PART II

Types of computing

8

CLASSICAL COMPUTATIONAL MODELS

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Introduction

In this chapter I discuss a familiar class of computational models that have played an influential role in artificial intelligence, computational psychology, and cognitive science – what are often called "classical" or "symbolic" models. In Section 1, I characterize such models, and discuss their relationship to some closely associated ideas. In Section 2, I sketch some putative virtues of classical models. In Section 3, I discuss some of the dimensions along which these models vary, and provide brief illustrations. Finally, in Section 4, I mention some of the more prominent criticisms levelled against the classical modeling paradigm.

1 What is a classical computational model?

The expression "classical computation" only became common currency in cognitive science during the 1980s, largely as a means of contrasting the symbolic tradition with the burgeoning field of connectionist cognitive science (Rumelhardt, McClelland, and the PDP Research Group, 1986; Fodor and Pylyshyn, 1988). However, the conception of computation it designates had been influential in AI since the 1950s, and has roots tracing back at least as far as Turing's research in the 1930s (Boden, 2006). Because of this complex history, it is important to distinguish the notion of a classical computational model from a range of associated ideas.

1.1 Classical computational models as a species of process model

There are many kinds of models in cognitive science. For example, some seek to characterize the evolution of a given psychological capacity (Barrett, 2014; Henrich and Tennie, 2017); others model statistical or causal dependencies between salient variables (Ratcliff et al., 2016); and still others seek to make precise some psychologically significant relationship, such as similarity (Tversky, 1977). However, the *prototypical* kind of cognitive models – what are often called "process models" – are primarily oriented towards addressing a kind of how-question. Roughly put: How does human performance (in a particular domain of cognition) come about? Further, the manner in which they seek to answer such questions can be characterized in terms of their target phenomena, and the sorts of features they attribute to their targets:

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- *Targets*: Such models purport to explain human performance by characterizing the processes involved in the exercise of some cognitive capacity, or the operation of some functionally characterized cognitive system(s).
- *Features*: These targets are characterized in terms of a combination of familiar kinds of psychological construct: representations; cognitive operations that effect transitions between representational states; and cognitively salient resources, such as memory space, attention, and time (Weiskopf, 2017).

As with all scientific models, cognitive models can be formulated with the aid of quite different representational resources. That is, the modeling *vehicles* can be of different sorts. For example, there are verbal models formulated in a natural language, no doubt supplemented by various pieces of jargon. There are mathematical models, which paradigmatically take the form of an equation or inequality in some mathematical formalism. There are diagrammatic models; and most importantly for our purposes, there are *computational models* where the target aspects of cognition are modeled by a computational system that permits dynamic simulation of the target phenomena. *Classical* computational models (CCMs), in the sense most relevant to the present chapter, are a species of computational, process model.[1]

1.2 Core characteristics of classical models

CCMs are best construed as a broad family of process models that share a core set of characteristics. Perhaps the most obvious is that, *qua* modeling *vehicles*, CCMs are computational systems – paradigmatically, suites of programs run on an ordinary, commercial, digital computer. Yet this is not, of course, a distinctive feature of CCMs, since all computational models take this form. What is distinctive of CCMs is that they characterize their *targets* as computational systems of a particular sort. Slightly more precisely, CCMs represent cognitive processes and systems as involving a kind of *algorithmically* specifiable *symbol manipulation*. What follows is a fairly typical way of spelling out the core aspects of this sort of computation.

Symbolic. If cognition is to be characterized in terms of symbol *manipulation*, one needs *symbols*. Symbols are representations in that they have semantic properties – e.g. they denote or refer. In addition, however, they are much like natural language expressions in that they possess formal or syntactic properties, and belong to a *system* of representations, akin to a language. Such symbol systems invariably contain both primitive symbols, which have no other symbols as parts, and complex symbols built up from primitive ones. Further, these symbol systems are characterized by sets of rules – typically recursive in form – that specify which combinations of symbols are well-formed or grammatical, and also assign meanings to both primitive and well-formed combinations of symbols. In short, symbolic representational systems of the sort relevant to classical computation possess a combinatorial syntax and semantics in much the same way as logical systems, and natural languages, do (Fodor and Pylyshyn, 1988). For this reason, classical models are sometimes called *language of thought* models (Fodor, 1975).

Algorithmic. In addition to syntactic and semantic rules, which define a symbol system, classical computation also presupposes a set of rules, or instructions, which specify *algorithms* for how these symbols are to be manipulated. An algorithm is a precise, stepwise procedure for doing something – of performing a mapping from a class of inputs (the domain) to a class of outputs (the range).[2] That is, it's a way of *computing* a function. Thus, CCMs seek to characterize cognitive processes as algorithmically specifiable processes for computing functions.

Formal. A third assumption embodied in CCMs is that the relevant class of algorithms are *formal* in that the operations specified by the algorithm are defined with respect to the syntactic, as opposed to semantic, properties of symbols. In this regard, they are akin to familiar grade-school algorithms for multiplication and long division, which are formulated in terms of operations on formally characterized items – Arabic *numerals* – and not the things they represent – i.e. numbers. One consequence of this is that the task of classically modeling a cognitive process is entwined with the task of specifying the formal properties of the representations involved. Change the symbol system – e.g. from Arabic to Roman numerals – and one must typically change the algorithm as well.

