Bilateralism: Negations, Implications and some Observations and Problems about Hypotheses

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Abstract

This short paper has two loosely connected parts. In the first part, I discuss the difference between classical and intuitionist logic in relation to different the role of hypotheses play in each logic. Harmony is normally understood as a relation between two ways of manipulating formulas in systems of natural deduction: their introduction and elimination. I argue, however, that there is at least a third way of manipulating formulas, namely the discharge of assumption, and that the difference between classical and intuitionist logic can be characterised as a difference of the conditions under which discharge is allowed. Harmony, as ordinarily understood, has nothing to say about discharge. This raises the question whether the notion of harmony can be suitably extended. This requires there to be a suitable fourth way of manipulating formulas that discharge can stand in harmony to. The question is whether there is such a notion: what might it be that stands to discharge of formulas as introduction stands to elimination? One that immediately comes to mind is the making of assumptions. I leave it as an open question for further research whether the notion of harmony can be fruitfully extended in the way suggested here. In the second part, I discuss bilateralism, which proposes a wholesale revision of what it is that is assumed and manipulated by rules of inference in deductions: rules apply to speech acts – assertions and denials – rather than propositions. I point out two problems for bilateralism. First, bilaterlists cannot, contrary to what they claim to be able to do, draw a distinction between the truth and assertibility of a proposition. Secondly, it is not clear what it means to assume an expression such as '+ A that is supposed to stand for an assertion. Worse than that, it is plausible that making an assumption is a particular speech act, as argued by Dummett. Bilaterlists accept that speech acts cannot be embedded in other speech acts. But then it is meaningless to assume + A or − A.

1 Harmony: Negation, Implication and Hypotheses

In standard systems of natural deduction, the rules for intuitionist negation are *in harmony*. The grounds for and consequences of asserting $\neg A$ balance each other:

$$\neg I: \quad \dfrac{\begin{array}{c}\overline{A}^{\,i}\\ \Pi\\ \bot\end{array}}{\neg A}\,i \qquad\qquad \neg E: \quad \dfrac{\neg A \qquad A}{\bot} \qquad\qquad \bot E: \quad \dfrac{\bot}{B}$$

By applying $\neg E$, we only get out of an assertion of $\neg A$ what is required for an application of ¬I: a deduction of $\bot$ from A. Everything follows from $\bot$, so it has no grounds for its assertion: $\bot E$ is harmonious with the lack of an introduction rule for $\bot$.

The rules for classical negation are not in harmony, as we need to add, e.g., one of the following:

$$\dfrac{\begin{array}{c}\overline{\neg A}^{\,i}\\ \Pi\\ \bot\end{array}}{A}\,i \qquad\qquad \dfrac{\neg\neg A}{A} \qquad\qquad \dfrac{\begin{array}{c}\overline{A}^{\,i}\\ \Pi\\ C\end{array} \qquad \begin{array}{c}\overline{\neg A}^{\,i}\\ \Xi\\ C\end{array}}{C}\,i \qquad\qquad \dfrac{}{A \vee \neg A}$$

This creates a misbalance between the consequences of asserting $\neg\neg A$ and the grounds for asserting it: we get more out of $\neg\neg A$ than we put in.[1]

There are axioms and rules involving neither $\bot$ nor $\neg$ that also have the effect of resulting in classical logic when added to intuitionist logic, such as Pierce's Law $\vdash ((A \supset B) \supset A) \supset A$, $\vdash A \vee (A \supset B)$, or $\vdash (A \supset B) \vee (B \supset C)$.[2] Putting these axioms into rule form eliminates the appeal to disjunction:

$$\dfrac{\begin{array}{c}\overline{A \supset B}^{\,i}\\ \Pi\\ A\end{array}}{A}\,i \qquad\qquad \dfrac{\begin{array}{c}\overline{\mathrm{A}}^{\,i}\\ \Pi\\ C\end{array} \qquad \begin{array}{c}\overline{A \supset B}^{\,i}\\ \Sigma\\ C\end{array}}{C}\,i \qquad\qquad \dfrac{\begin{array}{c}\overline{A \supset B}^{\,i}\\ \Pi\\ D\end{array} \qquad \begin{array}{c}\overline{B \supset C}^{\,i}\\ \Sigma\\ D\end{array}}{D}\,i$$

If we define $\neg A$ as $A \supset \bot$, then the misbalance in the classical rules is one between the grounds and the consequences of assertions of formulas of the form $A \supset B$: the classical rules allow us to get more out of some assertions of the form $A \supset B$ than we put into them, given the introduction rule for $\supset$. The problematic classical rules for negation and implication have in common that they introduce additional options for the *discharge of hypotheses*.

