Ming Hsiung

Some Open Questions about Degrees of Paradoxes

Abstract. We can classify the (truth-theoretic) paradoxes according to their degrees of paradoxicality. Roughly speaking, two paradoxes have the same degrees of paradoxicality, if they lead to a contradiction under the same conditions, and one paradox has a (non-strictly) lower degree of paradoxicality than another, if whenever the former leads to a contradiction under a condition, the latter does so under the very condition. This paper aims at setting forth the theoretical framework of the theory of paradoxicality degree, and putting forward some basic open questions about paradoxes around the notion of paradoxicality degree.

Keywords: Degree of Paradoxicality, Digraph, Paradox, Revision Sequence, Truth.

1. Paradoxes and Degrees of Paradoxicality

We work in the 'standard language' for the Liar and the like paradoxes, that is, the language obtained from the first-order language of the arithmetic by augmenting a distinguished unary predicate symbol T. Let $\mathscr{L}$ be the first-order language of the arithmetic, which includes S, $\bar{+}$, $\bar{\cdot}$ and $\bar{0}$ as its non-logical symbols. Let $\mathscr{L}^+$ be the language obtained from $\mathscr{L}$ by augmenting a distinguished unary predicate symbol T. Unless otherwise claimed, when we say a formula, we mean a formula of $\mathscr{L}^+$. We will also use $\mathscr{L}^+$ to denote the set of all sentences, and so by $A \in \mathscr{L}^+$, we mean A is a sentence of $\mathscr{L}^+$. The intended model of the language $\mathscr{L}$ is $\mathfrak{N} = \langle \mathbb{N}, ', +, \cdot, 0 \rangle$, that is, the structure of natural numbers. Correspondingly, for $\mathscr{L}^+$, we will only consider those models of the form $\langle \mathfrak{N}, X \rangle$, where $X \subseteq \mathbb{N}$ is the extension of T. We can routinely define $\mathcal{V}_{\mathfrak{N},X}(A)$, i.e., the truth value of A in the model $\langle \mathfrak{N}, X \rangle$. Since the ground model $\mathfrak{N}$ is always fixed, we use $\mathcal{V}_X(A)$ instead of $\mathcal{V}_{\mathfrak{N},X}(A)$. When $\mathcal{V}_X(A) = \mathrm{T}(\mathrm{F})$, we will say A is true (false) for X. Sometimes, we also use $X \models A$ for $\mathcal{V}_X(A) = \mathrm{T}$. For brevity, we use $A \equiv B$ to denote that $A \leftrightarrow B$ is true for all $X \subseteq \mathbb{N}$.

For a sentence A, we use $\ulcorner A \urcorner$ for the Gödel's number of A, and $\overline{\ulcorner A \urcorner}$ for the corresponding numeral to the number $\ulcorner A \urcorner$. But, to avoid too many complications, we will often identify $\ulcorner A \urcorner$ with $\overline{\ulcorner A \urcorner}$, and identify a set Σ of sentences with the set of the Gödel's number of all sentences in Σ. For example, we will use $T\ulcorner A \urcorner$ instead of $T\left(\overline{\ulcorner A \urcorner}\right)$, and use $\mathcal{V}_\Sigma(A)$ instead of $\mathcal{V}_{\{\ulcorner B \urcorner \mid B \in \Sigma\}}(A)$. For any $n \geq 0$, define inductively $T^n \ulcorner A \urcorner$ as follows: $T^0 \ulcorner A \urcorner = A$ and $T^{n+1} \ulcorner A \urcorner = T\ulcorner T^n \ulcorner A \urcorner \urcorner$ for $n \geq 0$.

Our method of constructing the paradoxes is the standard one via Gödel's diagonal lemma. For instance, by use of Gödel diagonalization, we can construct the Liar sentence λ, which satisfies the equivalence $\lambda \equiv \neg T\ulcorner \lambda \urcorner$.

