Chapter 4
The Clock Paradox: Luise Lange's Discussion

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Abstract In her articles on the clock paradox and the relativity of time Luise Lange (1891–1978) defends the theory of relativity against philosophical refutations, by showing that the apparent clock paradox is not a paradox, but merely conflicts with common sense and is based on a misunderstanding of the theory. The following study explores, contextualizes and analyzes Lange's clear and sophisticated contribution to the debate on the clock paradox for the first time.

Keywords Clock paradox · Twin paradox · Theory of relativity · Time dilation

4.1 The Clock Paradox: Luise Lange's Solution and Its Context

At the beginning of the 1920s great interest in Einstein's theory of relativity flared up in broad circles. Amidst this "Relativitätsrummel," as it was called by Arnold Sommerfeld (qtd. Hentschel 1990, 67), two papers on "The Clock Paradox of the Theory of Relativity" (Lange 1927a) and "On a Misconception of the Relativity of Time" (Lange 1927b) were published. The author was Luise Lange (1891–1978).[1]

[1]Luise Lange was born in Braunschweig on April 14, 1891. She studied at the Braunschweig University of Technology and at the Göttingen University and completed her doctorate in 1923. The examiners were Peter Josef William Debye (physics), Adolf Windaus (chemistry), and Constantin Carathéodory (mathematics). She also prepared David Hilbert's *Vorlesung zur Statistisches Mechanik* of the summer semester 1914 (Hilbert 1914). After her emigration to the USA, she worked at the Oxford Female College, a small women's college in Oxford, Ohio and later at the Woodrow Wilson College, Chicago. I gratefully acknowledge Katie Ehrlich, Adjunct Archivist for Access and Outreach at New York University Archives, who provided me insight

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In both works Luise Lange argued that the clock paradox does not invalidate that theory, but arises "by one or several misapplications of the theory of relativity" (Lange 1927a, 25).

Consider clocks brought together in the same inertial reference frame and synchronized. What happens if one clock moves away in a spaceship and then returns? The answer is well known today: The clock which had undergone the traveling would be found to lag behind the clock which stays put. This phenomenon is a necessary consequence of the relativity of simultaneity and time dilation. As Albert Einstein put it (Einstein 1905, 904 f.):

> If there are two synchronous clocks at A, and one of them is moved along a closed curve with constant velocity until it has returned to A, which takes, say t seconds, then this clock will lag on its arrival at A by ½ $t(v/V)^2$ seconds behind the clock that has not been moved.[2]

The core of Einstein's 1905 paper is the derivation of Lorentz invariance on just the two basic principles of relativity and light-speed invariance. It is often claimed that special relativity has replaced the conventional notion of an absolute universal time with the notion of time that is dependent on reference frame and spatial position. However, it is not a new concept of time, but a new conception and conceptual framework of spatiotemporal relations that emerges from special relativity, i.e. the Lorentz transformations preserve the space-time interval, an algebraic combination of space and time intervals when viewed from any inertial reference frame, not the invariant time interval between two events. Suppose an observer measures two events as being separated by a time Δt and a spatial distance Δx. Then the space-time interval Δs^2 between the two events that are separated by a distance Δx in space and a duration Δt in time is $\Delta s = \sqrt{c^2 + \Delta t^2 - \Delta d^2}$.

In a lecture to the *Naturforschende Gesellschaft* on 16 January 1911, Einstein noted that the same effect of time dilation would also apply to living organisms travelling at very high velocities (Einstein 1911, 12):

> We must add that what holds true for this clock we have introduced as a simple representative of all physical progress also applies to any closed physical system. If we placed a living organism in a box [...] one could arrange that the organism, after any arbitrary lengthy flight, could be returned to its original spot in a scarcely altered condition, while corresponding organisms which had remained in their original positions had already long

into the Richard Courant Papers MC 150, including the correspondence between Luise Lange and Richard Courant from 1939 to 1942. I would also like to express my grateful thanks to Ulrich Hunger from the Göttingen University Archive and Renate Tobies, Friedrich Schiller University of Jena, for their helpful comments and hints. A special thanks is also extended to Edgar E. Enochs from University of Kentucky Lexington, KY USA, who got to know Luise Lange during his time at the University of Chicago, from 1958 to 1960. The publication of my research results on Luise Lange is in preparation.

