Mathematical electron model and the SI unit 2017 Special Adjustment

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Following the 26th General Conference on Weights and Measures are fixed the numerical values of the 4 physical constants (h, c, e, k_B). This is premised on the independence of these constants. This article discusses a model of a mathematical electron from which can be defined the Planck units as geometrical objects (mass M=1, time T=2π ...). In this model these objects are interrelated via this electron geometry such that once we have assigned values to 2 Planck units then we have fixed the values for all Planck units. As all constants can then be defined using geometrical forms (in terms of 2 fixed mathematical constants, 2 unit-specific scalars and a defined relationship between the units kg, m, s, A), the least precise CODATA 2014 constants $(G, h, e, m_e, k_B...)$ can then be solved via the most precise $(c, \mu_0, \alpha, R_\infty)$, with numerical precision limited by the precision of the fine structure constant α. In terms of this model we now for example have 2 separate values for elementary charge, calculated from (c, α, R_∞) and the 2017 revision.

Table 1	Calculated using $(c^*, \mu_0^*, \alpha^*, R_\infty^*)$	CODATA 2014 values
Fine structure constant	α^* = 137.035999139	α = 137.035999139(31)
Speed of light	c^* = 299792458 u^{17}	c = 299792458
Permeability	$\mu_0{}^* = 4\pi/10^7\ u^{56}$	$\mu_0 = 4\pi/10^7$
Rydberg constant	$R_\infty{}^*$ = 10973731.568 508 u^{13}	R_∞ = 10973731.568 508(65)
Planck constant	h^* = 6.626 069 134 e-34 u^{19}	h = 6.626 070 040(81) e-34
Elementary charge	e^* = 1.602 176 511 30 e-19 u^{-27}	e = 1.602 176 6208(98) e-19
von Klitzing (h/e^2)	R_K^* = 25812.807 45559 u^{73}	R_K = 25812.807 4555(59)
Electron mass	m_e^* = 9.109 382 312 56 e-31 u^{15}	m_e = 9.109 383 56(11) e-31
Electron wavelength	λ_e^* = 2.426 310 2366 e-12 u^{-13}	λ_e = 2.426 310 2367(11) e-12
Boltzmann's constant	k_B^* = 1.379 510 147 52 e-23 u^{29}	k_B = 1.380 648 52(79) e-23
Gravitation constant	G^* = 6.672 497 192 29 e-11 u^6	G = 6.674 08(31) e-11
Planck length	l_p^* = .161 603 660 096 e-34 u^{-13}	l_p = .161 6229(38) e-34
Planck mass	m_P^* = .217 672 817 580 e-7 u^{15}	m_P = .217 6470(51) e-7
Gyromagnetic ratio	$\gamma_e/2\pi^*$ = 28024.953 55 u^{-42}	$\gamma_e/2\pi$ = 28024.951 64(17)e-7

1 Background

The mathematical electron model (see [1]) uses a unit-less (units = 1) formula for an electron f_e (eqs. 1, 20) to construct the Planck units as geometrical objects. In this model MLTA are interrelated (see units, eq.1), thus we need only assign numerical values to any 2 Planck units, for example M = m_P and T = t_p, in order to fix the numerical values for all Planck units and from these the dimensioned constants $(G, h, c, e, m_e, k_B...)$ where α (fine structure constant) and Ω = 2.0071349496... are dimensionless mathematical constants.

$$f_e = 4\pi^2(2^6 3\pi^2 \alpha \Omega^5)^3 = .23895453...x10^{23} \tag{1}$$

$$units = \frac{(AL)^3}{T} = \sqrt{\frac{L^{15}}{M^9 T^{11}}} = 1$$

We find this set of geometries for MLTA;

$$M = (1) \tag{2}$$

$$T = (2\pi) \tag{3}$$

$$L = (2\pi^2\Omega^2) \tag{4}$$

$$A = (\frac{2^6\pi^3\Omega^3}{\alpha}) \tag{5}$$

To convert to CODATA 2014 values requires 2 dimensioned scalars. In this example I use a mass scalar k and time scalar t with associated units u^{15} = mass, u^{-30} = time. Assigning appropriate numerical values to (k, t)