Next we define the revision sequence, which is a basic notion from the revision theory of truth. Note that the revision sequence was originally defined for arbitrarily large ordinals by Gupta and Herzerger. But for the present purpose, we only need to consider the revision sequences of length ω.

Definition 1.1 ([Gupta(1982)], p. 10; [Herzberger(1982)], p. 68). For a set Σ of sentences, define $\Sigma^r = \{A \in \mathscr{L}^+ \mid \Sigma \models A\}$. Define a sequence $\Sigma_0, \ldots, \Sigma_k, \ldots$ as follows: $\Sigma_0 = \Sigma$, and $\Sigma_{k+1} = \Sigma_k^r$ for all $k \geq 0$. This sequence is called the *revision sequence* starting from Σ.

We will generalize the notion of the revision sequence. To motivate the generalization, we recall that to say a set of sentences is paradoxical is to say there is no interpretation of T such that Tarski's scheme $T\ulcorner A\urcorner \leftrightarrow A$ holds for all A in this set. A precise definition is as follows.

Definition 1.2. A set Σ of sentences is *paradoxical*, if there is no Γ satisfying the condition: $\Gamma \cap \Sigma = \Gamma^r \cap \Sigma$. That is, there is no Γ such that for any $A \in \Sigma$, $\mathcal{V}_\Gamma(T\ulcorner A\urcorner) = \mathcal{V}_\Gamma(A)$.

From now on, we always use $\mathcal{K}$ to denote the digraph $\langle W, R\rangle$ unless otherwise claimed. An *assignment* in $\mathcal{K}$ is a mapping from W to the powerset $\mathscr{P}(\mathscr{L}^+)$.

Definition 1.3 ([Hsiung(2009)], pp. 243-244). Let Σ be a set of sentences. An assignment in $\mathcal{K}$, say t, is *admissible* for Σ, if for all $u, v \in W$ satisfying $u\,R\,v$,

$$t(v) \cap \Sigma = t(u)^r \cap \Sigma \tag{1}$$

Σ is *paradoxical* in $\mathcal{K}$, if there is no admissible assignment for Σ in $\mathcal{K}$.

When Σ is the set of all sentences, W is the set of natural numbers and R is the successor relation between natural numbers, an admissible assignment t for Σ in $\mathcal{K}$ is a revision sequence starting from the set $t(0)$. And so the revision sequence is a special instance of the admissible assignment. And the notion of being paradoxical in a digraph is also a generalization of being paradoxical. Actually, Σ is paradoxical, iff it is paradoxical in the minimal reflexive digraph. Note also that (1) is equivalent to

$$\text{for all } A \in \Sigma,\ \mathcal{V}_{t(v)}(T\ulcorner A\urcorner) = \mathcal{V}_{t(u)}(A).$$

And so the biconditional (1) is a formal representation of biconditional (2) in $\mathscr{L}^+$:

$$T\ulcorner A\urcorner \text{ (holds) at } v, \text{ iff } A \text{ (holds) at } u, \tag{2}$$

(where u and v are any points in the domain of a digraph such that u bears the binary relation of the digraph to v.) Hence, when a set of sentences is paradoxical in a digraph, we can think that it is impossible to evaluate these sentences (without contradiction) in the digraph such that scheme (2) holds for all of these sentences.

The idea behind the notion of paradoxicality in a digraph is that paradoxes are conditionally contradictory. As we all know, paradoxical sentences lead to a contradiction, but unlike those contradictory sentences such as 'the snow is white and it is not white', they are not absolutely contradictory otherwise there is no way to ban them from our cherished theories.

Definition 1.4 ([Hsiung(2009)], pp. 248, 254). Let Σ, Γ be two sets of sentences. Define $\Sigma \leq_P \Gamma$, if for any digraph $\mathcal{K}$, whenever Σ is paradoxical in $\mathcal{K}$, Γ is also paradoxical in $\mathcal{K}$. Define $\Sigma \equiv_P \Gamma$, if $\Sigma \leq_P \Gamma$ and $\Gamma \leq_P \Sigma$. Define $\Sigma <_P \Gamma$, if $\Sigma \leq_P \Gamma$ but $\Sigma \not\equiv_P \Gamma$.

