Exceeding Expectations: Stochastic Dominance as a General Decision Theory

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Abstract

The principle that rational agents should maximize expected utility or choiceworthiness is intuitively plausible in many ordinary cases of decision-making under uncertainty. But it is less plausible in cases of extreme, low-probability risk (like Pascal's Mugging), and intolerably paradoxical in cases like the St. Petersburg and Pasadena games. In this paper I show that, under certain conditions, stochastic dominance reasoning can capture most of the plausible implications of expectational reasoning while avoiding most of its pitfalls. Specifically, given sufficient background uncertainty about the choiceworthiness of one's options, many expectation-maximizing gambles that do not stochastically dominate their alternatives "in a vacuum" become stochastically dominant in virtue of that background uncertainty. But, even under these conditions, stochastic dominance will not require agents to accept options whose expectational superiority depends on sufficiently small probabilities of extreme payoffs. The sort of background uncertainty on which these results depend looks unavoidable for any agent who measures the choiceworthiness of her options in part by the total amount of value in the resulting world. At least for such agents, then, stochastic dominance offers a plausible general principle of choice under uncertainty that can explain more of the apparent rational constraints on such choices than has previously been recognized.

1 Introduction

Given our epistemic limitations, every choice you or I will ever make involves some degree of risk. Whatever we do, it might turn out that we would have done better to do something else. If our choices are to be more than mere leaps in the dark, therefore, we need principles that tell us how to evaluate and compare risky options.

The standard view in normative decision theory holds that we should rank options by their *expectations*. That is, an agent should represent the various possibilities over which she's uncertain as each assigning cardinal degrees of utility or choiceworthiness to each of her options, and evaluate each option by taking a probability-weighted sum of these values (an expectation). Call this view *expectationalism*.

Expectational reasoning provides seemingly indispensable practical guidance in many ordinary cases of decision-making under uncertainty.[1] But it encounters serious difficulties

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[1] Throughout the paper, I assume that agents can assign precise probabilities to all decision-relevant possibilities. Since there is little possibility of confusion, therefore, I use "risk" and "uncertainty" interchangeably, setting aside the familiar distinction due to Knight (1921). I default to "uncertainty" (and, in particular, speak of "background uncertainty" rather than "background risk") partly to avoid the misleading connotation of "risk" as something exclusively negative.

in many cases involving extremely large finite or infinite payoffs, where it yields conclusions that are either implausible, unhelpful, or both. For instance, expectationalism implies that: (i) Any positive probability of an infinite positive or negative payoff, no matter how minuscule, takes precedence over all finitary considerations (Pascal, 1669). (ii) When two options carry positive probabilities of infinite payoffs of the same sign (i.e., both positive or both negative), and zero probability of infinite payoffs of the opposite sign, the two options are equivalent, even if one offers a much greater probability of that infinite payoff than the other (Hájek, 2003). (iii) When an option carries any positive probabilities of both infinite positive and infinite negative payoffs, it is simply incomparable with any other option (Bostrom, 2011). (iv) Certain probability distributions over finite payoffs yield expectations that are infinite (as in the St. Petersburg game (Bernoulli, 1738)) or undefined (as in the Pasadena game (Nover and Hájek, 2004)), so that options with these prospects are better than or incomparable with any guaranteed finite payoff.[2] (v) Agents can be rationally required to prefer minuscule probabilities of astronomically large finite payoffs over certainty of a more modest payoff, in cases where that preference seems at best rationally optional (as in "Pascal's mugging" (Bostrom, 2009)).

The last of these problem cases, though theoretically the most straightforward, has particular practical significance. Real-world agents who want to do the most good when they choose a career or donate to charity often face choices between options that are fairly likely to do a moderately large amount of good (e.g., supporting public health initiatives in the developing world or promoting farm animal welfare) and options that carry much smaller probabilities of doing much larger amounts of good (e.g., reducing existential risks to human civilization (Bostrom, 2013) or trying to bring about very long-term "trajectory changes" (Beckstead, 2013)). Often, naïve application of expectational reasoning suggests that we are rationally required to choose the latter sort of project, even if the probability of having any positive impact whatsoever is vanishingly small. For instance, based on an estimate that future Earth-originating civilization might support the equivalent of 10^{52} human lives, Nick Bostrom concludes that, "[e]ven if we give this allegedly lower bound...a mere 1 per cent chance of being correct...the expected value of reducing existential risk by a mere one billionth of one billionth of one percentage point is worth a hundred billion times as much as a billion human lives" (Bostrom, 2013, p. 19). This suggests that we should pass up opportunities to do enormous amounts of good in the present, to maximize the probability of an astronomical long-term payoff, even if the probability of success is on the order of, say, 10^{-30}—meaning, for all intents and purposes, *no matter how small* the probability.

Even hardened utilitarians who think that we should normally do what maximizes expected welfare may find this conclusion troubling and counterintuitive. We intuit (or so I take it) not that the expectationally superior long-shot option is *irrational*, but simply that it is *rationally optional*: We are not rationally required to forego a high probability of doing a significant amount of good for a vanishingly small probability of doing astronomical amounts of good. And we would like decision theory to vindicate this judgment.

The aim of this paper is to set out an alternative to expectational decision theory that outperforms it in the various problem cases just described—but in particular, with respect to tiny probabilities of astronomical payoffs. Specifically, I will argue that under plausible epistemic conditions, *stochastic dominance reasoning* can capture most of the ordinary,

[2] As is common in discussions of the St. Petersburg paradox, I assume we can extend the strict notion of an expectation to allow that, when the probability-weighted sum of possible payoffs diverges unconditionally to $+/-\infty$, the resulting expectation is infinite rather than undefined. Nothing essential will depend on this assumption.

attractive implications of expectational decision theory—far more than has previously been recognized—while avoiding its pitfalls in the problem cases described above, and in particular, while permitting us to decline expectationally superior options in extreme, "Pascalian" choice situations.

Stochastic dominance is, on its face, an extremely modest principle of rational choice, simply formalizing the idea that one ought to prefer a given probability of a better payoff to the same probability of a worse payoff, all else being equal. The claim that we are rationally required to reject stochastically dominated options is therefore on a strong *a priori* footing (considerably stronger, I will argue, than expectationalism). But precisely because it is so modest, stochastic dominance seems too weak to serve as a final principle of decision-making under uncertainty: It appears to place no constraints on an agent's *risk attitudes*, allowing intuitively irrational extremes of risk-seeking and risk-aversion.

But in fact, stochastic dominance has a hidden capacity to effectively constrain risk attitudes: When an agent is in a state of sufficient "background uncertainty" about the choiceworthiness of her options, expectationally superior options that would not stochastically dominate their alternatives in the absence of background uncertainty can become stochastically dominant *in virtue of* that background uncertainty. It is significantly harder for this to happen, however, in situations where the balance of expectations is determined by inuscule probabilities of astronomical positive or negative payoffs. Stochastic dominance thereby draws a principled line between "ordinary" and "Pascalian" choice situations, and vindicates our intuition that we are often permitted to decline gambles like Pascal's Mugging or the St. Petersburg game, even when they are expectationally best. Since it avoids these and other pitfalls of expectational reasoning, if stochastic dominance can also place plausible constraints on our risk attitudes and thereby recover the attractive implications of expectationalism, it may provide a more attractive general theory of rational choice under uncertainty.

I begin in §2 by introducing some conceptual and formal apparatus. In §3, I say a little more about standard expectational decision theory, as motivation and point of departure for my main line of argument. §4 introduces stochastic dominance and describes the standard arguments for rejecting stochastically dominated options as a necessary but insufficient condition of rationality. In §5, I establish two central results: (i) a sufficient condition for stochastic dominance which implies, among other things, that whenever O_i is expectationally superior to O_j, it will come to stochastically dominate O_j given sufficient background uncertainty; and (ii) a necessary condition for stochastic dominance which implies, among other things, that it is harder for expectationally superior options to become stochastically dominant under background uncertainty when their expectational superiority depends on smaller probabilities of more extreme payoffs. In §6, I argue that the sort of background uncertainty on which these results depend is rationally appropriate at least for any agent who assign normative weight to *aggregative consequentialist* considerations, i.e., who measure the choiceworthiness of her options at least in part by the total amount of value in the resulting world. §7 offers an intuitive defense of the initially implausible conclusion that an agent's background uncertainty can make a difference to what she is rationally required to do. §8 describes two modest conclusions we might draw from the preceding arguments, short of embracing stochastic dominance as a general decision theory. In §9, however, I survey some additional advantages of stochastic dominance over expectational reasoning and argue that, insofar as stochastic dominance can recover plausible constraints on risk attitudes and hence capture the intuitively desirable implications of expectationalism, we have substantial reason to prefer it as a general theory of rational choice under uncertainty. §10 is the conclusion.

2 Preliminaries

Practical rationality (hereafter, "rationality") involves responding correctly to one's beliefs about practical reasons.[3] Following others in the recent literature (e.g. Wedgwood (2013, 2017), Lazar (2017b), MacAskill and Ord (forthcoming)), I will speak of the total, all-things-considered strength of an agent's reasons for or against choosing a particular option as the *choiceworthiness* of that option. Reasons and choiceworthiness, in the sense we're concerned with, are objective in the sense of being "fact-relative" rather than "belief-relative" (Parfit, 2011)—e.g., the fact that my glass is poisoned gives me a reason against drinking from it, and thereby makes the option of drinking less choiceworthy, even if I neither believe nor have any evidence that it is poisoned. I take no stance on whether an option's choiceworthiness depends on the agent's motivational states (desires, preferences, etc.), on acts of will (e.g. willing certain ends for herself), or on external normative/evaluative features of the world (e.g. universal moral obligations). In other words, choiceworthiness is objective in the sense of being *belief*-independent, but may or may not be objective in the sense of being *desire*- or *preference*-independent.[4]

Any expectational decision theory must assume that degrees of choiceworthiness can be represented on an interval scale (i.e., can be given a real-valued representation that is unique up to positive affine transformation), and I will adopt this assumption as well (except briefly in §9): Although stochastic dominance itself depends only on ordinal choiceworthiness relations, the main line of argument I advance in this paper assumes that choiceworthiness is amenable to a certain kind of cardinal representation (as explained below). I remain neutral, though, on whether cardinal choiceworthiness should be understood as primitive or as a representation of an underlying ordinal relation.

Let's now introduce some formal apparatus. A *choice situation* is an ordered triple $S = \langle A, \mathbf{O}, \beta \rangle$, where A is an agent, $\mathbf{O}$ is a set of *options* $\{O_1, O_2, ..., O_m\}$, and β is a probability density function (PDF) over the real numbers that represents the agent's *background uncertainty* in S. We identify each option $O_i \in \mathbf{O}$ with its *simple prospect*, a finite set of ordered pairs $O_i = \{\langle v_1^i, p_1^i \rangle, \langle v_2^i, p_2^i \rangle, ..., \langle v_n^i, p_n^i \rangle\}$, where $v_j^i \in \mathbb{R}$ is a possible *simple payoff* and $p_j^i \in (0, 1]$ is the probability of obtaining that simple payoff associated with O_i. (I will generally omit the superscripts on payoffs and probabilities, where there is no risk of confusion.) The p_j are all positive (i.e., we ignore simple payoffs with probability 0) and sum to 1.

I remain neutral on the interpretation of these probabilities, in two ways. First, I leave it unspecified whether p_j^i represents the causal probability $Pr(O_i \mathrel{\Box\!\!\rightarrow} v_j^i)$ or the conditional probability $Pr(v_j^i | O_i)$, and hence remain neutral between causal and evidential decision theory. Second, I leave it unspecified whether these probabilities are subjective or epistemic.

Intuitively, the simple payoff of an option is what the option itself yields. The crucial assumption of this paper, however, is that an option's *overall* payoff depends not just on its simple payoff, but also on what I will call a *background payoff.* A background payoff is, roughly, *what the agent starts off with*, or *the component of the overall outcome/payoff*

[3] I don't claim that this is all there is to practical rationality—some rational requirements, like the requirement against forming inconsistent intentions, may have a different source. But the decision-theoretic aspect of practical rationality with which this paper is concerned does, I assume, consist in responding correctly to reason-beliefs.

[4] I use the term "choiceworthiness" rather than "value" or "utility" to avoid two possible confusions: (i) "Value" suggest an evaluative rather than a normative property of options. (ii) "Utility" is often understood as a measure of preference satisfaction, while I wish to remain neutral on whether or to what extent an agent's reasons depend on her preferences.

that does not depend on her choice. As a mundane illustration: Suppose that a young person is deciding how to invest some money in her retirement account, and that her only concern in this context is her net worth when she retires at age 65. Her options are various funds that she might invest in. The simple payoff of buying some shares in fund F_i (call this option O_i) is the value those shares will have when she retires. But the *overall* payoff of O_i—the thing she ultimately cares about—is her total net worth at retirement, if she now invests in F_i. This overall payoff is the sum of her simple payoff (the future value of her F_i shares) plus a background payoff (the value of all her other assets).

Just as an agent may be uncertain about an option's simple payoff, she may be uncertain about her background payoff. This is what I will call *background uncertainty.* The defining feature of background uncertainty is its independence from other features of the choice situation: In particular, A's background payoff in S is probabilistically independent of (i) which option she chooses and (ii) which simple payoff she receives from her chosen option. Thus, A's background uncertainty captures uncertainties that apply to *all* the options in situation S, rather than uncertainties about any one option in particular. We will describe A's background uncertainty by means of a continuous random variable—her *background prospect*—with probability density function β, such that the probability of a background payoff in the interval $[n, m]$ is given by $\int_n^m \beta(x)\, dx$.[5]

I have already mentioned one possible source of background uncertainty (concerning financial decisions), but my primary focus will be on a different source: I will assume that agents should assign at least some normative weight to *aggregative consequentialist* considerations, i.e., they should measure the choiceworthiness of an option *at least in part* by the total amount of value in the resulting world. Such agents will be in a state of background uncertainty because they are uncertain how much value there is in the world *to begin with*, independent of their present choice. In this case, we can understand β as giving the probability that, excluding the outcome of A's present choice, the world contains value equivalent to between n and m units of choiceworthiness, via $\int_n^m \beta(x)dx$.

It might seem that background uncertainty has no bearing on what an agent ought to do, since it does not affect the *relative* choiceworthiness of her options. In what follows, however, I will make the case that background uncertainty can have a great deal of practical significance, and so needs to be included in our representation of choice situations.

The *payoff* of option is simply its overall degree of objective choiceworthiness, as determined by the combination of its simple and background payoffs. Specifically, I will assume that an option's payoff can be represented as the *sum* of its constituent simple and background payoffs—i.e., that there is some way of assigning real numbers to simple and background payoffs such that one overall payoff is at least as good as another if and only if the sum of the real numbers assigned to its constituent simple and background payoffs is at least as great. Call this *additive separability* between simple and background payoffs.

Additive separability is not as strong an assumption as it might sound: In particular, it does not require us to assume that payoffs have any primitive cardinal structure. Suppose there is a set S of possible simple payoffs and a set B of possible background payoffs, and that the set of possible overall payoffs $S \times B$ is totally preordered by a relation $\succcurlyeq_p$. Then additive separability amounts to the assumption that $\langle S, B, \succcurlyeq_p \rangle$ forms an *additive conjoint structure.* This involves satisfying a number of purely ordinal axioms, the most important of which is an ordinal separability condition to the effect that, if we know that two overall

[5] Again, these probabilities can be interpreted as either causal or conditional, and as either subjective or epistemic. The stipulation that background payoffs are independent of which option the agent chooses and of what simple payoff it yields means that they are probabilisticalaly independent in terms of the decision-relevant probabilities, whatever those may be.

payoffs p_i and p_j have one component in common (i.e., involve the same simple background payoff or the same background payoff), we can learn whether $p_i \succcurlyeq_p p_j$ by learning the distinctive component of each payoff.[6] If an additively separable representation of payoffs exists, then it is relevantly unique (unique up to choice of zero elements in S and B and a unit element in *either* S or B). Thus, the real numbers used to designate simple, background, and overall payoffs can be understood either as given independently (e.g., by purely ethical considerations) or as representing an underlying ordinal relation on overall payoffs, construed as ordered pairs of simple and background payoffs.

