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Evaluation of Multivariate GARCH Models in an Optimal Asset Allocation Framework

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1 Research highlights

- A large class of different advanced multivariate GARCH models including *VAR*, *ARMA, DCC, aDCC, FDCC, GOGARCH* and *Copula GARCH* models incorporating static and dynamic copulas in modelling the mean returns and variance-covariance matrices involving a symmetric and an asymmetric of GARCH models using multivariate Normal and Student distributions.
- We construct optimal portfolios using minimum variance, mean-variance, maximising Sharpe ratio, mean-CVaR and minimising Sortino ratio.
- We evaluate the out-of-sample performance for each model to determine the best model for each optimisation strategy.
- Our results suggest that the dynamic models are more capable of delivering better performance gains than the static models. These models reduce portfolio risk and improve the realised return in the out-of-sample period.
- By adding copula functions to the model, it does not give a better performance models when compared to the dynamic correlation models.

1. Introduction

With today's challenging environments and highly volatile markets, the study of asset management has become important for organisations in managing their assets to obtain the best possible returns. Asset allocation aims to balance the risk and return by adjusting the percentage of each asset in an investment portfolio based on the investor's specific goals, investment objectives, risk tolerance, and investment horizon. A strategic investment decision that comprises well-diversified portfolio is essential to ensure that the investment objectives can be achieved. The goal is to create a balanced mix of assets which has the growth potential that satisfies the investor's risk preferences and investment objectives. As a result, asset management studies, especially relating to asset pricing, portfolio selection and risk management are growing in importance.

The future forecast is highly dependent on the choice of the volatility modelling. It is known that volatility is not directly observable, which makes it important to have a good model to predict future volatilities. Obtaining an optimal portfolio requires estimating and forecasting very large conditional covariance matrices of the asset returns which depend on many parameters (Billio et al., 2006; Lee et al., 2006). Over the past years, several studies have developed methods and approached to examine the dynamics of covariance of assets. Previous studies on asset allocation, mainly in asset management, focus on a limited or specific econometric model to model its asset return covariances, see for example; Ferstl and Weissensteiner (2011), Hoevenaars et al. (2013), and Koivu et al. (2005). One of the widely used models is the Vector Autoregressive (VAR) model which is an extension of the univariate autoregressive model to dynamic multivariate time series. It is useful to consider different multivariate econometric models that can capture different characteristics of the data in selecting the best model to create optimal portfolios.

To deal with a large number of parameters in multivariate models, Bollerslev et al. (1998) suggest a Constant Conditional Correlation model (CCC) such that the conditional correlations are assumed to be constant. This model reduces the number of parameters and thus simplifies the estimations considerably. However, the assumption of a CCC model may not be realistic in empirical applications of multivariate GARCH models because the conditional shocks are correlated only in the same market, and not across markets (Chang et al., 2013). Engle (2002) proposes a generalisation of CCC model, by allowing the correlations to change over time, known as the Dynamic Conditional Correlation model (DCC). This model is estimated using a two-steps approach - the estimation of mean and variance by a series of univariate GARCH models and the correlation estimation.

Alternative DCC models also have been proposed in the literature which are aiming to solve problems associated with the basic DCC model. The limits of the DCC model are constrained by the equal dynamics for the correlations of all the assets (Billio et al., 2006). To avoid this problem, Billio et al. (2006) propose the Flexible DCC (FDCC) model such that the correlation dynamic is constrained to be equal only between w groups of variables, providing flexible dynamics. Another study by Otranto (2010) examines the performance of optimal asset allocation strategies using FDCC models with regime switching as compared to alternative models. Recently, Aielli (2013) suggests a more tractable dynamic conditional correlation model, known as a corrected DCC model or cDCC model, which involves the three-step approach that is feasible with large systems and provides unbiased estimations. Recent proposals of multivariate GARCH models include the asymmetric DCC model (aDCC) of Cappiello et al. (2006), the Copula GARCH of Patton (2004), the dynamic equicorrelation (DECO) model of Engle and Kelly (2012) and the smooth transition conditional correlation (STCC-GARCH) of Silvennoinen and Terasvirta (2015).

The copula theory was introduced by Sklar (1959). It states that any multivariate distribution function can be decomposed into its marginal distributions and a copula function. The application of copula function was only being introduced in the late 1990's in the actuarial sciences and finance field by Li (2000), Embrechts et al. (1999) and more recently by Hurlimann (2014) and Wu and Lin (2014). Over the years, copula function has been popular in the financial research, especially because of its application to risk management and asset allocation. However, there are very few works assessing the out-of-sample performance of a portfolio based on the copula model (see for example; Patton (2004), Ricetti (2013), Wu and Lin (2014) and Kresta (2015)). The first paper that applies asset allocation with the copula function is by Patton (2004). He uses the copula theory to construct models of the time-varying dependence structure which allow for different dependence during good and bad market conditions, by evaluating the asset allocation in terms of investor's utility. Weib (2013) performed an analysis of the accuracy of the parametric copula-GARCH and DCC models so as to ascertain whether the DCC model outperforms the copula model. He found that the parametric copula models are so easily outperformed by the correlation based model.

There is considerable literature on asset modelling and optimisation of portfolio allocation strategies using different empirical approaches; see, for instance; Vrontos et al. (2013), Koivu et al. (2005), Billio et al. (2006), Boubaker and Sghaier (2013) and Kalotychou et al. (2014). One of the earliest approaches of the portfolio theory was developed by Markowitz (1952) - a well-known approach known as Mean-variance optimisation. It is a myopic strategy which assumes that the decision maker has a mean-variance criterion defined over the single period rate of return on the portfolio. Other studies that are related are Sharpe and Tint (1990), Engle and Colacito (2006), and Platanakis and Sutcliffe (2014). Alexander et al. (2006), Rockafellar and Uryasev (2000) and Bogentoft et al. (2001) have approached the optimisation problem using the minimisation of mean Conditional Value at Risk (CVaR). Bogentoft et al. (2001) use linear programming and introduce a new approach to model asset return by combining a CVaR risk management technique with optimal decisions using sample paths. Rockafellar and Uryasev (2000) showed that CVaR can be efficiently minimised using the linear programming and non-smooth optimisation techniques. By minimising CVaR, the VaR is also reduced since CVaR is greater or equal than VaR. See recent papers by Boubaker and Sghaier (2013) and Huang and Hsu (2015) which propose minimising the CVaR assuming that the dependence structure is modelled by the copula parameter.

Alternative optimisation portfolio strategies include the maximisation of the Sharpe ratio, maximisation of the expected utility and the minimisation of mean absolute deviation (MAD). A study by Luo et al. (2015) compare the performance of each optimal portfolio in an out-of-sample period and found that orthogonal GARCH (OGARCH) model outperformed the other models, such as Markov switching and the EWMA model, in producing an optimal portfolio. Wu and Lin (2014) propose copula-based GARCH models to describe the time varying dependence structure of stock-bond returns while Kinoshita (2015) considers asset allocation problems under higher moments with GARCH effects using the expected utility maximisation, and uses a bootstrap method to measure the performance of the portfolio. Recently, Fulga (2016) proposes a bi-objective portfolio optimisation model involving efficient portfolios of a disutility-based risk measure (DCVaR), known as Mean-DCVaR that constitutes an improvement over Mean-CVaR or Mean-Variance model. It is important to develop an optimisation model for supporting the decision-making concerning the allocation of assets so that the investment goals are achieved.

This paper makes several contributions to the literature. First, this paper involves a large class of different advanced multivariate GARCH models. We use 26 different model specifications using multivariate GARCH processes in modelling the mean returns and variance-covariance matrices. Specifically, we use a symmetric GARCH model and an asymmetric version of it (GJR-GARCH) such that the models are implemented with the multivariate Normal and Student distributions. For the conditional mean dynamics, this study allows a constant, univariate AR, ARMA or VAR model to be fit. In general, the model specifications to model covariances includes DCC models, aDCC models, FDCC models, Generalised Orthogonal GARCH (GOGARCH) and Copula GARCH models. For the DCC models, the conditional mean is jointly estimated with the first stage GARCH while, for the GO-GARCH models, the dynamics for the conditional mean are defined from the general ARFIMAX model of Engle et al. (1987) assuming constant variance to obtain the parameter estimates. The Copula-GARCH models are implemented using the static and dynamic (DCC) estimation of the correlation. By employing different model specifications, we are able to explore the empirical applicability of the multivariate GARCH model when estimating large conditional covariance matrices. Second, from the above modelling, we construct optimal portfolios using different optimisation models, namely mean-variance, maximisation of the Sharpe ratio and mean-CVaR. By using different econometric models, we are able to capture different characteristics in the data which improve the portfolio performance and are useful for the construction of optimal portfolios. We design a comparative study that considers the results of each asset return covariances model to see which model is the best in each optimisation strategy. This paper differs from previous studies by assesing the out of sample performance of large groups of multivariate GARCH models in different optimisation schemes considering portfolios with and without short sales. Most of the existing literature compares the performance of limited GARCH models, and they are based on a specific optimisation model. Several studies have tried to examine the effectiveness of using parametric copula in estimating portfolio risk measures, but they have been inconclusive. It is still not clear whether or not it is optimal to use Copula GARCH over the sophisticated DCC model. This study, therefore, will provide useful insights for those wishing to explore the different GARCH models that are available and use a dynamic approach for asset allocation and portfolio construction purposes.

The remainder of the paper is organised as follows: Section 2 discusses the econometric models we use to model the asset returns covariances. Section 3 describes the portfolio optimisation strategies used to construct optimal asset portfolios and look at how the models evolve. Section 4 presents the data, empirical analysis and results of the proposed models and methods while Section 5 provides concluding remarks and identifies the shortcomings and future implications of the study.

2. Econometric Models for Asset Return

In this section, we consider different econometric methodologies to model mean asset return and covariances. Volatility has some characteristics that are not directly noticeable but are commonly seen in asset returns; i.e presence of volatility clustering in the data, a volatility jump is rare, stationary, and has a leverage effect (Tsay, 2013). Various volatility models were introduced, particularly to correct the weaknesses of their inability to capture the characteristics mentioned previously.

Consider the vector stochastic process, $\boldsymbol{x_t}$ which is the $T \times 1$ vector of log returns of n assets at time t and $\boldsymbol{\mu_t}$ is the $T \times 1$ vector of the expected value of conditional $\boldsymbol{x_t}$ such that $\boldsymbol{\mu_t}$ may be modeled as a constant vector or a time series model. The multivariate GARCH models are written as:

$$\boldsymbol{x_t} = \boldsymbol{\mu_t} + \boldsymbol{\epsilon_t}, \qquad \boldsymbol{\epsilon_t}|\boldsymbol{\Phi_{t-1}} \sim N(0, \boldsymbol{H_t}) \tag{1}$$

where $\boldsymbol{\epsilon_t}$ is the residuals of the process which follows a conditionally multivariate normal distribution with mean 0 and time varying conditional covariance matrix $\boldsymbol{H_t}$ given $\boldsymbol{\Phi_{t-1}}$ is the information set at time $t-1$. The residuals are modelled as,

$$\boldsymbol{\epsilon_t} = \boldsymbol{H_t^{\frac{1}{2}} z_t} \tag{2}$$

where $\boldsymbol{H_t}$ is $T \times n$ positive definite matrix of conditional variances of $\boldsymbol{x_t}$ at time t. $\boldsymbol{H_t^{\frac{1}{2}}}$ is the Cholesky factorisation of the time varying conditional covariance matrix of $\boldsymbol{H_t}$. The symbol $\boldsymbol{z_t}$ is $T \times 1$ vector of independent and identically distributed random errors such that $E[\boldsymbol{z_t}] = 0$ and $E[\boldsymbol{z_t z_t^{'}}] = \boldsymbol{I_T}$, whereby $\boldsymbol{I_T}$ denotes the identity matrix of order T. The conditional covariance matrix $\boldsymbol{H_t}$ of $\boldsymbol{x_t}$ may be defined as,

$$\begin{aligned} Var[\boldsymbol{x_t}|\mathcal{F}_{t-1}] &= Var_{t-1}[\boldsymbol{x_t}] = Var_{t-1}[\boldsymbol{\epsilon_t}] \\ &= \boldsymbol{H}_t^{\frac{1}{2}} Var_{t-1}(\boldsymbol{\epsilon_t})(\boldsymbol{H}_t^{\frac{1}{2}})^{'} = \boldsymbol{H}_t \end{aligned} \tag{3}$$

where $\mathcal{F}_t = \{x_t, x_{t-1}, \dots\}$. We use different specifications for multivariate GARCH processes of forecasting mean asset return and covariances. By using different econometric models, we are able to capture different characteristics in the data which improve the portfolio performance and are useful for the construction of optimal portfolios. We then compare the results of each model to see their ability in optimising portfolio.

2.1. Modelling mean returns

There are various models for time series which are oftenly divided into the autoregressive (AR) models and the moving average (MA) models. The AR(m) model can be written as:

$$\boldsymbol{x}_t = \sum_{i=1}^{m} \boldsymbol{\phi}_i \boldsymbol{x}_{t-i} + \boldsymbol{\epsilon_t} \tag{4}$$

For the moving average, the MA (n) refers to:

$$\boldsymbol{x_t} = \boldsymbol{\epsilon}_t - \sum_{j=1}^{n} \boldsymbol{\theta}_j \boldsymbol{\epsilon}_{t-j} \tag{5}$$

These models are commonly used to generate new models such as the Autoregressive Moving Average (ARMA) or Vector Autoregressive (VAR) model. We model the mean returns using different estimation processes, either using a constant mean, AR, ARMA or VAR model.

2.1.1. Autoregressive Moving Average (ARMA)

The ARMA model provides a parsimonious parameterisation and further simplification in modelling multivariate time series. It has both stationary stochastic processes of the autoregression and moving average methods which are applied to a multivariate time series data. The ARMA (m,n) model refers to the m autoregressive terms and n moving average terms which includes the AR (m) and MA (n) models. The mean of the process which is modeled by ARMA (m,n) is written as:

$$\boldsymbol{x_t} = \boldsymbol{\phi_0} + \sum_{i=1}^{m} \boldsymbol{\phi_i x_{t-i}} + \sum_{j=1}^{n} \boldsymbol{\theta_j \epsilon_{t-j}} + \boldsymbol{\epsilon_t} \tag{6}$$

where $\boldsymbol{\phi_0} \in \mathbb{R}$ and $\boldsymbol{\alpha_i}$ and $\boldsymbol{\beta_i}$ are $T \times n$ diagonal matrices. The autoregressive coefficients is denoted by $\boldsymbol{\phi_i}$ and moving average coefficients is denoted by $\boldsymbol{\theta_j}$ if there exist real coefficients $\boldsymbol{\phi_1}, \ldots, \boldsymbol{\phi_m}$ and $\boldsymbol{\theta_1}, \ldots, \boldsymbol{\theta_n}$ such that $\boldsymbol{\epsilon_t}$ is the linear innovation process of $\boldsymbol{x_t}$. $\boldsymbol{\phi_i}$ and $\boldsymbol{\theta_j}$ are $T \times n$ matrices with $\boldsymbol{\phi_i} \neq 0$ and $\boldsymbol{\theta_j} \neq 0$.

2.1.2. Vector Autoregressive (VAR)

The most commonly used multivariate econometric model is the VAR model. We allow the conditional mean to follow a VAR structure such that the model for the process can be represented by,

$$\boldsymbol{x_t} = \boldsymbol{c} + \sum_{i=1}^{m} \boldsymbol{\phi_i x_{t-i}} + \boldsymbol{\epsilon_t} \tag{7}$$

where $\boldsymbol{x_t}$ is vectors and $\boldsymbol{c}$ denotes $T \times 1$ dimensional vector of constants. $\boldsymbol{\phi_1, \ldots, \phi_m}$ is parameter matrices. This model has an important characteristic which is its stability. It generates stationary time series with time invariant means, variances and covariances.

2.2. Modelling covariances matrix

We use different specifications for multivariate GARCH processes of forecasting a variance-covariances matrix namely as (i) Dynamic Conditional Correlation (DCC), (ii) Asymmetric Dynamic Conditional Correlation (aDCC), (iii) Flexible Dynamic Condition Correlation (FDCC), (iv) Generalised Orthogonal GARCH (GOGARCH) and (v) Copula GARCH (C-GARCH).

2.2.1. Dynamic Conditional Correlation (DCC)

The DCC model was proposed by Engle (2002) which is a generalisation of the Constant Conditional Correlational (CCC) model from Bollerslev et al. (1998) to allow for the time varying correlation matrix of multiple asset returns. In the CCC model, the conditional covariance matrix is decomposed into conditional standard deviations and a constant correlation as:

$$\boldsymbol{H_t} = \boldsymbol{D_t R D_t} \tag{8}$$

where $\boldsymbol{D_t}$ is the $T \times n$ diagonal matrix of time varying standard deviations from univariate GARCH models, $\boldsymbol{D_t} = diag(\sqrt{h_{1t}}, \ldots, \sqrt{h_{nt}})$ and $\boldsymbol{R} = \rho_{ij}$ is the positive definite constant conditional correlation matrix with $\rho_{ii} = 1$ for $i = 1, \ldots, n$. The off diagonal elements of $\boldsymbol{H_t}$, are given by:

$$[\boldsymbol{H_t}]_{\boldsymbol{ij}} = \boldsymbol{\rho_{ij}} \sqrt{\boldsymbol{h_{it}}} \sqrt{\boldsymbol{h_{jt}}}, \quad i \neq j \tag{9}$$

This model is computationally attractive and simple because of the constant correlation. However, the assumptions of constant conditional correlations may be unrealistic in practice and it may be too restrictive in some cases. The DCC model was introduced by Engle (2002) to allow the time varying correlation dynamics, $\boldsymbol{R} = \boldsymbol{R_t}$. It is defined as:

$$\begin{aligned} \boldsymbol{x_t} &= \boldsymbol{\mu_t} + \boldsymbol{\epsilon_t} \\ \boldsymbol{\epsilon_t} &= \boldsymbol{H_t^{\frac{1}{2}} z_t} \\ \boldsymbol{H_t} &= \boldsymbol{D_t R_t D_t} \end{aligned} \tag{10}$$

such that $\boldsymbol{x_t}$ is a $T \times 1$ vector of log returns of n assets at time t. $\boldsymbol{\epsilon_t}$ is a $T \times 1$ vector of mean corrected returns of n assets at time t such that $E[\boldsymbol{\epsilon_t}] = 0$ and $Cov[\boldsymbol{\epsilon_t}] = \boldsymbol{H_t}$ and $\boldsymbol{\mu_t}$ is a $T \times 1$ vector of the expected value of the conditional $\boldsymbol{x_t}$. $\boldsymbol{H_t}$ is a $T \times n$ matrix of conditional variances of $\boldsymbol{\epsilon_t}$ at time t and $\boldsymbol{D_t}$ is a $T \times n$ diagonal matrix of conditional standard deviation of $\boldsymbol{\epsilon_t}$ at time t. Note that the conditional correlation matrix of $\boldsymbol{\epsilon_t}$ is time varying now and denoted by symbol $\boldsymbol{R_t}$. While $\boldsymbol{z_t}$ is $T \times 1$ vector of independent and identically distributed random errors such that $E[\boldsymbol{z_t}] = 0$ and $E[\boldsymbol{z_t z_t'}] = \boldsymbol{I_T}$.