Note that $\equiv_P$ is an equivalence relation. When $\Sigma \equiv_P \Gamma$, we will say Σ and Γ have the same *degree of paradoxicality*. When $\Sigma <_P \Gamma$, we say Σ has a (strictly) lower degree of paradoxicality than Γ.

2. Boolean Paradoxes

By the Gödel diagonal lemma, we can construct a large number of paradoxes in $\mathscr{L}^+$. Here are some examples:

Example 2.1. (a) (Cliche) the n-cycle liar $\lambda^n = \{\lambda_i^n \mid 1 \leq i \leq n\}$, where $\lambda_1^n \equiv \neg T \ulcorner \lambda_n^n \urcorner$ and $\lambda_{i+1}^n \equiv T \ulcorner \lambda_i^n \urcorner$ $(1 \leq i < n)$.

(b) ([Herzberger(1982)], pp. 74-75 and [Yablo(1985)], p. 340) the ω-cycle liar $\lambda^\omega = \{\lambda_\alpha^\omega \mid 1 \leq \alpha \leq \omega\}$, if $\lambda_1^\omega \equiv \neg T \ulcorner \lambda_\omega^\omega \urcorner$, $\lambda_{i+1}^\omega \equiv T \ulcorner \lambda_i^\omega \urcorner$ $(i \geq 1)$ and $\lambda_\omega^\omega \equiv \forall x\,(x > 0 \to T \ulcorner \lambda_{\dot{x}}^\omega \urcorner)$.

(c) ([Yablo(1985)], p. 340 and [Yablo(1993)]) Yablo's paradox $\nu = \{\nu_1, \nu_2, \nu_3, \ldots\}$, where for any $n > 0$,

$$\nu_n \equiv \forall x\,(x > \bar{n} \to \neg T \ulcorner \nu_{\dot{x}} \urcorner)$$

Definition 2.2. Let Δ be a finite set of sentences, say $\{\delta_1, \delta_2, \ldots, \delta_m\}$.

(1) Δ is a *Boolean system*, if for all $1 \leq i \leq m$,

$$\delta_i \equiv f_i \left(T \ulcorner \delta_{j_1} \urcorner, \ldots, T \ulcorner \delta_{j_{n_i}} \urcorner\right),$$

where f_i is a Boolean function which has $T \ulcorner \delta_{j_1} \urcorner, \ldots, T \ulcorner \delta_{j_{n_i}} \urcorner$ as its arguments.

(2) A *Boolean paradox* is a paradoxical Boolean system.

Remark: Roughly speaking, Boolean paradoxes are those paradoxes in which there is no occurrence of quantifiers, whose scope covers at least an occurrence of the truth predicate T. For any positive n, the n-cycle liar is a Boolean paradox, but neither the ω-cycle liar nor Yablo's paradox is Boolean.

Example 2.3. A typical Boolean paradox is Wen's paradox ([Wen(2001)], p. 44)

$$\begin{cases} \delta_1 & \equiv & T \ulcorner \delta_2 \urcorner \wedge \neg T \ulcorner \delta_3 \urcorner \\ \delta_2 & \equiv & \neg T \ulcorner \delta_1 \urcorner \vee T \ulcorner \delta_3 \urcorner \\ \delta_3 & \equiv & T \ulcorner \delta_1 \urcorner \wedge T \ulcorner \delta_2 \urcorner \end{cases}$$

We can give a complete characterization of the Boolean paradoxes with respect to their degrees of paradoxicality. The key to characterize them is the revision periods.

For the revision sequence $\mathcal{X} = \langle X_k \mid k \geq 0 \rangle$, we also use $X_k(A) = \mathrm{T}$ instead of $X_k \models A$ (and $X_k(A) = \mathrm{F}$ instead of $X_k \not\models A$). When $X_k(A) = \mathrm{T}$, we can say A is true at stage k of $\mathcal{X}$.