The canonical ground for the assertion of $A \supset B$ is that under assumption A, I can derive B. A derivation of B from A allows discharge of A and derivation of $A \supset B$. In their antecedents, conditionals contain information about the discharge of assumptions. Pierce's Rule can be understood in the following way: if A can be derived under the assumption that A can be discharged, that is if A is the premises of some deduction, where any B will do as the conclusion, then infer A and discharge the assumption that it can be discharged. More pithily: If A can be derived under the assumption that it can be discharged, then A is true regardless.[3] The other two rules allow for analogous interpretations: if C can

[1]Each of these four classical negation rules has its champion. Gentzen (1934): 190 opts for the fourth, later Gentzen (1936): 515 opts for the second, Prawitz (1965): 20 chooses the first, Tennant (1978) the third. Kürbis (2015) discusses intuitionist and classical negation from the perspective of a theory of meaning in Dummett's sense.

[2]Another option is $\vdash (A \supset (B \vee C)) \supset ((A \supset B) \vee C)$.

[3]This reading of Pierce's Rule was suggested to me by Wilfried Meyer-Viol in conversation.

be derived from A and the assumption that A can be discharged, then C is true regardless; if D can be derived on the assumption that B can be concluded and that B can be discharged, then D is true regardless.

We could say that the difference between classical and intuitionist logic is located in the notion of discharge of hypotheses. This raises a question: harmony is a relation between the grounds and consequences of formulas. Grounds and consequences are complementary notions, related by the notion of harmony. Harmony of rules for a connective $*$ is a relation between $*I$ and $*E$. What the characteristically classical rules add to the harmonious intuitionist ones are further options for the *discharge of hypotheses* with $*$ as main operator. It is this wider notion of discharge that is captured by the classical conditional and principles such as Pierce's Rule. The intuitionist logician recognises only two ways of manipulating formulas with a main operator $*$ in deductions for which harmony is salient. There are, however, at least three ways: introduction, elimination and discharge of formulas with $*$ as main operator. This observation opens up a path that allows the classicist to resist the charge that classical principles governing $\supset$ such as Pierce's rule upset the harmony that holds between its introduction and elimination rules: to demand an extension of the notion of harmony such that it relates not only the introduction and elimination rules for a connective $*$, but also rules allowing the discharge of formulas with $*$ as main operator with a suitable complementary way of manipulating formulas in deductions. Can we extend the notion of harmony in such a way that it lets us specify something harmonious to rules such as Pierce's Law? The question is whether there is such a notion: what might it be that stands discharge of formulas as introduction stands to elimination? This requires a fourth way of manipulation formulas in deductions. One that immediately comes to mind is the making of assumptions. I leave it as an open question for further research whether the notion of harmony can be fruitfully extended in the way suggested here.

2 Classical Bilateralism

In *bilateral logic* we find a wholesale revision of what it is that is assumed and manipulated by rules of inference in deductions: rules apply to speech acts – assertions and denials – rather than propositions. In a bilateralist system of natural deduction, motivated by Price (1983) (see also Price (1990) and Price (2016)) and formalised by Rumfitt (2000), the meanings of the logical constants are specified in terms of two primitive speech acts, assertion and denial. Now the situation appears to be reversed: the rules for classical negation are in harmony, and the misbalance occurs between the grounds for and consequences of denying negated formulas in intuitionist bilateral logic.

