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DETERMINATION OF LOCAL CRYSTAL SYMMETRY IN COMPLEX, MULTIELEMENT, FERROELECTRIC PEROVSKITES AND ALLOYS

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## DISSERTATION

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#### Abstract

Structural change arising from correlated lattice and charge interaction in complex, multi-element, crystals has a profound effect on their physical properties. Examples include charge ordering in complex oxides, polarization nanodomains in ferroelectrics, and vortex matter. A common scheme to these systems is that there exists a distinction between local crystal symmetry and the average, macroscopic symmetry imposed by fluctuations in the crystal lattice. Before we can control and exploit these fluctuationinduced emergent properties, the crucial first step is to fully characterize any correlation that may exist.

This thesis explores the crystallographic aspect of local charge, polarization, and lattice interactions in complex, multi-element, crystals by developing scanning convergent beam electron diffraction (SCBED) based techniques. The applications of SCBED characterization demonstrated here include: ferroelectric $\mathrm{BaTiO}_{3}$ single crystal, relaxor-ferroelectric ( $1-\mathrm{x}) \mathrm{Pb}\left(\mathrm{Zn}_{1 / 3} \mathrm{Nb}_{2 / 3}\right) \mathrm{O}_{3}-\mathrm{xPbTiO}_{3}(\mathrm{x}=0.08)$ single crystal, and multi-principal-element alloy $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$.

First, we show the local crystal symmetry and polarization fluctuations in $\mathrm{BaTiO}_{3}$ single crystals as determined using SCBED. An improved algorithm for CBED symmetry quantification is used to map the ferroelectric domains and local symmetry across the ferroelectric phase transition temperatures. The symmetry in $\mathrm{BaTiO}_{3}$ was found inhomogeneous; regions of a few tens of nanometers retaining almost perfect symmetry are interspersed in regions of lower symmetry. The SCBED results suggest the coexistence of displacive and order-disorder phase transition, affected upon by the local


structure.

Next, we examine the local symmetry, polarization nanodomains, and the domain wall (DW) structures in relaxor-ferroelectrics. Nanometer-sized domains having the monoclinic $P m$ symmetry in PZN-8\%PT single crystals are identified by performing SCBED along the [100], [001], and [111] zone axes. Intensity distribution in the (000) disks in the CBED patterns is used to determine lattice-rotation at the precision of $\pm 0.012^{\circ}$ by performing SCBED on a standard Si sample. A careful examination of the polarization DWs revealed the presence of lattice-rotation vortices of $\sim 15 \mathrm{~nm}$ in diameter in PZN-8\%PT, which can be attributed to bound charge discontinuity and depolarization fields.

The lattice distortion effect in high entropy alloys (HEAs) is explored as a model of multi element alloys. Lattice distortion is one of the four core effects of HEAs, which results from different atom sizes and influences solid solution hardening. However, so far quantification of lattice distortion effects by X-ray and neutron diffraction has provided contradictory results. Using SCBED, we visualize the sub-nanometer strain fluctuations and local symmetry breaking in single phase $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$. Our results reveal $10 \pm 3 \mathrm{~nm}$, disc-shaped, clusters having $\sim 7.1 \%$ tensile displacements along $<110>$ directions distributed throughout the specimen; local strain, on the contrary, was found to be fluctuating within $\pm 1.3 \%$ and slow-varying over $\sim 50 \mathrm{~nm}$. The observed inhomogeneous lattice distortion using scanning electron diffraction thus provides a new perspective on structure and property relations in multi-principal-element systems.

To my parents Syou and Thomas, my sisters Annie and Joy.

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## TABLE OF CONTENTS

CHAPTER 1: INTRODUCTION ..... 1
1.1 Ferroelectricity and crystallographic symmetry ..... 4
$1.2 \mathrm{BaTiO}_{3}$ ..... 9
$1.3(1-x) \mathrm{Pb}\left(\mathrm{Zn}_{1 / 3} \mathrm{Nb}_{2 / 3}\right) \mathrm{O}_{3}-\mathrm{xPbTiO}_{3}$ ..... 10
1.4 $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$ ..... 13
1.5 Motivation and organization of this thesis ..... 15
1.6 References ..... 17
CHAPTER 2: SCANNING CONVERGENT BEAM ELECTRON DIFFRACTION. ..... 23
2.1 Transmission electron diffraction. ..... 23
2.2 Illumination semi-convergence angle $\alpha$ ..... 24
2.3 Probe size ..... 27
2.4 Beam scanning system ..... 29
2.5 Operation procedures ..... 31
2.6 Symmetry quantification. ..... 36
2.7 CBED simulation ..... 41
2.8 References ..... 43
CHAPTER 3: NANOSCALE SYMMETRY FLUCTUATIONS IN FERROELECTRIC BARIUM TITANATE, $\mathrm{BaTiO}_{3}$ ..... 45
3.1 Introduction ..... 46
3.2 Experimental methods ..... 49
3.3 Results ..... 52
3.4 Discussion ..... 58
3.5 Conclusions ..... 61
3.6 References ..... 62
CHAPTER 4: LATTICE ROTATION VORTEX AT THE MONOCLINIC DOMAIN BOUNDARY IN RELAXOR FERROELECTRIC CRYSTAL ..... 66
4.1 Introduction ..... 66
4.2 Experimental methods ..... 68
4.3 Results ..... 71
4.4 Discussion ..... 77
4.5 Conclusions ..... 80
4.6 References ..... 81
CHAPTER 5: DIRECT IMAGING OF DISTORTED LATTICE IN A HIGH ENTROPY
ALLOY ..... 85
5.1 Introduction ..... 86
5.2 Experimental methods. ..... 88
5.3 Results ..... 91
5.4 Discussion ..... 101
5.5 Conclusions ..... 104
5.6 References ..... 105
CHAPTER 6: SUMMARY AND OUTLOOKS ..... 109
6.1 Summary ..... 109
6.2 Outlooks ..... 110
6.2.1 Quantitative refinement of SCBED: SQCBED ..... 110
6.2.2 Rocking beam SCBED: SRCBED. ..... 111
6.2.3 Time-resolved SCBED ..... 112
6.3 References ..... 113
APPENDIX A: DM SCRIPT FOR SCBED ..... 114
A. 1 Scanning electron diffraction acquisition ..... 114
A. 2 Symmetry quantification. ..... 122
APPENDIX B: PYTHON SCRIPT FOR CBED SIMULATION. ..... 143
B. 1 CBED simulation of $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$ with random displacements ..... 143
APPENDIX C: INPUT DATA FILES FOR CBED SIMULATION ..... 145
C. 1 Data file of $(1-\mathrm{x}) \mathrm{Pb}\left(\mathrm{Zn}_{1 / 3} \mathrm{Nb}_{2 / 3}\right) \mathrm{O}_{3}-\mathrm{xPbTiO}_{3}(\mathrm{x}=0.08)$ ..... 145
C. 2 Data file of tetragonal $\mathrm{BaTiO}_{3}$ ..... 145
C. 3 Data file of $\mathrm{FCC} \mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$. ..... 146

## CHAPTER 1

## INTRODUCTION

This thesis focuses on nanoscale characterization of correlated local order or disorder in the relaxor-ferroelectric and high entropy alloy crystals based on the improved scanning convergent beam electron diffraction (SCBED) techniques, as implemented in a transmission electron microscope (TEM). Diffraction in general forms the basis of crystal structures determination, where diffraction patterns are obtained by scattering of X-rays, neutrons or electrons. Scattering from a ordered structure, whether it is a crystal, a quasicrystal, or even an incommensurately modulated crystal, produces a set of sharp diffraction spots; while diffraction patterns recorded from liquids or glasses show continuous broad rings with no sharp diffraction spots at all. In between the aforementioned two extremes, scattering from crystals with correlated order/disorder gives rise to diffraction patterns having discrete Bragg reflections as well as continuous diffuse scattering. The diffuse scattering reflects the correlations of local structural order. However, the interpretation of such patterns is challenging (1).

The local structural disorder results from correlated lattice, spin, and charge interactions in complex, multi-element crystals, and often gives rise to exceptional properties. Examples include charge ordering in complex oxides (2), spontaneous polarization in ferroelectrics (3, 4), vortex matter (5), and unconventional superconductivity (6). These interactions must be understood in order to optimize and
design new functional materials based on the emergent properties. For example, barium titanate was discovered as ferroelectric and used for sonar during the World War II, yet the exact phase transition mechanism features local crystal symmetry breaking is still not fully understood; the relaxor-ferroelectric with polarization nanodomains show giant electromechanical response that is important for transducers, medical imaging, and nanoelectronic devices applications; the multi-principal element high entropy alloys are promising materials for their excellent thermomechanical properties and irradiation resistance. Thus, the outstanding experimental challenge is to image and characterize quantitatively the local structure and order parameters of materials with correlated disorder.

Ferroelectric perovskites are characterized by a non-volatile electric dipole moment, the spontaneous polarization $P_{s}$ vector, whose direction can be switched under the constraints of the crystal space group. Analogous to ferromagnets, ferroelectric samples often comprise domains with different orientation of the polarization depending on the local electromechanical boundary conditions. The interface between domains, known as domain walls, can contribute significantly to ferroelectric switching as well as adopt novel properties such as the formation of conducting planes in insulators (7, 8). Relaxor ferroelectrics are solid solutions between relaxors and ferroelectric crystals, characterized by the high dielectric permittivity over a broad temperature range. The relaxorferroelectrics exhibit exceptional piezoelectric response (9), which their nm-sized domains
and disordered polar nanoregions have been regarded as "hopeless mess" for structural characterization (10, 11).

Multi-principal element alloys, including high entropy alloys (HEAs), is another interesting class of material with local disorder due to their exceptional mechanical properties at cryogenic and elevated temperatures (12,13). HEAs are composed of five or more elements of equal or near-equal molar percentage, resulting in a large contribution of mixing entropy in the Gibbs free energy that stabilizes a single solid solution phase (14, 15). Due to the varying sizes of the constitutive elements coordinated at the same crystallographic site, HEAs are expected to have severe lattice distortion. However, systematic and quantitative studies of lattice distortion are still missing due to lack of the suitable probe (13, 16).

In what follows, we introduce the key concepts of ferroelectricity and crystallography, as well as the materials presented in this thesis: $\mathrm{BaTiO}_{3},(1-\mathrm{x}) \mathrm{Pb}\left(\mathrm{Zn}_{1 / 3} \mathrm{Nb}_{2 / 3}\right) \mathrm{O}_{3}-\mathrm{xPbTiO}_{3}$, and $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$. The central focus of this thesis, whether one can determine the local crystal structure in complex multi-element crystals, is raised and discussed in terms of local crystal symmetry associated with physical order parameters. Finally, this chapter ends with a brief summary of the organization of this dissertation and the findings within this document.

### 1.1. Ferroelectricity and Crystallographic Symmetry

When subjected to an applied stress, crystals undergo a small change in dimensions and develop a small strain. Some crystals develop electric polarization when strained under the applied stress, which is known as the 'direct' piezoelectric effect. Piezoelectric crystals also show a 'converse' effect, which the strain would develop proportional to an applied electric field. Crystals in general are classified by their symmetry. There are 32 point groups and thus 32 classes of crystals according to their point symmetry. Among the 32 crystal classes, 11 possess center of symmetry (Fig. 1.1). For centrosymmetric crystals, the applied stress results in symmetrical atomic displacements, in which the center of positive charge coincides with the center of negative charge, hence exhibit no polar properties. Of the remaining 21 non-centrosymmetric crystal classes, 20 of these possess polar axis and exhibit the piezoelectric effect. Of the 20 piezoelectric crystal classes, 10 of these have a unique polar axis, which mean they meet the necessary conditions to be pyroelectric. These crystals are called polar because they possess a spontaneous polarization $\left(P_{s}\right)$. Therefore, if pyroelectric crystals have a spontaneous polarization along unique polar axes they will exhibit temperature dependent changes in the spontaneous polarization. Finally, a small subset of crystals within the pyroelectric crystal class have two or more orientation states in the absence of external electric field, and can be switched between these states by an applied electric field are ferroelectric (17).

| Crystal system | Triclinic | Monoclinic | Orthorhombic | Tetragonal | Trigonal | Hexagonal | Cubic |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 222 | 4 | 3 | $6$ | 23 |
|  | $\overline{1}$ | $m$ | 2 mm | $\overline{4}$ | $\overline{3}$ | $\overline{6}$ | $\frac{2}{m} 3$ |
|  |  | $\frac{2}{m}$ | mmm | $\frac{4}{m}$ | 322 | $\frac{6}{m}$ | 432 |
|  |  |  |  | 422 | $3 m$ | 622 | $\overline{4} 3 m$ |
|  |  |  |  | $4 m m$ | $\overline{3} m$ | $6 \mathrm{~mm}$ | $\frac{4}{m} \overline{3} \frac{2}{m}$ |
|  | :Piezo :Polar | ctric Cryst <br> ystals | Class | $\begin{gathered} \overline{4} 2 m \\ \frac{4}{m} m m \end{gathered}$ |  | $\begin{gathered} \overline{6} 2 m \\ \hline \frac{6}{m} m m \\ \hline \end{gathered}$ |  |

Figure 1.1. Hierarchy of the 32 crystallographic classes. Blue box and red circles indicate piezoelectric and polar crystal classes, respectively.

A more rigorous definition of the ferroelectricity is based on the pyroelectric coefficient, i.e., the change of polarization with respect to temperature. When an electric field $\boldsymbol{E}$ is applied to a polar material the total electric displacement $\boldsymbol{D}$ is given by

$$
\begin{equation*}
\boldsymbol{D}=\varepsilon_{0} \boldsymbol{E}+\left(\boldsymbol{P}_{\boldsymbol{s}}+\boldsymbol{P}_{\text {induced }}\right)=\varepsilon \boldsymbol{E}+\boldsymbol{P}_{\boldsymbol{s}} \tag{1.1}
\end{equation*}
$$

Therefore

$$
\begin{equation*}
\frac{\partial \boldsymbol{D}}{\partial T}=\frac{\partial \boldsymbol{P}_{s}}{\partial T}+\boldsymbol{E} \frac{\partial \varepsilon}{\partial T} \tag{1.2}
\end{equation*}
$$

, where $\overrightarrow{\boldsymbol{p}}=\partial \boldsymbol{P}_{s} / \partial T$ is the pyroelectric coefficient, a vector. The crystallographic symmetry places strict restrictions on the orientation of $\vec{p}$. Ferroelectric phase transition is defined by the crystal undergoing from a high-temperature, high symmetry phase to a lowtemperature, low symmetry phase in which the vector of pyroelectric coefficients acquires
new components which were zero in the high-symmetry phase (18). Similarly, a crystal is considered ferroelastic if the phase transition results in a low symmetry phase in which the second-rank thermal dilatation tensor $\alpha_{i j}$ changes the number of its independent components with respect to those in the high symmetry phase (19). Therefore, the change in the tensor $\alpha_{i j}$ is associated with the change of the unit cell dimension. In the phenomenological theory of phase transition (Landau theory), the physical quantity which appears in the lower symmetry phase and is fully responsible for the symmetry change is called the order parameter of the transition.

Perovskite oxides are widely studied and technologically important because the versatile structure can accommodate a wide variety of cations, which leads to a wide range of properties, including ferroelectricity, ferromagnetism and colossal magnetoresistance, piezoelectricity, multiferroicity, and metal-insulator transitions (20). Perovskite ferroelectrics of the form $\mathrm{ABO}_{3}$ can be represented by 8 corner sharing A-site cations, 6 oxygen atoms at face centers, and a B-site cation at body center (inside an oxygen octahedron). The high-temperature paraelectric phase has a primitive cubic structure of $P m \overline{3} m$, as shown in Fig. 1.2. Upon cooled down from high temperature through the Curie temperature $T_{c}$, the material undergoes a structural distortion where the ions are displaced, resulting in the symmetry lowering and a net dipole moment. Figure 1.2 shows an example of the tetragonal distortion, where the A-site displacement produces a tetragonal distortion
in the unit cell, and the displacements of B-site and oxygen atoms lead to a non-zero polarization along the c -axis.


Figure 1.2. Schematic drawing of the perovskite structure in the cubic paraelectric phase above Curie temperature (left) and the tetragonal distortion occurring below the Curie temperature (right).

The perovskite structure is extremely flexible enabling a wide variety of A- and B-site cations with various valence states and defect chemistries of up to a few percent (21). From a geometric perspective, the resulting structure depends on the atomic radii of the A- and B-site cations, which can be predicted by calculating the Goldschmidt tolerance factor

$$
\begin{equation*}
t=\frac{R_{0}+R_{a}}{\sqrt{2}\left(R_{0}+R_{b}\right)} \tag{1.3}
\end{equation*}
$$

, where $R_{o}, R_{a}$ and $R_{b}$ are the atomic radii of the oxygen, A-site and B-site cations, respectively (22). Goldschmidt has shown that the perovskite structure is formed when $t \approx 1$. For $t>1$, the unit cell dimension is imposed by the distance $R_{o}+R_{a}$ and the B -site cation is too small for the octahedron, which the structure would develop a small polar
distortion. For $t<1$, the A-site cation is small compared to the hole in oxygen octahedra, meaning the A-site cation cannot effectively bond with all 12 neighboring oxygens. In perovskites with low tolerance factors, the octahedral rotations help to optimize the coordination environment of the A-site cations which suppress ferroelectricity (23). An example is the Pnma space group, which is a commonly adopted structure for $\mathrm{ABO}_{3}$ perovskites with octahedral rotations.

Another form of ferroelectric distortions is a result of symmetry breaking displacements of the B-site cation within the oxygen octahedra. The B-site driven ferroelectricity can be described by a second-order Jahn-Teller distortion, which involves a delicate balance between short-range forces and long-range Coulomb interactions. The long-range Coulomb interactions favor ferroelectric state, while short-range repulsions between charged ions are minimized for centrosymmetric structures. In the case of $\mathrm{BaTiO}_{3}$, the hybridization of Ti $3 d$ and $\mathrm{O} 2 p$ orbital is essential for stabilizing the ferroelectric distortion, while the $\mathrm{Ba} 5 p$ orbital remains stereochemically inactive. On the other hand, $\mathrm{PbTiO}_{3}$ is a more robust ferroelectric with A-site driven ferroelectric distortion, where the hybridization of $\mathrm{Pb} 6 s$ orbital lone-pair electrons with the $\mathrm{O} 2 p$ orbitals resulted an increase of the tetragonality of the unit cell (24).

## 1.2. $\mathrm{BaTiO}_{3}$

Barium titanate $\left(\mathrm{BaTiO}_{3}\right)$ is the first perovskite compound being identified as ferroelectric (25). Macroscopically, it has a cubic structure of $\operatorname{Pm} \overline{3} m$ in the high temperature paraelectric phase. Upon cooling down, it successively transforms from a cubic phase to tetragonal (T) phase of $P 4 m m$ at 393 K , then to orthorhombic (O) structure of $A m m 2$ at 278 K , and to the low-temperature rhombohedral ( R ) phase of $R 3 m$ at 183 K (26).


Figure 1.3. Schematic drawing of the domain states of $\mathrm{BaTiO}_{3}$ in (a) tetragonal phase 4 mm , (b) orthorhombic phase mm2, and (c) rhombohedral phase 3 m . The red arrows with numbers represent the $P_{s}$ vectors (redrawn after ref (18)).

The symmetry permissible polarization orientations in each ferroelectric phase of $\mathrm{BaTiO}_{3}$ are different, hence the different domain configurations. For example, Fig. 1.3 shows the polarization along the $\langle 100\rangle,<110\rangle$ and $<111\rangle$ directions corresponding to the directions of the atomic displacements in $\mathrm{T}, \mathrm{O}, \mathrm{R}$ phases, respectively. The domain configuration can be characterized by the angles between the respective $P_{s}$ vectors, for $T$
phase the $180^{\circ}(1,2)$ and $90^{\circ}(1,3)$ domains; in O phase $180^{\circ}(1,2), 90^{\circ}(1,3), 60^{\circ}(11,5)$, and $120^{\circ}(11,7)$ domains; and for R phase the $180^{\circ}(1,2), 71^{\circ}(1,2)$, and $109^{\circ}(1,4)$ domains [Fig.1.3]. In real crystals, the domain configuration is subjected to the mechanical (strain) and electrical (charge) boundary conditions (27, 28). The details of domain structures characterization and the phase transition mechanism will be discussed in Chapter 3.

## 1.3. (1-x) $\mathrm{Pb}\left(\mathrm{Zn}_{1 / 3} \mathrm{Nb}_{2 / 3}\right) \mathrm{O}_{3}-\mathrm{xPbTiO}_{3}$

Relaxor ferroelectrics are site- and charge-disordered solid solutions between relaxors and ferroelectric crystals. The name 'relaxor' came from the high dielectric permittivity over a broad temperature range, whereas in conventional ferroelectrics the permittivity is peaked around the Curie temperature (29). Lead-based relaxor ferroelectric perovskites having the formula $(1-x) \operatorname{Pb}\left(B_{1 / 3}^{+2} B_{2 / 3}^{\prime \prime+5}\right) O_{3-x}-x P b\left(B^{\prime \prime \prime+4}\right) O_{3}\left(B^{\prime}, B^{\prime \prime}, B^{\prime \prime \prime}=\mathrm{Zn}, \mathrm{Nb}\right.$, Ti for PZN-PT, and $\mathrm{Mg}, \mathrm{Nb}, \mathrm{Ti}$ for PMN-PT) exhibit exceptional piezoelectric properties (9) near the morphotropic phase boundary (MPB), which is useful for transducer and ultrasound applications. Fig. 1.4 show the dielectric constant $d_{33}$ as a function of composition, where the piezoelectric constant is defined as $d_{i j}=$ $\frac{\text { Induced strain along direction } i}{\text { Applied field along direction } j}$. The MPB refers to a chemically driven change in phase, and is defined in a narrow composition region between the rhombohedral ( R ) $R 3 m$ and tetragonal (T) P4mm phases, at lower x and higher x , respectively. For PMN-xPT, the MPB lies in the region of $x=28 \sim 35 \%(30,31)$, and the reported $x=8 \sim 11 \%$ for PZN-xPT (32-34).


Figure 1.4. Piezoelectric constant $d 33$ as a function of crystal composition and orientation in PZN-PT published in ref (34).

For relaxor-ferroelectric such as PZN-xPT and PMN-xPT, it is still unclear how underlying polar structure gives rise to their properties, including unusually large electromechanical response and high permittivity over a broad temperature range. A model of local inhomogeneities known as polar nanoregions (PNRs) inside a non-polar matrix has been widely used to describe the structure of relaxor-ferroelectrics. The inhomogeneities arise from chemical and valence mixing at the B cation site in the perovskite structure. While the PNR is evidenced by strong X-ray diffuse scattering (35) and piezoresponse force microscopy measurements (36), recent studies have questioned the validity of this model $(37,38)$. The butterfly shape in the diffuse scattering pattern can be reproduced by molecular dynamics simulations of a multi-domain structure, consisted of domains having sizes of 2-10 nanometers (11).

Nanometer-sized monoclinic domains has been reported by X-ray diffraction $(39,40)$, neutron diffraction (41-45), and electron microscopy $(46,47)$ for PZN-xPT in the MPB region. Suggested by the polarization model, the presence of monoclinic phases ease the polarization rotation from rhombohedral to tetragonal phases (48, 49). However, recent CBED results suggested the local structure of $\mathrm{PMN}-31 \% \mathrm{PT}$ is triclinic, but showed monoclinic $C m$ symmetry when averaged over a volume of larger than $\sim 4.7 \times 10^{3} \mathrm{~nm} 3$ (47, 50, 51).


Figure 1.5. Schematic drawing of the possible domain states of monoclinic phase Pm. The red arrows with numbers represent the $P_{s}$ vectors (redrawn after ref (52)).

For the monoclinic phase $P m$, there are 24 symmetry permissible polarization orientations. For example, Fig. 1.5 shows the polarization along the representative 12
<u0v> directions corresponding to the directions of the atomic displacements. The directions between adjacent domains usually satisfy the mechanical compatibility and electrical neutrality conditions. In real crystals, the domain walls may be misoriented relative to the permissible directions predetermined by mechanical conditions $(28,52)$. The characterization of monoclinic nanodomains and their disinclination strain will be detailed in Chapter 4.

## 1.4. $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$

High-entropy alloys (HEAs) are a new class of materials characterized by their unconventional compositions, which comprise 5 or more principal alloying elements in equimolar or near equimolar fraction. The rationale for the alloy design is that the large contribution of mixing entropy to the Gibbs free energy stabilizes a single solid solution phase (14, 15). HEAs have attracted much research attention for their interesting mechanical, electronic, and magnetic properties (14, 15, 53). For example in $\mathrm{Al}_{\mathrm{x}} \mathrm{CrFeCoNi}$ system, the Al contents have a large effect on the crystal structure. At low Al contents, the as-cast $\mathrm{Al}_{\mathrm{x}} \mathrm{CrFeCoNi}$ alloy adopt fcc crystal structure, which changes to bcc structure as Al contents increase (54-59). HEAs such as $\mathrm{Al}_{\mathrm{x}} \mathrm{CrFeCoNi}$ alloys also show exceptional structural stability under irradiation (12, 60-62), suggesting self-healing properties such as local melting and recrystallization happen easily under irradiation, wiping out much of structural defects.

The severe lattice distortion effect is one of the core effects for describing HEAs (63). Due to the various atomic sizes of elements in a concentrated solution, one would speculate that the constituent atoms would inevitably be displaced away from their ideal lattice positions. The lattice distortion is claimed to be more severe than in conventional alloys, contribute to excess entropy, and impede dislocation movements and lead to pronounced solid solution strengthening (13, 16, 64-66).

Due to the lack of suitable structural characterization approaches, however, the exact nature of lattice distortion effect in MPEAs is still unknown (13, 16, 65, 60). For single phase solid solution HEAs, the reported X-ray and neutron diffraction studies have shown sharp Bragg peaks belonging to a simple Bravais lattice (53,55,58). The Debye-Waller factors determined by the Rietveld refinement analysis, however, are noticeably larger than that expected for the thermal Debye-Waller factors, consistent with local lattice distortions (65, 67, 68). Furthermore, the X-ray or neutron total scattering and extended X-ray absorption fine structure (EXAFS) analysis suggests short-range ordering (SRO) and strong distortion away from the average Bravais lattice (69-74). While X-ray and neutron diffraction are highly quantitative, the main drawback of such studies is the difficulty in distinguishing between ordered and disordered phases with the same base crystal structure. The characterization of local symmetry and strain will be detailed in Chapter 5.

### 1.5. Motivation and organization of this thesis

The above discussions introduces the structure of ferroelectric crystals, their symmetry and structure-property relations. Local structures of ferroelectrics such as polarization nanodomains play an important role on their piezoelectric properties. Determination of local structure requires a general and robust method having spatial resolution at nanometer scale. To meet the challenges in characterizing local crystal structure, the major motivation associated with this research are 1) to develop a general method, or a 'local probe', for the determination of crystallographic symmetry at nanometer resolution, 2) to automate the data analysis for extracting structural information, and 3) to correlate local crystal symmetry variations with underlying materials physics.

In this thesis, we report on the development of scanning convergent beam electron diffraction (SCBED) for probing local structure. SCBED is a technique based on CBED, which takes the advantages of the dynamical diffraction effects, or strong interaction between electron and crystal potential, and the high resolution nature due to the small wavelength of high energy electrons. We have further improved this technique for local symmetry and strain analysis in complex, multi-element crystals.

The thesis is organized in 6 chapters. Chapter 2 introduces the principles of the SCBED technique that is used throughout this thesis as the main characterization technique. Experimental details and algorithms for extracting structural information are presented in Ch. 2. Chapters 3-5 describe the applications of SCBED to different materials. Chapter 3
investigates the nanoscale fluctuations of the ferroelectric phase transition order parameter in single crystal $\mathrm{BaTiO}_{3}$. Chapter 4 discusses about the symmetry determination of nanodomains in PZN-8\%PT and the discovery of a new type of defect, lattice-rotation vortex. Chapter 5 describes the study of local lattice distortion in $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$. The local crystal symmetry and strain are directly visualized using SCBED and a sub-nanometer electron probe. Conclusions along with discussions on future perspectives are described in Chapter 6.

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## CHAPTER 2

## SCANNING CONVERGENT BEAM ELECTRON DIFFRACTION

This chapter discusses about the experimental acquisition and data analysis methods for the scanning convergent beam electron diffraction (SCBED) technique. We include details about the scanning diffraction system, TEM operation procedures, and the symmetry quantification algorithm. The applications of SCBED and the symmetry quantification algorithm to probe local polarization domains and symmetry breaking in ferroelectric perovskites and multi-principalelement alloys are reported in Chapters 3-5.

### 2.1 Transmission electron diffraction

In a transmission electron microscope (TEM), the structure of materials can be determined by utilizing the electrons that traveled through a specimen. The electrons interact with the nuclei and the electron cloud in a specimen, and are scattered multiple times due to the strong Coulomb forces. As a result of destructive and constructive interference of scattered electron waves, diffraction may occur because the wavelength of high energy electrons (e.g., $0.0251 \AA$ for 200 keV electrons) is much smaller than the crystal lattice parameters. Electron diffraction in a TEM can be obtained from a small region/volume of the specimen (e.g., ranging from a few $\mathrm{nm}^{3}$ to $\mu \mathrm{m}^{3}$ ), which is useful for the determination of crystal orientation, local lattice parameter or strain, phase identification, and crystal defect analysis.

### 2.2 Illumination semi-convergence angle $\alpha$

The commonly recorded electron diffraction patterns in TEMs are spot patterns using a beam that is approximately parallel. Convergent beam electron diffraction (CBED), on the other hand, is obtained by focusing the electron beam into a probe onto the sample. Because of the convergent beam, the transmitted and diffracted spots in a diffraction pattern broaden into disks. The illumination convergence angle, or the size of the disk, is an important parameter in CBED, because it determines the range of excitation errors $\left(\mathrm{S}_{\mathrm{g}}\right)$ for each reflection. Along a zone axis, the ideal CBED disk size is twice the Bragg angle ( $2 \theta_{\mathrm{B}}$ ) of the lowest order zero-order Laue zone (ZOLZ) reflection, in order to fill the diffraction space as much as possible without disk overlap, as shown in Fig. 2.1 (1).


Figure 2.1. Schematic ray diagram for convergent beam electron diffraction. Source point $P$ gives rise to conjugate (diffracted) points $P^{\prime}$. Symbols $\alpha$ and $\theta_{B}$ denote semi-convergence angle and Bragg angle, respectively.

