ = 1 , the random variables X and Y are strictly monotone function on X and Y,
- $\mathcal{M}_{X,Y}$ = Zero, then the random variables X and Y are independent.

2.10.2 Correlation

The link between the dependent variable and the independent variable is expressed as a linear regression. Linear correlation is used to measure the relationship of bivariate data X and Y; where X is a predictor or independent and Y is referred to as the dependent or response variable. Correlation is a statistical tool which is used to measure the strength of dependence between two variables. The Pearson's linear correlation coefficient assumes that variables are normally distributed. The Pearson's linear correlation is the most desirable measure of dependence for its properties and characteristics. Although such a measure is commonly used, it may present significant drawbacks for non-linear distributions and may not favour the occurrence of extreme events, such as market crashes (Embrechts et al., 2003). In addition, several studies have reported that Pearson's correlation may present various shortcomings. Prakash et al. (2003) and Embrechts et al. (2003) reported that Pearson's coefficient of correlation does not capture the dependence structure for non-linear distributions. Patton (2006) provided evidence of asymmetry in the return distributions of exchange rates. Wu et al. (2012) stated that returns distributions are skewed, leptokurtic and display asymmetric tail dependence.

In the case where the distribution has a non-linear dependence structure, it may become impossible to draw some realistic conclusions. In light of this, it becomes essential to find dependence models, which are more flexible in specifying the dependence structure for different types of distribution including fat tails and skewed distributions.

Some of the limitations of Pearson's coefficient of correlation are (Embrechts et al., 2003):

- Independence of variables always implies that linear correlation is equal to zero but the opposite is only true for elliptical distributions,
- Linear correlation is not invariant under strict monotonic transformations of margins but invariant only under strictly increasing linear transformation. For example log (A) and log (B) do not have the same correlation as A and B,
- Correlation is only defined when the variances of variables are known,
- Correlation is a scalar measure of dependency, it does not provide all the information needed about dependence,
- A perfect positive correlation require having a correlation of one.

2.10.3 Concordance Measure

The concordance denoted by $m(X,Y)$ is a measure of association between two random variables X and Y. For n pairs of observations (X_i, Y_i) and (X_j, Y_j), the pairs are said to be concordant if the following equation holds:

$(X_i - X_j)\ (Y_i - Y_j) > 0$, and discordant if: $(X_i - X_j)\ (Y_i - Y_j) < 0$. (2.5)

Characteristics of concordances measures are: (references)

- $m(X,Y) \in [0,1]$,
- $m(X,Y) = m(Y,X)$ and $m(X,-Y) = -m(X,Y)$,
- if X and Y are independent, then $m(X,Y) = 0$,
- Let $F(X,Y)$ and $G(X,Y)$ be the joint-distribution of X and Y and $m_F\ (X,Y)$ and $m_G\ (X,Y)$ the concordance measure of the two distribution, then: X and Y are concordant if the following condition holds:
  $F(X,Y) \geq G(X,Y)$.

2.11 Rank Correlation

The rank correlations measure the association of different ranked random variables which belong to the same set. The most used measures of rank correlations are the Kendall's Tau and the Spearman's Rho.

2.11.1 Kendall Tau

Let $(X_i,\ Y_i)$ and (X_j, Y_j) be a pair of random variables for $i, j = 1, 2, \dots, n$ with distribution function F, then the Kendall"s Tau is given by (Nelsen, 1999):

$$\tau = P_r\ [(X_1 - X_2)(Y_1 - Y_2) > 0] - P_r\ [(X_1 - X_2)(Y_1 - Y_2) < 0], \quad (2.6)$$

also given by: $\frac{\sum_{i<j}[(X_i - X_j)(Y_i - Y_j)]}{\binom{n}{2}}$.

The Following are the characteristics of the Kendall Tau

- $\tau\ (-1 \leq \tau \leq 1)$,

- τ does not change under strictly increasing transformation, linear or nonlinear transformation of X and Y,
- (X_i, Y_i) and $\left(X_j, Y_j\right)$ for $i \neq j$ are concordant if the following equation holds:
  $X_i < X_j$, and $Y_i < Y_j$, or $X_i > X_j$, when $Y_i > Y_j$, and discordant if the following equation holds:
  $X_i > X_j$ When $Y_i < Y_j$, Or $X_i < X_j$, when $Y_i > Y_j$,
- Kendall's Tau is less sensitive to outliers (Kendall,1962).

2.11.2 The Spearman Rho

For (X_1, Y_1), (X_2, Y_2) and (X_3, Y_3) random variables with a common joint-distribution function J. The spearman's Rho associated with X and Y, and distributed with J is given by:

$$\rho = 3\left[P_r[(X_1 - X_2)(Y_1 - Y_3) > 0]\right] - P_r[(X_1 - X_2)(Y_1 - Y_3) < 0] \qquad (2.7)$$

The following are the characteristics of the Spearman Rho

- ρ $(-1 \leq \rho \leq 1)$
- It is invariant under strictly increasing linear or non-linear transformation of X and Y,
- The correlation between the random variables X and Y is positive if $\rho > 0$, and negative if $\rho < 0$,
- ρ is the difference between the likelihood of concordance and the likelihood of discordance for (X_i, Y_i) and $\left(X_j, Y_k\right)$ for $i \neq j \neq k$.
- The Spearman Rho does not depend on the marginal distribution, if the Spearman value is equal to one, then a strictly increasing function: $H: \mathcal{R} \rightarrow \mathcal{R}$ exist such that $X = H(Y)$.

2.12 Definitions of Time Series Concepts

2.12.1 Time Series

A time series is defined as an ordered series $\{x_t\}_{t=-\infty}^{\infty}$ of random variables observed at equal interval of time or probability space. The time series model can be described by the return as follows:

$r_t = \mu_t + \varepsilon_t,$

where $\varepsilon_t = \sigma_t Z_t$ is the noise terms and μ_t is the conditional mean.

2.12.2 Autocovariance and Autocorrelation

For time series defined by $(X_t)_{t\in T}$, if $Var\ (X_t) < \infty,\ \forall\ t \in T$, then autocovariance of the function $(X_t)_{t\in T}$ is given by:

$$\xi_y(k,t) \ := Cov(X_k, X_t) = \mathrm{E}\left[\left(X_k - E(X_k)\right)\left(X_t - E(X_s)\right)\right], \tag{2.8}$$

and the autocorrelation function is given by:

$$Corr(X_k, X_t) = \frac{Cov(X_k, X_t)}{\sqrt{Var(X_k)Var(X_t)}} \tag{2.9}$$

2.12.3 Stationary

The process $\{x_t\}$ is strictly stationary if the joint distributions are the same for a given time. In other word, the joint distribution of the sequence $(x_{n1}, \ldots, x_{nk})$ is the same as the joint distribution of $(x_{n1+\,h}, \ldots, x_{nk+h})$ for all choices of time point's numbers $n_1, \ldots, n_k$, and every choice of time lag h.

The time series is weakly stationary (covariance stationary) if:

- E $[X_n]$ = μ exists and is constant for all n,
- Var $[X_n] = \ \sigma_X^2 < \ \infty$, and
- The autocovariance E $[(x_n - u)\ (x_{t-r} - \mu)] = \ S_r$ that relies only on the lag, r is strictly stationary, together with the assumption of finite first and second moments, and implies weak stationarity.

2.12.4 White Noise Process

A process $\{\varepsilon_n\}$ is said to be white noise process if it is a series of uncorrelated random variables from a static distribution with:

- Constant mean; E $[\varepsilon_n] = 0$,
- Constant variance; E $[\varepsilon_n^2]$= $\sigma_\varepsilon^2 < \ \infty$, and
- Constant covariance E $\left[\varepsilon_n\,, \varepsilon_\mu\right] = 0$ if $n \ \neq \ \mu$.

The white noise process is not linearly forecastable, in the same that the best linear forecast of ε_{n+1} based on $\varepsilon_n, \varepsilon_{n-1}, \ldots$, is simply the series mean (zero), and does not depend on the present and the past observations.

2.13 GARCH Models

The Generalized Autoregressive Conditional Heteroskedastic (*GARCH)* model, which was introduced by Bollerslev (1986), is a generalisation of the *ARCH* model. It describes the volatility and captures the current and future level of volatility over a period.

Let y_t be the univariate time series and Ψ_{t-1} , be the data available at period $t-1$, then

$$y_t = E(y_t|\Psi_{t-1}) + \varepsilon_t, \qquad (2.10)$$

where $E(y_t|\Psi_{t-1})$ is the conditional expectation and ε_t is the innovation process.

According to Engle (1982), the Autoregressive Conditional Heteroskedastic (*ARCH*) process is expressed as:

$$\varepsilon_t = z_t \sigma_t, \qquad (2.11)$$

where z_t is an independently and indentically distributed (i.i.d) process,

$$E(z_t) = 0,$$

$$\mathrm{Var}(z_t) = 1.$$

In addition, the *ARCH* model of order q is defined as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 \qquad (2.12)$$

A process (r_t) is defined as a *GARCH* (p, q) model with p autoregressive lag and q the moving average lag if:

$r_t = \sigma_t \, \varepsilon_t$,

where ε_t is a Gaussian white noise,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \, \sigma_{t-j}^2 \qquad (2.13)$$

$$= \alpha_0 + \alpha(Z)\, \varepsilon_t^2 + \beta(Z)\, \sigma_t^2,$$

If $\alpha(Z)$ and $\beta(Z)$ are polynomials operators, then:

$$\alpha(Z) = \alpha_1 \mathrm{Z} + \alpha_2 \mathrm{Z} + \ldots + \alpha_q C^q , \qquad (2.14)$$

$$\beta(Z) = = \beta_1 \mathrm{Z} + \beta_2 \mathrm{Z} + \ldots + \beta_p \, p , \qquad (2.15)$$

with $\alpha_i \geq 0$, $\beta_i \geq 0$, for $i = 1, 2, \ldots, q \;\; and \, , j = 1, 2, \ldots, p$.

On condition that *GARCH* (p,q) is a second order-stationary,

$$\begin{aligned} \text{Var}\ (\varepsilon_t) &= E(\varepsilon_t^2) \\ &= E\ (\sigma^2\ \ \varepsilon_t^2) \\ &= \alpha_0 + \alpha(Z)\ \sigma^2 + \beta(Z)\ \sigma_t^2 \\ &= \frac{\alpha_0}{1 - \sum_{i=1}^{q} \alpha_i - \sum_{j=1}^{p} \beta_j}. \end{aligned} \qquad (2.16)$$

2.13.1 Some Advantages of *GARCH* model:

- Weak Stationarity,
- Common used for its applicability and proprieties,
- It tracks volatility over time,
- Garch is also used for model diagnostic,
- It captures a number of characteristics of time series.

2.13.2 Some of the limitations of the *GARCH* model:

- One of the drawbacks of the *GARCH* model is that: it cannot account for the leverage effects,
- As a symmetric model, it is unable to differentiate negative shocks from positive shocks, as the impact of negative shock in the market always differ from the effect of positive shock in the market. This makes it impossible to model asymmetric volatility using *GARCH*,
- With *GARCH*, it is impossible to identify the root of the variation of volatility.

The limitations of *GARCH* model makes a way for flexible models that account for the characteristics of time series such as asymmetry and different types of shock subject to financial distributions. As such, Asymmetry *GARCH* models such as *EGARCH* and *APARCH* are used in combination with Copula in the analysis of dependence between Rand/U.S.D exchange rate and prices of gold and platinum.

2.14 Previous Work on Currencies and Commodity Prices

This section of the literature reviews is largely centred on the relationship between commodity prices and exchange rates. The end of the Bretton Woods financial system stimulated the move from fixed exchange rate regime to floating exchange rate regimes, fluctuations in exchange rates, and researches on the relationship between exchange rates and fundamentals.

Meese and Rogoff (1983) investigated different exchange rates models including the economic and structural models of exchange rates. They noted that random-walk forecast model of exchange rates provided better results compared to others exchange rates models. They further submitted that no financial fundamental to currencies exist.

Contrary to Meese and Rogoff (1983), several other authors including but not limited to Chen et al. (2010) investigated the co-movement between the exchange rates and commodity prices of large commodity producers. Using Granger causality and out sample forecasting test, they analysed five currencies: the Canadian, Australian, New Zealand Dollars, the Chilean Peso and the South African Rand with their respective commodity prices. Their result showed evidence of a vigorous relationship between exchange rates and commodity prices rates once time changing parameters are delimited. They noted that the exchange rates of commodity currencies are good estimators of commodity prices. Furthermore, they established the link between exchange rates and macro-economic variables particularly commodity prices. Since then, the relationship between metals, oil, energy, food commodities prices and currencies has been discussed and reported (see for example: Zhu et al., 2014; le Roux & Els, 2013 & Sari et al., 2010).

Cashin et al. (2004) employed Cointegration techniques to examine the cross-sectional dependence between commodity prices and the exchange rates of developing countries. They used Canada, Australia and New Zealand as case studies. Their results showed evidence of a long term relationship between commodity prices and real exchange rates. They argued that there are fundamentals to the exchange rates.

Similarly, Chen and Rogoff (2003) investigated the direction of the co-movement of currencies and commodities and specifically the price of commodities exported in U.S.D by large commodity producers such as Australia, Canada and New Zeeland. Using Cointegration analysis, they claimed that commodities that are priced in U.S.D positively affect New Zeeland and Australia real exchange rates. However, Chen and Rogoff reported that an increase in one gold unit price has positive consequence on the value of the exchange rate. In projecting the prices of commodity, Chen et al. (2010) used the Granger Causality and Out Sample

Forecasting techniques to examine the link between commodities and currencies. They found that exchange rates of commodity-producing countries have an impact on future commodity prices.

Clements and Fry (2008) used Kalman Filter to measure the joint relationship between currencies and commodities. They analysed the commodity currencies of Australia, New Zealand and Canada from 1775 to 2005. Their results show that commodity prices are affected by currencies, though not enough evidence was found to suggest that commodity prices contributed significantly to the currency movements.

Bodart et al. (2013) used the Non-Stationary Panel technique to investigate the long term relationship between the exchange rates, exchange rates regimes, commodity prices, trade openness and export diversification for commodity producing countries, including developing and emerging markets countries for a period of 27 years. They reported evidence of the long terms relationship between commodity prices and real exchange rate. They inferred that the dependence is justified for commodity countries that export more than 20% of the country total export.

Sari et al. (2010) used the Autoregressive Distributed Lag approach to study the dependence structure of exchange rates and commodity prices. They analysed daily spot prices of gold, silver, platinum and palladium, oil price and the exchange rates of the U.S.D quoted against the Euro, from April 1999 to October 2007. They found evidence of a strong relationship between gold and platinum, palladium, silver return and the exchange rates fluctuations while no correlation was reported between fluctuations of the exchange rates and oil return prices.
In an attempt to forecast the nominal Canadian Dollar and U.S.D exchange rate using oil prices, Ferraro et al. (2015) analysed the daily frequency data of commodity currencies and found that forthcomings commodity prices could be used to forecast currency volatility for a larger exporter of commodities. They suggested that their findings could be extended to different pairs of commodity currencies including the South African Rand/U.S.D and gold price.

Jain and Ghosh (2013) studied the long term relationship between commodity prices and exchange rates. They examined the direction and the causality between oil, gold, platinum and silver priced in U.S.D and the Rupee/U.S.D exchange rate. The Autoregressive Distributed Lag Bounds Testing, Granger Non-Causality test and the Generalised Error Variance

Decomposition used in the study showed that currencies have an influence on commodity prices.

Jan et al. (2014) applied the Vector Autoregressive model to estimate the relationship between commodity prices and exchanges rates. They analysed monthly oil, gold prices and the United States Dollar/ Pakistani Rupee exchange from January 1997 to January 2012. They found that gold price and the exchange rate of the U.S Dollar quoted against the Rupee are negatively correlated. The results showed that commodity prices played a crucial role on the volatility of the exchange rate of the U.S .D/ Pakistani Rupee.

More recent studies on the dependence structure between commodity prices and exchange rates employed Copulas as well as *GARCH* as a model for marginal distribution. Pettersson (2010) used Copula-*GARCH* to investigate the relationship between 10 sectors indices of the global industry classification standard. He analysed 10 indexes: energy, telecommunication, health care, materials, consumer discretionary, consumer staples, and financials, industrial, information technology and utilities indices over a period of 5 years, from 2005 to 2010. His results show that Joe-Copula and Student-t Copula delivered a better fit for dependence. He reported a strong dependence between consumer discretionary, financial, industrial and material and a weak dependence on health care and utilities.

Shams and Zarshenas (2014) used Copula-*GARCH* methods to study the relationship between oil, gold prices and exchange rates fluctuations for a period of 8 years. Using Iran as a case study, they pointed out that exchange rates are well described by the Normal Copula while the Student-t Copula provides a good representation of oil and gold prices. They found two different results: evidence of a weak positive correlation between commodity prices and exchange rates before 2007 and a weak negative relationship between exchange rates and commodity prices after 2007.

Similarly, Reboredo (2012) used correlation, Copulas and T-*GARCH* models to investigate the dependence between exchange rates and commodity prices for a number of developed and emerging economies. He analysed the co-movement of crude oil prices and exchange rates of the Australian, Canadian Dollars, and Euro, the Pound, the Japanese currency (the Yen), the currencies of the Norway and Mexico. He suggested that there is a weak relationship between currencies and commodities before and after the 2008 financial crises.

Aloui et al. (2013) studied the conditional dependence structure between exchanges rates and oil prices using Copula-*GARCH*. They analysed the daily prices of WTI Cushing, Brent prices indices and the nominal exchange rates of U.S.D versus the Euro, the Canadian, and the British, the Swiss, the Japanese currencies and crude oil spot prices from 2000 to 2001. They suggested that a rise in the WTI and Brent price is connected to the devaluation of the U.S.D.

Wu et al. (2012) employed component Copula-*GARCH* model to investigate the link between the weekly closing oil prices and U.S.D exchange rate from 2 January 1990 to 28 December 2009. They found a negative and a declining dependence structure between crude oil prices and U.S.D exchange rate. In addition, their results indicated that the long-term volatility was bigger compared to the short-term volatility for crude oil prices.

Ignatieva and Ponomareva (2016) used Copula-*GARCH* techniques to examine the association between commodity prices and exchange rates of main commodity exporters. They analysed the daily nominal observations of the exchange rates of the Australia, Canadian and New Zealand Dollars, and the Norwegian Krone versus the U.S.D and their respective commodity prices indices. Their results showed evidence of a positive dependence structure between exchange rates and commodity prices. They alleged that a rise in time varying dependence was associated with beginning of the 2008 financial crisis.

Most research on the dependence between commodities and currencies focused on oil prices as a commodity and developed countries with only a few in emerging countries. This study argues that the relationship between currencies and commodities can be extended to others commodities such as precious metals and the exchange rates of emerging markets economies, such as and particularly South Africa.

Studies have been centred on dependence of commodities on exchange rate, most studies focused on Cointegration relationship and causality relatively. However, little attention has been paid to the relationship between the gold, platinum prices and the Rand/ U.S.D exchange rate for South Africa. Therefore, this research presents a more improved approach to the analysis relationship between the exchange rate of the Rand quoted against the U.S.D and the daily prices of gold and platinum mineral resource prices using a combination of Copulas *APARCH* and *EGARCH*.

In the case of South Africa, few studies have been reported on the dependence structure between commodity prices and exchange rates. MacDonald and Ricci (2004) investigated the relationship between the gold, coals, iron, copper and platinum prices and the real exchange rate of the South African Rand (ZAR) using Cointegration. They claimed that the real price of the commodity tends to affect the long term performance of the real exchange rate.

In an attempt to find the root of 1998 and 2001 South Africa Rand crises, Bhundia and Ricci (2004) investigated the South African Rand behaviour during the crises. They suggested that one of the leading causes of the Rand crises was the fluctuation of commodity prices. Moreover, they stated that as the real price of commodities exported by South Africa decreased by 1%, the real exchange rate of the Rand depreciated by 0.5%.

Ngandu (2005) conducted a study on the association between the real exchange rate, the commodity prices, and the effect of such relation to employment in the South African manufacturing sector. He reviewed the literature on the relationship between commodity prices and exchange rates using a number of economic variables such as; terms of trade, determinants of real exchange rate, exports and commodity prices. His findings indicate that there is a strong correlation between real exchange rates and the price of gold in a South African Rand. He concluded that commodity prices contributed significantly to the exchange rate.