The *prospect* of O_i is the probability distribution it yields over payoffs. Given the assumptions of independence and additive separability, we can express prospects as follows: Where $O_i = \{\langle v_1, p_1\rangle, \langle v_2, p_2\rangle, ..., \langle v_n, p_n\rangle\}$ and A's background uncertainty is described by β, the *prospect* of O_i is described by $\beta_i(x) = p_1\beta(x - v_1) + p_2\beta(x - v_2) + ... + p_n\beta(x - v_n)$. Formally, $\beta_i(x)$ is a *mixture distribution*, a convex combination of n copies of the background prospect β, each corresponding to a possible simple payoff $\langle v_i, p_i\rangle$, and therefore translated along the x axis by the value of that simple payoff (v_i) and weighted by the probability of receiving that simple payoff (p_i). Since the p_i sum to 1, β_i is a probability density function.

3 Expectationalism

3.1 Two kinds of expectationalism

Before embarking on the main line of argument, we should say a bit more about expectationalism, which will serve as its foil. First, we should distinguish two versions of expectationalism. One view, which I will call *primitive expectationalism*, holds that cardinal degrees of choiceworthiness are specified independently of any ranking of prospects or options under uncertainty—e.g., by purely ethical criteria.[7] Primitive expectationalism then holds that agents should maximize the expectation of these independently specified values. Another view, which I will call *axiomatic expectationalism*, holds that cardinal choiceworthiness is simply a representation of some ranking of uncertain prospects—e.g., an agent's preference ordering. This ranking is required to satisfy a set of axioms which guarantee that it can be represented as maximizing the expectation of *some* assignment of cardinal values to outcomes or options under certainty.

3.2 Arguments for expectationalism

There are two standard arguments for expectationalism, corresponding to primitive and axiomatic expectationalism respectively: *long-run arguments* and *representation theorems*.

Long-run arguments invoke the law of large numbers which implies that, as the length of a series of probabilistically independent risky choices goes to infinity, the probability that an expectation-maximizing decision rule will outperform any given alternative converges

[6] The other axioms that characterize additive conjoint structures are mainly technical—e.g. (i) requiring that the sets S and B are sufficiently rich that for any $s_i, s_j \in S$, $b_k \in B$ there is a $b_l \in B$ such that $\langle s_i, b_k\rangle \sim_p \langle s_j, b_l\rangle$, and (ii) requiring that no payoff is infinitely better than another, in the sense that we can always "get from" one payoff to another by repeatedly substituting a more preferred component for a less preferred component (e.g., repeatedly substituting s_i for s_j, where $\forall b \in B(\langle s_i, b\rangle \succ_p \langle s_j, b\rangle)$, to create an ascending series of overall payoffs), in a finite number of steps. For a full characterization of additive conjoint structures and a proof that all such structures have an additively separable representation, see Krantz et al. (1971, pp. 245-266).

[7] For defense of this "cardinalist" approach, see for instance Ng (1997). For one illustration of how cardinal values can be specified independent of a ranking of prospects, see Skyrms and Narens (forthcoming).

to certainty (Feller, 1968). If successful, long-run arguments justify a version of primitive expectationalism: Their conclusion is that the agent should maximize the expectation of a cardinal choiceworthiness function whose values to not represent or depend on the agent's antecedently specified preferences toward risky prospects. There is an extensive literature on long-run arguments, but the general consensus is that they are unsuccessful.[8] Among other objections, it's unclear what force long-run arguments have for agents who don't in fact face the relevant sort of long run. And since the standard long-run arguments presuppose an *infinitely* long run of independent gambles, it's therefore unclear what force they have for any actual agent, who will face only a finite series of choices in her lifetime.

Thus, the standard defense of expectationalism in contemporary decision theory appeals instead to *representation theorems*. Representation theorems in decision theory show that, if an agent's preferences satisfy certain putative consistency constraints, then there is some assignment of cardinal values to outcomes (a *utility function*) such that the agent can be accurately represented as maximizing its expectation. The two best-known such theorems are due to von Neumann and Morgenstern (1947) and Savage (1954). The axioms that figure in these theorems are subject to ongoing debate, but the axiomatic approach nevertheless retains the status of decision-theoretic orthodoxy.[9]

3.3 Expectationalism and risk attitudes toward objective value

My main interest in this paper is in what risk attitudes we should should adopt toward objective goods that have some natural cardinal structure—e.g., lives saved or lost. And the two versions of expectationalism have very different things to say about this question. Primitive expectationalism implies that, insofar as an option's choiceworthiness increases linearly with the quantity of objective value it produces, we should be exactly risk-neutral toward objective goods. But axiomatic expectationalism and the representation theorems that are its foundation do not have this implication.

For instance, suppose you are in a situation where many lives are at risk. Suppose that (i) the only thing you care about in this situation is saving lives, (ii) you always prefer saving more lives to saving fewer, and (iii) you value all the lives at stake equally, in the sense that all else being equal, you are always indifferent between saving one life or another. But you do not yet know how to compare risky prospects. If you accept primitive expectationalism, you might infer that your options in this situation have degrees of cardinal choiceworthiness that increase linearly with the number of lives saved (though this is not a *logical* consequence of (i)–(iii)), in which case primitive expectationalism implies that you should simply maximize the expected number of lives saved—in other words, you should be *risk-neutral* with respect to lives saved.

But suppose instead you merely believe that you should rank prospects in a way that satisfies, say, the on Neumann-Morgenstern (VNM) axioms. Even given (i)–(iii), and even given the assumption that the value of saving n lives increases linearly with n, the VNM axioms do not imply that you should maximize the expected number of lives saved. Rather, they merely imply that you should maximize the expectation of *some increasing function* of lives saved. This function can be arbitrarily concave or convex, meaning that you can be arbitrarily risk-averse or risk-seeking with respect to lives saved.[10] More generally, given any antecedently specified ranking or assignment of cardinal choiceworthiness to options

[8]For recent critical treatments, see Buchak (2013, pp. 212–8) and Easwaran (2014, pp. 3–4).

[9]For a survey of axiomatic approaches and objections to the standard axioms, see Briggs (2017). For criticism of the axiomatic approach more generally, see Meacham and Weisberg (2011).

[10]I am here referring to what are sometimes called "actuarial" risk attitudes, as opposed to the sort of risk attitudes that figure in generalized expected utility theories like Buchak's (2013) REU.

under certainty, VNM and the other standard axiom systems merely imply that you should maximize the expectation of some increasing function of that ranking or assignment.

This permissiveness has its advantages. For instance, consider the "Pascalian" conclusion imputed to expectationalism in the §1 that, if there is even a one percent chance of a future in which Earth-originating civilization supports 10^{52} happy lives, then the "the expected value of reducing existential risk by a mere one billionth of one percentage point is worth a hundred billion times as much as a billion human lives" (Bostrom, 2013, p. 19). Primitive expectationalism supports this kind of reasoning. Axiomatic expectationalism, on the other hand, can disclaim this reasoning and the seemingly-fanatical conclusions it entails—but only because it places no constraints at all on our risk attitudes toward goods like happy lives. And in more ordinary cases, this looks like a drawback. For instance, axiomatic expectationalism cannot tell you that you should save ten lives with probability .5 rather than one life for sure. Pushing the point to more counterintuitive extremes, it cannot tell you that you should save 1000 lives with probability .5 rather than 10 lives with probability .51; nor that you should save 1000 lives for sure rather than 1001 lives with probability .01.

Is this a defect in standard axiomatic decision theory? It's not obvious. Some decision theorists will say that it is not the job of decision theory to tell you what your risk attitudes should be toward objective goods like lives saved—rather, that's a job for ethics, or some other branch of normative philosophy. But it's pretty clearly a job for *someone*, wherever we place it on the disciplinary org chart: The complete normative theory of choice under uncertainty should tell us that, in a situation where all that matters is saving lives and all the lives at stake have equal value, one should prefer to save 1000 lives with probability .5 rather than 10 lives with probability .51. So even if these questions are beyond its intended remit, axiomatic expectationalism seems to be *incomplete* as a normative theory of decision-making under uncertainty.

In summary, there are two problems for expectationalism that I am hoping to remedy: First, neither version of expectationalism offers a compelling justification for choosing the option that maximizes the expectation of objective values in ordinary cases where it seems clear that this is what we should do. Primitive expectationalism relies on the dubious appeal to hypothetical long runs, while axiomatic expectationalism does not attempt to justify this conclusion in the first place. Second, insofar as expectationalism *does* offer a justification for maximizing expected objective value, it goes too far, committing us to Pascalian fanaticism in cases involving minuscule probabilities of astronomical payoffs.[11] I aim both to provide a stronger justification for choosing options that maximize expected objective value in ordinary cases, and in so doing to draw a principled line between those ordinary cases and extreme, Pascalian cases.

It is important to note, however, that the arguments I advance below will interact very differently with primitive and axiomatic expectationalism. Specifically: I will propose that first-order stochastic dominance can provide a sufficient criterion of rational choice under uncertainty. This view is a rival to both primitive and axiomatic expectationalism. The primary motivation for this view will be the results in §5. And while the primitive expectationalist cannot take any advantage of these results, the axiomatic expectationalist can: As we will see in §8.2, those who accept the standard axioms can interpret these results as furnishing a friendly "add-on" to standard axiomatic decision theory. The main advantages of my proposed view over axiomatic expectationalism will be that it can recover

[11] This is true of primitive expectationalism, and also of the most natural strategy for placing constraints on risk attitudes toward objective value within the axiomatic framework—namely, an appeal to "aggregation theorems" like that of Harsanyi (1955).

strong practical conclusions about choice under uncertainty from something much weaker and less controversial than the standard axiom systems, and that it better handles the range of problem cases surveyed in §1.

4 Stochastic dominance

Option O *first-order stochastically dominates* option P if and only if

1. For any payoff x, the probability that O yields a payoff at least as good as x is equal to or greater than the probability that P yields a payoff at least as good as x, and

2. For some payoff x, the probability that O yields a payoff at least as good a x is strictly greater than the probability that P yields a payoff at least as good as x.

In notation: $O \succ_{sd} P \leftrightarrow \forall x(\int_x^\infty \beta_o(y)dy \geq \int_x^\infty \beta_p(y))dy \land \exists x(\int_x^\infty \beta_o(y)dy > \int_x^\infty \beta_p(y))dy$.

There are also second- and higher-order stochastic dominance relations, which are less demanding than first-order stochastic dominance. (For a survey of these higher-order relations, see Ch. 3 of Levy (2016).) But since we will only be concerned with the first-order relation, I will henceforth omit the qualifier use "stochastic dominance" to mean "*first-order* stochastic dominance".

Stochastic dominance is a generalization of the familiar *statewise* dominance relation that holds between O and P whenever O yields at least as good a payoff as P in every possible state, and a strictly better payoff in some state. To illustrate: Suppose that I am going to flip a fair coin, and I offer you a choice of two tickets. The Heads ticket will pay \$1 for heads and nothing for tails, while the Tails ticket will pay \$2 for tails and nothing for heads. The Tails ticket does not *statewise* dominate the Heads ticket because, if the coin lands Heads, the Heads ticket yields a better payoff. But the Tails ticket does *stochastically* dominate the Heads ticket. There are three possible payoffs: winning \$0, winning \$1, and winning \$2. The two tickets offer the same probability of a payoff at least as good as \$0, namely 1. And they offer the same probability of an payoff at least as good as \$1, namely .5. But the Tails ticket offers a greater probability of a payoff at least as good as \$2, namely .5 rather than 0.

Stochastic dominance is generally seen giving as a necessary condition for rational choice:

Stochastic Dominance Principle (SDP) An option O is rationally permissible in situation S only if it is not stochastically dominated by any other option in S.

This principle is on a strong *a priori* footing. Various formal arguments can be made in its favor. For instance, if O stochastically dominates P, then O can be made to *statewise* dominate P by an appropriate permutation of equiprobable states in a sufficiently fine-grained partition of the state space (Easwaran, 2014; Bader, 2018). So if one is rationally required to reject statewise dominated options, and if the rational permissibility of an option depends only on its prospect and not on which payoffs are associated with which states, then one is rationally required to reject stochastically dominated options as well. The claim that an option's rational permissibility depends only on its prospect reflects the idea that all normatively significant features of an outcome are captured by the payoff value assigned to that outcome, so that as a conceptual matter an agent must be indifferent between receiving a given payoff in one state or another. If, say, you prefer winning \$0 with a Heads ticket to winning \$0 with a Tails ticket, then this should be reflected in the values assigned to the payoffs, in which case the Tails ticket would no longer stochastically dominate the Heads ticket.

More informally, it is unclear how one could ever *reason* one's way to choosing a stochastically dominated option P over the option O that dominates it. For any feature of P that one might point to as grounds for choosing it, there is a persuasive reply: However choiceworthy P might be in virtue of possessing that feature, O is equally or more likely to be at least that choiceworthy. And conversely, for any feature of O one might point to as grounds for rejecting it, there is a persuasive reply: However unchoiceworthy O might be in virtue of possessing that feature, P is equally or more likely to be at least that unchoiceworthy. To say that O stochastically dominates P is in effect to say that there is no feature of P that can provide a *unique* justification for choosing it over O.

For reasons like these, SDP is almost entirely uncontroversial in normative decision theory. In particular, it is much less controversial than the standard axioms of expected utility theory: The most widely discussed alternatives to and generalizations of axiomatic expectationalism, which give up one or more of those axioms (e.g., *rank-dependent expected utility* (Quiggin, 1982) and *risk-weighted expected utility* (Buchak, 2013)) all satisfy stochastic dominance. In fact, to my knowledge, no normative decision theories that has been widely discussed in philosophy or economics violates stochastic dominance.[12]

My aim, however, is to defend stochastic dominance as not just a necessary but also a *sufficient* criterion for rational permissibility. Let's call this the *stochastic dominance theory of rational choice.*

Stochastic Dominance Theory of Rational Choice (SDTR) An option O is rationally permissible in situation S if and only if it is not stochastically dominated by any other option in S.

What is the relationship between SDTR and expectationalism? In a broad range of cases, stochastic dominance is simply a weakening of expectational reasoning: In particular, whenever the expected choiceworthiness of all options is finite (i.e., neither infinite nor undefined), O stochastically dominates P only if it has greater expected choiceworthiness. So in these cases, SDTR is more permissive than expectationalism. But as we will see in §9, there are other cases where SDTR can deliver guidance that expectational reasoning cannot, and is therefore less permissive.

Like axiomatic expectationalism, SDTR does not constrain an agent's risk attitudes toward objective goods (in the absence of background uncertainty): In a situation where all that matters is saving lives, saving more lives is always better than saving fewer, and all the lives at stake have equal value, stochastic dominance does not require you to save three lives with probability .5 rather than one life for sure, or even to save 1000 lives with probability .5 rather than 10 lives with probability .51.[13] This means that *primitive* expectationalism has an apparent advantage over both axiomatic expectationalism and

[12]In descriptive decision theory, the original version of prospect theory allowed stochastic dominance violations, and largely for that reason was superseded by *cumulative* prospect theory (Tversky and Kahneman, 1992), which satisfies stochastic dominance.