[2]"Befinden sich in A zwei synchron gehende Uhren und bewegt man die eine derselben auf einer geschlossenen Kurve mit konstanter Geschwindigkeit, bis sie wieder nach A zurückkommt, was t Sek. dauern möge, so geht die letztere Uhr bei ihrer Ankunft gegenüber der unbewegt gebliebenen um ½ $t(v/V)^2$ Sek. nach." Unless otherwise noted, all translations in the text are my own.

> since given way to new generations. For the moving organism, the lengthy time of the journey was a mere instant, provided the motion took place with approximately the speed of light.[3]

In his talk « L'evolution de l'espace et du temps » on 11 April 1911, Paul Langevin used a similar example, supposing two arbitrary events in the history of an element of matter (Langevin 1911, 48):

> Suppose that two portions of matter meet for the first time, separate and meet again. We can say that observers attached to the portions during separation, will not evaluate that duration in the same way, as some have not aged as much as the others.[4]

Langevin's considerations stimulated Einstein to consider time delays as more than just affecting clocks. In an unpublished manuscript written after Langevin's talk Einstein assumed that relativity theory described "the temporal course of no matter which process" (qtd. Canales 2015, 57). Hermann Weyl, among others, supported this assertion in his famous book *Raum. Zeit. Materie. Vorlesungen über allgemeine Relativitätstheorie*: "The life processes of mankind may well be compared to a clock" (Weyl 1919, 187). Weyl was probably the first who mention the twins in the context of the so-called paradox (ibid., 158):

> Suppose we have two twin-brothers who take leave one another at a world-point A, and suppose one remains at home (that is, permanently at rest in an allowable reference-space), whilst the other sets out on voyages, during which he moves with velocities (relative to "home") that approximate to that of light. When the traveler returns home in later years he will appear appreciably younger than the one who stayed at home.[5]

Langevin's identification of the time of clocks with human life times triggered a heated debate which reached its climax in the early 1920s, when Henri Bergson attacked Albert Einstein at a meeting of the *Société française de philosophie* in Paris. Bergson found Einstein's definition of time in terms of clocks completely inappropriate. Clocks, by themselves, could not explain either simultaneity or time.

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[3] "Man muss hinzufügen, dass das, was für diese Uhr gilt, welche wir als einen einfachen Repräsentanten alles physikalischen Geschehens eingeführt haben, auch gilt für ein in sich abgeschlossenes physikalisches System irgendwelcher anderer Beschaffenheit. Wenn wir z.B. einen lebenden Organismus in eine Schachtel hineinbrächten [. . .], so könnte man es erreichen, dass dieser Organismus nach einem beliebig langen Fluge beliebig wenig geändert wieder an seinen ursprünglichen Ort zurückkehrt, während ganz entsprechend beschaffene Organismen, welche an den ursprünglichen Orten ruhend geblieben sind, bereits längst neuen Generationen Platz gemacht haben. Für den bewegten Organismus war die lange Zeit der Reise nur ein Augenblick, falls die Bewegung annähernd mit Lichtgeschwindigkeit erfolgte!"

[4] «Supposons que deux portions de matière se rencontrent une première fois, se séparent, puis se retrouvent. Nous pouvons affirmer que des observateurs liés à l'une et à l'autre pendant la séparation n'auront pas évalué de la même manière la durée de celle-ci, n'auront pas vieilli autant les uns que les autres.»

[5] "Von zwei Zwillingsbrüdern, die sich in einem Weltpunkt A trennen, bleibe der eine in der Heimat (d.h. ruhe dauernd in einem tauglichen Bezugsraum, der andere aber unternehme Reisen, bei denen er Geschwindigkeiten (relativ zur Heimat) entwickelt, die der Lichtgeschwindigkeit nahe kommen; dann wird sich der Reisende, wenn er dereinst in die Heimat zurückkehrt, als merklich jünger herausstellen als der Seßhafte."

Bergson argued that the concept of universal time arises from our own "proper" experienced time which he called "real duration" in contrast to the mechanistic time of science (see Bergson 1922). Einstein replied that the notion of simultaneity of individual perception should not be confused with the simultaneity of objective events independent of individuals as it is used in physics.