The conditional variances, $\boldsymbol{H_t}$ can be estimated separately by a simple univariate GARCH specification of,

$$\boldsymbol{H_t} = \boldsymbol{\sigma^2_{i,t}} = \boldsymbol{g_i} + \sum_{l=1}^{q} \boldsymbol{\beta_i \sigma^2_{i,t-l}} + \sum_{m=1}^{p} \boldsymbol{\alpha_i \epsilon^{(2)}_{i,t-m}} \tag{11}$$

where $\boldsymbol{g_i}$ is $T \times 1$ vector of constant, $\boldsymbol{\alpha_i}$ and $\boldsymbol{\beta_i}$ are $T \times n$ diagonal matrices. $\boldsymbol{\epsilon^{(2)}_{t-m}} = \boldsymbol{\epsilon_{t-m}} \bigodot \boldsymbol{\epsilon_{t-m}}$ is the Hadamard product which is the element by element product. $\boldsymbol{H_t}$ is a positive definite matrix such that $\boldsymbol{g_i} > 0$ and $\boldsymbol{\alpha_i, \beta_i} \geq 0$. $\boldsymbol{R_t}$ is also a positive definite conditional correlation matrix of the standardised disturbances of $\boldsymbol{z_t}$ such that:

$$\boldsymbol{z_t} = \boldsymbol{D_t^{-1} \epsilon_t} \sim N(\boldsymbol{0}, \boldsymbol{R_t}) \tag{12}$$

So the elements of $\boldsymbol{H_t} = \boldsymbol{D_t R_t D_t}$ with $\rho_{ii} = 1$ is now written as:

$$[\boldsymbol{H_t}]_{ij} = \boldsymbol{\rho_{ij,t} \sqrt{h_{it} h_{jt}}} \tag{13}$$

The conditions for the positivity of the covariance matrix $\boldsymbol{H_t}$ requires $\boldsymbol{R_t}$ to be positive definite, $\boldsymbol{g_i}$ and all diagonal elements of matrices $\boldsymbol{\beta_i}$ and $\boldsymbol{\alpha_i}$ are all positive. Therefore, to ensure that, we need to decompose $\boldsymbol{R_t}$ into:

$$\begin{aligned} \boldsymbol{R_t} &= (\boldsymbol{Q_t^*})^{-1} \boldsymbol{Q_t} (\boldsymbol{Q_t^*})^{-1} \\ \boldsymbol{Q_t} &= (1 - a - b)\boldsymbol{\bar{Q}} + a\boldsymbol{z_{t-1} z_{t-1}'} + b\boldsymbol{Q_{t-1}} \end{aligned} \tag{14}$$

$\bar{\boldsymbol{Q}}$ =Cov$[\boldsymbol{z_t}\boldsymbol{z_t}^{'}] = E[\boldsymbol{z_t}\boldsymbol{z_t}^{'}]$ is a $T \times n$ unconditional matrix of the standardised errors $\boldsymbol{z_t}$ such that $\boldsymbol{z_t} = \boldsymbol{D_t^{-1}}\boldsymbol{\epsilon_t}$. $\boldsymbol{Q_t^*} = diag(\sqrt{q_{1t}}, \sqrt{q_{2t}}, \ldots, \sqrt{q_{nt}})$ and a and b are non-negative parameters to be estimated such that $a+b < 1$ to ensure stationarity and positive definiteness of $\boldsymbol{Q_t}$. Engle (2002) estimates $\bar{\boldsymbol{Q}}$ using the following equation:

$$\bar{\boldsymbol{Q}}_{\boldsymbol{t}} = \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{z_t}\boldsymbol{z_t}^{'} \tag{15}$$

In general, the DCC GARCH model is written as in Engle (2002):

$$\boldsymbol{Q}_t = (1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n)\bar{\boldsymbol{Q}}_{\boldsymbol{t}} + \sum_{m=1}^{M} a_m \boldsymbol{z_{t-1}}\boldsymbol{z_{t-1}^{'}} + \sum_{n=1}^{N} b_n \boldsymbol{Q}_{t-1} \tag{16}$$

This model is estimated using a two-step approach - the first implies the estimation of univariate GARCH and the second step is the correlation estimation. The number of the estimated parameters in the correlation process of DCC GARCH is independent of the number of series correlated, hence allowing for a potentially large correlation matrices to be feasibly estimated. The limitation of this model is to hypothesise the same correlation dynamics of all the assets (Billio et al., 2006; Otranto, 2010).

2.2.2. Asymmetric Dynamic Conditional Correlation (aDCC)

Cappiello et al. (2006) introduce aDCC model to investigate whether conditional variances, covariances, and correlations of assets in a portfolio are sensitive to the sign of past innovations. As compared to the DCC model, this model further explores whether the positive and negative shocks are of same magnitude or have different impacts. Similarly to DCC models, the matrix $\boldsymbol{H_t}$ is decompose into:

$$\boldsymbol{H_t} = \boldsymbol{D_t}\boldsymbol{R_t}\boldsymbol{D_t} \tag{17}$$

The conditional variances, $\boldsymbol{H_t}$ is assumed to follow the GJR-GARCH models given by:

$$\boldsymbol{\sigma_{i,t}^2} = \boldsymbol{g_i} + \sum_{l=1}^{q} \boldsymbol{\beta_i}\boldsymbol{\sigma_{i,t-l}^2} + \sum_{m=1}^{p} (\boldsymbol{\alpha_i} + \boldsymbol{\gamma_i}\boldsymbol{\Psi_{i,t-m}})\boldsymbol{\epsilon_{i,t-m}^{(2)}} \tag{18}$$

where $\boldsymbol{w_i}, \boldsymbol{\alpha_i}, \boldsymbol{\beta_i}, \boldsymbol{\gamma_i} > 0$, and $\Psi_{i,t-m}$ is an indicator function such that $\Psi_{i,t-m} = 1$ if $\boldsymbol{\epsilon_{i,t}} < 0$ and 0 otherwise. In this model, the equation (14) can be extended to consider the asymmetries which may be written as following:

$$\boldsymbol{Q_t} = (\bar{\boldsymbol{Q}} - \boldsymbol{A'}\bar{\boldsymbol{Q}}\boldsymbol{A} - \boldsymbol{B'}\bar{\boldsymbol{Q}}\boldsymbol{B} - \boldsymbol{G'}\bar{\boldsymbol{N}}\boldsymbol{G}) + \boldsymbol{A'}\boldsymbol{z_{t-1}}\boldsymbol{z_{t-1}^{'}}\boldsymbol{A} + \boldsymbol{B'}\boldsymbol{Q_{t-1}}\boldsymbol{B} + \boldsymbol{G'}\boldsymbol{z_t}\boldsymbol{z_t'}\boldsymbol{G} \tag{19}$$

where $\bar{\boldsymbol{N}} = E[\boldsymbol{z_t}\boldsymbol{z_t'}]$ and $\boldsymbol{n_t} = \Psi[\boldsymbol{z_{t-1}} < 0] \bigodot \boldsymbol{z_t}$ such that the $\bigodot$ symbol is the element by element product of the residuals if shocks are negative, and $\boldsymbol{n_t} = \boldsymbol{0}$, otherwise. The asymmetric term is denoted by symbol $\boldsymbol{G}$ which captures the period when both markets experience negative shocks, resulting $[\boldsymbol{nn'_t}] = \Psi_t$. The symbols $\boldsymbol{A}, \boldsymbol{B}$, and $\boldsymbol{G}$ represent the diagonal parameter matrices such that $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{G} > 0$ to ensure the positiveness and stationarity of $\boldsymbol{Q_t}$. $\bar{\boldsymbol{Q}} = E[\boldsymbol{z_t z'_t}]$ is a $T \times n$ unconditional matrix of the standardised errors $\boldsymbol{z_t}$.

2.2.3. Flexible Dynamic Condition Correlation (FDCC)

Billio et al. (2006) introduce a Flexible Dynamic Condition Correlation (FDCC) model that allows for equal correlation dynamics between w groups of assets, providing a flexible parameterisation of correlation dynamics. The FDCC model, which is the extension of the DCC model, introduces a block diagonal structure to solve the problem of equal correlations dynamics in the assets. This model can parsimoniously be written as in Billio et al. (2006):

$$
\begin{aligned}
\boldsymbol{x_t} &= \boldsymbol{\mu_t} + \boldsymbol{\epsilon_t} \\
\boldsymbol{H_t} &= \boldsymbol{D_t R_t D_t} \\
\boldsymbol{R_t} &= (\boldsymbol{Q_t^*})^{-1} \boldsymbol{Q_t} (\boldsymbol{Q_t^*})^{-1} \\
\boldsymbol{Q}_t &= \boldsymbol{cc}' + \boldsymbol{aa}' \bigodot \boldsymbol{\eta_t \eta_t'} + \boldsymbol{bb}' \bigodot \boldsymbol{Q_{t-1}}
\end{aligned}
\tag{20}
$$

where $\boldsymbol{\eta_t}$ is the standardised residuals, $\boldsymbol{\eta_t} = \boldsymbol{D_t^{-1} \epsilon_t}$ and the variables $\boldsymbol{c}, \boldsymbol{a}$ and $\boldsymbol{b}$ are partitioned n-dimensional vectors of groups of assets with the following form:

$$
\boldsymbol{a} = [\boldsymbol{a_1} \times \boldsymbol{i'_{m_1}}, \boldsymbol{a_2} \times \boldsymbol{i'_{m_2}}, \ldots, \boldsymbol{a_k} \times \boldsymbol{i'_{m_k}}]'
$$

such that $m_i (i = 1, \ldots, k)$ is the number of assets in the group i (and similarly for $\boldsymbol{b}$ and $\boldsymbol{c}$). i_h is an h-dimensional vector of ones. The coefficients must satisfy these constraints: $\boldsymbol{a_i a_j} + \boldsymbol{b_i b_j} < 1 (i, j = 1, \ldots, k)$ such that k is the number of blocks or asset classes. The dynamics is equal only for group of assets and not for the whole correlation matrix due to the block structure of the coefficients matrices. For the purpose of our analysis, we divided the assets into two blocks - stock indices group and bond indices group. This is reasonable since the correlation dynamics within the stock group is almost similar from one asset to another and this is also the same for bond indices.

2.2.4. Generalised Orthogonal GARCH (GOGARCH)

Another type of multivariate GARCH known as Orthogonal GARCH (OGARCH) was first proposed by Ding (1994) and Alexander and Chibumba (1996). The observed time series can be linearly transformed to a set of uncorrelated time series using a principal component analysis. This model has commonly been used in much research to model the conditional covariance of financial time series due to its feasibility in estimating a large covariance matrices, see for example, Luo et al. (2015) and Weide (2002). For non-Gaussian data, the independent component analysis (ICA) is used to perform the orthogonal transformation. Weide (2002) applies the concept of ICA to propose the generalisation of the OGARCH (GOGARCH) model for volatility modeling. It consists of a set of conditionally uncorrelated univariate GARCH and a linear map that allows the linkage between these components and the observed data (Boswijk and Weide, 2006).

Consider a set of n assets with returns $\boldsymbol{x_t}$ which are observed for T periods, with conditional mean of $E[\boldsymbol{x}_t|\mathcal{F}_{t-1}] = \boldsymbol{m}_t$. $\mathcal{F}_t$ is the information set at time t, which is the σ-algebra generated by the lagged values of the outcome process of $\boldsymbol{x_t}$, that is $\mathcal{F}_t = \boldsymbol{\sigma(x_{t-1}, x_{t-2}, \ldots)}$. The GOGARCH model of Weide (2002) maps $\boldsymbol{x}_t - \boldsymbol{m}_t$ onto a set of unobserved independent factors $\boldsymbol{f_t}$. The process $\boldsymbol{x_t}$ satisfies the representation:

$$\boldsymbol{x_t} = \boldsymbol{m_t} + \boldsymbol{\epsilon_t} \tag{21}$$

$$\boldsymbol{\epsilon_t} = \boldsymbol{C f_t} \tag{22}$$

where $\boldsymbol{f}_t = (f_{1t}, \ldots, f_{nt})^{'}$. $\boldsymbol{C}$ is a non singular matrix which is invertible and constant over time. It may be decomposed into the de-whitening matrix $\boldsymbol{\Sigma}_t^{1/2}$ which is the square root of unconditional covariance and an orthogonal matrix $\boldsymbol{U}_0$. $\boldsymbol{U}_0$ is estimated using a computational method for separating multivariate mixed signals into additive statistically independent and non-Gaussian components using ICA (see Broda and Paolella (2009) and Zhang and Chan (2009)).

Let $\boldsymbol{C}$ be the map that links the uncorrelated components with the unobserved process, so that:

$$\boldsymbol{C} = \boldsymbol{\Sigma}_t^{1/2} \boldsymbol{U_0} \tag{23}$$

The factors are represented as:

$$\boldsymbol{f_t} = \boldsymbol{H}_t^{1/2} \boldsymbol{z_t} \tag{24}$$

such that $\boldsymbol{H_t} = E[\boldsymbol{f}_t \boldsymbol{f}_t^{'}|\mathcal{F}_{t-1}]$ is a diagonal matrix for all t with elements $(h_{1t}, \ldots, h_{nt})$ which are the conditional variances of $\boldsymbol{f_t}$. The symbol $\boldsymbol{z_t}$ is $T \times 1$ vector of independent and identically distributed random errors such that $E[\boldsymbol{z_t}] = 0$ and $E[\boldsymbol{z_t z_t^{'}}] = 1$ which implies that $E[\boldsymbol{f}_t|\mathcal{F}_{t-1}] = \boldsymbol{0}$ and $E[\boldsymbol{\epsilon}_t|\mathcal{F}_{t-1}] = \boldsymbol{0}$. The returns for the GOGARCH model may be expressed as:

$$\boldsymbol{x_t} = \boldsymbol{m} + \boldsymbol{C H}_t^{1/2} \boldsymbol{z_t} \tag{25}$$

For the conditional covariance matrix, it may be written as:

$$\boldsymbol{H_t} = E[(\boldsymbol{x}_t - \boldsymbol{m}_t)(\boldsymbol{x}_t - \boldsymbol{m}_t)'|\mathcal{F}_{t-1}] = \boldsymbol{C}\boldsymbol{H}_t\boldsymbol{C}' \tag{26}$$

2.2.5. Copula GARCH (C-GARCH)

Similar to the idea of DCC models, it is better to describe the multivariate random variable by using a Copula function which capture and model non-linear relationships between the asset returns. In the multivariate GARCH, the model assumes that stock and bond returns follow a multivariate normal or student distribution with linear correlation, and these assumptions are normally disregarded in many empirical finance studies. This study proposes various copula-GARCH based models considering the static version of copulas and dynamic copulas (more realistic in describing time varying dependence structure between assets returns). We then examine the out-of-sample performance and compare it with other models discussed earlier.

Let $F_1(x_1), \ldots, F_n(x_n)$ be the marginal distributions with a random vector $\boldsymbol{X} = (x_1, \ldots, x_n)$. The random vector has uniform marginal distributions when we apply the probability integral transform to each of the component $(U_1, U_2, \ldots, U_n) = F_1(x_1), \ldots, F_n(x_n)$. Sklar (1959) showed that the copula can be depicted as:

$$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)) \tag{27}$$

such that n-dimensional copula $C(u_1, \ldots, u_n)$ is an n-dimensional random vector on $[0,1]^d$ with uniform marginals. The copula can be deduced from equation 27 directly as:

$$C(u_1, \ldots, u_n) = F(F_1^{-1}(u_1), F_2^{-1}(u_2), \ldots, F_n^{-1}(u_n)) \tag{28}$$

The copula of $(X_1, X_2, \ldots, X_n)$ is defined as the joint cumulative distribution function of the continuous marginal distributions which may be written as:

$$C(u_1, \ldots, u_n) = P[F_1(x_1) \leq u_1, F_2(X_2) \leq u_2, \ldots, F_n(x_n) \leq u_n] \tag{29}$$

The density function may be obtained as:

$$f(x_1, \ldots, x_n) = c(F_1(x_1), \ldots, F_n(x_n)) \prod_{i=1}^{n} f_i(x_i) \tag{30}$$

such that f_i are marginal densities and F_i^{-1} is the quantile function of the margins. The density function of a copula is given by:

$$c\left(u_i,\ldots,u_n\right)=\frac{f\left(F_i^{-1}\left(u_i\right),\ldots,F_n^{-1}\left(u_n\right)\right)}{\prod_{i=1}^{n} f_i\left(F_i^{-1}\left(u_i\right)\right)}. \tag{31}$$

We use elliptical copulas which have been used widely in the literature for multivariate volatility modelling, i.e Gaussian and the Student-t copulas. The d-dimensional Gaussian copula $C^{GA}(u_1, u_2, \ldots, u_n)$ is an n-dimensional distribution over the unit hypercube $[0,1]^n$ with uniform margins. The dependence structure is determined by the standardised correlation matrix $\boldsymbol{R}$ such that the dipersion parameter, $\rho_{1,n}$ is estimated using Kendall's τ method. The Gaussian copula is represented by:

$$C^g(u_1,u_2,\ldots,u_n;\boldsymbol{R})=\int_{-\infty}^{\Phi^{-1}(u_1)}\cdots\int_{-\infty}^{\Phi^{-1}(u_n)}\frac{1}{2\pi^{n/2}|\boldsymbol{R}|^{1/2}}\cdot e^{\left\{-\frac{1}{2}\boldsymbol{x}'\boldsymbol{R}^{-1}\boldsymbol{x}\right\}}dx_1,\ldots,dx_n \tag{32}$$

The density of the Gaussian Copula, of the d-dimensional random vector $\boldsymbol{X}$ may be written as in E. Bouye and Roncalli (2000):

$$C^g(u_1,u_2,\ldots,u_n;\boldsymbol{R})=\frac{1}{|\boldsymbol{R}|^{1/2}}e^{\left\{-\frac{1}{2}\boldsymbol{\varsigma}'(\boldsymbol{R}^{-1}-\boldsymbol{I})\boldsymbol{\varsigma}\right\}} \tag{33}$$

where $\varsigma=(\phi^{-1}(u_1),\ldots,\phi^{-1}(u_n))'$ representing the quantile of the Probability Integral Transformed (PIT) values of $\boldsymbol{X}$, f^g is the multivariate density of the normal distribution, f_i is the density of the margin and $\boldsymbol{I}$ is the identity matrix. The Gaussian copula is not able to account for tail dependence. Student-t copula model allows for joint fat tails and an increased probability of joint extreme events as compared to Gaussian copula. This copula may be represented as:

$$C^T(u_1,u_2,\ldots,u_n;\boldsymbol{R},\boldsymbol{\nu})=\int_{-\infty}^{t_\nu^{-1}(u_1)}\cdots\int_{-\infty}^{t_\nu^{-1}(u_n)}\frac{\Gamma(\frac{\nu+n}{2})|\boldsymbol{R}|^{-1/2}}{\Gamma(\frac{\nu}{2})(\nu\pi)^{n/2}}\cdot(1+\frac{1}{\nu}\boldsymbol{x}'\boldsymbol{R}^{-1}\boldsymbol{x})^{-\frac{\nu+n}{2}}dx_1\ldots dx_n \tag{34}$$

and the density of the Student-t copula as:

$$C^T(u_1,u_2,\ldots,u_n)=|\boldsymbol{R}|^{-1/2}\frac{\Gamma(\frac{\nu}{2})}{\Gamma(\frac{\nu+1}{2})}\left(\frac{\Gamma(\frac{\nu+n}{2})}{\Gamma(\frac{\nu}{2})}\right)^n\frac{(1+\frac{1}{\nu}\boldsymbol{\varsigma}'\boldsymbol{R}^{-1}\boldsymbol{\varsigma})^{-\frac{\nu+n}{2}}}{\prod_{j=1}^{n}(1+\frac{\varsigma_j^2}{\nu})^{-\frac{\nu-n}{2}}} \tag{35}$$

where $\varsigma=(t_\nu^{-1}(u_1),\ldots,t_\nu^{-1}(u_n))'$. In Student-t Copula, the dependence structure introduces an additional parameter which is the degree of freedom ν. As the value of ν increases, the tendency to exhibit extreme co-movements decreases. In our study, the Copula-GARCH models are implemented using the multivariate