It is worth reiterating that we have not specified whether the decision-relevant probabilities are causal or conditional. In Newcomb-like cases, causal decision theories tell you to choose an option that is stochastically dominated in terms of your conditional probabilities, and evidential decision theories tell you to choose an option that is stochastically dominated in terms of your causal probabilities. But both views agree that you should reject options that are stochastically dominated *in terms of the decision-relevant probabilities*, whatever kind of probabilities those are.

[13]In fact, in this sort of case, SDTR and axiomatic expectationalism are very closely related: Given a fixed ordering of payoffs, it is possible to prefer O to P while satisfying the VNM axioms iff P does not stochastically dominate O (i.e., iff SDTR permits you to choose O over P). The difference is that axiomatic

SDTR: It can explain why, in ordinary cases, you ought to maximize the expectation of objective goods like lives saved.

But, I will argue, this advantage is only apparent, for once account for background uncertainty, things change dramatically: Given sufficiently background uncertainty, SDTR can effectively constrain an agent's risk attitudes, recovering many of the attractive practical implications of primitive expectationalism—while still avoiding its fanatical implications in Pascalian cases. In the next section, we will see how this can happen.

5 Stochastic dominance under background uncertainty

This section describes the general phenomenon of background uncertainty generating new stochastic dominance relations among options, and states two central results. The first, which I call the Upper Bound Theorem, gives a sufficient condition for stochastic dominance in the presence of background uncertainty: In effect, for any option O, it establishes an upper bound in the partial ordering $\succ_{sd}$ on the set of options to which O may be permissibly preferred (i.e., the set of options that do not stochastically dominate O). The second result, which I call the Lower Bound Theorem, gives a necessary condition for stochastic dominance: It establishes a lower bound in $\succ_{sd}$ on the the set of options that stochastically dominate O. The first result shows that, under sufficient background uncertainty, options whose simple prospects are expectationally best will eventually come to stochastically dominate their alternatives. The second result shows that, when the expectational superiority of an option depends on minuscule probabilities of astronomical payoffs, it requires much more background uncertainty to achieve stochastic dominance, so that SDTR is more permissive in more Pascalian choice situations.

First, we should define more precisely the phenomenon to be investigated, which I have described as "background uncertainty generating new stochastic dominance relations among options". What this means, more precisely, is that sufficient background uncertainty can make it the case that the *prospect* of O stochastically dominates the prospect of P, even though the *simple* prospect of O does not stochastically dominate the simple prospect of P.[14] Similarly, when I say that SDTR "constrains an agent's risk attitudes" under background uncertainty, I mean that it constrains her risk attitudes toward simple prospects: That is, under sufficient background uncertainty, an agent who satisfies stochastic dominance must rank options in a way that closely approximates the risk-neutral expectational ranking of their simple prospects.

The crucial condition for this phenomenon—and therefore, the condition under which the results below become interesting—is that the agent's background prospect must have *exponential or heavier tails.* I will abbreviate this to *large tails.*[15] I define this condition in a slightly unorthodox way, the utility of which will become apparent: Let us say that β has exponential or heavier tails iff its decay rate $\frac{|\beta'(x)|}{\beta(x)}$ is bounded above by the decay rate of some member of the *Laplace* (or *double-exponential*) family of distributions. Laplace distributions have PDFs of the form $\frac{1}{2\rho}e^{-\frac{|x-\mu|}{\rho}}$, where μ is a *location parameter* that determines where the distribution is centered, and ρ is a *scale parameter* that determines

expectationalism imposes global coherence requirements on an agent's preferences (e.g., Independence and Continuity) that SDTR does not.

[14]This general phenomenon has been noticed independently, under a slightly different description, by Pomatto et al. (2018). To my knowledge, it has not been noted or discussed elsewhere.

[15]To my knowledge, there is no standard term for distributions with exponential or heavier tails. It is more common to distinguish *heavy-tailed* distributions, whose tails are heavier-than-exponential. Large-tailed distributions, then, are either exponential-tailed or heavy-tailed.

the rate of tail decay. Since Laplace distributions have a constant decay rate equal to $\frac{1}{\rho}$, requiring that the decay rate be bounded above by that of a Laplace distribution is equivalent to requiring that it be bounded above by a finite constant. Thus, β has large tails iff $\exists r \forall x (\frac{|\beta'(x)|}{\beta(x)} \leq r)$.

Large tails are not strictly a necessary condition for background uncertainty to generate stochastic dominance. (In particular, local violations of the large tails condition, e.g. by a vertical asymptote in β, do not always substantially weaken the stochastic dominance constraints that β imposes.) But it is a very good approximate criterion (as far as I have been able to discover, anyway) for the circumstances in which stochastic dominance can strongly constrain risk attitudes toward simple prospects, and as we will see, has an important connection with the sufficient condition for stochastic dominance identified by the Upper Bound Theorem below. It is therefore a natural condition on which to focus.

While large tails are a sufficient condition for background uncertainty to generate *some* new stochastic dominance relations among options, the strength of the constraints it imposes on an agent's risk attitudes also depends on the *scale* of β. As we will see, given a large-tailed β, stochastic dominance approximates the ranking of options by the expectations of their simple prospects ever more closely under increasing rescalings of β. A rescaling of β is a transformation of the form $\beta_s(x) = \frac{1}{s}\beta(\frac{x-a}{s})$, for some constants a and s. By increasing s, we "stretch" β horizontally along the x axis, while otherwise preserving its shape. So that we can talk in a general way about the scale of distributions with various shapes, let us say that the *scale factor* of a distribution is its 50% confidence interval (that is, the length of the shortest interval $[x_1, x_2]$ such that $\int_{x_1}^{x_2} \beta(x)\, dx = .5$). (This should not be confused the with scale *parameter* in a parameterized family of distributions like the Laplace family.) "Increasing the scale factor of β" means transforming it to $\beta_s(x) = \frac{1}{s}\beta(\frac{x-a}{s})$ for $s > 1$ (which increases the scale factor of β by a factor of s). In §6, I will argue that large-tailed β with a large scale factor is rationally warranted, in particular for agents who assign normative weight to aggregative consequentialist considerations. For now, I take it for granted.[16]

I begin in §5.1 with an intuitive description of the target phenomena: how background uncertainty generates new stochastic dominance relations. In §§5.2–5.3, I state the Upper and Lower Bound Theorems respectively and draw out their implications. Finally, in §5.4, I give a toy example that shows how tightly SDTR constrains our risk attitudes with respect to ordinary gambles in the presence of moderate background uncertainty, and how much looser those constraints become for more Pascalian gambles.

5.1 How background uncertainty generates stochastic dominance

Suppose you face a risky option that will either save two lives (with probability .5) or cause one death (with probability .5). Suppose that the lives at stake all have equal value and there are no other normatively relevant considerations (e.g., deontological constraints) that should influence your choice besides maximizing the number of lives saved. Call this option the *Basic Gamble*.

Basic Gamble (G) $\{\langle -1, .5\rangle, \langle 2, .5\rangle\}$

Suppose that your only other option is what we will call the *Null Option*.

Null Option (N) $\{\langle 0, 1\rangle\}$

[16] As we will see in §5.4, though, the scale factor of β need not be particularly large to generate fairly strong constraints.

Intuitively, the Null Option can be thought of as the option of "doing nothing", and simply accepting your background payoff as your overall payoff.

In the absence of background uncertainty, neither of these options is stochastically dominant: G gives a greater probability of a payoff ≥ 2, but N gives a greater probability of a payoff ≥ 0. But suppose you are in a state of background uncertainty, described by a PDF β. N's prospect, then, is simply given by $\beta_N(x) = \beta(x)$. G's prospect is given by $\beta_G(x) = .5\beta(x-2) + .5\beta(x+1)$. Visually, we can think of G's prospect as follows (Fig. 1): We make two half-sized copies of β, corresponding to the two possible outcomes of G, each of which has probability .5. We then translate one of those copies two units to the right (representing a gain of 2, relative to the background payoff) and the other one unit to the left (representing a loss of 1, relative to the background payoff). Finally, we add these two half-PDFs together, obtaining the new PDF $\beta_G(x)$.

This means that, for each possible payoff x, choosing G rather than N makes both a positive contribution and a negative contribution to the probability of a payoff $\geq x$.

- Positive contribution: If β yields a background payoff in the interval $[x-2, x)$ and G yields the simple payoff $+2$, then G results in a payoff $\geq x$ where N would have resulted in a payoff $< x$. The probability of a background payoff in the interval $[x-2, x)$ is given by $\int_{x-2}^{x} \beta(y)dy$, and the probability that G yields a simple payoff of $+2$ is .5. Since these probabilities are independent, we can multiply them. So the possibility of a positive simple payoff from G increases the probability of an overall payoff $\geq x$ by $.5 \int_{x-2}^{x} \beta(y)dy$.

- Negative contribution: If β yields a background payoff in the interval $[x, x-1)$ and G yields the simple payoff -1, then G results in a payoff $< x$ where N would have resulted in a payoff $\geq x$. The probability of a background payoff in the interval $[x, x+1)$ is given by $\int_{x}^{x+1} \beta(y)dy$, and the probability that G yields a simple payoff of -1 is .5. So the possibility of a negative simple payoff from G decreases the probability of an overall payoff $\geq x$ by $.5 \int_{x}^{x+1} \beta(y)dy$.

Thus, G offers a greater probability than N of a payoff $\geq x$ iff $.5 \int_{x-2}^{x} \beta(y)\, dy > .5 \int_{x}^{x+1} \beta(y)\, dy$. If this inequality holds for every x, then G stochastically dominates N (see Fig. 2). Formally:

$$\forall x \left(.5 \int_{x-2}^{x} \beta\left(y\right)\, dy > .5 \int_{x}^{x+1} \beta(y)\, dy \right) \rightarrow G \succ_{sd} N$$

If β is unimodal (i.e., strictly decreasing in either direction away from a central peak), then this condition will be trivially satisfied for values of x in the right tail: Since β is decreasing in the right tail, $\int_{x-2}^{x} \beta(y)\, dy$ will clearly be greater than $\int_{x}^{x+1} \beta(y)\, dy$, being both "wider" and "taller". The interesting question is whether it holds in the left tail. A sufficient condition for it to do so is that the value of β never decreases by more than a factor of 2 in an interval of length 3: In this case, $\int_{x-2}^{x} \beta(y)\, dy$ is everywhere greater than $\int_{x}^{x+1} \beta(y)\, dy$, since it is twice as "wide" (i.e., the interval $[x-2, x]$ is twice as long as the interval $[x, x+1]$) and everywhere at least half as "tall" (i.e., the maximum value of β on the interval $[x-2, x+1]$ is no more than twice the minimum value). This guarantees that by choosing G, at every point x on the horizontal axis, you move more probability mass from the left of that point to the right (increasing the probability of a payoff $\geq x$) than from the right to the left (decreasing the probability of a payoff $\geq x$), which means that G stochastically dominates N.

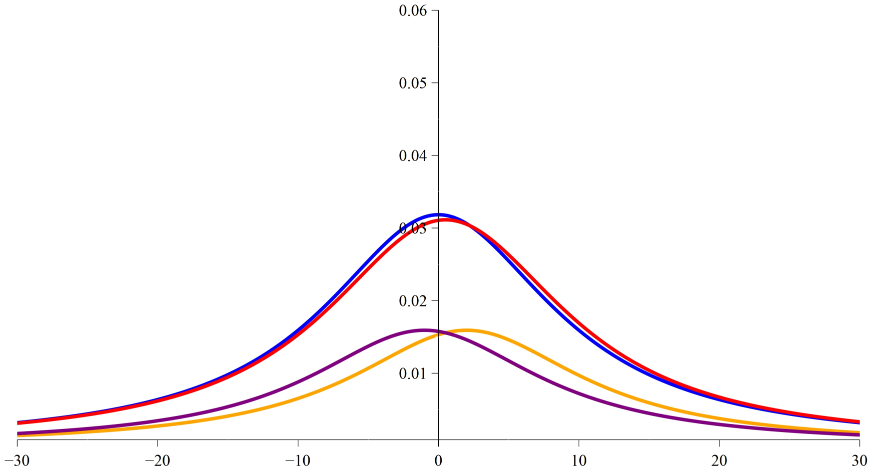

Figure 1: PDFs representing the prospects of the Null Option (blue) and the Basic Gamble (red), given a background prospect described by a Cauchy distribution with a location parameter of 0 and a scale parameter of 10. Purple and orange curves are "half PDFs" representing the two possible outcomes of the Basic Gamble: They are obtained from the background distribution β by multiplying by .5 (representing the .5 probabilities of each simple payoff), then translating by $+2$ and -1 respectively (representing the magnitudes of the simple payoffs). The prospect of the Basic Gamble is then obtained by summing the orange and purple curves. [Blue: $\beta(x) = \beta_N(x) = \left(10\pi(1+(\frac{x}{10})^2)\right)^{-1}$. Purple: $\beta_1^G(x) = .5\beta(x+1)$. Orange: $\beta_2^G(x) = .5\beta(x-2)$. Red: $\beta_G(x) = \beta_1^G(x) + \beta_2^G(x)$.]

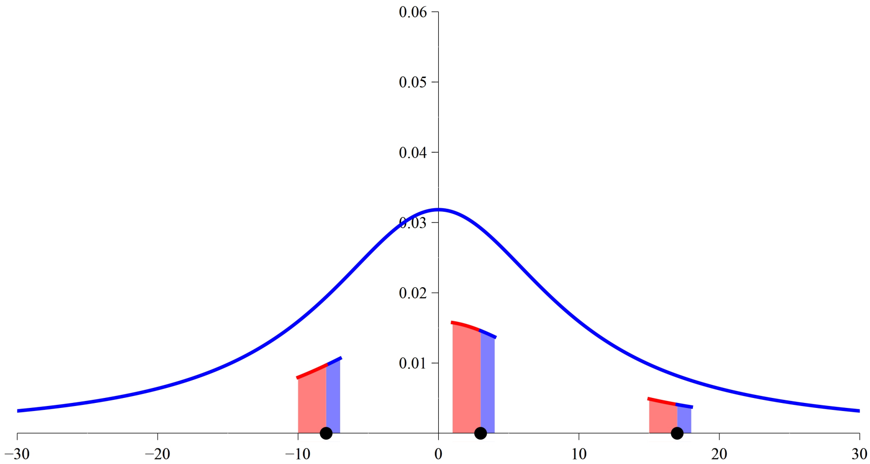

Figure 2: G stochastically dominates N if for every x, $.5\int_{x-2}^{x}\beta(y)\,dy$ (red area, corresponding to the possibility of a simple payoff of $+2$ and a background payoff in $[x-2,x)$) is greater than $.5\int_{x}^{x+1}\beta(y)\,dy$ (blue area, corresponding to the possibility of a simple payoff of -1 and a background payoff in the interval $[x,x+1)$).

