Against Preservation

Matthew Mandelkern* and Justin Khoo†

July 22, 2018

Penultimate draft; to appear in *Analysis*

Abstract

Richard Bradley offers a quick and convincing argument that no Boolean semantic theory for conditionals can validate a very natural principle concerning the relationship between credences and conditionals. We argue that Bradley's principle, *Preservation*, is, in fact, invalid; its appeal arises from the validity of a nearby, but distinct, principle, which we call *Local Preservation*, and which Boolean semantic theories can non-trivially validate.

1 Introduction

Bradley (2000) shows that no Boolean semantic theory for conditionals can validate the following natural *Preservation* condition on rational credences towards conditionals:[1] roughly, that if A is possibly true and C not possibly true, then the conditional $\ulcorner A \to C \urcorner$ is not possibly true. *Pace* Bradley, who takes this result to be a problem for standard semantic approaches to the conditional,

*All Souls College, Oxford, OX1 4AL, United Kingdom, matthew.mandelkern@all-souls.ox.ac.uk
†MIT, Cambridge, MA 02141, United States, jkhoo@mit.edu

[1] A Boolean semantic theory is any semantic theory which assigns to sentences denotations which are elements of a Boolean algebra; most theories of the conditional do just this, typically by identifying the denotations of conditionals with the elements of a set algebra whose ur-elements are possible worlds. In subsequent work, Bradley defends a non-Boolean semantics for conditionals that validates *Preservation* (Bradley, 2002). (Bradley (2012) defends a Boolean aproach that validates *Preservation* restricted to a natural subset of probability functions, a move that in some ways parallels our proposal concerning *Local Preservation*.) A broad range of other non-Boolean approaches designed to validate *The Thesis* (discussed briefly below) also validate *Preservation*, which is a corollary of *The Thesis*; see e.g. Adams 1975; Edgington 1995. See also Douven 2007; Rumfitt 2013 for other arguments against *Preservation*.

we argue that this preservation condition is invalid, and thus that this result poses no challenge to standard semantic theories—and, conversely, that non-standard approaches which validate *Preservation* are going in the wrong direction. We argue against *Preservation* first on the basis of intuitions about natural language, which we suggest do not actually conform to the predictions of *Preservation*; and, second, on the grounds that *Preservation* on its own has absurd consequences which should be unacceptable to any theorist of the conditional. We argue that one reason *Preservation* may have seemed so plausible is that there is a nearby, but weaker, principle, which we argue *is* valid; we call this principle *Local Preservation*. *Local Preservation* requires only that, for each context, the proposition expressed by $\ulcorner A \to C \urcorner$ at that context obeys the *Preservation* condition for the probability measure determined by that context. Boolean semantic theories of conditionals have no problem validating *Local Preservation*; indeed, we show that one prominent semantic theory, namely that given in Stalnaker 1968, 1975, does just that.

2 Bradley's result

Bradley's argument is simple and striking. We follow Bradley in using capital italics to range over "factual" sentences (sentences that do not contain modals or conditionals), and, like him, we restrict our attention to conditionals with factual sentences as antecedents and consequents. Let '$\to$' abbreviate the indicative conditional 'If…then…'. Let L be a set of sentences and P any probability measure on L, meant to correspond to an agent's credence function.[2] Then for factual sentences A and C such that $\{A, C, \ulcorner A \to C \urcorner\} \subseteq L$, we define the following constraint on P:

Preservation: If $P(A) > 0$ and $P(C) = 0$, then $P(\ulcorner A \to C \urcorner) = 0$.

[2]We follow Bradley here in taking probability measures to be defined over sentences; we assume that the set of sentences meets appropriate formal constraints, i.e. is a σ-algebra. Later we will take the more standard approach of treating probability measures as defined over propositions rather than sentences. Bradley states his result in a more general fashion, letting P fail to be a probability measure provided it still meets certain formal conditions; we present a slightly simplified version which is weaker in ways irrelevant for our purposes. Since Bradley formulates *Preservation* in terms of *sentences*, rather than contents (or sentences relative to contexts), we assume that he is restricting his attention to sentences without context-sensitive terms (notwithstanding his use of pronouns in example sentences, which we will follow here). We can easily restate *Preservation* in terms of contents as follows: for all contexts c and probability measures P, if P makes the content of A at c greater than 0 and the content of C at c equal to 0, then it must make the content of $\ulcorner A \to C \urcorner$ at c equal to 0.