Frankel (2007) used regression to study the determining factor of the real value of the Rand exchange rate. He analysed the weighted prices index of South African minerals exports commodities including gold and platinum, iron ores, coal, petroleum oil, aluminium and the nominal exchange rates of the U.S.D and the Rand from 1984 to 2006. He found that real exchange rate and real minerals prices were correlated: the lower price of mineral exports contributed to an unanticipated devaluation of the Rand. However, he argued that mineral price is one of the contributing factors to the real exchange rate of the Rand.

Similarly, Le Roux and Els (2013) utilised the Regression and Correlation to examine the relationship between the exchange rates of Australia, Canada, Chile, China and South Africa against the U.S.D and the copper price. They found evidence of a strong relationship between the Australia, Canada, Chile and South Africa exchange rate against the U.S Dollar and copper prices. However, an inverse relationship was reported for the copper price and the exchange rate of the U.S.D versus the Rand. In addition, they could not establish a link between the China exchange rate quoted against the U.S.D and copper price.

Through the Vector Error Correction model, Arezki et al. (2014) studied the causal relationship between the effective exchange rate of the Rand against the U.S.D, the fluctuations of the gold price and the magnitude at which the liberalisation of the capital account affect the correlation. They analysed monthly observations of gold prices and exchange rates for 30 years. Their findings show that currencies are affected by fluctuations in the real price of gold and unbalanced share of lesser movements, which countries experienced through the liberation of the capital account.

Schaling et al. (2014) applied Cointegration Analysis to study the causality of the association between the U.S.D/Rand exchange rate and commodity prices. They analysed the nominal data of the U.S.D/Rand exchange rate and the international monetary fund non-fuel commodity prices including vegetable oils, meat, seafood, sugar, fruits, beverages, and agricultural raw materials and metals indexes from 1996 to 2010. Their results show evidence of a negative weak relationship between the nominal Rand exchange rates and commodity prices in South Africa in comparison to developing commodity countries such as Australia and New Zealand, and no evidence of causality from the nominal exchange rate of U.S.D/Rand to the commodity prices indices was found.

Kim and Courage (2014) used the Generalised Autoregressive Conditional Heteroscedasticity model to investigate the association between commodity prices (oil) and nominal exchange rate in South Africa. They analysed monthly data over a period of 8 years starting from 1994 to 2012 and found an inverse relationship between the oil prices and currency. In addition, they noted that the depreciation of the Rand is associated with the rise in oil prices.

Findings and conclusions from studies on the link between commodities and currencies vary across countries and according to the methods employed, type of commodities (metals, agriculture), and type of variables (daily, monthly and quarterly) data, and therefore producing different results.

Findings have established a correlation between commodity prices and exchange rates for states, which produce a large quantity of the commodities (Chen et al., 2010). However, not enough evidence was found to suggest the existence of a currency/commodity relationship in other case studies. The currencies of countries, which produce and export large quantities of raw materials such as Australia, tend to be sensitive to the world price of raw material, as these are generally priced in U.S.D.

Chapter Three: Methodology

This chapter is subdivided into six sections, the section 3.1 explains and summarises the data, section 3.2 introduces the models for marginal distributions including the *EGARCH* and *APARCH*, section 3.3 describes the copulas model of dependence, section 3.4 describes the types of Copulas, section 3.5 introduces the measures of dependence associated with Copulas, and section 3.6 introduces the Copulas estimation methods.

3.1 Data

The observations consists of daily prices of gold, platinum per ounce in U.S.D and real exchange rates of the South African currency (ZAR) versus the United States Dollar for two years as listed on the Johannesburg Stock Exchange. The data were extracted from the JSE online database and organised in an excel spreadsheet. The return of gold and platinum prices and the exchange rate of the U.S.D against the Rand at a time t symbolised by R_t and computed as follows:

$$R_t = 100\,(\ln P_t) - (\ln P_{t-1}). \tag{3.1}$$

Packages and codes including R and Matlab software, as well as fGARCH, Tseries, Copulas packages, and Matlab codes were developed to carry out inference for *APARCH*(p,q), *EGARCH*(p,q) and Copulas estimation. Generally, financial observations are known to be correlated, as such different tests including the Ljung-box tests for residuals, Lagrange Multiplier test for *ARCH* tests and Akaike information criterion *(AIC*) and Bayesian information criterion (*BIC)* criteria were used to ascertain the occurrence of the *ARCH* properties on the time series. Returns of commodity prices and exchange rate were filtered with an *ARMA*(p,q)- *APARCH*(p,q) and *ARMA*(p,q)- *EGARCH*(p,q) model in order to obtain standard residuals.

Copulas splits the marginal distribution and the dependence structure, which is described by a Copula function (Nelsen, 1999). In this study, the research assumed that marginal distribution followed *ARMA*$(1,1)$-*APARCH*$(1,1)$ and *ARMA*$(1,1)$-*EGARCH* $(1,1)$ models. Different Copulas were fitted to the residuals. Copulas that provide the best fit of the data were selected using the goodness of fit test, the *AIC* and *BIC* informations criteria. Through the Sklar properties, the estimated marginal distribution and Copulas were joined in order to determine the dependence structure. Ultimately, Copulas maximum likelihood and canonical maximum estimation provided the results of Copulas. More theoretical details on the marginal distribution are presented in Appendix A.

3.2 Model for marginal distribution

Generally marginal distributions of Copulas are unkown. In this study, it is assumed that marginal distributions are characterised by Autoregressive Moving Average *(ARMA)* of order (p, q) with *APARCH* (p, q) and the Autoregressive Moving Average *(ARMA)* of order (p, q) with *EGARCH* (p, q).

3.2.1 Autoregressive Process

The process $\{x_t\}$ is autoregressive of order p (*AR* (p) if there exist constants $b_1, \dots, b_p$ such that

$$x_n = \sum_{t=1}^{p} b_t \; x_{m-t} + \varepsilon_n \;, \tag{3.2}$$

where ε_n $\{\varepsilon_n\}$ zero-mean white is noise, and ε_n is uncorrelated with $x_{n-1}, x_{n-2} \cdots$

The $AR(p)$ process exists and is weakly stationary if and only all the roots of the polynomial $P(Z)$ = 1- $a_1 Z, \dots, -a_p Z^p$ does not lie inside the unit circle.

3.2.2 Moving Average Models

$\{x_t\}$ is a moving average of order q $\big(MA(q)\big)$ if there exist constant a_1,…, a_q such that:

$$x_t = \sum_{k=0}^{q} a_k \; \varepsilon_{t-k}, \tag{3.3}$$

$b_0 = 1$. The $\big(MA(q)\big)$ process has a finite memory, the autocorrelation function of the $\big(MA(q)\big)$ cuts off outside lag q.

$\{x_t\}$ is an autoregressive moving average process of order *(p, q)* symbolised by $ARMA(p, q)$ if there exist constants $a_1, \dots, a_p, b_1, \dots, b_q$ such that

$$x_t = \sum_{j=1}^{p} a_j \; x_{t-j} + \sum_{j=1}^{q} b_j \; \varepsilon_{t-j} + \varepsilon_t, \tag{3.4}$$

where ε_t is zero mean white noise, and ε_t is uncorrelated with x_{t-1},x_{t-2},… The $ARMA(p, q)$ process exist and is weakly stationary if and only if all the roots of the polynomial

$P(Z)$= 1- $a_1 Z, \dots, -a_p Z^p$ are not in the unit circle.

3.2.3 APARCH (p, q)

$$y_t = X_k \varphi + \varepsilon_k \tag{3 .5}$$

$$\sigma_k^\delta = \alpha_0 + \sum_{j=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}\right)^\delta + \sum_{i=1}^{p} \beta_j \left(\sigma_{t-j}\right)^\delta, \tag{3 .6}$$

where $\alpha_0 > 0$, $\delta \geq 0, \beta_j \geq 0$ $(j = 1, \dots, p)$, $\alpha_i \geq 0, -1 < \gamma_i < 1(i = 1, \dots, q)$ and δ is the leverage effect.

APARCH ties varying exponent with the asymmetric coefficient and include *ARCH* extensions.

The covariance stationarity condition is:

$\sum_{i=1}^{p} \alpha_i \left(1 + \gamma_i^2\right) + \sum_{j=1}^{q} \beta_j < 1.$

According to Ding et al. (1993), a stationary solution for the *APARCH* model defined by

E $\left(\sigma_t^\delta\right) = \frac{\alpha_0}{1 - \sum_{i=1}^{q} \alpha_i (|z| - \gamma_i z)^\delta - \sum_{j=1}^{p} \beta_j}$ exists if $\sum_{i=1}^{q} \alpha_i E(|z| - \gamma_i z)^\delta + \sum_{i=1}^{p} \beta_j < 1.$

APARCH models fat tails distributions, excess kurtosis and capture the leverage effects (Ding et al., 1993).

3.2.4 EGARCH (p, q)

$$\log\left(\sigma_t^2\right) = \omega + \sum_{i=1}^{p} h(z_{t-i}) + \sum_{j=1}^{q} \alpha_j \log\left(\sigma_{t-j}^2\right) \tag{3 .7}$$

where h $(z_{t-i}) = \gamma_i z_{t-i} + \beta_i[|z_{t-i}| - E|z_{t-i}|]$, and

$z_t = \frac{\epsilon_{t-i}}{\sigma_{t-i}}$.

Then, the log conditional variance is given by:

$$\log\left(\sigma_t^2\right) = \omega + \sum_{j=1}^{q} \alpha_j \log \sigma_{t-j}^2 + \sum_{i=1}^{p} \beta_i \left(\left|\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right| - E\left(\left|\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right|\right)\right) + \sum_{i=1}^{p} \gamma_i \left(\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right), \tag{3 .8}$$

with β the symmetric effect, α is the lagged condition variance and γ indicates the asymmetric performance.

- *EGARCH* uses lagged conditional variance as a result there is no restriction on the parameter of the model,

- *EGARCH* is an asymmetrical model which is able to differentiate positive and negative lagged value of the innovation,
- Model volatility persistence and mean reversion.

This study used *APARCH* and *EGARCH* because they are appropriate for modelling commonly asymmetric effects in a time series. One of the advantages of using *APARCH* over *GARCH* is that it distinguishes between bad from good news, making it possible for volatility to responds asymmetrically to bad and good news.

3.2.5 Distribution of Error Terms

3.2.5.1 The Normal Distribution

The density function of Y_t of the normal distribution is given by:

$$\text{F}(Y_t|X_t, \phi_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_t^2}}\ exp\left\{\frac{-(Y_t - X_t\ \omega)^2}{2\sigma_t^2}\right\}, \quad -\infty < Y_t < \infty. \qquad (3.9)$$

The log-likelihood function of the standard normal distribution is given by:

$$G_T = -\frac{1}{2}\sum_{t=1}^{T}[ln(2\pi)\ + \ln(\alpha_t^2)\ +\ z_t^2],$$

where T is the sample size.

3.2.5.2 The Student-t Distribution

The density function of the student-t distribution is defined by:

$$\text{f}(Y_t) = \frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\Gamma\left(\frac{\upsilon}{2}\right)\sqrt{(\upsilon-2)\pi}}\left(1 + \frac{Z_t^2}{\upsilon-2}\right)^{-\frac{1}{2}(\upsilon+1)}, -\infty < Y_t < \infty. \qquad (3.10)$$

where υ is the number of degree of freedom $2 \leq \upsilon \leq \infty$, Γ is the gamma function.

The log-likelihood function of the student-t distribution is given by:

$$G_T = ln\left[\Gamma\left(\frac{\upsilon+1}{2}\right)\right] - ln\left[\Gamma\left(\frac{\upsilon}{2}\right)\right] - 0.5\ ln[\pi(\upsilon-2)] - 0.5\ \sum_{t=1}^{T}\left[\ln\sigma_t^2 + (1+\upsilon)ln\left(1+\frac{z_t^2}{\upsilon-2}\right)\right], \quad (3.11)$$

where υ is the degree of freedom, $2 < \upsilon \leq \infty$ and $\Gamma(.)$ is the gamma function.

3.2.5.3 The Skew Student-t Distribution

The skewed student-t distribution describes the characteristic of returns distributions including the skewness and kurtosis. The density function of the standardized skewed generalized error distribution is given by:

$$\mathrm{f}(Y_t|\upsilon) = \frac{\upsilon}{\left(2A\ \Gamma\left(\frac{1}{\upsilon}\right)\right)} \exp\left(-\frac{|Y_t - A|^{\upsilon}}{[1-sign(Y_t-A)\ \rho]^{\upsilon}.B^{\upsilon}}\right), \quad (3.12)$$

where $B = \Gamma\left(\frac{1}{\upsilon}\right)^{0.5}\ \Gamma\left(\frac{3}{\upsilon}\right)^{-0.5}\ \mathrm{U}(\rho)^{-1}$

$A = 2\rho.F.\mathrm{U}(\rho)^{-1}$

$U(\rho) = \sqrt{1+3\rho^2-4F^2\rho^2}$

$F = \Gamma\left(\frac{2}{\upsilon}\right)\Gamma\left(\frac{1}{\upsilon}\right)^{0.5}\Gamma\left(\frac{3}{\upsilon}\right)^{-0.5}$,

where ρ is a shape parameter of the distribution. The shape is positive and describes the degree of asymmetry.

The log-likelihood function of the skewed student-t distribution is:

$$G_T = ln\left[\Gamma\left(\frac{\upsilon+1}{2}\right)\right] - ln\left(\Gamma\left(\frac{\upsilon}{2}\right)\right) - 0.5ln[\pi(\upsilon-2)] + ln\left(\frac{2}{\mathfrak{I}+\frac{1}{\mathfrak{I}}}\right) + ln(s) - 0.5\ \sum_{i=1}^{T}\left[\ln\sigma_t^2 + (1+s)\ ln\left(1+\frac{s\ z_t + w}{\upsilon-2}\ \mathfrak{I}^{-\mathrm{I}_t}\right)\right], \quad (3.13)$$

where, $\mathrm{I}^t = f(x) = \begin{cases} -1\ if\ \ z_t < -\frac{w}{s} \\ 1\ if\ \ z_t \geq -\frac{w}{s} \end{cases}$

and $w = \frac{\Gamma\left(\frac{\nu+1}{2}\right)\sqrt{\nu-1}}{\sqrt{\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(\mathfrak{I} - \frac{1}{\mathfrak{I}}\right)$, and $s = \sqrt{\Im^2 + \frac{1}{\Im^2} - 1 - w^2}$

3.2.6 AIC and BIC Informations Criteria

To choose a model that provides a better fit to the normal, student-t and skewed student-t distribution, the study make use of the AIC (Akaike Information Criteria), BIC ((Bayes information criteria) and the log-likelihood. The value with the lowest value of the AIC and BIC indicates that the distribution gives a better fit for the model.

$$AIC_k = ln\left(\frac{SSR}{T}\right) + (k+1)\frac{2}{T} \quad (3.14)$$

$$BIC_k = Ln\left(\frac{SSR}{T}\right) + (k+1)\frac{lnT}{T}, \quad (3.15)$$

where SSR represents the total number of squared residuals. .

3.3 Model for Dependence

Some preliminaries concepts are defined before providing the definition of Copula.

Let $Q_i \subset \overline{\mathcal{R}}$, $i = 1, 2$ with $Q_i \neq 0$ and $\overline{\mathcal{R}} = \mathcal{R} \cup \{-\infty, +\infty\}$.

The domain J is given by: $Q_1 \ X \ Q_2 \longrightarrow \mathcal{R}$ function. We then define the J-volume of the function by:

$B = [y_{1,} y_2] \, X[x_1, x_2]$ as follows:

$V_J(B) = J(y_2, x_2) - J(y_2, x_1) - J(y_1, x_2) + J(y_1, x_1)$, where J is a 2-increasing function of $V_J(B)$, for every element B that belongs to the domain $Q_1 \ X \ Q_2$.

We have $V_J(B) \geq 0$, for every B ∈ $Q_1 \ X \ Q_2$. (3.16)

3.3.1 Definition

Let us denote max Q_i , $i = 1,\ 2$ by S_i , $i = 1,\ 2$. Since max Q_i are known. It is assume that and S_1 = max Q_1 and S_2 = max Q_2. The margins M and G of the function J are expressed as follows:

$$M: Q_1 \longrightarrow \mathcal{R}, M(x) + J(y, b_2), G: Q_2 \longrightarrow \mathcal{R}, G(x) + J(b_1, x), \tag{3.17}$$

where $b_i, i = 1, 2$ can take + ∞.

3.3.2 Definition

We set P_i = min Q_i, $i = 1,\ 2$.

J is grounded on condition that $J(P_1, x) = J(y, P_2) = 0$ for all $(x, y) \in Q_1 X Q_2$.

3.3.3 Remark

1. P_i can take -∞.

2. Given that J is increasing, $J(y_2, x_2) - J(y_1, x_2) \geq J(y_2, x_1) - J(y_1, x_1)$ (3.18)

and $J(y_2, x_2) - J(y_2, x_1) \geq J(y_1, x_2) - J(y_1, x_1)$ (3.19)

$\forall\ [y_1, y_2] * [x_1, x_2] \subset Q_1 X Q_2$.

With this in mind, the following result is obtained:

3.3.4 Lemma 1

Let $J : Q_1 X Q_2 \longrightarrow \mathcal{R}$ be grounded and a two –increasing function. Then J is non-decreasing in both arguments for every element in Q_1 and Q_2, with the properties: $x_1 \leq x_2$ and $y_1 \leq y_2$.

(3.20)

3.3.5 Lemma 2

Under the same hypotheses of Lemma1 above, we have

$|J(y_2, x_2) - J(y_1, x_1)| \leq |M(y_2) - M(y_1)| + |G(x_2) - G(x_1)| \quad \forall \quad [y_1, y_2] \quad X\ [x_1, x_2] \subset Q_1 X Q_2.$

(3.21)

3.3.6 Definition

Let $Q_i \subset [0,1], i = 1,2$ and let $C^/: Q_1 \, X \, Q_2 \longrightarrow \mathcal{R}$ be a grounded and 2-increasing function. $C^/$ is a 2-dimensional sub-Copula if for any $(u, v) \in Q_1 \, X \, Q_2$. We have

- $C^/(u, 1) = u,$
- $C^/(1, v) = v.$

The definition of the sub-Copula will be used to describe and define Copula in the section.

The bivariate Copula is referred to as, a sub-Copula with domain $[0, 1] \, X \, [0, 1]$.

Having defined Copula, the researcher should now introduce the main theorem about the existence of Copula.

3.3.7 Sklar's theorem

Sklar (1959) introduced the Sklar's theorem, the theorem states that any cumulative distribution function of random variable can be described by the marginal distribution functions and the dependence structure between the variable can be expressed as a Copula.

Let K be a bivariate distribution function with marginal distributions M and G, a Copula C exist if $K(x, y) = C\left(M(x) \, X \, G(y)\right)$. (3.22)

- If M and G are continuous marginal distribution functions, then the above Copula is unique satisfying:

$$C(u, v) = K\left(M^{-1}(u), G^{-1}(v)\right), \tag{3.23}$$

- On condition that X and Y are continuous random variable with marginal distribution function M and G, then C is the joint distribution function for uniformly distributed random variables and

$$U = M(X) \text{ and } V = G(Y) \tag{3.24}$$

Assuming that the random vector $Y = (Y_1, \dots, Y_k)^n$ has a joint distribution M with continuous marginals distributions $M_1, \dots, M_k$, to find the Copula function that measure the dependence, the standard uniform transformations was utilised as:

$$M^{-1}(\alpha) = \inf\{y | M(y) \geq \alpha\}, \alpha \in (0, 1), \text{ then} \tag{3.25}$$

1. For any standard uniformly distributed $U \sim U(0, 1)$, $M^{-1}(U) \sim M$,

2. If M is continuous then the random variable $M(Y) \sim U(0,1)$.

The density of the Copula is given by:

$$F(X_1, \dots, X_n) = C[F_1(X_1), \dots, F_n(X_n)]. \tag{3.26}$$

For the joint cumulative distribution, the function implies that the joint probability density function satisfies the following:

$$\frac{f(X_1,\dots,X_n)}{f_1(X_1),\dots,f_n(X_n)} = C[F_1(X_1), \dots, F_n(X_n)], \tag{3.27}$$

where $C(.)$ is the probability density function of the Copula.