$k = m_P$ = .217672817... x 10^{-7}, unit = u^{15} (kg)
$t = t_p/2\pi$ = .171585512... x 10^{-43}, unit = u^{-30} (s)

$$M = m_P = (1)k,\ unit = u^{15}\ (kg) \tag{6}$$

$$T = t_p = (2\pi)t,\ unit = u^{-30}\ (s) \tag{7}$$

We can now calculate scalars (l, a) and the Planck units LA

$$l = k^{9/15} t^{11/15},\ unit = u^{9/15*15+11/15*(-30)=-13}\ (eq.1) \tag{8}$$

$$L = l_p = (2\pi^2\Omega^2)l,\ unit = u^{-13}\ (m) \tag{9}$$

$$\frac{l^{15}}{k^9 t^{11}} = \frac{(.203...x10^{-36})^{15}}{(.217...x10^{-7})^9 (.171...x10^{-43})^{11}} . \frac{u^{-13*15}}{u^{15*9} u^{-30*11}} = 1 \tag{10}$$

$$a = \frac{1}{k^{9/15} t^{6/15}},\ unit = u^{9/15*(-15)+6/15*30=3} \tag{11}$$

$$A = \left(\frac{64\pi^3\Omega^3}{\alpha}\right)a,\ unit = u^3\ (ampere) \quad (12)$$

$$\frac{a^3l^3}{t} = \frac{(.126...x10^{23})^3(.203...x10^{-36})^3}{(.171...x10^{-43})}.\frac{u^{3*3}u^{-13*3}}{u^{-30}} = 1 \quad (13)$$

The physical constants in terms of MLTA (V = 2L/T);

$$c^* = V = (2\pi\Omega^2)\ \frac{k^{3/5}}{t^{4/15}},\ u^{9/15*(15)+8/30*(-30)=17} \quad (14)$$

$$G^* = \frac{V^2L}{M} = (8\pi^4\Omega^6)\ k^{4/5}t^{1/5},\ u^{12-6=6} \quad (15)$$

$$e^* = AT = \left(\frac{128\pi^4\Omega^3}{\alpha}\right)\ \frac{t^{3/5}}{k^{3/5}},\ u^{-18--9=-27} \quad (16)$$

$$h^* = 2\pi LVM = (8\pi^4\Omega^4)\ k^{11/5}t^{7/15},\ u^{33-14=19} \quad (17)$$

$$k_B^* = \frac{\pi VM}{A} = \left(\frac{\alpha}{32\pi\Omega}\right)\ k^{11/5}t^{2/15},\ u^{33-4=29} \quad (18)$$

$$\sigma_e = \frac{3\alpha^2 AL}{\pi^2} = 2^7 3\pi^3\alpha\Omega^5\ t^{1/3},\ u^{-10} \quad (19)$$

$$f_e = \frac{\sigma_e^3}{T} = \frac{(2^7 3\pi^3\alpha\Omega^5 t^{1/3})^3}{2\pi t},\ units = \frac{(u^{-10})^3}{u^{-30}} = 1 \quad (20)$$

$$m_e^* = \frac{M}{f_e},\ units = u^{15} \quad (21)$$

$$\lambda_e^* = 2\pi L f_e,\ units = u^{-13} \quad (22)$$

$$\mu_0^* = \frac{\pi V^2 M}{\alpha L A^2} = \left(\frac{\alpha}{2^{11}\pi^5\Omega^4}\right)\ \frac{k^{14/5}}{t^{7/15}},\ u^{42+14=56} \quad (23)$$

$$R_\infty^* = \frac{m_e}{4\pi l_p \alpha^2 m_P} = \frac{1}{(2^{23}3^3\pi^{11}\alpha^5\Omega^{17})}\ \frac{1}{k^{3/5}t^{11/15}},\ u^{13} \quad (24)$$

For example this common equation for alpha reduces to;

$$\alpha = \frac{2h^*}{\mu_0^*(e^*)^2c^*} = \alpha,\ u^{19-56+54-17=0} \quad (25)$$

Inserting the values for (α, Ω, k, t) in the above will give the calculated results listed in table 1.