Interpretable. A final core feature of CCMs is that they characterize cognitive processes assemantically *interpretable*. Although cognitive processes are modeled by formal procedures, the symbols involved have semantic properties as well. As a consequence, it is possible to make sense of – to interpret – the process, not merely as the manipulation of formal tokens, but as mappings from meaningful states to other meaningful states. To return to the example of grade-school arithmetic, although the algorithm for multiplication is specified in term of operations on Arabic numerals, the symbol transitions that occur can be systematically interpreted in terms of the numbers represented – i.e. as finding the product of two or more *numbers*.

1.3 Classical models and "Turing computation"

In addition to the above four features, CCMs are sometimes attributed other characteristics that are incorrectly presumed to be essential. Some of these false presumptions are, I suspect, a consequence of misunderstanding the idea that classicists seek to model "the mind as a Turing machine", or that they "attempt to fit the facts of human cognition to the classical, Turing account of computation" (Fodor, 2001, p. 5).

Turing's research is, of course, extraordinarily influential, in part because it provides a theoretically perspicuous model of (classical) computation. However, CCMs need not – and should not – attribute all the properties of Turing machines to cognitive systems. First, CCMs invariably make different assumptions about the nature of memory. The memory in a Turing machine – the tape – is infinite and unaddressable. In contrast, and for obvious reasons, CCMs do not characterize human memory as infinite; and they almost invariably assume – though often only tacitly – that cognition relies on addressable memory systems (Gallistel and King, 2010).

Second, Turing machines are serial processors, but CCMs need not characterize their targets in this way.[3] Admittedly some highly influential classical cognitive scientists have viewed seriality as an important property of human cognition (e.g. Simon, 1962). Nevertheless, many acknowledge that classical computations can have parallel implementations (Fodor and Pylyshyn, 1988; Gallistel and King, 2010). So, for example, more recent versions of Newell and Laird's SOAR implements symbolic processes, such as production rule activation, in parallel (Ritter, 2005).

Finally, a Turing machine is deterministic in the sense that at any point in a procedure, there is at most one unique next operation it can perform. In contrast, CCMs need not characterize cognition as deterministic. For example, they may posit stochastic processes that involve random number generation, or concurrent operations – both of which suffice for a process being nondeterministic in the relevant sense.

1.4 Classical models and Marr's levels

There is a longstanding tradition in cognitive science of characterizing models or explanations with reference to some hierarchy of levels. The best-known is David Marr's tri-level hierarchy, which I discuss here because it is often construed as a framework for classical modeling (Marr, 1982).

Suppose we seek to characterize some cognitive process or system. Then, each of Marr's level can be characterized by proprietary research questions that require the provision of distinct sorts of descriptions:

- *C-Level*: Computational level descriptions seek to characterize *what* function, or mapping, the system computes, and *why*.
- *A-Level*: Algorithmic level descriptions seek to characterize *how* the computation is performed by specifying (a) the class of symbols that are inputs to and outputs from the system; and (b) the algorithm(s) by which this transformation is accomplished.
- *I-Level*: Implementation level descriptions seek to characterize the physical organization which enables the cognitive process or system to be implemented.

Whether Marr's hierarchy is exhaustive is a matter of some theoretical debate (Peebles and Cooper, 2015). However, the tri-level hierarchy provides a useful way of bringing out certain typical characteristics of classical models.

First, of the three levels, CCMs are clearly best construed as A-level descriptions since they aim to characterize cognitive systems in terms of the algorithmic manipulation of symbols. Second, and relatedly, this observation allows us to see that classical and non-classical cognitive models need not always be incompatible with each other. For example, it may be that some connectionist models are best construed as I-level descriptions, which purport to explain how classical processes might be implemented in the brain (Pinker and Prince, 1988).

Third, since Marr's time, it has become widely accepted by modelers that C-level analyses are critically important to the development of process models – especially CCMs. Among other things, this is because, as Marr stressed, the appropriateness of any given algorithm depends crucially on the nature of the computational problem to be solved (e.g. Griffiths, Lieder, and Goodman, 2015; Bringsjord, 2008b). This was not, however, always a dominant view amongst classicists. Indeed, Marr's motivation for introducing the tri-level hypothesis was to correct what he saw as a major deficiency in the research of his day: a tendency to produce process models without any serious effort at developing rigorous C-level analyses.

Finally, it is worth noting that, as a matter of fact, classical modelers have given far less attention to I-level considerations than to C-level ones. In some cases, this is due to a paucity of relevant I-level information. In other cases, it is a consequence of adopting formalisms that do not readily map onto extant neuroscience (Bringsjord, 2008b).

1.5 Classical models and the computational theory of mind

Historically, a central motivation for classical modeling is the endorsement of the *classical computational theory of mind* (Samuels, 2010; Rescorla, 2015). Though formulated in different ways, the rough idea is this:

CCTM: The mind is literally a classical computational system – an interpretable, formal, symbol manipulator – of some sort; and cognitive processes, such as reasoning and visual perception, just are classical computational processes of some sort.