Definition 2.4. Let Δ be a set of sentences.

(1) A number $m \geq 1$ is a (*revision*) *period* of Δ on a revision sequence $\mathcal{X} = \langle X_k \mid k \geq 0 \rangle$, if there exists a number $N \geq 0$ such that $X_{k+m}(A) = X_k(A)$ for all $k \geq N$ and for all $A \in \Delta$. m is a period of Δ, if m is a period of Δ on some $\mathcal{X}$.

(2) A period p of Δ is said to be *primary*, if $m \nmid p$ (i.e., p is not divisible by m) for any period m of Δ with $m \neq p$.

Example 2.5. (a) the n-cycle paradox has the unique primary period 2^{i+1}, where $n = 2^i(2j+1)$

(b) Yablo's paradox has the unique primary period 2 (the same as the Liar does)!

(c) the ω-cycle paradox (and other transfinite cycle paradoxes) has no periods at all.

Remark: all the known paradoxes in the field of truth theory, without exception, have a unique primary period if they have at least one.

Proposition 2.6. *Every Boolean paradoxes has only finitely many but non-zero primary periods.*

The following graph-theoretical notion is also crucial for our characterization of Booleans paradoxes.

Definition 2.7. Let $\mathcal{K} = \langle W, R \rangle$ be a frame. A sequence $\xi = u_0\, u_1\, \ldots\, u_l$ is *a walk from u_0 to u_l*, if either $u_i\, R\, u_{i+1}$ or $u_{i+1}\, R\, u_i$ holds for any $0 \leq i < l$. ξ is *a closed walk*, if $u_0 = u_l$. ξ is *a cycle*, if none of the points in ξ is repeated except that $u_0 = u_l$.

Define a mapping h on the set of walks of $\mathcal{K}$ as follows: for any world $u \in W$, $h_{\mathcal{K}}(u) = 0$; and for any walk $\xi = u_0\, u_1\, \ldots\, u_l\, u_{l+1}$ ($l \geq 0$),

$$h_{\mathcal{K}}(\xi) = \begin{cases} h_{\mathcal{K}}(u_0\, u_1\, \ldots\, u_l) + 1, & \text{if } u_l\, R\, u_{l+1}; \\ h_{\mathcal{K}}(u_0\, u_1\, \ldots\, u_l) - 1, & \text{otherwise.} \end{cases}$$

$h_{\mathcal{K}}(\xi)$ is called the *height* of ξ in $\mathcal{K}$. The subscript $\mathcal{K}$ will be suppressed if no confusion arises.

Our characterization theorem about Boolean paradoxes is as follows:

Theorem 2.8. *([Hsiung(2017)], p. 885) For any Boolean paradox Δ and for any digraph $\mathcal{K}$, the following three conditions are equivalent:*

(a) Δ *is non-paradoxical in $\mathcal{K}$.*

(b) *For each connected component of $\mathcal{K}$, there exists a primary period of Δ, which divides the height of any cycle in this component.*

(c) *For each closed walk in $\mathcal{K}$, its height is a period of Δ.*

The following is a converse of Proposition 2.6, by which we can construct as we like.

THEOREM 2.9. *([Hsiung(2017)], p. 885) For any finite non-empty set P of numbers greater than 1, if no element of P is a multiple of any other, then there exists a Boolean paradox such that the set of its primary periods is just P.*