Preservation is *prima facie* a very plausible constraint on rational credence functions. Bradley motivates it like this:

> You cannot... hold that we might go to the beach, but that we certainly won't go swimming and at the same time consider it possible that if we go to the beach we will go swimming! To do so would reveal a misunderstanding of the indicative conditional (or just plain inconsistency). (Bradley, 2000, 220)

Semantically ascending: if you have non-zero credence in 'We go to the beach' and zero credence in 'We will go swimming', it seems incoherent to have non-zero credence in 'If we go to the beach, we will go swimming'. At first blush, this seems obviously right; certainly it would offend common sense if you said: 'There's a chance that we'll go to the beach today; there's no chance that we'll swim there; and yet there's a chance that we'll go swimming if we go to the beach'.

But Bradley observes that there is no non-trivial language L and Boolean semantic theory for indicative conditionals such that *Preservation* is guaranteed to hold for all sentences in L relative to all probability measures over L. By "non-trivial language," Bradley means a language L together with an entailment relation over L such that there are factual sentences $A, C \in L$ such that $\ulcorner A \to C \urcorner \in L$ and neither A nor $\ulcorner A \to C \urcorner$ entails C. Any plausible logic of natural language will be non-trivial in this sense. Now let L be a set of sentences closed under conjunction, disjunction, and negation, and partially ordered by an entailment relation $\vdash$. Assume that *Preservation* holds for all probability measures on L. Then L is trivial. For if L contains factual sentences A and C and conditional $\ulcorner A \to C \urcorner$, with $A \nvdash C$ and $\ulcorner A \to C \urcorner \nvdash C$, then there exists a probability measure P on L such that $P(A) > 0, P(C) = 0$, and $P(\ulcorner A \to C \urcorner) \neq 0$, contrary to *Preservation* (intuitively, this is because only entailment can force every probability measure to assign at least as great probability to one sentence as to another). Thus there is no way to give a Boolean semantics for the indicative conditional which guarantees that *Preservation* holds for all probability functions on that language. In other words, there is no way to encode *Preservation* in the meaning of the indicative conditional if we are working in a Boolean semantic framework.

3 A counterexample

Preservation is *prima facie* plausible, and in light of Bradley's observation that it cannot be non-trivially validated by any Boolean semantic theory of the conditional, it may seem that we have a deep problem for any Boolean semantic theory of the conditional; this, in any case, is Bradley's interpretation. But, on closer examination, we think *Preservation* is in fact invalid. We begin our argument for this conclusion by offering an intuitive counterexample to *Preservation*.

To have a counterexample to *Preservation*, we need a sentence of the form $\ulcorner A \rightarrow B \urcorner$, with A and B factual, such that it is rational to have non-zero credence in A and $\ulcorner A \rightarrow B \urcorner$ and zero credence in B. Here is a case which seems to have just that property:

> **Fundraiser:** It is Thursday night at the company fundraiser. 100 scratch card tickets are being sold; 99 of the tickets grant the ticket-holder Friday off, while one is worthless. Ginger arrives at the fundraiser late, and there is only one ticket left to purchase.
>
> Mark and Jim know all this, and they are wondering whether Ginger will buy the last ticket and whether she will win (i.e., get permission to take Friday off). They know that Ginger would take Friday off if she won. But they also know that the tickets are expensive and so Ginger probably won't buy the last ticket. Jim makes a prediction, saying:
>
> (1) If Ginger buys the last ticket, she will win.
>
> Mark thinks about Jim's claim. Since Mark knows that almost all of the tickets are winning tickets, he concludes that what Jim says is very likely, but not certainly, true.
>
> On Friday, Jim and Mark see that Ginger is at work. They now know that it's not the case that Ginger bought the ticket and won. Two possibilities remain open: (i) Ginger bought the ticket and lost; and (ii) Ginger didn't buy the ticket. Since Mark knows that almost all of the tickets were winners, and since he thought from the start that it was

unlikely Ginger would buy a ticket, he thinks it is much more likely that Ginger simply declined to buy the ticket than that she bought it and lost.

Given what Mark has learned, we now ask: should Mark now think that Jim's claim that (1) is certainly false? We think the answer to this question is *no*: in fact, there is good reason to think that Mark should be slightly less confident in Jim's claim, but overall still confident that it is true. If our answer here is correct, then this provides a counterexample to *Preservation*: Mark may rationally have non-zero credence in the claim that Ginger bought the last ticket, zero credence in the claim that Ginger won the fundraiser, and non-zero credence in Jim's claim in (1).