So construed, CCTM is a kind of empirically motivated, *metaphysical* doctrine, in that it provides a general characterization of what it is to be a mind, or cognitive process (Fodor, 1975; though see Piccinini, 2010, for an alternative construal of CCTM). Moreover, it is a view that has had some highly influential advocates – e.g. Fodor, Pylyshyn, and Gallistel explicitly endorse the

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view, and Newell and Simon's physical symbol systems hypothesis is a close cousin (Newell and Simon, 1976).

The relationship between CCTM and the classical modeling strategy is a complex one. Clearly, they dovetail with each other. Historically, adherence to CCTM has been a major motivation for developing classical models. Moreover, various kinds of predictive and explanatory success in developing classical models may, in turn, provide support for the doctrine itself.

Nevertheless, it is important to see that classical modelers need not incur a commitment to CCTM – they need not be doctrinal in this way. CCTM is a general thesis regarding the nature of mind and cognition, and CCMs might be scientifically useful even if this general thesis is false. First, it might be that our minds are *hybrid* systems, as some dual process theorists have claimed, where only some cognitive systems are as classicists suppose (Sloman, 1996; see also Anderson, 2007). Alternatively, even if CCTM is entirely inadequate as a metaphysics of mind, CCMs might still be (causal) explanatory at some appropriate level of granularity. (Compare: electrical circuit theory is explanatory in cellular neuroscience, even though no one maintains that neurons *just are* electrical circuits.) Finally, even if CCMs fail to explain human cognition, they might still be useful for other purposes, such as addressing "how-possibly" questions of various sorts.

To summarize: Though the success of the classical modeling enterprise obviously fits well with CCTM, modelers need not be committed to this doctrine since the provision CCMs might be scientifically valuable even if CCTM is false. In Section 3, I will suggest that more recent classical modeling is sometimes of this non-doctrinal variety.

2 Virtues of classical models

Why suppose the goals of cognitive science are fruitfully pursued by developing CCMs? For heuristic purposes, I divide the reasons into two sorts: (a) general methodological virtues, and (b) respects in which CCMs appear peculiarly suited to characterizing aspects of cognition. Whether any of these putative virtues are *unique* to CCMs is an issue of longstanding and ongoing disagreement, which I won't take up here.

2.1 General methodological virtues

Over the past sixty years or so, researchers have claimed that computational models in general, and CCMs in particular, can play a significant role in addressing various methodological concerns in the cognitive and behavioral sciences, including the following:

Prediction and testability. Computational models in general, and CCMs in particular, are useful in that they can generate testable predictions. In particular, they allow for the generation of predictions under different input conditions that can be tested against behavior data.

Avoiding under-specification. Computational modeling requires that researchers be explicit about the assumptions they make. This is especially important in the context of psychological science where, as behaviorists were fond of stressing, much of what passes for theory can be woefully underspecified. Of particular concern is the risk of "explanations" that posit undischarged "homunculi" – sub-processes or sub-systems – that do not so much explain as presuppose the target phenomenon. CCMs help ameliorate this concern because they require the specification and implementation of an algorithmic process. In doing so, they leave nowhere for homunculi to hide.

Avoiding vacuity. Another common behaviorist concern is that intentional psychological explanations may sometimes appear empty because they are too easily generated. Whatever

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the behavior we seek to understand, it's effortless to retrofit the unobserved mental causes to "explain" it. As Pylyshyn and others note, however, if one demands that psychological explanations take the form of precisely articulated procedures – implementable in the form of programs – this complaint no longer seems plausible. Indeed, far from being too easy, the problem is often that it is too *hard* to develop such models (Pylyshyn, 1984).

Addressing "how"-questions. Many of the most pressing issues in the cognitive and behavioral sciences concern the explanation of *capacities* with which we are already intimately familiar. Put a neurotypical subject in front of a tree under normal lighting conditions, and there's a good chance that they will *see* a tree. Ask such a subject what they see, and there's a good chance that they'll be able to tell you. The principle challenge for psychology is not merely to document such regularities, but to explain *how* instances of such regularities reliably occur. Process models quite generally seek to address such questions, by characterizing the state transitions between initial conditions (e.g. sensory inputs) and a given cognitive or behavioral outcome. CCMs are process models that achieve this goal by specifying a precise stepwise procedure for effecting such transitions. To that extent, they are appropriate for addressing the core explanatory challenge of cognitive science.

Addressing "how-possibly"-questions. There's a well-known and philosophically deeper reason for finding CCMs attractive. Contemporary research on human behavior and cognition occurs within the context of some widespread assumptions that are not easily reconciled. On the one hand, it is almost universally assumed amongst behavioral scientists that human beings are complex physical systems. On the other hand, there is widespread consensus that human behavior is at least partially explained by *representational* processes. In view of this, perhaps the deepest motivation for the classical modeling strategy is that it provides a framework within which these commitments can be reconciled (Haugeland, 1989; Pinker, 2000). CCMs provide *existence proofs* of complex, semantically interpretable physical systems, and in doing so suggest answers to longstanding questions regarding how it is possible for physical systems to exhibit the sorts of psychological capacities we possess.