The size of the disk is two times the semi-convergence angle $\alpha$, which is usually defined by the condenser aperture in a TEM. The use of the condenser minilens (CM), placed immediately above the objective prefield lens as shown in Fig. 2.2, allows additional change in convergence angles. The "alpha-selectors knob" in JEOL TEMs is used to adjust the CM lens strength (2). The NBD- and CBD-modes are under similar C2-CM configurations for JEOL microscopes installed in the Center for Microanalysis of Materials (CMM) in the Materials Research Laboratory (MRL). However, CBD-mode offers 9 selections of $\alpha$, while NBD-mode offered 5.


Figure 2.2. Schematic ray diagram for a two condenser lenses system. Focused electron probes with a small and large convergence angle obtained using different settings for the CL and CM lenses (3).

The Thermo Fisher Themis Z STEM installed recently in the CMM, MRL at University of Illinois has the three condenser lenses setup. The C1-C2 zoom controls the spot size, and the C2C3 zoom controls the beam convergence angle (4). Using the "MF-Y knob" under "probe mode", it is possible to adjust the convergence angle continuously, by tuning the ratio of C 2 and C 3 lenses excitations, i.e., changing the position of the intermediate image between C 2 and C 3 lenses as shown in Fig. 2.3. Specifically, the switching from "nanoprobe" to "microprobe" mode is associated with the switching on of the condenser minilens, where the beam convergence angles are five times smaller when the CM is switched on. The SCBED experiments can be performed in microprobe-STEM mode, under spectrum-imaging tab. In practice, while varying the convergence angle with MF-Y knob, it is important to check in the imaging mode whether the probe is wellfocused or not, since no TEMs are truly hysteresis-free (yet). The probe can be re-focused by changing the objective pre-field excitations, or "intensity list". However, note that by changing the objective excitations would lead to slight changes in the image magnification, which requires further calibration for careful experiments.


Figure 2.3. Schematic ray diagram for a three condenser lenses system. Focused electron probes with a small and large convergence angle can be varied continuously by changing the position of the intermediate image between C2 and C3 lenses, as indicated by the red arrows.

### 2.3 Probe size

A small electron probe is essential for studying local structures such as symmetry, strain, and defect characterization. The small probe it allows the diffraction patterns to be taken from local regions in the sample. In scanning CBED or scanning transmission electron microscopy (STEM), the probe size often defines the spatial resolution in the obtained images. To form a focused probe, the crossover from the last condenser lens is imaged by the objective prefield lens onto the specimen, as shown in Figs. 2.2 and 2.3. Under "incoherent" conditions (i.e., lateral coherence 2L is less than condenser aperture diameter $2 R$ ), the total probe diameter $d_{0}$ is a result of a series of broadening effects from spherical aberration in the probe forming lens $d_{s a}$, diffraction by aperture
$d_{d}$, chromatic aberration of probe forming lens $d_{c}$, and focusing error $d_{f}$, which can be approximately expressed as:

$$
\begin{equation*}
d_{o}^{2}=d_{s}^{2}+d_{d}^{2}+d_{s a}^{2}+d_{c}^{2}+d_{f}^{2}=\left(\frac{0.6 \lambda}{\theta_{c}}\right)^{2}+\left(0.5 C_{s} \theta_{c}^{3}\right)^{2}+\left(\frac{\Delta E_{0}}{E_{0}} C_{c} \theta_{c}\right)^{2}+\left(2 \theta_{c} \Delta f\right)^{2} \tag{2.1}
\end{equation*}
$$

, where $\lambda$ is the electron wave length, $\theta_{c}$ is the beam convergence angle, $C_{s}$ is the spherical aberration coefficient, $C_{c}$ is the chromatic aberration coefficient, and $\Delta f$ is the defocus value (3).

The size of electron probes in a TEM with a $\mathrm{LaB}_{6}$ thermionic source is largely determined by the size of the demagnified source image. Experimentally the probe size can be measured by the full-width at half-maximum (FWHM) of the electron intensity in the Gaussian distribution. With the JEOL $2100 \mathrm{LaB}_{6}$ TEM, the probe can vary from $50 \mathrm{~nm}, 30 \mathrm{~nm}, 20 \mathrm{~nm}$ at FWHM in TEMmode, and 10 nm , 5 nm , 2 nm at FWHM in CBD-mode. With the JEOL 2010F or 2200FS having a Schottky field-emission source, the probe can easily be tuned from 5 nm to 1 nm at FWHM. The advantages of the 2010F and 2200FS microscopes at MRL are not only the smaller probes due to the FEG, but also the energy-filtering capability (GIF and Omega filters, respectively). Energyfiltering is extremely crucial for CBED experiments, especially for thick specimen where the inelastic scattered electron would significantly blur out the fine features within the disks (predominantly plasmon-loss).

While FEG offers a smaller probe size which gives a better spatial resolution for SCBED experiments, $\mathrm{LaB}_{6}$ works better for beam-sensitive materials due to lower brightness. For example, low-dose SEND has been demonstrated to identify dendritic phases in $\mathrm{LiCoO}_{2}$ (5). The experiments were done on JEOL $2100 \mathrm{LaB}_{6}$ TEM with a small condenser aperture of $20 \mu \mathrm{~m}$, under NBD-mode $\alpha 1$, spot 3 nm , which gives a probe size of $\sim 2.6 \mathrm{~nm}$ in FWHM.

### 2.4 Beam scanning system

Electron nanodiffraction was first demonstrated by Prof. John M. Cowley using STEM and a TV camera for recording diffraction patterns (6). At University of Illinois, Dr. Jing Tao and Prof. Jian-Min Zuo developed the technique of "scanning electron nanodiffraction (SEND)" (7) for imaging nanoscale phase separation of charge ordering phases in $\mathrm{La}_{0.55} \mathrm{Ca}_{0.45} \mathrm{MnO}_{3}$ (8). The principle of scanning electron diffraction is based on controlling (1) scanning coils which deflects/shifts the electron beam, and (2) pixelated detectors such as CCD/K2 camera for automated acquisition, where a diffraction pattern is recorded at every probe position in a defined region of interest (see Fig. 2.4). A similar technique, scanning precession electron diffraction (SPED) $(9,10)$, was implemented by NanoMEGAS (11) using an external scanning system, which the electron beam is rocked at each probe position for reducing dynamical effects. The NanoMEGAS system works by using an additional hardware to control the scanning deflectors (beam shift/tilt, and post objective lens de-scan) with electrical signal, while capturing the diffraction patterns on fluorescent screen with a camera for fast acquisition. With the development of fast cameras which can easily acquire more than 1000 frames per second (e.g. K2 (12), EMPAD (13), medipix3 (14), just to name a few), the development of scanning diffraction is getting attention, which serves as a technique providing structural information complementary to STEMHAADF, EDS, or EELS spectrum imaging (15).


Figure 2.4. Schematic illustration of the SEND system.(16)

The DigitalMicrograph ${ }^{\oplus}$ script, Scanning_ED, developed at the University of Illinois is used to drive the beam scanning and image recording (17). DM is a common software used for acquisition, process, and analysis for TEM images or data. The software is programmable, known as DM script, which can be used to customize tasks. For example, different parts of TEM can be controlled using DM script, including the beam or image shift/tilt deflectors and the CCD camera. Useful database of DM script can be found online, see http://www.dmscripting.com/index.html or https://www.felmi-zfe.at/dm-script/dm-script-database/. The script Scanning_ED interfaces with
several main function that are used to control the microscope (EMGetBeamShift, EMSetBeamShift, and SSCGainNormalizedBinnedAcquire). The values set in functions EMGetBeamShift and EMSetBeamShift are digital numbers, and they are calibrated and converted into the values in terms of sample coordinates in nanometers. The calibration is done using a function built in the script, which the user would perform to establish a reliable current-to-nanometer conversion for each experiment. During acquisition, the script will shift the probe in a raster scan, from left to right and top to bottom. A parameter to rotate the raster scanning rectangle was also implemented, which is useful for aligning the scanning coordinates with the x - and y - axis of the region of interest (18).

To make the beam scan into large regions of interest and being able to handle a large dataset, we have implemented a thread-based acquisition. The thread object in DM allows the script to be run in 'background', not blocking user interaction, as well as the option to stop a long acquisition halfway. To enable SCBED acquisition in STEM mode, we have implemented the DM script using the DigiScan to drive the scanning coils. The main functions used in the STEM mode are DSSetBeamPosition, DSCalcImageCoordFromDS. The script has been tested on following models: JEOL 2100 LaB6, 2100F, 2010F, 2200FS, ARM 200F, and Thermo Fisher Themis Z. The DM script Scanning_ED used for acquiring scanning diffraction data is listed in Appendix A.

### 2.5 Operation procedures

The microscope alignments to consider when performing SCBED experiments are: (1) tiltshift purity, (2) beam shift-induced diffraction shift, (3) diffraction focus, (4) probe focus. The tiltshift purity is of prime importance since a CBED pattern is extremely sensitive to change in diffraction condition ( $<0.012^{\circ}$ in crystal tils or lattice-rotation). Ideally, the beam is not tilted
during the beam scanning for SCBED. Not only does the tilt affect the orientation of the CBED pattern, it also affects the position of CBED pattern. Second, the diffraction pattern also shifts during beam scanning if the diffraction focus and shift alignments are non-ideal. The beam shiftinduced diffraction shift alignment is crucial for measuring deflections due to the presence of electric/magnetic fields as well as for successful data processing. Third, incorrect diffraction focus will impose Fresnel fringes to disks in a CBED pattern, which will affect intensity analysis such as quantitative CBED refinements. Fourth, incorrect probe focus will give rise to a mixture of imaging plane information with the CBED pattern, which will complicate the CBED analysis. The full alignment procedure is listed as following:

1) Switch to CBD mode and the desired alpha, spot size selection. Press "standard focus".
2) With the largest condenser aperture inserted, perform the standard TEM alignments including gun tilt/shift, bright tilt, and pivot point. It is not uncommon that the alignments in CBD modes are far from ideal condition. One practical trick is to align the stigmators from top to bottom, while checking their HEX values. For example, the optimum HEX values for "Beam shift" on JEOL microscopes are around 8000/8000, so the alternative deflector for shifting the beam would be "spot align", which the latter is associated with "bright tilt".
3) Switch to the desired condenser aperture. The condenser aperture can be aligned by wobbling back and forth the brightness knob until the circles on the phosphorous screen are concentric.
4) Focus the beam in vacuum, and switch to diffraction mode. Align the diffraction focus (IL1) until the edge of the disk is sharp and its line profile has an inflection point near the middle, as shown in Fig. 2.5. In some cases, one would observe the irregular shape
on the disk edges, which results from imperfect circular machining of the condenser diaphragm. This step is crucial for quantitative CBED analysis for measuring low-order structure factors (19).


Figure 2.5. Image of the condenser aperture, or CBED disk in vacuum, with the right diffraction focus. The line profile shows an inflection point near the middle.
5) Check the alignments of "shift" and "shift $x /$ shift $y$ ". Align the diffracted disk until the movement is minimized. This step reduces the shift in diffraction pattern during beam scanning.
6) Check the alignments of "tilt" and "tilt $x / t i l t y "$. The pivot point alignment is subject to the change in bright tilt and focus, hence iterations in this step is needed. For JEOL 2200 FS in CBD-mode, the pivot-point may be way off and out of the deflector range. This may be attributed to the different objective lens setting from the "standard focus". A trick to get around this problem is to strongly defocus/over-focus the objective lens to about 3.28 V , which is close to the factory setting of the objective lens. However, note
that by doing so would require an iterative alignment of the stigmators above, such as "CL stigmator" or "bright tilt".
7) Bring the specimen to the correct z-height. This can be checked by either the "image wobbler", or more precisely, the image of a focused probe on specimen. To check if the specimen is not at the correct z-height, we should expect a well-focused round spot in imaging mode, otherwise there will be "diffracted spots/streaks" in the image mode, as shown in Fig. 2.6. If the CBED pattern is slightly defocused, the disks will contain image information, known as "shadow images", which will complicate the CBED analysis.


Figure 2.6. Real space images of a focused electron probe at the incorrect z-height on a Si [110] specimen with larger (left) and less (right) defocus. The defocused probe led to the mixture of diffraction information in the imaging plane, or vice versa.
8) Energy-filtering is strongly recommended for recording high quality CBED patterns, especially for thick specimens (see Fig. 2.7). A10eV energy selecting window is placed at the center of the zero-loss peak in the electron energy loss spectrum (EELS). For Omega filters on the JEOL 2200FS, two parameters are critical: isochromaticity and slit
position. For post column GIF, be sure to tune the low-order stigmatism (e.g. $\mathrm{f}_{\mathrm{x}}, \mathrm{f}_{\mathrm{y}}, \mathrm{s}_{\mathrm{x}}$, sy) to minimize image distortions.


Figure 2.7. Energy-filtered CBED patterns of Al [001] using an in-column Omega filter with 10 eV energy selecting window centered around zero-loss peak (left) and plasmon loss (right).
9) Perform the SEND calibration mentioned above. This step calibrated the DAC current into unit x - and y -vectors in nm .
10) Determine the scanning step size, offset angle, scanning area, and exposure time. In most cases, the step size is about the same order of magnitude as the probe size; the scanning area and exposure time is a trade-off between total acquisition time versus specimen stage stability/drift and CBED intensity/noise level.

The detailed procedures for performing SCBED experiments is described above. However, in some cases one would intentionally defocus the CBED pattern by either changing the z-height or the objective lens focus. This is referred as the large-angle CBED (LACBED) technique. The LACBED technique was shown to be useful for analyzing burgers vectors of dislocations, or
symmetry determination for large unit-cell crystals. Detailed description of the LACBED technique can be found in Refs. $(20,21)$.

### 2.6 Symmetry quantification

In real crystals, the local symmetry often breakdown depending on its boundary conditions. The symmetry breaking may be due to crystallographic defects, epitaxial strain, ferroelectric polarization, or magnetic symmetry. The symmetry recorded CBED pattern, to a column approximation, is averaged over the volume being illuminated by the electron probe, as shown in Fig. 2.6. When using SCBED for local symmetry determination, it is helpful to quantify the amount of symmetry breaking recorded in CBED patterns $(22,23)$. The symmetry quantification algorithm proposed by Dr. Kyou-Hyun Kim and Prof. Jian-Min Zuo (24) is based on the normalized cross correlation coefficient $\gamma(25)$, which measures the similarities between two images/diffracted disks:

$$
\begin{equation*}
\gamma=\frac{\sum_{x, y}\left\{\left[I_{A}(x, y)-\overline{I_{A}} \cdot\left[I_{B}(x, y)-\overline{I_{B}}\right]\right\}\right.}{\left.\sqrt{\left\{\sum_{x, y}\left[I_{A}(x, y)-\overline{I_{A}}\right]^{2}\right\}}\right\}\left\{\left\{\sum_{x, y}\left[I_{B}(x, y)-\overline{I_{B}}\right]^{2}\right\}\right\}} \tag{2.2}
\end{equation*}
$$

, where $\overline{I_{A}}$ and $\overline{I_{B}}$ are the mean values of two diffracted disks that are being compared. From the equation, the correlation coefficient is close to 1 as the two images become identical.


Figure 2.8. Schematic ray diagram for CBED pattern detecting symmetry in real crystals. For column approximation, the highlighted region indicates the volume being illuminated by the electron beam.

While the DM script implemented by Dr. Kyou-Hyun Kim works efficiently for quantifying a pair of disks in a CBED pattern, the algorithm itself has some drawbacks. First, it requires manual placing circular annotations onto the experimental CBED pattern, which introduces uncertainties to the pixel-to-pixel operation such as cross correlation. Second, in experimental CBED patterns, it is possible to have different cross correlation values $\gamma_{\mathrm{g}}$ for different pairs of reflections.

To further improve the robustness of symmetry quantification using SCBED, we have implemented a modified algorithm in DM script to quantify the CBED whole pattern. An array of circular annotations will be generated based on the two basis vectors. To provide one normalized cross-correlation vale for each CBED pattern, our new algorithm weights each $\gamma_{g}$ value with
respect to the disk intensities, namely $\gamma_{\text {representative }}=\frac{\sum_{g} I_{g} \gamma_{g}}{\sum_{g} I_{g}}$. For example, Fig. 2.5 shows the procedure for quantifying the symmetry of a Si [110] CBED pattern. Pairs of diffraction discs related by mirror are selected and marked by the orange circles from ( $A, A^{\prime}$ ) to ( $G, G^{\prime}$ ). The diffraction discs on the mirror plane are marked by blue circles from ml to m 4 . The table list out the $\gamma$ values and the corresponding summed intensities, which gives the weighted cross-correlation value of this pattern $\gamma=98.1 \%$. The script also quantifies the symmetry in the bright-field, or the transmitted (000) disk. The DM script QSymm for symmetry quantification of CBED whole pattern is listed in Appendix A.


|  | V | Sum intensity |
| :---: | :---: | :---: |
| $\left(A, A^{\prime}\right)$ | 0.961096 | 206236 |
| $\left(B, B^{\prime}\right)$ | 0.925561 | 150530 |
| $\left(C, C^{\prime}\right)$ | 0.993897 | $1.45153 e+006$ |
| $\left(D, D^{\prime}\right)$ | 0.968451 | 393573 |
| $\left(E, E^{\prime}\right)$ | 0.992349 | $1.42561 e+006$ |
| $\left(F, F^{\prime}\right)$ | 0.926832 | 153510 |
| $\left(G, G^{\prime}\right)$ | 0.966375 | 194898 |
| m1 | 0.920665 | 147888 |
| m2 | 0.988652 | 952739 |
| m3 | 0.989302 | 978776 |
| m4 | 0.914988 | 148226 |

Figure 2.9. Procedure used for quantifying mirror symmetry of a Si [110] CBED pattern. Disks m1-m4 indicate the reflections on the mirror plane.

The procedures for symmetry quantification in the DigitalMicrograph ${ }^{\odot}$ software using the DM script are listed as follows:

1) Open the results window and make sure it is visible. The cross correlation coefficients will be output into this panel.
2) Use the ROI tool to draw a rectangle. This limits the region for symmetry quantification, since the reflections outside the ZOLZ are usually weak.
3) Draw two line annotations on the CBED pattern. This defines its two basis vectors, $g$ and $h$.
4) Use keyboard " 0 " and arrow keys for adjusting the center coordinate; " $g$ ", " $h$ ", and arrow keys for adjusting the two vectors.
5) Use keyboard "T" for switching on/off indexing for each reflection.
6) Use keyboard "r/R" for adjusting size of the circle. Use "-/+" for adjusting increments.
7) Press " $m$ " for mirror symmetry quantification. Press " $M$ " for changing the mirror plane. The mirror plane has the index ( $\mathrm{m}, \mathrm{n}$ ), which came from $\vec{M}=m \vec{g}+n \vec{h}$.
8) Press " 2 ", " 3 ", " 4 ", " 6 " for quantifying 2-, 3-, 4 -, 6 -fold rotational symmetry.
9) Repeat steps 3-6 as if playing video games -- until you've got the highest score!
10) Save the templates for each symmetry element containing annotations. This is useful for quantifying the whole SCBED image stack.

A possible improvement of the current symmetry quantification algorithm is to include image processing procedures such as circular Hough transform (CHT) for disk detection. This will automate the step of finding disk positions in replacement of interactive trial and error using keyboard. However, implementing CHT in DM scripts would be more troublesome than in high level languages such as Matlab or Python.

We now give a few examples using the quantification algorithm for CBED patterns obtained from two sets of centrosymmetric and noncentrosymmetric crystals, $\mathrm{Si}, \mathrm{GaSb}, \mathrm{SrTiO}_{3}$, and $\mathrm{BaTiO}_{3}$. For clarity, regions outside the disks being quantified in the CBED pattern are being masked. For
example, the Si [110] possess 2 mm symmetry, as shown in real space. The corresponding CBED pattern has two mirror planes of $\gamma_{\mathrm{m}}=98.2 \%$ and $\gamma_{\mathrm{m}}=97.4 \%$, and two-fold rotation symmetry of $\gamma_{2}=98.2 \%$. On the other hand, noncentrosymmetric GaSb [110] possess $m$ symmetry, with a mirror plane of $\gamma_{\mathrm{m}}=97.4 \%$. Similarly, $\mathrm{SrTiO}_{3}$ is a centrosymmetric perovskite at room temperature, and possess 4 mm symmetry along [001] incident direction, which the corresponding CBED pattern shows 4 mirror planes and four-fold rotation symmetry of $\gamma_{4}=97.4 \%$. A polar perovskite, for example tetragonal $\mathrm{BaTiO}_{3}$ shows a mirror plane of $\gamma_{\mathrm{m}}=97.2 \%$. As shown in Fig. 2.10, the polarization lies in the mirror plane of CBED patterns along certain orientations.


Figure 2.10. Symmetry quantification of CBED patterns obtained from centrosymmetric (left column) and noncentrosymmetric (right column) crystals.

### 2.7 CBED simulation

One of the benefits of SCBED and symmetry quantification is to obtain the experimental CBED pattern (best if energy-filtered) with the highest symmetry. This allows one to compare the best experimental CBED patterns with simulated patterns, and thus determine the structural parameters in the specimen such as thickness, orientation, polarization, or lattice parameters. The intensity versus Sg , or rocking curve information, recorded in CBED patterns can be used for quantitative structure factor refinement using pattern matching (26). There are three algorithms used in this thesis for dynamical diffraction simulation: Bloch-wave, multislice, and scatteringmatrix method, and all of the simulation software packages were written by Prof. Jian-Min Zuo in Fortran and $\mathrm{C}(1,27)$.

The Bloch wave method is often used for CBED simulation, which assumes the translational symmetry. Bloch-wave method is used for its accuracy, flexibility and the ability to calculate diffraction pattern in any orientation including high order Laue zone (HOLZ) reflections. The problem of electron diffraction is based on solving the time-independent Schrödinger equation by expanding the electron wave function inside the crystal using a set of Bloch waves. Important reviews of the Bloch wave method include (1, 28-30).

The multislice method was first proposed by Cowley and Moodie and others (31, 32), which has the advantage for calculating diffraction patterns or exit-wave functions from both crystals and non-periodic structures. The multislice method solves the time-dependent Schrödinger equation by forward-scattering approximation, with the specimen potential divided into a sequence of thin rectangular slices. The slice thickness must be thin enough to be a phase object, and be perpendicular to the optical axis of the electron microscope. Appendix B has listed the example
python script for 1) generating a $\mathrm{BaTiO}_{3}$ supercell with random polarization clusters and 2) simulating SCBED of the supercell using multislice method (zmult.exe).

The scattering matrix method is used for simulating CBED patterns or images of crystals with defects (3, 33). The crystal potential with defects is constructed using the deformable ion approximation

$$
\begin{equation*}
U(\vec{r})=\sum_{g} U_{g} \exp (-2 \pi i \vec{g} \cdot \vec{R}(\vec{r})) \exp (-2 \pi i \vec{g} \cdot \stackrel{\rightharpoonup}{r}) \tag{2.3}
\end{equation*}
$$

where $\mathrm{U}_{\mathrm{g}}$ is the Fourier coefficient of the perfect crystal potential and $\vec{R}(\vec{r})$ the atomic displacement. The constructed potential takes account of the modifications in the structure factors by strain, not composition. An example using the scattering-matrix method for simulating the HOLZ line splitting due to strain in $\mathrm{GaAs} / \mathrm{In}_{\mathrm{x}} \mathrm{Ga}_{1-\mathrm{x}} \mathrm{As}$ quantum wells can be found in Ref. (34). Appendix B contains the python script for 1) generating an fcc high entropy alloy supercell being divided into small blocks, each block having displacements follows the Gaussian distribution with specified mean and standard deviation of displacements; 2) simulating SCBED of the supercell using scattering-matrix method (blochz.exe and scbed.exe); and 3) output a binary file which can be imported into DigitalMicrograph ${ }^{\oplus}$ as an image stack for SCBED analysis.

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## CHAPTER 3

## NANOSCALE SYMMETRY FLUCTUATIONS IN FERROELECTRIC BARIUM TITANATE, $\mathrm{BaTiO}_{3}{ }^{1}$

Crystal charge density is a ground state electronic property. In ferroelectrics, charge is strongly influenced by lattice, or conversely, leading to a range of interesting temperature dependent physical properties. However, experimental determination of charge in ferroelectrics is challenging because of the formation of ferroelectric domains. Here, we demonstrate the scanning convergent beam electron diffraction (SCBED) technique that can be simultaneously used for imaging ferroelectric domains and identifying crystal symmetry and its fluctuations. Results from SCBED confirm the acentric tetragonal, orthorhombic, and rhombohedral symmetry for the ferroelectric phases of $\mathrm{BaTiO}_{3}$. However, the symmetry is not homogeneous; regions of few tens nm retaining almost perfect symmetry are interspersed in regions of lower symmetry. While the observed highest symmetry is consistent with the displacive model of ferroelectric phase transitions in $\mathrm{BaTiO}_{3}$, the observed nanoscale symmetry fluctuations are consistent with the predictions of the order-disorder phase transition mechanism.

[^0]
### 3.1. Introduction

Charge density varies spatially across ferroelectric domains. The domain sizes vary from tens of nanometers ( nm ) in relaxor-ferroelectrics to hundreds of nm or microns in ferroelectric crystals, such as $\mathrm{BaTiO}_{3}$. The transition from paraelectric to ferroelectric phases in perovskite ferroelectrics (including $\mathrm{BaTiO}_{3}$ ) can be described by a second-order Jahn-Teller distortion, which involves a delicate balance between short-range forces and long-range Coulomb interactions. The long-range Coulomb interactions favor ferroelectric distortion, while on the other hand, short-range repulsions between charged ions are minimized for centrosymmetric structures, which are preferred in the high temperature, paraelectric phase (1). The second-order Jahn-Teller effects give rise to the offcentre displacement of the small cation in the perovskite ferroelectrics such as $\mathrm{BaTiO}_{3}$ and $\mathrm{Pb}(\mathrm{Zr}, \mathrm{Ti}) \mathrm{O}_{3}$, for example, $\mathrm{BaTiO}_{3}$ successively transforms from the high-temperature paraelectric cubic (C) phase to three low-temperature ferroelectric phases with tetragonal (T), orthorhombic (O), and rhombohedral (R) symmetries (2). Phase transitions in $\mathrm{BaTiO}_{3}$ are generally considered as a classic example of displacive soft-mode-type transitions, which results from the instability of the lattice against a soft polar phonon at the center of the Brillouin zone (3-5). Upon cooling below the Curie temperature, the ions are displaced from high-symmetry sites, which in turn generate a spontaneous dipole moment accompanied by a breaking of crystal symmetry (6). In the displacive model, the equilibrium position of each Ti atom is at the center of an oxygen octahedron for C , and displaced along <111>, <011>, <001> polarization directions for $\mathrm{R}, \mathrm{O}$, and T phases,
respectively. For the ferroelectric phases, the displacements are identical for all unit cells. However, this model fails to explain the heavily damped soft phonons, and contradicts experimental results obtained by x-ray fine structure (XAFS) (7), x-ray diffuse scattering (8, 9), and infrared reflectivity (IR) (10).

The spontaneous symmetry breaking, order-disorder, eight-site model was thus proposed to explain the failures of the displacive model $(8,11,12)$. In this model, all Ti atoms in each unit cell are located in one of the eight <111> directions for all crystal phases. In the R-phase, this model assumes all the Ti atoms are displaced in the same direction. As temperature increases, the polarization can have one or more crystal orientations, which results in the breaking of local crystal symmetry in O-, T-, and C-phases (13-18). Nevertheless, this model fails to predict the heavily damped soft phonon modes near the paraelectric-ferroelectric phase transitions observed by inelastic neutron scattering (19, 20), IR (10), and hyperRaman scattering experiments (21).

A model combining the displacive and order-disorder models proposed by Girshberg and Yacoby successfully explained the soft mode overdamping near paraelectric-ferroelectric phase transitions (22). To explain the atomic origins of this model, Zhang et al. suggest there exists antiferroelectric coupling based on first-principles calculations (23). Their calculations successfully reproduced the x-ray diffuse scattering results, as well as providing an explanation of the overdamping of soft modes. However, the observed entropy changes ( $\leq 0.52 \mathrm{~J} / \mathrm{mol}$ ) for each transition in $\mathrm{BaTiO}_{3}$ is far smaller than the entropy change in an order-disorder transition (5.76
$\mathrm{J} / \mathrm{mol}$, see (8)and (11)). To explain this, a short correlation length between 5 to 10 nm has been postulated (23).

Experimental clarification of the phase transition character in the model ferroelectric, $\mathrm{BaTiO}_{3}$, requires 1) the identification of polarization domains, 2) a determination of the local crystal symmetry within a single domain, and 3) identification of the nature of polarization, structural distortions and chemical bonding. All three tasks are experimentally challenging.

Here, we describe an energy-filtered scanning convergent beam electron diffraction (EFSCBED) study of the ferroelectric structures in single crystal $\mathrm{BaTiO}_{3}$. In what follows, we first demonstrate the determination of $90^{\circ}$ tetragonal domains and their polarizations. We then identify nanometer-sized regions of $\mathrm{BaTiO}_{3}$ exhibiting mirror symmetry in accordance with the $\mathrm{T}-$, $\mathrm{O}-$, and R-symmetry in each of the corresponding phases at three different temperatures. The regions exhibiting high symmetry are surrounded by regions with broken symmetries, with the latter in agreement with the findings by Tsuda et al. (18) and Tsuda \& Tanaka (17). The inhomogeneity in symmetry is explained by the anharmonicity in soft phonon modes and the population fluctuation of <111> nanopolar regions, with large implications for the origin of phase transitions in ferroelectric perovskites. Further, the identification of high symmetry regions in $\mathrm{BaTiO}_{3}$ paves a way for future study of charge density in $\mathrm{BaTiO}_{3}$.

### 3.2. Experimental methods

Top-seeded solution grown single crystal $\mathrm{BaTiO}_{3}$ (substrate-grade, MaTecK MaterialTechnologie \& Kristalle GmbH ) was selected for this study. Thin TEM specimens were prepared along pseudocubic axes of $[100]_{\mathrm{C}},[110]_{\mathrm{C}}$, and $[111]_{\mathrm{C}}$ using a focused ion beam (FIB) following the method described previously (24).

The principle of domain identification is based on determining the crystal symmetry using convergent beam electron diffraction (CBED). CBED is a well-established and powerful technique for determining crystal point and space group symmetry at the nanoscale (25-30). With the flexibility of illumination optics in a modern transmission electron microscope (TEM), local symmetry can be determined using a convergent electron probe of sizes ranging from subnanometer to several micrometers. Since the electron probe is convergent, the diffracted beams appear as disks instead of sharp spots in the recorded diffraction patterns. The crystal point group can thus be determined by a careful analysis of intensity patterns, also known as rocking curves, in the transmitted and diffracted disks.