This is our intuitive judgment about the case, anyways, and the judgment of others we have run the case by. We offer two arguments to bolster this judgment. First, intuitively, the probability of Jim's claim is the same as the probability that the final remaining ticket is a winning ticket, since Jim's claim—that Ginger will win if she buys the last ticket—is, in this context, intuitively equivalent to the claim that the final ticket is a winner. Mark has learned nothing over the course of the scenario that suggests that these probabilities should in fact diverge. And, although Mark's credence that the last ticket is a winner should go down slightly upon learning that the Ginger did not in fact win, he should still think it is overwhelmingly likely that the final ticket is a winner (in the Appendix, we argue for this by assigning numerical probabilities to the relevant claims and showing that this conclusion follows in the probability calculus). So, he should continue to think that Jim's claim is very likely true.

The second argument comes from considerations about foresight and updating. Before Mark learns that Ginger didn't win, it seems that he should think that Jim's claim—that if Ginger buys the last ticket, she will win—is likely true. Now, suppose at this point he is asked, 'But what if Ginger doesn't win?' It seems that Mark should reason as follows: 'In fact, I think it is pretty likely that Ginger won't win, since I think it's quite likely that Ginger won't buy a ticket at all. So, I still think Jim's claim is likely to be true, given that Ginger doesn't win. Thus, I would stand by my judgment that Jim's claim is likely true, even if I learned that Ginger didn't win.' On Friday, when Mark learns that Ginger didn't win, he should then just update his credences according to these commitments, and thus continue to believe that Jim's claim is likely.[3]

4 A triviality result

In this section we provide our second argument against *Preservation*: not only does *Preservation* face counterexamples like the one just sketched, but adopting it *on its own*—whether in a standard Boolean semantic framework or not—leads to absurd results.

Bradley's *Preservation* principle is a corollary of a well-known claim about the probabilities of conditionals, namely that the probability of a conditional equals the corresponding conditional probability of its consequent given its antecedent:[4]

$$\underline{\text{The Thesis}}\text{: } P(\ulcorner A \to C \urcorner) = P(C|A), \text{ if } P(A) > 0.$$

It is easy to see that *Preservation* follows immediately from *The Thesis*. However, it is now known, on the basis of both triviality results and intuitive counterexamples, that *The Thesis* does not hold in full generality.[5] It is worth noting that, since *Preservation* is a corollary of *The Thesis*, our counterexample to *Preservation* is also a further counterexample to *The Thesis*.

More pertinent for present purposes, however, is the question of whether the triviality results which showed that *The Thesis* cannot hold in full generality can be adapted to show the same for

[3]Yet another argument comes from hindsight judgments. Suppose that, after learning Ginger didn't win, Mark is asked whether he stands by his earlier claim. It seems perfectly reasonable for Mark to respond: 'Yes, I still think Jim's claim is likely'. He might go on: 'Before I learned that Ginger didn't win, it was likely that if Ginger had bought the last ticket, she would have won; and this is still likely, since it is still likely that the last ticket was a winning ticket.' One corollary of this point is that *Preservation* does not hold for counterfactuals (nor does the *Local Preservation* principle we propose below). Some have argued that, when it comes to 'will'-conditionals, the indicative/counterfactual distinction does not clearly obtain, and thus one might think that, given that there are independent reasons to think that *Preservation* does not hold for counterfactuals, our example could be dismissed by arguing that (1) is not an indicative conditional at all. But note that we can set up an identical case with a past-oriented version of (1), by having Jim say 'If Ginger bought the last ticket, she won' after the fundraiser is over, but before they come into work the next morning and discover Ginger didn't win. Intuitions about this variant, which is uncontroversially indicative, seem to pattern with our original case. Thanks to Richard Bradley and Paolo Santorio for helpful discussion on these points.

[4]$P(C|A)$ is the conditional probability of C given A. For our purposes, we will define this in the standard way as follows: $P(C|A) =_{def} \frac{P(AC)}{P(A)}$, if $P(A) > 0$.

[5]Something like *The Thesis* was first defended in Ramsey (1931), and has been extensively discussed in e.g. Stalnaker (1970); Adams (1975); Edgington (1995); Douven (2013). Lewis (1976) is responsible for the first in a long series of results showing that adopting *The Thesis* in full generality leads to triviality; see McGee 2000; Kaufmann 2004; Rothschild 2013 for some apparent counterexamples to *The Thesis*.