Normal and Student-t distributions, with static and dynamic (DCC) estimation of the correlation. The margins and PIT estimation are performed using a parametric density approach following Joe (1997). For the dependence measures, we use Kendall's τ as this method are based on order statistics of the sample which makes no assumption about the marginal distribution but depends only on copula C. Kendall's τ is defined as:

$$\tau(X_i, X_j) = Pr[(X_i - X_j) - (Y_i - Y_j) > 0] - Pr[(X_i - X_j) - (Y_i - Y_j) < 0] \tag{36}$$

where $(X_i, Y_i)'$ and $(X_j, Y_j)'$ are vectors of random variables. Kendall's τ measures the difference between the probability of concordant and discordant pairs. The pairwise measure of concordance may be represented in terms of copula functions as:

$$\begin{aligned} \tau(X_i, X_j) &= 4E[C(F_i(X_i), F_j(X_j))] - 1 \\ &= 4\int_0^1\int_0^1 C(u_i, u_j) dC(u_i, u_j) - 1 \end{aligned} \tag{37}$$

We also extend our analysis to include the dynamic copula models as investigated by Patton (2006) such that the vector stochastic process of financial returns be $x_t = x_{it}, \ldots, x_{nt}$ follows a copula GARCH model with $\boldsymbol{\mu_t}$ modeled as a time series model given by:

$$F(x_t|\mu_t, h_t) = C(F_1(x_{1t}|\mu_{1t}, h_{1t}), \ldots, F_n(x_{nt}|\mu_{nt}, h_{nt})) \tag{38}$$

Similar to other DCC models, the conditional variances, $\boldsymbol{H_t}$ can be estimated separately by a simple univariate GARCH specification. For the conditional density, it is given by:

$$C_t(u_{it}, \ldots, u_{nt}|\boldsymbol{R}, \eta) = \frac{f_t\left(F_i^{-1}(u_{it}|\eta), \ldots, F_n^{-1}(u_{nt})|\boldsymbol{R}, \eta\right)}{\prod_{i=1}^{n} f_i\left(F_i^{-1}(u_{it}|\eta)|\eta\right)} \tag{39}$$

where $u_{it} = F_{it}(r_i t|\mu_{it}, h_{it}, \xi_i, \nu_i)$ is the PIT transformation of each series which are estimated using parametric approach via the first stage GARCH process. The symbol $F_i^{-1}(u_{it}|\eta)$ represents the quantile function of the margins subject to the common shape parameter, ν, of the multivariate density function.

3. Optimal Asset Portfolios

Modern portfolio theory proposes how investors should diversify their investments to optimise their portfolio of risky assets. We employ five different optimisation strategies which are the minimum variance, mean-variance, maximising Sharpe ratio, mean-CVaR and minimising Sortino ratio. In the following portfolio problem, we construct efficient portfolios with and without short sales which sets box and group constraints on the weights such that the weights for each assets and the weights of groups of selected assets are restricted by lower and upper bounds; i.e. (i) portfolio without short sales where no more than 30% is invested in each asset (that is, $0 \leq x_i \leq 0.3, i = 1, \ldots, n$) and (ii) portfolio with short sales; where a percentage between -0.3% to 0.3% is invested in each asset (that is, $-0.3 \leq x_i \leq 0.3, i = 1, \ldots, n$) and 50% is the maximum percentage to invest in bonds and stocks.

3.1. Minimum Variance

The asset returns are assumed to be normally distributed and each investor wants to maximise their portfolio return at a minimal risk. This portfolio problem involves quadratic programming with linear constraints such that we want to construct an efficient portfolio with the lowest possible risk. We determine the optimal proportion allocation w_i to the ith asset, where

$$\sum_{i=1}^{n} w_i = 1 \tag{40}$$

with the returns $\boldsymbol{x_i}$ such that $i = 1, 2, \ldots, N$ and $\boldsymbol{x_i} \sim N(\mu_i, \sigma_i^2)$ is an independent and identically distributed random vector. To characterise the portfolio, the expected return of a portfolio is written as:

$$E[R_p] = \sum_{i=1}^{n} \boldsymbol{x_i w_i}$$

and the variance of the portfolio is given by:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \boldsymbol{\sigma_{ij} w_i w_j} = \boldsymbol{w^T Q w}$$

where $\boldsymbol{Q}$ is a $T \times n$ estimate of the covariance matrix of the assets returns. The vector $\boldsymbol{w}$ denotes the weight of the asset subject to the condition of $0 \leq \boldsymbol{w_i} \leq 1$ for portfolio without short sales. To solve the optimisation model, we choose to construct a portfolio of minimal risk:

$$\text{Minimise } \boldsymbol{w^T Q w} \tag{41}$$

subject to $R_p = \sum_{i=1}^{n} \boldsymbol{r_i w_i}$ and $\sum_{i=1}^{n} \boldsymbol{w_i} = \mathbf{1}$.

3.2. Mean-Variance

In a mean variance optimisation model, we want to construct an efficient portfolio with the lowest possible risk such that the return of the portfolio is greater than the target return. The mean-variance portfolios is constructed based on the following optimisation problem:

$$\text{Minimise } \boldsymbol{w^T Q w} \tag{42}$$

subject to $R_p = \sum_{i=1}^{n} \boldsymbol{r_i w_i}$, $R_p \geq r_{target}$ and $\sum_{i=1}^{n} \boldsymbol{w_i} = \boldsymbol{1}$.

3.3. Maximising the Sharpe Ratio

Now, we consider another alternative of optimisation strategy that instead of minimising the risk, we maximise the Sharpe ratio for a given risk free rate. Let r_f be the risk free rate. We consider,

$$\text{Maximise } \frac{\boldsymbol{\hat{\mu}^T w} - r_f}{\boldsymbol{w^T Q w}} \tag{43}$$

subject to $R_p = \sum_{i=1}^{n} \boldsymbol{x_i w_i}$ and $\sum_{i=1}^{n} \boldsymbol{w_i} = \boldsymbol{1}$. The p-dimensional vector $\boldsymbol{\hat{\mu}}$ is the estimates of the expected mean of the assets.

3.4. Mean-CVaR

The mean-CVaR portfolio can be solved using a standard linear programming solver. Given a confidence level β and a fixed $x \in X$, VaR is defined as the smallest number l such that the probability of a loss L is not more than $1 - \beta$ for losses greater than l.

$$VaR_\beta(x) = inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \beta\} = inf\{l \in \mathbb{R} : F_L(l) \geq \beta\} \tag{44}$$

where F_L is the distribution function of the losses. The mean-CVaR optimisation model is formulated as:

$$\text{Minimise CVaR}_\beta(x) = \text{Minimise } E[R | R \leq VaR_\beta(x)] \tag{45}$$

such that $R \geq r_{target}$.

3.5. Minimisation of the Sortino ratio

The last optimisation strategy is the max return/risk portfolio which is calculated by minimising the Sortino ratio for a given risk free rate. The Sortino ratio is a modification to the Sharpe ratio such that this ratio uses downside deviation rather than standard deviation as the measure of risk. The sortino ratio, S is defined as the ratio of the target return lowered by the risk free rate and the CVaR risk. The risk free rate is set at $r_f = 0$.

4. Data and Empirical Results

In this study, we consider 12 assets consisting of eight stocks and four bond indices of 10 years maturity, in the United States (US), United Kingdom (UK), Germany, Japan, Netherlands, Canada and Hong Kong. In particular, our data set consists of monthly observations on eight stock indices, which are, FTSE100 Index, MSCI Europe Excluding UK Index (MSEXUK), S&P 500 composite index (S.PCOMP), DAX30 Index (DAXINDX), AEX Index (AMSTEOE), TOPIX Index (TOKYOSE), Hang Seng Index (HNGKNGI) and TSX composite index (TTOCOMP). We also consider four bond indices, which are, UK Benchmark 10 Year Government Index (BMUK10Y), US Benchmark 10 Year Government Index (BMUS10Y), Germany Benchmark 10 Year Government Index (BMBD10Y), and FTSE Britain Government Linked Bond Index (BGILALL). All time series data were collected from Datastream for 30 years from January 1985 to December 2014 (yielding to 360 observations).

The historical monthly returns of these stocks from the preceding 240 months (from January 1985 to December 2004) are used as the in-sample period to estimate the models. The monthly returns for the considered time period are given as in Figure 1. From the figure, it is evident that the presence of the volatility clustering in the data whereby a low volatility period was followed by a low volatility period for a prolonged time and so does when it is in a high volatility period. Volatility clustering indicates the presence of serial correlation in the asset returns. It is also obvious that the volatility of the series changes over time.

Table 1 presents the basic statistical characteristics of the time series. There are substantial differences in the characteristics for each of the analysed asset return series, as expected; the stock indices have high average returns with a high volatility, while the bond returns have relatively lower average returns and volatilities.

Table 1: Statistical characteristics and Ljung-Box of historical monthly returns for the analysed assets from January 1985 to December 2014. The Ljung-box test is computed using 12 lags.

Assets	Rate of Returns, R_t					Sq. returns, R_t^2	Abs. returns, $\|R_t\|$
	Mean	Stdev	Skewness	Kurtosis	Q(12)	Q(12)	Q(12)
FTSE100	0.0046	0.0455	-1.1387	5.3324	7.1110	8.1196	34.587**
MSEXUK	0.0073	0.0563	-0.8630	1.8616	14.1310	55.498**	72.312**
S.PCOMP	0.0070	0.0444	-1.1008	3.6350	7.7062	20.4750	51.275**
DAXINDX	0.0069	0.0631	-0.9101	2.6948	7.3127	23.453**	35.758**
AMSTEOE	0.0044	0.0581	-1.3154	4.2702	15.0920	43.91**	57.092**
TOKYOSE	0.0012	0.0564	-0.3667	1.2278	16.2010	30.392**	11.320
HNGKNGI	0.0083	0.0782	-1.3354	8.8759	22.723**	3.5307	31.804**
TTOCOMP	0.0050	0.0437	-1.4879	6.1225	15.1270	7.1863	30.113**
BMUK10Y	0.0021	0.0206	-0.1586	1.3692	19.9310	40.983**	37.633**
BMUS10Y	0.0015	0.0218	0.0523	0.9303	16.3820	15.2260	9.5042
BMBD10Y	0.0015	0.0162	-0.2619	0.4050	27.532**	8.5521	11.2980
BGILALL	0.0044	0.0196	0.5009	2.4422	15.7770	23.109**	19.5850

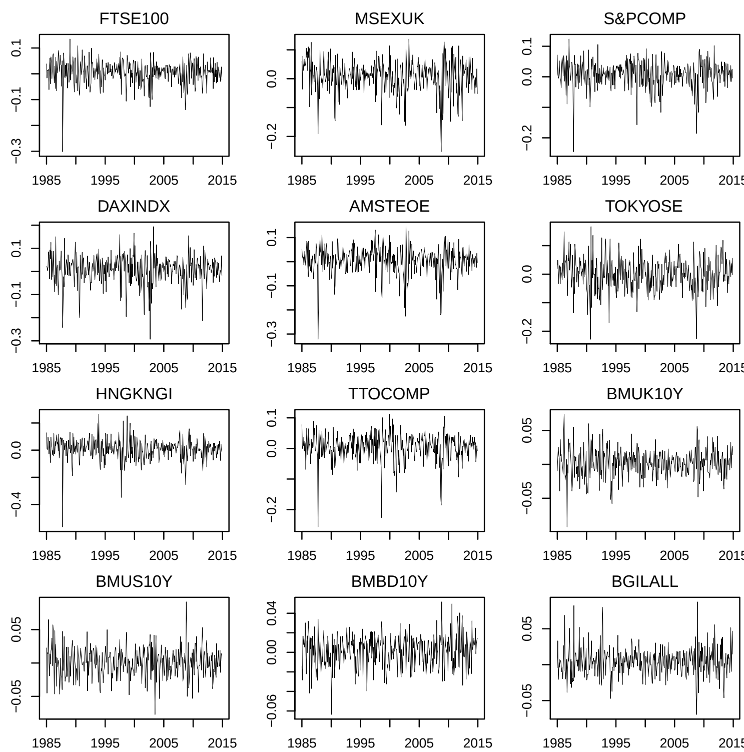

Figure 1: Plots of the monthly analysed asset returns series from January 1985 to December 2014.

The kurtosis for all of the assets ranges from 0.4 to 8.9, indicating fat tails in the asset return distributions. For the skewness, most of the assets are negatively skewed, indicating a distribution with an asymmetric tail extending toward more negative values. The results indicate that the asset returns exhibit skewed distributions, large variance and they are not normally distributed which means that a time varying conditional volatility exists. A normal distribution must be symmetric with excess kurtosis of zero. Note that ** represents the p-value of < 0.05, indicating that the null hypothesis of no autocorrelation is rejected at 95% confidence level (Critical value of 21.026).

We also examine if there is evidence for serial correlations in the asset returns by using the Ljung-Box Q(m) test statistics for the rates of return R_t, absolute rates of return $|R_t|$ and squared rates of return R_t^2 (see Table 1). There is evidence for a high level of autocorrelation in the absolute returns and the squared returns in which the null hypothesis of no autocorrelation is rejected at 5% level of significance for almost all of the assets series (** represents the significant p-value).

To test for normality, we conducted Shapiro test and obtained a p-value of $2.2e^{-16}$ which again rejects the null hypothesis, indicating that the distribution is not normal. Statistical tests on these data indicate that the hypotheses for normality cannot be accepted for the majority of the assets. We use the covariance matrix of the portfolio to quantify the deviation from the expected return and to capture the investment risk, given the standard deviations and the covariance, the correlation can be determined from $\rho_{ij} = \frac{\sigma_i \sigma_j}{\sigma ij}$. Table 2 reports the pairwise correlation coefficients for the analysed asset returns.

Table 2: Correlations for the analysed asset returns series from January 1985 to December 2014.

Assets	FTSE100	MSEXUK	S&PCOMP	DAXINDX	AMSTEOE	TOKYOSE	HNGKNGI	TTOCOMP	BMUK10Y	BMUS10Y	BMBD10Y	BGILALL
FTSE100	1.0000	0.6990	0.7950	0.6840	0.7560	0.4300	0.6300	0.6850	0.1480	0.0736	0.0709	0.2200
MSEXUK	-	1.0000	0.7360	0.7830	0.7270	0.4920	0.5770	0.6530	0.0008	0.0586	0.0772	0.1450
S&PCOMP	-	-	1.0000	0.6910	0.7320	0.4630	0.6100	0.7820	0.0080	0.0259	0.1140	0.1410
DAXINDX	-	-	-	1.0000	0.8140	0.4220	0.5240	0.5970	0.0499	0.1820	0.0997	0.0579
AMSTEOE	-	-	-	-	1.0000	0.4430	0.5680	0.6660	0.0806	0.2210	0.1580	0.0808
TOKYOSE	-	-	-	-	-	1.0000	0.3570	0.4480	0.0409	0.1120	0.0836	0.0668
HNGKNGI	-	-	-	-	-	-	1.0000	0.6500	0.0210	0.0918	0.1180	0.1040
TTOCOMP	-	-	-	-	-	-	-	1.0000	0.0375	0.0776	0.1290	0.1590
BMUK10Y	-	-	-	-	-	-	-	-	1.0000	0.5820	0.7090	0.5820
BMUS10Y	-	-	-	-	-	-	-	-	-	1.0000	0.6930	0.3010
BMBD10Y	-	-	-	-	-	-	-	-	-	-	1.0000	0.3920
BGILALL	-	-	-	-	-	-	-	-	-	-	-	1.0000

It is evident that the correlations within the stock indices and bond indices are relatively high. However, the pairwise correlation coefficients for the remaining pair asset returns exhibit low to medium correlations, indicating a potential for risk diversification in the constructed portfolio. The results confirm the presence of the stylised facts such as heavy tails, volatility clustering and heteroskedasticity in the asset returns distributions. The multivariate GARCH models are suitable to use to deal with these kinds of data.

4.1. Empirical Analysis

The objective of this study is to investigate which econometric model gives the best estimates for the asset return and covariances in constructing optimal portfolio. By using different econometric models, we expect that this approach will be able to capture different characteristics in the data and will have economic impacts to optimal asset portfolio construction. As mentioned in Section 4, we split the data set of 360 observations period into 240 initial in-sample estimation period, for the parameter estimation and model selection to get the forecast for asset return and covariance matrix. Then, we have an initial out-of-sample period of 120 evaluation period, to evaluate the performance of the portfolio over the period. We generate each new forecast by adding new observations and re-estimating the model with the new observations as the data become available. We employ recursive forecast (expanding-window) approach where we use data from January 1985 to December 2004, to make the first estimation in January 2005, data from January 1985 to January 2005 to make the second estimation in February 2005 and so on. We repeat the steps for each of the optimisation strategies namely minimum variance, mean-variance, maximising the Sharpe ratio, Mean-CVaR and minimising Sortino ratio.

4.2. Evaluating the out-of-sample portfolio performance

To measure the performance of the portfolio, we evaluate 120 out-of-sample periods and perform the optimal asset allocation with a difference covariance estimator depending on the time period for each of the optimisation models. We repeat the exercise several times using twenty six different model specifications below while considering efficient portfolios with and without short sales.

- DCC GARCH Normal *(DCC-MVN)*
- DCC GARCH Student *(DCC-MVT)*
- Asymmetric DCC GJR-GARCH Normal *(aDCC-MVN)*
- Asymmetric DCC GJR-GARCH Student *(aDCC-MVT)*
- FDCC GARCH Normal *(FDCC)*
- VAR DCC GARCH Normal *(VAR-MVN)*
- VAR DCC GARCH Student *(VAR-MVT)*
- ARMA DCC GARCH Normal *(ARMA-MVN)*
- ARMA Student *(ARMA-MVT)*
- GOGARCH-MVN *(GG-MVN)*
- ARMA GOGARCH-MVN *(ARMA-GG-MVN)*
- VAR GOGARCH-MVN *(VAR-GG-MVN)*
- Static Copula-MVN *(SCop-MVN)*
- Static Copula-MVT *(SCop-MVT)*
- Static ARMA Copula-MVN *(ARMA-SCop-MVN)*
- Static ARMA Copula-MVT *(ARMA-SCop-MVT)*
- Static VAR Copula-MVN *(VAR-SCop-MVN)*

- Static Asymmetric Copula-MVN *(a-SCop-MVN)*
- Static Asymmetric Copula-MVT *(a-SCop-MVT)*
- Dynamic Copula-MVN *(DCop-MVN)*
- Dynamic Copula-MVT *(DCop-MVT)*
- Dynamic ARMA Copula-MVN *(ARMA-DCop-MVN)*
- Dynamic ARMA Copula-MVT *(ARMA-DCop-MVT)*
- Dynamic VAR Copula-MVN *(VAR-DCop-MVN)*
- Dynamic Asymmetric Copula-MVN *(a-DCop-MVN)*
- Dynamic Asymmetric Copula-MVT *(a-DCop-MVT)*

4.2.1. Efficient minimum variance portfolio without short sales

Table 3 presents the out-of-sample portfolio performance of minimum variance portfolio without short sales and restricting the portfolio weights to be $0 \leq x_i \leq 0.3, i = 1, \ldots, n$ and the investment must be 50% in stocks and 50% in bonds. In general, the empirical results suggest that the dynamic models are able to deliver performance gains over the static models. The best model is the *aDCC-MVT* which accrues significant average monthly return of 0.38%, cumulative return of 45.48% and Sharpe ratio of 0.2013. The next best model is the *VAR-SCop-MVN* model which recorded average monthly return of 0.33%, cumulative return of 39.36% and Sharpe ratio of 0.1766 but with a lower risk (1.88%) as compared to *aDCC-MVT* (1.91%).