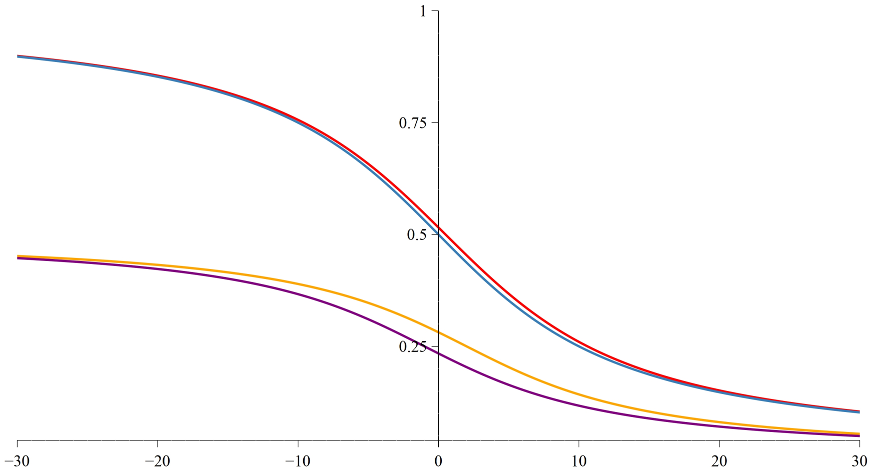

Figure 3: CCDFs (and "half CCDFs") corresponding to the PDFs (and "half PDFs") in Fig. 1. The blue curve gives the probability that the Null Option will yield a payoff $\geq x$. The red curve gives the probability that the Basic Gamble will yield a payoff $\geq x$. Purple and orange curves again represent the two possible simple payoffs of the gamble. $\bar{B}_G$ (red) is everywhere slightly greater than $\bar{B}$ (blue), indicating that the Basic Gamble yields a greater probability of a payoff $\geq x$ than the Null Option for every x, and hence is stochastically dominant. [Blue: $\bar{B}(x) = \frac{1}{\pi} tan^{-1}\left(\frac{x}{10}\right) + .5$. Purple: $\bar{B}_1^G(x) = .5\bar{B}(x+1)$. Orange: $\bar{B}_2^G(x) = .5\bar{B}(x-2)$. Red: $\bar{B}_G(x) = \bar{B}_1^G(x) + \bar{B}_2^G(x)$.]

In other words:

$$\forall x \forall y \left(|x-y| \leq 3 \rightarrow \frac{\beta(x)}{\beta(y)} \leq 2 \right)$$

implies that...

$$\forall x \left(.5 \int_{x-2}^{x} \beta(y)\, dy > .5 \int_{x}^{x+1} \beta(y)\, dy \right)$$

which in turn implies that...

$$G \succ_{sd} N$$

For β to never decrease by more than a factor of 2 within an interval of length 3, it is sufficient that β has large tails and a large enough scale factor: If a distribution has large tails, then for any finite l, there is *some* finite r such that β never decreases by more than a factor of r within an interval of length l. And if for $l = 3$ this factor is greater than 2, we can decrease it by "stretching" β (increasingly rescaling it), so that its tails decay more slowly.

The implication of stochastic dominance can be made more visually perspicuous by representing each prospect not by its probability density function, but by the corresponding *complementary cumulative distribution function* (CCDF). The *cumulative distribution function* (CDF) of a prospect gives the probability of it taking a value less than or equal to x: $B_i(x) = \int_{-\infty}^{x} \beta_i(y)\, dy$. The CCDF, $\bar{B}(x)$, is the complement of the CDF: $\bar{B}(x) = 1 - B(x)$. When prospects are continuous (as we have assumed), the CCDF gives the probability of a payoff $\geq x$. Thus, the Basic Gamble stochastically dominates the Null Option iff its CCDF is everywhere greater (Fig. 3).

5.2 Upper Bound Theorem

We have now seen how background uncertainty can generate stochastic dominance. But how general is this phenomenon—does it depend on special and improbable conditions? In this section, we will partially answer that question by identifying a sufficient condition for O_i to stochastically dominate O_j under background uncertainty, that depends only on (i) a measure of the expectational superiority of O_i to O_j and (ii) the rate at which the tails of β decay, relative to the range of possible simple payoffs from O_i and O_j.

To state the result, we need to introduce some new expressions. First, we introduce a function that, for options O_i and O_j, gives the difference between the probability that O_i yields a simple payoff $\geq x$ and the probability that O_j yields a simple payoff $\geq x$.

$$\Delta_{ij}(x) := Pr(O_i \geq x) - Pr(O_j \geq x)$$

Δ_{ij} can be understood as the difference of the CCDFs of the simple prospects of O_i and O_j (Fig. 4). We also define the positive and negative parts of Δ_{ij}:

$$\Delta^+_{ij}(x) := \max(\Delta_{ij}(x), 0)$$

$$\Delta^-_{ij}(x) := \max(-\Delta_{ij}(x), 0)$$

The integral of Δ_{ij} gives the difference in expected choiceworthiness between O_i and O_j. If Δ_{ij} is nowhere negative and somewhere positive, then the simple prospect of O_i stochastically dominates that of O_j, which guarantees that O_i will stochastically dominate O_j in any state of background uncertainty (Pomatto et al., 2018, pp. 2–3). On the other hand, if the simple prospect of O_j has a greater expectation than that of O_i, then it is impossible for O_i to stochastically dominate O_j in any state of background uncertainty (Pomatto et al., 2018, p. 3). Thus, the cases of interest to us are those where Δ_{ij} is somewhere positive and somewhere negative, and where the integral of Δ_{ij} is positive.

Second, we introduce a function $rate(O_i, O_j, \beta)$ that gives the maximum ratio between values of β, for arguments that differ by no more than the range of the support of Δ_{ij}, denoted $|\operatorname{supp}(\Delta_{ij})| = \max(\operatorname{supp}(\Delta_{ij})) - \min(\operatorname{supp}(\Delta_{ij}))$. (In general, $|\operatorname{supp}(\Delta_{ij})|$ is the difference between the best and worst possible simple payoffs in O_i and O_j.)

$$rate(O_i, O_j, \beta) := \max_{x,y:|y|<|\operatorname{supp}(\Delta_{ij})|} \frac{\beta(x+y)}{\beta(x)}$$

This notation in hand, we can now state the first result.

Theorem 1 (Upper Bound Theorem). *For any options O_i, O_j and background prospect β,*

$$\frac{\int_{-\infty}^{\infty} \Delta^+_{ij}(x)\,dx}{\int_{-\infty}^{\infty} \Delta^-_{ij}(x)\,dx} > rate(O_i, O_j, \beta) \rightarrow O_i \succ_{sd} O_j$$

The proof has been consigned to the appendix. But as an intuitive sketch: Consider a possible payoff p and an option O_i. Given a background payoff $p - x$, O_i yields a payoff $\geq p$ if and only if its simple payoff is $\geq x$. We could therefore calculate the probability that O_i yields a payoff $\geq p$ as $\int_{-\infty}^{\infty} \beta(x) \times Pr(O_i \geq p - x)\,dx$. And this means that for rival options O_i and O_j, we could calculate *difference* between O_i's and O_j's probabilities of a payoff $\geq p$ as $\int_{-\infty}^{\infty} \beta(x) \times (Pr(O_i \geq p - x) - Pr(O_j \geq p - x))\,dx$—or in other words, $\int_{-\infty}^{\infty} \beta(x) \times \Delta_{ij}(p - x)\,dx$. If β were uniform on the relevant interval around p (specifically,

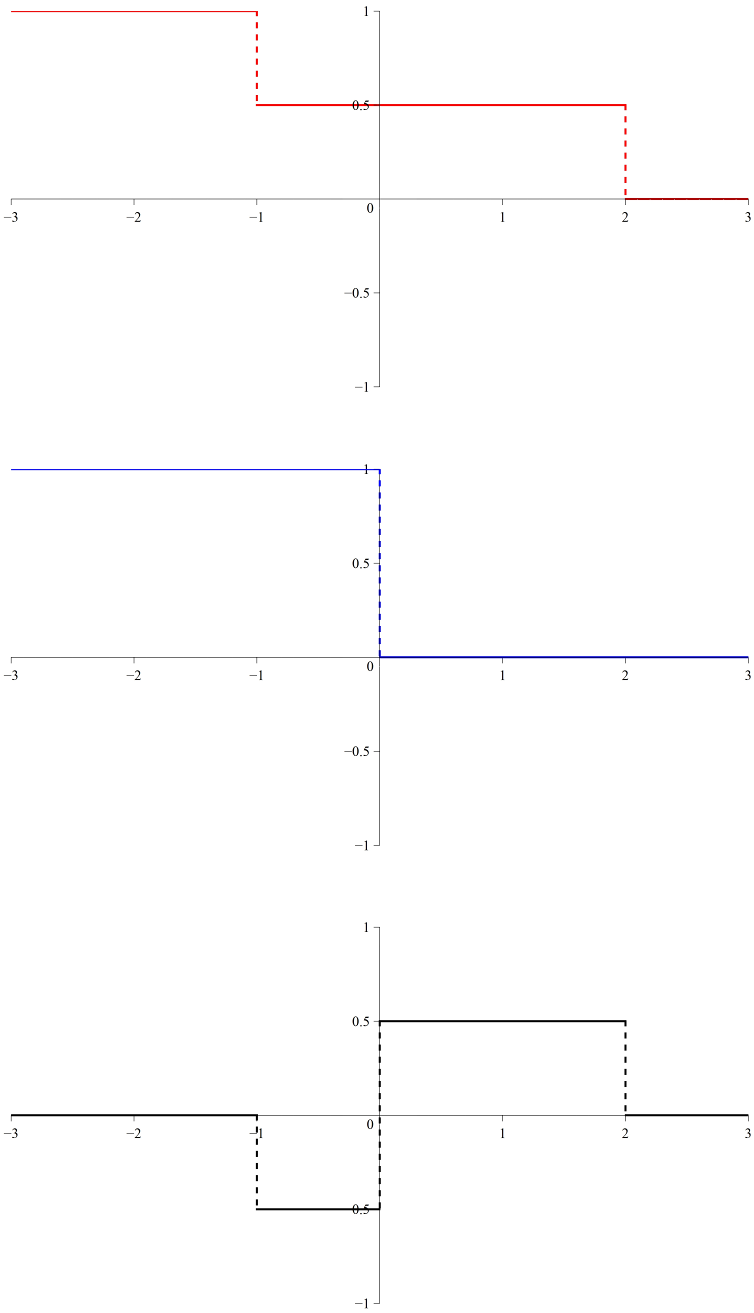

Figure 4: The CCDFs of the simple prospects of the Basic Gamble G (top) and the Null Option N (middle), and the difference Δ_{GN} of the two CCDFs (bottom). $\int_{-\infty}^{\infty} \Delta_{GN}(x)\,dx = \int_{-\infty}^{\infty} \Delta_{GN}^{+}(x)\,dx - \int_{-\infty}^{\infty} \Delta_{GN}^{-}(x)\,dx$ gives the difference in expected choiceworthiness between G and N.

$[p+\min_k(v_k^{i/j}), p+\max_k(v_k^{i/j})]$), then choosing O_i over O_j would change the probability of a payoff $\geq p$ by an amount proportionate to $\int_{-\infty}^{\infty} \Delta_{ij}(x)\,dx = \Delta_{ij}^{+}(x)\,dx - \int_{-\infty}^{\infty} \Delta_{ij}^{-}(x)\,dx$, and O_i would offer a better probability than O_j of a payoff $\geq p$ iff $\Delta_{ij}^{+}(x)\,dx > \int_{-\infty}^{\infty} \Delta_{ij}^{-}(x)\,dx$. Given that β is *not* uniform, it imposes a "discount" on some parts of Δ_{ij} relative to others. But if the ratio between values of β within intervals of size $|\operatorname{supp}(\Delta_{ij})|$ is never greater than some finite r, then this discount factor can never be greater than r. And so if $\frac{\int_{-\infty}^{\infty} \Delta_{ij}^{+}(x)\,dx}{\int_{-\infty}^{\infty} \Delta_{ij}^{-}(x)\,dx} > r$, it is guaranteed that for any arbitrary p we consider, O_i will yield a greater overall probability than O_j of a payoff $\geq p$.

If β has large tails, then for any O_i, O_j, $rate(O_i, O_j, \beta)$ will be finite.[17] Moreover, if we "stretch" β along the x-axis (i.e., increasingly rescale it), $rate(O_i, O_j, \beta)$ converges to 1 (for any O_i, O_j,).[18] So if β has large tails and the simple prospect of O_i is expectationally superior to that of O_j (so that $\frac{\int_{-\infty}^{\infty} \Delta_{ij}^{+}(x)\,dx}{\int_{-\infty}^{\infty} \Delta_{ij}^{-}(x)\,dx} > 1$), O_i will stochastically dominate O_j given only that β has a large enough scale factor. This means that, as we increasingly rescale β, the partial ordering of options given by SDTR asymptotically approaches the ordering of options by the expectations of their simple prospects.

5.3 Lower Bound Theorem

The Upper Bound Theorem also offers some suggestion that it is harder for background uncertainty to generate stochastic dominance in Pascalian contexts: All else being equal, increasing the range of simple payoffs increases $rate(O_i, O_j, \beta)$, and so the condition $\frac{\int_{-\infty}^{\infty} \Delta_{ij}^{+}(x)\,dx}{\int_{-\infty}^{\infty} \Delta_{ij}^{-}(x)\,dx} > rate(O_i, O_j, \beta)$ becomes more demanding. But since this is a sufficient rather than a necessary condition for stochastic dominance, this is only a suggestion.

The suggestion is confirmed, however, by the following *necessary* condition for stochastic dominance:

Theorem 2 (Lower Bound Theorem)**.** *For any options O_i, O_j and background prospect β,*

$$O_i \succ_{sd} O_j \rightarrow \max_x \Delta_{ij}(x) > \max_x \int_{-\infty}^{\infty} \Delta_{ij}^{-}(x-y) \times \beta(y)\,dy$$

The proof is again left for the appendix. But as an intuitive sketch: We saw above that, for a possible payoff p, the difference between O_i's and O_j's probabilities of a payoff $\geq p$ is given by $\int_{-\infty}^{\infty} \beta(x) \times \Delta_{ij}(p-x)\,dx$. So O_i gives an equal or greater probability of a payoff $\geq p$ iff $\int_{-\infty}^{\infty} \beta(x) \times \Delta_{ij}^{+}(p-x)\,dx \geq \int_{-\infty}^{\infty} \beta(x) \times \Delta_{ij}^{-}(p-x)\,dx$. Since β integrates to 1, $\int_{-\infty}^{\infty} \beta(x) \times \Delta_{ij}^{+}(p-x)\,dx$ cannot be greater than $\max_x \Delta_{ij}^{+}(x)$, which is equal to $\max_x \Delta_{ij}(x)$. So if $\int_{-\infty}^{\infty} \beta(x) \times \Delta_{ij}^{-}(p-x)\,dx$ is greater than $\max_x \Delta_{ij}(x)$ for any p, O_i cannot stochastically dominate O_j.

This result tells us two things: First, whenever the simple prospect of O_i does not stochastically dominate that of O_j (so that $\max_x \int_{-\infty}^{\infty} \Delta_{ij}^{-}(x-y) \times \beta(y)\,dy$ is non-zero),

[17] Since $\frac{|\beta'(x)|}{\beta(x)}$ is bounded above by r, the ratio between values of β separated by less than $|\operatorname{supp}(\Delta_{ij})|$ (i.e., $rate(O_i, O_j, \beta)$) cannot be greater than $e^{r|\operatorname{supp}(\Delta_{ij})|}$. This follows from Grönwall's inequality.