The density of the Copula is given by:

$$C(u_1, \dots, u_n) = \frac{\partial n}{\partial u_1 \dots \partial u_n} C(u_1, \dots, u_n). \tag{3.28}$$

3.3.8 Invariance (Embrechts et al., 2003:6)

The dependence structure of Copulas is invariant under monotone transformation of the marginal distribution. Let X_1 and X_2 random variables with distribution functions F_i and a Copula C. Let the functions: Y_1= h_1 (X_1), Y_2= $h_2(X_2)$ be increasing functions of random variables X_1 and X_2.

Let Y_1^{-1}, Y_2^{-1} be the inverse function of Y_1 and Y_2 , then the random variable Y_1= $h_1(X_1)$, $Y_2 = h_2(X_2)$ have the same Copula. Unlike correlation, which changes under strict transformation of margins, this theorem demonstrates that Copula is invariant under strictly increasing modification of the margin.

3.3.9 Frechet-Hoeffding bounds

Let Y_1, Y_2 be random variables with marginal distribution functions M and G, the Frechet-Hoeffding bounds are given by:

$$\max\{M(x) + G(y) - 1, 0\} \leq F(x,y) \leq \min\{M(x), G(y)\} \tag{3.29}$$

The upper bound described the perfect positive dependence while the lower bound corresponds to a perfect negative dependence.

3.3.10 Comonotonicity

Comonotonicity is referred to as the upper limit of the Frechet-Hoeffdings bound. For a given(x_1, y_1),(x_2, y_2) a comonotonicity set met the following criteria:

$$\begin{cases} x_1 \leq y_1 \\ x_2 \leq y_2 \end{cases} \text{or} \begin{cases} x_1 \geq y_1 \\ x_2 \geq y_2 \end{cases} \tag{3.30}$$

The upper limit of the Frechet-Hoeffdings bound is given by:

$M(x_1, x_2) = \min\{x_1, x_2\}$.

3.3.11 Countermonotonic

The random variables are countermonotonic if they have the lower limit of the Frechet-Hoeffdings bounds:

$W(x_1, x_2) = \max\{x_1 + x_2 - 1, 0\}$.

3.3.12 Independence

For random variables (X, Y) with margins and a Copula C, X and Y are independent if:

$$C\big(F_1(x), F_2(y)\big) = F_1(x)\, F_2(y). \tag{3.31}$$

These characteristics of Copulas including invariance, Frechet-Hoeffding bounds, comonotonicity, countermonotonic and independence demonstrate that Copulas methods are more flexible in the analysis of dependence compared to Correlation. Correlation as one of the measures is not invariant.

3.4 Types of Copulas

The well-known families of Copulas include the Cuadras–Ange Copulas, elliptical and Archimedean Copulas. Elliptical Copulas which are constructed using the multivariate normal distribution and include the Normal and Student-t Copula while Archimedean Copulas include Frank, Gumbel and Clayton Copulas are determsined by the generator functions.

3.4.1 Cuadras-Ange Copulas

$$C_{\alpha,\beta}\ (u,v) = \min\left(u^{1-\alpha}\ v,\ u\ v^{1-\beta}\right) \text{ with } \alpha,\beta \in [0,1]. \tag{3.32}$$

3.4.2 Elliptical Copula

Elliptical Copulas which includes the Gaussian (Normal) Copula and the Student-t Copula. These Copulas are derived from the Sklar theorem through the Gaussian multivariate distribution and multivariate t distribution respectively. This class of Copula have the same characteristics as Gaussian distribution. Elliptical Copulas are generated from the multivariate elliptical distribution. Distribution that has the following density function is referred to as elliptical distribution:

$f(x)$ = $|\Sigma|^{\frac{1}{2}}$ g$\left[(x\ \ u)^T\ \Sigma^{/}\ (x\ \ u)\right]$, x $\epsilon\ \mathcal{R}^n$, Σ is the symmetric positive semi-define matric, g is the density generator and $u\ \epsilon\ \mathcal{R}^n$ is the location.

If g(x) = $\left(1+\frac{t_x}{v}\right)^{\frac{2+v}{2}}$ yields the student-t distribution with v degree of freedom,

moreover, if g$(x) = \frac{1}{2\pi}\, exp^{\left(\frac{x}{2}\right)}$ leads to the normal distribution. The most know elliptical family of Copulas include normal and student-t Copulas. Elliptical Copulas are constructed using the probability integral transform to margins of the given multivariate elliptical distribution.

3.4.2.1 Normal Copula

The Gaussian or normal Copula simulates the dependence structure using the multivariate normal distribution. The Gaussian Copula accounts for dependence between Gaussian distribution, it is a symmetric Copula and does not have tail dependence, which is zero unless the correlation matrix is 1 (Cheung, 2009). Therefore, it is unable to describe asymmetric dependence. However, the Gaussian Copula capture positive and negative correlation.

The bivariate normal Copula is given by:

$$C(u_1,u_2) = \emptyset_p\left(\theta^{-1}(u_1),\theta^{-1}\ (u_2)\right)$$

$$= \int_{-\infty}^{\emptyset^{-1}(u_1)} \int_{-\infty}^{\emptyset^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\theta^2}} \; exp\left(\frac{-S^2-2\theta St+t^2}{2(1-\theta^2)}\right) dsdt, \qquad (3.33)$$

$w$here θ is the correlation coefficient and $\emptyset$ is the univariate standard normal distribution.

3.4.2.2 Student-t Copula

The Student-t Copula is an alternative to Gaussian Copula. It is a symmetric Copula which has two parameters including the correlation parameter and the degree of freedom. The Student-t Copula accounts for positive tail dependence and negative dependence between variables.

Let R be a symmetric, positive matrix with Copula $dia(R) = (1,1,1,\dots,1)^T$ and $t_{R,\nu}$ the standard student-t distribution with correlation matrix (R) and the degree of freedom ν is given by (Cherubini et al., 2004) ;

$$C(u_1, u_2) = t_{\nu,p}\ \left(t_v^{-1}(u_1), t_v^{-1}(u_2)\right) \qquad (3.34)$$

$$= \int_{-\infty}^{t_v^{-1}(u_1)} \int_{-\infty}^{t_v^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-p^2}} \exp\left\{1 + \frac{x^2-2pxy+y^2}{(1-p^2)}\right\}^{-\frac{v+2}{2}} dxdy$$

where $t_v^{-1}$the quantile is function from the t distribution and ν is the degree of freedom.

3.4.3 Archimedean's Copulas

An Archimedean Copula is defined as follows:

$$Q(u_1, \dots, u_n) = \begin{cases} \varphi^{-1}\left(\varphi(u_1) + \cdots + \varphi(u_n)\right) \\ 0 \end{cases} \qquad (3.35)$$

If $\sum_{k=1}^{n} \varphi(u_k) \le \varphi(0)$ otherwise, $\varphi(u)$ is Q^2 function with $\varphi(1) = 0$,

where $\varphi(n)$ represents the generator of the Copula.

Characteristics of Archimdeans Copulas are:

- $\varphi^{/}(u) < 0$: (φ is decreasing),

- $\varphi^{//}\ \ (u) > 0$: φ is convex for all $0 \leq n \leq 1$.

For all continuous and descrasing convex function $: [0,1] \rightarrow [0,\infty)$, $\varphi(1) = 0$ is refered to as a generator for an Archimedean Copula. The density of an Archimedean Copula with a generator is given by:

$$D^A(u_1,u_2) = \frac{-\theta^{//}(C(u_1,u_2))\varphi^{/}(u_1)\varphi^{/}(u_2)}{[\theta^{/}(C(u_1,u_2))]^3} \tag{3.36}$$

A number of Copulas including Clayton, Gumbel and Frank belong to the family of Archimedeans Copulas.

3.4.3.1 Clayton Copula

Clayton Copula, which is an asymmetric and one parameter Archimedean Copula was introduced by Clayton in 1978. Clayton Copula describes lower tail dependence and capture only positive dependence (Trivedi & Zimmer, 2005). The bivariate Clayton Copula is given by:

$$C(u_1,u_2) = \left(u_1{}^{-\vartheta} + u_2{}^{-\vartheta} - 1\right)^{-1/\vartheta}, \tag{3.37}$$

with $\vartheta \in (0,\infty)$.

3.4.3.2 Gumbel Copula

Named after Gumbel (1960), this Archimedean Copula captures positive right tail dependence and does not account for negative tail dependence). The bivariate Gumbel Copula is expressed by:

$$C(u_1,u_2) = Exp\left\{-\left[(-\ln u_1)^{\vartheta} + (-\ln u_2)^{\vartheta}\right]^{1/\vartheta}\right\}, \tag{3.38}$$

with $\vartheta \in (1,\infty)$. ϑ is the parameter of the Gumbel Copula. When $\vartheta \longrightarrow 1$, the marginal become indenpedent, but when the parameter goes to infinity, the Gumbel Copula tends to be the Frechet-hoeffding upper bound.

3.4.3.3 Rotated Gumbel Copula

The rotated gumbel Copula is obtained by extended the range of dependence of the Gumbel copula using its density. For a given density of Gumbel Copula, a rotation of 90 degrees, 180 degrees and 270 degreees are as follows:

$$GC_{90}(u_1, u_2) = C(1 - u_1, u_2), \tag{3.39}$$

$$GC_{180}(u_1, u_2) = C(1 - u_1, 1 - u_2), \text{ and} \tag{3.40}$$

$$GC_{270}(u_1, u_2) = C(u_1, 1 - u_2). \tag{3.41}$$

3.4.3.4 Frank Copula

Developed by Frank (1979), Frank Copula is also an asymmetric Copula function of which the dependence parameter has a wide range of dependence. It is appropriate for modelling weak tail dependence and has been used in many applications including the dependence structure between the last survivor annuity contracts and the mortality of annuities (Frees & Valdez, 1998). The bivariate Gumbel Copula is defined as:

$$C(u_1, u_2) = -\frac{1}{\vartheta}\ln\left(1 + \frac{exp(-\vartheta u_1) - 1)\exp(-\vartheta u_2) - 1)}{exp(-\vartheta) - 1}\right), \tag{3.42}$$

with $\theta \in (-\infty, +\infty)$, the parameter of the Frank Copula.

3.4.3.5 Archimedean Copula and dependence measure

For a given X, Y continuous random variable with Archimedean Copula C and generator φ, the Kendall's Tau is as follows (Joe, 1997):

$$\rho_\tau(X, Y) = 1 + 4\int_0^1 \frac{\varphi(t)}{\varphi^{/}(t)}\, dt. \tag{3.43}$$

3.4.4 Plackett Copula

Plackett introduced the Plackett Copulas in 1965. The family of plackett Copula is a one-parameter Copula function and is similar to elliptical Copulas. The Plackett Copula is given by:

$$c(u_1, u_2) = \frac{1 + (\theta - 1)(u_1 + u_2) - \sqrt{[1 + (\theta - 1)(u_1 + u_2)]^2 - 4\theta(\theta - 1)u_1 u_2}}{2(\theta - 1)},$$

(3.44)

for $\theta > 0$.

3.4.5 Symmetric Joe-Clayton Copula

The symmetric is a combination of Joe (1993) and Clayton Copulas (Patton, 2001). The symmetric Joe-Clayton Copula accounts for upper and lower tail dependence.

$$C(u,v) = \left\{ {u_1}^{-x} + {u_2}^{-x} - 1 - [({u_1}^{-x} - 1)^{-y} + ({u_2}^{-x} - 1)^{-y}]^{-\frac{1}{y}} \right\}.^{-\frac{1}{x}} \quad (3.45)$$

For $x \geq 0$ and $y > 0$.

3.5 Measures of Dependence Associated with Copulas

For a given pair of continuous random variables (X, Y) with continuous marginal distribution and the Copula C, the Spearman's Rho and Kendall's tau by means of Copulas are expressed respectively by (Cherubini et al., 2004):

3.5.1 Spearman

$$SPEARMAN(X,Y) = 12 \int_0^1 \int_0^1 \{C(u_1,u_2) - u_1 u_2\}\, du_1 du_2 \quad (3.46)$$

$$= 12 \int_0^1 \int_0^1 C(u_1,u_2)\, du_1 du_2 - 3.$$

3.5.2 Kendall

$$KENDALL(X,Y) = 4 \int_0^1 \int_0^1 C(u_1,u_1)\; dC\,(u_1,u_2) - 1. \quad (3.47)$$

Where $C(u_1, u_2)$ is the Copula of the bivariate distribution function of the random variables X and Y. These measures are also referred to as measures of the degree of monotonic dependence between the random variables X and Y.

3.5.3 Tail Dependence

The tail dependence is a measure of dependence that account for the concordance on the tail of the joint distribution of the random variables. It captures the dependence in the lower and upper quadrant of the joint distribution.

The coefficients of upper tail and lower tail dependence for continuous random variables with marginal distribution functions $M_1$and M_2 are given by (Nelsen, 2006: 214):

$$\text{Upper limit (UL)} = \lim_{i\to 1} Pr[X_2 > M_2^{-1}(i)|X_1 > M_1^{-1}(i)] \quad (3.48)$$

$$= \lim_{i\to 1} Pr[X_1 > M_1^{-1}(i)|X_2 > M_2^{-1}(i)]$$

$$= \lim_{i\to 1} \frac{(1-2i+C(i,i))}{1-i},$$

with UL $\in$ $[0,1]$ and $M^{-1}(i) = \inf \{x|M(x) \geq i\}$, $i \in (0,1)$.

$$\text{Lower limit (LL)} = \lim_{i\to 0} Pr[X_2 \leq M_2^{-1}(i)|X_1 \leq M_1^{-1}(i)] \quad (3.49)$$

$$= \lim_{i\to 0} Pr[X_1 \leq M_1^{-1}(i)|X_2 \leq M_2^{-1}(i)]$$

$$= \lim_{i\to 0} \frac{C(i, \ i)}{i}$$

With LL$\in$ $[0,1]$.

The tail dependence measures do not depend on the marginal distribution of the random variables and the tail dependence does not vary under strictly monotone transformations of random variables X and Y. The random variables X and Y are asymptotically independent if the Upper limit (UL) = Lower limit (LL) = 0.

3.6 Copula Estimation

For Copula estimation, three estimation methods including the exact maximum likelihood estimation and Canonical Maximum likelihood and the maximum likelihood estimation are used.

3.6.1 Maximum likelihood Estimation Method

The maximum likelihood estimation method is one of the most used estimation method for Copulas.

From the Sklar theorem, it is known that:

$$F(x_1, \dots, x_k) = C\{F_1(x_1), \dots, F_k(x_k)\} \quad (3.50)$$

in addition, the density of the random vector $X = (X_1, \dots, X_d)^T$ is given by:

$$f(x_1, \dots, x_K: \lambda_1, \dots, \lambda_K, \theta) = C\{F_{X1}(x_1, \lambda_1), \dots, F_{Xk}(x_k, \lambda_k); \theta\} . \prod_{j=1}^{d} f_j\left(x_j, \lambda_i\right), \quad (3.51)$$

where f_i is the density of the marginal distribution M_i and C is the density of the Copula given by:

$$c(u_1, \dots, u_k; \vartheta) = \frac{\partial^d C\,(u_1, \dots, u_d; \vartheta)}{\partial u_1, \dots, \partial u_d} \quad (3.52)$$

ϑ is the vector of the Copula parameters.

If the parameter $= (\lambda_1, \dots, \lambda_k, \theta)^T \in \mathcal{R}^{k+1}$, then the likelihood function can be express as:

$$L(\alpha;\ x_1, \dots, x_T) = \prod_{t=1}^{T} f\left(x_{1,t}, \dots, x_{k,t}; \lambda_1, \dots, \lambda_k, \theta\right) \quad (3.53)$$

The combination of (3.51) and (3.53) gives the corresponding likelihood function

$$l(\alpha; x_1, \dots, x_T) = \sum_{t=1}^{T} ln\left[C\left\{F_{x1}\left(x_{1,t}: \lambda_1\right), \dots, F_{xk}\left(x_{k,t}: \lambda_k\right); \theta\right\}\right] + \sum_{t=1}^{T}\sum_{j=1}^{k} ln\left[f_j\left(x_{j,t}: \lambda_j\right)\right]$$

(3.54)

$$\hat{\lambda}_j = arg\max_{\lambda} L_j\left(\lambda_j\right), \quad (3.55)$$

where $l_j\left(\lambda_j\right) = \sum_{i=1}^{T} \ln f_j\ \left[x_{j,t}: \lambda_j\right]$ is the likelihood function of every marginal distribution for $j = 1, 2, \dots, k$.

The estimated parameters are given by:

$$l\left(\theta, \hat{\lambda}_1, \dots, \hat{\lambda}_k\right) = \sum_{t=1}^{T} ln\left[C\left\{F_{X1}\left(x_{1,t};\ \hat{\lambda}_1\right), \dots, F_{Xk}\left(x_{k,t}; \hat{\lambda}_k; \theta\right)\right\}\right], \quad (3.56)$$

with $j = 1, \dots, k$.

$\hat{\alpha}_{IFM} = \left(\hat{\lambda}_1, \dots, \hat{\lambda}_k, \hat{\theta}\right)^T$ is found by solving the

$\frac{\partial l_1}{\partial \lambda_1}, \dots, \frac{\partial l_k}{\partial \lambda_k}, \frac{\partial l}{\partial \theta} = 0.$

3.6.2 Inference for Margins Method (IMF)

The Log-likelihood function in (3.54) has two parts, Joe and Xu (1996) proposed the inference for the margins (IMF), an estimation method that consists in estimating the optimal set of parameters by the margins parameters and the Copula respectively. The inference for margins estimate the parameters as follows:

$$\theta_1 = \underset{\theta_2}{\text{argmax}} \sum_{t=1}^{T} \sum_{i=1}^{n} \log f_i \, (x_{it}; \theta_1). \tag{3.57}$$

$$\theta_2 = \underset{\theta_1}{\text{argmax}} \sum_{i=1}^{T} Log \; C(F_1(x_{1t}), \dots, F_n \, (x_{nt}); \; \theta_1, \theta_2). \tag{3.58}$$

Set $\theta_{IFM} := (\theta_1, \theta_2)$ to be IFM estimators.

3.6.3 The Canonical Maximum Likelihood Method

The Canonical maximum likelihood estimation process is carried out as follows:

1. Transforming the observations $(x_{1s}, \dots, x_{Ns})$, s = 1...S into uniform variates via the empirical distribution
$\hat{F}_n(.)$ given by:

$$\hat{F}_n(.) = \frac{1}{S} \sum_{s=1}^{S} 1_{\{X_{ns} \leq .\}}, \tag{3.59}$$

where $1_{\{X_{ns} \leq .\}}$ is the indicator function

2. Estimating the Copula parameters by the following:

$$\hat{\vartheta} = \text{arg max} \sum_{s=1}^{S} Log \; \left(c\left(\hat{u}_{1,s}, \dots, \; \hat{u}_{n,s}\right); \; \vartheta\right). \tag{3.60}$$

3.6.4 Non-Parametric Estimation Method

Introduced by Deheuvels (1979), this approach is based on the theory of the population subset.. One of the advantages of this estimation method is that no assumptions are made on the margin distributions, but it is based on the multivariate distribution of the parameters.

A random vector X_i, for $i = 1, \dots, n$ has a Copula C with continuous margins F_j,

Let $X = \{(x_1^t, \dots, x_n^t)\}_{t=1}^{T}$ be ordered statistics with rank given by: $\left(x_1^{(t)}, \dots, x_n^{(t)}\right)$, (3.61)

with $t = 1, \dots, T$. then the empirical Copula is defined on a function called lattice.

The lattice function is given by:

$$\left\{\left(\frac{t_1}{T},\dots,\frac{t_n}{T}\right): 1 \geq j \leq n, t_j = 0,1,\dots,T\right\} \quad (3.62)$$

In addition, the estimate of the Copula (Deheuvels, 1979) function is given:

$$\left(\frac{t_1}{T},\dots,\frac{t_n}{T}\right) = \frac{1}{T}\sum_{t=1}^{T}\prod_{j=1}^{n} 1\left(r_j^t \leq t_j\right), \quad (3.63)$$

where 1 is an indicator function.