2.2 The peculiar suitability of classical models

In addition to their general methodological virtues, CCMs appear peculiarly suitable for modeling some apparently pervasive psychological phenomena.

Modeling inferential processes. Closely related to the last point in Section 2.1., many psychological processes appear both causal *and* inferential. This is perhaps most obvious in the case of reasoning, where earlier beliefs are not only causally related to later ones, but also *semantically* related in such a way that the former provide premises for the latter. Yet this phenomenon seems not to be restricted to reasoning. For example, much of our best perceptual psychology proceeds on the assumption that vision, audition, and the like involve "unconscious inference" (Helmholtz, 1867; Scholl, 2005; Olshausen, 2014).

Historically, the inferential character of many psychological processes was perceived as posing a serious challenge: a version of the notorious *homunculus regress*. To explain such rational-cum-causal relations, it seems that meanings themselves must be causally efficacious, which in turn appears to require some inner interpreter – an intelligent subsystem, or homunculus – for which thoughts have meanings. But then the same problem of coordinating semantic and causal relations recurs for the homunculus, resulting in a regress of interpreters. Classical modelers address this problem by rejecting the assumption that rational causation is explicable only if meanings are causally efficacious. Instead they invoke an idea familiar to logicians, that inferences can be characterized in terms of formal rules. (*Modus ponens* is a simple example.) When applied to the task of understanding cognition, the idea is that mental processes are inferential not because of any unexplained sensitivity to meanings, but because they depend on formal rules which, though defined over the syntax of representations, are like logical rules in that they preserve semantic relations. Moreover, since CCMs characterize cognitive processes algorithmically, they are ultimately decomposable into combinations of operations the execution of which requires no intelligence at all. We are thus able to explain the inferential character of cognitive processes without succumbing to regress.

Productivity and systematicity. CCMs are often regarded as suited to modeling aspects of cognition that are productive or systematic (Fodor and Pylyshyn, 1988). This plausibly includes language comprehension and production, planning, deductive reasoning, and perhaps perceptual capacities, such as vision. Roughly put, such cognitive capacities are *productive* at least in the sense that they permit the production of a great many distinct thoughts, many of which are novel. Further, such regions of cognition seem *systematic*, in roughly the sense that the salient representational capacities come in coherent packages. In particular, the ability to be in some cognitive states appears to reliably covary with the ability to be in other semantically related states. To use a well-worn example: so far as we know, there is no-one who can understand the sentence "Mary loves John" and yet lacks the ability to understand the sentence "John loves Mary". The capacity to understand the one, reliably covaries with the capacity to understand the other. *Mutatis mutandis* for a great many cognitive states. Or so it would appear.

Classical modelers have a general approach to modeling systematicity and productivity. In brief, part of the solution is that CCMs are specified relative to a recursively defined combinatorial system of syntactically structured representations. Further, the algorithmic processes defined over this system invariably involve combinatorial operations, sensitive only to the syntax of these representations. In view of this, it is relatively easy to accommodate productive processes because, under minimal assumptions, the system of representations is potentially infinite, and the model has the resources to generate increasingly more complex symbolic structures via the combination of simpler ones. Systematicity is similarly easy to accommodate. If combinatorial operations are defined over syntactic forms, syntactically similar representations will be treated in similar fashion, even where they differ semantically. To return to the well-worn example of "John" and "Mary": Assuming that "John loves Mary" and "Mary loves John" involve the same symbols and share the same syntax, if one of them is producible by the model, then, (given minimal assumptions) so too will the other. This is because the very same computational resources are required for the production of either.

Variables and quantification. Human beings engage in a wide array of cognitive tasks that are readily modeled in terms of operations on variables, often bound by quantifiers. This is perhaps most obvious in the case of natural language, but it appears to occur in a great many other tasks as well, including deductive reasoning, planning, and mathematical cognition. Further, much inductive learning appears to consist in learning the relationships between variables. For example, we are able to learn what Marcus (2001) called "universally quantified, one-to-one mappings", such as identity. These tasks are naturally modeled within the classical framework because variables and quantifiers are readily accommodated within a symbol system, and much classical computation involves operations over such variable structures.

One-shot learning. Human beings engage in various forms of one-shot learning that can exert a significant influence on both our overt behavior and inferential tendencies. Most obviously, this occurs when we acquire new factual information via natural language. As Griffiths et al.

(2010) note, "to a child who believes that dolphins are fish, hearing a simple message from a knowledgeable adult ('dolphins might look like fish but are actually mammals') might drastically modify the inferences she makes". This sort of phenomenon is readily modeled as relying on explicit representations of the sort assumed by classical models. In contrast, it is far from obvious how to accommodate this phenomenon within other modeling frameworks, such as connectionism.