[18] Rescaling β by a factor of s means transforming it to $\beta_s = \frac{1}{s}\beta(\frac{x-a}{s})$, for some constant a. Comparing the corresponding points in the original and transformed distributions (x and $\frac{x-a}{s}$), we find that $\beta(x)$ is reduced by a factor of s, but $\beta'(x)$ is reduced by a factor of s^2. So if $\frac{|\beta'(x)|}{\beta(x)}$ is bounded above by r, then $\frac{|\beta_s'(x)|}{\beta_s(x)}$ is bounded above by $\frac{r}{s}$, and $rate(O_i, O_j, \beta_s)$ is bounded above by $e^{\frac{r|\operatorname{supp}(\Delta_{ij})|}{s}}$. This implies that as s goes to infinity, $rate(O_i, O_j, \beta_s)$ goes to 1.

there is some probability threshold such that sets of simple payoffs with total probability below that threshold cannot generate stochastic dominance, no matter their magnitude. To illustrate, suppose we make the choice between O_i and O_j more Pascalian by taking each positive simple payoff of O_i and negative simple payoff of O_j, and replacing its simple payoff-probability pair with a smaller probability of a proportionately larger simple payoff, plus a complementary probability of 0—that is, replacing $\langle v_k^{i/j}, p_k^{i/j} \rangle$ with $\langle cv_k^{i/j}, \frac{p_k^{i/j}}{c} \rangle, \langle 0, p_k^{i/j} - \frac{p_k^{i/j}}{c} \rangle$, for some constant c. This "Pascalian transformation" preserves the expectations of both options. But as c goes to infinity, $\max_x \Delta_{ij}(x)$ goes to 0, while $\max_x \int_{-\infty}^{\infty} \Delta_{ij}^-(x-y) \times \beta(y)\, dy$ does not, so O_i must eventually cease to stochastically dominate O_j. More generally, holding other features of a choice situation fixed, SDTR will eventually cease to require the expectationally superior option as the source of its expectational superiority becomes increasingly Pascalian (reliant on very small probabilities of extreme simple payoffs).[19]

But second, the probability threshold established by the Lower Bound Theorem—namely, $\max_x \int_{-\infty}^{\infty} \Delta_{ij}^-(x-y) \times \beta(y)\, dy$—is sensitive to the scale of β. As we increase the scale of β, we spread its fixed budget of probability mass more thinly, so that $\max_x \int_{-\infty}^{\infty} \Delta_{ij}^-(x-y) \times \beta(y)\, dy$ must shrink, approaching zero in the limit. Thus, the greater the scale of β, the more Pascalian a choice situation can become while preserving stochastic dominance.

5.4 Some illustrations

The Upper and Lower Bound Theorems give separate sufficient and necessary conditions for stochastic dominance. If we fill in some details, though, we can find necessary-and-sufficient conditions for stochastic dominance in restricted contexts. This lets us see just how tightly SDTR constrains risk attitudes in particular choice situations, both "ordinary" and "Pascalian".

First, let's specify a background prospect: a Laplace distribution with a mean of zero and a scale parameter of $-\frac{500}{\ln(.05)}$ (≈ 166.9).[20] A Laplace distribution has exponential tails, and is therefore as light in the tails as any large-tailed distribution can be. The scale parameter of $-\frac{500}{\ln(.05)}$ is chosen because it yields a 95% confidence interval of $[-500, +500]$, which gives an intuitive sense of the scale of the distribution. If we assume that units represent lives saved/lost (or more precisely, the choiceworthiness of saving a typical life), then this means the agent is 95% confident that her background payoff will fall in an interval whose magnitude is the value of 1000 human lives. For an agent who attaches normative weight to the total value of the world that results from her choices, this scale parameter is implausibly small (as I will argue in §6). But I assume this very modest degree of background uncertainty in order to emphasize how easily background uncertainty can generate can generate very strong stochastic dominance constraints on an agent's choices.

To see the strength of these constraints, consider the following:

[19]In a very specific and limited sense, therefore, SDTR vindicates the oft-mooted idea that it is permissible to ignore outcomes with sufficiently small probabilities (described, for instance, as "*de minimis*" (Whipple, 1987) or "rationally negligible" (Smith, 2014, 2016)). But the Pascalian threshold drawn by SDTR differs from these previous ideas in important ways: First, it applies to *sets* of outcomes rather than individual outcomes (so it does not face problems of individuation). Second, it is sensitive in precise ways to other features of the choice situation (e.g., to the magnitude of any high-probability simple payoffs that must be weighed against the more improbable outcomes, and to the scale of the agent's background prospect), so it does not establish any *general* threshold below which probabilities can be ignored.

[20]Stochastic dominance is invariant under translations of the background prospect (i.e., transformations of the form $\beta'(x) = \beta(x-a)$ for some constant a), since translations of β only result in identical translations of each option's overall prospect. Thus, in a parameterized distribution like Laplace, the choice of a mean (or, more generally, location parameter) makes no difference for our purposes.

Generalized Basic Gamble (G') $\{\langle -1, .5\rangle, \langle 0, .5-p\rangle, \langle 2, p \in [0, .5]\rangle\}$

We can interpret G' as an option that will save two lives with probability p, cause one death with probability .5, and have no consequences with probability $.5 - p$.

Given a choice between G' and the Null Option N, G' has greater expected choiceworthiness than N iff $p > .25$. But for what values of p is G' stochastically dominant? To figure this out, let's consider the CDF of the background prospect, expressed as:

$$B(x) = \begin{cases} .5\exp(\frac{\ln(.05)x}{500}) & x \leq 0 \\ 1 - .5\exp(-\frac{\ln(.05)x}{500}) & x > 0 \end{cases}$$

For any x, G' improves the probability of a payoff $\geq x$ (relative to N) by $p(B(x) - B(x-2))$, and worsens the probability of a payoff $\geq x$ (relative to N) by $.5(B(x+1) - B(x))$. Thus, G' stochastically dominates N iff:

$$\forall x (p(B(x) - B(x-2)) > .5(B(x+1) - B(x)))$$

or equivalently

$$\forall x \left(p > \frac{.5(B(x+1) - B(x))}{B(x) - B(x-2)}\right)$$

The function on the right side of this inequality is bounded above at $\sim .25226$. This means that, while G' has greater expected choiceworthiness than N iff $p > .25$, G' stochastically dominates N iff $p > \sim .25226$. So, even given a relatively light-tailed background prospect with a small scale factor, stochastic dominance imposes *extremely* tight constraints on the choice between G' and N—nearly as tight as those imposed by expectationalism.

But now consider a more Pascalian version of G':

Generalized Pascalian Gamble (G'') $\{\langle -1, .5\rangle, \langle 0, 1-p\rangle, \langle 2000, p \in [0, .5]\rangle\}$

G'' has greater expected choiceworthiness than N iff $p > .00025$. By reasoning parallel to the case of G', G'' stochastically dominates N iff:

$$\forall x \left(p > \frac{.5(B(x+1) - B(x))}{B(x) - B(x-2000)}\right)$$

The function on the right side of *this* inequality, however, is bounded above at $\sim .0030047$. That is, G'' only comes to stochastically dominate N when the probability of a positive simple payoff is *more than ten times greater* than the probability at which G'' becomes expectationally superior. So at least here, stochastic dominance places very tight constraints on choices involving intermediate probabilities of modest simple payoffs, but gives much more latitude when faced with very small probabilities of very large simple payoffs.[21]

What does this mean for potentially Pascalian real-world choices—e.g., the choice between short-term interventions that do moderate amounts of good with high probability

[21] Notably, given that β has large tails, it seems to matter very little precisely how heavy its tails are. For instance, suppose we replace the Laplace distribution with a Cauchy distribution (which has *much* heavier tails) with a scale parameter of $-500(\cot(.525\pi))$ (≈ 39.35)—which yields the same 95% confidence interval of $[-500, +500]$. Now we find that G' stochastically dominates N iff $p >\sim .25969$ (as opposed to $\sim .25226$ for the Laplace distribution), and G'' stochastically dominates N iff $p >\sim .009452$ (as opposed to $\sim .0030047$ for the Laplace distribution). That is, at least in these two cases, moving to a much heavier tailed distribution with a roughly equivalent scale parameter does not change the conditions for stochastic dominance very much, and in fact makes those conditions somewhat *more* demanding, rather than less.

and interventions that try to influence the far future, doing potentially astronomical good, but with (plausibly) very low probability of success? Fully answering this question is a large project unto itself (requiring, among other things, a plausible model of our actual background uncertainty and of the probabilities and payoffs involved in the interventions we wish to compare). But as a first approximation, let's consider another stylized case in which we must choose between a "sure thing" option that saves one life for certain ($S = \{\langle 1, 1 \rangle\}$) and a "long shot" option that tries to prevent existential catastrophe, thereby enabling the existence of astronomically many future lives, but has only a very small probability of making any difference at all ($L = \{\langle 0, 1-p \rangle, \langle a, p \rangle\}$, where a is astronomically large, p is very small, and $ap > 1$).

First, we might ask what is the threshold on p imposed by the Lower Bound Theorem below which, no matter the magnitude of a, L cannot stochastically dominate S. To a good approximation, the answer turns out to be: the inverse of the scale factor of β.[22] If one's background uncertainty reflects uncertainty about the total amount of value in the Universe, it seems safe to assume that its scale factor should be at least on the order of 10^9. (Indeed, the arguments in the next section will suggest that it should be much larger.) If so, then the "Pascalian threshold" below which L cannot stochastically dominate S will likely be no greater than 10^{-9}.

This threshold applies to L *no matter the magnitude* of the astronomical simple payoff a. How much do things change if we consider some particular value of a, like Bostrom's 10^{52}? The short answer is: not much. For these purposes, payoffs much larger than the scale factor of the background prospect can, to a very close approximation, be treated as infinite.[23] Thus, the probability threshold at which a payoff like 10^{52} can generate stochastic dominance over S will be roughly the inverse of the scale factor of β, as long as that scale factor is significantly less than 10^{52}.

This suggests a strategy for proponents of Bostrom-style arguments (and "longtermists" more generally) to allay concerns about Pascalian fanaticism: If we can increase the probability of astronomically positive payoffs like 10^{52} future lives by significantly more than the inverse of the scale factor of our background uncertainty (e.g., conservatively, by more than 10^{-9}), then aggregative consequentialists at least should have no decision-theoretic reservations about favoring such interventions (assuming they are expectationally superior to their alternatives), since they are required by stochastic dominance. But if the decision-relevant probabilities are on the order of, say, 10^{-30}, then even though

[22] If β has a scale factor (i.e., 50% confidence interval) of 1, then the threshold given by the Lower Bound Theorem in the choice between L and S is $.5 - .5p$. (Assuming, without loss of generality, that the 50% confidence interval of β is centered at 0, $\max_x \int_{-\infty}^{\infty} \Delta^-_{LS}(x-y) \times \beta(y)\, dy = \int_{-\infty}^{\infty} \Delta^-_{LS}(.5-y) \times \beta(y)\, dy = \int_{-.5}^{.5} (1-p) \times \beta(y)\, dy$. And if the scale factor of β is 1, then $\int_{-.5}^{.5} \beta(y)\, dy = .5$.) As the scale factor s of β increases, the values of β shrink, but $\max_x \int_{-\infty}^{\infty} \Delta^-_{LS}(x-y) \times \beta(y)\, dy$ is also able to draw increasingly from the very peak of β. So in the limit as s goes to infinity, $\max_x \int_{-\infty}^{\infty} \Delta^-_{LS}(x-y) \times \beta(y)\, dy$ converges to $(.5 - .5p) \times r \times \frac{1}{s}$, where r is the ratio between the maximum value of β and its average value over its 50% confidence interval. Since r must be greater than 1, and will typically be well under 20, the probability threshold given by the Lower Bound Theorem will typically be within an order of magnitude of $\frac{1}{s}$ for large values of s.

[23] To see this, consider $L_1 : \{\langle 0, 1-p \rangle, \langle 10^{52}, p \rangle\}$ and $L_2 : \{\langle 0, 1-p \rangle, \langle +\infty, p \rangle\}$. Will L_2 stochastically dominate S for much smaller values of p than L_1? $L_1 \succ_{sd} S$ iff $\forall x p \int_{x-10^{52}}^{x} \beta(y)dy \geq \int_{x-1}^{x} \beta(y)dy$. As long as $p > 10^{-52}$, this condition will be satisfied for values of x in the right tail of the background prospect (in particular, for all values of x that exceed the mean of the background prospect by at least $.5 \times 10^{52}$). For all other values of x, $\int_{x-10^{52}}^{x} \beta(y)dy$ is only *very* slightly smaller than $\int_{-\infty}^{x} \beta(y)dy$, assuming the scale factor of the background prospect is much smaller than 10^{52}, since the cutoff $x - 10^{52}$ will be far out in the left tail of the distribution. Thus, the value of p required for L_1 to stochastically dominate S is only very slightly greater than than the value required by L_2.

the expectations may still be astronomical (using Bostrom's number, 10^{22}), it becomes more plausible that we are rationally permitted to prefer expectationally inferior "sure thing" interventions.[24]

Given the above results, it seems to me that the greatest intuitive worry about SDTR in the presence of large-tailed background uncertainty is not that it will capture too little of expectational reasoning (failing to recover intuitive constraints on our choices), but rather that it will capture too much—requiring us to accept many gambles that seem intuitively Pascalian (e.g., where the probability of any positive payoff is on the order of 10^{-9} or less). But really, this is not a worry at all: Unlike primitive expectationalism, SDP is supported by *a priori* arguments far more epistemically powerful than our intuitions about Pascalian gambles. If some gambles that seem intuitively Pascalian turn out to be stochastically dominant once we account for our background uncertainty, we should not conclude that stochastic dominance is implausibly strong. Rather, we should conclude that there is a *much more compelling argument* for choosing the expectation-maximizing option in these cases than we had previously realized. This would be not a *reductio* but rather an unexpected and practically important discovery.

6 Sources of background uncertainty

The results above are practically significant only if some agents are (or ought to be) in a state of background uncertainty described by a large-tailed distribution with at least a moderately large scale factor. In this section, I give three arguments that, at least for agents who assign normative weight to aggregative consequentialist considerations (and perhaps more generally), this sort of background uncertainty is rationally required.

First, an intuitive argument: The level of background uncertainty required by the arguments in the last section is in fact extremely modest. Setting aside some contrived exceptions, a distribution has exponential or heavier tails as long as there is some finite bound on the ratio between probabilities assigned to adjacent intervals of a fixed length, like $[x-1, x]$ and $[x, x+1]$.[25] In our context, this means that there is some finite upper bound on *how much more probable I take it to be* that the Universe contains between $x-1$ and x units of value than that it contains between x and $x+1$ units of value (or vice versa). The only way there could fail to be such a bound (given that my β is supported everywhere) is if the ratio increased without bound in one or both tails of β. But this implies that I become arbitrarily confident about the relative probability of *very* similar hypotheses, in a domain where I seem to have virtually no grounds for distinguishing between those hypotheses. It would mean, for instance, that I find it *vastly* more probable that the Universe contains between $-18,946,867,974,834$ and $-18,946,867,974,835$ units of value than that it contains between $-18,946,867,974,835$ and $-18,946,867,974,836$ units of value. And as the numbers get larger, my relative confidence only gets (boundlessly) greater. But it seems obvious that, if anything, my confidence in these relative probabilities should *diminish* as the numbers get larger. None of my evidence provides any serious support for the first of the above hypotheses ($[-...5, -...4]$) over the second ($[-...6, -...5]$), at least not in any way that I am capable of identifying.

[24] For a general exposition of the case for longtermism (roughly, the thesis that what we ought to do is primarily determined by the effects of our choices on the far future) based on the potentially astronomical scale of future human civilization, see Beckstead (2013)and Greaves and MacAskill (ms). For discussion of the worry that these "astronomical stakes" arguments involve a problematic form of Pascalian fanaticism, see Chapters 6–7 of Beckstead (2013).

[25] The constant 1 is arbitrary. If the ratio between the probabilities of $[x-l, x]$ and $[x, x+l]$ is bounded for some value of l, then it is bounded for any value.

The second argument is a bit more concrete: Attempting to model our actual background uncertainty, even on fairly conservative assumptions, yields tails significantly heavier than exponential. Assume that the total amount of value in the world is, at least in part, a function of the total welfare of all morally statused beings. In this case (unless other normative considerations are systematically anti-correlated with total welfare), my background uncertainty should be at least as great as my uncertainty regarding the total amount of welfare in the Universe. This uncertainty is determined by my uncertainty about (1) the number of morally statued beings and (2) their average welfare.