3.7 Copula Selection

Different tests are used to choose the Copula which give a btter fit to the observations including the goodness of fit tests, Akaike information criteria *(AIC*) and the Bayesian information criterion *(BIC*).

3.7.1 Goodness of Fit Test

In this study, the suitability of the parameters of Copula is tested based on the two hypothesis;

Null Hypothesis: $H_0: B \in \{B_\theta, \theta \in \Theta\}$ versus

The complement Hypothesis: $H_1: B \notin \{B_\theta, \theta \in \Theta\}$.

The aim is to compute the distance between a parametric estimate and a non-parametric estimate that is given by the following:

$$B_n(p) = \sqrt{n}\left(\hat{B}_n(p) - B_{\hat{\theta}_n}(p)\right) = (p_1,\dots,p_d) \in [0,1]^d, \quad (3.64)$$

where $\hat{B}_n(p)$ denotes the so-called empirical Copula. As stated by Segers (2012) and Tsukahara (2005), the empirical Copula is an unbiased estimator of B under negligible circumstance.

$$\hat{B}_n(p) = \frac{1}{n}\sum_{i=1}^{n} \mathrm{I}(\hat{p}_{i1} \leq p_1,\dots,\hat{p}_{id} \leq p_d) \quad (3.65)$$

$\hat{P}_1 = \left(\hat{P}_{11},\dots,\hat{P}_{id}\right),\dots,\hat{P}_n = \left(\hat{P}_{n1},\dots,\hat{P}_{nd}\right)$ designates the pseudo-observations inferred from ranks:

$$\hat{P}_{ij} = \frac{1}{(n+1)} K_{ij} = \frac{n}{n+1}\hat{F}_j\left(Y_{ij}\right), i \in \{1,\dots,n\}, j \in \{1,\dots,d\}. \quad (3.66)$$

According to Genest et al. (2009), the pseudo-observations is a sample from the underlying Copula.

A rank-based version of the Cramer-Von is as follows:

$$S_n = \int_{[0,1]^d} \mathbb{C}_n \ (n)^2 \ d \ \hat{C}_n(p). \qquad (3.67)$$

A big value of S_n indicates a large disparity between the parameter and the empirical Copula.

3.7.2 Information Criteria

Two information criteria are used for Copula selection including AIC and BIC

$$AIC = -2\{Log(Likelihood) + k\}, \qquad (3.68)$$

where k is the number of the parameter of the model

$$BIC = -2 \ Log(Likelihood) + k. Log(n), \qquad (3.69)$$

where n is the sample size and k is the number of the parameter of the model.

The Copula that provides the smallest value of AIC or BIC is considered as the Copula that provides the better fit of the distribution.

3.8 Conditional Copula

According to Patton (2006), the conditional Copula of random variables X, Y given $Z = z$, where $X|Z = Z \backsim M_{X|Z} \ (.|Z)$ and $Y|Z = z \ \backsim M_{Y|Z} \ (.|Z)$ is the conditional joint distribution of $U_1 \equiv M_{X|Z} \ (X|Z)$ and $U_2 \equiv M_{Y|Z} \ (Y|Z)$, given that the variables U_1 and U_2 are conditional probability integral transform of X and Y.

3.8.1 Sklar Theorem for Conditional Copula

Let $M_{X|Z} \ (.|Z)$ be the conditional distribution of $X|Z = z$, let $M_{Y|Z} \ (.|Z)$ be the conditional distribution of $Y|Z = z$, and let $M_{XY|Z} \ (.|Z)$ be the joint conditional distribution of (X, Y)|Z = z, moreover, Z is the support of Z. Let assumed that $M_{X|Z} \ (.|Z)$ and $M_{Y|Z} \ (.|Z)$ are continuous in x and y ∀ z ∈ Z. Then a Copula $C(.|Z)$ exist such that:

$F_{XY|Z} \ (X, Y|Z) = C\big(F_{X|Z} \ (x, y), F_{Y|Z} \ (y, z)|z\big)$, for every $(x, y) \in \overline{\mathfrak{R} \ X} \ \overline{\mathfrak{R}}$ and ∀ z ∈ Z. (3.70)

3.8.2 The Time-Varying Normal Copula

The time varying Gaussian Copula is defined (Patton; 2006);

$$C(u, v\,|\rho) = \int_{-\infty}^{\emptyset^{-1}(u)} \int_{-\infty}^{\emptyset^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}}\, exp\left(\frac{-S^2-2\rho St+t^2}{2(1-\rho^2)}\right) dsdt. \qquad (3.71)$$

Following Patton (2006), it should be assumed that the Copula parameter follows an *ARMA* (1, 10) with a regressor to capture any persistence in the dependence parameter and the mean of the product of the last 10 observations of the transformed variables .

$\phi^{-1}\,(u)$ and $\phi^{-1}(v_{\,t-j})$. (3.72)

$$\delta_k = \overline{\mathrm{T}}\left\{W\,\delta + \beta\,\delta\,\delta_{k-1} + \,a\,\delta\,\frac{1}{10}\sum_{j=1}^{10}[\,\varphi^{-1}(U_{k-j})\,\varphi^{-1}(V_{k-j})]\,\right\}, \qquad (3.73)$$

where $\overline{\mathrm{T}}\,(x) \equiv (1-e^{-x})\;(1+e^{-x})^{-1} = \tan h\left(\frac{x}{2}\right)$ is the modified logistic transformation, δ_{k-1} account for the persistence effects, W, δ, β are the parameters of the Copula and $\varphi^{-1}(U_{k-j})\;\varphi^{-1}(V_{k-j})$ describes the variability in the dependence structure.

For time varying Copula, the marginal distribution are estimated using a non-parametric estimation method while Copula parameter are estimated through the empirical cumulative density function. The empirical cumulative density function for the random variable *X* can be estimated by (Patton, 2006):

$\hat{F}_x\,(X) = \frac{1}{n+1}\sum_{i=1}^{n} 1\;\,(X_t\;\leq X)$, where 1 is the indicator of the function. (3.74)

Let $U = \hat{F}_x\,(X)$ and $V = \hat{F}_y\,(y)$ be the empirical cumulative density function of the continuous random variables X and Y. The joint distribution function can be defined as:

$$C(u, v, \theta) = C\big(\hat{F}_x\,(X), \hat{F}_y\,(y){:}\,\theta\,\big), \qquad (3.75)$$

the Copula density is expressed as:

$$C(u, v, \theta) = \frac{\partial^2\,C(u,v,\theta)}{\partial u\,\partial v}, \qquad (3.76)$$

the log-likelihood function is given by:

$$L(x, y; \theta) = ln\big[C(u, v; \theta), \hat{F}_x\,(X), \hat{F}_y\,(y)\,\big], \qquad (3.77)$$

and the time-varying dependence parameter is given by:

$$\hat{\theta} = \text{arg max}\; L_c\left(x, y{:}\,\theta, \hat{F}_x\,(X), \hat{F}_y\,(y)\right). \qquad (3.78)$$

Chapter Four: Presentation and Discussion of Results

This Chapter presents the summary statistics including graphical representations of the prices of gold, platinum and the Rand/U.S.D exchange rate. It provides the statistical description of the return distribution of gold, platinum prices and Rand/U.S.D exchange rate including the descriptives as well as the Jacque Bera test and the Quantile-Quantile (Q-Q) plots. The Chapter also presents the marginal distributions, the estimated parameters as well as the statistics including the p-values of the Ljung-box standard residual tests and Lagrange multiplier *ARCH* tests. The Chapter then discusses the results of dependence parameters of the Copulas-*EGARCH* and Copulas-*APARCH*.

4.1 Summary Statistics

4.1.1 Graphical Representation of the Data

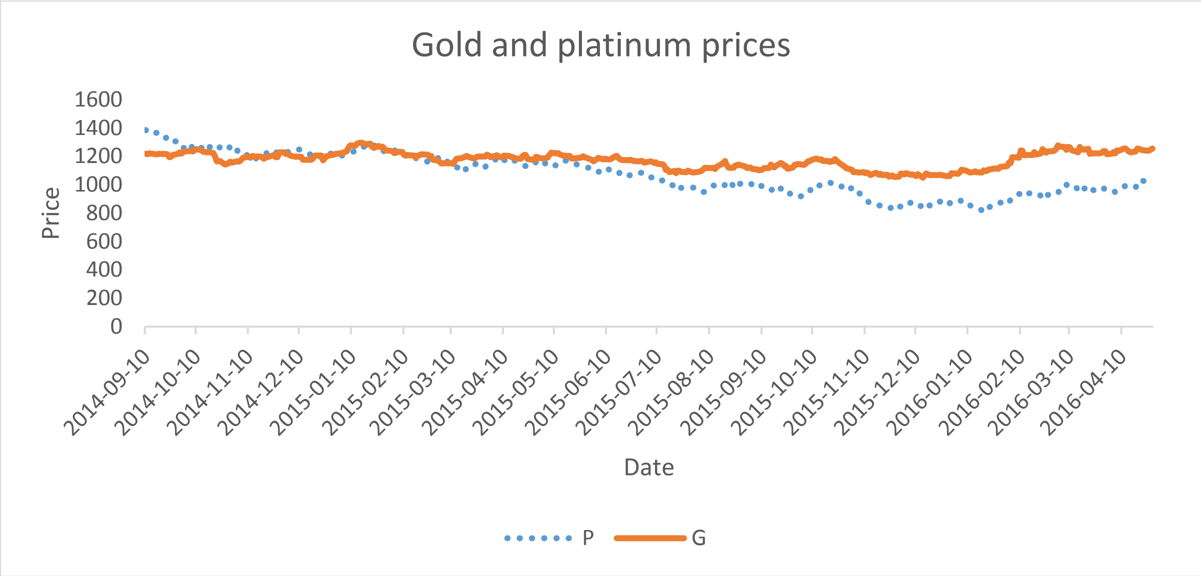


***Figure 4.1**: Daily Prices of Gold and Platinum from 2014 to 2016*

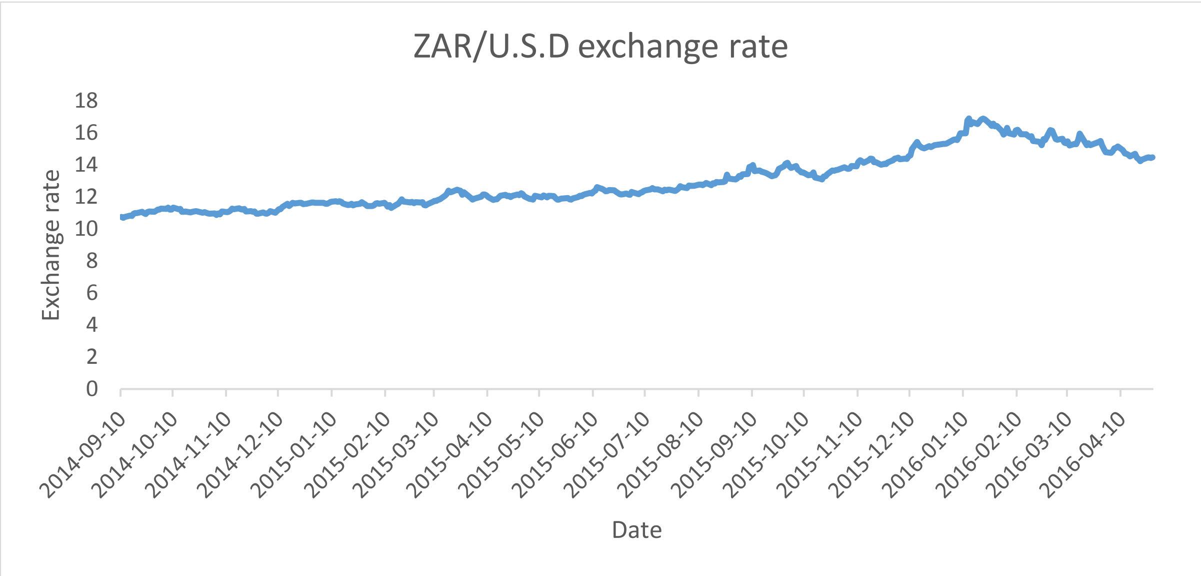


***Figure 4.2**: Daily ZAR/U.S D exchange rate from 2014 to 2016*

The summary of the data in Figures 4.1 and 4.2 show that the variance of prices of gold and platinum, and the ZAR/U.S.D exchange rate varies over time; an indication that the series are non-stationary. Therefore, the prices of gold, platinum, and the ZAR/U.S.D exchange rate are converted into returns using the logarithm. The return is then calculated as follows:

$$R_t = 100 (\ln P_t) - (\ln P_{t-1}), \tag{4.1}$$

where R_t is return prices at a time t, P_t is the prices at a time t, and P_{t-1} is the the prices at a time $t - 1$. From equation (4.1). the prices of platinum are used to compute the return prices of platinum. The same procedure was followed for the return prices of gold and the return on the Rand/U.S.D exchange rate.

4.1.2 Returns Prices of Gold, Platinum, and the Rand/U.S.D Exchange Rate.

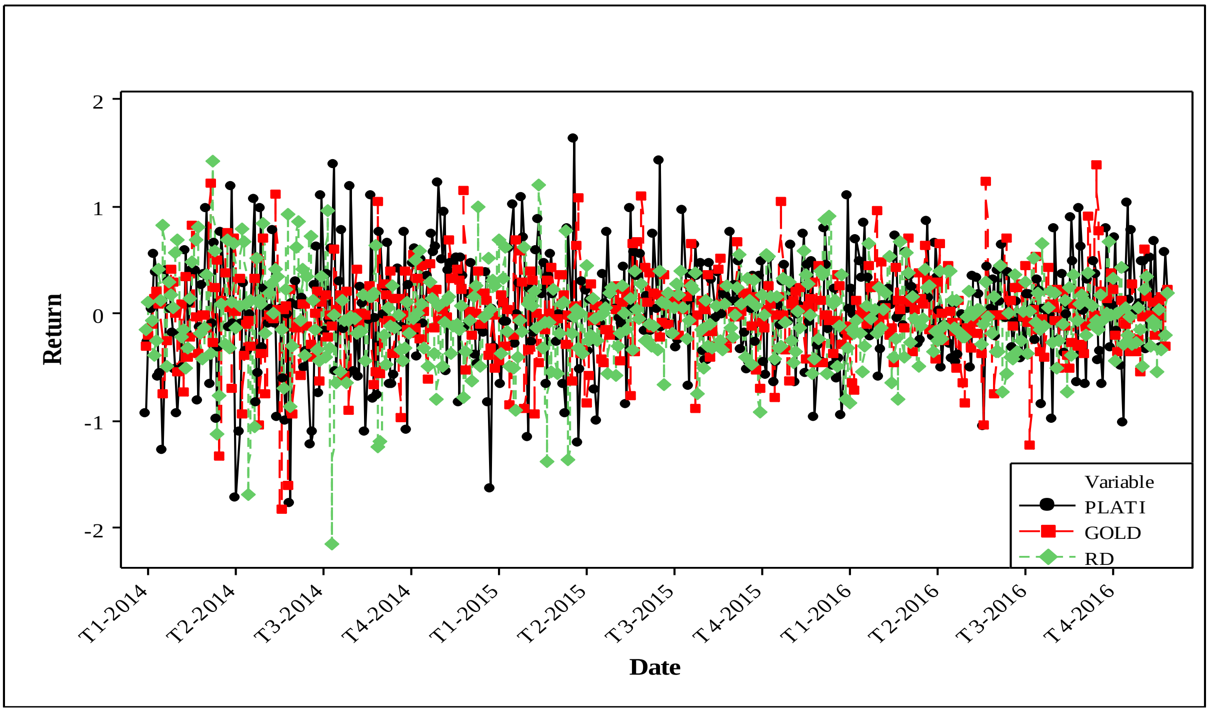


Figure 4.3: *Returns Prices of Gold, Platinum and the ZAR/U.S.D Exchange Rate.*

4.1.3 Summary statistics

Table 4.1: *Summary Statistics: Returns Prices of Gold, Platinum and the Rand/U.S.D Exchange Rate.*

	Gold	Platinum	Exchange rate
Mean	-0.0030	0.0294	-0.0310
Mode	1.3839	1.6358	1.4266
Median	0.0000	0.0383	-0.0335
SD	0.4168	0.5292	0.4192
Skewness	-0.2253	-0.2112	-0.3858
Kurtosis	4.8877	3.4021	5.3048
Sum	-1.2701	12.3165	12.9763
Jacques Bera	65.761	5.9374	103.1349
Probability	0.0000	0.0514	0.0000
Sample size	419	419	419

Table 4.1 shows that the mean, the mode and the median of the returns prices of gold, platinum and the ZAR/U.S.D exchange rate are not the same. The result implies that returns prices of gold, platinum and the ZAR/US.D exchange rate may not have the same distribution. The summary statistics revealed that skewness and the kurtosis values are respectively: -0.2253 and

4.8877 for return prices of gold, -0.2112 and 3.4021 for return prices of platinum and -0.3858 and 5.3048 for ZAR/U.S.D exchange rate. These results illustrate that returns distributions of the prices of gold, platinum and the exchange rate of the ZAR/U.S.D are skewed and exhibit asymmetric distributions. Furthermore, the values of the kurtosis are greater than 3, which is an indication that return prices have heavy tails characteristics. In addition, the Jacques Bera test shows that returns prices of gold and the Rand/U.S.D exchange rate are not normally distributed. The Jacque Bera test for platinum return prices indicates that the return distribution is asymmetric but closer to a symmetric distribution.

4.1.4 Q-Q Plots of Returns Prices of Gold, Platinum, and the ZAR/U.S.D Exchange Rate

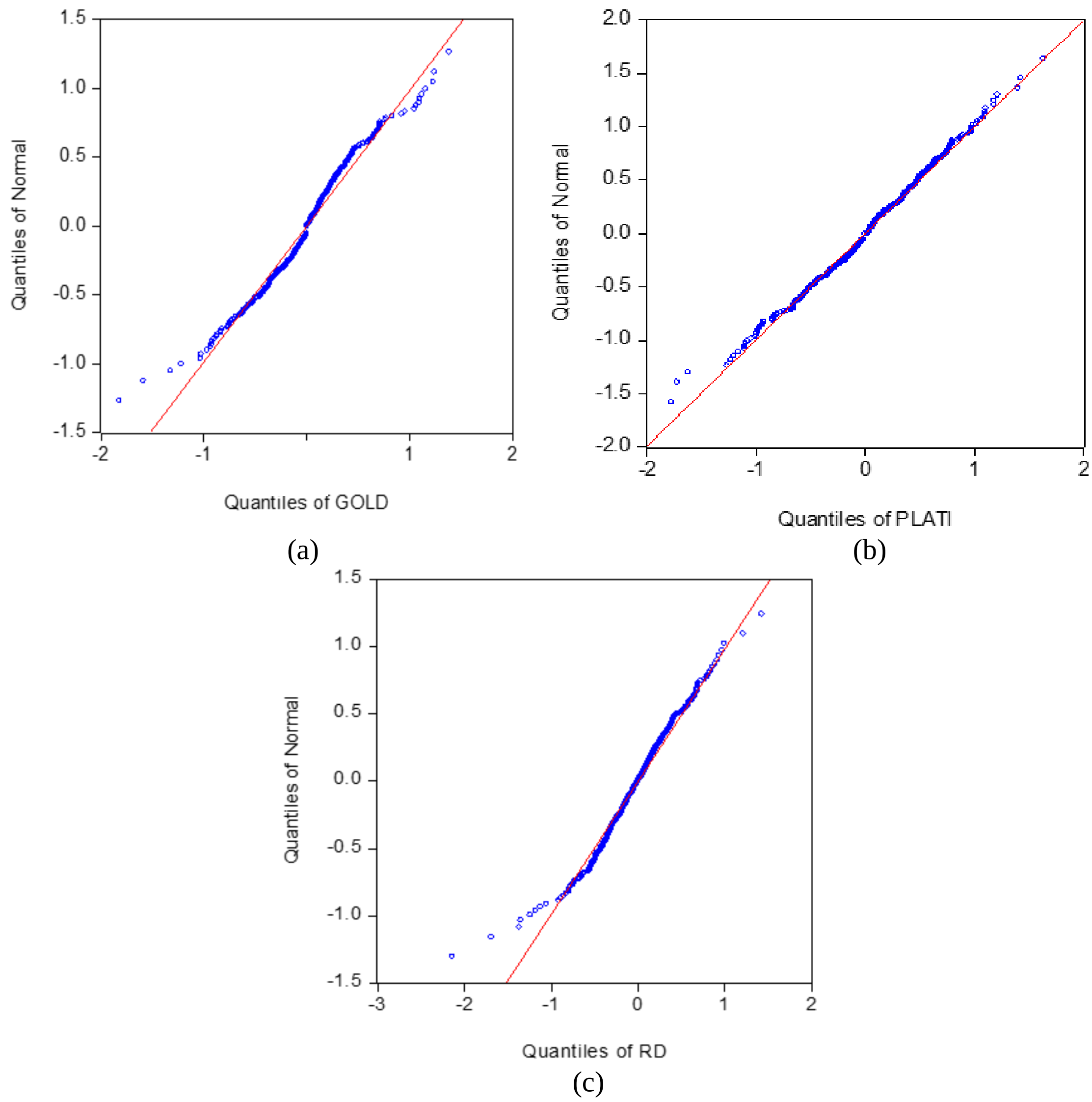


***Figure 4. 4**: Q-Q Plots of Returns Prices of Gold, Platinum and the Rand/U.S.D Exchange rate.*

Figures 4.4 (a) and (c) indicate that the distribution of the returns price of gold and the Rand/ U.S.D exchange rate are asymmetric and exhibits heavy tail distributions. However, Figure 4.4 (b) indicates that the platinum return distribution is asymmetric but approaching a normal distribution. The summary statistics, the Jacque Bera test and the Q-Q plot indicate that the hypothesis of normality can be rejected for returns prices of gold and Rand/U.S.D exchange rate. The skewness values indicate that return distributions are negatively skewed. However, for the distribution of return prices of platinum, the skewness value illustrate that the distribution is skewed but close to a normal distribution.