EXAMPLE 2.10. A paradox of primary periods 2 and 3: $\{\delta_1, \delta_2, \delta_3\}$, such that

$$\begin{cases} \delta_1 & \equiv & (T\ulcorner\delta_1\urcorner \wedge T\ulcorner\delta_2\urcorner) \vee (T\ulcorner\delta_1\urcorner \wedge \neg T\ulcorner\delta_2\urcorner \wedge T\ulcorner\delta_3\urcorner) \\ \delta_2 & \equiv & (T\ulcorner\delta_1\urcorner \wedge T\ulcorner\delta_3\urcorner) \vee (\neg T\ulcorner\delta_2\urcorner \wedge \neg T\ulcorner\delta_3\urcorner) \\ & & \vee(\neg T\ulcorner\delta_1\urcorner \wedge T\ulcorner\delta_2\urcorner \wedge T\ulcorner\delta_3\urcorner) \\ \delta_3 & \equiv & (T\ulcorner\delta_1\urcorner \wedge \neg T\ulcorner\delta_3\urcorner) \vee (\neg T\ulcorner\delta_1\urcorner \wedge T\ulcorner\delta_2\urcorner \wedge \neg T\ulcorner\delta_3\urcorner) \end{cases}$$

By the above main theorems, we can get a description of the structure of degrees of Boolean paradoxes.

THEOREM 2.11. *([Hsiung(2017)], p. 885) The set of Boolean paradoxes ordered by the binary relation $\leq_P$ is an unbounded dense lattice.*

EXAMPLE 2.12. An example for Denseness:

- The periods of the Liar: $\{\mathit{2}, 4, 6, \ldots, 2n, \ldots\}$
- The periods of the Joudain's card: $\{\mathit{4}, 8, 12, \ldots, 4n, \ldots\}$

$$\text{the Liar } <_P \text{ the Jourdain's card.}$$

Now find a paradox, namely Δ, such that

$$\text{the Liar } <_P \Delta <_P \text{ the Jourdain's card.}$$

- The periods of Δ: $\{\mathit{4}, \mathit{6}, 8, 12, \ldots, 4n, 6n, \ldots\}$

EXAMPLE 2.13. An example for Greatest Lower Bound:

- The periods of the Liar: $\{\mathit{2}, 4, 6, \ldots, 2n, \ldots\}$
- The periods of Wen's paradox: $\{\mathit{3}, 6, 9, \ldots, 3n, \ldots\}$

$$\text{the Liar } |_P \text{ Wen's paradox.}$$

Find a paradox Δ, such that its degree is the greatest lower bound of degrees of the Liar and Wen's paradox.

- The periods of Δ: $\{2, 3, 4, 6, \ldots, 2n, 3n, \ldots\}$

Example 2.14. An example for Least Upper Bound:

- The periods of the Liar: $\{2, 4, 6, \ldots, 2n, \ldots\}$
- The periods of Wen's paradox: $\{3, 6, 9, \ldots, 3n, \ldots\}$

$$\text{the Liar } |_P \text{ Wen's paradox.}$$

Find a paradox Δ, such that its degree is the least upper bound of degrees of the Liar and Wen's paradox.

- The periods of Δ: $\{6, 12, 18, \ldots, 6n, \ldots\}$

Even we only consider Boolean paradoxes, and even they are only a small part of the paradoxes and have relatively simple syntactical structures, the area of our study is proved to be rich in mathematical structures and properties.

Now we raise a question about Boolean paradoxes. Wen's paradox (as shown in Example 2.3) is evidently not directly self-referential. But the paradoxes we construct by the methods of Theorem 2.9 are usually directly self-referential (an typical example is shown in Example 2.10). Our question is whether we can construct a non-directly-self-referential Boolean paradox for a given finitely many primary periods.

Problem 2.15. ([Hsiung(2017)], p. 898-899) **Is there a Boolean paradox $\{\delta_1, \delta_2, \delta_3\}$ of the periodicity set $\{2, 3\}$ which is not directly self-referential in the following sense: for any $1 \leq i \leq 3$, there is no occurrence of $T\ulcorner\delta_i\urcorner$ in the right side of definitional equivalence of δ_i? For any Boolean paradox, is there a Boolean paradox which has the same primary periods as the known one, but is not directly self-referential in the similar sense we just explain for the Boolean paradox of the periodicity set $\{2, 3\}$?**

3. The Algebraic Structure for Degrees of Paradoxes

Theorem 2.11 provides us a description of the algebraic structure for degrees of Boolean paradoxes. A natural question is to ask about the algebraic structure for degrees of all paradoxes. For instance, can Theorem 2.11 be extended to all the paradoxes, no matter they are Boolean or not.