The most obvious source of large tails is uncertainty about the number of statused beings. Playing on the Drake equation, we can approximate the number of statused beings in the Universe as the product of (1.1) the total number of galaxies in the Universe, (1.2) the average number of stars (that will ever exist) per galaxy, (1.3) the average number of populations of statused beings per star, (1.4) the average longevity of those populations, in generations, and (1.5) the average number of individuals in a generation.

Any of factors (1.1), (1.4), and (1.5) might be a source of large-tailed background uncertainty, but (1.1) is the most straightforward.[26] One might think that (1.1) is a limiting factor, since there is an upper bound on the number of galaxies given by the size of the observable universe (thought to contain between 200 billion and a few trillion galaxies in total (Gott III et al., 2005; Conselice et al., 2016)). But the *observable* universe is only a small part of the Universe as a whole: It is now known, in fact, that the Universe as a whole must be many times larger than the part we can observe. And more importantly, there is no known upper bound on its size, assuming that it is finite.[27]

Of course, the absence of an upper bound on the size of the Universe is not enough for our purposes: We need a probability distribution. This requires a choice of prior, a fraught endeavor whose philosophical difficulties we will not be able to resolve here. But the best we can do is to choose a reasonable and conservative prior and see where it leads us. Vardanyan et al. (2009) suggest a physically motivated prior which they call the *astronomer's prior*. Conditional on a finite universe, the astronomer's prior is uniform over values of Ω_k in the interval $(0, 1]$, where Ω_k is the curvature parameter in the standard ΛCDM cosmology (smaller values of Ω_k indicating less curvature and

[26] With respect to (1.4) and (1.5), it could be that interstellar civilizations are extremely long-lived, or that they reach extremely large populations. Setting aside speculative physics, however, there seem to be fairly hard upper bounds on both these factors, given by the impending heat death of the Universe and the light-speed limit on a civilization's rate of expansion.

[27] Assuming that the Universe has the simplest (viz., simply connected) topology, it is finite if and only if it has positive curvature, with larger curvature implying a smaller Universe. Current cosmological data constrain the curvature of the Universe to a fairly small interval around zero (Gong et al., 2011; Jimenez et al., 2018). Based on this data, Vardanyan et al. (2011) find a lower bound on the size of the Universe of 251 Hubble volumes (roughly 7.7 times larger than the observable universe), with 99% confidence. Much larger numbers have been suggested as well: Greene (2004) notes that in many inflationary models, the Universe is so large that "[i]f the entire cosmos were scaled down to the size of earth, the part accessible to us would be much smaller than a grain of sand" (p. 285). From one such inflationary model, Page (2007) extrapolates (though without fully endorsing) a lower bound of roughly $10^{10^{10^{122}}}$ Hubble volumes.

To my knowledge, no cosmologist has proposed an *upper* bound on the size of the Universe as a whole. Vardanyan et al. (2009) give a probability distribution that is bounded above at roughly 10^8 Hubble volumes (p. 438). But this is an artifact of their choice of categories: Because a universe larger than that bound is observationally indistinguishable from a flat (infinite) universe, they group larger finite universes together with infinite universes for purposes of model comparison (see §3.3, pp. 435-6).

I am setting aside, as overkill, various multiverse hypotheses according to which the result of the Big Bang (our observable universe, and what lies beyond it) is only a small part of the Universe as a whole. But these hypotheses of course add to our uncertainty about the size of the Universe and the total amount of value it contains.

hence a larger Universe).[28] This implies a prior over the present curvature radius of the Universe, a_0, where $Pr(a_0) \propto a_0^{-3}$, which in turn implies a prior over the present *volume* of the Universe, V, where $Pr(V) \propto V^{-\frac{5}{3}}$. This distribution is extremely heavy-tailed—much heavier than exponential.[29] And it implies an equally heavy-tailed distribution with respect to the number of galaxies, which is directly proportional to V.

Given such a heavy-tailed distribution for the size of the Universe, a large-tailed distribution for total welfare in the the Universe (or, in the part of the Universe unaffected by our choices) is nearly a foregone conclusion. The remaining factors are (1.2-5) the number of statused beings per galaxy and (2) the average welfare of statused beings. A large-tailed background prospect then just requires two modest assumptions: first, that the product of these remaining factors is not strongly anti-correlated with the number of galaxies in the Universe, and second, that we assign positive probability to both positive and negative values for average welfare. The second assumption looks unassailable, and I cannot think of any reason to question the first.

The most serious objection I can see to the preceding line of argument is that the Universe, and the number of morally statused beings it contains, may well be *infinite* rather than finite (Knobe et al., 2006; Vardanyan et al., 2009; Carroll, 2017). I will, unfortunately, have little to say about this objection (though I say a bit in §9.6 below). I take it for granted that the true axiology can make non-trivial comparisons between infinite worlds, so that even if we were certain that the Universe was infinite, we could still be uncertain about its overall value. But how (if at all) we extend the arguments of this paper to the infinite context depends very much on what sort of infinite axiology we adopt, and there is as yet no agreement even on very basic questions about how to formulate an infinite axiology.[30] Perhaps more to the point (though no more satisfying), expectational reasoning is if anything *more* threatened by infinite worlds than stochastic dominance reasoning (see for instance Bostrom (2011, pp. 13ff), Arntzenius (2014)), so even if the arguments in this paper suffer in an infinitary context, that is not likely to generate much support for expectationalism over SDTR.

The third and final argument for large-tailed background uncertainty is the simplest: When I am uncertain which of several probability distributions best characterizes some phenomenon, the resulting "mixture distribution" (the probability-weighted average of the distributions over which I'm uncertain) inherits the tail properties of the heaviest distribution in the mixture: The further out we go in the tails of the mixture distribution, the more the heaviest-tailed distributions dominate the probability-weighted sum. So, sup-

[28] For motivation of the astronomer's prior, see Vardanyan et al. (2009, p. 436). Vardanyan et al also consider a second prior, which is log-uniform over Ω_k. But the plausibility of this prior depends significantly on their decision to group models with $|\Omega_k| \leq 10^{-5}$ together with $\Omega_k = 0$, since a log-uniform prior on the full interval $(0, 1]$ would be improper. If we were willing to entertain this improper prior, it would yield an even heavier-tailed distribution with respect to the size of the Universe than the astronomer's prior.

[29] Specifically, the astronomer's prior corresponds to the following prior over V:

$$f_V(x) = \begin{cases} \frac{2^{\frac{5}{3}}}{3}\pi^{\frac{4}{3}}c^2 H_0^{-2} x^{-\frac{5}{3}} & x \geq 2\pi^2 H_0^{-3} c^3 \\ 0 & \text{otherwise} \end{cases}$$

where H_0 is the Hubble constant and c is the speed of light.

Of course, this is only a prior, and what we are really care about is the posterior, i.e., the probability distribution we should actually adopt given our current evidence. But since observational evidence cannot measure Ω_k to a precision greater than $\sim 10^{-4}$, it cannot discriminate within the tail of very large finite universes (corresponding to values of Ω_k asymptotically approaching zero from below), and hence cannot significantly change the tail properties of the distribution.

[30] For some of the many extant proposals, see for instance Vallentyne and Kagan (1997), Mulgan (2002), Bostrom (2011), and Arntzenius (2014).

pose I am unsure what background prospect is justified by my evidence, or that I assign credence to multiple physical theories/models that imply different objective probability distributions over background payoffs. As long as I assign positive credence to any distribution with exponential or heavier tails, the resulting background prospect will itself have exponential or heavier tails. The hypothesis that the correct objective or epistemic probability distribution over, say, the size of the Universe should have exponential or heavier tails pretty clearly merits at least *some* positive credence. And this alone essentially guarantees large-tailed background uncertainty for agents who assign normative weight to aggregate consequentialist considerations. At the least, this "argument from higher-order uncertainty" puts the burden of proof on skeptics to give an argument for thinner-than-exponential tails compelling enough to set our credence in large-tailed hypotheses to zero.

This final argument also suggests that the phenomenon of large-tailed background uncertainty, and the significance of the results in §5, are not limited to agents who assign normative weight to aggregative consequentialist considerations. The observation that even minimal higher-order uncertainty can beget heavy-tailed "first-order" uncertainty applies regardless of the considerations one is uncertain about. So, for instance, even an agent whose concerns are exclusively prudential (i.e., who measures the choiceworthiness of an option solely by her resulting utility/welfare) plausibly has reason to be in a state of large-tailed background uncertainty.

It is also important to remember that even the total-welfare-based arguments for large-tailed background uncertainty do not only apply to agents who are out-and-out aggregative consequentialists, i.e., who measure the choiceworthiness of their options *exclusively* by the total amount of value in the resulting world. It is sufficient that aggregative consequentialist considerations make an additive contribution to the choiceworthiness of an option. In this case, the agent's background uncertainty will still have the same shape as is would if she attached normative weight only to aggregate consequences (unless she has additional non-consequentialist sources of background uncertainty). Non-consequentialist considerations, plausibly, will be mainly captured by simple prospects. Depending on the weight of these considerations, they could make stochastic dominance relationships among the agent's options less likely—in particular, if their contribution to choiceworthiness is very large relative to the scale factor of the agent's background prospect. But even if one assigns very little relative weight to consequentialist considerations, the weight of ordinary non-consequentialist considerations is still likely to be small relative to the scale of one's background uncertainty. For instance, I suggested above that 10^9 is a conservative lower bound on the scale factor of our uncertainty concerning the amount of value in the Universe (where units represent the value of happy lives). And only the most extreme deontological views assign ordinary non-consequentialist considerations a weight anywhere near that order of magnitude.

7 The rational significance of background uncertainty

An initially counterintuitive feature of the preceding arguments is their implication that what an agent rationally ought to do can depend on her uncertainties about seemingly irrelevant features of the world. To put the point as sharply as possible: Whether I am rationally required, for instance, to take a risky action in a life-or-death situation can depend on my uncertainties about the existence, number, and welfare of sentient beings in distant galaxies, perhaps outside the observable universe, with whom I will never and can never interact, on whom my choices have no effect, and whose existence, number, welfare, etc, make no difference to the local effects of my choices.

Surprising and counterintuitive though this conclusion may seem, however, I think it is fully intelligible on reflection. In this section, I will try to dispel (or at least mitigate) the feeling of counterintuitiveness. To do that, I will first describe a simple case where the rational relevance of background uncertainty is intuitively clear, then argue that what is true of this simple case is true of more complex cases as well.

Here, then, is the simple case:

Methuselah's Choice Methuselah is, and knows himself to be, the only sentient being in the Universe (past, present, or future). He came into existence finitely long ago, and has so far been in a neutral state. He now faces a choice—the only choice he will ever make. He can choose either O_1, which yields 100 years of happy life for sure, or O_2, which yields 1500 years of happy life with probability .1, or zero years of happy life with probability .9.

If these years of happy life are the only potential source of value in the Universe, it seems intuitively obvious to me that Methuselah is rationally permitted to make either choice. Even if he is rationally required to satisfy the VNM axioms, say, these alone do not tell him which option to choose. And long run arguments for expectationalism are irrelevant as well, since Methuselah knows for certain that there is no long run.

But now suppose that we add a source of background uncertainty:

Methuselah's Box In addition to Methuselah, Methuselah's universe contains a magic box, which contains a real-number generator. Methuselah will make his choice between O_1 and O_2 at time t. Then, at time t', the random number generator inside the box will generate a number, from a Cauchy distribution centered at zero with a scale parameter of 10,000, and open itself to reveal this number to Methuselah. In addition to the simple payoff from his choice, Methuselah will receive a number of happy or unhappy life-years equal to the number generated by the box.

(To avoid comparisons between happy and unhappy life-years, assume that whatever total payoff Methuselah receives, it will come in the form of exclusively happy or exclusively unhappy life-years. Thus, for instance, if he receives $+1500$ from his choice and -2000 from his box, he will experience 500 years of unhappy life. If he gets $+1500$ from his choice and -200 from his box, he will experience 1300 years of happy life.)

In virtue of Methuselah's uncertainty about the background payoff he will receive from his box, O_2 now stochastically dominates O_1 (assuming only that Methuselah regards happy life as better than unhappy life, more happy life as better, and more unhappy life as worse). And for precisely this reason, it now seems clear that Methuselah rationally ought to choose O_2. Absent the uncertainty that his box introduces, Methuselah could have reasoned his way to choosing O_1 on the grounds that if he chooses O_1, he will certainly receive at least 100 years of happy life, while if he choose O_2, he very probably will not. And there is no compelling defeater to this reasoning, provided that (as I claimed above) there is no compelling argument in this case for risk-neutrally maximizing expected happy life-years. But once we introduce the box, there *is* a compelling defeater to the original justification for O_1: First, Methuselah is *not* guaranteed to experience at least 100 years of happy life if he chooses O_1. Second, in fact, he has a *better* chance of experiencing at least 100 years of happy life if he chooses O_2. And third, the same is true for *any other possible payoff*: Whatever payoff he chooses to focus on, Methuselah has a better chance of a payoff at least that good if he choose O_2. Thus, Methuselah's background uncertainty gives him conclusive grounds for choosing O_2.

But what does this have to do with more ordinary choice situations? Let's generalize the lesson of Methuselah's case in two steps. First, consider an agent Alice who is certain that total hedonistic utilitarianism is true and faces a choice between O_1, which will do an amount of good equivalent to 100 happy life-years with probability 1, and O_2, which will do an amount of good equivalent to 1500 happy life-years with probability .1, and do nothing with probability .9. Suppose that Alice's beliefs about total welfare in the Universe, apart from the effects of her present choice, are described by a Cauchy distribution centered at zero with a scale parameter of 10,000 happy-life-year-equivalents.

Because Alice is a total hedonistic utilitarian, her situation is in every relevant respect equivalent to Methuselah's: It makes no difference, from a utilitarian standpoint, whether the welfare at stake is the agent's own, whether it belongs to a single welfare subject or to many, whether those subjects are near to the agent in space or time, etc. Just as in the case of Methuselah, therefore, we should conclude that (i) if there were nothing of moral significance in the Universe apart from the simple payoff of O_1 or O_2, then there would be no decisive justification for choosing O_2, but (ii), by making O_2 stochastically dominant over O_1, her background uncertainty gives her just such a decisive justification.

Does this line of reasoning apply only to rigorously orthodox utilitarians, who are committed to universal impartiality and the interpersonal fungibility of welfare? No. All that our reasoning in the cases of Methuselah and Alice really depended on was the fungibility of *choiceworthiness*, which is a conceptual truth so trivial that it is hardly worth stating. Suppose that Bob accepts a commonsense, pluralistic theory of practical reasons, and suppose he faces a choice between O_1 and O_2, where O_1 has a simple prospect of $\{\langle 100, 1\rangle\}$ and O_2 has a simple prospect of $\{\langle 0, .9\rangle, \langle 1500, .1\rangle\}$. And suppose his background uncertainty is a Cauchy distribution with a scale parameter of 10,000. Then for any degree of choiceworthiness, O_2 gives Bob a better chance of performing an action at least that choiceworthy, which provides a uniquely decisive justification for choosing O_2. The difference with the cases of Methuselah and Alice is simply that the various ways in which, say, O_2 might turn out to have choiceworthiness ≥ 500 are much more complex and qualitatively diverse. This makes the force of the stochastic dominance argument harder to see, but not any weaker. Just as, for classical utilitarians, any world that contains welfare equivalent to x happy life-years is equally good regardless of how that welfare is distributed across time, space, or sentient beings, so on *any* normative theory, any option with a choiceworthiness of x is just as choiceworthy as any other, no matter how complex or multifarious the considerations that determine its choiceworthiness. The case of Bob is therefore no different, from the standpoint of rational choice, than the case of Methuselah.

8 Two modest conclusions

What decision-theoretic conclusions should we take away from the preceding arguments? In this section, I describe two relatively moderate conclusions we might draw. In the next section, I make the case for my own more ambitious conclusion.