Cognitive flexibility: Amongst the most striking features of human cognition is its *flexibility*. To a first approximation, we appear capable of performing an indefinite range of qualitatively distinct tasks. Or, as Allen Newell once put it: we exhibit a kind of *unlimited qualitative adaptation* (Newell, 1990). One early and powerful motivation for the classical modeling paradigm is that it suggests an elegant account of this phenomenon. Classical computational systems can exhibit this sort of flexibility in that they can execute different sets of instructions designed for different tasks. Faced with the task of explaining human cognitive flexibility, some researchers suggest that a similar account may hold for human cognition as well – that much of our flexibility results from our possession of cognitive mechanisms that are capable of exploiting different bodies of task-relevant procedural knowledge (Newell, 1990).

3 Varieties of classical computational modeling: some illustrations

So far, we have discussed the characteristics, and general virtues, of CCMs. However, different families of classical models have been developed within research programs that vary considerably in their methodological and empirical commitments. As a consequence, there is considerable variation in the sorts of models – and broader modeling practices – which exist within classical paradigms. In what follows, I briefly discuss some of these approaches.

3.1 Heuristic search: early exemplars of classical modeling

Early research by Newell, Simon, and their collaborators aimed not only to provide workable bits of technology, but also to model how human beings solve various cognitive tasks. In addition to possessing the core features of CCMs, programs such as the Logic Theorist and the General Problem Solver (or GPS) incorporated a pair of additional assumptions that exerted a profound influence on subsequent research in AI and cognitive science (Newell and Simon, 1956; 1961).

The first of these assumptions is that much human cognition involves a kind of mental *search* through a space of options – a search space – in order to find a solution to the task at hand. Moreover, since the search space for interesting tasks is almost invariably too large to permit exhaustive search, Newell and Simon further proposed that search needs to be *heuristically* constrained. That is, the model needs to encode various guidelines or "rules of thumb" which constrain the range of options that need be considered in the course of solving the task at hand. For example, GPS used a kind of search heuristic known as means-end analysis, which aims to produce convergence on solution to a problem by successively reducing the difference between the current state of the system and the goal state.

Within AI the ideas found in Newell and Simon's early work spurred extensive research on heuristic search (see Russell and Norvig, 2010, chs. 3–5), and was instrumental in the development of planning systems, such as STRIPS, which relied on the notions of means-ends analysis and search (Fikes and Nilsson, 1971). Planning research, though dramatically transformed, remains a highly active region of AI (Ghallab, Nau and Traverso, 2016).

From the vantage of contemporary cognitive science, models such as GPS may appear quaint. Nevertheless, the idea that cognition relies on heuristic methods remains an influential one. For example, the "Ecological Rationality" research program, initiated by Gerd Gigerenzer and Peter Todd, relies heavily on the notion of heuristic processes in order to explain our capacity to make decisions in a computationally efficient manner (Todd, Gigerenzer, and the ABC Research Group, 2012). Moreover, whilst not doctrinally committed to a classical view of cognition, many of their models possess the core characteristics of CCMs. This is true, for example, of Gigerenzer and Goldstein's well-known model of the Take-the-Best heuristic, which (roughly) decides between two options – e.g. which of two cities is larger – by using the most valid available discriminating cue, and ignoring the rest (Gigerenzer and Goldstein, 1999).

3.2 Logicism

A second illustration of classical modeling is the logic-based or *logicist* approach. Though it played a prominent role in the development of AI and continues to spur research in applied logic, its influence in contemporary cognitive science is diminished. (For overviews, see Minker, 2000; Bringsjord, 2008a; and Thomason, 2016.)

Starting in the 1950s with the seminal work of John McCarthy and his collaborators, logicists proposed that many of the cognitive problems we confront are fruitfully construed as problems of logical inference (McCarthy, 1959). In slightly more detail, logicists maintain that propositional attitudes – mental states, such as beliefs, judgments, and intentions – are central to human cognition, and that many cognitive processes consist in inferential transitions between such attitudes. In paradigmatic instances of reasoning, for example, one starts with a set of premise beliefs, and infers a conclusion; and when planning one infers new intentions from prior goals, intentions, and beliefs.

If one thinks of cognition in the above way, then formal logic appears directly relevant. In particular, proof theory promises to provide the relevant resources for formally characterizing cognitive tasks in terms of inferential relations between propositional – or declarative – representations.

In terms of Marr's levels, logicism is naturally characterized by the following division of labor. At the C-level, logical formalization is used in order to provide precise specifications of the inferential problems that we solve – e.g. narrative understanding, or spatial reasoning (Thomason, 2016).[4] In view of the range and complexity of the problems we solve, this has resulted in significant developments in formal logic itself, including nonmonotonic logics (Antonelli, 2012), logics for spatial reasoning (Stock, 1997), and logics for temporal reasoning (Ghallab, Nau, and Traverso, 2016).

At the A-level, the core task for logicists is to provide computationally efficient implementations of the solutions specified at the C-level. Though different kinds of implementation are possible, in practice they typically consist in a kind of mechanized proof theory, where relevant information is represented by formulae in a logical language, and computation proceeds by the operation of a theorem-prover, so that new representations can be derived from existing ones, via the application of proof-theoretic rules (Chater and Oaksford, 1991).