Ferroelectric crystals can be characterized by an order parameter, the spontaneous polarization $P_{s}$ vector, whose direction can be switched under the constraints of the crystal space group. The charge redistribution associated with polarization in ferroelectric crystals leads to the breakdown of Friedel's law when electron multiple scattering is involved. Thus, from CBED patterns taken along certain incident beam directions, we can determine the projected polarization direction,
which lies in a mirror plane. Furthermore, by matching with dynamic diffraction simulations using the Bloch wave method (31), we can retrieve the directions of the polarizations, and map the domain configurations and local symmetry variations in real space (32, 33).

Recent studies have demonstrated that the local crystal symmetry and polarization nanodomains can be determined using SCBED (29, 32, 33). By rastering the convergent electron probe over a region of the crystal, symmetry fluctuations and ferroelectric domains can be identified by the change in CBED patterns. A similar technique called STEM-CBED was used by Tsuda et al. for the same purpose (18). To quantify the symmetries of the CBED patterns, normalized cross-correlation values $(\gamma)$ of a pair of diffraction discs expected to have symmetry relations were computed using the algorithm previously proposed by Kim et al. (34). For the convenience of having just one $\gamma_{\text {representative }}$ value for one CBED pattern, the $\gamma$ values of each pair of discs were weighted with respect to intensity of the discs, or $\gamma_{\text {representative }}=\frac{\sum_{g} I_{g} \gamma_{g}}{\sum_{g} I_{g}}$ (35). To deal with intensity redistribution due to a small amount of lattice-rotations or a slight tilt of the incident electron probe, a DM (DigitalMicrograph ${ }^{\circ}$ ) script was developed to interactively adjust the radius and positions of the disks being used for symmetry quantification.

The electron diffraction experiments reported here were carried out using two TEMs. A JEOL 2200FS FEG TEM was operated at 200 kV and 80 kV with a convergent beam having a probe size at the crossover of 1.1 nm in FWHM (full-width at half-maximum) along pseudocubic axes [100]c and $[110]_{\text {c }}$ through an in-column Omega energy filter with a window of 10 eV centered on the zero-
loss peak (ZLP) in the electron energy-loss spectrum (EELS). The second TEM was a JEOL 2010F FEG TEM operated at 200 kV with a beam of 1.6 nm in FWHM along [111]c through a post-column GIF energy filter with a window of 10 eV centered on the EELS ZLP. EF-SCBED was performed by positioning the focused electron probe on a selected area of a $25 \times 25$ grid, with step size of 1nm. A double-tilt Gatan liquid-nitrogen-cooling specimen holder was used for low temperature studies. Various SCBED experimental conditions in this text and the corresponding figures are summarized in Table 1 below.

| Acceleration voltage <br> $(\mathrm{kV})$ | Zone axis | Temperature (K) | Corresponding figures |
| :---: | :---: | :---: | :--- |
| 80 | $[100]_{C}$ | 293 | 3(c) |
| 80 | $[110]_{C}$ | 293 | 3(d) |
| 200 | $[100]_{C}$ | 293 | 1(b); 2(a); 3(a); 3(b) |
| 200 | $[100]_{C}$ | 263 | 2(b) |
| 200 | $[100]_{C}$ | 95 | 2(c) |
| 200 | $[111]_{c}$ | 293 | 4(a) |

Table 3.1. Summary of experimental conditions and corresponding CBED figures. The subscript $C$ indicates the incident beam direction in the pseudocubic coordinate.

The CBED simulation is based on the Bloch-wave method (Zuo 1992), using the neutral atomic scattering factors of Doyle and Turner (36), absorption parameters of Bird and King (37), a total of 120 beams and 3209 incident beam directions within the CBED disk.

### 3.3. Results



Figure 3.1. Identification of $90^{\circ}$ ferroelectric domains using scanning CBED. (a) Bright-field image of the sample viewed along [100]c. The red box indicates the scanned region of $360 \mathrm{~nm} x$ 6 nm , whereas the orange arrows indicate polarization directions. The symmetry of CBED patterns were quantified along two different mirror plane directions, m1 and m2. (b) Representative experimental CBED patterns of three $90^{\circ}$ domains, from left to right, with high cross-correlation coefficient ( $\gamma$ ) values of $\gamma_{m 1}=98.2 \%, \gamma_{m 2}=95.9 \%, \gamma_{m 1}=96.2 \%$, respectively.

Three adjacent head-to-tail $90^{\circ}$ domains in $\mathrm{BaTiO}_{3}$ are identified at room temperature in Fig. 1(a), which are distinguished by the CBED patterns in Fig. 1(b). The CBED patterns were recorded along the [100]C (c for the pseudo-cubic direction) incident direction [Fig. 1(b)]. The SCBED was performed in the scanned region marked as the red rectangular box in Fig. 1(a), which was sampled by 180 by 3 points, with a step size of 2 nm . We observed the tetragonal $90^{\circ}$ domains, where the
alternating mirror plane directions rotated by 90 degrees, as shown in Fig. 1(b). Symmetry in the CBED patterns are quantified along two mirror plane directions, ml and m 2 . The highest $\gamma \mathrm{m}$ values in each domain, from left to right, are $\gamma_{\mathrm{m} 1}=98.2 \%, \gamma_{\mathrm{m} 2}=95.9 \%$, and $\gamma_{\mathrm{m} 1}=96.2 \%$, respectively. From the change in $\gamma \mathrm{ml}$ and $\gamma \mathrm{m} 2$ values, we found the domain wall width is about $2-4 \mathrm{~nm}$, which is consistent with previous studies using x-ray powder diffraction (38) or high-resolution electron microscopy $(39,40)$.


Figure 3.2. CBED patterns of $\mathrm{BaTiO}_{3}$ single crystal having highest symmetry along [100]c at various temperatures. Mirror planes are marked by lines, with quantified mirror symmetry of (a) $\gamma_{m}=98.5 \%$ in the T-phase (b) $\gamma_{m}=98.7 \%$ in the $O$-phase and (c) $\gamma_{m}=97.5 \%$ in the $R$ phase. The reflections on the mirror plane marked by yellow arrows show the breaking of Friedel's law due to dynamic diffraction effects.

Figure 2 shows the representative CBED patterns of $\mathrm{BaTiO}_{3}$ for the tetragonal, orthorhombic, and rhombohedral phases, upon cooling from room temperature to approximately 95 K . The EFSCBED scans were acquired near the same sample regions along the [100]c zone-axis incidence.

The mirror planes are indicated by the lines marked in the CBED patterns. The arrows mark the reflections on the mirror plane showing the breakdown of Friedel's law due to dynamic diffraction effects. The quantified mirror symmetry for the three representative CBED patterns are $\gamma_{\mathrm{m}, \text { Tetragonal }}=98.5 \%, \gamma_{\mathrm{m}, \text { Orthorhombic }}=98.7 \%$, and $\gamma_{\mathrm{m}, \text { Rhombohedral }}=97.5 \%$, with mirror directions parallel to $[001]_{\mathrm{T}},[011]_{\mathrm{o}}$, and $[011]_{\mathrm{R}}$, respectively. The mirror plane directions for CBED patterns taken at 298,263 , and 95 K are consistent with the space groups of $P 4 m m, A m m 2, R 3 m$, respectively.


Figure 3.3. Experimental and simulated CBED patterns of the tetragonal-phase at various thicknesses. Representative experimental CBED patterns possessing the highest mirror symmetry obtained along [100]c with cross-correlation coefficients of (a) $\gamma_{m}=98.7 \%$ (b) $\gamma_{m}=98.6 \%$ (c) $\gamma_{m}=98.6 \%$, and (d) along [110]c with $\gamma_{m}=98.7 \%$. The simulated patterns of tetragonal phase $\mathrm{BaTiO}_{3}$ (P4mm) using the Bloch-wave method with thicknesses of (e) 110 nm (200kV), (f) $50 \mathrm{~nm}(200 \mathrm{kV}),(\mathrm{g}) 35 \mathrm{~nm}(80 \mathrm{kV})$, and (h) $36 \mathrm{~nm}(80 \mathrm{kV})$ correspond to the experimental (a), (b), (c), and (d) patterns, respectively.

To investigate the specimen thickness effects on local symmetry, CBED patterns were recorded from regions of different specimen thickness, zone-axis incidence, and accelerating voltage [Figs. 3(a-d)]. For the thinner TEM specimen, we operated the 2200FS FEG TEM at 80kV for a longer wavelength and stronger multiple scattering effects in order to enhance the contrast in the recorded CBED patterns. The simulated CBED patterns [Figs. 3(e), 3(f), 3(g), and 3(h)] correspond to the experimental CBED patterns [Figs. 3(a), 3(b), 3(c), and 3(d)], respectively. Lattice parameters were set to the tetragonal phase (41), and neutral atomic scattering factors were used as inputs for CBED simulations. Figures 3(a) and 3(b) show experimental patterns recorded with the 200 kV electrons, with the mirror symmetry measured at $\gamma_{\mathrm{m}}=98.7 \%$ and $\gamma_{\mathrm{m}}=98.6 \%$, respectively. The corresponding simulated CBED patterns are along [100] $]_{\text {t }}$ incidence, shown in Figs. 3(e) and 3(f), with the best matching specimen thicknesses of 110 nm and 50 nm , respectively. Figures 3(c) and 3(d) show experimental patterns recorded with the 80 kV electrons along [100] $]_{\mathrm{C}}$ and [110] $]_{\mathrm{C}}$ incidences, with mirror symmetry measured at $\gamma_{\mathrm{m}}=98.4 \%$ and $\gamma_{\mathrm{m}}=98.5 \%$, respectively. The corresponding simulated CBED patterns are along $[100]_{\mathrm{T}}$ and $[101]_{\mathrm{T}}$ incidence, as shown in Figs. $3(\mathrm{~g})$ and $3(\mathrm{~h})$ with the best matching specimen thickness at 35 nm and 36 nm , respectively. For a probe of about 1 nm at FWHM , Figure 3 shows that the $\mathrm{BaTiO}_{3}$ crystal still preserves tetragonal symmetry with the probed crystal volumes from $\sim 110 \mathrm{~nm}^{3}$ down to $\sim 35 \mathrm{~nm}^{3}$ at room temperature.

Figure 4 shows the representative CBED patterns and the symmetry maps obtained by EFSCBED taken along the [111]c zone-axis. The scanned region consists of 12 by 25 points, with a step size of 2 nm . Figure 4(a) shows the experimental CBED pattern with highest symmetry with $\gamma_{\mathrm{m}(1,1)}=95.4 \%$ compared to the average value of $\gamma_{\mathrm{m}(1,1)}=83.1 \%$ over the entire scanned area. Six possible projected 2D mirror plane directions can be expressed by ( $\mathrm{m}, \mathrm{n}$ ) as the linear combination of two basis vectors $\vec{g} \& \vec{h}$, namely $\vec{M}=m \vec{g}+n \vec{h}$. Possible mirror plane directions along [111] ${ }_{\mathrm{C}}$ incidence for tetragonal and rhombohedral symmetries are labeled in with orange solid lines. The corresponding simulated CBED pattern is shown in Fig. 4(d), with incident beam along the [1 $\overline{1} 1]_{\mathrm{T}}$, with a mirror plane parallel to $[\overline{1} 12]_{\mathrm{T}}$, and specimen thickness of 135 nm . The maps of spatial variations in mirror symmetry quantified along $(1,1),(1, \overline{1}),(1,0)$, and $(0,1)$ are shown in Figs. $4(b)$, 4(c), 4(e), and 4(f), respectively. Figure 4(b) shows the tetragonal symmetry is preserved in regions of $\sim 15 \mathrm{~nm}$ in the lateral direction.


Figure 3.4. Symmetry maps of tetragonal-phase $\mathrm{BaTiO}_{3}$ along [111]c zone axis. The scanned region is $24 n m \times 50 n m$. (a) The six possible mirror plane directions can be expressed by (m,n), where $\vec{M}=m \vec{g}+n \vec{h}$ and $\vec{g} \& \vec{h}$ being the two basis vectors. Mirror planes corresponding to tetragonal and rhombohedral phases are labeled with orange solid lines. Representative experimental CBED pattern possessing highest mirror symmetry with cross-correlation coefficients of $\gamma_{m(1,1)}=95.4 \%$, which corresponds to region-I in (b). Figure (d) shows the corresponding simulated tetragonal phase CBED pattern along [1 $\overline{1} 1]$ zone axis, with the thicknesses of 135 nm , and mirror plane $m / /[\overline{1} 12]_{T}$. Figures (c), (d), (e), and (f) show the symmetry maps quantified along mirror planes parallel to (1,1), (1, $\overline{1}),(1,0)$, and $(0,1)$, respectively.

### 3.4. Discussion

The symmetry information obtained from the experimental CBED patterns reflects the ferroelectric polarizations, averaged over all unit cells illuminated by the electron beam. In the column approximation (31), the illuminated volume of crystal is roughly the product of probe diameter squared and specimen thickness. The above results using $\mathrm{BaTiO}_{3}$ as a model ferroelectric demonstrate that SCBED can be used to identify the ferroelectric domains and determine polarization direction at various temperatures. The highest symmetry identified by SCBED is consistent with the T-, O-, and R-symmetry previously reported. Further, we observed that the tetragonal symmetry is well preserved within a crystal volume ranging from $35 \mathrm{~nm}^{3}$ to $110 \mathrm{~nm}^{3}$, as shown Figs. 3(a-d). However, the regions with the high tetragonal symmetry of $\sim 15 \mathrm{~nm}$ in diameter are surrounded by regions with symmetry breaking, as shown in Fig. 4(b).

Ferroelectric phase transition is traditionally explained by the displacive phase transition model (42). Within this model, local symmetry breaking can be explained by the anharmonicity of soft phonons (43-46). The interference of different phonon frequencies leads to a vibrating lattice wave packet, which causes structural distortions via strong electron-phonon coupling (47, 48).

An alternative explanation is the order-disorder model. Previously, Tsuda et al. suggested that the tetragonal structure consists of rhombohedral nanoclusters using the STEM-CBED technique $(16,18)$. Within the order-disorder interpretation, the observed tetragonal symmetry can possibly be a result of $<111\rangle$ polarizations averaged over $35 \mathrm{~nm}^{3}$, or $\sim 540$ unit cells, assuming a column of

1 nm in diameter and 35 nm thick. For example, Figs. 5(b) and 5(c) show the polarization directions projected along the $[100]_{\mathrm{C}}$ viewing direction for the order-disorder model and anti-ferroelectric coupling model, respectively. The polarization components along [100] $]_{C}$ and $[010]_{\mathrm{C}}$ cancel each other, which effectively results in an averaged polarization along [001]c.


Figure 3.5. Structural models of the tetragonal phase corresponding to various ferroelectric phase transition mechanisms. The structure models in the first row are viewed along the [100]c direction, and the second row is viewed along [111]c. Figures $(a, d),(b, e)$, and $(c, f)$ correspond to displacive, order-disorder, and anti-ferroelectric type ferroelectric phase transition mechanisms, respectively. The arrows indicate the polarization directions in a unit cell. Different colors of arrows indicate the polarizations lying in different diagonal planes.

A hint of <111> polarizations averaging is provided by the symmetry maps obtained from SCBED along the [111] $]_{\text {c }}$ zone axis, as shown in Figs. 4(e) and 4(f). In both order-disorder (Fig. 5(e)) and anti-ferroelectric coupling models (Fig. 5(f)), the four red <111> polarizations lie in the [110]c plane, which results in the observed highest mirror symmetry along m(1,1) (Fig. 4(b)). The orange lines $(\mathrm{m}(1,1), \mathrm{m}(2, \overline{1})$, and $\mathrm{m}(1, \overline{2}))$ in Figure 4 represent the 3D projected mirror planes, while the red dashed lines $(\mathrm{m}(0,1), \mathrm{m}(1,0)$, and $\mathrm{m}(1, \overline{1}))$ do not have corresponding 3D mirror planes. We noticed that region-I possesses a higher mirror symmetry along $m(0,1)$ than $m(1,0)$, while it is the opposite case for region-II (Figs. 4(e) and 4(f)). Since $m(0,1)$ and $m(1,0)$ have no corresponding 3D mirror planes, the observed "pseudo-mirror" of $m(0,1)$ and $m(1,0)$ can be a result of superimposed $m(1,1)$ with $m(1, \overline{2})$ and $m(2, \overline{1})$, respectively. The projected symmetry fluctuation thus may be explained by the population difference of $\langle 111\rangle$ polarizations within the volume illuminated by the electron beam. A knowledge of population fluctuations along the electron beam direction is then required to understand the observed 2D symmetry. Unfortunately, symmetry analysis alone does not provide such information.

It has been suggested that the simultaneous presence of soft phonon modes and Ti ion offcenter displacements leads to a coexisting displacive and order-disorder character (49). The temperature-dependent structural distortion and bonding of $\mathrm{BaTiO}_{3}$ was recently investigated using electron energy-loss spectroscopy (EELS) by studying the fine structure in the $\mathrm{O}-\mathrm{K}$ edge and Ti- $\mathrm{L}_{23}$ edge (50). The study of Bugnet et al. suggested that the paraelectric phase is not cubic
but tetragonal-like, which is in good agreement with the work of Zalar et al. (43, 44). However, first principles calculations do not reproduce the overall shape of O-K edge, hence hinders the quantitative interpretation of the FSs.

We noticed the difference in experimental and simulated CBED patterns using neutral atomic scattering factors, as shown in Fig. 3. The difference is most obvious in diffracted disks nearest to the transmitted disk, which is most sensitive to chemical bonding. The simulated CBED patterns using ionic scattering factors did not resemble the experimental patterns, which implies the bonding to be more covalent. To further understand the microscopic origin of ferroelectric phase transitions, there is a need to investigate the local crystal bonding in $\mathrm{BaTiO}_{3}$, with even smaller illuminated volumes than the present study. The SCBED study reported here paves the way for a future investigation of the local chemical bonding and charge density distributions by measuring low-order structure factors using quantitative CBED (51-53) and using the multipole-model that Prof. Coppens and his collaborators have pioneered (54, 55).

### 3.5. Conclusions

In conclusion, we have identified regions of $\sim 15 \mathrm{~nm}$ of $\mathrm{BaTiO}_{3}$ possessing high symmetry surrounded by symmetry breaking regions using the EF-SCBED technique. The crystal still preserves tetragonal symmetry within a local volume of $35 \mathrm{~nm}^{3}$. The symmetry breaking can be
explained by both anharmonicity in soft phonon modes or Ti ion off-centering averaged over the crystal volume probed by the electron beam.

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## CHAPTER 4

## LATTICE ROTATION VORTEX AT THE MONOCLINIC DOMAIN BOUNDARY IN RELAXOR FERROELECTRIC CRYSTAL ${ }^{1}$

We present evidences of lattice rotation vortices having an average radius of $\sim 7 \mathrm{~nm}$ at the ferroelectric domain boundary of $(1-\mathrm{x}) \mathrm{Pb}\left(\mathrm{Zn}_{1 / 3} \mathrm{Nb}_{2 / 3}\right) \mathrm{O}_{3}-\mathrm{xPbTiO}_{3}(\mathrm{x}=0.08)$. Maps of crystal orientations and domain symmetry breaking are obtained using scanning convergent beam electron diffraction (SCBED), which show fractional rotation vortices near the $50^{\circ}$ monoclinic domain walls. The merging of 2D and 1D topological defects is consistent with inhomogeneous boundary charge and expected to have a large impact on the domain-switching mechanisms in relaxor ferroelectric crystals and ferroelectric devices.

### 4.1. Introduction

Topological defects, such as ferroelectric domain walls (DWs), exhibit emergent physical properties with potential applications in electronic devices. For example, charged $\mathrm{DWs}^{\text {of }} \mathrm{BiFeO}_{3}$ (1) and $\mathrm{Pb}(\mathrm{Zr}, \mathrm{Ti}) \mathrm{O}_{3}$ (2) exhibit a significant conductivity increase compared to bulk materials, which is movable and can be advantageous for device applications (3). DWs involve a change in the polarization direction and small lattice distortions. Having a large density of mobile DWs also

[^1]facilitates domain switching and therefore dramatically enhances the susceptibility of ferroelectrics and piezoelectric coupling coefficients (4).

Extensive studies of ferroelectric domains by microscopy (5,6) and diffraction (7-18) have demonstrated that ferroelectric DWs can be categorized by the dipole transition behaviors across the boundary, which are non-chiral DWs (Ising-like), chiral DWs (Bloch- or Neel-like), or mixtures of both (19). However, recent atomic resolution electron imaging revealed polarization rotation vortex in tetragonal $\mathrm{Pb}(\mathrm{Zr}, \mathrm{Ti}) \mathrm{O}_{3}(20,21)$, rhombohedral $\mathrm{BiFeO}_{3}$ (22) crystals and the superlattices of $\mathrm{SrTiO}_{3}$ and $\mathrm{PbTiO}_{3}(23,24)$. Furthermore, in lead-based complex perovskite oxides having the chemical formula $(1-\mathrm{x}) \mathrm{Pb}\left(\mathrm{B}_{1 / 3}^{+2} \mathrm{~B}_{2 / 3}^{+5}\right) \mathrm{O}_{3-\mathrm{x}}-\mathrm{xPb}\left(\mathrm{B}^{\prime+4}\right) \mathrm{O}_{3}\left(B^{\prime}, B^{\prime \prime}, B^{\prime, \prime}=\mathrm{Zn}, \mathrm{Nb}\right.$, Ti for PZN-PT, and $\mathrm{Mg}, \mathrm{Nb}$, Ti for PMN-PT), exceptional piezoelectric properties (25) are obtained at the morphotropic phase boundary (MPB), where nanometer-sized monoclinic domains have been reported by X-ray diffraction (5, 26), neutron diffraction (9, 10, 12, 17, 18), and electron microscopy $(7,11,14)$. On the other hand, we know little about the structure and properties of DWs in monoclinic crystals. Theory predicts mechanically permissible but slightly charged DWs parallel to 100 or 110 planes in addition to $180^{\circ}$ DWs $(27,28)$. The structural determination of DWs in general requires 1) identifying two neighboring polarization domains, 2) determining the transition structure between the domains, and 3) identifying the nature of the polarization in the transition region. All three tasks are experimentally difficult for determining monoclinic, and nanometer-sized, domains, and consequently there was no good experimental means to study in
relaxor ferroelectrics the coupling between electric dipoles and strain, which can induce chirality at the DWs (29).

Here, we describe a scanning convergent beam electron diffraction (SCBED) study of the DWs in the relaxor-based ferroelectric crystal of PZN-8\%PT. Using energy-filtered (EF) SCBED, we have identified nm-sized domains having monoclinic (M) Pm symmetry in single crystal PZN$8 \% \mathrm{PT}$. A careful examination of the DWs revealed the presence of lattice rotation vortices near DWs. These vortices involve continuous lattice rotation across length scales of $\sim 15 \mathrm{~nm}$ in diameter.

### 4.2. Experimental methods

Single crystal PZN-8\%PT (unpoled flux-grown single crystal, Microfine Materials Technologies Pte. Ltd., Singapore) was selected for study. Thin crystals were prepared along pseudocubic axes of $[100]_{C},[001]_{C}$, and $[111]_{C}$ (C for pseudo-cubic axes) using the method described previously (30). The same sample preparation procedure was applied successfully for the determination of symmetry in single crystal $\mathrm{BaTiO}_{3}(31)$.

The principle of domain identification is based on CBED determination of crystal symmetry. A focused electron probe is rastered across a region of the sample and used to record diffraction patterns. Because of the convergent beam, the diffracted beam appears as a disk instead of a sharp diffraction spot and CBED has the spatial resolution ranging from few to hundreds of nanometer
(32-34). The crystal point group can be obtained at different sample points by careful study of 2D features in transmitted disk and diffracted disks. Specifically, the ferroelectric polarization direction which lies in the mirror plane can be determined from multiple scattering effects for certain incident beam directions. Thus, ferroelectric domains can be identified by the change of CBED pattern symmetry (Fig. 1). For example, the mirror direction can be used to determine the $60^{\circ}$ domains in PMN-31\%PT with the aid of dynamic diffraction simulation using the Bloch wave method (35, 36). The change in CBED pattern symmetry is quantified using the normalized crosscorrelation $\left(\gamma_{\mathrm{m}}\right)$ value of a pair of diffraction discs related by mirror symmetry using the algorithm previously proposed by Kim et al. (37). For convenience of having just one $\gamma_{\mathrm{m}}$ value for one CBED pattern, the $\gamma \mathrm{m}$ values of three pairs of discs with the highest intensity were averaged, noted as $\gamma_{\mathrm{m}, \text { average }}$ shown in Fig. 1(a). By scanning the electron probe over a region of crystals and recording and quantifying CBED patterns for scanning CBED, the crystal symmetry can be mapped. Figure 1(d) shows an example. CBED patterns of different $\gamma_{\mathrm{m} \text {,average }}$ values are indicated by different colors in Fig. 1(d).

For determining the crystal rotation, we use the BF disk (transmitted beam) of CBED, which possesses the center of symmetry belonging to the Laue diffraction group according to Buxton et al. (32). The location of the center of symmetry changes when crystal rotates as illustrated in Fig. 1(b) and (c).


Figure 4.1. Principles of using CBED for determining mirror symmetry and crystal rotation. Figure (a) shows an example for the mirror symmetry quantification, while crystal rotation along the $x$ - and $y$-axes leads to a shift in the center of the CBED (000) pattern as shown in (b) and (c). The average of the cross-correlation coefficients of three pairs of discs labeled in orange in (a) is taken as $\gamma_{m, a v e r a g e, ~ w h o s e ~ v a l u e s ~ a r e ~ s h o w n ~ i n ~(d) ~ f o r ~ a ~ s c a n ~ o f ~} 15 x 15$ points or 225 CBED patterns. Here each color represents a different CBED pattern, whereas similar CBED patterns are shown in the same color.

The SCBED experiments were carried out using a JEOL 2010F FEG TEM operated at 200kV with a convergent beam of 2.6 nm in FWHM (full-width half-maximum). Energy-filtering (EF), which improves the contrast of CBED patterns, was performed using a Gatan imaging filter (GIF). EF-SCBED was performed by scanning the focused electron probe over a selected area on a 15 x

15 grid, step size of 2 nm , and through a post-column GIF energy window of 10 eV . The shift and tilt of diffraction patterns during beam scanning were minimized and calibrated using a silicon single crystal (38). Following the procedures described in (35), the symmetry of PZN-8\%PT was determined as monoclinic Pm, which agrees with the X-ray diffraction result (10).

### 4.3. Results



Figure 4.2. Distribution of two nanodomains using SCBED. (a), (b) and (c) map out the $\gamma_{m, a v e r a g e}$ variations across two types of domains. The red dashed line indicates the domain boundary. The orange arrows indicate the projected polarization directions for each type of domain.

Nanodomains are observed using EF-SCBED. Symmetry variations across these domains in three EF-SCBED datasets from three different sample areas are shown in Figs. 2(a), (b), and (c).

The scan consists of 15 by 15 points, with a step size of 2 nm . The $\gamma_{\mathrm{m}, \text { average }}$ of the representative CBED patterns in each region are shown in greyscale. We identified type-1 and type-2 nanodomains with different mirror symmetry. The boundaries between these two domains are indicated as dashed lines in the figures.


Figure 4.3. Maps of distribution of two nanodomains and lattice rotation vortices. Figs. (a), (b), and (c) show the crystal rotation at each pixel, superimposed with the domain walls indicated by the blue dashed lines. Figs. (d), (e), and (f) illustrates how the crystal rotates across the domain boundaries schematically.

We noticed that the center-of-mass of the intensity distribution within the BF disc of each pattern in the EF-SCBED dataset is not always located at the exact center. This observation could have two possible explanations: microscope optics and local crystal tilting. First, the hysteresis in the scanning coils or the lens in the microscope could lead to imperfect optical alignment while scanning the beam, which results in an effective beam-tilt and a consequent intensity redistribution in the BF disc. Second, if the crystal is not oriented on the exact zone axis, this small angular deviation could also lead to an intensity redistribution in the BF disc. This is shown schematically in Figs. 1(b) and 1(c). If small bending or buckling exists during specimen preparation, the intensity redistribution in the BF disc would mainly be shifting in a way consistent with bending. We excluded the effects of microscope optics by performing EF-SCBED on a Si single crystal. This measurement defines the maximum electron beam tilt and the lattice rotation measurement precision at $\pm 0.012$ degrees. In an effort to quantify how much the crystal is deviated from the exact zone axis, we calculated the displacements (in pixels) of the center-of-mass of each BF disc and converted these displacements into crystal rotations (in degrees).

By measuring the shift in the BF disc of a CBED pattern using this method, we determined the rotation of the crystal and represented this rotation as a vector over nanometer-sized sample regions. The vector at each data point indicates the crystal rotation averaged over a volume of $\sim 280 \mathrm{~nm}^{3}$. Figs. 3(a), (b), and (c) show the crystal rotation map derived from the same EF-SCBED datasets as Figs. 2(a), (b), and (c), respectively. Figure 3(a) shows a vortex-like pattern with the
vortex center near the domain boundary, and a radius of curvature of $\sim 7 \mathrm{~nm}$ calculated from the discrete points. On the other hand, the vortex feature is not as distinct in Figs. 3(b) and (c). The continuous crystal rotation is shown schematically in Figs. 3(d), (e), and (f).

The type-1 and type-2 domains identified in Fig. 2 are associated with two distinguishable CBED patterns that were observed along the $[100]_{C}$ incident direction (Figs. 4(a) and 4(b)). Figs. 2(a), (b), and (c) show the symmetry maps where these two patterns were detected. The highest $\gamma_{\mathrm{m}}$ values of type-1 and type-2 patterns are detected along two different directions (A and B) as shown in Figs. 4(a) and 4(b). The A and B directions are rotated by $45^{\circ}$ along the $[100]_{C}$ zone axis. The corresponding simulated patterns for type-1 and type-2 domains are along monoclinic Pm zone axis $[100]_{P m}$ and $[010]_{P m}$, as shown in Figs. 4(c) and 4(d), respectively. In the $P m$ structure model, the polarization direction is $\vec{P}_{S}=[u, 0, v]_{P m}=[3,0,4]_{P m}$, which lies in the mirror plane of Pm symmetry [12]. Along the $[100]_{P m}$ incident direction, the mirror plane is superimposed on $(001) /(00 \overline{1})$ reflections, which is parallel to the A direction in Fig. 4(a). This mirror is not observed along the $[010]_{P m}$ incident direction. The projection of the polarization lies approximately on the $(101) /(\overline{1} 0 \overline{1})$ reflections, which is parallel to the B direction in Fig. 4(b). The highest mirror symmetry in this case is detected along direction B in the simulated pattern (Fig. 4(d)) with $\gamma_{m, \text { simulated }}=60 \%$.