8.1 A decision theory for consequentialists?

In recent years, there has been a great deal of activity at the intersection of ethics and decision theory, and considerable interest in the idea of "moral/ethical decision theory"—a decision theory distinct from expected utility theory that either governs ethical decision-making in general or serves as the decision-theoretic component of particular ethical theories. Along these lines, the results in §5 might be seen as laying the foundation for a "utilitarian decision theory", analogous to recent attempts to develop a "deontological decision theory" (Colyvan et al., 2010; Isaacs, 2014; Lazar, 2017a,b). Though I have argued that the assumptions on which these results depend generalize well beyond purely consequentialist theories like classical utilitarianism, they are clearly *easiest* to justify in the context of such a theory: The additive separability of simple and background payoffs is trivial for classical utilitarians (the total welfare that results from an option can be expressed as the sum of, say, welfare inside and outside the agent's future light cone), and as we saw in §6, uncertainty about total welfare in the Universe provides an especially strong source of background uncertainty. We might conclude from the preceding arguments, then, that SDTR is an attractive ethical decision theory for classical utilitarians and other aggregative consequentialists.

At minimum, though, we have found that accounting for background uncertainty gives aggregative consequentialists a new and powerful basis for choosing options whose simple prospects maximize expected objective value (and not just the expectation of some increasing function of objective value) in most ordinary choice situations—even if they are also subject to decision-theoretic requirements besides stochastic dominance. That is, we have reached an important practical conclusion for aggregative consequentialists which requires no decision-theoretic assumptions besides the almost entirely uncontroversial SDP. *A fortiori*, this conclusion applies to any aggregative consequentialist who satisfies any of the standard axiom systems like VNM or Savage, or even non-standard axiom systems like that of Buchak's (2013) REU (which, like VNM and Savage, satisfies stochastic dominance). Any such agent must, in practice, rank options almost exactly by the expectations of their simple prospects, even if she is extremely risk-averse or risk-seeking with respect to objective value (except in Pascalian situations where, as we have seen, she may enjoy greater latitude).

8.2 An add-on to standard decision theory?

Building on the last observation, we can understand the results in §5 as a friendly "add-on" to axiomatic expectationalism: At least for some agents, the presence of background uncertainty coupled with the stochastic dominance requirement implied by the standard axioms imposes strict constraints on the agent's preferences over simple prospects, constraints that don't follow from those axioms in the absence of background uncertainty. Specifically, agents can be rationally required to rank options in a way that closely approximates the expectational ranking of their simple prospects *under a particular, privileged assignment of cardinal values to payoffs*—namely, the assignment that satisfies additive separability between simple and background payoffs.[31]

Plausibly, this privileged cardinalization will match the natural cardinal structure of the phenomena in the world to which the agent attaches normative weight. For instance, suppose that I only care about my lifetime income, always preferring more income to less. The only assignments of cardinal values to outcomes that allows additive separability between simple payoffs (the monetary reward of my present choice) and background payoffs (the remainder of my lifetime income) will be those that are positive affine transformations

[31]Remember that this assignment, if it exists, is unique up to positive affine transformation. So any non-affine transformation of this assignment will break the additive separability condition on which the results in §5 depend. Perhaps more to the point, stochastic dominance relations only depend on the ordinal ranking of payoffs, so the same stochastic dominance relations will hold under a positive monotone but non-affine transformation of the privileged cardinal choiceworthiness assignment. These relations will no longer be accurately described by the Upper and Lower Bound Theorems, however, so we cannot link stochastic dominance with expectational superiority under the transformed assignment, but only by adverting to the original, privileged assignment.

of the monetary value of payoffs, as measured in a currency like dollars or euros. So under sufficient background uncertainty, SDP and any axiomatic theory that implies it will require me to rank my options approximately by the *expected monetary value* of their simple prospects.

To put the point a little differently: Under sufficient background uncertainty, the standard axioms (by way of SDP) let us derive strong decision-theoretic conclusions merely from the agent's ranking of payoffs (or, equivalently, her ranking of options under certainty), without any information about her ranking of uncertain prospects. The add-on to standard decision theory here is not stochastic dominance, which was already implied, but rather the idea that agents often are or ought to be in a state of large-tailed background uncertainty. Recognizing this sort of background uncertainty does not impose any new constraints on the agent's utility function *per se*: given a ranking of overall payoffs, she may still maximize the expectation of any utility function that is increasing with respect to that ranking. But background uncertainty forces all these utility functions to agree much more than they otherwise would on the ranking of options, in a way that makes it *look as if* the agent was simply maximizing her expected simple payoff on a privileged cardinal scale.

As promised in §3, I haven't given any novel arguments for rejecting any of the standard axioms of expected utility theory, except by showing that we can derive robust decision-theoretic conclusions without appeal to those axioms. If you are inclined to accept the standard axioms, therefore, it is natural to adopt this "add-on" interpretation of the arguments in the last three sections, as supplementing rather than replacing the implications of axiomatic expectationalism.[32]

9 Stochastic dominance as a general decision theory

But I will advance a more ambitious conclusion: that SDTR rather than expectationalism is the true theory of rational choice under uncertainty. In other words, I will argue that rejecting stochastically dominated options is a sufficient as well as a necessary condition for rational choice—or at least, I will argue that this is a position worth exploring. My argument, in short, in this: The major disadvantage of SDTR relative to expectationalism is its apparent failure to place plausible constraints on risk attitudes. On the other hand, SDTR has a number of advantages over expectationalism, some of which we've already seen and others of which will be introduced in this section. These advantages are significant enough that, if stochastic dominance *can* in fact recover constraints on our risk attitudes that are as strong, or nearly as strong, as our decision-theoretic intuitions demand, then it deserves to be treated as a serious competitor to expectationalism.

We have already seen two possible advantages of SDTR: First, its requirements rest on

[32]There is another closely related way in which the results in §5 might be welcome news to orthodox decision theorists: They lend support to the already widely recognized idea that, if we adopt a "grand world" rather than a "small world" framing of decision problems and account for the level of background risk or uncertainty that the grand world context implies for real-world agents, non-standard decision theories like rank-dependent utility (Quiggin, 1982) and risk-weighted expected utility (Buchak, 2013) are likely to end up in close practical agreement with standard decision theory. For existing arguments to this effect, see for instance Quiggin (2003), Thoma and Weisberg (2017) and Thoma (forthcoming). The existing literature tends to assume that the grand world context generates background uncertainty with only bounded support or thin tails, and that the agent's (non-EU-compliant) risk attitude comes from some narrowly constrained class (e.g., a transformation $f : [0,1] \mapsto [0,1]$ on cumulative probabilities of the form $f(x) = x^c$ for some constant c). But when background uncertainty is sufficient to generate stochastic dominance, it constrains the implications of a much wider class of risk attitudes: viz., any risk attitude that satisfies stochastic dominance, including any risk attitude permitted by RDU or REU.

stronger *a priori* foundations than those of expectationalism (if nothing else, because they are strictly weaker). Second, unlike primitive expectationalism, it can both constrain our risk attitudes in ordinary situations and avoid fanaticism in Pascalian situations (without resource to ad hoc devices like excluding "*de minimis* probabilities"). In this section, I will briefly survey some other cases where SDTR outperforms primitive and/or axiomatic expectationalism. Some of these are still problem cases for SDTR, where it is not obvious what stochastic dominance reasoning will imply or where it gives less guidance than we would like. But in all of them, SDTR delivers better answers than expectationalism seems capable of providing.

In this survey, I will mainly ignore the effects of background uncertainty. Incorporating background uncertainty into each of the cases discussed below is (at least) a paper unto itself, and my aim is only to illustrate that there is a broad range of problem cases in which SDTR outperforms expectationalism.[33]

9.1 Infinite payoffs

The simplest problem cases for expectational decision theory are those involving possibilities of infinite positive and/or negative payoffs, as exemplified by Pascal's Wager (Pascal, 1669). In these cases, expectational reasoning delivers either implausible advice or no advice at all. On the other hand, even in the absence of background uncertainty, stochastic dominance can often deliver plausible verdicts. To illustrate, let's consider a few variants of the Wager.

Case 1: Pascal's Wager (Costly)

O_1 $\{\langle 10, 1\rangle\}$

O_2 $\{\langle 9, .99\rangle, \langle +\infty, .01\rangle\}$

Here, expectationalism implies that O_2 is rationally required, while SDTR implies that either option is rationally permissible.

Case 2: Pascal's Wager (Costless)

O_1 $\{\langle 10, 1\rangle\}$

O_2 $\{\langle 10, .99\rangle, \langle +\infty, .01\rangle\}$

Here, both SDTR and expectationalism imply that O_2 is rationally required.

Case 3: Pascal's Wager (Regular)

O_1 $\{\langle 10, .99\rangle, \langle +\infty, .01\rangle\}$

O_2 $\{\langle 10, .9\rangle, \langle +\infty, .1\rangle\}$

Here, expectationalism implies that both options are equally good, and hence rationally permissible. SDTR implies O_2 is rationally required.[34]

[33] As far as I have been able to discover, the presence of background uncertainty only ever favors stochastic dominance (in particular, because background uncertainty can only ever generate new stochastic dominance relations among options as identified by their simple prospects, never undo existing relationships (Pomatto et al., 2018, pp. 2–3)) and only ever disfavors expectationalism (in particular, by generating undefined expectations), though of course this is an imprecise and speculative claim in need of further support.

[34] SDTR thus furnishes a simple reply to the "mixed strategies" objection to Pascal's Wager raised in Hájek (2003), while also allowing that one is not *always* rationally required to accept the Wager.

Case 4: Pascal's Wager (Angry God)

O_1 $\{\langle -\infty, .09\rangle, \langle 9, .9\rangle, \langle +\infty, .01\rangle\}$

O_2 $\{\langle -\infty, .01\rangle, \langle 9, .9\rangle, \langle +\infty, .09\rangle\}$

Here, the expected choiceworthiness of both option is undefined, so insofar as expectationalism yields any practical conclusions at all, it implies that both options are rationally permissible. SDTR implies that O_2 is rationally required.[35]

9.2 The St. Petersburg game

In the St. Petersburg game (Bernoulli, 1738), you are offered the chance to pay some finite price for a lottery ticket that pays +2 with probability $\frac{1}{2}$, +4 with probability $\frac{1}{4}$, +8 with probability $\frac{1}{8}$, and so on. Since the ticket has infinite expected value, expectationalism implausibly implies that you should be willing to pay any finite price for it. Once again, SDTR can do better.

Case 5: St. Petersburg

O_1 $\{\langle 100, 1\rangle\}$

O_2 $\{\langle 2, .5\rangle, \langle 4, .25\rangle, \langle 8, .125\rangle, ...\}$

Here, expectationalism implies that O_2 is rationally required. SDTR implies that both options are rationally permissible.

Case 6: St. Petersburg, St. Petersburg +1

O_1 $\{\langle 100, 1\rangle\}$

O_2 $\{\langle 2, .5\rangle, \langle 4, .25\rangle, \langle 8, .125\rangle, ...\}$

O_3 $\{\langle 3, .5\rangle, \langle 5, .25\rangle, \langle 9, .125\rangle, ...\}$

Here, expectationalism implies that O_2 and O_3 are both rationally permissible, but O_1 is rationally prohibited. SDTR implies that O_1 and O_3 are both rationally permissible, but O_2 is rationally prohibited.[36]

[35] From the results in previous sections, we can infer a few conclusions about infinite payoffs under large-tailed background uncertainty. First, it is always permissible under SDTR to prefer an option that increases the probability of an infinite positive payoff (or decreases the probability of an infinite negative payoff) relative to its alternatives. Second, just as with finite payoffs, large-tailed background uncertainty will sometimes generate new stochastic dominance relations between options whose simple prospects involve infinite payoffs. Among other things, this means we can put a *minimum price* on Pascal's Wager: that is, if accepting the Wager increases the probability of an infinite positive payoff by p, then there is some finite threshold t such that the Wager stochastically dominates any sure payoff less than t. If Pascal's Wager has the simple prospect $\{\langle 0, 1-p\rangle, \langle +\infty, p\rangle\}$, then this threshold can be expressed as $t : \min_x(B(x-t) - (1-p)B(x)) = 0$, where B is the CDF of the agent's background prospect. If, for instance, $p = .01$ and the agent's background prospect is described by a Laplace distribution with a scale parameter of 1000, then the minimum price she is required to pay for Pascal's Wager is slightly greater than 10.

[36] As with Pascal's Wager, large-tailed background uncertainty lets us put a minimum price on the St. Petersburg game, which increases under increasing rescalings of the background prospect. The fact that the St. Petersburg game can stochastically dominate sure-thing payoffs > 2 under background uncertainty follows from the fact that its finite truncations can do so. The fact that it can *fail* to stochastically dominate finite sure-thing payoffs follows from the Lower Bound Theorem: Where O_i is a St. Petersburg gamble and O_j yields a sure payoff of t, as t goes to infinity, $\max_x \Delta_{ij}(x)$ goes to 0, while $\max_x \int_{-\infty}^{\infty} \Delta^-_{ij}(x-y) \times \beta(y)\, dy$ is non-zero and increasing. These facts together imply the existence of a minimum price.

9.3 The Pasadena game

The Pasadena game (Nover and Hájek, 2004) is a gamble in which the probability-weighted sums of both positive and negative simple payoffs diverge to infinity. This means that the expected choiceworthiness of the gamble is not infinite but undefined.[37] In the original version of the game, we toss a fair coin until it lands heads, and receive a payoff of $(-1)^{n-1} \times \frac{2^n}{n}$, where n is the number of flips.

We can say more or less the same things about the Pasadena game as we said about the St. Petersburg game.

Case 7: Pasadena

O_1 $\{\langle 100, 1\rangle\}$

O_2 $\{\langle 2, .5\rangle, \langle -2, .25\rangle, \langle \frac{8}{3}, .125\rangle, -4, .0625\rangle, ...\}$

Here, SDTR and expectationalism agree: The expectation of O_2 is undefined, and so incomparable with the expectation of O_1, so expectationalism implies that both options are permissible. SDTR straightforwardly implies that both options are permissible, since neither is stochastically dominant.

But now consider...

Case 8: Pasadena, Altadena

O_1 $\{\langle 100, 1\rangle\}$

O_2 $\{\langle 2, .5\rangle, \langle -2, .25\rangle, \langle \frac{8}{3}, .125\rangle, -4, .0625\rangle, ...\}$

O_3 $\{\langle 3, .5\rangle, \langle -1, .25\rangle, \langle \frac{11}{3}, .125\rangle, -3, .0625\rangle, ...\}$

Here, since both O_2 and O_3 have undefined expectations, expectationalism implies that neither of them is comparable with O_1, and all three options are rationally permissible. SDTR, on the other hand, yields the intuitively correct verdict that O_1 and O_3 are rationally permissible but O_2 is not.[38]

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[37] Strictly, the expectation is given by an infinite series of probability-weighted payoffs whose sum can conditionally converge to any finite or infinite value, depending on how the terms are ordered. Here I assume that simple payoffs have no privileged ordering, so expected choiceworthiness is defined only when their probability-weighted sum converges absolutely. For discussion of possible extensions of expectational decision theory to handle cases like the Pasadena game, see for instance Easwaran (2008), Colyvan (2008), Bartha (2016), Colyvan and Hájek (2016) and Lauwers and Vallentyne (2016).