4.2 Copulas

The Copulas-*EGARCH* and Copulas-*APARCH* of the dependence structure analyses are subdivided into 3 sections: the marginal distributions using the *ARMA-EGARCH* and *ARMA-APARCH* filters, the leverage effects and the bivariate dependence structure using Copulas parameters.

4.2.1 Marginal Distributions

A Copula allows the joint distribution of random variables to be described as function of marginal distribution. Furthermore, a Copula separates marginal distributions from the dependence structure (Patton, 2006). In this case, a marginal distribution that describes the characteristics of returns prices of gold, platinum and the ZAR/U.S Dollar exchange rate is needed.

Previous studies including the work of Shams and Zarshenas (2014), and Pettersson (2010) adopted the *GARCH* (1, 1) model as a marginal distribution of Copulas when investigating the relationship between commodities and currencies. This study extends the work of Shams and Zarshenas (2014), and Pettersson (2010) by using two asymmetric *GARCH* models including the *EGARCH* and *APARCH* models, most likely models that account for asymmetric conditional heteroscedasticity and leverage effects (Nelson, 1991; Ding et al., 1993).

Financial times series are known to be autocorrelated and often heteroscedastic. Therefore, an appropriate filtration of the data is necessary. To make use of Copulas for the analysis of the dependence structure between gold, platinum prices and ZAR/U.S.D exchange rate, it is assumed that marginal distributions followed an *ARMA* (p, q)-*EGARCH* (1, 1) and *ARMA* (p, q)-*APARCH* (1, 1) under three different errors terms: the normal, student-t and the skewed student-t distributions. The three error terms are used for their capabilities to capture the characteristic and the aspect of distribution including heavy tail and asymmetric behaviour.

4.2.1.1 Selection of *ARMA (p, q)* Model

To find an adequate autoregressive moving average (*ARMA*) model, the *ARMA* (p,q) was fitted to the returns prices of gold, platinum and the exchange rate of the Rand/U.S.D. The information criteria *AIC* and *BIC,* were used to select the *ARMA* (p,q) model that provides a better fit for the mean equation. Comparing the *AIC* and *BIC*, it was found from Tables: 4.2(a), (b) and (c) that the model *ARMA (1, 1)* shows a good fit for the mean equation and is an adequate model for returns prices of gold , platinum and exchange rate.

Table 4.2 (a): ARMA Selection (Return Prices Platinum)

ARMA (p, q)	**AIC**	**BIC**
ARMA (1, 0)	2.2385	2.2399
ARMA (0, 1)	2.2274	2.2344
ARMA (1, 1)	**2.2097**	**2.2151**
ARMA (0, 2)	2.2104	2.2207
ARMA (1, 2)	2.2203	2.2306
ARMA (2, 1)	2.2308	2.2209
ARMA (2, 2)	2.2304	2.2343

Table 4.2(b): ARMA Selection (Return Prices Gold)

ARMA (p, q)	**AIC**	**BIC**
ARMA (1, 0)	2.2285	2.2195
ARMA (0, 1)	2.2174	2.2246
ARMA (1, 1)	**2.2097**	**2.2053**
ARMA (0, 2)	2.2104	2.2106
ARMA (1, 2)	2.2313	2.2305
ARMA (2, 1)	2.2418	2.2304
ARMA (2, 2)	2.2414	2.2341

Table 4.2(c): ARMA Selection Return on RAND/U.S.D Exchange Rate

ARMA (p, q)	AIC	BIC
ARMA (1, 0)	2.2273	2.2195
ARMA (0, 1)	2.2175	2.2246
ARMA (1, 1)	**2.2107**	**2.2147**
ARMA (0, 2)	2.2109	2.2296
ARMA (1, 2)	2.2293	2.2310
ARMA (2, 1)	2.2378	2.2304
ARMA (2, 2)	2.2334	2.2352

4.2.1.2 Estimation of *ARMA (1, 1)-EGARCH (1, 1)* model: Platinum

Having selected the appropriate *ARMA* model, two different filters: *EGARCH* (1, 1) and *APARCH* (1, 1) with the purpose of obtaining models that specify the characteristics of returns prices of gold, platinum and Rand/U.S.D exchange rate. To obtain standard residual and remove the autocorrelation effects from the returns distributions, the *ARMA (1, 1)-EGARCH (1, 1)* and *ARMA (1, 1)-APARCH* (1, 1) were fitted to the returns prices of gold, platinum and the Rand/US.D exchange rate. The study made use of the R.X64 3.2.1 software and maximum likelihood estimation method to estimate the parameters of the *ARMA (1, 1)-EGARCH (1, 1)* and *ARMA (1, 1)-APARCH (1, 1)* models.

Copulas require margins to be identical and independently distributed (*i.i.d*) observations (Boero et al., 2011). In this study, different tests including the Ljung-box test for standardized residuals, Lagrange Multiplier(LM) tests for *ARCH* effects, the *p*-value, the AIC and BIC criteria were used to compare the *ARMA (1, 1)* with *APARCH (1, 1)* and the *ARMA (1, 1)* with the *EGARCH (1, 1*) under the normal, student-t and skewed student-t distributions. Tables 4. 3, 4.4, 4.7, 4.8, 4.11 and 4.12 provide a summary of the estimated parameters of the marginal distributions while Tables 4.5, 4.6, 4.9 and 4.10 give the statistics and the *p*-values of the Lagrange Multiplier for ARCH effects and the Ljung-box statistics for standardized residual test obtained from models *ARMA (1, 1)-EGARCH (1, 1)* and *ARMA (1, 1)-APARCH (1, 1)* for returns prices of gold, platinum and the *ZAR/U.S.D* exchange rate. The values in parentheses in the tables are *p*-values of the estimated parameters, which are followed by the standard error values. The adequate marginal distribution was selected using the information criteria *AIC* and *BIC*, the significance of the parameters, the Lagrange multiplier test and the standardized residuals test.

***Table 4.3**: Estimated Parameters: ARMA (1, 1)-EGARCH (1, 1): Return Prices Platinum*

	Normal	**Student-t**	**Skew Student-t**
Mu	0.0243 (0.0000) 0.0000	0.4096 (0.0000) 0.0000	0.0100 (0.0000) 0.0000
AR1	0.2247 (0.0000) 0.0000	0.6351(0.0000) 0.0047	0.0139 (0.0000) 0.0000
MA1	0.2921 (0.0000) 0.0000	0.6892 (0.0000) 0.0000	0.0375 (0.0000) 0.0000
Omega	0.0170 (0.0000) 0.0000	0.0187 (0.0000) 0.0001	0.0119 (0.0000) 0.0000
Alpha 1	-0.7150 (0.0000) 0.0000	0.0751 (0.0000) .0000	0.0615 (0.0000) 0.0000
Beta 1	0.9897 (0.0000) 0.0000	0.9900(0.0000) 0.0004	0.9897 (0.0000) 0.0000
Gamma 1	0.0078 (0.0000) 0.0000	0.0762 (0.0000) 0.0000	0.0796 (0.0000) 0.0000
Shape		46.242 (0.0000) 0.3613	30.1499 (0.0000) 0.0022
Skew			0.9309 (0.0000) 0.0022
AIC	1.5018	1.5101	1.5113
BIC	1.5693	1.5876	1.5981
Log-likelihood	-307.633	-308.449	-307.6229

Table 4.3 shows that the p-values of all the estimated parameters are less than 0.05. These results imply that the estimated parameters of the model under the normal, student-t and skewed student-t distributions are statistically significant; hence, the estimates are good for the return prices of platinum. Furthermore, the table shows that the *AIC* and *BIC* values for *ARMA (1, 1) - EGARCH (1, 1)* under the normal error terms are the least when compared to those under student-t and skewed student-t error terms. There is thus enough evidence to suggest that the *ARMA (1, 1)-EGARCH (1, 1)* under normal error terms provides the best fit for the distribution of platinum return prices. It is thus a candidate for the marginal distribution.

4.2.1.3 Estimation of the *ARMA (1, 1)-APARCH (1, 1)* model (Platinum)

***Table 4.4**: Parameters for ARMA (1, 1)-APARCH (1, 1) Return Prices Platinum*

	Normal	Student-t	Skew Student-t
Mu	0.0357 (0.1683) 0.0259	0.0366 (0.1822) 0.0274	0.0325 (0.2142) 0.0262
AR1	-0.2918 (0.6388) 0.6217	-0.2961 (0.5164) 0.4562	-0.2527 (0.7172) 0.6977
ma1	0.3459 (0.5700) 0.6089	0.34923 (0.4349) 0.4473	0.3023 (0.6594) 0.6860
Omega	0.0005 (0.4064) 0.0006	0.0005 (0.5055) 0.0008	0.009 (0.4349) 0.0088
Alpha 1	0.0013 (0.6897) 0.0033	0.0014 (0.7740) 0.0050	0.0038 (0.6675) 0.0088
Beta 1	0.9783 (0.0000) 0.0045	0.9781 (0.0000) 0.0050	0.9751 (0.0000) 0.0042
Gamma 1	1 (0.0000) 0.0010	0.9618 (0.0000) 0.0239	0.9999 (0.0000) 0.0013
Delta	3.2899 (0.072) 1.8286	3.2884 (0.1142) 2.0820	2.6858 (0.1351) 1.7973
Shape		61.5594 (0.6767) 147.628	30.1499 (0.0000) 0.0670
Skew			0.9301 (0.0000) 0.0670
AIC	1.5423	1.5467	1.5491
BIC	1.6194	1.6334	1.6455
Log-likelihood	-315.1029	-315.036	-314.5333

Table 4.4 shows that the *p*-values of some of the parameters are greater than 0.05, thus, non-significant coupled with the fact that the *AIC* and *BIC* values in Table 4.3 are smaller than those in Table 4.4. It can therefore be concluded that there is not enough evidence to suggest that *ARMA (1, 1)-APARCH* (1, 1) under the normal, student-t and skewed student-t error terms is an adequate model for the return prices distribution of platinum. As a result, the model *ARMA (1, 1)-APARCH (1, 1)* is not selected as the marginal distribution.

4.2.1.4 Lagrange Multiplier and Standardized residual Tests: *ARMA-EGARCH (1, 1)* and *ARMA-APARCH (1, 1)* (Platinum)

Having selected the appropriate model for platinum return prices, the Lagrange Multiplier *ARCH* tests and Ljung box tests for standardized residuals were used to test for *ARCH* effects and correlation. These are displayed respectively in Tables 4.5 and 4.6 for *ARMA (1, 1)-EGARCH (1, 1)* and *ARMA (1, 1)-APARCH (1, 1)* under the normal, student-t and skewed student-t error terms. The values in parentheses are the p-values of the statistics.

Table 4. 5: ARCH-LM Tests: Return Prices Platinum

	Lag	**Normal**	**Student –t**	**Skew Student-t**
ARMA(1,1)-EGARCH (1, 1)	10	0.9549 (0.3285)	0.6393 (0.4239)	0.9549 (0.2277)
	15	2.5032 (0.3704)	2.1220 (0.4449)	3.009 (0.2884)
	20	3.3974 (0.4418)	2.8882 (0.5349)	4.002 (0.3181)
ARMA(1,1)-APARCH(1, 1)	10	1.365 (0.2426)	1.366 (0.2425)	1.601 (0.2057)
	15	2.406 (0.3883)	2.416 (0.3864)	2.889 (0.3064)
	20	4.402 (0.2932)	4.407 (0.2917)	4.890 (0.2359)

Table 4.5 shows that all the *p*-values of the *ARCH* tests are greater than 0.05, which means that all the parameters of the Lagrange Multiplier *ARCH* tests are not significant. The results imply that the hypothesis of presence of the *ARCH* effects is not accepted and there is no evidence of correlation. These findings illustrate that the conditional variance fits well the data.

Table 4. 6: Standardized Residual Tests: Return Prices Platinum

	lag	Normal	Student-t	Skew Student-t
ARMA(1,1)-EGARCH(1, 1)	10	0.0021 (0.9634)	0.0376 (0.8462)	0.0960 (0.7566)
	15	1.3922 (0.9992)	1.9772 (0.9622)	1.5982 (0.9957)
	20	4.2785 (0.6247)	4.5893 (0.5498)	4.6005 (0.5471)
ARMA(1,1)-APARCH(1, 1)	10	0.0913 (0.8432)	0.0321 (0.8579)	0.00429 (0.9478)
	15	1.5372 (0.9973)	1.5332 (0.9974)	1.6050 (0.9955)
	20	5.0178 (0.4496)	5.0083 (0.4517)	4.9514 (0.4647)

The analysis of the residuals for the *ARMA (1,1)-EGARCH (1, 1)* and *ARMA (1, 1)-APARCH (1, 1)* under the normal, student-t and skewed student-t error terms reported in Table 4.6 reveals that all the *p*-values are greater than 0.05, an indication that the statistics are not significant. The *p*-values provide enough evidence to infer that residuals obtained from the two representations under normal, student-t and skewed student-t error terms are white noise.

4.2.1.5 Estimation of the ARMA (1, 1)-EGARCH (1, 1) and ARMA (1, 1)-APARCH (1, 1) models (Gold)

Table 4.7: Estimated Parameters: ARMA (1, 1)-EGARCH (1, 1): Return Prices Gold

	Normal	Student-t	Skew Student-t
Mu	0.0021(0.9136) 0.0197	0.0101(0.6047) 0.0195	0.0083(0.7303) 0.0241
AR1	0.4772(0.0000) 0.0719	0.8478(0.0000) 0.0868	0.8520(0.0014) 0.2661
Ma1	-0.4952(0.0000) 0.0702	-0.8306(0.0000) 0.0901	-0.8333(0.0027) 0.2781
Omega	-0.0650(0.0000) 0.0031	-0.5261(0.5304) 0.8386	-0.4399(0.5691) 0.7727
Alpha 1	-0.0249(0.0000) 0.0060	0.0424(0.6492) 0.0931	0.3079(0.7581) 0.0999

Table 4.7: Estimated Parameters: ARMA (1, 1)-EGARCH (1, 1): Return Prices Gold (continued)

	Normal	Student-t	Skew Student-t
Beta 1	0.9780(0.0000) 0.0028	0.6702(0.1923) 0.5141	0.7204(0.1334) (0.4801)
Gamma 1	0.0462(0.1522) 0.3229	0.0329(0.0000) 0.1281	-0.0359(0.7525) 0.1139
Shape		3.5322(0.0000) -0.7958	3.4347(0.0000) 0.773
Skew			0.9225(0.0000) 0.5626
AIC	1.1024	1.0347	1.0351
BIC	1.1698	1.1118	1.1219
Log-likelihood	-223.944	-208.776	-207.8586

Table 4.8: Estimated Parameters: ARMA (1, 1)-APARCH (1, 1): Return Price Gold

	Normal	Student-t	Skew Student-t
Mu	0.0024 (0.8992) 0.0196	0.0085(0.0008) 0.0192	0.0072(0.7643) 0.0239
AR1	0.4011(0.7203) 1.1193	0.8320(0.0000) 0.1548	0.8412(0.0000) 0.1700
ma1	0.4077(0.7145) 1.1145	0.8105(0.0000) 0.16147	0.8165(0.0000) 0.1784
Omega	0.0004(0.11638) 0.0001	0.0003(0.0000) 0.1327	0.0003(0.1690) 0.0002
Alpha 1	0.0000(1.000) 0.0033	0.0000(0.0000) 0.0003	0.0000(1.0000) 0.0002
Beta 1	0.9862 (0.0000) 0.0036	0.9910(0.0000) 0.0022	0.9919(0.0000) 0.0020
Gamma 1	0.9998(0.0000) 0.0023	0.9996(0.0000) 0.0023	0.9994(0.0000) 0.0023
Delta	3.4998(0.0000) 0.3101	3.4988(0.0000) 0.529462	3.4988(0.0000) 0.5052
Shape		3.8648(0.0000) 0.9890	3.7254(0.0000) 0.9445
Skew			0.9278(0.0000) 0.0566
AIC	1.0937	1.0263	1.0274
BIC	1.1708	1.1131	1.1237
Log-likelihood	-221.1218	-206.017	-205.2362

The results displayed in Tables 4.7 and 4.8 show the estimated parameters, the p-value (in bracket) and the standard error of the estimates. The *AIC* and *BIC* criteria for the *ARMA (1,* *1)-APARCH (1, 1)* under the student-t error terms outperforms those with normal and skewed student-t error terms and all of the ARMA *(1, 1)-EGARCH (1, 1*) as it has the most minimum values. All the parameter estimates are significant to in *ARMA (1, 1)-APARCH (1, 1*) under student-t error terms. It can thus be concluded that the *ARMA (1, 1)-APARCH (1, 1)* with student-t error terms is the appropriate model for the distribution of gold price returns and thus selected for the marginal distribution. The next Table provides the Lagrange Multiplier *ARCH* test for and *ARMA (1, 1)-APARCH (1, 1).*

4.2.1.6 Lagrange Multiplier and Standardized Residual Tests for ARMA (1, 1)-EGARCH (1, 1) and ARMA (1, 1)-APARCH (1, 1) (Gold)

Table 4.9: ARCH-LM Tests: Return Price Gold

	Lag	Normal	Student- t	Skew Student-t
ARMA(1,1)-EGARCH (1, 1)	10	6.1006 (0.0134)	14.8100 (0.0002)	14.8500 (0.0002)
	15	6.263 (0.0521)	15.64 (0.0003)	15.68 (0.0002)
	20	6.564 (0.1076)	15.84 (0.0007)	15.90 (0.0007)
ARMA(1,1)-APARCH(1, 1)	10	7.513 (0.0062)	7.740 (0.0540)	7.622 (0.0576)
	15	7.693 (0.0238)	7.936 (0.0208)	7.824 (0.0222)
	20	8.690 (0.0367)	8.908 (0.3281)	8.782 (0.0350)

Table 4.9 indicates that the *p*-values (in brackets) are greater than 0.05 at lags 15 and 20, for the *ARMA (1, 1)-EGARCH (1, 1)* model under the normal error terms, at lags 10 and 20, for ARMA *(1,* 1)*-APARCH (1, 1)* under the student-t error terms and at lag 10 under the skewed student-t distribution of *ARMA (1, 1) –APARCH (1, 1)* model. These show no evidence of *ARCH* effects. However, at others lags, error terms and distributions, there is evidence of *ARCH* effects.