Luise Lange defended the theory of relativity against such philosophical refutations, by showing that Bergon's criticism is based on a misunderstanding of the theory. Her clear and sophisticated contribution to the debate is completely forgotten and was never discussed nor mentioned in the literature. Lange writes (1927a, 24):

> In 1911 P. Langevin formulated and solved the problem in the since well-known way: Peter stands on the earth, Paul, enclosed in a bullet, is fired away with tremendous velocity, but due to a suitable encounter with some star his path is directed back to the earth. On both journeys from and to the earth his clock is slow in comparison to those at rest to the earth; (as say the L. E.); hence in returning his time is behind earth-time in the ratio $(1 - v^2c^{-2})^{-1/2} : 1$. If he stayed away 2 years according to his time that elapsed on the earth is, say, two hundred years, if the velocity with which he performed his cosmic journey was such that $(1 - v^2c^{-2})^{-1/2} : 0.01$. While Paul has aged only slightly during his absence, the sixth generation of Peter's descendants are already populating the planet.

One might think that each twin should see the other aging more slowly, and so the paradox arises that each believes the other should be younger at their reunion. However, whereas time dilation is symmetrical, the travelling twin's trajectory involves two different inertial frames, and so there is no symmetry between the space-time paths of the two twins. The paradox centers, as Luise Lange rightly stated, on the assertion that, in relativity, either twin could regard the other as the traveler, in which case each should find the other younger – a logical contradiction. The logical fallacy lies in the incorrect assumption that the twins' situations are symmetrical and interchangeable which is indeed false. To quote Luise Lange (ibid.):

> This result is frequently spoken of as Langevin's Paradox, but unjustly; for there is nothing self-contradictory in it, contrary as it may be to customary notions. It turned paradox only in the later development.

In this context Luise Lange attacked the "fanciful stories" of Ernst Gehrcke (1920) and Karl Vogtherr (1923) who advocated an anti-relativity propaganda campaign (Lange 1927b, 501):

> We now want to show that, no matter what revolutions in our concepts of space and time the theory of relativity may have brought about it is wholly innocent as regards the above fanciful stories. That it neither demands nor even admits the dependence of sense perception on the relative state of motion of the observer.

Luise Lange pointed out that the notion "observer" is a technical one. It has nothing to do with the sense perception of human beings, but refers to measurement procedures, clocks and measuring rods, based on the invariance of the speed of light, whereby Lorentz transformation is a method of measurement of a distance in space-time. Thus, the concept of proper time does not imply the claim that just the present moment experienced by the observer is real (ibid., 506):

> Observers then in relative motion disagree on the position of the clock hands of a distant clock not because they receive different visual impressions from it, but because they account differently for the time of light transmission. The misunderstanding of this point seems to have arisen because this one fact is not kept in mind: the time of a distant event is never – in classical mechanics as little as in relativity mechanics – directly observed or experienced; but it is determined only by a combination of measuring (which involves perception) and *computation*. In the terminology of the theory of relativity, it is true, the computation is frequently disguised as a "nothing but observation," due namely to the underlying scheme if imagining observers at every place "right next to clock" synchronized with all others by means of a light signal. That way the computation is simply performed in advance and once for all by all the observers who at places x set their clocks on $t = \frac{x}{c}$ on receiving the signal sent from $x = 0$ at $t = 0$. According to our theory it is only this computation, this dating back into the past of an event observed in the present, which contains the relative element, not the sense perception and thus we have come to see that the theory of relativity in no way implies the doctrine of solipsism.

To conclude, special theory of relativity is not inconsistent as it might appear at first glance. As Luise Lange put it, "there is nothing self-contradictory in it" (Lange 1927a, 24). The paradox arises from a false premise of the motional symmetry of the twins.

4.2 Two Incorrect Solutions

In her discussion of time dilation Luise Lange presented not only the correct solution of the clock/twin paradox. She also examined two false solutions: (i) acceleration is essential in order to explain this thought experiment; (ii) a complete solution of the paradox can just be found in the frame of general relativity. Luise Lange argued that both claims are false. Neither acceleration nor general relativity are necessary ingredients for the resolution of the twin paradox. Nevertheless, they are possible and interesting cases.