Table 3: Descriptive statistics and out-of-sample performance of minimum variance efficient portfolio without short sale for several econometrics models from January 2005 to December 2014.

Model	Return	Cumulative Return	Risk	Sharpe Ratio	VaR@90%	VaR@95%	VaR@99%
DCC-MVN	0.0027	0.3272	0.0191	0.1447	-0.0141	-0.0245	-0.0508
DCC-MVT	0.0031	0.3732	0.0191	0.1643	-0.0159	-0.0215	-0.0625
aDCC-MVN	0.0033	0.4007	0.0192	0.1723	-0.0137	-0.0238	-0.0627
aDCC-MVT	0.0038	0.4548	0.0192	0.2013	-0.0151	-0.0239	-0.0663
FDCC-MVN	0.0027	0.3223	0.0192	0.1420	-0.0142	-0.0244	-0.0507
VAR-MVN	0.0026	0.3081	0.0191	0.1329	-0.0136	-0.0246	-0.0681
VAR-MVT	0.0023	0.2780	0.0188	0.1207	-0.0141	-0.0252	-0.0611
ARMA-MVN	0.0027	0.3228	0.0194	0.1430	-0.0140	-0.0229	-0.0513
ARMA-MVT	0.0027	0.3286	0.0188	0.1452	-0.0138	-0.0235	-0.0673
GG-MVN	0.0031	0.3665	0.0188	0.1653	-0.0155	-0.0255	-0.0652
ARMA-GG-MVN	0.0026	0.3119	0.0188	0.1371	-0.0165	-0.0275	-0.0656
VAR-GG-MVN	0.0026	0.3104	0.0184	0.1428	-0.0160	-0.0299	-0.0612
SCop-MVN	0.0022	0.2696	0.0191	0.1209	-0.0195	-0.0331	-0.0605
SCop-MVT	0.0021	0.2526	0.0201	0.1012	-0.0212	-0.0320	-0.0663
ARMA-SCop-MVN	0.0020	0.2388	0.0189	0.1060	-0.0191	-0.0315	-0.0656
ARMA-SCop-MVT	0.0021	0.2482	0.0187	0.1109	-0.0187	-0.0285	-0.0689
VAR-SCop-MVN	0.0033	0.3936	0.0188	0.1766	-0.0184	-0.0299	-0.0623
a-SCop-MVN	0.0022	0.2639	0.0191	0.1148	-0.0183	-0.0333	-0.0613
a-SCop-MVT	0.0020	0.2453	0.0201	0.0975	-0.0209	-0.0303	-0.0673
DCop-MVN	0.0024	0.2848	0.0191	0.1253	-0.0180	-0.0333	-0.0645
DCop-MVT	0.0024	0.2915	0.0203	0.1196	-0.0207	-0.0334	-0.0718
ARMA-DCop-MVN	0.0025	0.2968	0.0193	0.1298	-0.0177	-0.0334	-0.0777
ARMA-DCop-MVT	0.0027	0.3265	0.0192	0.1379	-0.0163	-0.0314	-0.0707
VAR-DCop-MVN	0.0026	0.3147	0.0189	0.1373	-0.0194	-0.0311	-0.0670
a-DCop-MVN	0.0024	0.2841	0.0191	0.1248	-0.0118	-0.0333	-0.0644
a-DCop-MVT	0.0024	0.2917	0.0203	0.1196	-0.0207	-0.0334	-0.0717

Table 4 presents the average weight of an efficient portfolio without short sales using the minimum variance. We observe that all models invested mainly in *BMBD10Y* which is a bond index, ranging from 20% to 30% out of the overall investment. The next biggest allocation then goes mostly to stock indices such as *FTSE100, S&PCOMP* and *TTOCOMP* with around 10% to 22% of total investment. Note that, most of the models allocate only a very small percentage to *DAXINDX* with only 0 to 1% out of total investments.

4.2.2. Efficient minimum variance portfolio with short sales

The results presented in Table 5 are based on the out-of-sample performance of the constructed minimum variance efficient portfolio with short sales based on the constraints of the portfolio weights to be $-0.3 \leq x_i \leq 0.3, i = 1, \ldots, n$ and the investment must be made at most 50% in stocks and 50% in bonds.

Table 4: Average weights of the analysed assets of minimum variance efficient portfolio without short sale for several econometrics models from January 2005 to December 2014.

Model	FTSE100	MSEXUK	S&PCOMP	DAXINDX	AMSTEOE	TOKYOSE	HNGKNGI	TTOCOMP	BMUK10Y	BMUS10Y	BMBD10Y	BGILALL
DCC-MVN	0.1218	0.0041	0.1466	0.0098	0.0055	0.0795	0.0055	0.1271	0.0758	0.1074	0.2860	0.0309
DCC-MVT	0.0951	0.0043	0.1465	0.0132	0.0113	0.0660	0.0056	0.1580	0.0829	0.1067	0.2701	0.0403
aDCC-MVN	0.1360	0.0027	0.1400	0.0045	0.0109	0.0636	0.0124	0.1300	0.0980	0.1039	0.2695	0.0285
aDCC-MVT	0.0832	0.0097	0.1081	0.0107	0.0104	0.0815	0.0232	0.1732	0.1226	0.1111	0.2537	0.0125
FDCC-MVN	0.1225	0.0047	0.1468	0.0100	0.0063	0.0817	0.0058	0.1222	0.0706	0.1065	0.2868	0.0361
VAR-MVN	0.0869	0.0094	0.1553	0.0084	0.0046	0.0884	0.0042	0.1428	0.0565	0.1151	0.2974	0.0309
VAR-MVT	0.0970	0.0101	0.1571	0.0083	0.0039	0.0856	0.0013	0.1365	0.0712	0.1277	0.2916	0.0096
ARMA-MVN	0.1153	0.0051	0.1480	0.0101	0.0050	0.0786	0.0123	0.1256	0.0676	0.1016	0.2843	0.0464
ARMA-MVT	0.1078	0.0020	0.1463	0.0081	0.0061	0.0786	0.0045	0.1466	0.0873	0.1090	0.2906	0.0131
GG-MVN	0.2214	0.0009	0.0161	0.0001	0.0007	0.1070	0.0018	0.1519	0.0202	0.1395	0.2615	0.0789
AR-GG-MVN	0.2111	0.0016	0.0293	0.0000	0.0002	0.1104	0.0006	0.1469	0.0155	0.1254	0.2720	0.0871
VAR-GG-MVN	0.2117	0.0012	0.0211	0.0000	0.0002	0.0925	0.0000	0.1733	0.0012	0.1481	0.2972	0.0535
SCop-MVN	0.1292	0.0031	0.1407	0.0066	0.0185	0.0909	0.0066	0.1047	0.1186	0.0828	0.2875	0.0111
SCop-MVT	0.0897	0.0137	0.1913	0.0049	0.0461	0.1065	0.0008	0.0469	0.0890	0.0591	0.2834	0.0685
ARMA-SCop-MVN	0.1139	0.0000	0.1321	0.0083	0.0070	0.0901	0.0091	0.1393	0.0485	0.1367	0.2992	0.0155
ARMA-SCop-MVT	0.1071	0.0012	0.1238	0.0089	0.0100	0.0910	0.0006	0.1575	0.0475	0.1446	0.2993	0.0086
VAR-SCop-MVN	0.1271	0.0000	0.1939	0.0000	0.0049	0.0555	0.0000	0.1191	0.0270	0.0937	0.3000	0.0793
a-SCop-MVN	0.1297	0.0032	0.1407	0.0061	0.0184	0.0899	0.0061	0.1058	0.1218	0.0780	0.2902	0.0100
a-SCop-MVT	0.0896	0.0153	0.1913	0.0056	0.0443	0.1061	0.0005	0.0472	0.0896	0.0581	0.2837	0.0686
DCop-MVN	0.1410	0.0053	0.1225	0.0047	0.0226	0.0961	0.0055	0.1024	0.1053	0.0875	0.2962	0.0110
DCop-MVT	0.0917	0.0200	0.1720	0.0040	0.0532	0.1099	0.0014	0.0479	0.0732	0.0621	0.2903	0.0744
ARMA-DCop-MVN	0.1578	0.0056	0.1049	0.0058	0.0156	0.0947	0.0044	0.1112	0.0684	0.0798	0.2959	0.0559
ARMA-DCop-MVT	0.1318	0.0035	0.1081	0.0077	0.0245	0.0880	0.0024	0.1341	0.0734	0.1817	0.2111	0.0338
VAR-Cop-MVN	0.2021	0.0007	0.1151	0.0099	0.0000	0.1042	0.0000	0.0676	0.0567	0.1048	0.3000	0.0384
a-DCop-MVN	0.1409	0.0052	0.1233	0.0047	0.0221	0.0948	0.0054	0.1035	0.1078	0.0850	0.2958	0.0114
a-DCop-MVT	0.0919	0.0198	0.1723	0.0040	0.0532	0.1095	0.0014	0.0479	0.0737	0.0613	0.2903	0.0746

Table 5: Descriptive statistics and out-of-sample performance of minimum variance efficient portfolio with short sale for several econometrics models from January 2005 to December 2014.

Model	Return	Cumulative Return	Risk	Sharpe Ratio	VaR@90%	VaR@95%	VaR@99%
DCC-MVN	0.0029	0.3444	0.0187	0.1611	-0.0138	-0.0219	-0.0317
DCC-MVT	0.0039	0.4670	0.0187	0.2148	-0.0107	-0.0160	-0.0374
aDCC-MVN	0.0038	0.4567	0.0187	0.2100	-0.0114	-0.0167	-0.0462
aDCC-MVT	0.0048	0.5734	0.0189	0.2603	-0.0098	-0.0163	-0.0432
FDCC-MVN	0.0029	0.3474	0.0189	0.1596	-0.0139	-0.0205	-0.0317
VAR-MVN	0.0029	0.3439	0.0185	0.1569	-0.0139	-0.0199	-0.0409
VAR-MVT	0.0027	0.3269	0.0181	0.1508	-0.0118	-0.0188	-0.0404
ARMA-MVN	0.0030	0.3588	0.0191	0.1644	-0.0116	-0.0186	-0.0281
ARMA-MVT	0.0034	0.4025	0.0184	0.1870	-0.0114	-0.0192	-0.0392
GG-MVN	0.0039	0.4676	0.0186	0.2127	-0.0134	-0.0228	-0.0379
AR-GG-MVN	0.0038	0.4513	0.0189	0.1980	-0.0164	-0.0226	-0.0428
VAR-GG-MVN	0.0035	0.4213	0.0183	0.1915	-0.0125	-0.0242	-0.0364
SCop-MVN	0.0020	0.2380	0.0184	0.1133	-0.0199	-0.0242	-0.0534
SCop-MVT	0.0018	0.2147	0.0205	0.0817	-0.0194	-0.0311	-0.0624
ARMA-SCop-MVN	0.0021	0.2525	0.0181	0.1174	-0.0161	-0.0289	-0.0532
ARMA-SCop-MVT	0.0021	0.2482	0.0178	0.1170	-0.0169	-0.0248	-0.0550
VAR-SCop-MVN	0.0035	0.4162	0.0172	0.2002	-0.0167	-0.0274	-0.0610
a-SCop-MVN	0.0019	0.2251	0.0184	0.1069	-0.0193	-0.0237	-0.0559
a-SCop-MVT	0.0018	0.2156	0.0205	0.0806	-0.0203	-0.0289	-0.0645
DCop-MVN	0.0017	0.2088	0.0187	0.0988	-0.0166	-0.0243	-0.0630
DCop-MVT	0.0016	0.1917	0.0211	0.0761	-0.0205	-0.0352	-0.0793
ARMA-DCop-MVN	0.0022	0.2689	0.0195	0.1188	-0.0152	-0.0206	-0.0804
ARMA-DCop-MVT	0.0024	0.2921	0.0188	0.1248	-0.0164	-0.0224	-0.0642
VAR-Cop-MVN	0.0027	0.3261	0.0186	0.1389	-0.0201	-0.0262	-0.0487
a-DCop-MVN	0.0017	0.2082	0.0187	0.0984	-0.0166	-0.0245	-0.0624
a-DCop-MVT	0.0016	0.1906	0.0211	0.0755	-0.0205	-0.0353	-0.0794

By using these constraints, we observe that the multivariate *aDCC-MVT* once again outperforms the other models. The *aDCC-MVT* has the highest cumulative return of 57.34% with the Sharpe ratio of 0.2603 but at a higher risk of 1.89%. On the other hand, the *GOGARCH-MVN* and *DCC-MVT* models both produce a high average return at a lower risk. By incorporating the copula function, we can see that it does not bring any improvement to the model performance. For mean-variance portfolios with short sales, the *GOGARCH*, *DCC* and *aDCC* models perform well compared to the other models.

Similar to the portfolio without short sales, all models have invested mainly in bonds (the highest allocation is to asset *BMBD10Y* with about 30% of the investment), while a substantial fraction of the investment is allocated to stock indices (see Table 6). This is expected since bond indices are known to be 'safer' as they have medium volatilities with lower average returns. If the investor is risk-averse, then he or she will take the risk of investing in a higher risk portfolio with a hope of getting a better return from the investments. Note that the allocation to *DAXINDX*, *AMSTEOE*, *HNGKNGI*, *BMUK10Y* and *BGILALL* are different across the models.

Table 6: Average weights of the analysed assets of minimum variance efficient portfolio with short sale for several econometrics models from January 2005 to December 2014.

Model	FTSE100	MSEXUK	S&PCOMP	DAXINDX	AMSTEOE	TOKYOSE	HNGKNGI	TTOCOMP	BMUK10Y	BMUS10Y	BMBD10Y	BGILALL
DCC-MVN	0.1792	-0.0460	0.1446	0.0246	-0.0100	0.0860	-0.0188	0.1403	0.0618	0.1063	0.2954	0.0364
DCC-MVT	0.1409	-0.0582	0.1484	0.0371	0.0064	0.0726	-0.0179	0.1706	0.0567	0.1110	0.2784	0.0540
aDCC-MVN	0.1958	-0.0378	0.1504	-0.0013	-0.0185	0.0726	-0.0015	0.1402	0.0887	0.0864	0.2933	0.0316
aDCC-MVT	0.1155	-0.0346	0.1121	0.0211	0.0022	0.0866	0.0153	0.1819	0.1239	0.1004	0.2752	0.0005
FDCC-MVN	0.1810	-0.0389	0.1403	0.0267	-0.0134	0.0867	-0.0188	0.1363	0.0349	0.1096	0.2989	0.0566
VAR-MVN	0.1599	-0.0501	0.1360	0.0398	-0.0171	0.0920	-0.0241	0.1636	0.0401	0.1320	0.2982	0.0297
VAR-MVT	0.1573	-0.0530	0.1504	0.0359	-0.0129	0.0896	-0.0282	0.1608	0.0710	0.1379	0.2956	-0.0045
ARMA-MVN	0.1719	-0.0482	0.1441	0.0306	-0.0185	0.0879	0.0031	0.1292	0.0435	0.1037	0.2911	0.0616
ARMA-MVT	0.1610	-0.0552	0.1505	0.0208	0.0059	0.0860	-0.0191	0.1502	0.0767	0.1128	0.2969	0.0136
GG-MVN	0.2742	-0.0370	0.0238	-0.0377	-0.0072	0.1223	-0.0259	0.1875	-0.0827	0.1590	0.2930	0.1306
AR-GG-MVN	0.2648	-0.0192	0.0330	-0.0491	0.0034	0.1246	-0.0328	0.1753	-0.1040	0.1643	0.2940	0.1457
VAR-GG-MVN	0.2664	-0.0590	0.0402	-0.0299	0.0148	0.1027	-0.0295	0.1943	-0.1511	0.2207	0.3000	0.1304
SCop-MVN	0.1806	-0.0415	0.1245	0.0190	0.0148	0.0929	-0.0296	0.1395	0.1388	0.0876	0.2987	-0.0251
SCop-MVT	0.1367	0.0030	0.1753	-0.0122	0.0778	0.1059	-0.0306	0.0441	0.0117	0.0847	0.2866	0.1169
ARMA-SCop-MVN	0.1756	-0.0734	0.1250	0.0210	-0.0003	0.0998	-0.0040	0.1562	0.0332	0.1534	0.2998	0.0136
ARMA-SCop-MVT	0.1635	-0.0658	0.1181	0.0156	0.0221	0.0980	-0.0245	0.1731	0.0371	0.1693	0.2987	-0.0051
VAR-SCop-MVN	0.1845	-0.1332	0.2236	0.0108	0.0511	0.0700	-0.0443	0.1376	-0.0050	0.1143	0.3000	0.0908
a-SCop-MVN	0.1808	-0.0404	0.1261	0.0200	0.0127	0.0923	-0.0305	0.1390	0.1421	0.0820	0.2996	-0.0237
a-SCop-MVT	0.1366	0.0047	0.1758	-0.0116	0.0746	0.1057	-0.0308	0.0449	0.0108	0.0834	0.2872	0.1186
DCop-MVN	0.1936	-0.017	0.0956	-0.0103	0.0372	0.0988	-0.0301	0.1324	0.1460	0.0990	0.3000	-0.0450
DCop-MVT	0.1433	0.0325	0.1451	-0.0452	0.1045	0.1101	-0.0345	0.0441	-0.0079	0.0874	0.2917	0.1288
ARMA-DCop-MVN	0.2015	-0.0124	0.0771	0.0191	-0.0072	0.1017	-0.0074	0.1275	0.0503	0.0815	0.2986	0.0696
ARMA-DCop-MVT	0.1809	-0.0456	0.0755	0.0010	0.0584	0.0941	-0.0115	0.1471	0.0666	0.2071	0.2143	0.0120
VAR-Cop-MVN	0.2464	0.0008	0.0674	0.0491	-0.0675	0.1051	-0.0494	0.1482	0.0423	0.1007	0.3000	0.0570
a-DCop-MVN	0.1938	-0.0198	0.0965	-0.0091	0.0371	0.0981	-0.0308	0.1343	0.1493	0.0968	0.3000	-0.0461
a-DCop-MVT	0.1439	0.0321	0.1453	-0.0454	0.1048	0.1099	-0.0350	0.0444	-0.0072	0.0864	0.2918	0.1290

4.2.3. Efficient mean-variance portfolio without short sales

Similarly, using the same constraints, we obtained the results of the out of sample performance of mean-variance optimisation without short sales as presented in Table 7. Given a target return, $r_{target} = 0.33\%$, we observe that the multivariate *DCC-MVT* has the highest cumulative return of 40.99% and achieve a higher sharpe ratio of 0.1528. Next, we present the average weight of an efficient portfolio without short sales using the mean variance optimisation. On the basis of the results in Table 8, similarly to minimum variance portfolios, we observe that all models invested mainly in the bond index *BMBD10Y*, ranging from 26% to 30% out of the overall investment. *FTSE100* also has a high allocation at around 10% to 22% across all modeling approaches. Most of the model do not invest in the stock *HNGKNGI* which have high volatility while some model allocate a very small proportion (around 1% only).