Table 4.10: Standardized Residuals Test Return Prices of Gold

	lag	**Normal**	**Student-t**	**Skew Student-t**
ARMA(1,1)-EGARCH (1, 1)	10	0.1059 (0.7449)	0.720 (0.3961)	0.7763 (0.3783)
	15	1.9811 (0.9615)	3.831 (0.1007)	3.9637 (0.0729)
	20	6.564 (0.1076)	5.594 (0.3286)	5.7064 (0.3075)
ARMA(1,1)-APARCH (1, 1)	10	0.07931 (0.7782)	0.07235 (0.7879)	0.1096 (0.7406)
	15	1.6454 (0.9941)	2.45429 (0.8024)	2.6305 (0.7051)
	20	3.4682 (0.8077)	3.9440 (0.7041)	4.0897 (0.6699)

Table 4.10 shows the *p*-values (in brackets) of the standardized residual tests for the *ARMA (1, 1)-EGARCH (1, 1)* and the *ARMA (1, 1) -APARCH* (1, 1) under the normal, student-t and skew student-t error terms. These values indicate that there is no correlations, which means that the residuals of the models are white noise.

4.2.1.7 Estimation of the ARMA (1, 1)-EGARCH (1, 1) and ARMA (1, 1)-APARCH (1, 1) (Exchange rate)

Table 4.11: ***Estimated Parameters of the ARMA (1, 1)-EGARCH (1, 1): ZAR/U.S.D Exchange rate***

	Normal	**Student-t**	**Skew Student-t**
Mu	-0.0094(0.0000) 0.0000	-0.0171(0.0000) 0.0000	0.0015(0.0000) 0.0000
AR1	-0.7252(0.0000) 0.0001	-0.7989(0.0000) 0.0004	-0.8058(0.0000) 0.0002
Ma1	0.6932(0.0000) 0.0001)	0.7792(0.0000) 0.0006	0.7749(0.0000) 0.0002
Omega	-0.0091(0.0000) 0.0000	-0.0041(0.0000) 0.0000	-0.0066(0.0000) 0.0000
Alpha 1	0.0799(0.0000) 0.0000	0.0669(0.0000) 0.0000	0.0736(0.0000) 0.0000

Table 4.11: ***Estimated Parameters of the ARMA (1, 1)-EGARCH (1, 1): ZAR/U.S D Exchange Rate (continued)***

	Normal	**Student-t**	**Skew Student-t**
Beta 1	0.9950(0.0000) 0.0001	0.9976(0.0000) 0.0001	0.9945(0.0000) (0.0002)
Gamma 1	-0.0601(0.0000) 0.0000	-0.0461(0.0000) 0.0000	-0.0579(0.0000) 0.0000
Shape		11.1456(0.0000) 0.0066	13.1579(0.0000) 0.0032
Skew			0.9225(0.0000) 0.5626
AIC	0.9728	0.9782	0.9608
BIC	1.0404	1.0552	1.0377
Log-likelihood	-196.3174	-196.9216	-195.9975

Table 4.12: Estimated Parameters of ARMA (1, 1)-APARCH (1, 1): ZAR/U.S.D Exchange Rate

	Normal	**Student-t**	**Skew Student-t**
Mu	0.0228 (0.1307) 0.0512	0.0242(0.8349) 0.0140	0.0235(0.1530) 0.0164
AR1	0.7026(0.0000) 0.0743	0.7965(0.0000) 0.0665	0.7998(0.0000) 0.8471
ma1	0.6753(0.0000) 0.0768	0.7757(0.0000) 0.0692	0.7785(0.0000) 0.0880
Omega	0.0042(0.7029) 0.0112	0.0162(0.4512) 0.0214	0.0140(0.4772) 0.0196
Alpha 1	0.0415(0.1543) 0.0291	0.0612(0.1159) 0.0389	0.0574(0.1731) 0.0042
Beta 1	0.9497 (0.0000) 0.0540	0.9007(0.0000) 0.0774	0.5004(0.0000) 0.0842
Gamma 1	1.0000(0.0000) 0.0044	1.0000(0.0000) 0.0026	1.0000(0.0000) 0.0025
Delta	1.2149(0.0000) 0.3108	1.1670(0.0000) 0.5738	1.2914(0.1555) 0.9091
Shape		10.0629(0.0311) 0.7340	10.0888(0.0330) 4.7323
Skew			1.0159(0.0000) 0.0738
AIC	1.0184	1.0064	1.0141
BIC	1.0954	1.0932	1.1107
Log-likelihood	-204.7862	-201.8474	-201.9551

Tables 4.11 and 4.12 show that the estimated model results, the selection criteria: *AIC*, *BIC* information criteria and the log-likelihood. The AIC and BIC show that *ARMA (1, 1)-EGARCH (1, 1)* under the skewed student-t outperforms the result and those of *ARMA (1, 1)-APARCH (1, 1)*. Based on this finding, it was concluded that the *ARMA (1, 1) EGARCH (1, 1)* under the skewed student-t error terms provides a better fit of the ZAR/U.S.D exchange rate.

4.2.1.8 Lagrange Multiplier and Standardized Residual Tests for *ARMA (1, 1)-EGARCH (1, 1) and ARMA (1, 1)-APARCH (1, 1)*

Table 4.13: LM ARCH Tests ZAR/U.S D Exchange Rate

	Lag	**Normal**	**Student-t**	**Skew Student-t**
ARMA(1,)-EGARCH (1, 1)	*10*	3.217 (0.0729)	2.799 (0.0943)	2.9970 (0.0834)
	15	4.865 (0.1104)	4.503 (0.1337)	4.777 (0.1157)
	20	6.052 (0.1377)	5.807 (0.1547)	6.1200 (0.1333)
ARMA(1,)-APARCH (1, 1)	10	2.120 (1.1454)	1.655 (0.1982)	1.633 (0.2013)
	15	4.831 (0.1124)	4.489 (0.1347)	4.565 (0.1294)
	20	6.559 (0.1078)	6.651 (0.1031)	6.663 (0.1025)

The Lagrange Multiplier *ARCH* tests in Table 4.13 for *ARMA (1, 1)-EGARCH (1, 1)* and *ARMA (1, 1)-APARCH (1, 1)* under the normal, student-t and skewed student-t error terms indicate that the hypothesis of the presence of *ARCH* effects is rejected, since the values are greater than 0.05. Thus, we conclude that there are no arch effects in residuals obtained from *ARMA (1, 1)-EGARCH (1, 1)* and *ARMA (1, 1) - APARCH (1, 1)* for the exchange rate.

Table 4.14: Standardized Residual Tests Returns of Rand/U.S.D Exchange Rate

	Lag	Normal	Student-t	Skew Student-t
ARMA(1,)-EGARCH (1, 1)	10	0.0104 (0.9185)	0.0954 (0.7574)	0.0052 (0.9424)
	15	0.9451 (1.000)	1.1075 (1.0000)	0.9619 (1.0000)
	20	1.9174 (0.9877)	2.0687 (0.9815)	1.8896 (0.9887)
ARMA(1,1)-APARCH(1, 1)	10	0.1537 (0.6950)	0.0661 (0.7971)	0.0768 (0.7817)
	15	1.0027 (1.000)	1.0029 (1.000)	1.0348 (1.000)
	20	2.2888 (0.9726)	2.2402 (0.9719)	2.2610 (0.9705)

The analysis of the residual in Table 4.14 shows that the p-values of the Ljung box for the standardized residual tests are not significant for *ARMA(1, 1)- (1, 1)- EGARCH (1, 1)* and *ARMA(1, 1)-APARCH (1, 1)* under the normal, student-t and skewed student-t error terms. The results demonstrates that the residual are white noise and there is no evidence of heteroscedasticity in the residuals.

4.2.1.9 Summary of Findings for Marginal Distributions

For an appropriate margins for the distribution of returns prices of platinum, gold, and ZAR/U.S.D exchange rate, a wide range of methods were used including the standardized residual test, the Lagrange Multiplier *ARCH* tests, the information criteria, *AIC* and *BIC* and the log-likelihood using the *ARMA (1, 1)-APARCH (1, 1) and ARMA (1, 1) - EGARCH (1, 1)* with different error terms, including the normal, student-t and the skewed student-t distributions.

For return prices of platinum, the combined results of the estimated parameters of the *ARMA (1, 1)-APARCH (1, 1)* and *ARMA (1, 1) - EGARCH (1, 1)*, the *AIC* and *BIC*, the Ljung box test for standardized residual test and the Lagrange Multiplier test illustrate that *ARMA (1, 1)-EGARCH (1, 1)* with normal error terms are significant and suitable for residuals for Copulas marginal distributions.

For the return prices of gold, the *ARMA (1, 1) – APARCH (1, 1)* parameters are significant, the AIC and BIC values are the least, the residuals are white noise and there is no correlation. Therefore, it was concluded that the *ARMA (1, 1) - APARCH (1, 1)* under the student-t error terms is the adequate model for gold price return and a suitable model for the marginal distribution.

For returns on Rand /U.S D exchange rate, the *ARMA (1, 1) -EGARCH* (1, 1) under the skewed student-t error terms is an adequate model for return prices of ZAR/U.S.D exchange rate. Since it has the least *AIC* and *BIC* values. The residuals are white noise and they have no *ARCH* effects.

In conclusion, the *ARMA (1, 1)-EGARCH (1, 1)* under the normal error terms for the return prices of platinum. The *ARMA(1, 1)-APARCH (1, 1)* under the student-t error terms for return prices of gold and *ARMA(1, 1)-EGARCH (1, 1)* under the skewed student-t error terms for returns on the Rand/ U.S D exchange rate are selected as marginal distributions for Copulas.

4.2.2 Leverage Effects

The leverage effects on the selected models namely the *ARMA (1, 1)-EGARCH* (1, 1) with normal error terms for platinum, the parameter which describes the leverage effect is significant and positive. This means that volatility in platinum return price exhibits an asymmetric behaviour; the distribution of negative returns displays a heavier tail compared to the distributions of positive returns, and negative shocks influence platinum price more than positive shocks. For the gold return price, the coefficient of the leverage effects is positive and significant, an indication that gold price has an asymmetric behaviour and negative shockwaves affects strongly gold prices compared to positive shocks. For exchange rate, the coefficient of the leverage effect is also significant but negative, an indication that negative returns of ZAR/U.S.D exchange rate affects the future volatility more as compared to positive returns.

After selecting the appropriate *ARMA (1, 1)-EGARCH (1, 1) and ARMA (1, 1)-APARCH (1, 1)* models for marginal distributions, the marginal distribution are transformed into univariate distributions and various Copulas are used to model the dependence structure.

4.2.3 Results of the Dependence Structure

For the analysis of dependence structure between the return prices of gold, platinum, and the Rand/ U.S Dollar exchange rate, the constant Copulas including Normal, Student-t, Gumbel, Clayton, rotated Clayton, asymmetric Joe-Clayton, Plackett and time-varying normal Copula were used. The benefits of using these Copulas are that the normal Copula captures both negative and positive dependence; the Student-t Copula captures fat-tailed dependence including lower and upper tail. The Archimedeans Copulas that provides better properties than elliptical Copulas (Melchiori et al., 2003; Chen et al., 2007). Clayton Copula captures dependence in the upper tail when rotated; it captures dependence in the lower tail. The Plackett Copula accounts for positive dependence structure while Joe-Clayton Copula captures asymmetric tail dependence, including lower and upper tail dependence.

Bivariate Copulas (constant and time varying) were fitted to a pair of residuals obtained from return prices of platinum and Rand/ U.S.D exchange rate and return prices of gold and Rand/U.SD exchange rate through a vector of standardized residuals provided by:$(y_{1k}, y_{2k}, \dots, y_{nk})_{k=1}^{K}$.

The residuals were obtained from the *ARMA (1, 1)-EGARCH (1,* 1) with normal error terms for return prices platinum, *ARMA (1, 1)-APARCH (1, 1)* with student-t error terms from the return prices of gold and *ARMA (1, 1)-EGARCH (1, 1)* with skewed student-t error terms from Rand/ U.S D exchange rate.

Following Cherubini et al. (2004), the vectors of standard residuals were transformed into uniform variates$(u_{1k}, u_{2k}, \dots, u_{nk})_{k=1}^{K}$ using an empirical cumulative distribution function and unknown parameters of Copulas were estimated using the canonical maximum likelihood estimation method. The study took advantage of the benefit of the goodness of fit test (Gof) proposed by Genest et al. (2009), the *AIC* and the *BIC* to assess the performance of Copulas and choose Copulas that provide a better fit for the distribution. Smaller value of the goodness of fit test, the *AIC* and *BIC* criteria indicates that Copulas provide a good fit for the distribution.

4.2.3.1 Copulas Parameters Estimation

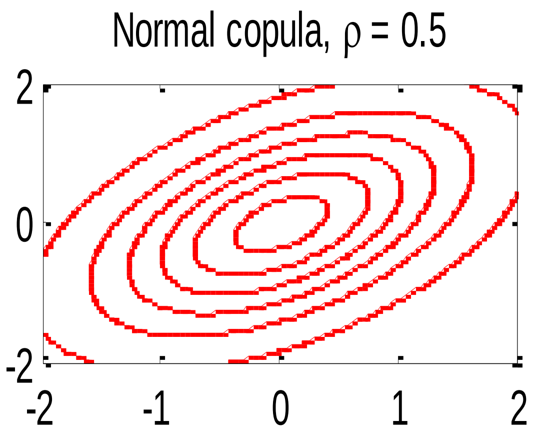
Bivariate Copulas including the Normal, the Student-t, the Gumbel, the Clayton, the rotated Clayton, the rotated Gumbel, the symmetric Joe- Clayton, the Plackett, and the time-varying normal were fitted to the pair return prices of platinum/ Rand/U.S.D exchange rate and returns prices of gold / the Rand/U.S.D exchange rate. Using *ARMA (1, 1)-EGARCH (1, 1)* with normal error terms for returns prices of platinum joined with the *ARMA (1, 1)-EGARCH (1, 1)* with skewed error terms return price for exchange rate. The *ARMA (1, 1)-APARCH (1, 1)* with student-t error terms for returns prices of gold is joined with *ARMA (1, 1)-EGARCH (1, 1)* skewed student-t error terms for ZAR/U.S D exchange rate summarized in Table 4.15.

Table 4. 15: **Commodities Prices and Exchange Rates**

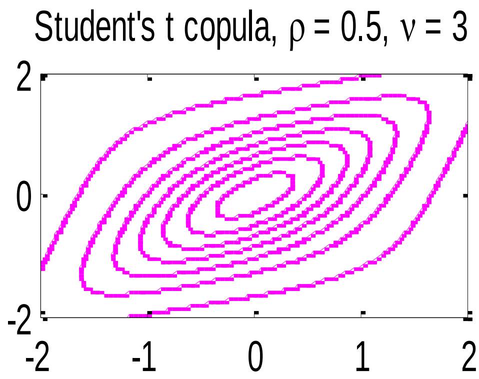
Commodities Prices	Exchange rates
Return Prices Platinum	Returns on ZAR/U.S D Exchange rate
Return Prices Gold	Returns on ZAR/U.S.D Exchange rate

4.2.3.2 Contours of Copulas

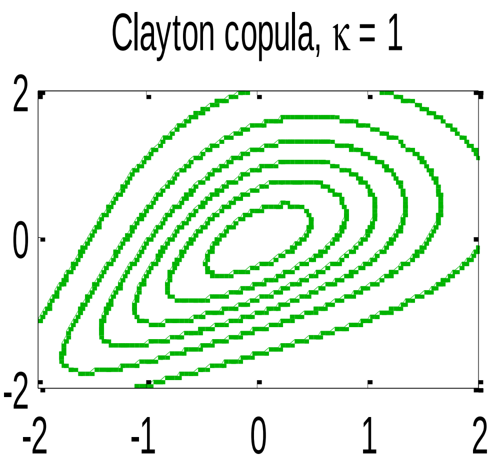
Contours bivariate Copula for *ARMA (1, 1)-EGARCH (1, 1)* with normal error terms for return prices of platinum and *ARMA (1, 1)-EGARCH (1, 1)* with skewed error terms for returns on the ZAR/U.S Dollar exchange rate. Copulas parameters and normally distributed margins were used to construct the contour plot of the Copulas.



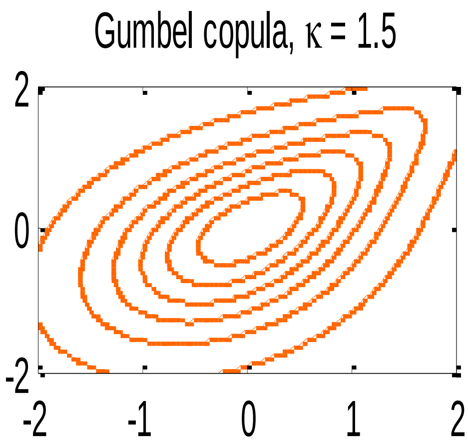
(a)



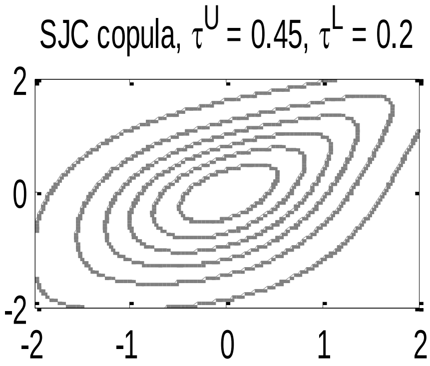
(b)



(c)



(d)



(e)

Figure 4.5**: **Contours of the Normal, Student-t, Clayton, Gumbel and Symmetrical Joe-Copulas

Notes: Figures 4.5: (a), (b), (c), (d) and (e) are the contours of the Normal, Student-t, Clayton, Gumbel and Symmetrical Joe-Clayton with their dependence parameters including the upper and lower tail dependence.

Copulas separate marginal distributions and the dependence structure and measure the dependence structure (Cherubini et al., 2004). The dependences are described by the parameter of the Copulas. The Tables 4.15 and 4.16 are estimated parameters of the Normal, Clayton, Rotated Clayton, Plackett, Gumbel, Rotated Gumbel, student-t and the symmetric Joe Copulas. The dependence parameters of the Copulas are given by θ, the p –values of the goodness of fit test is represented by Gof p-values, while the information criterion are given by *AIC* and *BIC*. Tau represent the Kendall tau value, λ^U and λ^L are respectively the upper of the lower tails of Copulas.

4.2.3.3 Dependence Structure for Constant Copulas

4.2.3.3.1 Return Prices Platinum/ Rand/U.S.D Exchange Rate

Table 4. 16: Platinum Return Prices and ZAR/U.S D Exchange Rate

Copulas	$\boldsymbol{\theta}$	**Gof-P values**	**AIC**	**BIC**	**Tau**	$\boldsymbol{\lambda^U}$	$\boldsymbol{\lambda^L}$
Normal	-0.4934	0.739	-3.6571	-3.6474	0.233	NA	NA
Clayton	1.00e-004	0.0027	0.0101	0.0198	0.033	NA	0.200
Rotated Clayton	1.00e-004	0.0024	0.0096	0.0193	0.0032	0.222	2.339
Plackett	0.7351	0.856	-4.277	-4.268	0.0021	0.210	NA
Gumbel	1.04	0.00510	17.70	17.7165	0.2231	2.0043	NA
Rotated G	0.99	0.00231	19.8558	19.8751	0.2317	0.0212	0.007
Student-t	3.4396	0.00455	-2.9094	1.1261	2.0003	0.212	0.004
Symmetric Joe-Clayton	1.0e-005 0.1907 0.1907	0.06701	3.3497	3.3690	2.0002	0.45	0.2

Table 4.16 indicates that Plackett Copula provides a good fit for the distribution. The Plackett Copula's Goodness of fit test, *AIC* and *BIC* information criteria are respectively: 0.856, -4.277 and -4.2680 which values are least as compared to Normal, Clayton, Rotated Clayton, Gumbel, Rotated Gumbel, student and Symmetric Joe Copulas. In terms of the estimated parameter of dependence, the Plackett Copula (θ = 0.7351) shows evidence of positive dependence structure between return prices of platinum and Rand/U.S D exchange rate. This implies that a rise in platinum prices affect positively the Rand/U.S.D exchange rate while a decrease in commodity(platinum) prices affect negatively the Rand/U.S.D exchange rate. There is a dependence structure between return prices of platinum and returns on the ZAR/U.S.D exchange rate.