2 CODATA 2014

By defining all the dimensioned constants in terms of (α, Ω), 2 scalars and the unit rule set u, we can define constants in terms of each other.

In the following the geometries of the least precise constants $(G^*, h^*, e^*, m_e^*, k_B^*)$ can be constructed using the geometries of the most precise $(c^*, \mu_0^*, R_\infty^*, \alpha^*)$;

$$(h^*)^3 = (8\pi^4\Omega^4k^{11/5}t^{7/15}u^{19})^3 = \frac{2\pi^{10}(\mu_0^*)^3}{3^6(c^*)^5\alpha^{13}(R_\infty^*)^2},\ unit = u^{57} \quad (26)$$

$$(e^*)^3 = \frac{4\pi^5}{3^3(c^*)^4\alpha^8(R_\infty^*)},\ unit = u^{-81} \quad (27)$$

$$(k_B^*)^3 = \frac{\pi^5(\mu_0^*)^3}{3^3 2(c^*)^4\alpha^5(R_\infty^*)},\ unit = u^{87} \quad (28)$$

$$(G^*)^5 = \frac{\pi^3(\mu_0^*)}{2^{20}3^6\alpha^{11}(R_\infty^*)^2},\ unit = u^{30} \quad (29)$$

$$(m_e^*)^3 = \frac{16\pi^{10}(R_\infty^*)(\mu_0^*)^3}{3^6(c^*)^8\alpha^7},\ unit = u^{45} \quad (30)$$

$$(l_p^*)^{15} = \frac{\pi^{22}(\mu_0^*)^9}{2^{35}3^{24}\alpha^{49}(c^*)^{35}(R_\infty^*)^8},\ unit = (u^{-13})^{15} \quad (31)$$

$$(m_P^*)^{15} = \frac{2^{25}\pi^{13}(\mu_0^*)^6}{3^6(c^*)^5\alpha^{16}(R_\infty^*)^2},\ unit = (u^{15})^{15} \quad (32)$$

$$\gamma_e/2\pi = \frac{g l_p^* m_P^*}{2k_B^* m_e^*},\ unit = u^{-13-29=3-30-15=-42} \quad (33)$$

$$(\gamma_e/2\pi)^3 = \frac{g^3 3^3(c^*)^4}{2^8\pi^8\alpha(\mu_0^*)^3(R_\infty^*)^2} \quad (34)$$

Inserting the above in the alpha formula

$$\alpha^3 = \frac{8(h^*)^3}{(\mu_0^*)^3(e^*)^6(c^*)^3} = \alpha^3,\ units = 1 \quad (35)$$

We can then replace $(c^*, \mu_0^*, R_\infty^*, \alpha^*)$ with the numerical CODATA 2014 values for $(c, \mu_0, \alpha, R_\infty)$ giving the calculated results listed in table 1.

3 Revision 2017

As eq.27 uses only $(c^*, R_\infty^*, \alpha)$ and as $e = 1.602176634x10^{-19}$ is the most precise of the revised values I shall discuss it here in terms of this model.

$$\alpha^8 = \frac{4\pi^5}{3^3(c^*)^4(e)^3(R_\infty^*)},\ unit = 1 \quad (36)$$

Using the CODATA 2014 R_∞ gives $\alpha = 137.03599520$

$$R_\infty^* = \frac{4\pi^5}{3^3(c^*)^4(e)^3(\alpha^8)},\ unit = 1 \quad (37)$$

Using the CODATA 2014 α gives $R_\infty = 10973729.04723$. In other words we now have 2 separate values for elementary charge, from $(c^*, R_\infty^*, \alpha)$ and from the revision.

Notes:

k_B does not agree with CODATA, however it can be used in eq.33 to solve the gyro-magnetic ratio.

G agrees with Rosi et al G = 6.67191(77)(62) x 10^{-11} [3].

A complete discussion of this model can be found here [1], the formulas used in this article downloaded in maple format [2], for convenience I use the commonly recognized value for alpha as $\alpha \sim 137$;

References

1. Macleod, Malcolm J. *"Programming Planck units from a virtual electron; a Simulation Hypothesis"* Eur. Phys. J. Plus (2018) 133: 278
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