Table 7: Descriptive statistics and out-of-sample performance of mean-variance efficient portfolio without short sale for several econometrics models from January 2005 to December 2014.

Model	Return	Cumulative Return	Risk	Sharpe Ratio	VaR@90%	VaR@95%	VaR@99%
DCC-MVN	0.0031	0.3672	0.0203	0.1530	-0.0199	-0.0240	-0.0867
DCC-MVT	0.0034	0.4099	0.0213	0.1528	-0.0192	-0.0338	-0.0742
aDCC-MVN	0.0033	0.4007	0.0192	0.1723	-0.0137	-0.0238	-0.0627
aDCC-MVT	0.0030	0.3629	0.0210	0.1356	-0.0182	-0.0288	-0.0801
FDCC-MVN	0.0031	0.1533	0.0203	0.1533	-0.0199	-0.0239	-0.0867
VAR-MVN	0.0038	0.4610	0.0200	0.1852	-0.0195	-0.0240	-0.0723
VAR-MVT	0.0036	0.4379	0.0205	0.1708	-0.0183	-0.0311	-0.0745
ARMA-MVN	0.0028	0.3400	0.0205	0.1457	-0.0193	-0.0258	-0.0857
ARMA-MVT	0.0022	0.2656	0.0189	0.1159	-0.0174	-0.0317	-0.0681
GG-MVN	0.0023	0.2816	0.0188	0.1282	-0.0175	-0.0295	-0.0743
ARMA-GG-MVN	0.0029	0.3440	0.0187	0.1534	-0.0171	-0.0293	-0.0719
VAR-GG-MVN	0.0021	0.2535	0.0184	0.1194	-0.0202	-0.0334	-0.0697
SCop-MVN	0.0022	0.2690	0.0191	0.1222	-0.0189	-0.0332	-0.0610
SCop-MVT	0.0022	0.2593	0.0201	0.1038	-0.0202	-0.0326	-0.0666
ARMA-SCop-MVN	0.0018	0.2157	0.0192	0.0991	-0.0186	-0.0332	-0.0737
ARMA-SCop-MVT	0.0021	0.2519	0.0186	0.1135	-0.0189	-0.0301	-0.0699
VAR-SCop-MVN	0.0033	0.3919	0.0201	0.1624	-0.0211	-0.0302	-0.0589
a-SCop-MVN	0.0022	0.2611	0.0191	0.1176	-0.0194	-0.0336	-0.0598
a-SCop-MVT	0.0022	0.2609	0.0201	0.1039	-0.0205	-0.0516	-0.0645
DCop-MVN	0.0026	0.3142	0.0194	0.1293	-0.0160	-0.0263	-0.0674
DCop-MVT	0.0024	0.2824	0.0202	0.1167	-0.0208	-0.0329	-0.0717
ARMA-DCop-MVN	0.0026	0.3075	0.0195	0.1210	-0.0164	-0.0270	-0.0659
ARMA-DCop-MVT	0.0027	0.3265	0.0192	0.1379	-0.0163	-0.0314	-0.0707
VAR-DCop-MVN	0.0023	0.2804	0.0189	0.1238	-0.0180	-0.0320	-0.0671
a-DCop-MVN	0.0026	0.3133	0.0194	0.1283	-0.0164	-0.0266	-0.0672
a-DCop-MVT	0.0024	0.2823	0.0202	0.1167	-0.0207	-0.0329	-0.0717

Table 8: Average weights of the analysed assets of mean-variance efficient portfolio without short sale for several econometrics models from January 2005 to December 2014.

Model	FTSE100	MSEXUK	S&PCOMP	DAXINDX	AMSTEOE	TOKYOSE	HNGKNGI	TTOCOMP	BMUK10Y	BMUS10Y	BMBD10Y	BGILALL
DCC-MVN	0.1044	0.0554	0.0666	0.0034	0.0284	0.0924	0.0000	0.1495	0.0312	0.0942	0.2948	0.0799
DCC-MVT	0.0996	0.0715	0.1185	0.1185	0.0237	0.0681	0.0001	0.1162	0.0085	0.0567	0.2646	0.1701
aDCC-MVN	0.1360	0.0027	0.1400	0.1400	0.0109	0.0636	0.0124	0.1300	0.0980	0.1039	0.2695	0.0285
aDCC-MVT	0.1224	0.0806	0.0739	0.0739	0.0176	0.0726	0.0005	0.1302	0.0202	0.1224	0.2402	0.1171
FDCC-MVN	0.1044	0.0553	0.0670	0.0670	0.0288	0.0921	0.0000	0.1490	0.0326	0.0937	0.2957	0.0779
VAR-MVN	0.1688	0.0470	0.0657	0.0657	0.0020	0.0914	0.0000	0.1237	0.0026	0.1059	0.2897	0.1017
VAR-MVT	0.1038	0.0640	0.0792	0.0792	0.0113	0.1033	0.0000	0.1312	0.0087	0.0953	0.2872	0.1088
ARMA-MVN	0.1102	0.0564	0.0666	0.0666	0.0249	0.0921	0.0008	0.1457	0.0270	0.0833	0.2894	0.1003
ARMA-MVT	0.1078	0.0020	0.1463	0.1463	0.0063	0.0788	0.0045	0.1461	0.0884	0.1076	0.2906	0.0135
GG-MVN	0.2235	0.0004	0.0146	0.0146	0.0004	0.1094	0.0011	0.1506	0.0224	0.1340	0.2611	0.0825
AR-GG-MVN	0.2087	0.0022	0.0290	0.0290	0.0003	0.1076	0.0007	0.1474	0.0147	0.1279	0.2697	0.0836
VAR-GG-MVN	0.2108	0.0011	0.0222	0.0222	0.0001	0.0924	0.0000	0.1733	0.1733	0.1459	0.2976	0.0551
SCop-MVN	0.1284	0.0035	0.1417	0.0071	0.0186	0.0908	0.0064	0.1037	0.1202	0.0824	0.2871	0.0103
SCop-MVT	0.0884	0.0152	0.1897	0.0059	0.0473	0.1074	0.0007	0.0454	0.0878	0.0594	0.2841	0.0687
ARMA-SCop-MVN	0.1503	0.0055	0.1071	0.0087	0.0174	0.0918	0.0049	0.1144	0.0821	0.0857	0.2879	0.0444
ARMA-SCop-MVT	0.1082	0.0013	0.1241	0.0082	0.0112	0.0908	0.0004	0.1558	0.0451	0.1483	0.2979	0.0087
VAR-SCop-MVN	0.0822	0.0000	0.2220	0.0451	0.0000	0.1436	0.0000	0.0072	0.0554	0.0525	0.3000	0.0922
a-SCop-MVN	0.1294	0.0031	0.1430	0.0067	0.0178	0.0895	0.0064	0.1042	0.1209	0.0783	0.2894	0.0114
a-SCop-MVT	0.0905	0.0145	0.1908	0.0046	0.0455	0.1064	0.0006	0.0471	0.0890	0.0572	0.2837	0.0702
DCop-MVN	0.1358	0.0027	0.1165	0.0083	0.0170	0.0918	0.0083	0.1197	0.0831	0.1763	0.2062	0.0344
DCop-MVT	0.0950	0.0187	0.1695	0.0044	0.0513	0.1095	0.0007	0.0509	0.0745	0.0633	0.2906	0.0715
ARMA-DCop-MVN	0.1342	0.0024	0.1174	0.0079	0.0166	0.0901	0.0141	0.1173	0.0777	0.1744	0.2073	0.0406
ARMA-DCop-MVT	0.1318	0.0035	0.1080	0.0077	0.0245	0.0880	0.0024	0.1341	0.0732	0.1815	0.2118	0.0335
VAR-Cop-MVN	0.0986	0.0001	0.1379	0.0137	0.0115	0.0838	0.0026	0.1519	0.0448	0.1274	0.3000	0.0278
a-DCop-MVN	0.1360	0.0027	0.1187	0.0084	0.0161	0.0868	0.0081	0.1232	0.0911	0.1694	0.2030	0.0364
a-DCop-MVT	0.0950	0.0187	0.1695	0.0044	0.0513	0.1095	0.0007	0.0509	0.0745	0.0633	0.2906	0.0715

4.2.4. Efficient mean-variance portfolio with short sales

For the optimal mean-variance portfolios with the weights constraints of $-0.3 \le x_i \le 0.3, i = 1, \ldots, n$, with a target expected return of 0.34%, the results is presented in 9. We can see that the multivariate *aDCC-MVT*, *VAR-MVT* and *VAR-MVN* outperform the other competing models. In general, these models have higher sharpe ratio of 0.2100 (aDCC-MVN), 0.1716 (VAR-MVN) and 0.1715 (VAR-MVT). These models also have lower values of the corresponding risk measures such as the volatility and the value at risk values.

Table 9: Descriptive statistics and out-of-sample performance of mean-variance efficient portfolio with short sale for several econometrics models from January 2005 to December 2014.

Model	Return	Cumulative Return	Risk	Sharpe Ratio	VaR@90%	VaR@95%	VaR@99%
DCC-MVN	0.0026	0.3081	0.0207	0.1286	-0.0184	-0.0287	-0.1004
DCC-MVT	0.0032	0.3837	0.0220	0.1336	-0.0193	-0.0370	-0.0583
aDCC-MVN	0.0038	0.4567	0.0187	0.2100	-0.0114	-0.0167	-0.0462
aDCC-MVT	0.0026	0.3095	0.0215	0.1095	-0.0220	-0.0270	-0.0767
FDCC-MVN	0.0026	0.3070	0.0207	0.1281	-0.0184	-0.0286	-0.1004
VAR-MVN	0.0038	0.4517	0.0208	0.1716	-0.0148	-0.0268	-0.0789
VAR-MVT	0.0038	0.4580	0.0209	0.1715	-0.0169	-0.0327	-0.0646
ARMA-MVN	0.0026	0.3123	0.0211	0.1568	-0.0180	-0.0279	-0.0942
ARMA-MVT	0.0021	0.2491	0.0184	0.1170	-0.0160	-0.0288	-0.0555
GG-MVN	0.0029	0.3497	0.0186	0.1558	-0.0192	-0.0303	-0.0682
AR-GG-MVN	0.0036	0.4297	0.0189	0.1916	-0.0189	-0.0278	-0.0634
VAR-GG-MVN	0.0030	0.3610	0.0183	0.1613	-0.0191	-0.0294	-0.0646
SCop-MVN	0.0020	0.2379	0.0184	0.1155	-0.0197	-0.0240	-0.0550
SCop-MVT	0.0018	0.2175	0.0206	0.0819	-0.0198	-0.0290	-0.0640
ARMA-SCop-MVN	0.0016	0.1861	0.0193	0.0871	-0.0169	-0.0263	-0.0751
ARMA-SCop-MVT	0.0023	0.2739	0.0178	0.1305	-0.0176	-0.0272	-0.0544
VAR-SCop-MVN	0.0042	0.5008	0.0192	0.2159	-0.0206	-0.0313	-0.0634
a-SCop-MVN	0.0019	0.2310	0.0184	0.1092	-0.0200	-0.0244	-0.0522
a-SCop-MVT	0.0019	0.2241	0.0205	0.0851	-0.0197	-0.0290	-0.0629
DCop-MVN	0.0023	0.2703	0.0189	0.1148	-0.0146	-0.0246	-0.0604
DCop-MVT	0.0015	0.1806	0.0211	0.0713	-0.0210	-0.0352	-0.0791
ARMA-DCop-MVN	0.0022	0.2652	0.0191	0.1112	-0.0160	-0.0225	-0.0606
ARMA-DCop-MVT	0.0024	0.2921	0.0188	0.1248	-0.0164	-0.0224	-0.0642
VAR-Cop-MVN	0.0024	0.2898	0.0181	0.1356	-0.0171	-0.0255	-0.0677
a-DCop-MVN	0.0023	0.2735	0.0189	0.1155	-0.0145	-0.0245	-0.0610
a-DCop-MVT	0.0015	0.1806	0.0211	0.0713	-0.0210	-0.0352	-0.0791

In general, when we allow short sales, we observe that all modeling approaches sell short the stock *HNGKNGI* ranging from -1% to -6%. On the other hands, all models take long positions of their portfolios to *FTSE100*, *S&PCOMP*, *TOKYOSE*, *TTOCOMP*, *BMUK10Y* and *BMUS10Y*.

Table 10: Average weights of the analysed assets of mean-variance efficient portfolio with short sale for several econometrics models from January 2005 to December 2014.

Model	FTSE100	MSEXUK	S&PCOMP	DAXINDX	AMSTEOE	TOKYOSE	HNGKNGI	TTOCOMP	BMUK10Y	BMUS10Y	BMBD10Y	BGILALL
DCC-MVN	0.1518	0.0625	0.0109	-0.0276	0.0640	0.0894	-0.0563	0.2054	-0.0810	0.1408	0.2959	0.1444
DCC-MVT	0.1564	0.0841	0.0793	-0.0355	0.0484	0.0652	-0.0555	0.1576	-0.1407	0.0960	0.2902	0.2546
aDCC-MVN	0.1958	-0.0378	0.1504	-0.0013	-0.0185	0.0726	-0.0015	0.1402	0.0887	0.0864	0.2933	0.0316
aDCC-MVT	0.1836	0.1064	0.0358	-0.0361	0.0187	0.0682	-0.0453	0.1687	-0.1181	0.1524	0.2781	0.1876
FDCC-MVN	0.1514	0.0626	0.0108	-0.0262	0.0638	0.0892	-0.0562	0.2047	-0.0814	0.1412	0.2971	0.1431
VAR-MVN	0.1931	0.0598	0.0103	-0.0088	0.0018	0.0899	-0.0557	0.2096	-0.1840	0.1611	0.2983	0.2246
VAR-MVT	0.1567	0.0733	0.0421	-0.0118	0.0019	0.0978	-0.0557	0.1959	-0.1375	0.1353	0.2993	0.2029
ARMA-MVN	0.1359	0.0660	0.0190	-0.0299	0.0542	0.0938	-0.0283	0.1893	-0.0778	0.1246	0.2953	0.1579
ARMA-MVT	0.1610	-0.0546	0.1505	0.0210	0.0059	0.0862	-0.0190	0.1489	0.0773	0.1113	0.2969	0.0146
GG-MVN	0.2784	-0.0376	0.0164	-0.0375	-0.0063	0.1251	-0.0263	0.1879	-0.0823	0.1581	0.2919	0.1323
AR-GG-MVN	0.2654	-0.0229	0.0347	-0.0506	0.0049	0.1225	-0.0331	0.1791	-0.1040	0.1661	0.2957	0.1422
VAR-GG-MVN	0.2653	-0.0566	0.0407	-0.0306	0.0150	0.1015	-0.0293	0.1941	-0.1553	0.2202	0.3000	0.1351
SCop-MVN	0.1790	-0.0407	0.1252	0.0199	0.0137	0.0927	-0.0297	0.1400	0.1390	0.0862	0.2990	-0.0242
SCop-MVT	0.1341	0.0060	0.1756	-0.0118	0.0775	0.1064	-0.0310	0.0422	0.0118	0.0840	0.2866	0.1176
ARMA-SCop-MVN	0.1980	-0.0307	0.0735	0.0363	-0.0019	0.0971	-0.0115	0.1393	0.0555	0.0916	0.2901	0.00629
ARMA-SCop-MVT	0.1647	-0.0650	0.1170	0.0152	0.0228	0.0977	-0.0252	0.1727	0.0349	0.1728	0.2981	-0.0059
VAR-SCop-MVN	0.1464	-0.0608	0.2402	0.1622	-0.1309	0.1350	-0.0489	0.0567	0.0210	0.0389	0.3000	0.1401
a-SCop-MVN	0.1798	-0.0400	0.1260	0.0188	0.0140	0.0916	-0.0311	0.1409	0.1406	0.0826	0.2999	-0.0231
a-SCop-MVT	0.1379	0.0039	0.1745	-0.0138	0.0788	0.1061	-0.0318	0.0444	0.0104	0.0834	0.2852	0.1210
DCop-MVN	0.1949	-0.0575	0.0868	0.0136	0.0364	0.0987	-0.0032	0.1302	0.0757	0.2006	0.2096	0.0141
DCop-MVT	0.1414	0.0269	0.1450	-0.0444	0.1025	0.1125	-0.0184	0.0343	-0.0063	0.0897	0.2945	0.1221
ARMA-DCop-MVN	0.1911	-0.0588	0.0842	0.0150	0.0330	0.0995	0.0131	0.1229	0.0712	0.1980	0.2122	0.0186
ARMA-DCop-MVT	0.1809	-0.0456	0.0755	0.0010	0.0584	0.0941	-0.0115	0.1471	0.0666	0.2071	0.2143	0.0120
VAR-Cop-MVN	0.1654	-0.0792	0.1199	0.0492	0.0055	0.0887	-0.0301	0.1805	0.0112	0.1494	0.3000	0.0394
a-DCop-MVN	0.1974	-0.0657	0.0898	0.0164	0.0366	0.0959	-0.0057	0.1354	0.0863	0.1944	0.2071	0.0123
a-DCop-MVT	0.1414	0.0269	0.1451	-0.0444	0.1025	0.1125	-0.0184	0.0343	-0.0063	0.0897	0.2945	0.1221

4.2.5. Efficient portfolio based on maximising Sharpe ratio without short sales

In this section, we construct an optimal portfolio based on maximising the Sharpe ratio for a portfolio without short sales. The results are presented in Table 11 assuming the risk free rate, r_f to be 0%. From the results, we observe that the multivariate *AR-GG-MVN* outperforms the other models, by having the highest value in three out of five of the portfolio measurements with a high risk of 2.49%. The other best optimal models are *ARMA-MVT* and *ARMA-MVN*. The results for the maximisation of Sharpe ratio show that it has a better portfolio in terms of higher return and Sharpe ratio as compared to the mean-variance optimisation. However, this optimisation strategy has a higher risk in all of the models when compared to the mean-variance strategy.

Table 11: Descriptive statistics and out-of-sample performance based on maximising Sharpe ratio without short sale for several econometrics models from January 2005 to December 2014.