Rules of bilateral logics apply to *signed formulas*: asserted formulas are signed by $+$, denied ones by $-$. Lower case Greek letters range over signed formulas. $\alpha*$ designates the *conjugate* of α, the result of 'reversing' its sign from $+$ to $-$, and from $-$ to $+$. Rumfitt's system contains rules that specify primitively, for each connective, the grounds for asserting/denying any complex formulas and the consequences of asserting/denying them. Call the following list of rules $\mathfrak{AD}$:

$$+\&I:\quad \frac{+A \qquad +B}{+A\&B} \qquad\qquad +\&E:\quad \frac{+A\&B}{+A} \qquad \frac{+A\&B}{+B}$$

$$-\&I:\quad \frac{-A}{-A\&B} \qquad \frac{-B}{-A\&B} \qquad\qquad -\&E:\quad \frac{-A\&B \qquad \begin{array}{c}\overline{-A}^{\,i}\\ \Pi\\ \phi\end{array} \qquad \begin{array}{c}\overline{-B}^{\,i}\\ \Sigma\\ \phi\end{array}}{\phi}\,i$$

$$+\vee I:\quad \frac{+A}{+A\vee B} \qquad \frac{+B}{+A\vee B} \qquad\qquad +\vee E:\quad \frac{+A\vee B \qquad \begin{array}{c}\overline{+A}^{\,i}\\ \Pi\\ \phi\end{array} \qquad \begin{array}{c}\overline{+B}^{\,i}\\ \Sigma\\ \phi\end{array}}{\phi}\,i$$

$$-\vee I:\quad \frac{-A \qquad -B}{-A\vee B} \qquad\qquad -\vee E:\quad \frac{-A\vee B}{-A} \qquad \frac{-A\vee B}{-B}$$

$$+\supset I:\quad \frac{\begin{array}{c}\overline{+A}^{\,i}\\ \Pi\\ +B\end{array}}{+A\supset B}\,i \qquad\qquad +\supset E:\quad \frac{+A\supset B \qquad +A}{+B}$$

$$-\supset I:\quad \frac{+A \qquad -B}{-A\supset B} \qquad\qquad -\supset E:\quad \frac{-A\supset B}{+A} \qquad \frac{-A\supset B}{-B}$$

$$+\neg I:\quad \frac{-A}{+\neg A} \qquad\qquad +\neg E:\quad \frac{+\neg A}{-A}$$

$$-\neg I:\quad \frac{+A}{-\neg A} \qquad\qquad -\neg E:\quad \frac{-\neg A}{+A}$$

The four rules for negation evidently exhibit some kind of harmony. An intuitionist must reject $-\neg E$. This creates a misbalance between the grounds for denying $\neg A$ as specified by $-\neg I$ and the consequences of denying it: we get less out of denying $\neg A$ than we put in.

Gibbard (2002) points out that $\mathfrak{AD}$ is constructive logic with strong negation: double negation elimination and DeMorgan's Laws hold, but the laws of excluded middle and non-contradiction do not. Rumfitt (2002) responds that $\mathfrak{AD}$ is intended to be supplemented by a form of *reductio* he names after Timothy Smiley:

Smiley: If $\Gamma, \alpha \vdash \beta$ and $\Gamma, \alpha \vdash \beta*$, then $\Gamma \vdash \alpha*$

Adding Smiley to $\mathfrak{AD}$ gives a system of classical bilateral logic which I call $\mathfrak{B}$.

Notice that, in line with observations of the previous section, what needs to be added to $\mathfrak{AD}$ in a formalisation of classical bilateral logic is once more a rule that allows further options for the discharge of hypotheses. Classical logic requires a stronger notion of discharge than $\mathfrak{AD}$, constructive logic with strong negation, allows. So once more, we can locate the difference between the two systems in a difference of the rules for the discharge of hypotheses.

Alternatively to adding Smiley, Rumfitt can appeal to a notion of incompatibility between the speech acts assertion and denial, registered by $\perp$, and add the following two rules to $\mathfrak{AD}$:

Non-Contradiction: From $\alpha, \alpha*$, infer $\bot$
Reductio: If $\Gamma, \alpha \vdash \bot$, then $\Gamma \vdash \alpha*$

Adding these two rules to $\mathfrak{AD}$ gives a system essentially equivalent to $\mathfrak{B}$.