Preservation. This is a substantive question, since *Preservation* is strictly weaker than *The Thesis*, and so formal arguments which show that the latter holds in general only in trivial cases do not necessarily show the same of the former. But it turns out that we can indeed adapt those arguments to *Preservation*. Here is one such argument, building on Lewis (1976)'s result. By the law of total probability, we know the following to be true for any A and C, assuming $P(C)$ and $P(\overline{C})$ are both non-zero:

$$P(\ulcorner A \to C \urcorner) = P(\ulcorner A \to C \urcorner | C) \cdot P(C) + \underbrace{P(\ulcorner A \to C \urcorner | \overline{C}) \cdot \; P(\overline{C})}_{x}$$

Suppose further that *Preservation* is valid, i.e. holds for all probability functions over our language. Then, since $P(\cdot|\overline{C})$ will be a probability function, it will satisfy *Preservation*. Assume that $P(A|\overline{C}) > 0$. Then, since (by the probability calculus) $P(C|\overline{C}) = 0$, by *Preservation*, $P(\ulcorner A \to C \urcorner | \overline{C})$ will have to be 0; so the right-hand term x in the equation above is 0. Then we get $P(\ulcorner A \to C \urcorner) = P(\ulcorner A \to C \urcorner | C) \cdot P(C) = P((\ulcorner A \to C) \wedge C \urcorner)$. Thus if *Preservation* holds in general, then, whenever your initial credence in a conditional's consequent and the consequent's negation are both non-zero, and the probability of the conditional's antecedent will remain non-zero if you learn the negation of the consequent, you must hold that the probability of the conditional as a whole is equal to the probability of the conjunction of the conditional and its consequent.

But this is absurd. For instance, in **Fundraiser** it is clear that, before Ginger decides whether to buy the ticket, you should think the probability of (1) is high, since the final ticket is likely a winner. But the conjunction of (1) with its consequent is much less probable, since you think it is quite unlikely Ginger will buy a ticket at all. Moreover, your prior credences in the consequent (that Ginger will win) and its negation are non-zero, and your credence in the conditional's antecedent (that Ginger buys the ticket) will remain non-zero if you learn the negation of the consequent (that she didn't win).

Not only is this corollary absurd, but it also leads to a kind of triviality very much like the kind Bradley himself identifies, as an anonymous referee for this journal helpfully points out to us. By the probability calculus, $P(\ulcorner(A \to C) \wedge C\urcorner) = P(\ulcorner A \to C\urcorner) \cdot P(C|\ulcorner A \to C\urcorner)$. So then if $P(\ulcorner(A \to C) \wedge C\urcorner) = P(\ulcorner A \to C\urcorner)$, it follows that $P(C|\ulcorner A \to C\urcorner) = 1$: in other words, if we come to learn $\ulcorner A \to C\urcorner$, we must thereby come to learn C! This is another obviously absurd corollary of *Preservation*. And not only is this result obviously false, it in fact shows that *Preservation on its own* comes very close to rendering our language trivial *in Bradley's own sense*.[6]

Note that the present result is very different from Bradley's triviality result. Bradley's triviality results showed that the *combination* of *Preservation* with the assumption that conditionals have a Boolean semantics leads to triviality. Bradley's result could thus be taken to tell against Boolean approaches to the semantics of conditionals (as Bradley took it) *or* against *Preservation*. By contrast, the present considerations show that *Preservation on its own*, given basic features of the probability calculus, leads to absurd consequences, and indeed nearly to triviality in Bradley's own sense. This means that we should not respond to this result by validating *Preservation* and rejecting the assumption that conditionals have a Boolean semantics: rather, the present considerations show that *Preservation* itself cannot be valid.[7]

[6] It doesn't *quite* render the language trivial in Bradley's sense—which would require that A or $\ulcorner A \to C\urcorner$ always entails C—since our result only holds when $P(A|\overline{C}) > 0$ and $P(C) > 0$; and, in infinite models, $P(C|\ulcorner A \to C\urcorner)$ can be 1 without $\ulcorner A \to C\urcorner$ entailing C, for reasons having to do with regularity. But we can formulate this result as rendering our language trivial in a manner which is the natural probabilistic parallel to Bradley's notion: as long as $P(C) > 0$, $P(\overline{C}) > 0$, and $P(A) > 0$, we have that $P(C|A) = 1$ or $P(C|\ulcorner A \to C\urcorner) = 1$, i.e. that C is probabilistically guaranteed by either A or by $\ulcorner A \to C\urcorner$.

[7] Another, more radical, response, in line with the non-propositional tradition in the theory of conditionals (e.g. Adams 1975; Edgington 1995; Bennett 2003), would be to argue that the probability calculus is not an appropriate model for rational credence towards conditionals, and in particular that the law of total probability does not hold for conditionals. On this view, strictly speaking, conditionals do not have classical probabilities at all (as Lewis (1976) points out). We are not very sympathetic to this response. First, we don't see the motivation to give up the probability calculus in order to hold onto a principle which, in light of the evidence given in the last section, is not intuitively valid. Second, it is hard to see how this response can make sense of a variety of natural judgments about conditionals. In particular, it is not obvious how it could explain comparative probability judgments between conditionals and non-conditionals, such as:

(i) It's more likely that the die landed on an odd than that it landed on an even if it landed on a prime.