Model	Return	Cumulative Return	Risk	Sharpe Ratio	VaR@90%	VaR@95%	VaR@99%
DCC-MVN	0.0041	0.4979	0.0235	0.1843	-0.0181	-0.0298	-0.0625
DCC-MVT	0.0049	0.5863	0.0239	0.1507	-0.0169	-0.0273	-0.0698
aDCC-MVN	0.0042	0.5091	0.0246	0.1796	-0.0156	-0.0320	-0.0706
aDCC-MVT	0.0053	0.6311	0.0248	0.2111	-0.0205	-0.0274	-0.0796
FDCC-MVN	0.0042	0.5047	0.0236	0.1864	-0.0181	-0.0300	-0.0625
VAR-MVN	0.0021	0.2469	0.0241	0.0783	-0.0212	-0.0400	-0.0861
VAR-MVT	0.0023	0.2744	0.0240	0.0870	-0.0229	-0.0400	-0.0846
ARMA-MVN	0.0075	0.9058	0.0243	0.3078	-0.0194	-0.0275	-0.0735
ARMA-MVT	0.0086	1.0312	0.0230	0.3474	-0.0153	-0.0215	-0.0518
GG-MVN	0.0047	0.5622	0.0256	0.1801	-0.0258	-0.0351	-0.0807
AR-GG-MVN	0.0132	1.5850	0.0249	0.5195	-0.0155	-0.0263	-0.0667
VAR-GG-MVN	0.0024	0.2938	0.0239	0.0934	-0.0247	-0.0400	-0.0835
SCop-MVN	0.0035	0.4151	0.0191	0.1842	-0.0191	-0.0437	-0.0689
SCop-MVT	0.0034	0.4034	0.0251	0.1354	-0.0220	-0.0428	-0.0674
ARMA-SCop-MVN	0.0049	0.5856	0.0236	0.2066	-0.0202	-0.0417	-0.0728
ARMA-SCop-MVT	0.0046	0.5551	0.0226	0.2088	-0.0202	-0.0373	-0.0826
VAR-SCop-MVN	0.0047	0.5588	0.0243	0.1890	-0.0247	-0.0354	-0.0551
a-SCop-MVN	0.0033	0.4020	0.0249	0.1326	-0.0177	-0.0360	-0.0875
a-SCop-MVT	0.0035	0.4236	0.0251	0.1409	-0.0212	-0.0413	-0.0686
DCop-MVN	0.0038	0.4571	0.0248	0.1508	-0.0181	-0.0426	-0.0915
DCop-MVT	0.0032	0.3844	0.0252	0.1318	-0.0189	-0.0406	-0.0730
ARMA-DCop-MVN	0.0035	0.4221	0.0236	0.1465	-0.0193	-0.0416	-0.0794
ARMA-DCop-MVT	0.0040	0.4763	0.0242	0.1608	-0.0250	-0.0436	-0.0700
VAR-Cop-MVN	0.0043	0.5184	0.0244	0.1906	-0.0223	-0.0341	-0.0763
a-DCop-MVN	0.0040	0.4768	0.0248	0.1567	-0.0166	-0.0385	-0.0982
a-DCop-MVT	0.0030	0.3566	0.0253	0.1245	-0.0196	-0.0403	-0.0700

Table 12 presents the average weight of an efficient portfolio without short sales based on maximising the Sharpe ratio. In mean-variance optimisation, the biggest proportion of investment goes to *BMBD10Y*, while in maximising the Sharpe ratio, all models invested mainly in *BGILALL* which is also a bond index, ranging from 20% to 30% of the overall investment.

Table 12: Average weights of the analysed assets based on maximising Sharpe ratio without short sale for several econometrics models from January 2005 to December 2014.

Model	FTSE100	MSEXUK	S&PCOMP	DAXINDX	AMSTEOE	TOKYOSE	HNGKNGI	TTOCOMP	BMUK10Y	BMUS10Y	BMBD10Y	BGILALL
DCC-MVN	0.0231	0.0677	0.1381	0.0941	0.0119	0.0000	0.0034	0.1617	0.0569	0.0648	0.0798	0.2985
DCC-MVT	0.0127	0.0412	0.1650	0.1063	0.0091	0.0000	0.0795	0.0862	0.0079	0.0130	0.1937	0.2853
aDCC-MVN	0.0206	0.0899	0.1829	0.0628	0.0191	0.0000	0.0313	0.0934	0.0898	0.1176	0.0000	0.2926
aDCC-MVT	0.0113	0.0551	0.1241	0.0770	0.0070	0.0001	0.1462	0.0791	0.0087	0.0000	0.2229	0.2684
FDCC-MVN	0.0224	0.0686	0.1402	0.0942	0.0110	0.0001	0.0037	0.1597	0.0608	0.0515	0.0891	0.2986
VAR-MVN	0.0736	0.0668	0.1135	0.0311	0.0650	0.0366	0.0518	0.0615	0.0922	0.1084	0.0906	0.2088
VAR-MVT	0.0779	0.0663	0.1085	0.0313	0.0684	0.0345	0.0519	0.0613	0.0933	0.1106	0.0932	0.2029
ARMA-MVN	0.0572	0.0295	0.1342	0.0398	0.0116	0.0340	0.1314	0.0623	0.0704	0.0871	0.1134	0.2292
ARMA-MVT	0.0556	0.0238	0.1329	0.0907	0.0160	0.0282	0.0592	0.0936	0.0774	0.0945	0.1178	0.2103
GG-MVN	0.0307	0.1390	0.2296	0.0014	0.0000	0.0000	0.0716	0.0277	0.0881	0.0605	0.0514	0.3000
AR-GG-MVN	0.0392	0.1275	0.1200	0.0085	0.0323	0.0249	0.0565	0.0913	0.0805	0.0718	0.0890	0.2587
VAR-GG-MVN	0.0953	0.0737	0.0909	0.0141	0.0704	0.0327	0.0509	0.0720	0.0756	0.1109	0.0978	0.2157
SCop-MVN	0.0068	0.0548	0.1782	0.0594	0.1200	0.0009	0.0257	0.0542	0.1053	0.0845	0.0103	0.2999
SCop-MVT	0.0165	0.0938	0.2418	0.0501	0.0531	0.0000	0.0226	0.0221	0.1189	0.0479	0.0332	0.3000
ARMA-SCop-MVN	0.0494	0.0189	0.1256	0.0473	0.0090	0.0492	0.1022	0.0984	0.0402	0.0817	0.1212	0.2569
ARMA-SCop-MVT	0.0392	0.0196	0.1559	0.0938	0.0055	0.0356	0.0524	0.0981	0.0362	0.0961	0.1491	0.2186
VAR-SCop-MVN	0.0927	0.1253	0.0681	0.0264	0.0466	0.0416	0.0384	0.0609	0.0859	0.0984	0.1385	0.1772
a-SCop-MVN	0.0047	0.0608	0.1772	0.0570	0.1186	0.0006	0.0253	0.0447	0.1038	0.0824	0.0138	0.3000
a-SCop-MVT	0.0163	0.0964	0.2376	0.0520	0.0519	0.0000	0.0228	0.0230	0.1179	0.0442	0.0379	0.3000
DCop-MVN	0.0061	0.0657	0.1590	0.0437	0.1456	0.0000	0.0249	0.0549	0.1074	0.0795	0.0461	0.2670
DCop-MVT	0.0205	0.1108	0.2228	0.0308	0.0660	0.0000	0.0193	0.0298	0.0894	0.0315	0.0792	0.3000
ARMA-DCop-MVN	0.0582	0.0461	0.1185	0.0101	0.0242	0.1986	0.0280	0.0164	0.0737	0.0784	0.1290	0.2189
ARMA-DCop-MVT	0.0394	0.0661	0.1476	0.1001	0.0276	0.0532	0.0609	0.0052	0.0558	0.0743	0.1593	0.2106
VAR-Cop-MVN	0.1005	0.0715	0.0797	0.0713	0.0238	0.0425	0.0751	0.0356	0.1141	0.1141	0.0944	0.1774
a-DCop-MVN	0.0066	0.0650	0.1538	0.0448	0.1442	0.0000	0.0289	0.0566	0.1116	0.0826	0.0428	0.2630
a-DCop-MVT	0.0170	0.1143	0.2208	0.0319	0.0672	0.0000	0.0159	0.0328	0.1002	0.0292	0.0706	0.3000

4.2.6. Efficient portfolio based on maximising Sharpe ratio with short sales

When we allow short selling, such that the constraints of the portfolio weights to be between $-0.3 \leq x_i \leq 0.3, i = 1, \ldots, n$, we obtained a more attractive portfolio as we have a higher monthly return, higher cumulative return and higher Sharpe ratio in most of the models as compared to the portfolio without short sales. The risk free rate, r_f is assumed to be 0%.

Table 13 presents the results obtained from constructing an efficient Sharpe ratio portfolio with short sales. From the results, we observe that, once again, the multivariate *AR-GG-MVN*, *ARMA-MVN* and *ARMA-MVT*, outperform the other models. The *AR-GG-MVN* and *ARMA-MVN* models have the highest risk of 2.63% among all models whereas *ARMA-MVT* has a slightly lower risk of 2.59%.

Table 13: Descriptive statistics and out-of-sample performance based on maximising Sharpe ratio with short sale for several econometrics models from January 2005 to December 2014.

Model	Return	Cumulative Return	Risk	Sharpe Ratio	VaR@90%	VaR@95%	VaR@99%
DCC-MVN	0.0047	0.5592	0.0249	0.1953	-0.0240	-0.0414	-0.0734
DCC-MVT	0.0064	0.7621	0.0252	0.2483	-0.0137	-0.0290	-0.0420
aDCC-MVN	0.0055	0.6627	0.0285	0.1932	-0.0179	-0.0321	-0.0545
aDCC-MVT	0.0072	0.8675	0.0270	0.2707	-0.0161	-0.0254	-0.0519
FDCC-MVN	0.0046	0.5510	0.0247	0.1945	-0.0130	-0.0221	-0.0554
VAR-MVN	0.0042	0.5033	0.0263	0.1608	-0.0196	-0.0306	-0.0713
VAR-MVT	0.0043	0.5216	0.0263	0.1681	-0.0181	-0.0312	-0.0702
ARMA-MVN	0.0159	1.9050	0.0263	0.5867	-0.0073	-0.0144	-0.0362
ARMA-MVT	0.0159	1.9046	0.0259	0.5961	-0.0039	-0.0145	-0.0292
GG-MVN	0.0061	0.7293	0.0279	0.2096	-0.0247	-0.0402	-0.0566
AR-GG-MVN	0.0244	2.9290	0.0263	0.9171	-0.0032	-0.0101	-0.0289
VAR-GG-MVN	0.0046	0.5485	0.0264	0.1748	-0.0238	-0.0364	-0.0734
SCop-MVN	0.0027	0.3229	0.0184	0.1508	-0.0211	-0.0524	-0.0764
SCop-MVT	0.0025	0.2992	0.0277	0.0965	-0.0257	-0.0513	-0.0758
ARMA-SCop-MVN	0.0053	0.6332	0.0258	0.2224	-0.0219	-0.0383	-0.0698
ARMA-SCop-MVT	0.0064	0.7665	0.0254	0.2668	-0.0223	-0.0423	-0.0632
VAR-SCop-MVN	0.0075	0.8961	0.0247	0.2939	-0.0177	-0.0246	-0.0496
a-SCop-MVN	0.0026	0.3094	0.0272	0.1013	-0.0188	-0.0497	-0.0953
a-SCop-MVT	0.0030	0.3620	0.0278	0.1098	-0.0246	-0.0471	-0.0755
DCop-MVN	0.0034	0.4033	0.0277	0.1143	-0.0174	-0.0452	-0.1005
DCop-MVT	0.0029	0.3462	0.0284	0.1089	-0.0281	-0.0496	-0.0810
ARMA-DCop-MVN	0.0045	0.5428	0.0262	0.1604	-0.0210	-0.0434	-0.0705
ARMA-DCop-MVT	0.0044	0.5320	0.0281	0.1516	-0.0221	-0.0489	-0.0718
VAR-Cop-MVN	0.0042	0.5087	0.0260	0.1802	-0.0262	-0.0344	-0.0748
a-DCop-MVN	0.0033	0.4000	0.0277	0.1180	-0.0222	-0.0403	-0.1063
a-DCop-MVT	0.0026	0.3175	0.0284	0.0997	-0.0279	-0.0500	-0.0816

The average weight of an efficient portfolio with short sales based on maximising the Sharpe ratio is presented in Table 14. Similarly as with the portfolio without short sale, the biggest asset allocation goes to *BGILALL* with about 18% to 30% out of the total investment. Note that some assets have different allocations across the models.

Table 14: Average weights of the analysed assets based on maximising Sharpe ratio with short sale for several econometrics models from January 2005 to December 2014.

Model	FTSE100	MSEXUK	S&PCOMP	DAXINDX	AMSTEOE	TOKYOSE	HNGKNGI	TTOCOMP	BMUK10Y	BMUS10Y	BMBD10Y	BGILALL
DCC-MVN	0.0185	0.0804	0.1548	0.1518	-0.0054	-0.0712	-0.0244	0.1954	0.0210	0.0627	0.1164	0.2999
DCC-MVT	0.0154	0.0245	0.1926	0.1771	-0.0440	-0.0891	0.1052	0.1183	-0.0643	-0.0074	0.2764	0.2953
aDCC-MVN	0.0367	0.1400	0.1783	0.1003	0.0221	-0.0766	0.0393	0.0599	0.2462	0.2258	-0.2691	0.2971
aDCC-MVT	-0.0552	0.0710	0.2029	0.1432	-0.0929	-0.0647	0.1783	0.1174	0.1170	-0.1962	0.3000	0.2792
FDCC-MVN	0.0226	0.0845	0.1551	0.1487	-0.0181	-0.0651	-0.0228	0.1951	0.0267	0.0297	0.1439	0.2996
VAR-MVN	0.0768	0.0411	0.1174	0.0900	0.0133	0.0109	0.0362	0.1143	0.0946	0.0518	0.1625	0.1910
VAR-MVT	0.0687	0.0403	0.1268	0.0888	0.0197	0.0079	0.0376	0.1102	0.0974	0.0561	0.1638	0.1827
ARMA-MVN	0.0556	-0.0205	0.1442	0.1400	-0.0481	0.0004	0.1373	0.0910	0.0470	0.0789	0.1354	0.2388
ARMA-MVT	0.0547	0.0007	0.1852	0.1779	-0.0443	-0.0519	0.0871	0.0905	0.0611	0.0809	0.1297	0.2283
GG-MVN	0.1135	0.2255	0.2815	0.0139	-0.1769	-0.1445	0.0778	0.1092	0.0302	-0.0229	0.1927	0.3000
AR-GG-MVN	0.1113	0.1326	0.1965	-0.0286	-0.0416	-0.0187	0.0420	0.1065	0.0073	0.0614	0.1668	0.2645
VAR-GG-MVN	0.1062	0.0746	0.1000	-0.0023	0.0355	0.0100	0.0394	0.1365	0.0243	0.0673	0.1988	0.2096
SCop-MVN	-0.1040	0.0657	0.1906	0.0932	0.1703	-0.0504	0.0290	0.1055	0.2410	0.1255	-0.1665	0.3000
SCop-MVT	0.0275	0.1408	0.2498	0.0608	0.0919	-0.0605	0.0305	-0.0408	0.1545	0.0832	-0.0376	0.3000
ARMA-SCop-MVN	-0.0068	-0.0230	0.1539	0.1603	-0.0611	0.0218	0.1010	0.1538	0.0234	0.0241	0.1752	0.2773
ARMA-SCop-MVT	-0.0090	-0.0151	0.2127	0.1914	-0.0399	-0.0395	0.0879	0.1114	0.0263	0.0462	0.1726	0.2548
VAR-SCop-MVN	0.0207	0.0146	0.1508	0.0593	0.1014	0.0136	-0.0118	0.1512	0.0183	0.0822	0.2258	0.1736
a-SCop-MVN	-0.0987	0.0682	0.1887	0.0957	0.1622	-0.0547	0.0274	0.1114	0.2329	0.1228	-0.1557	0.3000
a-SCop-MVT	0.0242	0.1447	0.2462	0.0608	0.0945	-0.0652	0.0295	-0.0346	0.1573	0.0778	-0.0351	0.3000
DCop-MVN	-0.1128	0.0938	0.1758	0.0703	0.2126	-0.0739	0.0357	0.0985	0.2158	0.0819	-0.0678	0.2701
DCop-MVT	0.0291	0.1860	0.2295	0.0137	0.1332	-0.0863	0.0272	-0.0325	0.1012	0.0415	0.0576	0.3000
ARMA-DCop-MVN	0.0783	0.0565	0.1603	0.0558	-0.0482	0.2113	0.0514	-0.0654	0.0722	0.0087	0.1641	0.2549
ARMA-DCop-MVT	0.0779	0.0689	0.1821	0.1506	-0.0038	0.0731	0.1182	-0.1669	0.0081	0.0663	0.1750	0.2506
VAR-Cop-MVN	0.1497	0.0790	0.1099	0.0246	-0.0335	0.0561	0.0344	0.0797	0.0837	0.0578	0.1864	0.1721
a-DCop-MVN	-0.1072	0.0984	0.1742	0.0633	0.2088	-0.0753	0.0373	0.1004	0.2038	0.1031	-0.0737	0.2668
a-DCop-MVT	0.0273	0.1901	0.2317	0.0155	0.1291	-0.0880	0.0220	-0.0277	0.1148	0.0199	0.0653	0.3000

4.2.7. Efficient portfolio Mean-CVaR portfolio without short sales

In the Mean-CVaR optimisation, we compute the optimised efficient portfolio which has the lowest risk for a given return. The portfolio is constructed by minimising the conditional value at risk using 95% probability level, with similar restrictions as in other models. When the portfolio is in the long position, the result is presented as in Table 19. The *aDCC-MVN* once again outperform other models with the highest average return of 1.75%, followed by *ARMA-MVN* which recorded average return of 1.73% with a similar volatility of 2.27%.

Table 15: Descriptive statistics and out-of-sample performance based on minimising Sortino ratio without short sale for several econometrics models from January 2005 to December 2014.

Model	Return	Cumulative Return	Risk	Sharpe Ratio	VaR@90%	VaR@95%	VaR@99%
DCC-MVN	0.0158	1.8981	0.0226	0.6754	-0.0090	-0.0184	-0.0495
DCC-MVT	0.0020	0.2420	0.0197	0.1071	-0.0214	-0.0319	-0.0645
aDCC-MVN	0.0175	2.1035	0.0227	0.7545	-0.0068	-0.0156	-0.0492
aDCC-MVT	0.0022	0.2663	0.0181	0.1147	-0.0181	-0.0256	-0.0622
FDCC-MVN	0.0104	1.2471	0.0210	0.4620	-0.0144	-0.0224	-0.0581
VAR-MVN	0.0147	1.7618	0.0226	0.6176	-0.0110	-0.0288	-0.0683
VAR-MVT	0.0102	1.2266	0.0230	0.4409	-0.0011	-0.0208	-0.0766
ARMA-MVN	0.0173	2.0812	0.0227	0.7163	-0.0086	-0.0160	-0.0439
ARMA-MVT	0.0118	1.4152	0.0214	0.4847	-0.0114	-0.0208	-0.0621
GOGARCH-MVN	0.0183	2.1956	0.0232	0.7568	-0.0047	-0.0137	-0.0504
AR-GOGARCH-MVN	0.0150	1.7944	0.0230	0.6033	-0.0124	-0.0207	-0.0513
VAR-GOGARCH-MVN	0.0167	1.9996	0.0234	0.6979	-0.0099	-0.0245	-0.0554
SCop-MVN	0.0023	0.2766	0.0233	0.1025	-0.0262	-0.0396	-0.0886
SCop-MVT	0.0057	0.6888	0.0207	0.2661	-0.0174	-0.0306	-0.0648
ARMA-SCop-MVN	0.0159	1.9050	0.0229	0.6513	-0.0113	-0.0188	-0.0520
ARMA-SCop-MVT	0.0099	1.1857	0.0207	0.4368	-0.0104	-0.0221	-0.0510
VAR-SCop-MVN	0.0150	1.7993	0.0230	0.6125	-0.0091	-0.0191	-0.0728
a-SCop-MVN	0.0142	1.7008	0.0221	0.6080	-0.0111	-0.0173	-0.0514
a-SCop-MVT	0.0062	0.7433	0.0208	0.2889	-0.0170	-0.0312	-0.0671
DCop-MVN	0.0033	0.4001	0.0225	0.1482	-0.0223	-0.0310	-0.0771
DCop-MVT	0.0032	0.3847	0.0210	0.1481	-0.0193	-0.0291	-0.0634
ARMA-DCop-MVN	0.0021	0.2483	0.0222	0.0953	-0.0196	-0.0309	-0.0867
ARMA-DCop-MVT	0.0033	0.3910	0.0236	0.1452	-0.0209	-0.0408	-0.0833
VAR-Cop-MVN	0.0033	0.4009	0.0242	0.1444	-0.0289	-0.0407	-0.0840
a-DCop-MVN	0.0031	0.3684	0.0225	0.1388	-0.0216	-0.0303	-0.0802
a-DCop-MVT	0.0030	0.3550	0.0211	0.1403	-0.0210	-0.0262	-0.0810

The highest asset allocation in this model, mainly goes to stock *TOKYOSE*, which dominating the portfolio with about 10% to 30% of allocation. The other main allocation goes to bond indices such as *BMBD10Y* and *BMUS10Y* with around 8% to 30%. The average weights for the analysed assets are presented in Table 20.