Quantification of mirror symmetry for Fig. 4(a) and (b) gives $\gamma_{m, A}^{1}=95 \%$ and $\gamma_{m, A}^{2}=$ $34 \%$, respectively (The superscript indicates the domain type, and the subscript denotes the mirror
plane direction). This shows that the mirror plane of type- 1 domains is along the A direction. For the type-2 domains, a good match is obtained with $[010]_{P m}$. The $\gamma_{\mathrm{m}}$ value along the B direction of the recorded patterns roughly agrees with the simulated value, with $\gamma_{m, \text { experimental }}=54 \%$.


Figure 4.4. Experimental and simulated CBED patterns along various zone axes. The mirror plane in the (a) type-1 and (b) type-2 domains is rotated by $45^{\circ}$. Figs. (c) \& (d) show simulated patterns of $M_{C}(P m)$ using the Bloch wave method and corresponding to the experimental $(a) \&(b)$ patterns, respectively. The indexing is based on simulated diffraction patterns.

Based on the best matching structural model of Pm, the orientation relationship between the type1 and 2 nanodomains with respect to the pseudocubic axes is shown schematically in Figure 5. For type- 1 domains, which belong to the $[100]_{P m}$ zone axis, the monoclinic axes of $a_{P m}$ and $b_{P m}$ are along $[100]_{P C}$ and $[010]_{P C}$, respectively. The $c_{P m}$ is slightly deviated away from the $[001]_{P C}$ direction with an angle $\left(90^{\circ}-\beta\right)$ in the $a_{P m}-c_{P m}$ plane. Type- 2 domains belong to the $[010]_{P m}$ zone axis, for which the monoclinic axes of $a_{P m}$ and $b_{P m}$ are rotated by $90^{\circ}$ with respect to the cubic $c$-axis. If converting the two polarization directions $[3,0,4]_{P m}$ and $[0,3,4]_{P m}$ from fractional coordinates into Cartesian coordinates, the polarization directions in Cartesian coordinates would be $[3.03,0.02,4.05]$ and $[0.02,3.01,4.05]$, respectively. The angle between the two vectors is $50^{\circ}$. The presence of $50^{\circ}$ polarization domains is also evidence which excludes the tetragonal (T)- or rhombohedral (R)-symmetries, since this type of domain is only permitted in crystals with orthorhombic or lower symmetries (27,28). Bokov \& Ye considered mechanically permissible domain configurations with monoclinic Pm symmetry. We observed regions with high symmetry, which implies no overlapping of $1 / 2$ type domains in the thickness direction. However, we cannot exclude the overlapping of $2 / 4$ domains in high symmetry region (27).


Figure 4.5. Orientation relationship between two nanodomains with respect to the pseudocubic axes. Figs. (a) and (b) correspond to type-1 and type-2 domains, respectively.

### 4.4. Discussion

Vortices form in ferroelectric crystals by two different mechanisms. Figure 6 (a) shows fluxclosure domain patterns associated with continuous dipole rotations have been reported in ferroelectric thin films (21-24, 39) or ferroelectric nanodots (40-42). These patterns involve continuous dipole rotations near the vertices of triangular or quadrantal domain boundaries. By reducing the thickness of the thin film, ferroelectric dipole vortex-antivortex pairs can be stabilized (24), as shown in Fig. 6(b). The continuous rotation of the dipoles in vortex structures can also result from depolarizing fields, which are created by incompletely compensated charges at the surfaces and interfaces (Fig. 6(c)). Experimental evidence for a continuous rotation of the dipoles
was reported by Jia et al. $(20,21)$ in $\mathrm{Pb}(\mathrm{Zr}, \mathrm{Ti}) \mathrm{O}_{3}$ at the $180^{\circ}$ domain walls. The organization of dipoles in vortex structures reduces depolarizing fields.


Figure 4.6. Schematic cartoon of various types of domain configurations, polarization orientation, and charge discontinuity. The reported (a) flux-closure quadrants, (b) dipole vortex-antivortex pairs, (c) $180^{\circ}$ charged domain wall, and (d) our observation of $50^{\circ}$ monoclinic charged domain wall. $E_{1}$ and $E_{2}$ represent the inhomogeneous electric field in type-1 and type-2 domains, respectively.

The rotation we observed is part of the lattice deformation matrix with displacement vector $u(\vec{r})$, defined by rigid body rotation tensor $\widetilde{w}_{i j}=1 / 2\left(e_{i j}-e_{j i}\right)$, where the strain tensor is $\varepsilon_{i j}=$ $1 / 2\left(e_{i j}+e_{j i}\right)$ and the quantity $e_{i j}=\frac{\partial u_{i}}{\partial x_{j}}$. For relaxor ferroelectric crystals with monoclinic symmetry, disinclination exists between two domains with an angular mismatch determined by unit cell parameters $(27,43)$. Strain accompanies the polarization rotation due to the strong electromechanical coupling ( $15,16,25$ ). We speculate that the crystal rotation vortex can be a result of accommodating disinclination strain and charge discontinuity. First, the disinclination strain can be estimated by calculating Lagrangian finite strain tensors (44). Lattice parameters of two neighboring monoclinic Pm unit cells, distorted along two directions as depicted in Figure 5, are input parameters for calculating the strain tensors. The maximum strain at the domain wall is $1.3 \%$, which is comparable to the $1.5 \%$ strain at the vertex core of rhombohedral $\mathrm{BiFeO}_{3}(23)$.

Second, the depolarizing fields and effects on local dipoles can be simplified by considering the polarization of adjacent domains since $\vec{\nabla} \cdot \vec{P}=\rho_{b}$, where $\rho_{b}$ is the bound charge density, and $\vec{\tau}=\vec{P} \times \vec{E}$, which is the torque acting on the dipoles by the depolarization field. At $180^{\circ}$ charged DWs, the depolarization fields induced by bound charge are symmetrical and the forces on the dipoles are opposite on the two sides of the DWs, which creates a flux-closure vortex. However, this symmetry is broken in the case of a slightly charged monoclinic $50^{\circ}$ DW, as shown in Fig. 6(d). The magnitude of inhomogeneous electric fields, $E_{1}$ and $E_{2}$ in type- 1 and type- 2 domains, respectively, are different due to strong dielectric anisotropy of the crystal (25). This torque is also
strongest when $\vec{P}$ is normal to $\vec{E}$, and this torque is weak in the type- 1 domain because its $\vec{P}$ is pointing out of plane. This is consistent with our observation shown in Figs. 3(a) and 3(b), which the average magnitude of lattice rotation in type-2 domains is larger than that of in type-1 domains.

The above observation raises important questions about the roles of the lattice rotation vortex in domain switching in ferroelectric systems. Previously, first-principles calculations have predicted an intermediate state having a coexisting toroidal moment and out-of-plane-polarization in ferroelectric nanoparticles $(45,46)$. The occurrence of lattice rotation vortices at the ferroelectric domain walls suggests the interaction between 2D and 1D topological defects. An analogy can be made with the presence of magnetic vortices, known as skyrmions. The interplay between spin, orbital, charge, and strain degrees of freedom associated with skyrmions suggests a complex landscape of topological defects in ferroelectrics that may be explored for new applications and functionalities.

### 4.5. Conclusions

In conclusion, we observed local crystal rotation vortex at the $50^{\circ}$ monoclinic domain boundary. The fractional crystal rotation vortex is attributed to depolarization fields due to charge discontinuity across the domain walls and their forces on local dipoles.

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## CHAPTER 5

## DIRECT IMAGING OF DISTORTED LATTICE IN A HIGH ENTROPY ALLOY

High entropy alloys (HEAs) feature severely distorted lattices arising from atomic size mismatch that has become a focal point of research for the study of their unusual properties. Lattice distortions can be measured by electron imaging or by diffraction through the study of pair distribution functions (PDF). However, the PDF results so far have only found small distortions within a small atomic radius. Here, we present quantitative imaging results of lattice distortions and local symmetry breaking in a single phase $\mathrm{HEA}, \mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$. Results revealed two types of lattice distortions in $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$. One features $10 \pm 3 \mathrm{~nm}$, disc-shaped, clusters having $\sim 7.1 \%$ tensile displacements along <110> directions distributed throughout the specimen, and the other gives rise to local fluctuating strain within $\pm 1.3 \%$, which is slow-varying over $\sim 50 \mathrm{~nm}$. Both results suggest the non-ideal solid solution nature of HEA. Furthermore, peak broadening with increasing diffraction orders is consistent with the paracrystal model having lattice distortion parameter of $2.12 \%$. Our observation is made possible using scanning convergent beam electron diffraction, and the results here thus demonstrate a new powerful experimental approach toward quantitative study of multi-principal element systems.

### 5.1. Introduction

Crystals possessing distorted lattice are ubiquitous in nature and often associated with functional material properties. Examples include polarization nanodomains in ferroelectrics and solid solution alloys. A common scheme to these systems is that there exists a distinction between local crystal symmetry and the average, macroscopic symmetry imposed by fluctuations in the crystal lattice. Understanding their local structures is a prerequisite for further understanding of the material's properties, however, requires robust and quantitative methods for probing crystallography at the nanoscale (1, 2). High entropy alloys (HEAs) represent an exemplar of crystals with distorted lattices. They are made of five or more elements in equal or near-equal molar percentage, and often forms a single solid solution phase (3-5). HEAs, including the more broadly defined multi-principal element alloys (MPEAs), have attracted much research attention for the discovery of outstanding mechanical properties at extreme temperatures and exceptional structural stability under irradiation (6-9). The lattice distortion is expected to be more severe than in dilute solid solution alloys, contributing to excess entropy, and impede dislocation movements leading to pronounced solid solution strengthening (10-14). Consequently, the severe lattice distortion effect is one of the core effects for describing MPEAs (15).

However, the characterization of lattice distortions in HEAs present a significant challenge to the materials research community (11-14, 16). The reported X-ray and neutron diffraction studies generally demonstrate sharp Bragg peaks (5, 17, 18), although the decrease in peak intensities may
result from effects such as lattice distortion, thermal vibration, and crystallographic texture (10, 12, 13, 19, 20). The lattice distortions in high-entropy alloys were further investigated based on pair distribution functions obtained from synchrotron X-ray or neutron scattering. However, the results of such analyses have provided contradictory pictures about lattice distortions in MPEAs. For example, a large deviation of the local structure from the average lattice was found in the bodycentered cubic (bcc) ZrNbHf and $\mathrm{Al1.3CoCrCuFeNi}(21,22$ ), whereas a negligible deviation was observed in the face-centered cubic (fcc) CrFeCoNi and $\mathrm{CrMnFeCoNi}(23,24)$.

Here, we describe a systematic study of lattice distortion quantification in fcc $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$ alloy using a new electron imaging technique, namely, scanning convergent beam electron diffraction (SCBED). Unlike scanning transmission electron microscopy (STEM) which uses scattered electron intensity for imaging, images or maps can be constructed in SCBED based on an analysis of information recorded in the diffraction patterns. We employed three types of analyses here: 1) measurement of the d-spacing for imaging nanoscale strain variations, 2) quantification of the peak broadening with increasing diffraction orders for lattice distortion mapping, and 3) a determination of local symmetry using CBED. The combination of these measurements identified two types of lattice distortions in $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$ : one features $10 \pm 3 \mathrm{~nm}$, disc-shaped, clusters having $\sim 7.1 \%$ tensile displacements along <110> directions distributed throughout the specimen, and the other gives rise to local fluctuating strain within $\pm 1.3 \%$, which
is slow-varying over $\sim 50 \mathrm{~nm}$. Furthermore, the peak broadening with increasing diffraction orders revealed $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$ as paracrystalline with lattice distortion parameter g of $2.12 \%$.

### 5.2. Experimental methods

Polycrystalline $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$ MPEA was prepared by vacuum induction melting and casting (Sophisticated Alloys Inc., Butler, PA). The as-cast samples were hot isostatic pressed at $1,100^{\circ} \mathrm{C}$ for 1 h under a 207 MPa ultra-high-purity argon pressure to reduce porosity, which resulted large grain sizes $\left(\sim 10^{2} \mu \mathrm{~m}\right)$. The composition and chemical homogeneity were checked by atom probe tomography analysis (25). Thin TEM specimen were carefully prepared along [001] axis using focused ion beam (FIB) with the method described previously (26). The SCBED experiments were carried out using a Themis Z STEM (Thermo Fisher Scientific) operated at 300 kV , and a JEOL 2200FS FEG TEM with an in-column Omega energy filter and operated at 200 kV for energy-filtered CBED. SCBED works by rastering the focused electron probe over a selected area, and recording the electron diffraction pattern over each probe position using a pixelated detector (Fig. 1a, also see ref (27, 28)). Two types of diffraction patterns were recorded, one uses a semi-convergence angle of 1.4 mrad and probe size of 1.7 nm in FWHM (full-width at half-maximum) for a spot-like diffraction pattern (Fig. 1b) and another uses a semi-convergence angle of 5.5 mrad and probe size of 0.8 nm in FWHM for CBED (Fig. 1c). The spot-like diffraction patterns was used for strain analysis following the procedures described in ref (29), while the CBED pattern was quantified for its mirror and rotational symmetry using a correlation based
analysis (30,31). Local lattice distortion can be identified by the breakdown of symmetry in CBED patterns deviating from perfect 4 mm symmetry for a fcc crystal projected along [001] direction.

With the aid of CBED simulations, the specimen thickness is determined as $\sim 186 \mathrm{~nm}$.


Figure 5.1. SCBED imaging. (a) Schematic of the SCBED technique with various probe semiconvergence angle $\alpha$. As the convergent electron probe rasters over a region of interest, a CBED pattern is recorded at each probe position. Figure (b) shows a representative CBED pattern with $\alpha$ of 1.4mrad, strain can be obtained by measuring relative shifts in disk positions. (c) Representative experimental CBED pattern with $\alpha$ of 5.5 mrad , having highest symmetry of $\gamma_{4}=95.9 \%, \gamma_{m l}=96.4 \%, \quad \gamma_{m 2}=95.4 \%$. (d) HAADF image acquired with illumination semiconvergence angle $\alpha$ of 5.5 mrad , inner collection angle of 60 mrad . R1 and $R 2$ indicate the regions of the SCBED experiments performed with convergence angles in ( $B$ and $C$ ), respectively. $D$ and D' indicate two types of defects, dislocation and "coffee bean", respectively. ( $\boldsymbol{E}$ ) Overlay of symmetry maps obtained by quantifying symmetry in CBED patterns for 4-fold rotation (green), and along two different mirror plane directions, $m 1$ (red) and m2 (blue).

### 5.3. Results

Two types of symmetry breaking are observed, "coffee bean" contrast and symmetry fluctuations. By quantifying each of the 40,000 CBED patterns taken from region R2 (Fig. 1D), symmetry maps can be reconstructed from the SCBED dataset (Figs. 1C, 1E, and S1) for 4-fold rotation (green), and along two mirror plane directions m1 (red) and m2 (blue). Symmetry in each CBED pattern reflects the distorted lattice averaged over the local volume illuminated by the electron beam. In column approximation, the illuminated volume of crystal is roughly the product of probe diameter squared and specimen thickness of 186 nm . Small probes are hence more sensitive to crystal symmetry breaking (fig. S3).

The defects D' showing "coffee bean" contrast (Figs. 1D and 2A) feature anisotropic symmetry breaking along $\langle 110\rangle$ directions, which is different from the diffraction contrast TEM micrographs of coherent spherical inclusions in conventional solid solution alloys (32,33). If the inclusion were spherical, the 4 -fold rotational symmetry would only be preserved when the electron beam be positioned at the very core of inclusion (34,35). Moreover, the shape of the symmetry breaking are found in close resemblance of the projected normal stress field around an oblate spheroid calculated from the Eshelby's model for ellipsoid inclusions (Fig. 2B, also see refs. (36-40)). The tensile displacement fields from the inclusion can be verified by looking into individual CBED patterns. The CBED patterns in this region (Fig. 2A) can roughly be grouped into 3 categories: core, left-hand-side (LHS) and right-hand-side (RHS). For example, the
experimental CBED pattern taken from RHS of the cluster (Fig. 2C) shows a higher mirror symmetry along m 2 but a broken mirror symmetry along ml , with $\gamma_{\mathrm{m} 2}=92.5 \%$ and $\gamma_{\mathrm{ml}}=65.2 \%$, respectively. CBED patterns of imperfect crystals having displacements were simulated using the Bloch-wave scattering matrix method (41-43). Along the electron beam path, the displacement fields was approximated as a column with a displaced block of 10nm (Fig. 2B). Figure 2D shows a simulated CBED pattern of a block located at 10 nm from the surface and displaced $\sim 7.1 \%$ along (2 $2 \overline{2} 0)$ from the column, which corresponds to the experimental CBED pattern (Fig. 2C). Within the 200nm x 200nm SCBED dataset (Region R2, Fig. 1D), 22 disc-shaped clusters were identified and found to be $10 \pm 3 \mathrm{~nm}$ in diameter, all having tensile displacements from the cluster along one of the <110> directions.


Figure 5.2. Displacement fields in $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi} \mathrm{HEA}$ provided by experimental and simulated CBED patterns. Virtual ADF images of $(\boldsymbol{A})$ selected and $(\boldsymbol{E})$ without "coffee bean" contrast. CBED patterns from imperfect crystals were simulated using the Bloch-wave scattering-matrix method. The lattice displacements along the electron beam path were approximated as a column having (B) a displaced block of 10nm, and ( $\boldsymbol{F}$ ) blocks of few nanometers with displacements following the Gaussian distribution. (B) The displacement fields from Eshelby's theory of ellipsoidal inclusions calculated using finite element analysis method. Figures ( $\boldsymbol{D}$ and $\boldsymbol{H}$ ) show simulated CBED patterns corresponding to the experimental ( $\boldsymbol{C}$ and $\boldsymbol{G}$ ) patterns, respectively.

Symmetry fluctuations in regions free of inclusions (Figs. 1E and 2E) can be attributed to variations in atomic displacement. To estimate the amount of displacement fluctuations in HEAs, we performed a series of CBED simulations by dividing the $186-\mathrm{nm}$-thick crystal column into nmsized blocks, each block having displacements ( $u_{x}, u_{y}, u_{z}$ ) which follows the Gaussian distribution
with zero mean displacement (Fig. 2E). 625 CBED patterns were simulated and quantified for each combination of block size and standard deviation of displacement, ranging from 1-10nm and 0.35$0.65 \%$, respectively. By comparing the distribution of 4 -fold rotational symmetry of 4,375 experimental CBED patterns from regions free of coffee bean contrast with simulated CBED patterns, local symmetry fluctuation can be estimated by $3-8 \mathrm{~nm}$ blocks having displacements following the Gaussian distribution with $0.55 \%$ standard deviation displacement (Fig. 5.3). For example, Fig. 2D shows a simulated CBED pattern of 5 nm blocks having $0.55 \%$ standard deviation of displacements, which corresponds to the experimental CBED pattern shown in Fig. 2C.


Figure 5.3. CBED symmetry fluctuations versus nm-sized blocks having displacements in Alo.l CrFeCoNi HEA. CBED patterns from imperfect crystals were simulated using the Blochwave scattering-matrix method. The lattice displacements along the electron beam path were approximated as a column having blocks of few nanometers with displacements following the Gaussian distribution. Each data point corresponds to the distribution of 4-fold rotational symmetry from 625 simulated CBED patterns. The red horizontal line (band) corresponds to the mean (standard deviation) of 4-fold rotational symmetry of 4375 experimental CBED patterns from regions without "coffee bean" contrast.

The strain maps along (220) and (2 $\overline{2} 0$ ) directions (Figs. 4 b \& 4 c , respectively) are obtained by measuring the relative shifts in disk positions at region R1 (Fig. 1d). The precision of strain is
$\pm 0.13 \%$, which is determined by the SCBED strain measurement on a Si [110] single crystal. The disc-shaped inclusions correspond to localized region with compressive strain in Fig. 2, B and C. In addition, no extra reflections were found from the SCBED dataset with semi-convergence angle $\alpha$ of 1.4 mrad , which suggests the absence of ordering of inclusions (Fig. 1B). The line profiles (Line 1 and Line 2 in Fig. 4, C and D ) taken from the strain maps show the strain to be slowvarying over $\sim 50 \mathrm{~nm}$ while having fluctuations within $\pm 1.3 \%$ in the range of a few nanometers.


Figure 5.4. Local strain fluctuations in $\mathrm{Al}_{0.1} \mathbf{C r F e C o N i}$ MPEA. Strain maps of region $R 2$ along the (A) (220) and (B) (2 $\overline{2} 0)$ directions were obtained by measuring relative shifts in disk positions. ( $\boldsymbol{C}$ and $\boldsymbol{D})$ Line profiles drawn from ( $A$ and $B$ ), respectively.

We noticed the peak broadening with increasing diffraction orders in the SCBED dataset (Fig. 5A). This observation could have two possible explanations: zone-axis effects and local lattice distortions. Along the zone axis orientation, all zero-order Laue zone (ZOLZ) reflections except (000) do not satisfy the Bragg conditions, which could lead to diffraction peak broadening resulting from the intersection of Ewald sphere and reciprocal lattice rods. In addition, the
inelastic, forward-scattered electrons would result in Ewald spheres with multiple radii and a consequent peak broadening. Second, the crystal lattices possessing nonuniform strain would also give rise to peak broadening in the diffraction patterns. For example, the paracrystal model, or lattice distortions of the second kind, suggests a quadratic increase in the integral breadths $\delta \mathrm{b}$ of successive diffracted orders $(\mathrm{h})(44,45)$. We excluded the peak broadening from the crystallites size effect since the region for performing SCBED is a single grain. The zone-axis effect was considered by performing SCBED on a Si single crystal along [110] zone axis, which defines the peak integral breadth measurement precision at $\pm 0.00315 \AA^{-1}$. By fitting the (200), (400), and (600) reflections from 10,000 CBED patterns taken from region R1 (Fig. 1D), lattice distortion parameter map can be reconstructed from the SCBED dataset (Fig. 5C). The mean lattice distortion parameter g in this region was determined as $2.12 \%$ (Fig. 5B).


Figure 5.5. Lattice distortions of paracrystalline $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$ HEA. (A) Example fit of an intensity profile with three orders of (200) reflections drawn from the averaged CBED pattern with $\alpha=1.4$ mrad. ( $\boldsymbol{B})$ Diagram of integral width $\delta b$ versus square of reflection orders $h^{2}$. Lattice distortion parameter $g$ of $2.12 \%$ can be derived from the slope of the $\delta b-h^{2}$ diagram. The error bars represent the standard deviation of 400 measurements on standard Si along [110] zone axis. (C) Lattice distortion parameter map of region $R 2$ obtained by measuring the peak broadening with increasing reflection orders.

The observed symmetry breaking in both transmitted and diffracted disks of experimental CBED patterns could have two possible explanations: microscope optics and local crystal imperfections. First, we exclude the effects of microscope optics by performing SCBED on a Si single crystal, which defines a figure of merit of $\gamma_{4} \geq 95 \%$ for a symmetrical CBED pattern. Secondly, symmetry in each CBED pattern reflects the imperfect lattice averaged over the local
volume illuminated by the electron beam. In column approximation, the illuminated volume of crystal is roughly the product of probe diameter squared and specimen thickness of 186 nm . By performing SCBED using various probe sizes of $50 \mathrm{~nm}, 10 \mathrm{~nm}, 5 \mathrm{~nm}, 1 \mathrm{~nm}$, the symmetry breaking in CBED patterns can be detected as the electron probe $\leq 10 \mathrm{~nm}$ (Fig. 6), or illuminated volume $\leq 18,600 \mathrm{~nm}^{3}$, which demonstrates the sensitivity of the SCBED technique.


Figure 5.6. Probe size dependent symmetry in $\mathrm{Al}_{0.1} \mathbf{C r F e C o N i}$ HEA. For each probe size, more than 1875 CBED patterns were quantified with 4-fold rotational symmetry. The red dashed line was drawn along $\Upsilon_{4}=95 \%$, indicating a figure of merit for symmetrical patterns.

### 5.4. Discussion

The distorted lattice in $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$ HEA can be understood from comprehensive SCBED results of symmetry, strain, and lattice distortion parameters. First, the symmetry maps show there being regions with and without coherent, disordered inclusions (Figs. 1-2). The inclusions with tensile displacements can be assumed as Al-rich, since Al has the largest atomic size mismatch. However, no obvious elemental segregation were found in K- $\alpha$ peak ratio maps obtained from EDS-SI performed at same region as SCBED [Fig. 7]. This may be explained by the 7nm cluster embedded in a 186 nm -thick MPEA specimen being well below the detection limits of our EDS system (46).


Figure 5.7. Energy dispersive $X$-ray spectrum images of $\boldsymbol{A l}_{0.1} \mathbf{C r F e C o N i}$ HEA. The EDSspectrum image, 180x180 with step sizes of 1nm, was acquired about the same region as for SCBED, with specimen tilted $\sim 6^{\circ}$ away from [001] zone axis. Each spectrum was acquired for $1 s$, spanning the total acquisition time of $\sim 11 h r s$. Every $K-\alpha$ peak signal for each element was first smoothed with Savitzky-Golay filter, fitted with a Gaussian peak to obtain the peak height, then normalized with respect to total $K$ - $\alpha$ peak heights in each spectrum. Each image were displayed with limits between $\pm 3$ times of standard deviation about the average.

Second, the strain maps correspond to the local d-spacing averaged over all unit cells being illuminated by the electron beam, as shown in Figs. 3(a)-3(b). In the column approximation, the volume being illuminated is roughly the product of probe diameter squared and specimen thickness. Third, the lattice distortion parameter reflects the variance of the d-spacing of the column (Fig. 4c). In the paracrystal model, the lattice distortion parameter $g$ was defined as $g^{2}=$ $\left\langle d_{h k l}^{2}\right\rangle /\left\langle d_{h k l}\right\rangle^{2}-1$, where $\left\langle\mathrm{d}_{\mathrm{hkl}}\right\rangle$ is the mean lattice d-spacing of the (hkl) plane. Putting things
together, besides inclusions, both the mean and the variance of the lattice d-spacing within a crystal volume of $537 \mathrm{~nm}^{3}$ are varying and inhomogeneously distributed at the nanometer-scale.

The lattice of HEAs was expected to be severely distorted based on the atomic size misfit.

Several models based on hard sphere packing have been proposed to estimate the size mismatch in HEAs. For example, the delta factor $\delta=\sqrt{\sum_{i=1}^{N} c_{i}\left(1-r_{i} / \bar{r}\right)^{2}}$, where N is the number of elements in HEA, $c_{i}$ and $r_{i}$ indicate the atomic fraction and radius of ith element, respectively. The magnitudes of strain and lattice distortion parameter obtained from SCBED are comparable with that of estimated from delta factors of $2-4 \%$ (47), however, are about 10 times larger than that of measured from X-ray or neutron scattering for single phase fcc HEAs (23, 24).

The above observation raises important questions about the roles of the inhomogeneous lattice distortion in thermodynamic stability and deformation mechanisms. The occurrence of clusters with displacements of $\sim 7.1 \%$ from the averaged lattice and slow-varying strain over $\sim 50 \mathrm{~nm}$ (Figs. 2 and 3) suggest the non-ideal solid solution nature in $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$ HEA system (11). The identified Al-rich clusters is in agreement with the $\mathrm{Pd}-\mathrm{Pd}$ pairs in PdCrFeCoNi observed by EXAFS studies, which the strain field follows Eshelby's inclusion theory (48); while the slowvarying strain is consistent with the spinodal decomposition of $\mathrm{Cr}, \mathrm{Fe}$-rich versus $\mathrm{Al}, \mathrm{Ni}, \mathrm{Co}$-rich segregations observed in $\mathrm{Al}_{\mathrm{x}} \mathrm{CrFeCoNi}$ systems with higher Al content $(49,50)$, as well as the Cr ordering in CrFeCoNi observed by quantitative atomic resolution imaging (51). Previously, first-
principles calculations have suggested the effect of lattice distortion on solid solution strengthening and phase stability by considering difference in atomic sizes $(52,53)$ and magnetism or electronic effects (10,51,54-57). Furthermore, the reported slow, intermittent dislocation motion in $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}$ observed using in-situ TEM can be attributed to the observed severe lattice distortion using SCBED, which is known to increase the Peierls-Nabarro stress (25).

### 5.5. Conclusions

In conclusion, we have used a recent-developed SCBED technique to directly visualize the local symmetry and strain in $\mathrm{Al}_{0.1} \mathrm{CrFeCoNi} \mathrm{HEA}$. Two types of symmetry breaking were found. One features $10 \pm 3 \mathrm{~nm}$, disc-shaped, clusters having $\sim 7.1 \%$ tensile displacements along <110> directions, and the other gives rise to local fluctuating strain within $\pm 1.3 \%$, which is slow-varying over $\sim 50 \mathrm{~nm}$. Furthermore, the diffraction peaks broadening is consistent with the paracrystal model having lattice distortion parameter of $2.12 \%$.

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## CHAPTER 6

## SUMMARY AND OUTLOOKS

### 6.1. Summary

In this study, we have further developed SCBED as a versatile and robust technique for characterization of local disorder in complex, multi-element crystals. Both local and average crystallographic symmetry can be determined by varying the illuminated sample volume, which can be achieved by using various electron probe sizes and specimen thicknesses.

We have further improved the symmetry quantification algorithm for SCBED, which demonstrated the sensitivity of detecting nanoscale symmetry fluctuations within a single domain in $\mathrm{BaTiO}_{3}$ single crystals. The interpretation of the observed symmetry fluctuation is twofold: 1) the local symmetry breaking can be explained by the anharmonicity of soft phonons within the displacive model of ferroelectric phase transition; or alternatively, 2) the population variations of the $\langle 111\rangle$ polarizations along the electron beam direction, which is consistent with the order-disorder phase transition mechanism.

For relaxor ferroelectric PZN-8\%PT, we have successfully determined the $50^{\circ}$ nanodomains of monoclinic Pm symmetry using SCBED. The local structure of PZN$8 \% \mathrm{PT}$ is drastically different from that of $\mathrm{PMN}-31 \% \mathrm{PT}$, which the latter is found to be triclinic at nanoscale (1, 2). Furthermore, by measuring the intensity redistributions within the transmitted (000) disk, we found a new type of defect, lattice-rotation vortex, at the boundary of nanodomains. The observed lattice-rotation vortices can be explained by disinclination strain and depolarization fields due to charge discontinuity.