Within contemporary AI, perhaps the most well-known example of a logic-based system is Doug Lenat's monumental CYC system, which aims to codify, in machine-usable form, the millions of items of information that constitute human commonsense (Lenat et al., 1990). Within contemporary cognitive science, logic-based models are less commonplace than they once were. Nevertheless, they are found in various fields of research, such as computational

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linguistics and the psychology of reasoning. For example, Lance Rips PSYCOP is a well-known logic-based model of human deductive reasoning (Rips, 1994). In addition, production systems, such as SOAR and ACT-R – of which more below – bear close ties to logic-based models (Bringsjord, 2008b).

3.3 Cognitive architectures

A third example of classical research concerns the characterization of *cognitive architecture*. Although such architectural models take the form of working computer systems, they possess a pair of features not typical of CCMs in general.

First, in contrast to many CCMs, which tend to target relatively narrow aspects of cognition, models of cognitive architecture seek to provide a comprehensive, detailed specification of how cognition operates across a wide range of domains and tasks. In Newell's (1990) memorable phrase, they are intended to provide "unified theories of cognition". Further, since such models are typically motivated by the idea that much human behavior is a product of complex interactions between systems, they typically specify a variety of systems – e.g. for different sorts of memory, and for different sorts of cognitive process.

Second, in contrast to many CCMs, architectural models pursue the ambitious project of specifying the core set of *basic* computational operations, structures, and resources on which cognition depends. To put the point metaphorically, they purport to specify the sorts of properties that would be described in a "user's manual" for the cognitive system (Fodor and Pylyshyn, 1988). Such properties are assumed to be basic in at least two senses. First, they are presumed to be relatively invariant over the lifespan of the agent. Second, they are typically assumed to be properties of the mind that are presupposed – but not explained – by one's classical account of cognition. As a consequence, it is typically assumed that the explanation of these properties requires recourse to some "lower"-level science, such as neurobiology or biochemistry. Again, none of this is typical of CCMs more generally.

Of course, not all architectural models are classical. For example, Randy O'Reilly's well-known *Leabra* model specifies a connectionist architecture (O'Reilly, Hazy, and Herd, 2017). Nevertheless, since the 1970s, there have been a number of notable efforts to specify cognitive architectures, which are either uniformly classical, or at least hybrid models with classical subcomponents. The SOAR architecture, developed by Allen Newell and John Laird is a prominent example of the former, and John Anderson's ACT-R is an influential version of the latter (Laird, 2012; Anderson, 2007). Versions of these architectures have been around since the 1980s, and have been used to simulate human performance on a broad array of tasks, including arithmetic, categorization, video game playing, natural language understanding, concept acquisition, verbal reasoning, driving, analogy making, and scientific discovery.

Although SOAR and ACT-R are too complex to describe here, it is interesting to note that both architectures contain *production systems* as a core component. Such systems operate on if-then rules, known as productions, in which the antecedent of the rule specifies a condition (e.g. the knight is on square 8), and the consequent an action to be performed when that condition obtains (e.g. move the knight to square 1). Production systems operate via the coordination of three sub-systems: a long-term memory in which the rules are stored, a working memory, and an inference engine. A production rule can be retrieved from long-term memory (or "fired") only if its antecedent condition is met by an element in working memory. The task of the inference engine is to determine which of the rules have all their conditions met, and then to decide which of these should be fired. Over the years, production systems have had various practical applications – e.g. in expert systems such as Mycin and Dendral. But more importantly for present purposes, they have proven exceedingly useful in modeling complex sequences of behavior, such as those that occur in problem solving.

3.4 Bayesian modeling

My final illustration of classical modeling may strike readers as an odd one in that Bayesian research in cognitive science is not ordinarily construed as a form of classicism. And, indeed, there are important differences between Bayesian approaches and the sorts of "good old fashioned" cognitive science mentioned above. For one thing, Bayesian cognitive scientists in general have no doctrinal commitment to CCTM. For another, in contrast to the classical models of old, it is absolutely central to the Bayesian approach that psychological processes are fruitfully construed in terms of *probabilistic* inference of the sort characterized by Bayesian statistics. For all that, some of the models and modeling strategies used by Bayesians bear a striking resemblance to more traditional classical ones; and for this reason, I suggest that some Bayesian modeling is appropriately construed as a non-doctrinal form of classical modeling.

Although there are currently different forms of Bayesianism in the brain and behavioral sciences, researchers in cognitive science typically adopt a methodology, reminiscent of Marr, which starts with C-level analyses. Thus, Griffiths et al. (2010) contrast their Bayesian approach to that of connectionism, and other "mechanism-first" approaches, by noting: "probabilistic models of cognition pursue a top-down or 'function-first' strategy, beginning with abstract principles that allow agents to solve problems posed by the world – the functions that minds perform (2010, p. 357). In other words, Bayesians typically start by providing a C-level analysis of the cognitive task, where they specify the function computed in terms of principles drawn from probability theory, such as those involved in sampling and model selection.