Table 16: Average weights of the analysed assets based on minimising Sortino ratio without short sale for several econometrics models from January 2005 to December 2014.

Model	FTSE100	MSEXUK	S&PCOMP	DAXINDX	AMSTEOE	TOKYOSE	HNGKNGI	TTOCOMP	BMUK10Y	BMUS10Y	BMBD10Y	BGILALL
DCC-MVN	0.0528	0.0205	0.0457	0.0137	0.0304	0.2093	0.0775	0.0500	0.1175	0.1552	0.1860	0.0412
DCC-MVT	0.0984	0.0004	0.0013	0.0000	0.0002	0.2998	0.0000	0.0602	0.0802	0.2983	0.1190	0.0000
aDCC-MVN	0.0469	0.0281	0.0405	0.0222	0.0294	0.1664	0.0707	0.0957	0.0840	0.1222	0.2147	0.0790
aDCC-MVT	0.0035	0.0004	0.0003	0.0003	0.0000	0.2833	0.0000	0.1237	0.0508	0.2532	0.1905	0.0055
FDCC-MVN	0.0620	0.0190	0.0171	0.0127	0.0164	0.2184	0.0175	0.1343	0.0513	0.1535	0.2605	0.0346
VAR-MVN	0.0415	0.0628	0.0618	0.0470	0.0687	0.0676	0.0506	0.0655	0.0769	0.1415	0.1454	0.1289
VAR-MVT	0.0542	0.0285	0.0440	0.0484	0.0615	0.1027	0.0662	0.0943	0.1084	0.1535	0.1623	0.0758
ARMA-MVN	0.0504	0.0520	0.0639	0.0477	0.0502	0.1109	0.0242	0.0946	0.0983	0.1170	0.1640	0.1204
ARMA-MVT	0.0411	0.0247	0.0302	0.0259	0.0411	0.1737	0.0288	0.1095	0.0927	0.1268	0.1858	0.0946
GOGARCH-MVN	0.0302	0.0297	0.0285	0.0540	0.0803	0.1581	0.0500	0.0694	0.0923	0.1291	0.1772	0.1015
AR-GOGARCH-MVN	0.0743	0.0281	0.0338	0.0578	0.0403	0.1258	0.0732	0.0610	0.1020	0.1448	0.1570	0.0962
VAR-GOGARCH-MVN	0.0389	0.0527	0.0536	0.0564	0.0576	0.0905	0.0656	0.0639	0.0992	0.1293	0.1292	0.1380
SCop-MVN	0.0014	0.0000	0.0281	0.0000	0.1685	0.3000	0.0000	0.0021	0.1439	0.0000	0.0000	0.2993
SCop-MVT	0.0719	0.0038	0.0030	0.0041	0.0105	0.2941	0.0021	0.1041	0.0476	0.2077	0.2077	0.2338
ARMA-SCop-MVN	0.0671	0.0557	0.0080	0.0424	0.0777	0.1152	0.0405	0.0869	0.1118	0.1494	0.1494	0.1514
ARMA-SCop-MVT	0.0795	0.0321	0.0297	0.0085	0.0344	0.1645	0.0335	0.0784	0.1234	0.1712	0.1712	0.1521
VAR-SCop-MVN	0.0468	0.0455	0.0807	0.0510	0.0294	0.0860	0.0672	0.0761	0.0982	0.1619	0.1619	0.0930
a-SCop-MVN	0.0758	0.0334	0.0271	0.0186	0.0186	0.2282	0.0394	0.0590	0.1067	0.1513	0.1513	0.1995
a-SCop-MVT	0.0719	0.0060	0.0046	0.0043	0.0110	0.2925	0.0036	0.0991	0.0536	0.1954	0.1954	0.2336
DCop-MVN	0.0042	0.0000	0.0000	0.0000	0.0089	0.3000	0.0000	0.1869	0.0194	0.0050	0.2600	0.2157
DCop-MVT	0.0304	0.0000	0.0000	0.0008	0.0024	0.3000	0.0000	0.1583	0.0079	0.1240	0.2818	0.0863
ARMA-DCop-MVN	0.0645	0.0051	0.0017	0.0060	0.0836	0.2017	0.0102	0.1248	0.1206	0.1194	0.1186	0.1396
ARMA-DCop-MVT	0.0029	0.0041	0.0032	0.0270	0.0663	0.2752	0.0000	0.1213	0.1262	0.0431	0.1068	0.2229
VAR-Cop-MVN	0.0241	0.0487	0.0036	0.0533	0.0620	0.2430	0.0102	0.0374	0.0492	0.0841	0.1260	0.2349
a-DCop-MVN	0.0014	0.0000	0.0000	0.0000	0.0082	0.2992	0.0000	0.1912	0.0149	0.0030	0.2650	0.2171
a-DCop-MVT	0.0315	0.0000	0.0000	0.0000	0.0041	0.3000	0.0000	0.1574	0.0112	0.1238	0.2770	0.0881

4.2.8. Efficient portfolio Mean-CVaR portfolio with short sales

Table 21 reports the monthly average returns of the constructed portfolios, the cumulative returns, risk and value at risk at 90%, 95% and 99% probability levels. The portfolio allows short selling, that is the portfolio is constructed when we restrict the portfolio weights to be $-0.3 \leq x_i \leq 0.3, i = 1, \ldots, n$, with a target expected return of 0.34%. The results shows that the multivariate *ARMA-SCop-MVN* has the highest average return of 4.81% with a higher risk of 3.20%. Note that, model such as a-SCop-MVN has a higher average return of 4.62% and highest sharpe ratio of 1.5803 with a much lower risk of 2.92%.

Table 17: Descriptive statistics and out-of-sample performance based on minimising Sortino ratio with short sale for several econometrics models from January 2005 to December 2014.

Model	Return	Cumulative Return	Risk	Sharpe Ratio	VaR@90%	VaR@95%	VaR@99%
DCC-MVN	0.0465	5.5828	0.0294	1.5766	0.0200	0.0159	0.0027
DCC-MVT	0.0399	4.7859	0.0266	1.4885	0.0135	0.0051	0.004
aDCC-MVN	0.0463	5.5511	0.0293	1.5780	0.0187	0.0091	0.0028
aDCC-MVT	0.0389	4.6720	0.0267	1.4563	0.0100	0.0045	-0.0006
FDCC-MVN	0.0446	5.3541	0.0287	1.5596	0.0187	0.0073	0.0000
VAR-MVN	0.0467	5.5993	0.0314	1.4776	0.0197	0.0051	-0.0056
VAR-MVT	0.0341	4.0925	0.0296	1.1401	0.0000	0.0000	0.0000
ARMA-MVN	0.0470	5.6432	0.0303	1.5548	0.0225	0.0123	-0.0054
ARMA-MVT	0.0443	5.3178	0.0293	1.5102	0.0194	0.0082	0.0014
GOGARCH-MVN	0.0468	5.6163	0.0296	1.5741	0.0202	0.0096	-0.0050
AR-GOGARCH-MVN	0.0464	5.5669	0.0296	1.5603	0.0197	0.0081	-0.0025
VAR-GOGARCH-MVN	0.0473	5.6746	0.0315	1.4939	0.0228	0.0076	-0.0068
SCop-MVN	0.0037	0.4487	0.0293	0.1294	-0.0350	-0.0443	-0.0739
SCop-MVT	0.0408	4.8908	0.0271	1.5015	0.0177	0.0046	-0.0019
ARMA-SCop-MVN	0.0481	5.7749	0.0320	1.5049	0.0242	0.0088	0.0001
ARMA-SCop-MVT	0.0428	5.1337	0.0285	1.4939	0.0171	0.0125	-0.0027
VAR-SCop-MVN	0.0477	5.7228	0.0305	1.5573	0.0249	0.0094	0.0019
a-SCop-MVN	0.0462	5.5412	0.0292	1.5803	0.0233	0.0105	-0.0019
a-SCop-MVT	0.0410	4.9226	0.0273	1.4946	0.0171	0.0062	-0.0068
DCop-MVN	0.0041	0.4881	0.0292	0.1343	-0.0334	-0.0405	-0.0771
DCop-MVT	0.0045	0.5391	0.0307	0.1388	-0.0350	-0.0500	-0.0806
ARMA-DCop-MVN	0.0008	0.0926	0.0299	0.0144	-0.0364	-0.0450	-0.1046
ARMA-DCop-MVT	0.0033	0.3964	0.0317	0.1126	-0.0363	-0.0547	-0.1073
VAR-Cop-MVN	0.0041	0.4975	0.0292	0.1368	-0.0353	-0.0391	-0.0625
a-DCop-MVN	0.0365	4.3811	0.0263	1.4062	0.0121	0.0072	0.0000
a-DCop-MVT	0.0045	0.5382	0.0306	0.1472	-0.0325	-0.0469	-0.1015

Table 22 indicate that the allocation to assets *MSEXUK*, *S&PCOMP*, *DAXINDX*, *AMSTEOE*, *HNGKNGI*, *BMUS10Y* and *BGILALL* are different across models. It is also interesting to note that, only the model *a-DCop-MVT* short sell the asset TOKYOSE when the other models remain in long position. In particular, all model invested heavily in bond indices such as *BMUK10Y* and *BMBD10Y*, with a range of 15% to 22% and 11% to 30% respectively.

Table 18: Average weights of the analysed assets based on minimising Sortino ratio with short sale for several econometrics models from January 2005 to December 2014.

Model	FTSE100	MSEXUK	S&PCOMP	DAXINDX	AMSTEOE	TOKYOSE	HNGKNGI	TTOCOMP	BMUK10Y	BMUS10Y	BMBD10Y	BGILALL
DCC-MVN	0.1293	-0.0114	0.1295	0.0172	0.0064	0.1174	0.0609	0.0508	0.1880	0.1391	0.1977	-0.0248
DCC-MVT	0.1636	-0.0582	0.1244	-0.0360	0.0561	0.2303	-0.1198	0.1394	0.2131	0.1557	0.1700	-0.0388
aDCC-MVN	0.1049	-0.0068	0.0966	0.0522	0.0298	0.1044	0.0265	0.0924	0.1702	0.1040	0.2103	0.0155
aDCC-MVT	0.1615	0.0035	0.0433	0.0606	0.0111	0.2030	-0.1608	0.1778	0.1550	0.1627	0.1793	0.0030
FDCC-MVN	0.1250	0.0336	0.1174	0.0924	-0.0293	0.1208	-0.0567	0.0969	0.1907	0.1445	0.2077	-0.0429
VAR-MVN	0.0565	0.0742	0.1030	0.1039	0.0336	0.0204	0.0174	0.0911	0.1274	0.1112	0.1499	0.1115
VAR-MVT	0.0775	0.0211	0.0334	0.1011	0.0295	0.1398	-0.0243	0.1220	0.1544	0.1076	0.1911	0.0468
ARMA-MVN	0.1012	0.0343	0.1149	0.0972	0.0504	0.0384	-0.0122	0.0758	0.1716	0.0932	0.1275	0.1078
ARMA-MVT	0.0988	0.0190	0.0794	0.0488	0.0221	0.1276	-0.0146	0.1188	0.1700	0.1091	0.1500	0.0708
GOGARCH-MVN	0.0706	0.0399	0.0826	0.0914	0.0490	0.1019	-0.0012	0.0659	0.1691	0.1095	0.1981	0.0233
AR-GOGARCH-MVN	0.1349	-0.0295	0.0977	0.1169	0.0167	0.0837	0.0150	0.0646	0.1492	0.1126	0.1479	0.0904
VAR-GOGARCH-MVN	0.0598	0.0708	0.1054	0.0965	0.0521	0.0419	0.0168	0.0566	0.1520	0.0976	0.1479	0.1009
SCop-MVN	0.2188	-0.2358	-0.3000	0.2218	0.2952	0.3000	-0.3000	0.3000	0.1928	-0.2874	0.3000	0.2946
SCop-MVT	0.1753	-0.0567	0.0515	0.0221	0.0488	0.2391	-0.0942	0.1141	0.1689	0.1434	0.1978	-0.0101
ARMA-SCop-MVN	0.1227	0.0680	-0.0883	0.1255	0.0670	0.0628	0.0646	0.0778	0.1808	0.0933	0.1268	0.0990
ARMA-SCop-MVT	0.1559	0.0071	0.1128	-0.0281	0.0594	0.1564	-0.0705	0.1069	0.1912	0.1450	0.1430	0.0208
VAR-SCop-MVN	0.1107	0.0022	0.1251	0.0720	0.0342	0.0660	0.0356	0.0541	0.1593	0.1342	0.1086	0.0979
a-SCop-MVN	0.1546	0.0036	0.0984	0.0623	-0.0217	0.1356	0.0037	0.0634	0.1916	0.1331	0.1908	-0.0156
a-SCop-MVT	0.1548	-0.0284	0.0525	0.0133	0.0284	0.2385	-0.0799	0.1208	0.1821	0.1349	0.2064	-0.0234
DCop-MVN	0.2190	-0.2200	-0.3000	0.2058	0.2953	0.3000	-0.3000	0.3000	0.2005	-0.2927	0.3000	0.2922
DCop-MVT	-0.0496	-0.426	-0.3000	0.2943	0.3000	0.2997	-0.3000	0.2982	0.2002	-0.2994	0.2992	0.3000
ARMA-DCop-MVN	0.0041	0.0412	-0.2949	0.2735	0.2309	0.2693	-0.2855	0.2613	0.0880	0.0201	0.1860	0.2059
ARMA-DCop-MVT	-0.1246	0.0721	-0.2962	0.2896	0.2987	0.2953	-0.3000	0.2651	0.1356	-0.1072	0.1782	0.2934
VAR-Cop-MVN	0.2273	-0.2441	-0.3000	0.2246	0.2930	0.3000	-0.3000	0.2992	0.1819	-0.2752	0.2992	0.2941
a-DCop-MVN	0.2250	-0.0129	0.1350	-0.0342	-0.1136	-0.1136	-0.0451	0.1162	0.2221	0.2073	0.2583	-0.1877
a-DCop-MVT	-0.0361	-0.0542	-0.3000	0.2939	0.3000	0.3000	-0.3000	0.2966	0.1959	-0.2942	0.2983	0.3000

4.2.9. Efficient portfolio based on minimising Sortino ratio without short sales

Now we consider the case of the efficient Sortino ratio portfolio without short sales. This portfolio is constructed by minimising the Sortino ratio for a given risk free rate with same weight restrictions as in previous models. The sortino ratio is the ratio of the target return lowered by the risk free rate and the CVaR risk. Table 19 reports the performance results for the portfolios. *DCC-MVT* has the highest average return of 1.94%, cumulative return and also the highest Sharpe ratio. The other best models are *aDCC-MVT*, *a-Dcop-MVT* and *SCop-MVT*.

Table 19: Descriptive statistics and out-of-sample performance based on minimising Sortino ratio without short sale for several econometrics models from January 2005 to December 2014.

Model	Return	Cumulative Return	Risk	Sharpe Ratio	VaR@90%	VaR@95%	VaR@99%
DCC-MVN	0.0171	2.0530	0.0233	0.7480	0.0000	-0.0131	-0.0440
DCC-MVT	0.0194	2.3319	0.0231	0.8408	0.0000	-0.0119	-0.0440
aDCC-MVN	0.0170	2.0407	0.0230	0.7624	0.0000	-0.0129	-0.0450
aDCC-MVT	0.0191	2.2893	0.0228	0.8322	0.0000	-0.0119	-0.0397
FDCC-MVN	0.0172	2.0604	0.0232	0.7497	0.0000	-0.0131	-0.0440
VAR-MVN	0.0087	1.0493	0.0229	0.3914	-0.0161	-0.0420	-0.0755
VAR-MVT	0.0031	0.3677	0.0242	0.1460	-0.0178	-0.0466	-0.0902
ARMA-MVN	0.0110	1.3225	0.0228	0.4989	0.0000	-0.0156	-0.0647
ARMA-MVT	0.0137	1.6495	0.0225	0.6074	0.0000	-0.0139	-0.0438
GOGARCH-MVN	0.0155	1.8607	0.0232	0.6801	0.0000	-0.0128	-0.0448
AR-GOGARCH-MVN	0.0149	1.7886	0.0229	0.6515	0.0000	-0.0144	-0.0576
VAR-GOGARCH-MVN	0.0080	0.9571	0.0236	0.3316	0.0000	-0.0254	-0.0629
SCop-MVN	0.0174	2.0886	0.0227	0.7775	0.0000	-0.0128	-0.0440
SCop-MVT	0.0186	2.2354	0.0228	0.8038	0.0000	-0.0119	-0.0440
ARMA-SCop-MVN	0.0118	1.4117	0.0226	0.5306	-0.0011	-0.0167	-0.0775
ARMA-SCop-MVT	0.0148	1.7713	0.0228	0.6398	0.0000	-0.0128	-0.0479
VAR-SCop-MVN	0.0083	0.9946	0.0238	0.3338	0.0000	-0.0200	-0.0538
a-SCop-MVN	0.0168	2.0194	0.0226	0.7585	0.0000	-0.0115	-0.0440
a-SCop-MVT	0.0190	2.2784	0.0231	0.8197	0.0000	-0.0119	-0.0440
DCop-MVN	0.0168	2.0131	0.0229	0.7536	0.0000	-0.0128	-0.0440
DCop-MVT	0.0187	2.2452	0.0229	0.8165	0.0000	-0.0119	-0.0440
ARMA-DCop-MVN	0.0104	1.2483	0.0221	0.4692	0.0000	-0.0142	-0.0618
ARMA-DCop-MVT	0.0103	1.2419	0.0212	0.4978	0.0000	-0.0156	-0.0446
VAR-Cop-MVN	0.0087	1.0434	0.0234	0.3681	-0.0012	-0.0173	-0.0643
a-DCop-MVN	0.0170	2.0385	0.0230	0.7669	0.0000	-0.0128	-0.0440
a-DCop-MVT	0.0188	2.2518	0.0229	0.8177	0.0000	-0.0119	-0.0440

The highest asset allocation in this model, mainly goes to bond indices such that the assets *BMUK10Y*, *BMUS10Y* and *BMBD10Y* dominate the portfolio with about 7% to 20% of investment in these assets. This is true for the entire model in mean-CVaR efficient portfolios without short sales. The average weights for the analysed assets are presented in Table 20.

Table 20: Average weights of the analysed assets based on minimising Sortino ratio without short sale for several econometrics models from January 2005 to December 2014.