3 Intuitionist Bilateralism

Rumfitt claims that bilateralism is superior to unilateralism, the usual approach to natural deduction, as it allows us to justify classical and rule out intuitionist logic. In bilateral logic, classical negation is governed by harmonious rules, while intuitionist negation is not governed by harmonious rules. Closer inspection of the resources for formulating rules of inference provided by the bilateral framework shows, however, that the claim does not stand.

It is undeniable that dropping $-\neg E$ or weakening it somehow while keeping $-\neg I$ creates a misbalance. However, if we weaken *both* rules we can formulate intuitionistically acceptable, harmonious rules for the denials of negated formulas. To formalise $\mathfrak{Int}$-$\mathfrak{B}$, an intuitionist system of bilateral logic, we adopt the assertive rules of $\mathfrak{AD}$, change the rejective rules for &, $\supset$ and $\neg$, restrict Smiley and add a version of *ex contradictione quodlibet*:

Reductio$_{Int}$: If $\Gamma, +A \vdash \beta$ and $\Gamma, +A \vdash \beta^*$, then $\Gamma \vdash -A$
ex contradictione quodlibet: $\alpha, \alpha^* \vdash \beta$ (more economically, $+A, -A \vdash +B$ suffices)

$$-\&I_{Int}:\quad \dfrac{\begin{array}{c}\overline{+A}^{\,i}\\ \Pi\\ -B\end{array}}{-A\&B}\,i \qquad\qquad -\&E_{Int}:\quad \dfrac{-A\&B \qquad +A}{-B}$$

$$-\supset I_{Int}:\quad \dfrac{-B \qquad \begin{array}{c}\overline{-A}^{\,i}\\ \Pi\\ \alpha\end{array} \qquad \begin{array}{c}\overline{-A}^{\,i}\\ \Xi\\ \alpha*\end{array}}{-A\supset B}\,i \qquad\qquad -\supset E_{Int}:\quad \dfrac{-A\supset B \qquad -A}{\beta} \qquad \dfrac{-A\supset B}{-B}$$

$$-\neg I_{Int}:\quad \dfrac{\begin{array}{c}\overline{-A}^{\,i}\\ \Pi\\ \alpha\end{array} \qquad \begin{array}{c}\overline{-A}^{\,i}\\ \Xi\\ \alpha*\end{array}}{-\neg A}\,i \qquad\qquad -\neg E_{Int}:\quad \dfrac{-\neg A \qquad -A}{\beta}$$

α and β can be restricted to atomic signed formulas in any rule. The rules exhibit harmony in Dummett's and Prawitz' sense: grounds and consequences of rejected formulas balance each other, and we can prove a normalisation theorem. They are also harmonious in Tennant's sense, approved by Rumfitt, where 'an introduction rule I is in *harmony* with an elimination rule E when (a) E's major premiss expresses the weakest proposition that can be eliminated when using E, with I taken as given, and (b) I's conclusion expresses the strongest proposition that can be introduced using I, with E taken as given.' (Rumfitt (2000): 790) Hence the claim that the negation rules of a bilateral intuitionist logic cannot be harmonious is incorrect. $\mathfrak{Int}$-$\mathfrak{B}$ shares all the meaning-theoretically relevant properties of $\mathfrak{B}$ and meets all formal requirements Rumfitt imposes on satisfactory systems of bilateral logic. (See Kürbis (2016) for details.) Thus, just as $\mathfrak{B}$ according to Rumfitt, $\mathfrak{Int}$-$\mathfrak{B}$ specifies the senses of the connectives bilaterally. This time, however, that sense is intuitionist. There is nothing specifically classicist about bilateralism.

Reductio may display a symmetry that $Reductio_{Int}$+*ex contradictione quodlibet* does not display. This is no objection. According to Rumfitt, *Reductio* is a structural rule and not subject to considerations of harmony. It holds by stipulation: 'as a matter of simple definition, then, quite independently of the soundness of double negation elimination, the double conjugate α^{**} is strictly identical with α itself.' (Rumfitt (2000): 804) The intuitionist can adopt an analogous attitude.