Besides polarization determination, we extend the application of SCBED by demonstrating the capability of imaging quantitatively the lattice distortions and local symmetry breaking in HEAs. Our results revealed two types of lattice distortions in $\left.\mathrm{Al}_{0.1} \mathrm{CrFeCoNi}: 1\right) 10 \pm 3 \mathrm{~nm}$, disc-shaped, clusters having $\sim 7.1 \%$ tensile displacements along $\langle 110\rangle$ directions distributed throughout the specimen, and 2) the local fluctuating strain within $\pm 1.3 \%$, which is slow-varying over $\sim 50 \mathrm{~nm}$. Both suggest the non-ideal nature of solid solutions and thermodynamic instabilities, which contradict with previous speculations based on the high entropy hypothesis. Our results, made possible using SCBED, thus demonstrate a new powerful experimental approach towards quantitative study of complex, multi-element crystals.

### 6.2. Outlooks

### 6.2.1. Quantitative refinement of SCBED: SQCBED

The refinement of diffracted intensity in experimental CBED patterns using Blochwave method, or quantitative CBED (QCBED), have been shown to measure low order structure factors and crystal bonding charge density with accuracy of $<0.09 \%$ (3, 4). The fundamental assumption of Bloch-wave method is based on the translational symmetry of a unit cell. Disordered crystals, unfortunately cannot be applied in this regime.

Alternatively, dynamical diffraction intensities can be calculated from non-periodic structures using the multislice or the Bloch-wave scattering matrix methods (5-7). The retrieval of structural information based on the Bloch-wave scattering matrix method has been demonstrated by Pennington \& Koch (8). Using simulated CBED data of $\mathrm{BaTiO}_{3}$, their algorithm was able to retrieve the 3D Ti displacements with lateral accuracy of 3pm
and depth resolution of $5-10 \mathrm{~nm}$. The success of this simulation experiment has demonstrated a promising possibility, the scanning quantitative CBED (SQCBED) technique, to retrieve the 3D structural information from every CBED pattern in the SCBED dataset, i.e., diffraction tomography without tilting. This is an ongoing effort in Zuo group in collaboration with Prof. Philip Nakashima's group (Monash University, Australia) to develop multislice-based SQCBED to study 3D structures of voids in metals.

### 6.2.2. Rocking beam SCBED: SRCBED

Another limitation of the SCBED technique is the disk size, or the convergence angle. To avoid the overlapping of disks, the largest disk radius, or semi-convergence angle $\alpha$, is restricted by the Bragg angle $\theta_{B}$. Hence for large unit cell crystals, there is limited angular range of data can be recorded in a CBED pattern. Large-angle CBED (LACBED) patterns obtained by the Tanaka method operates by defocusing the specimen plane, which drastically sacrifices the spatial resolution up to $\sim 100 \mathrm{~s} \mathrm{~nm}$. Recently, Koch (9) and Beanland (10) groups have used computer control to rock/tilt the electron beam, which provides LACBED patterns and keeps the probe within a few nm by patching many CBED disks from individual beam tilts. We propose to develop LACBED with scanning based techniques, scanning rocking CBED (SRCBED), to map nanometer-scale structure variations in large unit cell crystals such as unconventional superconductors such as $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-x}$. Furthermore, SRCBED can be extended by changing the electron beam into a low-dose, parallel illumination. By patching the spot-like Bragg reflections into disks allows the determination of local handedness and chirality variations in beam-sensitive
materials such as metal-organic framework or crystalline polymers. Effort is ongoing in Zuo group to develop SRCBED to study strongly correlated materials.

### 6.2.3. Time-resolved SCBED

The SCBED data in this thesis assumes that the specimen is static. The experimental acquisition time, hence the drift in specimen stage, is limited by the CCD acquisition speed of $\sim 1-10$ frames per second (fps). The temporal resolution of SCBED can be improved either by using fast cameras or by ultrafast pump-probe approach. First, recent developments of fast camera with speed of $>1000 \mathrm{fps}$ has enabled atomic-resolution in-situ TEM with millisecond time resolution (11). In combination with SCBED, in-situ deformation for strain measurements has been demonstrated by Gammer et al. where timeresolved 2D strain maps can be correlated with the load-displacement curve (12). For relaxor ferroelectrics, we propose to study the switching behavior of nanodomains as well as the temperature-dependent behavior of PNRs by combining in-situ biasing/heating with SCBED. Second, ultrafast electron pulses generated by photoemission has enabled the study of transient structures with temporal resolution of $n s$ to fs. Ultrafast CBED has been demonstrated for the determination of strain dynamics by utilizing the change in HOLZ lines (13) and Kikuchi bands (14). With the recently customized Hitachi 9500 environmental TEM at University of Illinois, we propose to study the phonon softening associated with structural phase transitions in the nanosecond to picosecond regime in correlated systems such as ferroelectrics.

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## APPENDIX A:

## DM SCRIPT FOR SCBED

## A.1. Scanning electron diffraction acquisition

/*
Scanning_ED, (Scanning electron diffraction acquisition)
Yu-Tsun Shao, Ph.D. Candidate, U of lllinois
This script is for scanning electron diffraction. 'EMGet-' or 'EMSet-' functions are used to control TEM.
Start development: 05.05.10
10.05.07-Dialog Box created
10.05.09- Calibration part is done.
10.05.10 - Calibration part is added.
10.05.10 - Starts 'Acquire Part'
10.05.17 - Experimental conditions are added into result.
10.11.07 - Centered scan and check scan options added
11.10.30 - Tested for Cryo TEM (Image size changed from 1024 to 2048)
16.12.16 - Added for scan rotation (Yifei Meng, Univ. of Illinois)
17.04.07 - (Background) Thread
17.04.09 - Tested for threading (Tianyu Liu, Monash Uni. \& Yu-Tsun Shao, Univ. of Illinois)
17.06.28 - Tested on JEOL ARM 200CF TEM-mode @NTNU Trondheim (Yu-Tsun Shao, Univ. of Illinois)
17.10.12 - debugged for scan rotation angle (Yu-Tsun Shao, Univ. of Illinois)
*/ 17.11.05 - debugged for "center" function; enable rectangular CCD (such as the camera on 2010 LaB6) (Yu-Tsun Shao, Univ. of Illinois)
*/
// Declaration of variables
TagGroup Binning, Exp // Binning and exposure time
TagGroup xNum, yNum // Data points along x and y direction
TagGroup xLength, yLength // Step length along x and y direction
TagGroup RealNum1, RealNum2, RealNum3, RealNum4, RealNum5, RealNum6, RealNum7, RealNum8, RealNum9, RealNum 10, RealNum11, RealNum10_x, RealNum 10 y
TagGroup Name1, Name2, Name3 // Specimen name, Mode, Mangnification
TagGroup ScanOption, CheckScan //Check scan option deleted on 2011/11/02
number abort=0 // mission aborted? for background thread
/* TagGroup description
RealNum1 = Binning
RealNum2 = Exposure time
RealNum3 $=$ Data points $X$
RealNum4 $=$ Data points $Y$
RealNum $5=$ Step Length $X$
RealNum6 $=$ Step Length $Y$
RealNum7 $=$ Spot Size
RealNum8 $=$ Alpha
RealNum10 = CCD pixel size
RealNum10_x, RealNum10_y (for rectangular CCD)
RealNum11 = tilt angle degree
*/
// Thread
Class AcquThread: Thread
\{
void f_Start ( object self)
\{
DLGGetValue(RealNum11,angleDegree)
number tiltAngle $=$ angleDegree*3.14159265359/180
/Gets experimental conditions
String Mode, Mag
Number Alpha, SpotSize
DLGGetValue (Name2, Mode)
DLGGetValue (Name3, Mag)
DLGGetValue (RealNum7, Alpha)
DLGGetValue (RealNum8, SpotSize)
//Gets a specimen name
String SpecimenName
DLGGetValue (Name1, SpecimenName)
/Gets a CCD pixel size
Number CCDPixelsX, CCDPixels $Y$
DLGGetValue (RealNum10_x, CCDPixelsX)
DLGGetValue (RealNum10_y, CCDPixelsY)
//Gets a binning value
number Binning
DLGGetValue (RealNum1, Binning)
//Gets an exposure time
number ExpTime
DLGGetValue (RealNum2, ExpTime)

```
    Gets a data poin
    number XPoints, YPoints
    DLGGetValue (RealNum3, XPoints)
    DLGGetValue (RealNum4, YPoints)
    //Gets a step length
    number StepL X,StepL Y, CorF
    DLGGetValue (RealNum5, StepL_X)
    DLGGetValue (RealNum6, StepL_Y)
    //Stores the setting values in TagBase("LastSettingValues")
    String TagBase=("LastSettingValues_SED")
    SetPersistentStringNote(TagBase+"Mode", Mode)
    SetPersistentStringNote(TagBase+"Mag",Mag)
    SetPersistentNumberNote(TagBase+"Alpha", Alpha)
    SetPersistentNumberNote(TagBase+"SpotSize", SpotSize)
    SetPersistentStringNote(TagBase+"SpecimenName", SpecimenName)
    SetPersistentNumberNote(TagBase+"CCDPixelsX", CCDPixelsX)
    SetPersistentNumberNote(TagBase+"CCDPixelsY", CCDPixelsY)
    SetPersistentNumberNote(TagBase+"Binning", Binning)
    SetPersistentNumberNote(TagBase+"ExpTime", ExpTime)
    SetPersistentNumberNote(TagBase+"XPoints", XPoints)
    SetPersistentNumberNote(TagBase+"YPoints", YPoints)
    SetPersistentNumberNote(TagBase+"StepL_X", StepL_X)
    SetPersistentNumberNote(TagBase+"StepL Y",StepL Y)
    SetPersistentNumberNote(TagBase+"CorF", CorF)
    / Recall the calibrated unit
    TagBase = ("SED_Calibration")
    String Units
    GetPersistentStringNote(TagBase + "CalUnits", units)
    / Recall the calibrated data
    TagBase = ("SED_Calibration")
    complexnumber c_Horz_UnitVector, c_Vert_UnitVector
    number real HorzScale, real VertScale
    GetPersistentNumberNote(tagBase + "HorzCal", real_HorzScale)
    GetPersistentComplexNumberNote(tagBase + "c_Horz_UnitVector", c_Horz_UnitVector)
    GetPersistentNumberNote(tagBase + "VertCal", real_VertScale)
    GetPersistentComplexNumberNote(tagBase + "c_Vert_UnitVector", c_Vert_UnitVector)
    // Store the initial value of deflector: This needs to move the beam back to the initial position after scanning function.
    number TDefX 1,TDefY 1
    EMGetBeamShift(TDefX - 1, TDefY 1)
    TagBase=("InitialDefValue0")
    SetPersistentNumberNote(TagBase+"xDefValue0", TDefX 1)
    SetPersistentNumberNote(TagBase+"yDefValue0", TDefy_1
    /Centered Scan option: The scanning rectangular area has the center with the initial beam position
    ComplexNumber StartDef
    Number DefaultScanOption
    DefaultScanOption=ScanOption.DlgGetValue()
    f (DefaultScanOption==1)
    If (D
    number radiusDef = SQRT((XPoints-1)**2+(YPoints-1)**2)/2
    number initAngle = pi() - atan((YPoints-1)/(XPoints-1))
    StartDef = complex(TDefX_1, TDefY_1) + c_Horz_UnitVector*real_HorzScale*StepL_X*radiusDef*cos(initAngle-tiltAngle)
c_Vert_UnitVector*real_VertScale*StepL_Y*radiusDef*sin(initAngle-tiltÄngle)
    //OLD; wrong
    //StartDef=complex(TDefX 1, TDefY 1)-c Vert UnitVector*(0.5*(Ypoints-1)*StepL Y*real VertScale)-
c Horz UnitVector*(0.5*(Xpoints-1)*real_HorzScale*StepL X
    SetPersistentComplexNumberNote(TagBase+"StartDef", StartDef)
    }
    Else
    StartDef=complex(TDefX_1, TDefY_1)
    SetPersistentComplexNumberNote(TagBase+"StartDef", StartDef
    // End
    GetPersistentComplexNumberNote(TagBase+"StartDef", StartDef)
    // Work Image
    Number TotSlice=XPoints*YPoints
    Image Img3D :=
IntegerImage(SpecimenName+" "++XPoints+"by"+YPoints+"pts_"+StepL_X+"by"+StepL_Y+units+"_"+Mode+"_"+Mag+"_Alpha("+Alpha+")"+" Spot("+SpotSize+")"
2, 1, CCDPixelsX/Binning , CCDPixelsY/Binning, TotSlice ) // Integerlmage ("Image name", # bytes, singed or not, pixels, pixels, total slice numbers)
    Img3D.ShowImage()
    Imagedisplay ImgDisp3D = Img3D.ImageGetImageDisplay(0)
    Number Slice n=0
    // Processing time measurement
    Number Start=GetHighResTickCount()
    // Shift a beam and record an image
    Number iY=0
    ComplexNumber ScanningDef
    while (iy<YPoints)
                            {
                            number ix=0
```



```
number width = img.ImageGetDimensionSize(0
number height = img.ImageGetDimensionSize(1)
number bin = CCDPixelsY/height
number scale = img.ImageGetDimensionScale(0)
string units = img.ImageGetDimensionUnitString(0)
/Creates ROI boxes on the front image
magedisplay imgdisp = img.ImageGetImageDisplay(0)
ROI ROI1 =NewRoi()
number top, left, bottom, right
top=height*0.2
eft=width*0.2
bottom=height*0.8
right=width*0.8
ROI1.ROISetRectangle(top, left, bottom, right)
ROI1.ROISetVolatile(0)
ROI1.ROISetMoveable(0)
mgDisplay.ImageDisplayAddROI( ROI1 )
/Calculate a real scale from the image
number r_width=abs((bottom-top)*scale
number r_height=abs((left-right)*scale)
result ("width="+r_width+" "+units+","+"r_height="+r_height+" "+units+"\n")
Shows ROls for beam position
/Beam box }
ROI ROI2=NewROI()
number ROI2_top=top-50/bin
number ROI2_left=left-50/bin
number ROI2_bottom=top+50/bin
number ROI2_right=left+50/bin
ROI2.ROISetRectangle (ROI2_top, ROI2_left, ROI2_bottom, ROI2_right)
ROI2.ROISetVolatile(0)
ROI2 ROISetMoveable(0)
ROI2.ROISetColor(0,0,1)
String Label_ROI2="Locate a beam center on this box"
ROI2.ROISetLabel(label_ROI2)
imgDisplay.ImageDisplay}
Get deflector's current - 
number DefX 1, DefY 1
f(!OKCanceIDialog("Use a beam shift knob to locate in the blue box.\nPress 'OK' when you are done."))
    exit(0)
EMGetBeamShift(DefX_1, DefY_1) //FasTEM_GetDefValue (Deflector number, xDefValue, yDefValue)
OKDialog ("Click to continue")
imgDisplay.ImageDisplayDeleteROI(ROI2)
Beam box }
ROI2_top=top+50/bin
ROI2_left=right-50/bin
ROI2 bottom=top-50/bin
ROI2_right=right+50/bin
ROI2.ROISetRectangle (ROI2_top, ROI2_left, ROI2_bottom, ROI2_right)
ROI2.ROISetRectangle
ROI2.ROISetVolatile(0)
ROI2.ROISetMoveable(0)
ROI2.ROISetLabel(label_ROI2)
mgDisplay.ImageDisplayAddROI(ROI2 )
//Get deflector's current - 2
number DefX 2 DafY 2
if(!OKCancelDialog("Use a beam shift knob to locate in the blue box.\nPress 'OK' when you are done."))
exit(0)
EMGetBeamShift(DefX_2, DefY_2)
OKDialog ("Click to continue")
//Beam box }
ROI2_top=bottom+50/bin
ROI2_left=right-50/bin
ROI2 bottom=bottom-50/bin
ROI2 right=right+50/bin
ROI2_right=right+50/bin
ROI2.-ROISetRectangle (ROI2_top, ROI2_left, ROI2_bottom, ROI2_right)
ROI2.ROISetVolatile(0)
ROI2.ROISetMoveable(0
ROI2.ROISetColor(0,0,1)
ROI2.ROISetLabel(label_ROI2)
imgDisplay.ImageDisplayAddROI( ROI2 )
//Get deflector's current - 3
number DefX 3, DefY 3
if(!OKCancelDialog("Use a beam shift knob to locate in the blue box.\nPress 'OK' when you are done."))
    exit(0)
EMGetBeamShift(DefX_3, DefY_3)
OKDialog ("Click to continue")
imgDisplay.ImageDisplayDeleteROI(ROI1)
imgDisplay.ImageDisplayDeleteROI(ROI2)
//Print results
result("\n----------------------------------------------------------------------------
result("\n -----------------------------------------
```

```
result("\n-------------------------------------------------------------------------------
/Convert deflector's current to a real unit
complexnumber c_Horz=(complex(DefX_2, DefY_2)-complex(DefX_1, DefY_1)) // Convert the beam current change to a horizontal
vector component
number real HorzScale=sqrt(norm(c Horz))/r width // Ratio of 'Beam current change' to 'Real unit' (# current/1 nm
complexnumber c_Horz_UnitVector=c_Horz/sqrt(norm(c_Horz)) // Calculate a horizontal unit vector
complexnumber c_Vert=(complex(DefX_3, DefY_3)-complex(DefX_2, DefY_2)) // Convert the beam current change to a vertical vector
component
number real_VertScale=sqrt(norm(c_Vert))/r_height // Ratio of 'Beam current change' to 'Real unit' (# current/1 nm)
complexnumber c_Vert_UnitVector=c_Vert/sqrt(norm(c_Vert)) // Calculate a vertical unit vector
OKDialog ("Please check calibration by using 'Test' button in Calibration box.")
string Time=GetTime(1),Date=GetDate(1)
//Save the calibration result in script
String tagBase = ("SED_Calibration")
SetPersistentStringNote(TagBase + "CalUnits", units)
SetPersistentNumberNote(tagBase + "HorzCal", real_HorzScale)
SetPersistentComplexNumberNote(tagBase + "c_Horz_UnitVector", c_Horz_UnitVector)
SetPersistentNumberNote(tagBase + "VertCal", real_VertScale)
SetPersistentComplexNumberNote(tagBase + "c_Vert_UnitVector", c_Vert_UnitVector)
SetPersistentStringNote(tagBase + "CalUnits", units )
SetPersistentStringNote(tagBase + "CalDate", Date)
SetPersistentStringNote(tagBase + "CalTime", Time)
GetPersistentStringNote(TagBase + "CalDate", Date)
GetPersistentStringNote(TagBase + "CalTime", Time )
result("\n*Last Calibration: "+Date+" "+Time+"\n")
//For check
result("\n*Calibrated area\n")
    result ("Width= "+r width+" "+units+","+"\nHeight= "+r height+" "+units+"\n")
    esult("\nHorizontal Scale= "+real HorzScale+" (# current/1 )"+units+"\n")
    result("Vertical unit vector= "+c_Horz_UnitVector+" (current)\n")
    result("\nVertical Scale= "+real_VertScale+" (# current/1 )"+units+"\n")
    result("Horizontal unit vector= "'+c_Vert_UnitVector+" (current)\n")
}
// End of scripts for 'Calibration button' in Control Box 2
Void Start(Object self) { thread.StartThread(); }
Void Stop(Object self) { thread.AcquStop(); }
Object Init(Object self, taggroup dialog, Number thread_id) {
                    thread = GetScriptObjectFromID(thread_id)
                    Return self.super.Init(dialog)
}
}
// Creates the buttons, fields and labels in the dialog
//Script Information
taggroup ScriptInfo()
    {
            taggroup ScriptInfo=dlgcreatelabel("\nScanning Electron NanoDiffraction (SEND)\n")
            ScriptInfo.dlgexternalpadding(10,0)
            return ScriptInfo
}
// Creates the buttons in Control Box 1
TagGroup ExperimentalCondition()
{
    TagGroup Box0_Items
    TagGroup Box0=DlgCreateBox(" Experimental Conditions ", box0_items)
    //Recall the last setting values
    String TagBase=("LastSettingValues_SED")
    String Mode, Mag
    Number Alpha, SpotSize
    GetPersistentStringNote(TagBase+"Mode", Mode)
    GetPersistentStringNote(TagBase+"Mag", Mag)
    GetPersistentNumberNote(TagBase+"Alpha", Alpha)
    GetPersistentNumberNote(TagBase+"SpotSize",SpotSize)
    String Name
    TagGroup Label11=DLGCreateLabel("Mode: ")
    Label11.DLGExternalPadding(0, 0)
    Name2=DLGCreateStringField (Name, 8)
    Name2.DLGExternalPadding(0, 0)
    DLGValue(Name2, Mode)
    TagGroup Label12=DLGCreateLabel(" Mag: ")
    Label12.DLGExternalPadding(0,0)
    Name3=DLGCreateStringField (Name, 10)
    Name3.DLGExternalPadding(0,0)
    DLGValue(Name3, Mag)
TagGroup Group11=DLGGroupltems(Label11, Name2, Label12, Name3)
Group11.DLGTableLayout(4, 1, 0)
Box0 items.DLGAddElement(Group11)
Group11.DLGExternalPadding(12,3)
```

```
        TagGroup Label14=DLGCreateLabel(" Spot Size : ")
        Label14.DLGExternalPadding(0, 0)
        RealNum8=DLGCreateRealField (0,7,0)
        RealNum8.DLGExternalPadding(0,0)
        DLGValue(RealNum8, SpotSize)
        TagGroup Label13=DLGCreateLabel("Alpha : ")
        Label13.DLGExternalPadding(1,1)
        RealNum7=DLGCreateRealField (0, 7, 0)
        Name2.DLGExternalPadding(0,0)
        DLGValue(RealNum7, Alpha)
    TagGroup Group12=DLGGroupltems(Label14, RealNum8, Label13, RealNum7)
    Group12.DLGTableLayout(4, 1, 0)
    Box0 items.DLGAddElement(Group12)
    Group12.DLGExternalPadding(19,0)
    Box0.DLGExternalPadding(0, 0)
    Return Box0
}
// Creates the buttons in Control Box 1
TagGroup MainControlBox1()
{
// Make 'other buttons' - Group 2 starts from here
taggroup box2_items //Control box 2 is grouped by 'box2_items//
taggroup box2=dlgcreatebox(" Control Box 1 ", box2_items)_// Label of 'Control Box 1'
//Recall the last setting values
String TagBase=("LastSettingValues_SED")
number CCDPixelsX, CCDPixelsY, Binning, ExpTime, Xpoints, YPoints, StepL_X, StepL_Y, CorF, angleDegree
String SpecimenName
GetPersistentStringNote(TagBase+"SpecimenName", SpecimenName)
GetPersistentNumberNote(TagBase+"CCDPixelsX", CCDPixelsX)
GetPersistentNumberNote(TagBase+"CCDPixeIsY", CCDPixelsY)
GetPersistentNumberNote(TagBase+"Binning", Binning)
GetPersistentNumberNote(TagBase+"ExpTime", ExpTime)
GetPersistentNumberNote(TagBase+"XPoints",XPoints)
GetPersistentNumberNote(TagBase+"YPoints", YPoints
GetPersistentNumberNote(TagBase+"StepL_X", StepL_X
GetPersistentNumberNote(TagBase+"StepL Y", StepL Y
GetPersistentNumberNote(TagBase+"CorF",
GetPersistentNumberNote(TagBase+"angleDegree", angleDegree)
// Specimen Name
String Name
TagGroup Label10=DLGCreateLabel("1. Specimen Name: ")
Label10.DLGExternalPadding(1,1)
Name1=DLGCreateStringField (Name, 25)
Name1.DLGExternalPadding(0,0)
DLGValue(Name1, SpecimenName)
TagGroup Group10=DLGGroupltems(Label10, Name1)
Group10.DLGTableLayout(2,1,0)
Box2_Items.DLGAddElement(Group10)
Group10.DLGExternalPadding(0,3)
//CCD pixel size: 2010F(1024), 2100 Cryo (2048)
TagGroup CCDLabelx=DLGCreateLabel("2. CCD pixel size X: ")
CCDLabelx.DLGExternalPadding(1,1)
RealNum10_x=DLGCreateRealField(0,9,0)
RealNum10 x.DLGExternalPadding(0,0)
TagGroup CCDLabely=DLGCreateLabel("pixel size Y: ")
CCDLabely.DLGExternalPadding(1,1)
RealNum10_y=DLGCreateRealField(0,9,0)
RealNum10_y.DLGExternalPadding(0,0)
DLGValue(RealNum10_x,CCDPixelsX)
DLGValue(RealNum10_y,CCDPixelsY)
TagGroup CCDGroup=DLGGroupltems(CCDLabelx, RealNum10_x, CCDLabely, RealNum10_y)
CCDGroup.DLGTableLayout(4,1,0)
Box2 Items.DLGAddElement(CCDGroup)
Box2_Items.DLGAddElement(CCDGroup)
//Binning and Exposure time
TagGroup label1 = DLGCreateLabel("3. Binning:")
label1.dlgexternalpadding(1,1)
RealNum1 = DLGCreateRealField(0, 7, 2)
RealNum1.dlgexternalpadding(0,0)
DLGValue(RealNum1,Binning)
TagGroup label2 = DLGCreateLabel(" 4. Exp time:")
label2.dlgexternalpadding(1,1)
RealNum2 = DLGCreaterealField(0,7,2)
RealNum2.dllgexternalpadding(0,0)
DLGValue(RealNum2,ExpTime)
TagGroup group1 = DLGGroupltems(label1, RealNum1,label2, RealNum2)
group1.DLGTableLayout(6, 1, 0)
box2 items.DLGAddElement(group1)
group1.DLGExternalPadding(0,3)
```

```
//2nd row - Step numbers and length
TagGroup label3=DLGCreateLabel("4. Data points "+"x:")
label3.dlgexternalpadding(1,1)
RealNum3 = DLGCreateRealField(0, 7, 0)
RealNum3.dlgexternalpadding(0,0)
DLGValue(RealNum3,XPoints)
TagGroup label4 = DLGCreateLabel(" y:")
label4.dlgexternalpadding(1,1)
RealNum4 = DLGCreaterealField(0, 7,0)
RealNum4.dlgexternalpadding(0,0)
dlgvalue(RealNum4,YPoints)
TagGroup group2 = DLGGroupltems(label3, RealNum3, label4, RealNum4)
group2.DLGTableLayout(7, 1, 0)
box2_items.dlgaddelement(group2)
group2.dlgexternalpadding(0,3)
//3rd row - Step length
TagBase = ("SED_Calibration")
String Units
GetPersistentStringNote(TagBase + "CalUnits", units)
TagGroup label5=DLGCreateLabel("5. Step Length ("+units+") "+"x:")
label5.dlgexternalpadding(1,1)
ReaINum5 = DLGCreateRealField(0,7,2)
RealNum5.dlgexternalpadding(0,0)
dlgvalue(RealNum5,StepL_X)
TagGroup label6 = DLGCreateLabel(" y:")
label6.dlgexternalpadding(1,1)
RealNum6 = DLGCreaterealField(0,7,2)
RealNum6.dlgexternalpadding(0,0)
DLGValue(RealNum6,StepL Y)
TagGroup group3 = DLGGroupltems(label5, RealNum5, label6, RealNum6)
group3.DLGTableLayout(4, 1, 0);
box2_items.dlgaddelement(group3)
group3.dlgexternalpadding(0,3)
TagGroup label7 = DLGCreateLabel("7. Scanning tilt angle (degree):").dlgexternalpadding(1,1)
RealNum11 = DLGCreaterealField(0,7,3).dlgexternalpadding(0,0)
DLGValue(RealNum11,angleDegree)
box2_items.dlgaddelement(dlggroupitems(label7,RealNum11).dlgtablelayout(2,1,0),"","West")
//Scan options
Number defaultscan=0
ScanOption=DLGCreateCheckBox("Check this box to set an initial position",defaultscan)
TagGroup ScanLabel=DLGCreateLabel(" as the center of scanning area.
ScanLabel.DLGExternalPadding(1,1)
/*
Number defaultcheckscan=0
CheckScan=DLGCreateCheckBox("Check Scan",defaultcheckscan)
*/
/*
TagGroup ScanOptionsGroup=DLGGroupltems(ScanOption, CheckScan)
ScanOptionsGroup.DLGTableLayout(2,1,0)
box2_items.dlgaddelement(ScanOptionsGroup)
ScanOptionsGroup.dlgexternalpadding(0,3)
*/
TagGroup ScanOptionsGroup=DLGGroupltems(ScanOption, ScanLabel)
ScanOptionsGroup.DLGTableLayout(1,2,0)
box2_items.dlgaddelement(ScanOptionsGroup)
ScanOptionsGroup.dlgexternalpadding(0,3)
//Start button
TagGroup StartButton = DLGCreatePushButton(" Start ", "Start")
StartButton.dlgexternalpadding(5,5)
//Stop button
TagGroup StopButton = DLGCreatePushButton(" Stop ", "Stop").dlgexternalpadding(5,5)
Box2_Items.DIGAddElement(DLGGroupltems(StartButton,StopButton).DLGTableLayout(2,1,0),"Center","")
Box2.DLGExternalPadding(10, 0)
Return Box2
}
// Creates the buttons in Control Box 2
// Control Box 3 starts here
// Control Box 3 starts here
{
taggroup box3_items //Control box 3 is grouped by 'box3_items//
taggroup box3=dlgcreatebox(" Control Box2 ", box3_items) // Label of 'Control Box 2'
String tagBase = ("SED_Calibration")
String Units
GetPersistentStringNote(TagBase + "CalUnits", units)
TagGroup label7 = DLGCreateLabel("Currently Calibrated Units: "+units)
box3_items.dlgaddelement(label7)
```

```
    //Calibration button
    TagGroup CalButton = DLGCreatePushButton(" Calibration ", "f_Cal").DLGSide("Center")
    CalButton.dlgexternalpadding(72,5)
    box3_items.dlgaddelement(CalButton)
    Box3.DLGExternalPadding(10,0)
    return box3
}
taggroup Mylnfo()
            {
                taggroup MyInfo=dlgcreatelabel("Yu-Tsun Shao, MatSE, U of Illinois")
                    MyInfo.dlgexternalpadding(5,5)
                return MyInfo
    }
// Function to assemble the compenents of the dialog into the dialog frame
void DialogLibraryScanningED()
{
// Configure the positioning in the top right of the application window
TagGroup position;
position = DLGBuildPositionFromApplication()
position.TagGroupSetTagAsTagGroup( "Width", DLGBuildAutoSize() )
position.TagGroupSetTagAsTagGroup( "Height", DLGBuildAutoSize() )
position.TagGroupSetTagAsTagGroup( "X", DLGBuildRelativePosition( "Inside", 1) )
position.TagGroupSetTagAsTagGroup( "Y", DLGBuildRelativePosition( "Inside", -1.0 ) )
TagGroup dialog_items;
TagGroup dialog = DLGCreateDialog("SED", dialog_items).dlgposition(position);
// Call each button
Dialog_Items.DLGAddElement(ScriptInfo())
Dialog_Items.DLGAddElement(ExperimentalCondition())
dialog items.DLGAddElement(MainControlBox1())
dialog_items.DLGAddElement(MainControlBox1())
dialog_items.DLGAddElement(MainControlBox2())
dialog_items.DLGAddElement(MyInfo())
```

object dialog_thread $=$ alloc(AcquThread)
object dialog_frame $=$ alloc(DialogLibraryTestClass4).init(dialog,dialog_thread.ScriptObjectGetID()) dialog_frame.display("Scanning Electron NanoDiffraction (SEND)");