In the standard formulation of the twin paradox acceleration is responsible for the asymmetric aging effect. But it is also possible to explain the scenario in another way, be switching inertial reference frames which slows the clock of the traveling twin down. There is a difference between the trajectories of the twins: the trajectory of the traveling twin is equally divided between two different inertial frames, while the Earth-based twin stays in the same inertial frame. Another way to look at it is in terms of what are known as word lines which are the path an object follows in space-time. The shift of the word line is such that the resulting world line of the travelling twin is shorter than the one of the stay at home twin.

Max von Laue, to whom Luise Lange refers, used this argumentation in order to elaborate on Langevin's explanation. He demonstrated that the world lines of the inertially moving bodies maximize the proper time elapsed between two events and concluded that the asymmetric aging is completely accounted for by the fact that the astronaut twin travels in two separate frames, while the Earth twin remains in one frame, and the time of acceleration can be made arbitrarily small compared with the time of inertial motion (Laue 1911).

Thus, the resolution of the paradox can be given purely in special relativity within a flat (Minkowski) space-time. However, Luise Lange also asked: “What about the general theory?” In his “Dialog über die Einwände gegen die Relativitätstheorie” (Einstein 1918), Einstein used gravitational time dilation to resolve the twin paradox. According to general theory of relativity, clocks low in a gravitational field run slower than clocks high in a gravitational field. Einstein invoked the equivalence principle and a homogenous gravitational field equivalent to the acceleration as the physical cause of asymmetrical time dilation.

In her discussion of Einstein’s dialogue, Luise Lange made clear that the compensating gravitational time dilation is due to the difference in gravitational potential at two points in the field rather than being an effect of the accelerating motion itself. In the case of special relativity, it is the difference in the paths that results in a time dilation for the accelerated twin; analogously, in the case of general relativity, the compensating gravitational time dilation is due to the difference in gravitational potential at two points in the field rather than being an effect of the accelerating motion itself. Luise Lange showed that Max Born (1924) and August Kopff (1923) misunderstood this fact.

4.3 Conclusion

The twin paradox, or, more general, the clock paradox has been a subject of a heated discussion for over hundred years and continues to attract physicists and philosophers presenting new insight or defending old theses. In his highly-regarded book *Relativity: Special, General and Cosmological* Wolfgang Rindler (2001, 67) writes:

> Reams of literature were written on it unnecessarily for more than six decades. At its root apparently lay a deep psychological barrier to accepting time dilatation is real. From a modern point of view, it is difficult to understand the earlier fascination with this problem or even to recognize it as a problem.

Even though nowadays many physicists may consider the matter only of educational interest, the debate is ongoing. Why? Is it just because of the unfamiliarity with the historical discussion and context? I don’t think so. The key to a solution of a paradox often lies in clarifying misconceptions and identifying mistaken premises. In a certain manner this is a never-ending process, beyond pure pedagogical challenges. To conclude with an example, namely the concept of “proper time.” Proper time, expressed as an integral over the world line, is invariant, i.e. it takes the same value in all inertial references. Luise Lange noted with regard to proper time (Lange 1927a, 29f.):

> Thus our survey has led us to the conclusion that the question regarding the difference in duration experienced in two relatively moving systems between two encounters is as yet

> unanswered. It may be, as many relativists assert, that in worlds of different state of motion the derôlement of time is different, like on motion pictures reeled through slower and faster, with the inert system being the fastest ones, but we do not see that the complete proof for it has yet been given.

The crucial point here is: Proper time is independent of coordinates, but depends on world lines. It is expressed as an integral over the world line. The time difference when the twins meet again does not come from comparing local rates of time along their world lines; it results from integrating the proper time along the world lines and concluding that their paths through space-time have different lengths. In other words, we evaluate a path integral along two different paths, and get two different results. An accelerated clock will measure a smaller elapsed time between two events than that measured by a non-accelerated (inertial) clock between the same two events. The twin paradox is an example of this effect. In this sense, the solution of the paradox is given by clarifying a common misunderstanding regarding the concept of proper time. This is exactly what Luise Lange did in her papers on the clock paradox and the relativity of time.

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