Bilateralism fails where unilateralism succeeds. On Dummett's and Prawitz' unilateralist account, classical logic is anomalous, but intuitionist logic is not, and so classical logic is ruled out by proof-theoretic considerations. On the bilateral account, intuitionist logic is not anomalous, hence not ruled out by proof-theoretic considerations. As a consequences, the methodological complications introduced by bilateralism cannot be justified by claiming that their introduction allows us to meet a well-known Dummettian challenge.

Adding the other half of Smiley to $\mathfrak{Int}$-$\mathfrak{B}$ gives a system equivalent to $\mathfrak{B}$. Thus it looks as if on the bilateral account, whether a logic is classical or intuitionist depends on which version *reductio* is adopted. Looking back to the discussion of the different roles of discharge of hypotheses in classical and intuitionist logic, Smiley allows additional cases of discharge of assumptions of the form $-A$ that $Reductio_{Int}$ does not allow. That lack of options for the discharge of denied formulas may give the impression of some kind of misbalance. Once more, however, harmony as it stands has nothing to say about what it is that might balance discharge of assumptions, and so an independent argument would be needed to establish that something is amiss about $Reductio_{Int}$.

An extended notion of harmony that also applies to the discharge of hypotheses might get the classical bilateralist on the way to addressing the issue of how to exclude $\mathfrak{Int}$-$\mathfrak{B}$. But we don't know until it's on the table.

Even if we had an argument for excluding $Reductio_{Int}$, whether it is based on an extended account of harmony or not, this would not yet show that there is something wrong with constructive logic with strong negation, if we do not add Smiley to $\mathfrak{AD}$, or an intuitionist version thereof, should anyone want it, if we drop $Reductio_{Int}$ and *ex contradictione quodlibet* from $\mathfrak{Int}$-$\mathfrak{B}$. There is a more general question whether there are principled reasons for deciding between these options from the bilateralist perspective. So far, no one has given any.

4 A Problem about the Status of Hypotheses in Bilateral Logic

The formal framework of bilateral logic has no advantage over the ordinary approach to proof-theory when it comes to the question whether we should adopt classical or intuitionist logic. In this section I will argue that it may in fact have disadvantages.

In bilateral logic, the premises, discharged assumptions and conclusions of rules of inference are supposed to be asserted or denied formulas. Many, and Rumfitt amongst them, accept the view that speech acts cannot be embedded in other speech acts. Thus, the formulas in Rumfitt's system cannot be understood as being prefixed by 'It is assertible that' and 'It is deniable that', as these are sentential operators that can be embedded.

Assertion and denial are *activities*. Ordinary proof-theory is normally understood not to be about activities but about *propositions*. Ordinary proof-theory is concerned with such activities only in a derivative sense. If I have asserted that A, and there is a deduction of B from A, then I can assert B, should assert B, or, failing that, retract my assertion of A or of some other proposition I asserted and that the deduction of B depends on. But deductions can equally be carried out independently of any assertions, even if, when all assumptions are discharged, we reach a propositions we should accept and assert as true.

How are we to understand the +'s and −'s of Rumfitt's logic? They cannot mean that some assertions or denials have actually been made. This is irrelevant for logic. Maybe no one ever asserted that he is being deceived by a most powerful and evil demon, but nevertheless we may assume that proposition and see what consequences it has. The making of assumptions is essential to logic. What is it to make an assumption in Rumfitt's system? Rumfitt often paraphrases $+ A$ as 'It is correctly assertible that' and $- A$ as 'It is correctly deniable that A'. Although Rumfitt accepts 'that whenever it is correct to assert a sentence, that sentence is true; and that whenever it is correct to deny a sentence, that sentence is false' (Rumfitt (2002): 314), he does not accept the converses. 'To say that it is (objectively) correct to assert (or to deny) a sentence A is to say that knowledge is (tenselessly) available which, were a speaker to apprehend it, would warrant him in asserting (or in denying) A' (Rumfitt (2002): 313) Thus to assume that it is correctly assertible that A is a stronger assumption than the assumption that A is true, and equally to assume that it is correctly deniable that A is a stronger assumption than the assumption that A is false. To assume that A is correctly assertible is to assume that something about our epistemic state in addition to the mere truth of A, and to assume that A is correctly deniable is to assume something about our epistemic state in addition to the mere falsehood of A. The problem now is that in Rumfitt's bilateral system, all formulas are prefaced with + or −. Thus it would appear that all assumptions in the system correspond to the stronger assumptions about our epistemic status, and nothing corresponds to the weaker assumptions of the mere truth or falsity of a formula. But it is those latter assumptions that logic is concerned with.