A basic result about the paradoxes is that their degrees are bounded above by the degree of ω-liar or by the degree of McGee's paradox ([McGee(1985)], p. 400).

Theorem 3.1. *([Hsiung(2018)]) The ω-liar and McGee's paradox have the highest degree of paradoxicality.*

Dually, we ask:

PROBLEM 3.2. ([Hsiung(2017)], p. 913) **Is there a paradox whose degree is the lowest (among all the degrees of paradoxes)?**

We conjecture that the answer to this question is positive. A reasonable candidate is the paradox whose primary periods are exactly the prime numbers. And, if it is so, the crucial point of this question is how to construct such a paradox.

There are some new questions we can ask of course. The first question we ask is about the completeness of degrees of paradoxes.

PROBLEM 3.3. **Is the structure for degrees of paradoxes complete? In other words, for countably infinite many paradoxes, is there always a paradox whose degree is least upper bound/greatst lower bound of the degrees of these paradoxes?**

Remark: Problem 3.3 is well-defined because of Theorem 3.1 and Problem 3.2.

PROBLEM 3.4. **For any paradox, is there any paradox such that the supremum of their degrees is just the degree of ω-liar (i.e., the greatest degree)? Or, For any paradoxicality, is there any degree of paradoxicality such that their supremum is just the greatest degree?**

Dually, we ask: for any paradox, is there any paradox such that the infimum of their degrees is just the smallest degree (assume it does exist)?

To sum up, we actually ask whether the degrees of paradoxicality can form a Boolean algebra.

Up to now, we can summarize the above observations and problems by Figure 1. Note, in Figure 1, the big diamond stands for the degrees of paradoxes, and the small diamond surrounded by the dashed line stands for the degree of Boolean paradoxes. The degrees of the Liar and its finitary variants occur in the middle line of the two diamonds, and the degree of Yablo's paradox occurs at the same level as that of the Liar (Theorem 5.1).

As far as the small diamond is concerned, Theorem 2.11 provides a lot of information. **The future main task is to investigate the whole big diamond.**

4. Some Implicitly defined Paradoxes

We can define semantically some paradoxes in $\mathscr{L}^+$ without seeing their syntactical specification. The following are some simplest examples.

DEFINITION 4.1 ([Hsiung(2009)], 248). For any positive number n, we use λ^n for a sentence such that for any frame $\mathcal{K}$ and for any admissible assignment t of $\mathcal{K}$, and for any points u and v of W such that $u\, R^n\, v$,

$$t(v) \models \lambda^n \iff t(u) \not\models \lambda^n. \tag{3}$$

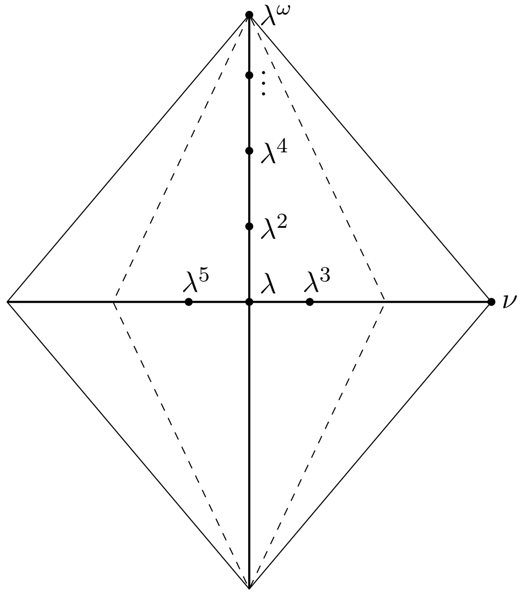

Figure 1: degrees of paradoxes

Here, note that $u\,R^n\,v$ denotes that there exist $u_0, \dots, u_n$ such that $u_0 = u$, $u_n = v$ and for $0 \le i < n$, $u_i R u_{i+1}$. λ^n is called 'n-jump liar'. Clearly, the 1-jump liar is the liar, which is the unique one that we have known its syntactical representation among all the jump liars up to now. Let us say the n-jump liar is implicitly defined.