## \}

// Call the main function which creates and displays the dialog
DialogLibraryScanningED()

## A.2. Symmetry quantification

```
/*
QSymm (Symmetry quantification)
Yu-Tsun Shao, Ph.D. Candidate, U of llinois
This script is used to quantify the symmetry of CBED whole pattern using normalized cross-correlation.
Some of the legacy functions from Dr. Kyou-Hyun Kim are used.
Usage:
1. circle annotations in an array of lattice; switch on/off indexing by pressing "T"
2. adjust circle positions interactively with "0", "g", "h", and arrow keys
3. adjust circle radius with "r", and "R"
4. calculate cross-correlation coefficient for mirror with "m"; rotational symmetry by pressing "2", "3", "4", "6"
5. press "space bar" to stop adjusting
6. Export annotations compatible with "Symm Quant." script initiated by Dr. K.-H. Kim
*/
number nint(number x) {
    number isign = 1
    if ( }x<0\mathrm{ ) {
    isign =-1
    x=abs(x)
}
return isign*(x-mod}(x,1)
}
number IsInBox(number x, number y, number xmin, number xmax, number ymin, number ymax) {
    if ( }x>x\operatorname{xmin}&&x<xmax && y>ymin && y< ymax
    return 1
    else
    return 0
}
void deleteovals(image img) {
    number anns, ann, type, n, i
    anns = CountAnnotations(img)
    n=anns
    i=0
    while (n>0)
    {
    ann = GetNthAnnotationID(img, i)
        type = AnnotationType(img,ann)
        if ((type == 6) || (type == 5) || (type == 13) || (type == 29))
    {
        DeleteAnnotation(img, ann)
            }
    else
        i+=1
            }
}
void ROIInfo(number TopX, number TopY, number BotX, Number &CenX, Number &CenY, Number &Radius)
{
        Radius=abs(TopX-BotX)*0.5
        CenX=TopX+Radius
        CenY=TopY+Radius
}
number DegToRad(number Degree)
{
} Return (pi()/180)*Degree
number RadToAng(number Radian)
{
}
number slope(number aX, number aY, number bX, number bY)
{
```



```
            If(Slope<0)
            {
                                    Angle=RadToAng(atan(slope))+90
    Angle=-DegToRad(Angle)
    Ang
    return Angle
}
Image Mask(image Template, number Bot, number Top, number xSize, number ySize, number Correction)
{
Image Template:=Reallmage("", 4, xSize, ySize)
    Number Radius=abs((Bot-Top))*0.5
Template=tert(iradius<Radius*Correction, 1, 0)
return Template
}
Image Rotation(number ImgID, number TopY, number TopX, number BotY, number BotX, number rotation)
{
Image Temporary:=GetImageFromID(ImgID)[TopY, TopX, BotY, BotX]
Image TempR_A:=Rotate(Temporary, Rotation)
ScrapCopy(TempR_A)
Number Width, Height
Number Radius=abs(BotX-TopX)*0.5
Get2Dsize(TempR A, Width, Height)
/Width = TempR_A.ImageGetDimensionSize(0)
//Height = TempR_A.ImageGetDimensionSize(1)
Image TempR_B:=Reallmage("", 4, Width, Height)
ScrapPaste(TempR_B)
ScrapMerge(TempR_B)
return TempR_B
}
// Cross-correlation coefficient calculation
Number XCorr(Image ImgA, Image ImgB, Image Mask, Number Pixels)
Nu
Number Numer, Denom, NCC
Number ImgAMean=sum(ImgA)/pixels
Number ImgBMean=sum(ImgB)/pixels
Numer=sum(((lmgA-ImgAMean)*(ImgB-ImgBMean))*Mask)
Denom=sum(((lmgA-ImgAMean)*Mask)**2)*sum(((lmgB-ImgBMean)*Mask)**2)
Denom=sqrt(Denom)
NCC = Numer/Denom
If (Denom == 0)
{
                    NCC=0
}
return NCC
}
// Crop Image function
Image ImageCrop(Image ResultImage, Number ROIRadius, number CutOff)
{
Image Temp
    Number Width, Height, Coeff=0
    Get2DSize(Resultlmage, Width, Height)
    /Width = ResultImage.ImageGetDimensionSize(0)
    //Height = ResultImage.ImageGetDimensionSize(1)
    ScrapCopy(Resultlmage[Height*0.5-ROIRadius, Width*0.5-ROIRadius, Height*0.5+ROIRadius+Coeff, Width*0.5+ROIRadius+Coeff])
    Temp:=Reallmage("', 4, 2*ROIRadius+Coeff, 2*ROIRadius+Coeff)
    ScrapPaste(Temp)
    ScrapMerge(Temp)
    \mathrm{ return Temp}
}
// creates an array of oval annotations
void DrawLattice(image front, Number x0, Number y0, Number gx, Number gy, Number hx, Number hy, Number r0, Number N_hkl, Number TextLabel)
{
Number x, y, i, j, Nx, Ny, Mx, My, xx, yy, xsize, ysize
    GetSize(front,xsize,ysize)
    Number N_root
    N_root = SQRT(N_hkl)/2-mod(SQRT(N_hkl)/2,1)
    For(i=-N_root; i<N_root+1; i++)
    For
    For (j=-N_root; j<N_root+1; j++)
    {
        y=x0+i*gx+j*hx
        x=y0+i*gy+j*hy
        if (x<xsize || y<ysize)
    {
        front.CreateOvalAnnotation(x-r0,y-r0,x+r0,y+r0)
        if (TextLabel ==1)
        {
        CreateTextAnnotation(front, x, y-r0, "("+i+","+(-j)+")")//top, left: label the order
        }
    }
}
// function to copy all annotations into a clone image
void CopyAnnotations(Image img, Image clonefront)
```

```
{
    Number ncol, nrow, value=0
    ImageDisplay Disp1, Disp2
    Disp1 = Disp1 = img.ImageGetlmageDisplay(0)
    GetSize(img, ncol, nrow)
    number AnnotN = CountAnnotations(img)
    number
    for (i=0; i<AnnotN; i++)
    {
    Number AnnotID = GetNthAnnotationID(img,i)
    number AnnotType=AnnotationType(img, AnnotID)
    Number a,b,c,d
    If (AnnotType == 6)
    {
        GetAnnotationRect(img, AnnotID, a,b,c,d)
        CreateOvalAnnotation(clonefront, a, b, c, d)
    }
    Else If (AnnotType == 5
    {
        GetAnnotationRect(img, AnnotID, a,b,c,d)
        CreateBoxAnnotation(clonefront, a, b, c, d)
    }
}
// Gauss ladder function
number gauss(number x) {
    number isign = 1
    if (x<0) {
    isign = -1
    x = abs(x-1)
    }
    return isign*(x-mod(x,1))
}
//create circular ROI
ROI cs_CreateCircleROI(Number cx, Number cy, Number radius, Number points)
    {
    Number px, py, phi, count
    ROI proi = NewRoi()
    proi.ROISetName("CircleROI_r"+radius+"_p"+points) // name the ROI including creation paramters
    If (points<3) points = Trunc(2* Pi()*radius / abs(points))+1 // calculate number of vertices on circle
    points=max(points,3)
    For (count=0; count<points; count++) // calculate vertices on circle
    For
    phi = count * 2* Pi()/points //points defines how smooth the ROI is
    px = cx+radius*Cos(phi)
    py = cy+radius*Sin(phi)
    proi.ROIAddVertex(px,py)
}
pROI.ROISetlsClosed(1) // connect first with last vertex
return pROI
}
// A simple calculation to sort out the indexing (m,n) by computing the nearest integer
//(x,y)=((gx,hx),(gy,hy))X(m,n)
//(m,n)=A^-1*((hy,-hx),(gy,hy))X(disX,dis Y)
void GetCircleCoor(Number x0, Number y0, Number x, Number y, Number gx, Number gy, Number hx, Number hy, Number &m, Number &n)
{
Number disX=x-x0
Number dis Y=y-y0
Number detA=abs(gx*hy-gy*hx)
If ( }\operatorname{det}A==0
    If (detA==0)
    detA=1
}
m=(1/\operatorname{det}A)*(hy*disX-hx*disY)
n=(1/detA)*(gx*disY-gy*disX)
m=gauss(m+0.5)
n=gauss(n+0.5)
}
//Determine mirror and counterpart
void Reflection(Number m,Number n, Number mX, Number mY, Number &m_prime, Number &n_prime)
void Reflection(
If (m)
    m_prime=n
    n_prime=m
    }
Else if (mY==0)
    Else
    m_prime=m
    n_prime=-n
    }
    Else if (mX==0)
    {
    m_prime=-m
    n_prime=n
}
Else if (mX==-mY)
    {
    m_prime=-n
```

```
    n_prime=-m
}
    {
    number angle
    angle = abs(abs(slope(m,n,0,0))-abs(slope(mX,mY,0,0)))
    m_prime =abs(m)*}\operatorname{cos}(\mp@subsup{2}{}{*}\mathrm{ angle) +abs(n)*}\operatorname{sin}(\mp@subsup{2}{}{*}\mathrm{ angle)
    n_prime =abs(m)**sin(2*angle)-abs(n)*}\operatorname{cos}(\mp@subsup{2}{}{*}\mathrm{ angle)
    m_prime = gauss(m_prime+0.5
    n_prime = gauss(n_prime+0.5)
}
// deletes the annotation outside of image
void DeleteOutBound(Image front)
{
Number i,n, N markers=CountAnnotations(front)
number xsize = front.ImageGetDimensionSize(0)
number ysize = front.ImageGetDimensionSize(1)
n=N_markers
i=0
while (n>0)
    {
    number AnnotID=GetNthAnnotationID(front,i)
    number A_AnnTy, A_AnnTx, A_AnnBy, A AnnBx
    GetAnnotationRect(front, AnnotID, A AnnTy, A AnnnTx, A\_AnnBy, A\_AnnBx)
    number A CenterX, A CenterY, RadiusA
    ROIInfo(A_AnnTx, A_AnnTy, A_AnnBx, A_CenterX, A_CenterY, RadiusA)
    If ((A_CenterX-RadiusA)<0)
        DeleteAnnotation(front,AnnotID)
    }
    Else if ((A_CenterY-RadiusA)<0)
    {
    DeleteAnnotation(front,AnnotID)
    }
    Else if ((A_CenterY+RadiusA)>ysize)
    DeleteAnnotation(front,AnnotID)
    Else if ((A_CenterX+RadiusA)>xsize)
    {
    \}
    {
    i+=1
    }
    n-=1
}
void All_ROltoMask(image front, image SavedID, image mask)
{
ImageDisplay disp
disp = front.ImageGetImageDisplay(0)
number N_markers=CountAnnotations(front)
number i, xsize, ysize
front.GetSize(xsize, ysize)
mask = 0
ROI tempROI
For (i=0; i<<(N_markers); i++)
{
    //function GetNthAnnotationID
    number AnnotID=GetPixel(SavedID,0,i+0)
    number AnnotType=AnnotationType(front, AnnotID)
    If (AnnotType!=2)
    {
    number A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx
    GetAnnotationRect(front, AnnotID, A_AnnTy, A_AnnTx, A_AnnBy, A\_AnnBx)
    number A_CenterX, A_CenterY, RadiusA
    ROIInfo(A AnnTx, A A nnnTy, A AnnBx, A CenterX, A CenterY, RadiusA)
    tempROI = cs_CreateCircleROI(A_CenterX,A_CenterY,RadiusA,-3)
    tempROI = CS_CreateCircleROI(A_C
    disp.ImageDisplayAddRoi(tempROI)
    //Result("\n ID= "+AIDSID+"
    l/put ROI into mask image 
    /delete temporary ROI
    number tempROI_ID=GetNthAnnotationID(front,0)
    DeleteAnnotation(front, tempROI_ID)
}
}
// An image that registers Annotation-ID for mirror counterparts
void MatchReflectionCircle(image front,image SavedID, image MirrorID, Number mX, Number mY, Number &Useful, Number Print)
{
number i, j, n,row, N_markers
    i=0
    row=0
    N markers=CountAnnotations(front)
    For (i=0; i<(N_markers); i++) //N_markers = 0 then there's bug
{
```

```
number AnnotID = SavedID.GetPixel(0,i)
number AnnotType=AnnotationType(front, AnnotID)
number A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx
GetAnnotationRect(front, AnnotID, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx)
number A_CenterX, A_CenterY, RadiusA
ROlInfo(A_AnnTx, A_A_nnTy, A_AnnBx, A_CenterX, A_CenterY, RadiusA)
number m = SavedID.GetPixel(\overline{6},i)
number n = SavedID.GetPixel(7,i)
number m_prime, n_prime
Reflection(m,n,mX,mY,m_prime,n_prime)
For (j=0; j<(N_markers); j++)
{
//search for second set of circles
    number AnnotID2 = SavedID.GetPixel(0,j)
    number AnnotType2=AnnotationType(front, AnnotID2)
    number A AnnTy2, A AnnTx2, A AnnBy2, A AnnBx2
    GetAnnotationRect(front, AnnotID2, A_AnnTy2, A_AnnTx2, A_AnnBy2, A_AnnBx2)
    GetAnnotationRect(front, AnnotID2, A_AnnTy2
    number A_CenterX2, A_CenterY2, RadiusA2 
    ROlInfo(A_AnnTx2, A_AnnTy2, A_A
    number n2 = SavedID.GetPixel(7,j)
    /this causes problem if no counterpart in image i.e., (m2=n2=0, but not BF disc)
    //delete un-matched circles
    If ((m!=0) && (n!=0) && (m_prime==0) && (n_prime==0))
    If ((
    DeleteAnnotation(front,AnnotID)
    }
    Else If ((m2==m_prime) && (n2==n_prime) && (m2!=100000))
    {
    MirrorID.SetPixel(0,row+0,AnnotID)
    MirrorID.SetPixel(1,row+0,A_CenterX)
    MirrorID.SetPixel(2,row+0,A_CenterY)
    MirrorID.SetPixel(3,row+0,m)
    MirrorID.SetPixel(4,row+0,n)
    MirrorID.SetPixel(5,row+0,AnnotID2)
    MirrorID.SetPixel(6,row+0,A_CenterX2)
    MirrorID.SetPixel(6,row+0,A_CenterX2)
    MirrorID.SetPixel(7,row+0,A_Cen
    MirrorID.SetPixel(9,row+0,n2)
    row+=1
    If (Print==1)
    {
    result("\n #="+(row)+"(m,n)=("+m+","+n+"),(m',n')=("+m_prime+","+n_prime+"),(m",n")=("+m2+","+n2+")")
    }
}
}}\mathrm{ Useful=row-1
If (Print==1)
{
result("\n Total of "+(Useful)+" discs used for mirror quantification.")
}
}
//An image that registers Annotation-ID for convenience of tracing
void AnnotIDSave(image front, image SaveID, Number x0, Number y0, Number gx, Number gy, Number hx, Number hy, Number Print)
    // (image with ROI, image to be saved)
    {
number i, N_square, N_circle, N_markers=CountAnnotations(front)
For (i=0; i<(N_markers); i++)
    {
    //function GetNthAnnotationID
    number AnnotID=GetNthAnnotationID(front,i)
    //function AnnotationType
    //5=box; 6=oval
    number AnnotType=AnnotationType(front, AnnotID)
        /*
    If (AnnotType == 5)
    {
    AnnotType = 5
}
    */
number A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx
GetAnnotationRect(front, AnnotID, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx)
number A_CenterX, A_CenterY, RadiusA
ROIInfo(A_AnnTx, A_AnnTy, A_AnnBx, A_CenterX, A_CenterY, RadiusA)
number m, n
GetCircleCoor(x0, y0, A_CenterX, A_CenterY, gx, gy, hx, hy, m, n)
//now save all values on image SaveID
SaveID.SetPixel(0, i+0, AnnotID) //for further Annot ID read-in
SaveID.SetPixel(0, i+0, AnnotID) //for further Annot ID read-in 
SaveID.SetPixel(1, i+0, AnnotType) //for further Annot Type read
SaveID.SetPixel(2, i+0, A_CenterX)
SaveID.SetPixel(3, i+0, A_CenterY) //for further Annot y read-in
SaveID.SetPixel(4, i+0, RadiusA) //for further Annot radius read-i
    If (AnnotType==6)
    {
    SaveID.SetPixel(6, i+0, m) //for further Annot radius read-in
    SaveID.SetPixel(7, i+0, n) //for further Annot radius read-in
    }
```

```
    {
    m=100000
    n=100000
    SaveID.SetPixel(6, i+0, m) //for further Annot radius read-in
    SaveID.SetPixel(7, i+0, n) //for further Annot radius read-in
    |f
    If (Print==1)
    Result("In #"+(i+1)+" ID= "+AnnotID+",Annotation Type= "+AnnotType+", (x,y)= ("+A_CenterX+","+A_CenterY+"), radius= "+RadiusA+", (m,n)= ("+m+","+n+")")
    }
}
// Re-generate in the mirror order
//on the mirror plane: exclude (0,0);
//others: generate 2 sets
void Regenerate(Image front, Image MirrorID, Number Useful, Number RadiusA, Number Box)
{
    DeleteOvals(front)
    mageDisplay disp
    disp = front.ImageGetImageDisplay(0)
    Image Panel:=Reallmage("Info", 4, 15, 200)
    Number TempID, TempID2, TempID3
    Number i, j, k, A CenterX, A CenterY, A CenterX2, A CenterY2, m,n,m2,n2, m3,n3
    Number B_CenterX, B_CenterY, B_CenterX2, B_CenterY2
    Useful+=1
    j=0
    Number BoxCount=0
ROI TempROI //for BF disc
//Box for mirror plane
For (i=0; i<Useful; i++)
{
    A_CenterX=GetPixel(MirrorID, 1,i+0)
    A CenterY=GetPixel(MirrorID, 2,i+0)
    A CenterX2=GetPixel(MirrorID,6,i+0)
    A_CenterX2=GetPixel(MirrorID, 6,i+0)
    A_CenterY2=GetPixel(MirrorID, 7,i+0)
    m=GetPixel(MirrorID,3,i+0)
    n=GetPixel(MirrorID,4,i+0)
    m2=GetPixel(MirrorlD,8,i+0)
    n2=GetPixel(MirrorID,9,i+0)
    Panel.SetPixel(0,i+0,m)
    Panel.SetPixel(1,i+0,n)
    Panel.SetPixel(2,i+0,m2)
    Panel.SetPixel(3,i+0,n2)
    Panel.SetPixel(3,i+0,n2))
    If (Box==1)
    {
        CreateBoxAnnotation(Front, (A_CenterY-RadiusA), (A_CenterX-RadiusA), (A_CenterY+RadiusA), (A_CenterX+RadiusA))
    }
    Els
    CreateOvalAnnotation(Front, (A_CenterY-RadiusA), (A_CenterX-RadiusA), (A_CenterY+RadiusA), (A_CenterX+RadiusA))
    }
        BoxCount++
    }
    If ((m==0) && ( }n==0)&& (Box==1)
    {
    TempID=GetNthAnnotationID(front,0)
    DeleteAnnotation(front, TempID)
    BoxCount-=1
    tempROI = cs_CreateCircleROI(A_CenterX,A_CenterY,RadiusA,-3)
    disp.ImageDisplayAddRoi(tempRŌI)
}
Result("In # of Boxes="+BoxCount)
If (mod(BoxCount,2)==1)
{
TempID=GetNthAnnotationID(front,0)
    DeleteAnnotation(front, TempID)
}
//Circle for off-mirror plane
For (j=0; j<Useful; j++)
{
B_CenterX=GetPixel(MirrorID, 1,j+0)
B_CenterY=GetPixel(MirrorID, 2,j+0)
B_CenterX2=GetPixel(MirrorID, 6,j+0)
B_CenterY2=GetPixel(MirrorID, 7,j+0)
m=GetPixel(MirrorlD,3,j+0)
n=GetPixel(MirrorlD,4,j+0)
n=GetPixel(Mirrorr,4,,+0)
m2=GetPixel(MirrorlD,8,j+0)
n2=GetPixel(MirrorlD,9,j+0
Panel.SetPixel(0,j+0,m)
Panel.SetPixel(1,j+0,n)
Manel.SetPixel(2,j+0,m2)
Manel.SetPixel(3,j+0,n2)
If ((m!=m2) || (n!=n2))
{
    CreateOvalAnnotation(Front, (B CenterY-RadiusA), (B CenterX-RadiusA), (B CenterY+RadiusA), (B CenterX+RadiusA))
    CreateOvalAnnotation(Front, (B_CenterY2-RadiusA), (B_CenterX2-RadiusA), (B_CenterY2+RadiusA), (B_CenterX2+RadiusA))
}
For (k=0; k<j; k++)
```

```
{
    m3=Panel.GetPixel(2,k+0)
    n3=Panel.GetPixel(3,k+0)
    If ((m==m3) && (n==n3))
    {
        TempID2=GetNthAnnotationID(front,0)
        TempID3=GetNthAnnotationID(front,1)
        DeleteAnnotation(front,TempID2)
        DeleteAnnotation(front,TempID3)
    }
}
// Calculate weighted cross-correlation of Mirror
Number Gamma(Image front, Number Delete)
{
//Get image info
ImageDisplay ImgDisp = front.ImageGetImageDisplay(0
Number xsize = front.ImageGetDimensionSize(0)
Number ysize = front.ImageGetDimensionSize(1)
//Temporary scrap image for rotate , mask & crosscorrelate
mage TempScrap:=Reallmage("Temp Scrap", 4, xsize, ysize)
TempScrap=ImgDisp.ImageDisplayGetDisplayedImage()
TempScrap=ImgDisp.ImageDisplayGetDispl
// Register IDs
// Assign annotations' IDs to AnnotA and AnnotB
Image AnnotIDSave:=Reallmage("Pair Info", 4, 10, 200)
Image AnnotIDSave2:=Reallmage("Mirror Plane Info", 4, 10, 200)
Number SetsCount=0
Number Sets=1
Number SetTemp
Number Row=0, Row2=0, i=0, ix=0
Number NoOfAnnot=CountAnnotations(front), TempAnnotID, TypeOFAnnot
Number AnnotA,A AnnTy,A AnnTx,A AnnBy,A AnnBx,A CenterX,A CenterY,RadiusA
Number AnnotA,A_AnnTy,A_AnnTX,A_AnnBy,A_AnnBX,A_CenterX,A_CenterY,RadiusA
While (i<NoOfAnnot)
{
TempAnnotID=GetNthAnnotationID(Front, i)
    TypeOFAnnot=AnnotationType(Front, TempAnnotID)
    TypeOFAnnot=AnnotationType(Front,
    |f(Ty
            If(TypeOfAnnot==6)
            { AnnotIDSave.SetPixel(0, row, TempAnnotID)
                row++
    lf(TypeOfAnnot==5)
    { AnnotIDSave2.SetPixel(0, row2, TempAnnotID)
                row2++
                ix++
            } i++
            Sets++
        }
            Else
            {
            i++
Sets=(Sets-1)/2+0.5*ix
Result("\n>"+Sets+" set(s) selected")
Result("\l>>"+ix+" discs are placed on the mirror plane")
//Some values for calculations
number CrossCorrelationCoeff=0
number Totallntensity=0
number Temp CrossCorrelation=0
Number SetsCountB=0
Number RotAngleTemp, RotAngle
Number nxx=0, nyy=0
Result("lnln X-corr r-Factor Intensity Sum")
While (SetsCount<Sets) //*****************************************************************************
{
// Allocate annotations' IDs and get annotaions' info
If (SetsCount<(Sets-ix))
    {
        AnnotA=AnnotIDSave.GetPixel(0, SetsCount*2+1)
        AnnotB=AnnotIDSave.GetPixel(0, SetsCount*2)
    GetAnnotationRect(Front, AnnotA, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx)
    GetAnnotationRect(Front, AnnotB, B_AnnTy, B_AnnTx, B_AnnBy, B_AnnBx)
    //Calculate Radius & Rotation Angle
    ROIInfo(A AnnTx, A AnnTy, A AnnBx, A CenterX, A CenterY, RadiusA)
    ROIInfo(B_AnnTx, B_AnnTy, B_AnnBx, B_CenterX, B_CenterY, RadiusB)
    If (RadiusA!=Radius隹)
```

```
    {
            B_AnnTx=B_centerX-RadiusA
            B_AnnTy=B_centery-RadiusA
            B_AnnBx=B_centerX+RadiusA
            B_AnnBy=B_centery+RadiusA
    }
    //CreateTextAnnotation(Front, A CenterY, A CenterX, "A"+(SetsCount+1))
    //CreateTextAnnotation(Front, B_CenterY, B_CenterX, "B"+(SetsCount+1))
    RotAngle=slope(A_CenterX, A_CenterY, B_CenterX, B_CenterY)
            /*
                Result ("\n\nRotation angle="+Rad2Ang(RotAngle)+"\n")
            */
    |
    If (SetsCount>=(Sets-ix))
    {
    If (nxx<1)//nx used to calculate rotatiion angle of disc lied on the mirror axis for one time
    {
                    AnnotA=AnnotIDSave2.GetPixel(0, SetsCountB+1)
                    AnnotB=AnnotIDSave2.GetPixel(0, SetsCountB)
                    GetAnnotationRect(Front, AnnotA, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx)
                    GetAnnotationRect(Front, AnnotB, B_AnnTy, B_AnnTx, B_AnnBy, B_AnnBx)
                    ROlInfo(A_AnnTx, A_AnnTy, A_AnnBx, A_CenterX, A_CenterY, RadiusA)
                    ROllnfo(B_AnnTx, B_AnnTy, B_AnnBx, B_CenterX, B_CenterY, RadiusB)
                    If (RadiusA!=RadiusB)
                    If (R
                B AnnTx=B centerX-RadiusA
                    B_AnnTy=B centery-RadiusA
                        B_AnnBx=B_centerX+RadiusA
                B_AnnBy=B_centery+RadiusA
            }
            //CreateTextAnnotation(Front, A_CenterY, A_CenterX, "m"+(SetsCountB+2))
            //CreateTextAnnotation(Front, B CenterY, B CenterX, "m"+(SetsCountB+1))
            RotAngleTemp=slope(A_CenterX, A_CenterY, B_CenterX, B_CenterY) //Calculate rotation angle first
            SetsCountB=SetsCountB+2
    }
    Image Temp_A, Temp_B, TempMask
    Image TempAAA, TempBBB
    Number xWidth, yHeight
    Mirror symmetry
    If (SetsCount>=(Sets-ix))
    {
        TempAAA=Rotation(ImageID, A AnnTy, A AnnTx, A AnnBy, A AnnBx, RotAngleTemp)
        TempBBB=Rotation(ImageID, B_AnnTy, B_AnnTx, B_AnnBy, B_AnnBx, RotAngleTemp)
        If (nyy==0)
            {
        Temp_A=TempBBB //I switched the order of TempAAA & TempBBB s.t. the indices in both "image"&"result" the same
        Temp_B=TempBBB
        nXX++
        nyy++
    }
    Else lf (nyy==1)
    {
        Temp A=TempAAA
        Temp_B=TempAAA
        nyy=0
            nxx=0
        }
    Else
    {
                Result("\n"+A_AnnTy+" "+A_AnnTx+""+A_AnnBy+" "+A_AnnBx)
    */
    Temp_A=Rotation(ImageID, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx, RotAngle)
    Temp_B=Rotation(ImageID, B_AnnTy, B_AnnTx, B_AnnBy, B_AnnBx, RotAngle)
    }
//Create a mask
    Get2Dsize(Temp_A, xWidth, yHeight)
    //xWidth = Temp_A.ImageGetDimensionSize(0)
    //yHeight = Temp A.ImageGetDimensionSize(1)
    TempMask=Mask(TempMask, A_AnnBx, A_AnnTx, xWidth, yHeight, 0.9)
    Number NoOfPixels=sum(TempMask) //Calculate a number of pixes inside the created mask (to be used for quantification)
    // Generate Mask Image -> generate mirror image -> apply
    //(4) Mask x Image
    Temp_A=Temp_A*TempMask
Image Temp_AA, Temp_BB, FlipTemp
If (SetsCount>=(Sets-ix))
{
    Temp_A.SetName("aligned m_Set "+(SetsCount+1))
    FlipVertical(Temp_B)
    Temp_B=Temp_B*TempMask
    Temp_B.SetName("mirror applied_Set "+(SetsCount+1))
}
Else
Temp_A.SetName("aligned A_Set "+(SetsCount+1))
    FlipHorizontal(Temp_B)
/*
```

```
    Temp_A.Showlmage()
        emp_B.Showlmage()
        TempMask.ShowImage()
        */
            Temp_B=Temp_B*TempMask
            Temp B.SetName("symm applied B set "+(SetsCount+1))
            Temp BB=Temp B
            FlipHorizontal(Temp_BB)
            Temp_BB.SetName("aligned B_set "+(SetsCount+1))
            }
            // Cross-correlation coefficient
            Number Temp_XCross=XCorr(Temp_A, Temp_B, TempMask, NoOfPixels
            /Sum of intensity of discs, for weighted gamma use
            Number IntensityA, IntensityB, IntensityAB
            IntensityA=sum(Temp A*TempMask)
            IntensityB=sum(Temp_B*TempMask)
            IntensityB=Sum(Temp_B*TempMask)
            IntensityAB=(IntensityA
            Temp_A.Deletelmage()
            Temp_B.Delete
            If (SetsCount<(Sets-ix))
            {
            Result("\nSet "+(SetsCount+1)+" "+" "+Temp_XCross+" "+IntensityAB)
            }
    { Result("\nSet "+"m"+(SetsCount-(Sets-ix)+1)+" "+Temp_XCross+" "+IntensityAB)
            } 
            CrossCorrelationCoeff=CrossCorrelationCoeff+Temp_XCross
            //numerator
            Temp_CrossCorrelation=Temp_CrossCorrelation+Temp_XCross*IntensityAB
            //denominator
            TotalIntensity=TotalIntensity+IntensityAB
            SetsCount++
    }
    Result("\n\nWeight "+Temp_CrossCorrelation/TotalIntensity)
    If (Delete==1)
    {
    DeleteOvals(front)
}
return RotAngleTemp
}
//Rotational Symmetry
void TwoFold(Number m, Number n, Number &m_prime, Number &n_prime)
{
    m_prime=-m
    n_prime=-n
}
void FourFold(Number m, Number n, Number &m2, Number &n2, Number &m3, Number &n3, Number &m4, Number &n4)
{
    m2=-n
    n2=m
    m3=-m
    n3=-n
    m4=n
    n4=-m
}
void SixFold(Number angle, Number m, Number n, Number &m2, Number &n2, Number &m3, Number &n3, Number &m4, Number &n4, Number &m5, Number
&n5, Number &m6, Number &n6)
&n5
//depends on how one drew the g, h lines
    If (angle==1) // acute angle
    { m3=n
    n3=-(m+n)
    }
    Else if (angle==0) // blunt angle
    {
        m3=n-m
        n3=-m
    }
    m2=m+m3
    n2=n+n3
    m4=-m
    n4=-n
    m5=-m2
    m5=-m2
    n5=-n2
    m6=-m3
    n6=-n3
}