To take a well-known example, suppose that we seek to understand the problem of learning the extension of new words in the absence of negative evidence. When a child learns the word "dog", for instance, it suffices for a teacher to point to a few positive examples – a Chihuahua, a Shetland Sheepdog, and a Boxer, for example – and call them each "dog", without also pointing to, say, a horse or a sunflower and saying "That's not a dog". Within the Bayesian framework, this is naturally construed as a task in which hypotheses are being prioritized according to some probabilistic criterion. Thus, Xu and Tenenbaum (2007) propose that this task and its solution can be partially characterized by a simple but powerful principle of probabilistic inference – the *size principle* – which says that if you are considering a set of nested hypotheses, you should prefer the smallest hypothesis is consistent with the available evidence (Perfors et al., 2011).

Most extant Bayesian research is similar to the above in that it aims to provide C-level analyses that specify ideal solutions to computational problems. Moreover, even where algorithmic issues arise, Bayesians are not generally committed to a classical modeling strategy. Nevertheless, some Bayesian models have a distinctly classical flavor. This is so for a pair of reasons. First, Bayesian C-level analyses frequently suggest a role in cognition for complex representational structures of the sort most readily accommodated by classical models – e.g. hierarchically structured category systems, or tree-structured representations (Griffiths et al., 2010). Second, when seeking to explain how ideal solutions might be efficiently approximated by human cognition, Bayesians have pursued a strategy of borrowing techniques from theoretical computer science – such as Monte Carlo methods – which possess characteristics readily accommodated within a classical computational framework – such as variable binding, and compositionally structured hypothesis spaces (Griffiths, Kemp, and Tenenbaum, 2008).

In view of the above, it is perhaps unsurprising that in recent years, some Bayesian researchers have described their own approach to cognitive modeling as relying on what the arch classicist, Jerry Fodor, called a *language of thought* (LOT). In its Bayesian incarnation, however, the LOT is (of course) probabilistic – a *pLOT*. Thus, Piantadosi and Jacobs conclude a recent paper by articulating a conception of Bayesianism that's continuous with, whilst improving upon, the more traditional symbolic approaches that preceded it:

> The pLOT is not a revolutionary new theory that promises to overthrow existing paradigms; it is a resurgent old theory that promises to integrate many approaches into a unitary framework … we argue that it provides one of the most promising frameworks for cognition, combining the compositionality of symbolic approaches with the robustness of probabilistic approaches, thereby permitting researchers to formulate and test theories that do not acquiesce to the poles of major debates.
>
> *(Piantadosi and Jacobs, 2016)*

4 Challenges to classical models

The classical paradigm has been subject to a bewildering array of objections; and though there isn't the space to consider them in detail here, I propose to briefly discuss some of the more prominent ones.

4.1 A priori *philosophical objections*

One sort of objection is not so much directed at CCMs as the metaphysics of mind with which they are associated. Specifically, such arguments purport to show on broadly *a priori* grounds that CCTM is false. Perhaps the most well-known of these objections is Searle's Chinese Room argument, which is sometimes taken to show that CCTM is false because performing the right computations is insufficient for such cognitive capacities as understanding. The argument proceeds via a thought experiment:

> A native English speaker who knows no Chinese [is] locked in a room full of boxes of Chinese symbols (a database) together with a book of instructions for manipulating the symbols (the program). Imagine that people outside the room send in other Chinese symbols which, unknown to the person in the room, are questions in Chinese (the input). And imagine that by following the instructions in the program the man in the room is able to pass out Chinese symbols which are correct answers to the questions (the output).
>
> *(Searle, 1999)*

From outside it seems the system understands Chinese. But according to Searle, no matter what program the man executes, he won't know what the symbols *mean*. Thus mastery of syntactic operations – of the program – is insufficient for semantics; and since understanding a sentence requires a grasp of what the sentence *means*, running a program is insufficient for understanding as well. Further, Searle maintains that the conclusion generalizes. What's true of natural language understanding is also true for cognition more generally. Running a program, no matter how good, is insufficient for cognition.

The critical discussion surrounding Searle's argument is too large to consider in detail here. (See Searle, 1980, and responses; and, Preston and Bishop, 2002, for further discussion.)

It should be noted, however, that even if Searle is correct about the inadequacy of CCTM, it is far from clear that this would render classical modeling scientifically moribund. For as we saw earlier, the two are logically independent. In addition, it is not obvious that Searle's argument undermines CCTM. One common response is that, as an objection to CCTM, it misses the mark. Classicists do not claim that executing the right program is, by itself, sufficient for thought. This would require the acceptance of a claim that classicists routinely deny: that computational role – the way the program uses a representation – determines its meaning. Rather, what classicists maintain is that cognitive processes are computational processes operating on semantically evaluable representations, whilst *leaving open* – indeed frequently endorsing – the option that semantic properties are determined by something other than computational role, such as causal relations to the environment (Fodor, 1990). Thus according to this response, the conclusion of Searle's argument is wholly compatible with the truth of CCTM.