Model	FTSE100	MSEXUK	S&PCOMP	DAXINDX	AMSTEOE	TOKYOSE	HNGKNGI	TTOCOMP	BMUK10Y	BMUS10Y	BMBD10Y	BGILALL
DCC-MVN	0.0661	0.0279	0.0417	0.0500	0.0322	0.1457	0.0810	0.0553	0.1250	0.1472	0.1665	0.0613
DCC-MVT	0.0522	0.0299	0.0403	0.0404	0.0456	0.1597	0.0522	0.0797	0.1142	0.1589	0.1433	0.0836
aDCC-MVN	0.0477	0.0328	0.0419	0.0520	0.0287	0.1406	0.0720	0.0844	0.0994	0.1300	0.2094	0.0611
aDCC-MVT	0.0510	0.0271	0.0452	0.0350	0.0597	0.1555	0.0418	0.0847	0.0992	0.1991	0.1300	0.0717
FDCC-MVN	0.0685	0.0272	0.0413	0.0486	0.0299	0.1472	0.0818	0.0553	0.1211	0.1524	0.1648	0.0617
VAR-MVN	0.0812	0.0491	0.0726	0.0199	0.0605	0.1177	0.0528	0.0462	0.1229	0.1731	0.1498	0.0541
VAR-MVT	0.0753	0.0586	0.0584	0.0376	0.0603	0.1047	0.0708	0.0343	0.1291	0.1319	0.1106	0.1284
ARMA-MVN	0.0890	0.0247	0.0620	0.0217	0.0358	0.1496	0.0759	0.0413	0.1294	0.1539	0.1705	0.0462
ARMA-MVT	0.0580	0.0292	0.0510	0.0253	0.0333	0.1711	0.0429	0.0892	0.1161	0.1453	0.1668	0.0718
GOGARCH-MVN	0.0410	0.0293	0.0280	0.0566	0.0770	0.1554	0.0502	0.0625	0.0945	0.1553	0.1820	0.0681
AR-GOGARCH-MVN	0.0679	0.0110	0.0303	0.0455	0.0522	0.1818	0.0658	0.0454	0.0733	0.1451	0.2070	0.0746
VAR-GOGARCH-MVN	0.0782	0.0543	0.0450	0.0700	0.0503	0.1050	0.0617	0.0354	0.1260	0.1476	0.1494	0.0769
SCop-MVN	0.0833	0.0346	0.0287	0.0331	0.0347	0.1542	0.0601	0.0712	0.1056	0.1531	0.1892	0.0521
SCop-MVT	0.0523	0.0308	0.0262	0.0345	0.0402	0.1760	0.0483	0.0917	0.0925	0.1505	0.1767	0.0803
ARMA-SCop-MVN	0.0713	0.0337	0.0550	0.0264	0.0379	0.1501	0.0560	0.0696	0.1289	0.1546	0.1633	0.0532
ARMA-SCop-MVT	0.0501	0.0438	0.0420	0.0265	0.0523	0.1477	0.0368	0.1006	0.1257	0.1463	0.1523	0.0758
VAR-SCop-MVN	0.0898	0.0876	0.0427	0.0414	0.0391	0.1112	0.0522	0.0360	0.1132	0.1525	0.1397	0.0947
a-SCop-MVN	0.0830	0.0297	0.0375	0.0425	0.0326	0.1541	0.0523	0.0683	0.1084	0.1547	0.1839	0.0530
a-SCop-MVT	0.0483	0.0368	0.0220	0.0396	0.0425	0.1738	0.0545	0.0825	0.0917	0.1590	0.1725	0.0530
DCop-MVN	0.0792	0.0357	0.0316	0.0368	0.0325	0.1498	0.0633	0.0712	0.1122	0.1561	0.1756	0.0561
DCop-MVT	0.0530	0.0300	0.0253	0.0380	0.0408	0.1716	0.0529	0.0883	0.0992	0.1448	0.1825	0.0735
ARMA-DCop-MVN	0.0829	0.0202	0.0387	0.0297	0.0926	0.0332	0.0524	0.1504	0.1432	0.1616	0.1306	0.0646
ARMA-DCop-MVT	0.0401	0.0076	0.0249	0.0180	0.0568	0.1170	0.0283	0.2073	0.1667	0.1742	0.1020	0.0571
VAR-Cop-MVN	0.0673	0.0401	0.0466	0.0801	0.0577	0.0977	0.0593	0.0513	0.1149	0.1508	0.1526	0.0818
a-DCop-MVN	0.0777	0.0326	0.0378	0.0501	0.0314	0.1429	0.0627	0.0648	0.1121	0.1455	0.1722	0.0702
a-DCop-MVT	0.0520	0.0298	0.0258	0.0421	0.0417	0.1746	0.0522	0.0818	0.0971	0.1517	0.1775	0.0737

4.2.10. Efficient portfolio based on minimising Sortino ratio with short sales

We construct an optimal portfolio based on minimising Sortino ratio with short sales such that the portfolio weight is restricted to be between $-0.3 \leq x_i \leq 0.3, i = 1, \ldots, n$ at 95% probability level. Table 21 reports the performance measure for this optimisation strategy. The results show that *DCC-MVT* has the highest monthly average return, highest cumulative return and highest Sharpe ratio. But this model has the highest risk as compared to the other models. On the other hand, models like *FDCC-MVN*, *aDCC-MVN*, *SCop-MVN* also have a high return with a high Sharpe ratio at a much lower risk.

The average weight of asset portfolios constructed based on minimising Sortino ratio is reported in Table 22. The asset allocation for assets *MSEXUK, DAXINDX, AMSTEOE, HNGKNGI,* and *BGILALL* are different across models. The allocation to *TOKYOSE* asset is high, around 10% to 25% of total investment in most of the models. It is interesting to note that only the model *ARMA-DCop-MVN* involves the short selling for this optimisation strategy when other models decided not to short sale this asset. This is also happening to asset *TTOCOMP* for the model *a-SCop-MVN*.

Table 21: Descriptive statistics and out-of-sample performance based on minimising Sortino ratio with short sale for several econometrics models from January 2005 to December 2014.

Model	Return	Cumulative Return	Risk	Sharpe Ratio	VaR@90%	VaR@95%	VaR@99%
DCC-MVN	0.0362	4.3470	0.0272	1.3446	0.0014	0.0007	0.0000
DCC-MVT	0.0540	6.4811	0.0325	1.6599	0.0285	0.0172	0.0052
aDCC-MVN	0.0333	3.9977	0.0263	1.2776	0.0090	0.0000	0.0000
aDCC-MVT	0.0283	3.3954	0.0252	1.1447	0.0091	0.0000	0.0000
FDCC-MVN	0.0369	4.4271	0.0274	1.3605	0.0140	0.0074	0.0000
VAR-MVN	0.0268	3.2168	0.0292	0.9214	0.0000	0.0000	0.0000
VAR-MVT	0.0164	1.9721	0.0304	0.5471	0.0000	0.0000	0.0000
ARMA-MVN	0.0229	2.7496	0.0276	0.8444	0.0000	0.0000	0.0000
ARMA-MVT	0.0149	1.7829	0.0259	0.5799	0.0000	0.0000	0.0000
GOGARCH-MVN	0.0326	3.9096	0.0256	1.2733	0.0095	0.0000	0.0000
AR-GOGARCH-MVN	0.0125	1.4958	0.0254	0.4953	0.0000	0.0000	0.0000
VAR-GOGARCH-MVN	0.0294	3.5317	0.0303	0.9507	0.0000	0.0000	0.0000
SCop-MVN	0.0359	4.3089	0.0262	1.3844	0.0114	0.0000	0.0000
SCop-MVT	0.0339	4.0714	0.0257	1.3242	0.0122	0.0000	0.0000
ARMA-SCop-MVN	0.0275	3.2972	0.0283	0.9672	0.0000	0.0000	0.0000
ARMA-SCop-MVT	0.0231	2.7711	0.0265	0.8740	0.0000	0.0000	0.0000
VAR-SCop-MVN	0.0317	3.8009	0.0301	1.0673	0.0000	0.0000	0.0000
a-SCop-MVN	0.0362	4.3458	0.0263	1.3909	0.0123	0.0000	0.0000
a-SCop-MVT	0.0339	4.0639	0.0259	1.3170	0.0142	0.0000	0.0000
DCop-MVN	0.0358	4.2960	0.0261	1.3823	-0.0117	0.0000	0.0000
DCop-MVT	0.0333	3.9993	0.0257	1.3110	-0.0130	0.0000	0.0000
ARMA-DCop-MVN	0.0254	3.0434	0.0287	0.8573	0.0000	0.0000	0.0000
ARMA-DCop-MVT	0.0197	2.3607	0.0253	0.7462	0.0000	0.0000	0.0000
VAR-Cop-MVN	0.0283	3.3995	0.0300	0.9370	0.0000	0.0000	0.0000
a-DCop-MVN	0.0365	4.3811	0.0263	1.4062	0.0072	0.0000	0.0000
a-DCop-MVT	0.0329	3.9475	0.0259	1.2847	0.0000	0.0000	0.0000

Table 22: Average weights of the analysed assets based on minimising Sortino ratio with short sale for several econometrics models from January 2005 to December 2014.

Model	FTSE100	MSEXUK	S&PCOMP	DAXINDX	AMSTEOE	TOKYOSE	HNGKNGI	TTOCOMP	BMUK10Y	BMUS10Y	BMBD10Y	BGILALL
DCC-MVN	0.1991	-0.0698	0.1564	-0.1126	-0.0068	0.2242	0.0680	0.0415	0.2354	0.2144	0.2539	-0.2037
DCC-MVT	0.0650	0.0433	0.1025	0.1075	0.0458	0.0225	0.0492	0.0642	0.1700	0.0942	0.1175	0.1183
aDCC-MVN	0.2131	-0.0979	0.1168	-0.1198	0.0107	0.2320	0.0021	0.1431	0.1948	0.1918	0.2800	-0.1666
aDCC-MVT	0.2462	-0.1167	0.1683	-0.1229	0.1099	0.2332	-0.1738	0.1558	0.2186	0.2624	0.1680	-0.1490
FDCC-MVN	0.1956	-0.0657	0.1536	-0.1112	-0.0011	0.2214	0.0660	0.0414	0.2317	0.2056	0.2531	-0.1904
VAR-MVN	0.0834	-0.0175	0.0734	0.0852	0.0356	0.1517	0.0077	0.0804	0.1650	0.1499	0.2140	-0.0289
VAR-MVT	0.0946	-0.0102	0.0522	0.1068	0.0461	0.0804	0.0694	0.0609	0.1833	0.0904	0.1756	0.0507
ARMA-MVN	0.1484	0.0351	0.0832	-0.0912	0.0595	0.2114	-0.0769	0.1305	0.2032	0.1973	0.2166	-0.1172
ARMA-MVT	0.1744	-0.0289	0.1341	-0.1509	0.1392	0.2364	-0.1649	0.1606	0.1885	0.2226	0.2248	-0.1360
GOGARCH-MVN	0.2068	-0.1463	0.0232	0.0168	0.1782	0.2391	-0.1600	0.1420	0.2014	0.1976	0.2718	-0.1708
AR-GOGARCH-MVN	0.1908	-0.1606	0.0165	0.0226	0.1341	0.2823	-0.1366	0.1510	0.1482	0.2364	0.2811	-0.1657
VAR-GOGARCH-MVN	0.0469	0.0322	0.0588	0.0725	0.0728	0.1162	0.0149	0.0857	0.1333	0.1221	0.1866	0.0580
SCop-MVN	0.2333	0.0016	0.1204	-0.0124	-0.1207	0.2284	-0.0702	0.1197	0.2215	0.2065	0.2586	-0.1865
SCop-MVT	0.2394	-0.0848	0.0298	-0.0546	0.0619	0.2381	-0.1284	0.1987	0.1959	0.2142	0.2476	-0.1578
ARMA-SCop-MVN	0.1553	0.0029	0.0955	-0.0235	0.0677	0.1316	-0.0155	0.0860	0.2334	0.1834	0.2092	-0.1260
ARMA-SCop-MVT	0.1806	-0.0136	0.0823	-0.1054	0.1270	0.2089	-0.1459	0.1660	0.2363	0.1676	0.2149	-0.1187
VAR-SCop-MVN	0.1010	-0.0067	0.0737	0.1073	0.0400	0.0730	0.0565	0.0552	0.1541	0.1320	0.1584	0.0555
a-SCop-MVN	0.2163	-0.0190	0.1268	-0.0339	-0.1074	0.2291	-0.0399	-0.0399	0.2198	0.2051	0.2649	-0.1899
a-SCop-MVT	0.2427	-0.0864	0.0337	-0.0573	0.0459	0.2352	-0.1189	0.2051	0.2001	0.2078	0.2565	-0.1644
DCop-MVN	0.2199	-0.0151	0.1426	-0.0364	-0.1089	0.2285	-0.0530	0.1224	0.2190	0.2059	0.2606	-0.1856
DCop-MVT	0.2504	-0.0868	0.0070	-0.0568	0.0635	0.2318	-0.1292	0.2201	0.2024	0.2026	0.2546	-0.1596
ARMA-DCop-MVN	0.1846	-0.0243	0.0593	0.0979	0.1117	-0.1358	-0.0393	0.2459	0.2052	0.2050	0.1494	-0.0596
ARMA-DCop-MVT	0.2256	-0.1252	0.0913	-0.1775	0.1283	0.1973	-0.1197	0.2799	0.2400	0.2318	0.1102	-0.0820
VAR-Cop-MVN	0.0554	0.0485	0.0415	0.0555	0.0731	0.1356	0.0054	0.0849	0.1614	0.1157	0.2006	0.0223
a-DCop-MVN	0.2250	-0.0129	0.1350	-0.0342	-0.1136	0.2295	-0.0451	0.1162	0.2221	0.2073	0.2583	-0.1877
a-DCop-MVT	0.2447	-0.0760	0.0084	0.0084	0.0608	0.2395	-0.1160	0.2197	0.2160	0.1955	0.2560	-0.1676

5. Conclusion

In this paper, we perform a comparative study on the performance of portfolios by constructing optimal portfolios that maximise the investor's return for the minimum level of risk. This study involves the use of advance econometric models in forecasting variances and covariances of asset returns and applies it to construct optimal portfolios based on several optimisation models; i.e., minimum variance, mean-variance, maximising Sharpe ratio, mean-CVaR and minimising Sortino ratio. We compare the performance of each model using different risk measures.

In the empirical application, we compare different multivariate GARCH models, *VAR*, *ARMA, DCC, aDCC, FDCC, GOGARCH* and *Copula GARCH* models by using the Normal and Student distributions involving static and dynamic copulas. The use of the multivariate DCC-GARCH family is particularly appealing as it preserves the ease of estimation with a small number of parameters involved, thus solving the problems of dealing with a large number of parameters in other multivariate models. We consider different models for the asset allocation process and also for risk management purposes to see which is the best model in optimising the portfolio return in a pension fund. The analysis focuses on 12 assets consisting of 4 bond and 8 stock indices. The results confirm the presence of heteroskedasticity, fat tails and volatility clustering in the asset returns of the data. We fit the multivariate GARCH models to the portfolio returns to get the estimation of the variance-covariance matrix. Then, for the asset allocation, we consider the box-group constrained portfolio whereby the weights for each asset and the weights of groups of selected assets are constrained by lower and upper bounds.

Specifically, for the minimum variance portfolios, we found that the multivariate *aDCC-MVT* outperforms the other models, by having the highest average monthly return, cumulative return and Sharpe ratio for both portfolio with or without short sales. When we construct the portfolio using the mean-variance such that we set the return of the portfolio to be greater than the target return, we found that, the multivariate *DCC-MVT*, *aDCC-MVN*, *VAR-MVN* and *VAR-SCop-MVN* have good performance measures as compared to the others. As for maximising the Sharpe ratio, the model *AR-GOGARCH-MVN* along with *ARMA-MVN* beats the other models. For the mean-CVaR optimisation, we found that the model *DCC-MVT, aDCC-MVT, FDCC* and *a-DCop-MVT* are comparatively the best models. Finally, when minimising the Sortino ratio, we observe that once again the model *DCC-MVT*, *aDCC-MVT*, *a-DCop-MVT* and *FDCC* provides the best out-of-sample performance.

Our results suggest that the dynamic models are more capable of delivering better performance gains than the static models. These models reduce portfolio risk and improve the realised return in the out-of-sample period. This paper concludes that by adding copula functions to the model, it does not give a better performance models when compared to the dynamic correlation models. Our results are consistent with Weib (2013) and Ricetti (2013), such that the DCC models are not easily outperformed by any of the parametric copula models. The copula model may not perform well when involving many related assets, hence we only use stock and bond indices in this study. These findings are useful for portfolio optimisation and risk measurement in pension schemes. It will help the fund manager to decide the best investment strategies that will ensure maximum benefit for its members.

This study can be extended in several ways. The problem has been solved using limited restrictions on the weights of the constructed portfolios; we only restrict the weights to be between $0 \leq x_i \leq 0.3$ for the case of the portfolio without short sales. Moreover, while allowing short sales, we considered the restrictions of $-0.3 \leq x_i \leq 0.3$. Future studies could incorporate different constraints to examine the robustness of the constructed portfolio, and make a comparison. There are also other possible ways to extend the study by looking at different situations in which the copula model may perform better (i.e by using other copula models or looking at different asset allocation horizons or using multiple asset class in the portfolio or considering other frequency of asset returns). It is also good to note that so far we have examined the asset management in a portfolio, and it would be more interesting to extend the research by including the liabilities. In this context, the methodology proposed could be useful to pension managers and asset allocation purposes in pension schemes to ensure that enough assets are available to support pension liabilities. This study, therefore, can be a good benchmark for future related research.

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*Abstract

ABSTRACT

This study critically analyses plethora of advanced multivariate econometric models, which forecast the mean and variance-covariance of the asset returns in order to create optimal asset allocation models. Most existing studies compare the performance of a limited number of Generalised Autoregressive Conditional Heteroscedasticity (GARCH) models, and they are only based on specific optimisation models. In this study, we provide an in depth knowledge of large asset modelling and optimisation strategies for solving a portfolio selection problem. Specifically, we use symmetric GARCH model and an asymmetric version of it (GJR-GARCH) such that the models are implemented with the multivariate Normal and Student distributions. Several studies have tried to examine the effectiveness of using parametric copula in estimating portfolio risk measures but their results have been inconclusive. We are interested in evaluating if copula-GARCH could be an optimal model in assessing the performance of a portfolio. This study, therefore, implemented various copula-GARCH based models using the static and dynamic (DCC) estimation of the correlation. By employing different model specifications, we are able to explore the empirical applicability of the multivariate GARCH model when estimating large conditional covariance matrices. In constructing the optimal portfolios, we evaluate the minimum variance, mean-variance, maximising sharpe ratio, mean-CVaR and minimisation of Sortino ratio. We compare the out-of-sample performance for each of the models based on the risk-adjusted performance for portfolio with and without short sales. Our results suggest that the dynamic models are more capable of delivering better performance gains than the static models. These models reduce portfolio risk and improve the realised return in the out-of-sample period. This paper concludes that by adding copula functions to the model, it does not give a better performance models when compared to the dynamic correlation models.

Keywords: asset management, multivariate GARCH, Copula GARCH, portfolio optimization, dynamic conditional correlation, asset allocation.

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Dear Chief Editor,

RE: Paper submission for Journal of Empirical Finance

Please consider for publication to the Journal of Empirical Finance, our research paper entitled "Evaluation of Multivariate GARCH Models in an Optimal Asset Allocation Framework".

Thank you for your consideration of our work. Please address all correspondence concerning this paper to nsbabd@essex.ac.uk. We look forward to hearing from you at your earliest convenience.

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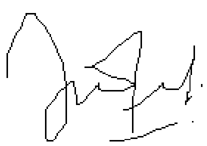

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