void MatchTwoFold(image front,image SavedID, image TwoFoldID, Number &Useful, Number Print)
{
number i, j, row, N_markers=CountAnnotations(front)
    i=0
j=0
For (i=0; i<N_markers; i++)
```

```
{
    number AnnotID = SavedID.GetPixel(0,i)
    number AnnotType=AnnotationType(front, AnnotID)
    number A_AnnTy, A AnnTx, A AnnBy, A AnnBx
    GetAnnotationRect(front, AnnotID, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx)
    number A CenterX, A CenterY, RadiusA
    ROIInfo(A_AnnTx, A_A_nnTy, A_AnnBx, A_CenterX, A_CenterY, RadiusA)
    numberm = SavedID.GetPixel(6,i)
    number n = SavedID.GetPixel(7,i)
    number m_prime, n_prime
    TwoFold(m,n,m_prime,n_prime)
    For (j=0; j<(N_markers); j++)
For
    //search for second set of circles
    number AnnotID2 = SavedID.GetPixel(0.j)
    number AnnotType2=AnnotationType(front, AnnotID2)
    number A AnnTy2, A AnnTx2, A AnnBy2, A AnnBx2
    GetAnnotationRect(front, AnnotID2, A_AnnTy2, A_AnnTx2, A_AnnBy2, A_AnnBx2)
    number A_CenterX2, A_CenterY2, RadiusA2
    ROllnfo(A_AnnTx2, A_A_-_nnTy2, A_AnnBx2, A_CenterX2, A_CenterY2, RadiusA2)
    number m2 = SavedID.GetPixel(6,j)
    number n2 = SavedID.GetPixel(7,j)
    If ((m!=0) && (n!=0) && (m_prime==0) && (n_prime==0))
    {
        DeleteAnnotation(front,AnnotID)
    Else lf ((m2==m_prime) && (n2==n_prime) && (m2!=100000))
    {
        TwoFoldID.SetPixel(0,row+0,AnnotID)
        TwoFoldID.SetPixel(1,row+0,A_CenterX)
        TwoFoldID.SetPixel(2,row+0,A_CenterY)
        TwoFoldID.SetPixel(3,row+0,m)
        TwoFoldID.SetPixel(4,row+0,n)
        TwoFoldID.SetPixel(5,row+0,AnnotID2)
        TwoFoldID.SetPixel(6,row+0,A_CenterX2)
        TwoFoldID.SetPixel(7,row+0,A_CenterY2)
        TwoFoldID.SetPixel(8,row +0,m2)
        TwoFoldID.SetPixel(9,row+0,n2)
        row+=1
        If (Print==1)
        {
        result("ln #="+(row)+"(m,n)=("+m+","+n+"),(m',n')=("+m_prime+","+n_prime+"),(m",n")=("+m2+","+n2+")")
    }
}
}seful=row
If (Print==1)
{
result("In Total of "+(Useful)+" discs used for 2-fold quantification.")
}
}
//Match the 4R circles
void FourR(image front, image SavedID, image FourFoldID, Number &Useful, Number Print)
{
image Temp:=Reallmage("Certified 4R", 4, 15, 2000)
number i, j, N_markers=CountAnnotations(front)
i=0
j=0
number set =0
//1st loop: use image Temp to record all the circles
// who has their full 4R counterparts
For (i=0; i<N_markers; i++)
{
number AnnotID = SavedID.GetPixel(0,i)
number AnnotType=AnnotationType(front, AnnotID)
number A_CenterX, A_CenterY, RadiusA
A_CenterX = SavedID.GetPixel(2,i)
A_CenterY = SavedID.GetPixel(3,i)
number m = SavedID.GetPixel(6,i)
number n=SavedID.GetPixel(7,i)
number m2, n2, m3, n3, m4, n4,m_dummy, n_dummy
number m2, n2, m3, n3,m4,n4,m_d
number count=0
    For (j=0; j<(N_markers); j++)
{
//search for following sets of circles
    number AnnotID2 = SavedID.GetPixel(0,j)
    number AnnotType2=AnnotationType(front, AnnotID2)
    number A CenterX2, A CenterY2, RadiusA2
    A CenterX\ = SavedID.GetPixel(2,j)
    A_CenterX2 = SavedID.GetPixel(2,j)
    A_CenterY2 = SavedID.GetPixel(3,j)
    number m_dummy = SavedID.GetPixel(6,j)
    If ((m!=0) && (n!=0) && (m2==0) && ( }n2==0) &&(m3==0) && (n3==0) && (m4==0) && (n4==0)
    {
    DeleteAnnotation(front,AnnotID)
    }lse if (m_dummy == m2 && n_dummy == n2)
    Els
    count+=1
```

```
    }lse if (m_dummy == m3 && n_dummy == n3)
    {
        count+=1
    }
    Else if (m_dummy == m4 && n_dummy == n4)
    {
        count+=1
    }
    If (count==3)
    {
        Temp.SetPixel(0,0+set,AnnotID)
        Temp.SetPixel(1,0+set,m)
    Temp.SetPixel(2,0+set,n)
    Temp.SetPixel(3,0+set,A CenterX)
    Temp.SetPixel(4,0+set,A_CenterY)
    set++
}
// 2nd-loop
// read image Temp and writes in the image FourFoldID
// with (m,n) & (m2,n2) s.t. it'll be easier to re-generate
For (i=0; i<set; i++)
For (
number m2,n2,m3,n3,m4,n4
    number AnnotID = Temp.GetPixel(0,i)
    number m = Temp.GetPixel(1,i)
    number n = Temp.GetPixel(2,i)
    number A_CenterX = Temp.GetPixel(3,i)
    number A CenterY = Temp.GetPixel(4,i)
    FourFold(m,n,m2,n2,m3,n3,m4,n4)
    For (j=0; j<set; j++)
    {
    number AnnotID2 = Temp.GetPixel(0,j)
    numberm_pr = Temp.GetPixel(1,j)
    number n_pr = Temp.GetPixel(2,j)
    number A_CenterX2 = Temp.GetPixel(3,j)
    number A_CenterY2 = Temp.GetPixel(4,j)
    If (m_pr == m4 && n_pr == n4)
    {
        FourFoldID.SetPixel(0,j+0,AnnotID)
    FourFoldID.SetPixel(1,j+0,m)
    FourFoldID.SetPixel(2,j+0,n)
    FourFoldID.SetPixel(3,j+0,A_CenterX)
    FourFoldID.SetPixel(4,j+0,A_CenterY)
    FourFoldID.SetPixel(5,j+0,AnnotID2)
    FourFoldID.SetPixel(6,j+0,m_pr)
    FourFoldID.SetPixel(7,j+0,n_pr)
    FourFoldID.SetPixel(8,j+0,A_CenterX2)
    FourFoldID.SetPixel(9,j+0,A_CenterY2)
    If (print==1)
    If (p
    result("\n 2 neighboring 4R-certified circles: (m,n)=("+m+","+n+"),(m2,n2)=("+m_pr+","+n_pr+")")
    }
}
Useful = se
}
//Match the 6R circles up to 20 circles (2016. April. 28th)
void SixR(image front, image SavedID, image SixFoldID, Number angle, Number Select, Number &Useful, Number Print)
void SixR(image front, image SavedID, image SixFoldID
image Temp:=Reallmage("Certified 6R", 4, 15, 2000)
i=0
j=0
number set = 0
//1st loop: use image Temp to record all the circles
// who has their full 6R counterparts
For (i=0; i<N_markers; i++)
For (i
number AnnotID = SavedID.GetPixel(0,i)
    number AnnotType=AnnotationType(front, AnnotID)
    number A_CenterX, A_CenterY, RadiusA
    A_CenterX = SavedID.GetPixel(2,i)
    A_CenterY = SavedID.GetPixel(3,i)
    number m = SavedID.GetPixel(6,i)
    number n = SavedID.GetPixel(7,i)
    number m2, n2,m3, n3,m4, n4,m5, n5, m6, n6, m_dummy, n_dummy
    Number m2, n2, m3, n3, m4, n4, m5, n5,m6, n6,m_dum
    SixFold(angle, m
    number count=0 
    Fo
    //search for following sets of circles
    number AnnotID2 = SavedID.GetPixel(0,j)
    number AnnotType2=AnnotationType(front, AnnotID2)
    number A CenterX2, A_CenterY2, RadiusA2
    A Center\}2=\mathrm{ SavedID.GetPixel(2,j)
    A-CenterX2 = SavedID.GetPixel(2,j)
    A_CenterY2 = SavedID.GetPixel(3,j)
```

```
number n_dummy = SavedID.GetPixel(7,j)
    If ((m!=0) && (n!=0) && (m2==0) && (n2==0) && (m3==0) && (n3==0) && (m4==0) && (n4==0) && (m5==0) && (n5==0) && (m6==0) && (n6==0))
    DeleteAnnotation(front,AnnotID)
    }
    Else if (m_dummy == m2 && n_dummy == n2)
    {
    ount+=1
    }
    Else if (m_dummy == m3 && n_dummy == n3)
    {
    count+=1
    }
    Else if (m_dummy == m4 && n_dummy == n4)
    {count+=1
    }
    Else if (m_dummy == m5 && n_dummy == n5)
    {
    count+=1
    }lse if (m_dummy == m6 && n_dummy == n6)
    {
    count+=1
    }
lf (count==5)
{
    Temp.SetPixel(0,0+set,AnnotID)
    Temp.SetPixel(1,0+set,m)
    Temp.SetPixel(2,0+set,n)
    Temp.SetPixel(3,0+set,A_CenterX)
    Temp.SetPixel(4,0+set,A_CenterY)
    set++
}
For (i=0; i<set; i++)
%
number AnnotID = Temp.GetPixel(0,i)
number m = Temp.GetPixel(1,i)
number n = Temp.GetPixel(2,i)
number A_CenterX = Temp.GetPixel(3,i)
number A CenterY = Temp GetPixel(4,i)
Number A_-CenterY = Temp.GetPixel(4,i)
Number m_select, n_select,m2,n2,m3,n3,m4,n4,m5,n5
SixFold(angle,m,n,m2,n2,m3,n3,m4,n4,m
// This is where
{
If (angle==1)
    { (a
    m_select=m2
    n_select=n2
}
Else if (angle==0)
    {
        m_select=m6
        n_select=n6
    }
Else if (Select==3) //3R
{
If (angle==1)
{
    m_select=m3
    n_select=n3
}
Else if (angle==0)
{
    m_select=m5
    n_select=n5
}
}
//result("\n m_select,n_select=("+m_select+","+n_select+")")
For (j=0; j<set; j++)
{
number AnnotID2 = Temp.GetPixel(0,j)
numberm_pr = Temp.GetPixel(1,j)
number n_pr = Temp.GetPixel(2,j)
number A_CenterX2 = Temp.GetPixel(3,j)
number A_CenterY2 = Temp.GetPixel(4,j)
If (m_pr == m_select && n_pr == n_select)
{
SixFoldID.SetPixel(0,j+0,AnnotID)
    SixFoldID.SetPixel(1,j+0,m)
    SixFoldID.SetPixel(2,j+0,n)
    SixFoldID.SetPixel(3,j+0,A_CenterX)
    SixFoldID.SetPixel(4,j+0,A_CenterY)
    SixFoldID.SetPixel(5,j+0,AnnotID2)
    SixFoldID.SetPixel(6,j+0,m_pr)
    SixFoldID.SetPixel(6,j+0,m_pr)
    SixFoldID.SetPixel(8,j+0,A_CenterX2)
    SixFoldID.SetPixel(8,j+0,A_CenterX2)
```

```
        If (print==1)
        result("\n 2 neighboring 6R-certified circles: (m,n)=("+m+","+n+"),(m2,n2)=("+m_pr+","+n_pr+")")
    }
    }}
    }seful = set
}
void TwoFoldRegenerate(Image front, Image TwoFoldID, Number Useful, Number RadiusA)
{
DeleteOvals(front)
ImageDisplay disp
disp = front.ImageGetImageDisplay(0)
Image Panel:=Reallmage("Info", 4, 15, 2000)
Image Panel:=Reallmage("Info", 4, 15
Number TempID, TempID2, TempID3 
Number i, j, k, A_CenterX, A_CenterY, A_CenterX2, A_Cent
j=0
//delete BF disc; not useful for QSymm
For (i=0; i<Useful; i++)
{
    A CenterX=GetPixel(TwoFoldID, 1,i+0)
    A CenterY=GetPixel(TwoFoldID, 2,i+0)
    A_CenterX2=GetPixel(TwoFoldID, 6,i+0)
    A_CenterY2=GetPixel(TwoFoldID, 7,i+0)
    m=GetPixel(TwoFoldID,3,i+0)
    n=GetPixel(TwoFoldID,4,i+0)
    m2=GetPixel(TwoFoldID,8,i+0)
    n2=GetPixel(TwoFoldID,9,i+0)
    TempID=GetNthAnnotationID(front,0)
    number TempType=AnnotationType(front,TempID)
    If (TempType == 6)
    If (TempType == 6)
    If ((m==0) && ( }\textrm{n}==0)
    {
        DeleteAnnotation(front, TempID)
    }
}
//other than BF disc
For (j=0; j<Useful; j++)
{
    B_CenterX=GetPixel(TwoFoldID, 1,j+0)
    B_CenterY=GetPixel(TwoFoldID, 2,j+0)
    B_CenterX2=GetPixel(TwoFoldID, 6,j+0)
    B_CenterY2=GetPixel(TwoFoldID, 7,j+0)
    m=GetPixel(TwoFoldID,3,j+0)
    n=GetPixel(TwoFoldID,4,j+0)
    m2=GetPixel(TwoFoldID,8,j+0)
    n2=GetPixel(TwoFoldID,9,j+0)
    Pa=Gel.SetPixel(0,j+0,m)
    Panel.SetPixel(0,j+0,m)
    Panel.SetPixel(1,j+0,n)
    Panel.SetPixel(2,j+0,m2)
    If ((m!=m2) || (n!=n2))
    {
    CreateOvalAnnotation(Front, (B_CenterY-RadiusA), (B_CenterX-RadiusA), (B_CenterY+RadiusA), (B_CenterX+RadiusA))
    CreateOvalAnnotation(Front, (B_CenterY2-RadiusA), (B_CenterX2-RadiusA), (B_CenterY2+RadiusA), (B_CenterX2+RadiusA))
    }
    For (k=0; k<j; k++)
For
    m3=Panel.GetPixel(2,k+0)
    n3=Panel.GetPixel(3,k+0)
    If ((m==m3) && ( }n===n3)
    {
        TempID2=GetNthAnnotationID(front,0)
        TempID3=GetNthAnnotationID(front,1)
        DeleteAnnotation(front,TempID2)
        DeleteAnnotation(front,TempID3)
    }
    }
}
void FourFoldRegenerate(image front, image FourFoldID, Number Useful, Number RadiusA)
{
DeleteOvals(front)
ImageDisplay disp
disp = front.ImageGetImageDisplay(0)
Image Panel:=Reallmage("Info", 4, 15, 2000)
Image Panel:=Reallmage("Info", 4, 15,
Number TempID, TempID2, TempID3 
Number i, j, k, A_CenterX, A_CenterY, A_CenterX2, A_Cent 
j=0
For (i=0; i<(Useful); i++)
{
    A CenterX=GetPixel(FourFoldID, 3,i+0)
    A CenterY=GetPixel(FourFoldID, 4,i+0)
    A_CenterX2=GetPixel(FourFoldID, 8,i+0)
```

```
    A_CenterY2=GetPixel(FourFoldID, 9,i+0)
    m=GetPixel(FourFoldID,1,i+0)
    n=GetPixel(FourFoldID,2,i+0)
    m2=GetPixel(FourFoldID,6,i+0)
    n2=GetPixel(FourFoldID,7,i+0)
    TempID=GetNthAnnotationID(front,0)
    number TempType=AnnotationType(front,TempID)
    If (TempType == 6)
    {
    If ((m==0) && (n==0))
    {
        DeleteAnnotation(front, TempID)
    }
}
For (j=0; j<(Useful); j++)
{
    B_CenterX = FourFoldID.GetPixel(3,j+0)
    B_CenterY = FourFoldID.GetPixel(4,j+0)
    B_CenterX2 = FourFoldID.GetPixel(8,j+0)
    B_CenterY2 = FourFoldID.GetPixel(9,j+0)
    If (B_CenterX!=0 && B_CenterY!=0 && B_CenterX2!=0 && B_CenterY2!=0)
    {
    CreateOvalAnnotation(Front, (B_CenterY-RadiusA), (B_CenterX-RadiusA), (B_CenterY+RadiusA), (B_CenterX+RadiusA))
    CreateOvalAnnotation(Front, (B_CenterY2-RadiusA), (B_CenterX2-RadiusA), (B_CenterY2+RadiusA),(B_CenterX2+RadiusA))
}
}
void SixFoldRegenerate(image front, image SixFoldID, Number Useful, Number RadiusA)
void
DeleteOvals(front)
ImageDisplay disp
disp = front.ImageGetImageDisplay(0)
Imp = front.ImageGetImageDisplay(0)
Image Panel:=Reallmage("Info", 4, 15,
Number TempID, TempID2, TempID3
Number i, j, k, A_CenterX, A_CenterY, A_CenterX2, A_CenterY2, m,n,m2,n2, m3,n3
Number B_CenterX, B_CenterY, B_CenterX2, B_CenterY2
j=0
For (i=0; i<(Useful); i++)
{
A_CenterX=GetPixel(SixFoldID, 3,i+0)
    A_CenterY=GetPixel(SixFoldID, 4,i+0)
    A_CenterX2=GetPixel(SixFoldID, 8,i+0)
    A_CenterX2=GetPixel(SixFoldID, 8,i+0)
    A_CenterY2=GetPixel(SixFol
    m=GetPixel(SixFoldID,1,i+0)
    n=GetPixel(SixFoldID,2,i+0)
    m2=GetPixel(SixFoldID,6,i+0)
    n2=GetPixel(SixFoldID,7,i+0)
    TempID=GetNthAnnotationID(front,0)
    number TempType=AnnotationType(front,TempID)
    If (TempType == 6)
    If (T
    If ((m==0) && ( }\textrm{n}==0)
    {
        DeleteAnnotation(front, TempID)
    }
}}
}
B CenterX = SixFoldID.GetPixel(3,j+0)
    B_CenterY = SixFoldID.GetPixel(4,j+0)
    B_CenterX2 = SixFoldID.GetPixel(8,j+0)
    B_CenterX2 = SixFoldID.GetPixel(8,j+0)
    B_CenterY2 = SixFoldID.GetPixel(9,j+0) 
    {
    CreateOvalAnnotation(Front, (B_CenterY-RadiusA), (B_CenterX-RadiusA), (B_CenterY+RadiusA), (B_CenterX+RadiusA))
        CreateOvalAnnotation(Front, (B_CenterY2-RadiusA), (B_CenterX2-RadiusA), (B_CenterY2+RadiusA), (B_CenterX2+RadiusA))
    }
}
// Calculate weighted cross-correlation of 2R,4R
void GammaRotate(Image front, Number Delete, Number RotateSelect)
{
//Get image info
ImageDisplay ImgDisp = front.ImageGetImageDisplay(0)
Number xsize = front.ImageGetDimensionSize(0)
Number ysize = front.ImageGetDimensionSize(1)
//Temporary scrap image for rotate , mask & crosscorrelate
Image TempScrap:=Reallmage("Temp Scrap", 4, xsize, ysize)
Image TempScrap:=Reallmage("Temp Scrap", 4, xsize, ysize()
Number ImageID=GetImageID(TempScrap)
// Register IDs
// Assign annotations' IDs to AnnotA and AnnotB
Image AnnotIDSave:=Reallmage("Pair Info", 4, 40, 2000)
Image AnnotIDSave2:=Reallmage("Mirror Plane Info", 4, 40, 2000)
Number SetsCount=0
Number SetsCou
Number Sets=1
```

```
Number Row=0, Row2=0, i=0, ix=0
Number NoOfAnnot=CountAnnotations(front), TempAnnotID, TypeOFAnno
Number AnnotA,A_AnnTy,A_AnnTx,A_AnnBy,A_AnnBx,A_CenterX,A_CenterY,RadiusA
Number AnnotB,B AnnTy,B AnnTx,B AnnBy,B AnnBx,B CenterX,B CenterY,RadiusB
While (i<NoOfAnnot)
{
    TempAnnotID=GetNthAnnotationID(Front, i)
    TypeOFAnnot=AnnotationType(Front, TempAnnotID)
    If(TypeOfAnnot==5||TypeOfAnnot==6)
    {
        If(TypeOfAnnot==6)
        AnnotIDSave.SetPixel(0, row, TempAnnotID)
        row++
    If(TypeOfAnnot==5)
    {
                AnnotIDSave2.SetPixel(0, row2, TempAnnotID)
                row2++
                ix++
    }
    Sets++
    }
    Else
    { i++
    }
Sets=(Sets-1)/2+0.5*ix
Result("\n>"+Sets+" set(s) selected")
Result("\n>"+ix+" discs are placed on the mirror plane")
//Some values for calculations
number CrossCorrelationCoeff=0
number Totallntensity=0
number Temp_CrossCorrelation=0
Number SetsCountB=0
Number RotAngleTemp, RotAngle
Number nxx=0, nyy=0
Result("\n\n X-corr r-Factor Intensity Sum")
While (SetsCount<Sets)
{
// Allocate annotations' IDs and get annotaions' info
    If (SetsCount<(Sets-ix))
        {
        AnnotA=AnnotIDSave.GetPixel(0,SetsCount*2+1)
        AnnotB=AnnotIDSave.GetPixel(0,SetsCount*2)
        GetAnnotationRect(Front, AnnotA, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx)
        GetAnnotationRect(Front, AnnotB, B_AnnTy, B_AnnTx, B_AnnBy, B_AnnBx)
        //Calculate Radius & Rotation Angle
        ROIInfo(A_AnnTx, A AnnTy, A AnnBx, A CenterX, A CenterY, RadiusA)
        ROIInfo(B_AnnTx, B_AnnTy, B_AnnBx, B_CenterX, B_CenterY, RadiusB)
        If (RadiusA
    {
        B_AnnTx=B_centerX-RadiusA
        B_AnnTy=B_centery-RadiusA
        B_AnnBx=B_centerX+RadiusA
        B_AnnBy=B_centery+RadiusA
    RotAngle=slope(A_CenterX, A_CenterY, B_CenterX, B_CenterY)
    }
If (SetsCount>=(Sets-ix))
    {
If (nxx<1)
    AnnotA=AnnotIDSave2.GetPixel(0, SetsCountB+1)
    AnnotB=AnnotIDSave2.GetPixel(0, SetsCountB)
    GetAnnotationRect(Front, AnnotA, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx)
    GetAnnotationRect(Front, AnnotB, B_AnnTy, B_AnnTx, B_AnnBy, B_AnnBx)
    ROlinfo(A AnnTx, A AnnTy, A Ann\overline{B}x, A CenterX, A CenterY, RadiusA)
    ROlInfo(B_AnnTx, B_AnnTy, B_AnnBx, B_CenterX, B_CenterY, RadiusB)
    If (Radius}\overline{\textrm{A}!}=\mathrm{ Radius }\overline{B}
    {
    B_AnnTx=B_centerX-RadiusA
    B_AnnTy=B_centery-RadiusA
    B_AnnBx=B_centerX+RadiusA
    B_AnnBy=B_centery+RadiusA
    }
    RotAngleTemp=slope(A_CenterX, A_CenterY, B_CenterX, B_CenterY) //Calculate rotation angle first
    SetsCountB=SetsCount}\overline{B}+
    }
}
Image Temp_A, Temp_B, TempMask
Image TempAAA, TempBBB
    Number xWidth, yHeight
    // Rotational Symmetry
    If (RotateSelect==2 || RotateSelect==3 || RotateSelect==4 | RotateSelect==6)
    {
    RotAngle=0
    Image TempSymm_A=Rotation(ImageID, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx, RotAngle)
    Temp_A=ImageCrop(TempSymm_A,RadiusA, 1)
```

```
        Image TempSymm_B=Rotation(ImageID, B_AnnTy, B_AnnTx, B_AnnBy, B_AnnBx, RotAngle)
        If (RotateSelect==2)
        RotAngle=DegToRad(180)
        }
    If (RotateSelect==3)
    RotAngle=DegToRad(120)
    }
    If (RotateSelect==4)
    RotAngle=DegToRad(90)
    }
    If (RotateSelect==6)
    RotAngle=DegToRad(60)
    }
    Temp_B:=ImageCrop(Rotate(TempSymm_B, RotAngle), RadiusA, 1)
    Temp_A.SetName("Disk A_Set "+(SetsCount+1))
    Temp_B.SetName("Disk B_Set "+(SetsCount+1))
}
//Create a mask
Get2Dsize(Temp_A, xWidth, yHeight)
TempMask=Mask(TempMask, A_AnnBx, A_AnnTx, xWidth, yHeight, 0.9)
Number NoOfPixels=sum(TempMask) //number of pixels inside the created mask
    Temp_A=Temp A*TempMask
    Image Temp_AA,
    Temp B=Temp B*TempMask
    Temp_B.SetName("Disk B_Set "+(SetsCount+1))
        // Cross-correlation coefficient
        Number Temp_XCross=XCorr(Temp_A, Temp_B, TempMask, NoOfPixels)
        //Sum of intensity of discs, for weighted gamma use
        Number IntensityA, IntensityB, IntensityAB
    IntensityA=sum(Temp_A*TempMask)
    IntensityB=sum(Temp - B*TempMask)
    IntensityAB=(IntensityA}+\mathrm{ Intensity }\textrm{B})/
    //Return values
    If (SetsCount<(Sets-ix)
    {
        Result("lnSet "+(SetsCount+1)+" "+" "+Temp_XCross+" "+IntensityAB)
    Else
{ Result("InSet "+"m"+(SetsCount-(Sets-ix)+1)+" "+Temp XCross+" "+IntensityAB)
    }
    CrossCorrelationCoeff=CrossCorrelationCoeff+Temp_XCross
    //numerator
    Temp_CrossCorrelation=Temp_CrossCorrelation+Temp_XCross*IntensityAB
    //denominator
    Total Intensity=Totallntensity+IntensityAB
    Temp_A.Deletelmage()
    Temp_B.Deletelmage()
    SetsCount++
}
Result("\n\nWWeight "+Temp_CrossCorrelation/Totallntensity)
    If (Delete==1)
    {
    DeleteOvals(front)
}
// This quantifies the BF symmetry
void BF(image front, Number x0, Number y0, Number r0, Number RotAngleTemp, Number mode)
{
ImageDisplay ImgDisp = front.ImageGetImageDisplay(0)
Number xsize = front.ImageGetDimensionSize(0)
Number xsize = front.ImageGetDimensionsize(0)
Number A_AnnTy,A_AnnTx,A_AnnBy,A_AnnBx
A_AnnTx = x0 - r0
A_AnnTy = y0 - rO
A-AnnBx = x0 + r0
A-AnnBy = y 0 + ro
//Temporary scrap image for rotate, mask & crosscorrelate
Image TempScrap:=Reallmage("Temp Scrap", 4, xsize, ysize)
TempScrap=ImgDisp.ImageDisplayGetDisplayedlmage()
Number ImageID=GetImageID(TempScrap)
Image TempAAA, TempBBB
Image TempMask, Temp_A, Temp_B
Number xWidth, yHeight, NoOfPixels, RotAngle
if (mode == 1)// mirror
{
TempAAA=Rotation(ImageID, A_AnnTy, A_AnnTx, A AnnBy, A_AnnBx, RotAngleTemp)
TempBBB=Rotation(ImageID, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx, RotAngleTemp)
xWidth = TempAAA.ImageGetDimensionSize(0)
yHeight = TempAAA.ImageGetDimensionSize(1)
TempMask=Mask(TempMask, A_AnnBx, A_AnnTx, xWidth, yHeight, 0.9)
NoOfPixels=sum(TempMask)
// Generate Mask Image -> generate mirror image -> apply
Temp_A=TempAAA*TempMask
Temp_A=TempAAA* \empMask
Temp_A.SetName(alig
```

```
                    Temp_B=TempBBB*TempMask
                    Temp_B.SetName("mirror applied BF")
                    Number Temp_XCross=XCorr(Temp_A, Temp_B, TempMask, NoOfPixels)
                    Result("\nBF disk "+Temp_XCross)
    }
    else // rotational symmetry
            RotAngle=0
            Image TempSymm_A=Rotation(ImageID, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx, RotAngle)
            Image TempSymm_B=Rotation(ImageID, A_AnnTy, A_AnnTx, A_AnnBy, A_AnnBx, RotAngle)
            Temp_A=ImageCrop(TempSymm_A, r0, 1)
            //Create a mask
            xWidth = Temp_A.ImageGetDimensionSize(0)
            yHeight = Temp_A.ImageGetDimensionSize(1)
            TempMask=Mask(TempMask, A AnnBx, A AnnTx, xWidth, yHeight, 0.9)
            Number NoOfPixels=sum(TempMask)
            f (mode==2)
            { RotAngle=DegToRad(180)
            If (mode==3)
            {
            If (mode==4)
            If (mode==4)
                    RotAngle=DegToRad(90)
            If (mode==6
            {
            Temp B:=ImageCrop(Rotate(TempSymm B, RotAngle), r0, 1)
                    Temp A=Temp A*TempMask
                    Temp A.SetName("Aligned BF"
            Temp_A.SetName("Aligned BF")
            Temp_B=Temp_B*TempMask
            Numbler Temp_XCross=XCorr(Temp_A, Temp_B, TempMask, NoOfPixels)
            Result("InBF "+Temp_XCross)
    }
}
// Two vectors drawn manually with line annotations:
// g= (gflag*gx, gflag*gy)
// h= (hflag*hx, hflag*hy)
image img
if( !GetFrontlmage(img) )
{
                OkDialog("You must have an image to operate on!")
                exit(0)
}
number nrow, ncol, zsize
ncol = img.ImageGetDimensionSize(0)
nrow = img.ImageGetDimensionSize(1)
nrow = img.ImageGetDimensionSize(1)
number i, J, ann, anntype, anntot, maskid 
number row1, col1, row2, col2, col
//find two base lines marked on the image
j=0
for (i = 0; i< anntot; i++) {
    ann = GetNthAnnotationID(img, i)
    anntype = AnnotationType(img, ann)
    if (anntype == 2) {// a line
    if (annty
    j += 1
    if (j==1) GetAnnotationRect(img, ann, row0, col0, row1, col1)
    if (j==2) GetAnnotationRect(img, ann, row2, col2, row3, col3)
    if (j==3) Result("Skipped lines after first two lines\n")
}
if (j<2) // must have two line2
{
    OkDialog("Must have two lines")
    exit(0)
}
Image mask := CreateFloatlmage( "mask", ncol, nrow )
mask = 0
imagedisplay imgdisp = img.ImageGetlmageDisplay(0)
if (imgdisp.ImageDisplayCountROIs() > 0)
{
        mask = 0
        imgdisp.ImageDisplayAccumulateROIsToMask(mask, 0, 0, nrow, ncol, 1)
        maskid = 1
} else {
        mask=1
maskid = 0
}
number d1, d2, id=0
d1=(row0-row2)**2+(col0-col2)**2
```

```
if ((d2=(row0-row3)**2+(col0-col3)**2) < d1) {
    id = 1
    d1 = d2
}
    ((d2=(row1-row2)**2+(col1-col2)**2) < d1) {
    id =2
    d1 = d2
}
if ((d2=(row1-row3)**2+(col1-col3)**2) < d1) {
    id=3
    d1 = d2
}
number gx, gy, hx, hy, x0, y0, gflag = 1, hflag = 1
if (id == 0) {
    x0 = (col0+col2)/2
    x0 =(col0+col2)/2
    y0 = (row0+row2)/2
    gx = col1-x0
    gy = row 1-y0
    hy = row3-y0
    } else if (id == 1) {
    x0 = (col0+col3)/2
    y0 = (row0+row3)/2
    gx = col1-x0
    gx = col1-x0
    gy = row 1-y0
    hx = col2-x0
    } else if (id == 2) {
    x0 = (col1+col2)/2
    y0 = (row1+row2)/2
    gx = col0-x0
    gy = row0-y0
    gy = row0-y0
    hy = row3-y0
    hy = row3-y0
    } else if (id == 3) {
    y0 = (row 1 +row3)/2
    gx = col0-x0
    gy = row0-y0
    hx = col2-x0
    hy = row2-y0
}
if (abs(gx) < abs(gy)) {
    row0 = gx
    col0 = gy
    gx = hx
    gy = hy 
    hy = colO
}
if (gx < 0) {
    gx*= 1
    gy * = -1
    gflag=-1
}
if (hy < 0) {
    hx*=-1
    hy *= -1
    hy =-1
}
// for 3R & 6R purposes
number tt = gflag*hflag*((gx*hx)+(gy*hy))
number tt = gflag
If (tt>0)
{
angleSix=1
}
Else if (tt<0)
angleSix=0
}
// Adjust circles interactively
number mag=1
gx/= mag
gy/= mag
gy/= mag
hy/= mag
hy/= mag
number dx = gx/4
number dy = hy/4
n0 = nint(1.5*(-x0)/dx)
n1 = nint(1.5*(ncol-x0)/dx)
m0 = nint(1.5* (-y0)/dy)
m1 = nint(1.5* (nrow-y0)/dy)
Result("Use following keys to adjust lattice, press Space to exit\n")
Result("0 to shift latticeln")
Result("g to adjust 1st base vector\n")
```