PROBLEM 4.2 ([Hsiung(2009)], 269). **For any number $n > 1$, is there any sentence of $\mathscr{L}^+$, which is a syntactical specification of λ^n? Or more briefly, can we always construct the n-jump liar?**

We can of course provide more implicitly defined paradoxes as we like. Here are some examples. For convenience, I will use

$$\psi \overset{P(u,v)}{\Longleftrightarrow} \varphi$$

to denote the statement that "$\psi \iff \varphi$" holds for all worlds u and v in the domain of a frame such that the condition $P(u, v)$ is satisfied.

DEFINITION 4.3. For any positive number n, let λ_i $(1 \le i \le n)$ be sentences such that for any frame $\mathcal{K}$ and for any admissible assignment t of $\mathcal{K}$, we have

$$t(v) \models \lambda_1 \overset{uRv}{\Longleftrightarrow} t(u) \not\models \lambda_n. \tag{4-1}$$

$$t(v) \models \lambda_2 \overset{uR^2v}{\Longleftrightarrow} t(u) \not\models \lambda_1. \tag{4-2}$$

$$t(v) \models \lambda_3 \overset{uR^3v}{\Longleftrightarrow} t(u) \not\models \lambda_2. \tag{4-3}$$

$$\cdots\cdots$$

$$t(v) \models \lambda_n \xLeftrightarrow{uR^n v} t(u) \not\models \lambda_{n-1}. \tag{4-n}$$

PROBLEM 4.4. **Find the characterization frames for the set of λ_i ($1 \leq i \leq n$). And if possible, construct these sentences in $\mathscr{L}^+$.**

5. Yablozation of Paradoxes

One interesting respect of Yablo's paradox is that it gives a method of eliminating self-reference of a paradox (see for instance [Schlenker(2007)]). More interesting, such a method seems not to damage the degree of paradoxes which removing the self-reference of a paradox.

THEOREM 5.1. *([Hsiung(2013)], p. 26) The Yablo's paradox has the same degree of paradoxicality as the Liar.*

More generally, for any n-cycle liar λ^n, we can define its Yablozation ν^n as follows: let $\nu^n = \{\nu^i_j \mid 1 \leq i \leq n, j \geq 1\}$ such that

(1) for all $j \geq 1$, ν^1_j is the sentence which says ν^n_k is untrue for all $k > j$;

(2) for all $1 \leq i < n$, $j \geq 1$, ν^{i+1}_j is the sentence which says ν^i_k is true for all $k > j$.

As is illustrated in the right column of table 1, all of these sentences form an infinite matrix. Note that the above procedure of Yablozation is also called "unwinding". See for instance [Cook(2004)], pp. 770-771.

λ^n	ν^n			
λ^n_1	ν^1_1	ν^1_2	ν^1_3	$\ldots$
λ^n_2	ν^2_1	ν^2_2	ν^2_3	$\ldots$
λ^n_3	ν^3_1	ν^3_2	ν^3_3	$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
λ^n_n	ν^n_1	ν^n_2	ν^n_3	$\ldots$

Table 1: The n-cycle liar and its Yablozation

THEOREM 5.2. *For any positive integer n, the n-cycle liar and its Yablozation have the same degree of paradoxicality.*

Clearly, the above procedure of Yablozation can be extended to any paradox. For instance, for Yablo's paradox itself, we can Yabloze it one more and get an even big monster paradox.

PROBLEM 5.3. **Does a paradox always have the same degree of paradoxicality as its Yablozation?**

There is no reason to resist conjecturing that the answer should be positive.

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Ming Xiong (Hsiung)
School of Politics and Administration
South China Normal University
Guangzhou 510631, P. R. China
`mingshone@163.com`