Weiss (2018) argues that Rumfitt's system does not allow him to draw a distinction between the truth and the assertibility of a sentence. That distinction, however, turns out to be crucial not only to Rumfitt's classicist allegiance, but to the entire bilateralist approach, on the basis of which the classicist allegiance was supposed to be justified. A core tenet of Rumfitt's approach is that denial and assertion conditions are independent of each other in the sense that they cannot be derived from each other: in a bilateralist theory of meaning that the denial and assertion conditions of a sentence must be stipulated independently. Weiss argues that as a consequence of there being no viable distinction between truth and assertibility in the system, 'denial conditions follow from failure of assertion conditions, or, more strictly, from assertion that assertion conditions fail'. Weiss appeals to very plausible principles governing the operators 'It is assertible that' and 'It is deniable that':

(A.1) From + A infer + (It is assertible that A)
(A.2) From + (It is assertible that A) infer + A

(D.1) From − A infer + (It is deniable that A)
(D.2) From + (It is deniable that A) infer − A

Weiss argues as follows. Suppose + (It is not assertible that A). If $-\neg A$, then, by $-\neg E$, $+ A$, so by (A.1), + (It is assertible that A). Hence $+\neg A$ by Smiley, and so $- A$, by $+\neg E$. Hence from (A.1), which encapsulates the lack of a distinction between truth and assertibility in Rumfitt's account, it follows that + (It is not assertible that A) entails $- A$.

Rumfitt should accept (A.2), as he accepts that if a sentence is assertible, then it is true, and (A.2) is the only possibility to express this in his logic. Rumfitt should also accept (D.2), as he accepts that if a sentence is deniable, then it is false, so its negation is true, and (D.2) is the only possibility to express this in his logic. Rumfitt wants to reject (A.1), as he accepts that there may be sentences that are true but not assertible, and reject (D.1), as he accepts that there are sentences that are false but not deniable. But, as Weiss points out, he can hardly do either of them, as his logic does not allow him to reason from sentences, but only from sentences that are asserted or denied.

It is possible to back up Weiss's account by the following observation. Presumably it is inconsistent to assert A and deny that it is assertible that A, and it is inconsistent to deny A and deny that it is deniable that A. We should expect to have:

$$\frac{+A \qquad -\,(\text{It is assertible that } A)}{\bot} \qquad\qquad \frac{-A \qquad -\,(\text{It is deniable that } A)}{\bot}$$

(A.1) and (D.1) now follow by *Reductio*. Thus we can prove Weiss's principles on the basis of what may be considered even simpler ones.

To draw the discussion to a close, there is something even worse for bilateralism than what has already been said. There lurks a danger for the coherence of Rumfitt's entire framework. It is essential to rules such as $+\supset I$ and *reductio* that their application allows the discharge of assumptions. We may wonder what it could mean to discharge a speech act? Is it like making an assertion that is no longer needed, like having informed someone yesterday that it looks like it is raining and it is best to take an umbrella, or an assertion that is retracted, like having informed someone yesterday when it looked like it was going to rain that it will be best to take an umbrella today, but now the sun is shining? If an assertion has been made, it is not possible to make it "unhappen". Assertions are activities that have effects external to any reasoning we might do on the basis of them that cannot be made undone. Making an assumption and discharging it in no way commits a reasoner to the proposition nor any consequences that the making of the assumption might have, apart from what follows by an application of a rule that discharges it. That's the point of making an assumption. But it is worse than that. What does it mean to assume $+ A$ and $- A$? It is plausible that making an assumption is a particular speech act, as argued by Dummett (1981): 309ff. $+ A$ and $- A$ are supposed to represent speech acts. Rumfitt accepts that speech acts cannot be embedded in other speech acts. But then it is meaningless to assume $+ A$ or $- A$.

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