Result("h to adjust 2nd base vector\n")
Result("Press arrow keys to make changes $\backslash n$ ")
Result("r and R to adjust circle diameter\n")
Result("+ and - to adjust increment\n")
Result("T to switch on/off index $\backslash \mathrm{n}$ ")
Result("m to check mirror cross-correlation values $\backslash n$ ")
Result("M to switch between mirror plane\n")
Result(" 2 to check Two-fold rotational cross-correlation values $\backslash n$ ")
Result("3 to check Three-fold rotational cross-correlation values $\ln$ ")
Result(" 4 to check Four-fold rotational cross-correlation values $\backslash n$ ")
Result(" 6 to check Six-fold rotational cross-correlation values $\backslash n$ ")
number px, py, r0 = 30, inc = 1
number key $=0$, value, LabelText=1
$i d=0$
//specify the mirror direction
Number mX , $\mathrm{m} Y$
if(!getnumber("Enter mirror plane: vector g component=("+gflag*gx+","+gflag*gy+")", 1, mX)) exit(0)
if(!getnumber("Enter mirror plane: vector h component=("+hflag*hx+","+hflag*hy+")", $1, \mathrm{mY}$ )) exit(0)
number N_hkl
if (!GetNumber("Estimated number of reflections? (odd squared)",25, N_hkl)) exit(0)
image AnnotationRegister:=Reallmage("Circle ID, type, x, y, radius, order, m, n", 4, 10, 2000)
image MirrorRegister:=Reallmage("Circle Order", 4, 15, 2000)
image TwoFoldRegister:=Reallmage("Two Fold Order", 4, 15, 2000)
image ThreeFoldRegister:=Reallmage("Three Fold Order", 4, 15, 2000)
image FourFoldRegister:=Reallmage("Four Fold Order", 4, 15, 2000)
image SixFoldRegister:=Reallmage("Six Fold Order", 4, 15, 2000)
Number Useful, RotAngleTemp=0
// Perform a while loop until space bar is hit
while(key!= 32) $\quad / /$ space bar
deleteovals(img)
DrawLattice(img, x0,y0,gx,gy,hx,hy,r0, N_hkl,LabelText)
DeleteOutBound(img)
DeleteOutBound(img)
key = GetKey()
if (key ==29) //right arrow key
\{ if
if (id $==0$ ) $\times 0+=$ inc
if (id $==1$ ) $g x+=$ inc
if (id $==2$ ) $h x+=$ inc
$\}$
else if (key ==28) //left arrow key
\{
if (id $==0$ ) $x 0-=$ inc
if (id $==1$ ) $g x-=$ inc
if (id == 2) hx-=inc
\}
else if (key $==30$ ) //up arrow key
\{
if (id $==0$ ) y0 $+=$ inc
if (id ==1) gy $+=$ inc
if (id $==2$ ) hy $+=$ inc
\}
else if (key $==31$ ) //down arrow key
\{ if
if (id $==0$ ) y0 -= inc
if (id $==1$ ) gy $-=$ inc
if (id $==2$ ) hy $-=$ inc
${ }^{\text {if }}{ }^{\text {if }}$
else if (key == 48) // "0" key
\{
$i d=0$
${ }_{\text {else }}$
else if (key == 103 ) //"g"
\{ ${ }_{i d}=$
id = 1
else if (key == 104 )//"h"
\{
$\}^{\mathrm{id}=}$
else if (key == 114 ) //"r"
\{ ${ }^{\text {els }}$
r 0 - $=\mathrm{inc}$
result("In Radius="+r0)
\}
else if (key == 82) //"R"
\{
$0+=$ inc
result("In Radius="+r0)
\}
else if (key ==43) //"+"
$\left\{\begin{array}{c}\text { inc }\end{array}\right.$
inc * $=2$
\}
else if (key == 45) //"-"
$\left\{\begin{array}{l}\text { inc } \\ \end{array}\right.$
inc * $=0.5$
\}

```
Else If (key == 77) //M
\{if!
if!!getnumber("Enter mirror plane: vector g component=("+gflag*gx+","+gflag*gy+")", 1, mX)) exit(0)
    if(!getnumber("Enter mirror plane: vector h component=("+hflag*hx+","+hflag*hy+")", 1, mY)) exit(0)
    OpenResultsWindow()
    Result("Inln ***Now mirror plane is along (m,n)=("+mX+","+mY+") \n")
\}
Else If (key == 109) //m
\{ Op
    OpenResultsWindow()
    Result("Inln \({ }^{* * *}\) mirror operation")
    AnnotIDSave(img, AnnotationRegister, x0,y0,gflag*gx,gflag*gy,hflag*hx,hflag*hy,0)
    MatchReflectionCircle(img,AnnotationRegister, MirrorRegister, mX, mY, Useful, 0)
    Regenerate(img, MirrorRegister, Useful, r0, 1)
    RotAngleTemp=Gamma(img,1)
        BF(img, x0, y0, r0, RotAngleTemp, 1)
Else
Else If (key == 50) //2R
\{
    OpenResultsWindow()
    Result("Inln \({ }^{* * *}\) Two-Fold Rotation operation")
    AnnotIDSave(img, AnnotationRegister, x0,y0,gflag*gx,gflag*gy,hflag*hx,hflag*hy,0)
    MatchTwoFold(img,AnnotationRegister, TwoFoldRegister,Useful, 0 )
    TwoFoldRegenerate(img, TwoFoldRegister, Useful, r0)
    GammaRotate(img,0,2)
            BF (img, \(\mathrm{x0}, \mathrm{y} 0, \mathrm{ro}\), RotAngleTemp, 2)
Els
\{
OpenResultsWindow()
    Result("Inln ***Three-Fold Rotation operation")
    AnnotIDSave(img, AnnotationRegister, x0,y0,gflag*gx,gflag*gy,hflag*hx,hflag*hy,0)
    SixR(img,AnnotationRegister, ThreeFoldRegister,angleSix,3,Useful,0)
    SixFoldRegenerate(img, ThreeFoldRegister, Useful, r0)
    GammaRotate(img, 0,3 )
            BF (img, x0, y0, r0, RotAngleTemp, 3)
    Else If (key == 52) //4R
\(\{\)
    OpenResultsWindow()
    Result("Inln ***Four-Fold Rotation operation")
    AnnotIDSave(img, AnnotationRegister, x0,y0,gflag*gx,gflag*gy,hflag*hx,hflag*hy,0)
    FourR(img,AnnotationRegister, FourFoldRegister,Useful, 0)
    FourFoldRegenerate(img, FourFoldRegister, Useful, r0)
    GammaRotate(img, 0,4 )
            BF (img, \(\mathrm{xo}, \mathrm{y} 0, \mathrm{r} 0\), RotAngleTemp, 4)
    Else If (key ==54) //6R
\(\{\)
    OpenResultsWindow()
    Result("Inln ***Six-Fold Rotation operation")
    AnnotIDSave(img, AnnotationRegister, x0,y0,gflag*gx,gflag*gy,hflag*hx,hflag*hy,0)
    SixR(img,AnnotationRegister, SixFoldRegister,angleSix,6,Useful,0)
    SixR(img,AnnotationRegister, SixfoldRegister,angle
SixFoldRegenerate(img, SixFoldRegister, Useful, rO )
    SixFoldRegenerate (img
GammaRotate(img \(, 0,6\) )
            BF(img, x0, y0, r0, RotAngleTemp, 6)
    Else
\(\{\)
LabelText \({ }^{*}=-1\)
\(\}^{\}}\)
Result("In Center, g and hin (col, row) "+x0+" "+y0+" "+gflag*gx+" "+gflag*gy+" "+hflag*hx+" "+hflag*hy+"\n");
number gl, alpha, gy2, hy2
\(\mathrm{gl}=\mathrm{sqrt}\left(\mathrm{g} x^{*} g x+g y^{*} g y\right)\)
gy2 = -gy //caution!
alpha \(=\) asin \((\mathrm{gy} / \mathrm{gl}))^{*} 180 / 3.141592654\)
Result("g length angle "+gl+" , "+alpha+"|n");
gl=sqrt(hx*hx+hy*hy)
hy \(2=-\) hy \(/ /\) caution
alpha \(=\) asin \((\) hy \(/ \mathrm{gl}) * 180 / 3.141592654\)
alpha=asin
if \((h x<0)\{\)
if (hy <0) alpha \(=-180\)-alpha
    if \((\) hy \(<0)\) alpha \(=-180\)-alpha
if \((\) hy \(>0)\) alpha \(=180\)-alpha
\}
Result("h length angle "+gl+" , "+alpha+"|n");
Image BasisVector = Reallmage("", 4,8,20+N_hkl)
BasisVector.SetName("Basis vectors for \(0, \mathrm{~g}\) and h ")
BasisVector.SetPixel ( \(0,0, \mathrm{x0}\) )
BasisVector.SetPixel ( 10,0 y0 \()\)
BasisVector.SetPixel( \(1,0, y 0\) )
BasisVector.SetPixel( \(0,1, \mathrm{gx}\) )
BasisVector.SetPixel( 1,1, gy)
BasisVector.SetPixel( \(0,2, \mathrm{hx}\) )
BasisVector.SetPixel(1,2,hy)
BasisVector.SetPixel( \((0,3\), ro)
BasisVector.SetPixel( \(\left.0,4, \mathrm{~N} \_\mathrm{hkl}\right)\)
BasisVector.Showimage()
BasisVector.SetDisplayType(5)
// Output all annotation info
```

```
// Oth-coloumn:ID
// 1st-coloumn:type
// 2nd-coloumn:x position
// 3rd-coloumn:y position
// 4th-coloumn:radius
// 5th-coloumn:order
// 6th-coloumn:m
// 7th-coloumn:n
// 8th, 9th-column:
// 1st-row:(x0,y0)
2nd-row:(gx,gy)
AnnotIDSave(img, AnnotationRegister, x0,y0,gflag*gx,gflag*gy,hflag*hx,hflag*hy,1)
AnnotationRegister.SetPixel(8,0,x0)
AnnotationRegister.SetPixel(9,0,y0)
AnnotationRegister.SetPixel(8,1,gflag*gx)
AnnotationRegister.SetPixel(9,1,gflag*gy)
AnnotationRegister.SetPixel(8,2,hflag*hx)
AnnotationRegister.SetPixel(9,2,hflag*hy)
Showlmage(AnnotationRegister)
AnnotationRegister.SetDisplayType(5)
// Copy Annotations and duplicate //
OkDialog("Proceed to duplicate images & annotations for image stack purpose. Press c to stop")
Image clonefront := img[0,0,0,ncol, nrow,1]
while(key != 99) = //"c" key
key = GetKey()
    deleteovals(img)
    DrawLattice(img,x0,y0,gx,gy,hx,hy,r0, N_hkl,LabelText)
    DeleteOutBound(img)
    If (key == 109) //m
    {
    AnnotIDSave(img, AnnotationRegister, x0,y0,gflag*gx,gflag*gy,hflag*hx,hflag*hy,0)
    if(!getnumber("Enter mirror plane: vector g component=("+gflag*gx+","+gflag*gy+")", 1, mX)) exit(0)
    if(!getnumber("Enter mirror plane: vector h component=("+hflag*hx+","+hflag*hy+")", 1,mY)) exit(0)
    MatchReflectionCircle(img,AnnotationRegister, MirrorRegister, mX, mY, Useful, 0)
    Regenerate(img, MirrorRegister, Useful, r0, 1)
    Image temp = clonefront
    CopyAnnotations(img, temp)
    Showlmage(temp)
    SetName(temp, "(clone)mirror ("+mX+", "+mY+")")
}
    Else lf (key == 50) //2R
{
    AnnotIDSave(img, AnnotationRegister, x0,y0,gflag*gx,gflag*gy,hflag*hx,hflag*hy,0)
    MatchTwoFold(img,AnnotationRegister, TwoFoldRegister,Useful,0)
    TwoFoldRegenerate(img, TwoFoldRegister, Useful, r0)
    Image temp = clonefron
    CopyAnnotations(img, temp)
    ShowImage(temp)
    SetName(temp, "(clone)Two-fold rotation")
    }
    Else If (key == 51) //3R
{
    AnnotIDSave(img, AnnotationRegister, x0,y0,gflag*gx,gflag*gy,hflag*hx,hflag*hy,0)
    SixR(img,AnnotationRegister, ThreeFoldRegister,angleSix,3,Useful,0)
    SixFoldRegenerate(img, ThreeFoldRegister, Useful, rO)
    Image temp = clonefront
    CopyAnnotations(img, temp)
    ShowImage(temp)
    SetName(temp, "(clone)Three-fold rotation")
    }
Else lf (key == 52)//4R
Else
    AnnotIDSave(img, AnnotationRegister, x0,y0,gflag*gx,gflag*gy,hflag*hx,hflag*hy,0)
    FourR(img,AnnotationRegister, FourFoldRegister,Useful,0)
    FourFoldRegenerate(img, FourFoldRegister, Useful, r0)
    Image temp = clonefront
    CopyAnnotations(img, temp)
    ShowImage(temp)
    SetName(temp, "(clone)Four-fold rotation")
    }
}
Else If (key == 54) //6R
{
    AnnotIDSave(img, AnnotationRegister, x0,y0,gflag*gx,gflag*gy,hflag*hx,hflag*hy,0)
    SixR(img,AnnotationRegister, SixFoldRegister,angleSix,6,Useful,0)
    SixFoldRegenerate(img, SixFoldRegister, Useful, r0)
    Image temp = clonefront
    CopyAnnotations(img, temp)
    ShowImage(temp)
    SetName(temp, "(clone)Six-fold rotation")
    }
}
```


## APPENDIX B: <br> PYTHON SCRIPT FOR SCBED SIMULATION

## B.1. CBED simulation of $\mathrm{Al}_{0.1} \mathbf{C r F e C o N i}$ with random displacements

## A simple script to automate CBED simulation using scattering matrix method

This example demonstrates the simulation of CBED patterns of AlO.1 CrFeCoNi HEA by:

1. Divide a crystal column into N blocks. Each block is randomly displaced from the ideal lattice,
2. Specify the mean, standard deviation of Gaussian distribution of the displacement (unit: fraction of unit cell)
3. Simulate CBED patterns and save into a single .img binary file (can be imported into DM)

Yu-Tsun Shao, PhD candidate,
University of Illinois at Urbana-Champaign, 2018

```
# -*- coding: utf-8 -*
import time
import numpy as np
import matplotlib.pyplot as plt
import subprocess
def safe_div(x,y):
    if }\textrm{y}==0\mathrm{ :
        return 1
    return x/y
def cmd_Bloch(commands):
    subprocess.call(commands, shell=True)
def loadimg(filename)
    """Load data matrix from .img file; Return 2d array and the file header. """
    with open(filename, 'rb') as fin:
        dt header = np.dtype(['ncol', 'i2'), ('nrow', 'i2'),
            ('bytesperpixel', 'i2'), ('type', 'i2')])
            header = np.fromfile(fin, dtype=dt_header, count=1)[0]
            data = np.fromfile(fin, dtype='44').reshape((header['ncol'],
                                    header['nrow']))
return data, header
```

\# Displace the blocks which follows the random distribution
def RandomStrainBox(disp_mean, disp_std, N_slab, dz, N_scan):
"' list of displacements along (column, probe_position) "
JazzRndmColumn = np.zeros((N_slab, 5, N_scan)) \#\# (row, col, probe_position); [t1, t1+dt, disp_x, disp_y, disp_z]
NameList = []
\#\# each probe position = 1 CBED pattern
for probe in range(N_scan):
rnd_dx = np.random.normal(disp_mean[0], disp_std[0], N_slab) \#\# a list of random numbers for each probe position
rnd_dy = np.random.normal(disp_mean[1], disp_std[1], N_slab) \#\# a list of random numbers for each probe position
rnd_dz = np.random.normal(disp_mean[2], disp_std[2], N_slab) \#\# a list of random numbers for each probe position
f_temp = "probe_"+str(probe) \#\# temporary filename
for mosaic in range(N_slab):
t1 = int(mosaic*dz) \#\# [A]
disp $x=$ = $\{0: .5 f\}$ ".format(rnd dx[mosaic] $)$
disp_y $=$ "\{0:.5f\}".format(rnd_dy[mosaic]
disp_z $=$ "\{0:.5f\}".format(rnd_dz[mosaic])
JazzRndmColumn[mosaic, ,., probe] = np.array([t1, t1+dz, disp_x, disp_y, disp_z])
NameList.append(f_temp)
print("probe position at: ",probe)
return NameList, JazzRndmColumn
\#\# Generate input files for scbed.exe
def TXYZFile(thickness, dz, NameList, JazzRndmColumn):
"'For each probe position,
generates a .rxy file according to NameList, JazzNBlues'"
N_slab, rxyz, N_scan = JazzRndmColumn.shape
for probe_position in range( N _scan):
JazzNBlues = JazzRndmColumn[:, ,, probe_position] \#\# the list for each probe position
f_temp = NameList[probe_position] \#\# temporary filename
\#\# writes the .dat file
with open (f_temp+".rxyz", 'w') as f:
LineOne = "0.0 $0.00000 .0 \mathrm{ln} "$
f.writelines(LineOne)
for i in range(len(JazzNBlues)):
t1_temp = JazzNBlues[ij[0] \#\# temporary t1
${ }^{\text {t1 }}$-prime $=$ JazzNBlues[ij[1] $\quad \# \#$ temporary t1+dt
disp_x = JazzNBlues[i][2]
disp_y = JazzNBluesin|3
disp_z = JazzNBlues[i][4]
Lines = str(t1_prime)+" "+str(disp_x)+" "+str(disp_y)+" "+str(disp_z)+" ${ }^{\prime \prime}$ "
f. writelines(Lines)

LastLine $=$ str(thickness) + " $0.00 .00 .0 ~ \ n^{\prime \prime}$ f.writelines(LastLine) f.close()

```
## call the program scbed.ex
def StrainCBED_Bloch(fname, NameList, x0, y0, g_length, g_angle, npix):
    print("\n**Generating a .bzt file using Blochz.exe ...\n")
    commands = "blochz "+fname+".dat"
    cmd Bloch(commands)
    print("**Generating .img files using SCBED.exe ...\n")
    for i in range(len(NameList)):
        f_temp = NameList[i] ## temporary filename
        ## now run the scbed.exe --> rename .img
        commandOne = "scbed "+fname+" "+f_temp+".rxyz "+str(x0)+" "+str(y0)+" "+str(g_length)+" "+str(g_angle)+" -s "+str(npix)
        cmd_Bloch(commandOne)
        commandTwo = "rename "+fname+".img "+f_temp+".img"
        cmd_Bloch(commandTwo)
        print("progress: "+str(i+1)+"/"+str(int(len(NameList)))+" images")
        ## removes the .rxyz files
    for i in range(len(NameList)):
        f_temp = NameList[i] ## temporary filename
        try:
            os.remove(f_temp+".rxyz")
        except OSError:
            pass
## Stack the CBED patterns into a binary file of image stack, which can be imported into DM with header of 8 bytes
def StrainCBED_stackIMG(fname_out, NameList, npix)
    CBED array = np.zeros((npix, npix, len(NameList)))
    print("\\\^**Generating .img stack from .img files...")
    temp_data = np.empty([npix, npix], dtype='f4')
    f_temp = NameList[0] + ".img"
    useless_data, header = loadimg(f_temp)
    ## writes the output .img (stack)
    with open(fname_out+'.img', 'wb') as fout:
        fout.write(header)
        for i in range(0, len(NameList)):
            f_temp = NameList[i] + ".img" ## temporary filename
            temp_data, header = loadimg(f_temp)
            CBED_array[:,,i] = temp_data
            temp_data = np.asarray(CBED_array[:,:,i], dtype='float32', order='C')
            fout.write(temp_data)
    fout.close()
    for i in range(len(NameList)):
        f_temp = NameList[i]
        try:
            os.remove(f temp+".img")
        except OSError:
            pass
    print(" Done! ")
    ##return CBED_array
## This binary image file saves the displacements of individual block
def SaveJazzRandom(fname_out, JazzRndmColumn):
    datatype = np.dtype([('ncol', 'i2'), ('nrow', 'i2'), ('bytesperpixel', 'i2'), ('type', 'i2')])
    header = np.array((512, 512, 4, 2), dtype=datatype)
    header = np.array((512, 512, 4, 2), dtype=datatype
    N_slab, rxyz, N_scan = JazzRndmColumn.shape
    temp_data = np.empty([N_slab, rxyz], dtype='f4')
        fout.write(header)
            for i in range(N_scan):
            temp_data = temp_data = np.asarray(JazzRndmColumn[:,.,i], dtype='float32', order='C')
            fout.write(temp_data)
    fout.close()
"' Input parameters "' \#\# thickness of TEM specimen [units: Anstrom]
thickness \(=1650\)
\begin{tabular}{ll} 
thickness = 1650 & \\
\begin{tabular}{ll} 
N_slab \(=33\) & \\
dz \(=50\) & \#\# thickness of TEM specimen [units: Anstrom] \\
disp_mean \(=[0.0,0.0,0.0]\) & \#\# number of blocks \\
disp_std \(=[0.0055,0.0055,0.0055]\) & \(\# \# 0.35 \%=0.0035\)
\end{tabular} & \begin{tabular}{l} 
\#\# thickness of each block [units: Angstrom]
\end{tabular} \\
\#\#, mean_y, mean_z] &
\end{tabular}
\begin{tabular}{l} 
disp_mean \(=[0.0,0.0,0.0]\) \\
\#\#[mean_x, mean_y, mean_z] \\
disp_std \(=[0.0055,0.0055,0.0055]\) \\
N scan \(=625\)
\end{tabular}\(\quad \# \# 0.35 \%=0.0035 \quad\) \#\# 25x25 images
\#\# file name
fname = "HEA" \#\# file name of the crystal data (.dat)
fname_out = fname+"_"+str(thickness)+"_slab_"+str(N_slab)+"_mean_"+str(disp_mean)+"_std_"+str(disp_std)
\#\# define output image format
\(x 0=256 \quad\) \#\# pixel \(x\) coordinate of (000) disk
\(y 0=256 \quad\) \#\# pixel y coordinate of (000) disk
g_length \(=90 \quad\) \#\# length of \(g\), diffracted beam, in pixels
g_angle =0 ## rotation angle of g
npix = 512
## pixel size of the image
"' Run the functions
time_start = time.clock()
NameList, JazzRndmColumn = RandomStrainBox(disp_mean, disp_std, N_slab, dz, N_scan)
TXYZFile(thickness, dz, NameList, JazzRndmColumn)
StrainCBED_Bloch(fname, NameList, x0, y0, g_length, g_angle, npix)
StrainCBED_stackIMG(fname_out, NameList, npix)
SaveJazzRandom(fname_out, JazzRndmColumn)
time_fin = time.clock()
print("time elapsed : "+str(time_fin - time_start))
```


## APPENDIX C:

## INPUT DATA FILES FOR CBED SIMULATION

## C.1. Data file of ( $1-x) \operatorname{Pb}\left(\mathrm{Zn}_{1 / 3} \mathrm{Nb}_{2 / 3}\right) \mathrm{O}_{3}-x \mathrm{PbTiO}_{3}(\mathrm{x}=\mathbf{0} .08)$ (see J.-M. Kiat et al., Phys.

## Rev. B 65, 064106 (2002) for atomic positions)

```
#!Written by QED, (C) JM ZUO
#!crystal
crystal PZN-8PT: dw = iso occ = par
cell 4.06080 4.00840 4.05290 90.000 90.250 90.000
atom Pb#1 0.000000 0.000000 0.000000 2.8000 1.00000
atom Zn#1 0.528900 0.500000 0.474800
atom Nb#1 0.528900 0.500000 0.474800 0.6300 0.60670
atom Ti#1 0.528900 0.500000 0.474800 0.6300 0.09000
atom O#1 0.588200 0.000000 0.456100 0.2600 1.00000
atom O#2 -0.006000 0.500000 0.528500 1.0300 1.00000
atom O#3 0.523500 0.500000 0.009000 0.1700 1.00000
spg 6 1
#! diffraction condition
hv 200.00
zone 1 1 0
norm 1 1 0
kt 0.000000 0.000000 0.000000
xaxi 0 0 1
kmax 0.115000
samp 16
# beam slection rules
sele 2.000000 3.5 0.1 0
sgmin 0.3
omeg 20.0
# define output
aper 1.5
abs 0.0
```


## C.2. Data file of tetragonal $\mathrm{BaTiO}_{3}$ (see A. Rother et al., Phys. Rev. B 74, 134116

## (2006) for atomic positions)

```
#!Written by QED, (C) JM ZUO
#!crystal
crystal tetragonl BTO: dw = uij occ = par
cell 3.994600 3.994600 4.038800 90.000000 90.000000 90.000000
atom Ba#1 0.000000 0.000000 0.000000 0.00560 0.00560 0.00630 0.00000 0.00000
0.00000 1.00000
atom Ti#1 0.500000 0.500000 0.488300 0.00760 0.00760 0.00760 0.00000 0.00000
0.00000 1.00000
atom O#1 0.500000 0.500000 0.024700 0.00770 0.00770 0.00550 0.00000 0.00000
0.00000 1.00000
atom N#1 0.500000 0.000000 0.513900 0.00740 0.00670 0.00860 0.00000
0.00000 1.00000
spg 99 1
#! diffraction condition
hv 200.00
zone 1 0 0
norm 1 0 0
```

```
kt 0.000000 0.000000 0.000000
xaxi 0 0 1
kmax 0.120000
samp }3
# beam slection rules
sele 2.000000 3.5 0.1 0
sgmin 0.3
omeg 20.0
# define output
aper 1.5
abs 0.0
```


## C.3. Data file of FCC Al ${ }_{0.1}$ CrFeCoNi (see Y. Hu et al., Comm. Phys. 1, 61 (2018) for composition)

```
#!Written by QED, (C) JM ZUO
#!crystal
crystal AlO.1CrFeCoNi : dw = iso occ = par
llllllllllllllll
atom Al 0.000000 0.000000 0.000000 0.800000 0.023600
atom Co 0.000000 0.000000 0.000000 0.800000 0.251500
atom Cr 0.000000 0.000000 0.000000 0.800000 0.244800
atom Fe 0.000000 0.000000 0.000000 0.800000 0.250400
atom Ni 0.000000 0.000000 0.000000 0.800000 0.229400
spg 225 1
#! diffraction condition
hv 200.00
zone 0 0 1
norm 0 0 1
kt 0.000000 0.000000 0.000000
xaxi 2 0 0
kmax 0.260000
samp }3
# beam slection rules
sele 2.000000 3.5 0.1 0
sgmin 0.3
omeg 20.0
# define output
aper 1.5
abs 0.0
```


[^0]:    ${ }^{1}$ Y.-T. Shao and J.-M. Zuo, Nanoscale symmetry fluctuations in ferroelectric barium titanate, $\mathrm{BaTiO}_{3}$, (2017) Acta Cryst. B73, 708. Copyright 2017 International Union of Crystallography.

[^1]:    ${ }^{1}$ Y.-T. Shao and J.-M. Zuo, Lattice-rotation vortex at the charged monoclinic domain boundary in a relaxor ferroelectric crystal, (2017) Phys. Rev. Lett., 118, 157601. Copyright 2017 American Physical Society.