The Clayton, the Rotated Clayton, the Plackett, the Gumbel, the Rotated Gumbel, the Student-t and the Joe-Clayton Copulas parameters are positive except for the normal Copula parameter that exhibits a negative dependence. These findings show evidence of a relationship between return prices of platinum and returns on the ZAR/U.S.D exchange rate. The dependence parameters of the Plackett, Student-t, Gumbel, and Rotated Gumbel Copulas show evidence of a positive strong relationship between fluctuation in platinum return prices and volatility in returns on the ZAR/U.S.D exchange rate. In terms of Copulas that provide a better fit of the distributions, the values of the goodness of fit test illustrates that Plackett Copula gives a better fit for the pair return prices of platinum and returns on ZAR/U.S.D exchange rate. Therefore, it can be concluded that there is a positive dependence structure between return prices of platinum and ZAR/U.S.D exchange rate returns.

4.2.3.3.2 Gold and Exchange Rate

Table 4. 17: Gold Return prices and ZAR/U.S.D Exchange Rate Return

Copulas	θ	Gof	AIC	BIC	Tau	λ^U	λ^L
Normal	0.0665	0.0254	-1.846	-1.836	0.002	NA	NA
Clayton	0.6541	0.068	-2.531	-2.419	0.124	NA	0.120
Rotated clayton	0.0597	0.308	-1.256	-1.247	0.0014	2.113	0.123
Plackett	1.1784	0.0123	-1.219	-1.210	0.124	3.016	NA
Gumbel	1.093	0.3418	0.6884	0.6980	1.003	1.2903	NA
Rotated G	1.1003	0.4825	0.9746	0.9939	1.017	0.923	0.201
Student-t	2.0599	0.762	-0.792	3.2431	3.012	0.81	0.012
Symmetric Joe-Clayton	0.7102 0.0048 0.0074	0.8621	-2.807	-2.787	2.997	0.95	071

Table 4.17 indicates that the symmetric Joe-Clayton Copula is AIC and BIC values are -2.807 and -2.787. The Joe-Clayton's p-values of the goodness of fit is 0.8621 that is the highest in all the estimated Copulas. The AIC and BIC values for Joe-Clayton Copula are the smallest in all the estimated Copulas. The p-values of the goodness of fit test implies that Joe-Clayton provides a better fit for the observations. When considering the dependence, Joe-Clayton Copula dependence parameter is 0.7102, an indication that return prices of gold and Rand/ U.S.D exchange rate are positively correlated and the dependence structure is more pronounced in the lower and upper tails.

These results suggest that there is a dependence structure between volatility in the return prices of gold and fluctuations in the Rand/U.S.D exchange rate. The dependence parameters of all the considered Copulas are positive. This is an indication that the volatility in return prices of gold and fluctuations in the returns on ZAR/U.S.D exchange rate are positively dependent.

4.2.3.4 Time Varying Copulas

The time varying Copulas account for the variation in the dependence structure over time. This study used the time-varying normal Copula, as the normal Copula accounts for both negative and positive dependence. The time-varying dependence parameters are estimated and generated from the time varying normal Copulas and are plotted for pairs *ARMA (1, 1)-EGARCH(1, 1)* under normal error terms for returns prices of platinum versus *ARMA(1, 1)-EGARCH(1, 1)* under skew error terms for the ZAR/U.S.D exchange rate and the *ARMA(1, 1)-APARCH (1, 1)* under student-t error terms for gold price return and the *ARMA(1, 1)-EGARCH (1,* 1) with skewed error terms for ZAR/U.S.D exchange rate returns,

The estimated parameters of the time-varying Copulas with margins plotted in Figures 4.6 and 4.7 show that the dependence is moving from one point to the other and the dependence is oscillating from positive value to a negative value.

4.2.3.4.1 Return Prices of Platinum and ZAR/U.S D Exchange Rate

Figure 4.6 shows the ttime-varying normal Copula and constant Copula for models *ARMA (1, 1)-EGARCH (1, 1)* under the normal error terms for return prices of platinum and *ARMA (1, 1) - EGARCH (1, 1)* under the skew error terms for returns on the ZAR/U.S.D exchange rate.

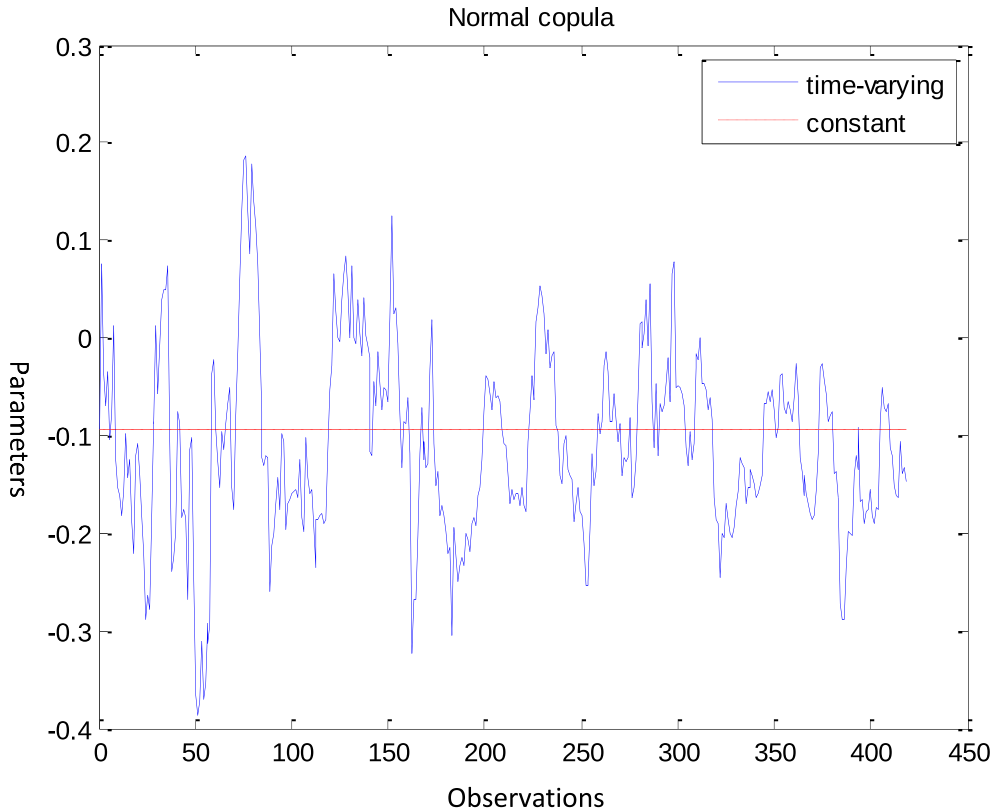


***Figure 4.6**: Time-Varying Parameter of Dependence Return Prices of Platinum versus Returns on ZAR/U.S D Exchange Rate.*

The plot shows that the constant Copula is actually constant. However, the time-varying Copula exhibits a co-movement in the dependence over time. The parameters of the dependence structure between the return prices of platinum and the returns on the ZAR/U.S.D exchange rate varies. An indication that the dependence between the returns prices of platinum and ZAR/U. S .D exchange rate is sensitive to changes in prices of commodities.

4.2.3.4.2 Return Prices of Gold and Return on the U.S.D/ZAR Exchange Rate.

The time- varying normal Copula and constant Copula for models *ARMA (1, 1)-APARCH (1, 1)* with student-t error terms for return price of gold and *ARMA (1, 1) - EGARCH (1, 1)* with skewed error terms for ZAR/U.S.D exchange rate is shown in figure 4.7.

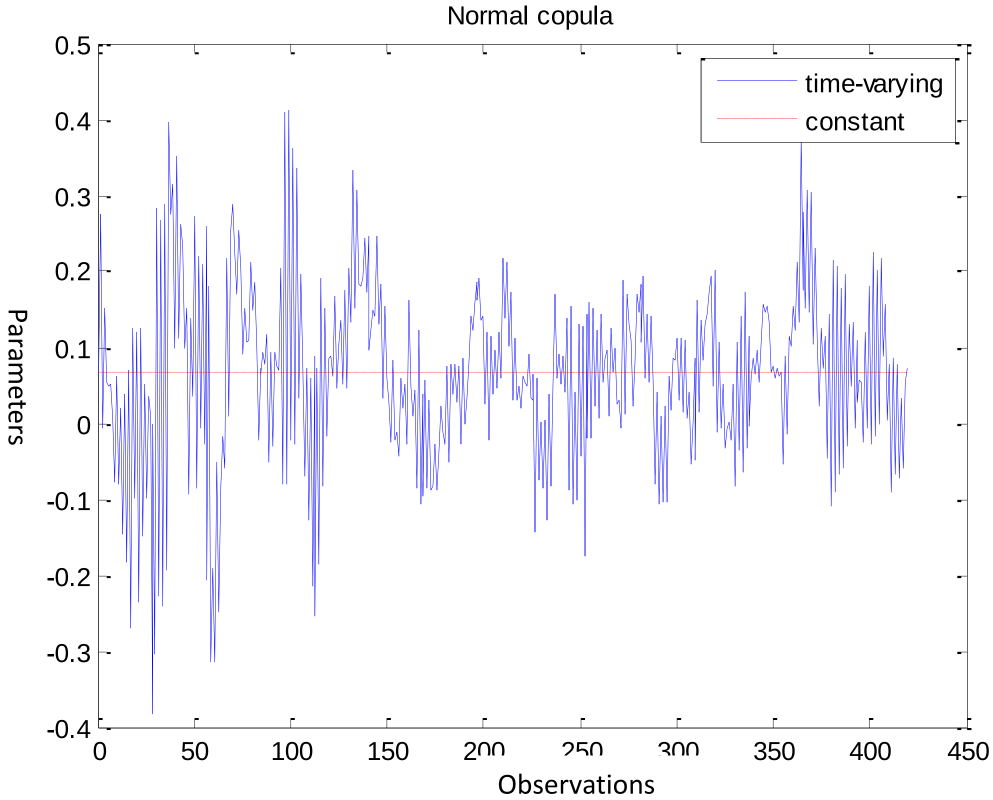


Figure 4.7: Time Varying Parameter of Dependence for Return Prices of Gold and ZAR/U.S Dollar Exchange Rate

The plot shows that the parameters of the time-varying Copula fluctuate from positive correlation values to negative correlations values. There are cases where the graph of the time-varying Copulas illustrates evidence of heavy positive and negative correlation. The values of the dependence structure change from positive to negative. However, the plot of the normal constant Copula does not change.

The graph of the parameter of the dependence of the constant Copula shows that the dependence structure between the return prices of platinum and the returns on the ZAR/U.S.D exchange rate is constant. Contrary to the parameter of the dependence of the constant Copula, the parameter of the dependence using the time-varying Copula show evidence of variation in the dependence structure between return prices of gold and the returns on the ZAR/U.S.D exchange rate over time. The graph of the time-varying Copula illustrates evidence of ascendant and descendent correlation from negative and positive dependence.

These results suggest that the dependence structure tends to move with the co-movement of the prices of gold and platinum. For South Africa, a larger exporter of gold and platinum, a rise in the global commodity prices may stimulate the appreciation of the Rand, whereas, a decline in global platinum and gold prices does not necessarily supports the local currency against the U.S. Dollar. The results indicate that fluctuations in return prices of gold and platinum contribute significantly to the return on the Rand/U.S Dollar exchange rate volatility.

Chapter Five: Conclusion

This study employs Copulas to investigate the relationship and the dependence structure between the return prices of mineral resources (gold and platinum) and ZAR/U.S.D exchange rate using the daily prices of gold and platinum (per ounce, denominated in the U.S.D) and the ZAR/U.S.D exchange rate over a period of 2 years.

Returns on the exchange rate of the Rand quoted against the U.S.D, the gold and the platinum prices were computed. Descriptive statistics including Jacque Bera test were also computed as well as the Q-Q plots. The *ARMA (1, 1)-EGARCH (1, 1)* and *ARMA (1, 1)-APARCH (1, 1)* were fitted to the returns prices of gold, platinum and Rand/U.S.D exchange rate. The information criteria *AIC* and *BIC*, standardized residuals tests and LaGrange Multiplier were used to select the appropriate *ARMA (1, 1)-EGARCH (1, 1)* and *ARMA (1, 1)-APARCH (1, 1)* models. The study selected the model *ARMA (1, 1)-EGARCH (1, 1)* under the normal error terms for the return prices of platinum, *ARMA (1, 1)-APARCH (1, 1)* under the student-t error terms for the return prices of gold and *ARMA (1, 1)-EGARCH (1, 1)* under the skewed student-t error terms for returns on the ZAR/U.S.D exchange rate with residuals extracted from the selected models. These models were employed as marginal distributions for the Copulas. Constant Copulas including the Normal, Student-t, Gumbel, rotated Gumbel, Plackett, Clayton, rotated Clayton and time-varying Normal Copulas were fitted to the residuals. The goodness of fit test, the AIC and BIC criteria were used to select Copula, which provided the best fit for the distribution. The parameters of the Copulas were used to measure the dependence structure between the return prices of gold, and platinum and the Rand/U.S.D exchange rate.

The leverage effects were significant, positive for gold and platinum return prices, and negative for the returns on the ZAR/U.S.D exchange rate. This implies that negative shocks affect strongly commodities return prices volatility compared to positive shocks and returns prices of gold and platinum exhibit an asymmetric trend while negative returns affect future volatility more than positive returns for the exchange rate. The Copulas-*EGARCH-APARC*H models revealed that Plackett, Student-t, and Joe-Clayton Copulas provided evidence of a positive dependence between the return prices of gold and the returns on the ZAR/U.S D exchange rate, and the return prices of platinum and the returns on the ZAR/U.S.D exchange rate. The positive dependency implies that higher gold and platinum return prices cause the appreciation of the

Rand while lower gold and platinum prices are associated with the depreciation of the Rand. In addition, the Normal time varying Copula shows a co-movement between the return prices of mineral resources (platinum and gold) and the ZAR/U.S.D exchange rate.

The results of this study corroborate the findings reported by Frank (2007), who noted that mineral resources prices are one of the determinants of the South African real exchange rate using an econometric model. The author found that ZAR/U.S D exchange rate depends on the gold and platinum prices. This view was also noted by Mpofu (2016) using the *GARCH* methods and Azerki et al. (2014) using the Vector Correction model. The two authors claimed that for South Africa, one of the main exporter of the commodities, the volatility of mineral resources prices have a significant influence on the exchange rate fluctuations.

The results of this study is not in agreement with the conclusion reached by Schaling et al. (2010) who found evidence of an inverse relationship between changes in the commodity prices and fluctuations in the ZAR/U.S.D exchange rate in the short term. They rejected the hypothesis that exchange rates and commodity prices are positively correlated using the Cointegration analysis. The results in this study are different from that of Schaling et al. perhaps because they used a different approach to the study of the dependence.

The results of this study show evidence of a dependence structure between the return prices of platinum, gold and the ZAR/U.S D exchange rate. The dependence can be explained by the fact that South Africa's commodities, including gold and platinum, are one of the main source of wealth. An increase in gold and platinum prices may generate more revenues for the economy and appreciation of the Rand against the U.S Dollar may have positive influence on the prices of gold and platinum. Cashin et al. (2004) stated that for larger exporters of commodities, commodity prices play a crucial role in the values of their exchange rates. This study concludes that platinum and gold prices contribute significantly to the Rand/U.S.D real exchange rate.

It should be noted that other variables including the supply and demand of commodities, policies and market sentiment might also explain the relationship between the return prices of gold, platinum and the Rand/U.S.D exchange rate.

This study is limited to asymmetric *GARCH* models associated with Copulas. Further work can be extended to non-linear *GARCH* models and Copulas to model the dependence structure between the returns prices of gold and platinum with the returns on the ZAR/U.S.D exchange rate.

References

Abeysinghe, T. & Yeok, T. L. (1998). Exchange rate appreciation and export competitiveness. The Case of Singapore. *Journal of Applied Economics, (30): 51-55.*

Alaba, O, B., (2003). Exchange rate uncertainty and foreign direct investment in Nigeria. A paper presented at the WIDER conference on sharing global prosperity, Helsinki, Finland, 6 – 7. September 2003.

Ali, D. A., Johari, F. & Alias, M. H. (2014). The effects of exchange rate movements on trade balance: A chronological theoretical review. *Economics Research International. Volume 2014*. Article ID893170,7. Http:/dx.doi.org/10.1155/2014/893170.

Aloui, R., Ben Aissa, M. S., & Nguyen, D. K., (2013). Conditional dependence structure between oil prices and exchange rates. A Copula-Garch approach. *Journal of International Money and Finance, (32): 719-738.*

Arezki, R., Dumitrescu, E., Freytag, A. & Quintyn, G. (2014). Commodity prices and exchange rate volatility: Lessons from South Africa's capital account liberalization. *Emerging Markets Review, (19):96-105.*

Aron, J., Elbadawi, I. & Kahn, B. (1997). Real and monetary determinants of the real exchange rate in South Africa, University of Oxford.

Aron, J., Elbadawi, I., & Kahn, B. (2000). The real and monetary determinant of real exchange rate in South Africa. In Elbadawi, I. A & Hartzenberg, T. (Eds.). Development Issues in South Africa, London, Africa Economic Research.

Avanzi, B, Cassar, L. C. & Wong, B. (2011). Modelling dependence in insurance claims processes with Levy Copulas. Paper presented at biennial convention, Institute of Actuaries of Australia, April 2011, Hilton, Sydney.

Bank for International Settlements, Triennila central bank survey(2016). Foreign exchange turnover, April 2016. www.bis.org/publ/spf_x16.htm.

Bhundia, A. & Ricci, L. (2004). The rand crises of 1998 and 2001. What we learned? International Monetary Fund, working paper, Number 03|252. Washington: International monetary fund.

Bleaney, M. (2008). Openness and real exchange rate volatility: In search of an explanation. *Open Economics Review, 19 (2):135-146.*

Bodart , V., Cadelon, B. & Carpantier, J.F., (2013). Real exchange rates in commodity producing countries : A reappraisal. *Journal of international Money and Finance, 31(6):2012.*

Boero, G., Silvapulle, P. & Tursunalieva, A. (2011). Modelling the bivariate dependence structure of exchange rates before and after the introduction of Euro: A semi-parametric approach. *International Journal of Finance and Economics, 16: 357-374.*

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics, 31: 307-327.*

Bouye, E. ., Durrlemann, A., Nikeghbali, A., Riboulet, G. & Roncalli, T. (2000). Copulas for finances, a reading guide and some applications. Groupe de recherché operationnelle, credit Lyonnais, Paris. Working paper.

Bova, E., (2009). Is the Zambian Kwacha a commodity currency? The co-movement between copper prices and exchange rate. NCCR Trade Regulations SWISS National Centre of competence in research. Working paper N0 2009/11, March 2009.

Byrne, P., Fazio, G., & Fiess, N. (2013). Primary commodity prices: Co-movement, common factors and fundamentals. *Journal of development economics, 101: 16-26.*

Catão. L.A.V. (2007). Why real exchange rates? Finance and development. A quarterly magazine of IMF. September 2007.

Carrera, J & Vuletin, G. (2002). The effects of exchange rate regimes on real exchange volatility. A dynamic panel data approach. University of la Plata and University of Maryland. April 2003. Unpublished paper.

Cashin, P. A., McDermott, C.J & Scolt, A. M. (1999). The myth of co-moving commodity prices: Discussions paper series G99/9. Reserve Bank of New Zeland.

Cashin, P. A., MCDermott. C. J (2002). The long run behaviour of commodity prices: small trends and big variability. IMF working paper: WP/01/68

Cashin, P., Cespedes, L., & Sahay, R. (2004). Commodity currencies and real exchange rate. *Journal of Development Economics. 75 (1): 239-268.*

Chamber of mines of South Africa (2015). Integrated annual review 2015, available at www.chamberofmines.org.za. Accessed October 2016.

Chen, Y-C., Rogoff, K. & Rossi, B., (2010). Can exchange rates forecast commodity prices? *Quarterly journal of Economics, 125 (3): 1145-1194.*

Chen, C, Y-H., Tu, A. H, & Wang, K. (2007). Dependence structure between the credit default swap return and the kurtosis of the equity return distribution: Evidence from Japan. *Journal of International Financial Markets, Institutions & money(18); 259-271.*

Chen, Y-C., & Rogoff, K. (2003). "Commodity Currencies". *Journal of International Economics, (60): 133-160.*

Cherubini, U., Luciano, E. and Vecchiato, W., (2004). Copula methods in finance, John Wiley & Sons, inc. ISBN:978-0-470-86344-2.