4.2 Arguments from mathematics

A second family of objections seeks to draw out implications for the classical paradigm from well-known results in mathematics – most famously Gödel's incompleteness theorems (Gödel, 1934; Lucas, 1961; Penrose, 1989). These arguments take a variety of forms; but the general idea is that the mathematical result implies limitations on computational systems, which the human mind allegedly exceeds. In the case of Gödel's results, the limitation is, roughly, that for any formal system – such as a classical computer – which is both consistent and capable of expressing the truths of arithmetic, there will be truths that are not provable in the system. Further, it is argued that since we are capable of appreciating such truths, human minds are not classical computers.

Again, this sort of argument has generated an extensive literature. (For an overview, see Franzén, 2005.) Once more, I restrict myself to two comments. First, even if sound, it is unclear how severe the consequences for the classical paradigm would be. This is because extant mathematical arguments, even if sound, would only show that some human cognitive capacities exceed those of computers. Yet this is wholly compatible with most of our capacities being amenable to classical modeling.

Second, the extant mathematical arguments invariably depend on empirical assumptions regarding the extent of our cognitive powers. But as many commentators have noted, these assumptions are at best highly idealized (Shapiro, 1998; 2003). Why, for example, suppose that we can always see whether or not a given formalized theory is consistent, or that we are capable of formulating our own Gödel sentence? It is, to put it mildly, unobvious that these are powers we possess.

4.3 Explanatory limitations

A third family of criticisms maintains that the classical paradigm lacks the resources to explain various important psychological phenomena. In some cases, the "objection" consists of little more than drawing attention to an obviously complex psychological phenomenon, and persuasively asserting that no mere computer model could explain, or exhibit, such a capacity. Claims regarding the prospects of modeling creativity often have this flavor (Boden, 2004). In other cases, the focus is on phenomenally conscious states – such as, perceptual experiences or emotions – where there is, in Nagel's memorable phrase, "something that it is like to be" in those states (Nagel, 1974). The claim, in brief, is that classical models cannot provide a satisfactory account of how organisms can have such states because phenomenal properties are not plausibly characterized in terms of computational or functional roles (Haugeland, 1989; Chalmers, 2004).

The above concerns are seldom met with consternation by proponents of the classical paradigm. In the case of creativity, whatever the ultimate prospects for a satisfactory computational explanation, it should be unsurprising that no such account currently exists, since we lack good explanations of the *prosaic* cognitive capacities on which creativity depends. If we currently lack good explanations of *humdrum* cognition, why on earth would we expect to possess good explanations for exceptional cognition?

In the case of conscious experience and emotion, researchers are similarly unperturbed, though for different reasons. For in these cases, many classicists already *accept* the criticism. Though some brave theorists suggest that a computational account of phenomenal consciousness might be in the offing (McDermott, 2001), a more typical response is to construe classicism, not as an approach to all mental phenomena, but only to what we might loosely term the "cognitive mind". On such a view, phenomenal consciousness as such is simply not a plausible target for classical modeling.

In contrast to the above, the final example of a putative explanatory limitation that I discuss here is of genuine concern to classical researchers. This is because it appears to challenge the prospects of modeling some of the core phenomena of cognitive science – reasoning, planning, and learning, for example. The issue in question is often subsumed under the heading of the *frame problem*, and concerns our ability to determine what information is *relevant* to the tasks we perform (Ford and Pylyshyn, 1996). In particular, when making plans or revising our beliefs, we somehow manage to identify what information is relevant to the task at hand and ignore the rest. How is this "relevance sensitivity" to be explained in classical terms? It is implausible that we survey *all* our beliefs since such a strategy would require more time and computational power than we possess. Some more computationally feasible process is required. Yet many doubt such a process can be specified in classical terms. It has been suggested, for example, that relevance is unlikely to be explicable in classical terms because it is a *holistic* property of thought, in roughly the sense that the relevance of a given thought depends on a broad array of "surrounding conditions", such as one's background beliefs and intentions (Fodor, 2001; Haugeland, 1989).

Conclusion

In this chapter, I outlined the core aspects of classical computational models, enumerated their main virtues, and provided brief illustrations, both historical and contemporary. Further, I briefly discussed the relationship between classical modeling, and a range of associated ideas; and I sketched some of the more common objections to the classical approach. Although the cognitive sciences have changed dramatically since the early work of Newell and Simon, classical modeling retains a significant role in contemporary research.

Notes

1 Two terminological points. First, talk of "models" is relatively recent. More typically, early exponents of classical modeling spoke of programs as "theories" or "explanations" as opposed to models. Moreover, in recent years, "classical computation" is most typically used by computer scientists in contradistinction to *quantum* computation (Yu et al., 2002). On this use of "classical", computational systems that would be categorized as non-classical by cognitive scientists – e.g. connectionist models – are classical (i.e. non-quantum) systems.

2 Of course, algorithms satisfy a number of constraints: (a) each step in the procedure is moronic in that it requires no ingenuity or intelligence to carry out; (b) no insight or ingenuity is required to determine what the next step in the procedure is; and (c) if each step is followed exactly, the procedure is guaranteed to produce some determinate outcome in a finite number of steps.

3 Of course, the model, *qua* program, is typically run on a serial computer. However, this is also true of PDP models.

4 Roughly put, the rules of the logical system are used to specify the function in intension that is computed in the course of performing a given inferential task.

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