[38] Lauwers and Vallentyne (2016) object to stochastic dominance reasoning in the context of gambles with undefined expectations like St. Petersburg and Pasadena. They describe a case involving two St. Petersburg lotteries, $SP1$ and $SP2$, with anti-correlated payoffs (each giving its minimum payoff in exactly those states where the other does not), along with a slightly improved St. Petersbury lottery $W+$. Although $W+$ stochastically dominates both $SP1$ and $SP2$, the lottery $\frac{SP1+SP2}{2}$ (which yields the average of $SP1$'s and $SP2$'s payoff in each state) *statewise* dominates (and hence stochastically dominates) $W+$. As I understand their argument, Lauwers and Vallentyne take it as an objection to stochastic dominance reasoning that there can be triples of options $O_{i/j/k}$ such that O_k stochastically dominates O_i and O_j, but $\frac{O_i+O_j}{2}$ stochastically dominates O_k (which implies that $\frac{O_i+O_j}{2}$ stochastically dominates O_i and O_j): It seems implausible that the average of two prospects can be strictly better than both the prospects it averages.

As far as I can see, though, this is simply a case of hasty generalization from finite to infinite cases. The real lesson of Lauwers and Vallentyne's case is that, when two options have infinite expectations, averaging their payoffs *can* result in an improvement over both options. This is wholly plausible when we consider the result: $SP1$ and $SP2$ each have a simple prospect of $\{\langle 2, .5\rangle, \langle 4, .25\rangle, \langle 8, .125\rangle, ...\}$, whereas $\frac{SP1+SP2}{2}$ has a simple prospect of $\{\langle 3, .5\rangle, \langle 5, .25\rangle, \langle 9, .125\rangle, ...\}$. The result of averaging the two anti-correlated St. Petersburg lotteries, in other words, is St. Petersburg +1. As long as we accept that this is an improvement over St. Petersburg, Lauwers and Valletyne's case is not a reason to question stochastic dominance.

9.4 Ordinality and lexicality

Philosophers have recently begun paying attention to decision-theoretic questions that arise when an agent is uncertain not only about the empirical state of the world but also about basic normative principles.[39] As has been noted in this literature (e.g. by Sepielli (2010) and MacAskill (2016)), a major difficulty for extending standard expectational decision theory to this "metanormative" context is that some normative theories appear to give only ordinal rankings of options, which cannot be multiplied by probabilities to compute expectations and which expectational reasoning is therefore unable to handle. This has led to the suggestion (e.g. in MacAskill (2014)) that fundamentally different decision procedures may be needed to handle different categories of normative theory. Stochastic dominance reasoning, however, can handle both ordinal and cardinal contexts, and may therefore offer a more unified theory of rational choice than expectationalism, given the existence of merely-ordinal normative theories. As a simple illustration, consider the following case, where Roman numerals represent ordinal ranks, with larger numerals representing greater degrees of choiceworthiness.

Case 9: Ordinal Risk

O_1 $\{\langle i, .3\rangle, \langle ii, .7\rangle\}$

O_2 $\{\langle ii, .7\rangle, \langle iii, .3\rangle\}$

Since the payoffs have only ordinal values, the expected choiceworthiness of both options is of course undefined, so expectationalism implies that both options are rationally permissible. SDTR correctly implies that O_2 is rationally required.

Another worry in the literature on normative uncertainty is that some normative theories rank options *lexically*, either regarding certain categories of action as absolutely required or prohibited (e.g., lying or intentionally killing the innocent), or regarding certain categories of normative consideration as taking absolute precedence over others (e.g., the welfare of the worse off over the welfare of the better off). I won't attempt to say how these cases should be represented, but at least the simplest such cases (involving straightforwardly absolutist theories) have the same structure as the "infinite payoff" cases described in §9.1, and so seem to favor SDTR over expectationalism.[40]

9.5 Incomparability and incompleteness

Another kind of problem case for expectationalism involves *incompleteness*. Normative incompleteness can take many forms, e.g., an incomplete ordinal ranking of options, total incomparability between different dimensions of normative evaluation (e.g., morality vs. prudence), or rough/imprecise comparability between different categories of goods that gives rise to phenomena like "parity" (Chang, 2002). As with ordinality and lexicality, the decision-theoretic problems associated with incompleteness are especially acute when we account for normative uncertainty: As MacAskill (2013) points out, an agent who has *any* positive credence in theories that posit incomparability between the possible payoffs of her options is likely to find that the expected choiceworthiness of those options is undefined.

Once again, however, stochastic dominance can recover intuitive verdicts in cases of incomparability that expectational reasoning cannot. For instance, as Bader (2018) points

[39] For a survey of this literature, see Bykvist (2017).

[40] For more on stochastic dominance reasoning in the context of uncertainty among merely-ordinal and/or lexical normative theories, see Tarsney (2018) and Aboodi (unpublished).

out, stochastic dominance gives the right result in the "opaque sweetening" case introduced by Hare (2010). Here, a and b represent incomparable payoffs, and a^+ and b^+ are improved versions of those payoffs, such that a^+ is preferable to a but incomparable to b and b^+ (and likewise b^+ is preferable to b but incomparable with a and a^+).

Case 10: Opaque Sweetening

O_1 $\{\langle a, .5\rangle, \langle b, .5\rangle\}$

O_2 $\{\langle a^+, .5\rangle, \langle b^+, .5\rangle\}$

Once again, the expected choiceworthiness of both options is undefined, so expectationalism implies that both options are rationally permissible. SDTR more plausibly implies that O_2 is rationally required.

9.6 Infinite worlds

As I admitted in §6, the possibility that the world is infinite, and contains infinitely much positive and negative value *regardless* of our choices, complicates my central line of argument. Generalizing the result from §5 to the infinitary context requires a satisfactory axiology for infinite worlds, which we don't yet have. But at least on face, SDTR seems much better equipped than expectationalism to handle infinite worlds. The simplest axiological representation of infinite worlds is given by the extended real number line (the reals, plus special elements ∞ and $-\infty$, ordered as you would expect). This is the worst case for consequentialist ethical reasoning, since it implies that no finite difference we can make to the world has any axiological effect. Nonetheless, even under this gloomy supposition, SDTR is able to provide useful practical guidance, so long as I have non-zero credence that the world is finite. Suppose, for instance, that I am nearly certain that the world is infinite and contains either infinite positive value or infinite negative value, but have some credence that it is finite, such that my actions can make an axiological difference.

Case 11: Heaven or Hell

O_1 $\{\langle -\infty, .45\rangle, \langle 10, .1\rangle, \langle +\infty, .45\rangle\}$

O_2 $\{\langle -\infty, .45\rangle, \langle 11, .1\rangle, \langle +\infty, .45\rangle\}$

Here, the expected choiceworthiness of both options is undefined, so expectationalism implies that both options are rationally permissible, while SDTR correctly implies that O_2 is rationally required.

This is just a simple illustration of a broader point: If O_i and O_j each carry the same probabilities of infinite positive and infinite negative payoffs, then O_i stochastically dominates O_j just in case its *finite* prospect is stochastically dominant. Thus, if we can't change the probabilities of infinite payoffs, SDTR (unlike expectationalism) allows us to simply ignore the infinite possibilities and condition our choice on the assumption of a finite payoff. In this way at least, the positive features of SDTR under background uncertainty established in §5 transfer straightforwardly to the infinite context.

Things get slightly tricker when we consider the more realistic possibility that the world, being infinite, contains infinitely much of both positive *and* negative value. Here it is not only the expectation but the cardinal value itself that is undefined. However, if we are willing to treat $\infty - \infty$ as a special degree of value, albeit one that is incomparable with any other finite or infinite degree of value, then the same conclusions will hold:

Case 12: Heaven + Hell

O_1 $\{\langle -\infty, .05\rangle, \langle 10, .1\rangle, \langle +\infty, .05\rangle, \langle \infty - \infty, .8\rangle\}$

O_2 $\{\langle -\infty, .05\rangle, \langle 11, .1\rangle, \langle +\infty, .05\rangle, \langle \infty - \infty, .8\rangle\}$

Here again, expectationalism is silent, while SDTR implies that O_2 is rationally required. Given that $-\infty < 10 < 11 < \infty$ and $\infty - \infty$ is incomparable with all four of these values, it is still the case that, for any possible payoff, O_2 offers at least an equal (and for some payoff, a strictly greater) probability of payoff at least that desirable.[41]

Of course, the extended real number line gives a supremely unsatisfying account of the axiology of infinite worlds, and much ink has been spilled trying to do better (see note 30). I won't try to survey these accounts or describe how stochastic dominance might interact with each of them. But I will point out that, if the correct axiology allows us to make ordinal comparisons between infinite worlds, then SDTR can derive practical conclusions from uncertainty over those ordinal values. And if the correct axiology lets us make ordinal but not cardinal comparisons (between some or all pairs of infinite worlds), then SDTR is here too at an advantage over expectationalism.[42]

Consider, for instance, a modified version of the ordinal case from §9.4, this time with Roman numeral subscripts used to represent better and worse ordinal ranks assigned to infinite worlds.

Case 13: Ordinal Heaven + Hell

O_1 $\{\langle 10, .15\rangle, \langle 15, .35\rangle, \langle (\infty - \infty)_i, .15\rangle, \langle (\infty - \infty)_{ii}, .35\rangle\}$

O_2 $\{\langle 15, .35\rangle, \langle 20, .15\rangle, \langle (\infty - \infty)_{ii}, .35\rangle, \langle (\infty - \infty)_{iii}, .15\rangle\}$

Once again, expectationalism is silent, while SDTR correctly implies that O_2 is rationally required.

10 Conclusion

At least for agents who give normative weight to aggregative consequentialist considerations, stochastic dominance can effectively constrain risk attitudes, recovering many of the plausible implications of expectational reasoning in a novel and unexpected way, while potentially avoiding the threat of Pascalian fanaticism. Stochastic dominance reasoning also handles a range of problem cases better than expectational reasoning. And it rests on stronger *a priori* foundations. These facts together, I have argued, put SDTR in the running as a general theory of rational choice under uncertainty.

[41] One might be tempted to think that we should treat the probability assigned to $\infty - \infty$ like pure Knightian uncertainty over the whole extended real number line, in which case we could not say for instance that O_2 offers a greater probability of a payoff at least as good as 11. But this would be a mistake: I am not uncertain whether $\infty - \infty$ is greater than, less than, or equal to 11. Rather, I am certain that the two values are incomparable.

[42] Cardinal comparisons have been treated as a desideratum in the infinite ethics literature largely in order to accommodate expectational decision theory (e.g. in Bostrom (2011, pp. 21–22) and Arntzenius (2014, p. 37)). If the correct decision theory does not require cardinality, therefore, this might make it easier to find a satisfactory axiology for infinite worlds.

A Proofs of theorems

Theorem 1 (Upper Bound Theorem)**.** *For any options O_i, O_j and background prospect β,*

$$\frac{\int_{-\infty}^{\infty} \Delta_{ij}^{+}(x)\,dx}{\int_{-\infty}^{\infty} \Delta_{ij}^{-}(x)\,dx} > rate(O_i, O_j, \beta) \to O_i \succ_{sd} O_j$$

Proof. Consider an arbitrary payoff p. Given a background payoff of x, option O_i yields a payoff $\geq p$ iff it yields a simple payoff $\geq p - x$. Therefore, where $Pr(O_i \geq x)$ is the probability that O_i yields a simple payoff $\geq x$, the total probability that O_i yields an overall payoff $\geq p$ is given by

$$\int_{-\infty}^{\infty} \beta(x) \times Pr(O_i \geq p - x)\,dx.$$

Therefore the *difference* between the probability that O_i yields a payoff $\geq p$ and the probability that O_j yields a payoff $\geq p$ is given by

$$\begin{aligned}
\bar{B}_i(x) - \bar{B}_j(x) &= \int_{-\infty}^{\infty} \beta(x) \times Pr(O_i \geq p - x)\,dx - \int_{-\infty}^{\infty} \beta(x) \times Pr(O_j \geq p - x)\,dx \\
&= \int_{-\infty}^{\infty} \beta(x) \times (Pr(O_i \geq p - x) - Pr(O_j \geq p - x))\,dx \\
&= \int_{-\infty}^{\infty} \beta(x) \times \Delta_{ij}(p - x)\,dx \\
&= \int_{-\infty}^{\infty} \beta(x) \times \Delta_{ij}^{+}(p - x)\,dx - \int_{-\infty}^{\infty} \beta(x) \times \Delta_{ij}^{-}(p - x)\,dx
\end{aligned}$$

From the definition of $rate(O_i, O_j, \beta)$, the value of β cannot vary over the support of Δ_{ij} by more than a factor of $rate(O_i, O_j, \beta)$. So we can conclude that:

$$\bar{B}_i(p) - \bar{B}_j(p) \geq \int_{-\infty}^{\infty} \Delta_{ij}^{+}(p - x)\,dx - \left(rate(O_i, O_j, \beta) \times \int_{-\infty}^{\infty} \Delta_{ij}^{-}(p - x)\,dx \right)$$

This implies:

$$\frac{\int_{-\infty}^{\infty} \Delta_{ij}^{+}(x)\,dx}{\int_{-\infty}^{\infty} \Delta_{ij}^{-}(x)\,dx} > rate(O_i, O_j, \beta) \to \bar{B}_i(p) - \bar{B}_j(p) > 0$$

And since this is true for arbitrary p, we can conclude that

$$\frac{\int_{-\infty}^{\infty} \Delta_{ij}^{+}(x)\,dx}{\int_{-\infty}^{\infty} \Delta_{ij}^{-}(x)\,dx} > rate(O_i, O_j, \beta) \to \forall x(\bar{B}_i(x) - \bar{B}_j(x) > 0),$$

which implies

$$\frac{\int_{-\infty}^{\infty} \Delta_{ij}^{+}(y)\,dy}{\int_{-\infty}^{\infty} \Delta_{ij}^{-}(y)\,dy} > rate(O_i, O_j, \beta) \to O_i \succ_{sd} O_j$$

□

Theorem 2 (Lower Bound Theorem)**.** *For any options O_i, O_j and background prospect β,*

$$O_i \succ_{sd} O_j \to \max_x \Delta_{ij}(x) > \max_x \int_{-\infty}^{\infty} \Delta_{ij}^{-}(x-y) \times \beta(y)\, dy$$

Proof. From the proof of the Upper Bound Theorem, we know that $O_i \succ_{sd} O_j$ only if

$$\forall x \left(\int_{-\infty}^{\infty} \beta(y) \times \Delta_{ij}^{+}(x-y)\, dy \geq \int_{-\infty}^{\infty} \beta(y) \times \Delta_{ij}^{-}(x-y)\, dy \right)$$

Suppose that Δ_{ij} was a constant function, with $\Delta_{ij}(x) = k$ for all x. Then, since $\int_{-\infty}^{\infty} \beta(y)\, dy = 1$, it would follow that $\int_{-\infty}^{\infty} \beta(y) \times \Delta_{ij}^{+}(x-y)\, dy = k$. From this we can infer that

$$\int_{-\infty}^{\infty} \beta(y) \times \Delta_{ij}^{+}(x-y)\, dy \leq \max_x \Delta_{ij}(x).$$

And in fact, given that the simple prospects of O_i and O_j (i) are non-identical (a necessary condition for stochastic dominance) and (ii) involve only finite payoffs (as stipulated in §2), Δ_{ij} cannot be constant, so the inequality is strict: $\int_{-\infty}^{\infty} \beta(y) \times \Delta_{ij}^{+}(x-y)\, dy < \max_x \Delta_{ij}(x)$.

From this it follows that:

$$O_i \succ_{sd} O_j \to \forall z \left(\max_x \Delta_{ij}(x) > \int_{-\infty}^{\infty} \beta(y) \times \Delta_{ij}^{-}(z-y)\, dy \right)$$

Or in other words:

$$O_i \succ_{sd} O_j \to \max_x \Delta_{ij}(x) > \max_x \int_{-\infty}^{\infty} \Delta_{ij}^{-}(x-y) \times \beta(y)\, dy$$

□

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