Cheung, W. (2009). Copula: a Primer for fund managers. Working paper. Available at htttp://papers.ssrn.com/sol3/papaers.cfm?abstract-id= 1456980. (accessed on Marh 2017).

Chinn, M. D., & Alquist, R. (2000). Tracking the Euro's progress. *Journal of International Finance, 3(3):357-373.*

Chipili, J. M. (2012). Modelling exchange rate volatility in Zambia. *The African Finance Journal, 14 (2):85-107.*

Clayton, D. G. (1978). A model for association in bivariate life tables and its applications in Epidemiological studies of familial tendency in chronic disease incidence. *Biometric, 1: 141-151.*

Clements, K. W. & Fry, R. (2008). Commodity currencies and currency commodities. *Resources Policy, 33(2): 55-73.*

Coudert, V., Couharde, C. & Mignon, V. (2008). "Do terms of trade drive real exchange rate? Comparing oil and commodity currencies". Working papers 2008:32. CEP Type equation here.

Cruz-Rodrigues, A. (2013). Choosing and assessing exchange rate regimes: A survey of the literature. *Revista de Analisis Economicos, 28, 2 :37-61.*

Constable, S., & Wright, R. E. (2011). The wall street Journal, guide to 50 economic indicators that really matter. Harper Business, New York. ISBN10:0062001388.

Copeland, L. S. (2005). Exchange rates and international finance. 5th edition. Prentice hall, ISBN: 0273683063, 9780273683063.

De Kock Commission (1985). Inquiry into monetary system and monetary policy in South Africa: Final report, Pretoria: Government Printer.

Deheuvels, P. (1979). La fonction de dependence empirique et ses proprieties. Un test non-parametrique d independence. *Bulletin Royal Belge de l academie des sciences, 65: 274-292.*

Devereux, M, B., & Engel, C. (1998). Fixed vs floating exchange rates: how price setting affects the optimal choice of exchange rate regime. NBER working paper number 6867. Cambridge: national bureau of economic research.

Ding, Z., Granger, C. W. J. & Engle, R. F, (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance, 1:83-106..*

Dubas, J. M., Lee, B-J. & Mark, N. C. (2005).” Effective exchange rate classifications and growth”. Working paper:11271, National Bureau of economic research.

Edwards, L. & Garlick, R. (2007). Trade flows and the exchange rate in South Africa. Trade and Policy Strategies. University of Cape Town, Unpublished work.

Egert, B., Halpern, L. & MacDonald, R. (2006). Equilibrium exchange rates in transition economies: taking stock of the issues. *Journal of Economics Surveys, 20 (2): 257-324.*

Embrechts, P., Lindskog, F. & McNeil, A. (2003). Modelling dependence with Copulas and applications to risk management: In handbook of heavy distribution in finance, S.Rachev edition 8: 329-384. Elsevier, Amsterdam, the Netherlands, 2003..

Enders, W. (2010). Applied econometrics time series. Third edition. John Wiley and Sons, 121-170.

Engle, R. F (1982). Autoregresive conditional heteroscedasticity with estimate of variance of the United Kingdom inflation. *Econometrica, 45: 987-1007.*

Engel, C. & West, K. D. (2005). Exchange rates and fundamentals. *Journal of Political Economic, 113(3): 485- 51719.*

Faulkner, D. & Makrelov, K. (2008). Determsinants of the equilibrium exchange for South Africa’s manufacture sector and implications for competitiveness. Paper presented at the 13th Annual conference on Econometric modelling in Africa. University of Pretoria, 9 to 11 July 2008.

Fedderke, J., & Flammand, P. (2005). Macroeconomic news “surprises” and the Rand/Dollar exchange rate. *Studies in Economics and Econometrics, 29(3): 1-16.*

Ferraro, D. Rogoff, K., & Rossi, B. (2015). Can oil prices forecast exchange rates? An empirical analysis of the relationship between commodity prices and exchange rates. *Journal of International Money and Finance, 54:116-141.*

Fouquin, K., Sekkat, K., Mansour, J. M., Mulder, N., & Nayman, L. (2001). Sector sensibility to exchange rate fluctuations. Working paper Number: 2001-11. CEP II Research center.

Fourie, C.V. N. (1997). How to think and reason in macroeconomics, 4 th edition. Cape Town: Jute & Co.

Frank, M. J. (1979). On the simultaneous associativity of f (x,y) and (x,y) - f(x,y), *Aequationes Mathematicae, 19(1): 194-226.*

Frankel, J. (2007). On the Rand determinants of the South Africa exchange rates. South *African journal of economics, Economic Society of South Africa, 75 (3):425-411.*

Frees, E. W. & Valdez, E. A. (1998). Understanding relationship using Copulas. *North America Actuarial Journal, 2(2):1-25.*

Galati, G. & Ho, C. (2003). Macroeconomic news and the Euro/Dollar exchange rate, " *Economics Notes" 32(3):371-398.*

Ganguly, S. , & Breuer, J. B. (2010). Nominal exchange rate volatility, relative price volatility and the real exchange rate. *Journal of International Money and Finance, 29(5):840-856.*

Genest, C., Remillard, B. & Beaudoin, D. (2009). Goodness-of -fits test for Copulas: A review and power study, *Insurance: Mathematics and Economics, 44 (2):199-213.*

Ghalayini. L. (2013). Modeling and forecasting the U.S Dollar/Euro exchange rate. *International Journal of Economics and Finance, 6(1):194-207.*

Ghosh, A. R., Gulde, A-M and Wolf, H. (2002). Exchange rate regimes: Choices and consequences. Cambridge, Masssachustts, The MIT press.

Giannellis, N. , & Papadopoulos, A. P. (2011). What causes exchange rate volatility? Evidence from selected EMU members and candidates for EMU membership countries. *Journal of International Money and Finance, 30, (1): 39-61.*

Goodman, B. (1956). The price of gold an international liquidity. *Journal of Finance, 1: 15-28.*

Gumbel, E. J. (1960). Bivariate exponential distributions. *Journal of the American Statistical Association, 5: 698-707.*

Harvey, D. I., Kellard, N.M., Madsen, J. B., & Wohar, M. C. (2010). Trends and cycles in real commodity prices: 1960-2010. Nottingham University, Essex University, Monash University and University of Nebraska at Omaha. Discussion paper number 2012-10.

Heap, A. (2005). China. The engine of commodities super cycle, Citygroup. New York city. IFM, 2007.

Ignatieva, K. & Ponomareva, N. (2016). Commodity currencies and commodity Prices: Modelling statistic and time-varying dependence. *Journal of Applied Economics, 49: 1491-1512.*

International Monetary Fund (2014). Annual report on exchange arrangments and exchange restrictions. 2014.

Ishfaq, M. (2010). Overview of different exchange rate regimes and preferred choice for UAE Economic paper series. I, 1-24.

Jabeen, M. and Khan, S. A. (2014). Modelling exchange rate volatility by macroeconomic fundamentals in Pakistan. *International Economic Review (IER). 6(2): 59-77.*

Jan, S.U., Wali, S. & Asif, M. (2014). The relationship between commodities prices and Pakistani currency exchange rate. *City University Research Journal, 4, 02 July 2014, Article 09.*

Jain, A. & Ghosh, S. (2013). Dynamics of global oil prices, exchange rates and precious metal prices in India. *Resources Policy, (38):88-93.*

Jhingan, M. L. (2003). Macroeconomic theory: 11th edition. New Delhi, 12th enlarged revised edition Keynes.

Joe, H., (1993). Parameter families of multivariate distributions with given margins. *Journal of Multivariate Analysis, 46: 262-282.*

Joe, H., (1997). Multivariate models and dependence. concepts, Chapman& Hall, London. Http://dx.doi.org/10.1201/b13150.

Joe, H., & Xu, J.J. (1996). The estimation method of inference functions for margins for multivariate models. Working paper, Department of Statistics, University of British Columbia.

Jondeau, E. & Rockinger, M. (2006). The Copula-Garch model of conditional dependencies. An international stock market application. *Journal of International Money and Finance, 25: 827-853.*

Johnson & Matthey (2017). PGM Market repot May 2017. Summary of Platinum 2018, Supply & Demand in 2016.

Jerrett, D. & Cuddington, J. (2008). Broadening the statistical search for metal price super cycles to steel and related metals. *Resources Policy, 33(4): 188-196.*

Kakkar, V., & Yan, I. (2014). Determinants of real exchanges rates: An empirical investigation, BOFIT discussions paper. Number 1/2014. http://www.bof.fit/bofit.

Kendall, MG (1962). Rank correlation methods. 3 rd edition. New York; Hafner publishing company, Locoi, 1962 Hardcover.

Kissi, D. (2013). South Africa's currency retains international appeal. A magazine published by CME group, August 14, 2013.

Kim, S., & Courage, M. (2014). The impact of oil prices on the exchanges rates in South Africa. *Journal of Economic, 5(2): 193-199.*

Klaasen, J.G.M (2005). Long swings in exchange rates: Are they really in the data? *Journal of Business and Economic Statistics, (23): 87-95.*

Kocenda, E. & Valachy, J. (2006). Exchange rate volatility and regime change. A visegard comparison. *Journal of Comparative Economics, 34(4): 727-753.*

Kodongo, O. & Ojah, K. (2014). Conditional pricing of currency risk in Africa's equity markets. *Emerging markets Review, (29): 133-135.*

Lane, P. (1999). What determines the nominal exchange rate? Some cross-sectional evidence " *the Canadian Journal of Economics'', 32, 1.*

Le Roux, C & Els, G. (2013). The co-movement between copper prices and the exchange rate of five major commodity currencies. *Journal of Economics and Financial Science, 6 (30): 773-794.*

Liu, D. Wang, D, Singh,V.P. , Wang, Y, Wu, J., Wang, L. Zou, X, Chen,Y. & Chen, Xi (2017). Principle of maximum entropy-Copula to model hydrometeorological dependence. *Journal of Advances in Water Resources; 105(2017):39-50.*

Lombardi, M., Osbat, C., & Schnatz, B. (2012). Global commodity cycles and linkages: A FAVAR approach. *Empirical Economics, 43 (2): 651-670.*

MacDonald, R. & Ricci, L A. (2004). Estimation of the equilibrium real exchange rate for South Africa. *Journal of Economics 72, (2):282-304.*

Mark, N. C. (1995). Exchange rates and fundamentals: Evidence on long-horizon predictability. *American Economic Review, (85): 202-18.*

Meese, R. & Rogoff, K. (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of International Reconomics, 14:3-24.*

Melchiori, M. R. (2003). Which Archimedean Copula is the risght one? Yiel curve, dependence structure analysis using Copula-garch models: An application to U. K. and U.S stock markets. 2003.

Meucci, A. (2011). A short, comprehensive, practical guide to Copulas. *Garp Risk Professional*, October 2011: 22-27.

Mpofu, T. (2016). The determinants of exchange rate volatility in South Africa. ERSA working paper number 604.

Mtonga, E. (2006). " The real exchange of the Rand and competitiveness of South Africa trade". School of economics, Unversity of Cape Town, August 15. Munich. Personal repec Archive, paper No 1192, Dec.

Mtonga, E. (2011). Did it matter? Monetary policy regime change and exchange rate dynamics in South Africa. Technical report World Bank.

Nelsen, R. B., (1999). An Introduction to Copulas, second edition. Springer-Verlag New York.

Nelsen, R. B., (2006). An Introduction to Copulas, second edition. New York. Springer, USA.

Nelson, D. B. (1991). Conditional heteroscedasticity in assets returns: A new approach. *Econometrica,59: 347-370.*

Ngandu, S. (2005). Mineral prices and exchange rate. What does the literature say, employement growth & development initative. Human Science Research Council.

Nguyen, C. & Nguyen, T. (2014). Analyzing dependence structure of equity, bond and money markets by using time varying Copulas. *International Journal of Economics and Finance, 6, 3. ISSN 1916-971X-ISSN.*

Nikoloulopoulos, A. K. and Karlis, D. (2008). Multivariate logit Copula model with an application to dental data. *Statistics in Medicine, 27, (30): 6393-6406.*

Opoko-Afari, M. (2004). Measuring the real effective exchange rate in Ghana. Credit research paper No: 04/11. Centre for research in economic development and International trade: School of Economics, University of Nottingham.

Patton, A. J. (2006). Modelling asymmetric exchange rate dependence. *International Journal of Economic Review, 47: 527-556.*

Patton, A. J. (2001). Modelling time-varying exchange rate dependence using the conditional Copula. Department of Economics, Univesity of California, San Diego.

Pettersson, J. (2010). The dependence structure in the Finnish stock market: A Copula Approach. Thesis, Hanken School of Economics. Department of Finance and Statistics, VAASA 2010.

Plourde, A. & Watkins, G.C. (1998). Crude oil prices between 1985 and 1994: How votalile in relation to other commodities? *Resource and Energy Economics, 20(3):245-262.*

Prakash, A. J. , Chang, C-H. , & Pactwa (2003). Selecting a portfolio with skewedness: Recent evidence form US, European and Latin America equity markets, *Journal of Finance and Banking, 27(7): 1375-1390.*

Raddatz, C. (2008). "Exchange rate volatility and trade in South Africa", second draft, September 2008, the World Bank. Unpublished Work.

Radetzki, M. 2006). The autonomy of 3 commodity booms. *Journal of Policy, 31(2006): 56-64.*

Raputsoane, L., & Todani, K. R. (2008). The exchange rate and macroeconomic fundamentals in South Africa. Working paper. South African Reserve Bank.

Reboredo, J.C. (2012). Modelling oil price and exchange rate co-movements. *Journal of Policy Modeling. 34(3):419-440.*

Ricci, L. A. (2005). South Africa's real exchange rate performance. Post-apartheid South Africa: the first ten years:142-155.

Rogers, J. (2004). Hot Commodites: How anyone can invest and profitability in the world's best market. Random House.

Roberts, M. (2009). Duration and characterictics of metal prices cycles. *Ressources Policy, 34(3):87-102.*

Salvatore, D. (2005). The Euro-dollar rate defies prediction African. *Journal of Policy Modeling, 27(4).*

Samuelson, P.A. & Nordhaus, W. D. (2001). Economics, 17 edition. McGraw-HILL. Higher education , Economics, 792 pages.

Sari. R., Hammoudeh, S., & Soytas, U., (2010). Dynamics of oil price, precious metal prices, and the exchange rate. *Energy Economics, 32:351-362.*

Scarsini, M. (1984). On measures of concordances. *Stochastica, 8: 201-218.*

Schaling, E., Ndlovu, X. , & Alagidede, P. (2014). Modelling the Rand and commodity prices: A granger causality and Co-integration analysis. *South Africa Journal of Economics and Management Science, 17, 5.*

Segers, J. (2012). Asymtotics of empirical Copulas processes under non-restrictive smoothness assumptions. *Bernoulli, 18(30): 764-782.*

Sephton, P. S. (1995). Modelling the link between commodity prices and exchanges rates. The tale daily of data. *Canadian Journal of Economics, 25: 156-71.*

Shams, S., & Zarshenas, M. (2014). Copulas approach for modelling oil and gold prices and exchange rate co-movement in Iran. *International Journal of Statistics and Application, 4 (3): 172-175.*

Sklar, A. (1959). "Functions de repartition a n dimension et leurs marges". *Publication, Institut de Statistics. University Paris, 8:229-231.*

Solt, M.E. and Swanson, P. J. (1981). On the efficiency of the markets for gold and silver. *Journal of Business. 54: 453-78.*

South Africa mining (2015). Annual report 2015. & 7 th edition Available at www.pwc.co.za. Accessed on 24 June 2016.

Statistics South Africa (2016). Quarterly labour force survey quarter 2, 2016. Available at www.statssa.gov.za, info@statssa.gov.za. Accessed July 2016.

Tsukahara, H. (2005). Semiparametric estimation in Copulas methods. *Canadian Journal Statistics,33: 357-375.*

Trivedi, P. K. & Zimmer, D. M. (2005). Copula modeling : An introduction for practitioners. *Foundations and Trends in Econometrics,1 (1): 1-111.*

Trivedi, P. K & Zimmer, D. M. (2009). Pitfalls in modeling dependence structures: explorations with Copulas, the methodology and practice of econometrics (the Festschrift volume in honour of David F. Hendry), Oxford University press, 150-174.

United States Geological Survey (2017). Mineral Commodity Summaries, January 2017: U.S Geological Survey. Minerals.usgs.gov/minerals/pub/2017/mcs2017pdf.

Vandenberghe, S., Verhoest, N.E. C. & De Baets, B. (2010). Fitting Bivariate Copulas to the dependence structure between storm characteristics: A detailed analysis based on 105 year 10 minutes rainfall. Water Resource Research, An AGU *Journal, 46(1).* Doi: 10:1029/2009WR007857.

Van der Merve, E. J. (1996). Exchange rate management policies in South Africa: Recent experiences and prospects. Occasional paper number 9.Pretoria: South Africa Reserve Bank.

Venables, W.N., Smith, D. M. & the R core team (2015). An introduction to R. Notes on R: A programming environment for data analysis and graphs. Version 3.2.1 (2015)-06-18.

Volcker, P. A & Bernstein, P. L., (2008). A Primer on money, banking and gold, third edition. Hoboken, NJ. ISBN798- 0-470-28758-3.

Wang, P. (2009). The Economics of foreign exchange and global finance. Second Edition Berlin: Springer-Verlag Berlin Heidelberg 2009. DOI 10.1007/978-3-64200/00-0.

Wilmott, P. (2007). Introduces quantitative finance. Second Edition: John Wiley & Sons, Ltd West Sussex, England. ISBN 978-0-470-31958-1.

Wu, C-C, Chung, H. & Chang, Y. H. (2012). The economic value of co-movement between oil price and exchange rate using Copula based GARCH models. *Energy Economic, 34: 270-282.*

GFMS ltd. (2015). Gold Survey 2015. London Thomson Reuters GFMS:8-110.

Zhu, H-M, Li, R., & Li, S. (2014). Modelling dynamic dependence between crude oil prices and Asian-pacific stocks markets returns. *International Review of Economics and Finance, (29):208-22.*

Appendix A

A.1 Probability Concepts

Joint-Distribution

The function $f(a, b)$ is a joint distribution function if it satisfies the followings:

1. $f(a, b) \geq 0$, with $-\infty < a < \infty, \quad -\infty < b < \infty$, (A.1)

2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(a, b)\, da\, db = 1$ (A.2)

Cumulative Density Function

Consider a function $F(a, b)$ such that

$F(a, b) = Pr(x \leq a, y \leq b)$ for $-\infty < x < \infty$ & $-\infty < y < \infty$ (A.3)

$$= \int_{-\infty}^{a} \int_{-\infty}^{b} f(x, y)\, dx\, dy.$$

Then the function F(a,b) is a joint cumulative density function.

A.2 Marginal distribution function

Marginalisation refers to the process of taking out or removing the impact or influence of one or more events from joint probability.

For random variables A and *B*, and *F(A, B)* their joint probability at (*A*, *B)*, then function given by:

> $h(a) = \sum_a f(a, b)$ and $g(b) = \sum_b f(a, b)$ are marginal distribution function of *A* and *B.*

The joint distribution of a random vector $Y = (Y_1, \ldots, Y_n)$ defined on $(\Omega, A, P(.))$ is given by $F_Y: \mathbb{R}^n \longrightarrow [0,1]$, defined by $F_y(y) = P(Y \leq y)$, $y \in \mathbb{R}^n$.

A.3 Properties of the Joint Distribution Function

The joint distribution function $F_{X,Y}$ of the random vector (X, Y) fulfils the following properties:

1. $\lim_{x,y \longrightarrow -\infty} F_{x,y}\ (x, y) = 0,$
2. $\lim_{x,y \longrightarrow \infty} F_{x,y}\ (x, y) = 1,$
3. If $(x_1, y_1) \leq (x_2, y_2)$ then $F_{X,Y}\ (x_1, y_1) \leq F_{X,Y}\ (x_2, y_2),$
4. $F_{X,Y}$ is continuous for $F_{X,Y}\ (x + u, \quad y + v) \longrightarrow F_{X,Y}(x, y)$ for $u, v \longrightarrow 0^+$