Thanks to Ian Rumfitt and Paolo Santorio for helpful discussion on these issues.

5 *Local Preservation*

On the basis of these considerations, we conclude that *Preservation* is in fact invalid. This deprives Bradley's result of its revisionary force. His result shows that no non-trivial Boolean semantic theory for the conditional can guarantee *Preservation*. But this turns out to be all to the good for standard truth-conditional approaches to the conditional: *Preservation* is invalid, and so our theory of conditionals had better not validate it.

This, however, leaves something unexplained: why did *Preservation* seem so plausible in the first place? On reflection, there is something special about our counter-example to *Preservation*: it involves the evaluation of a conditional *asserted at a different time, in light of different information*, than at the time of evaluation. Indeed, all the counterexamples to *Preservation* which we have been able to construct have just this form.

We propose to make sense of this situation by holding that the reason that *Preservation* seemed so plausible at first sight is because a different principle in the neighborhood is in fact valid. In order to state the principle, we will switch from thinking about probability measures over sentences to probability measures over *contents*, which for our purposes we can treat as possible-worlds propositions; where A is a sentence, $[\![A]\!]^c$ is the proposition A expresses relative to a context c, where a context is a centered world (a triple of a world, time, and location). The same sentence can express different contents in different contexts. This shift in formalism allows us to distinguish *Preservation* from a different principle which we call *Local Preservation*, and which, by contrast to *Preservation*, we maintain is valid. Given a context c, let P_c be the probability measure associated with the context: the probability measure which represents in some way what is commonly accepted in that context, and thus which assigns non-zero measure to all and only the elements of the context's common ground.[8] We hold that the correct principle in the neighborhood of *Preservation* is the following limited principle:

[8]This will not be plausible in the infinite case for reasons having to do with regularity, so we restrict attention to cases where the common ground is finite. Plausibly there are many probability measures associated with a given context, but it is obvious how to generalize this condition to that case, so we stick with a uniqueness assumption for simplicity.

Local Preservation: If $P_c(\llbracket A \rrbracket^c) > 0$ and $P_c(\llbracket B \rrbracket^c) = 0$, then $P_c(\llbracket \ulcorner A \rightarrow B \urcorner \rrbracket^c) = 0$.

This states, essentially, that *Preservation* always holds *as viewed from the context of assertion*, but leaves it open that it fails as viewed from other contexts. In other words, whenever someone asserts $\ulcorner A \rightarrow B \urcorner$ in a given context c, then, if the probability function associated with *that* context makes the content of the antecedent greater than zero and the content of the consequent zero, it must make the content of the conditional zero. But *Local Preservation* says nothing about how we must evaluate conditionals *in different contexts from those in which they are asserted*; in particular, it leaves it open that we sometimes assign non-zero credence to the content of the antecedent of a conditional, zero credence to the content of the consequent of the conditional, and non-zero credence to the content of the whole conditional, *provided we are doing so from the perspective of a different context than the context of assertion*.

We thus believe that *Local Preservation* makes good sense of the intuitions used to motivate *Preservation*, while remaining consistent with the counterexample to *Preservation* provided above: *Preservation* is always valid when we are thinking about a conditional in the same context in which it was asserted; it is not always valid when we are thinking about a conditional from a different context.

6 Validating *Local Preservation*

At this point, the essential question to ask is whether Bradley's triviality result can be generalized to show that only trivial languages can validate *Local Preservation*, in which case we would not have done much to vindicate truth-conditional approaches to the conditional.

Fortunately, it cannot. Bradley's result follows from the fact that, when A fails to entail B, there is a probability function which assigns A non-zero value and B zero, which entails that there is no non-trivial Boolean semantic theory for the conditional which validates *Preservation*. But there is no problem validating *Local Preservation*; to do this, we need only ensure that the content that a conditional expresses in a given context is connected in a suitable way to the probability measure associated with that context. And this is straightforward to do. In fact, one of the leading theories of indicative conditionals already does just this, namely the theory of Stalnaker 1968, 1975. Stalnaker's theory runs as follows. Let $f_c(\varphi, w)$ be a selection function provided by the context c which takes a proposition φ and world w to a φ-world w' (intuitively, the closest φ-world to w), with f_c meeting the following conditions, for all contexts and all propositions φ, ψ and worlds w (CG_c is the common ground of c—the set of worlds compatible with the common assumptions in the conversation):[9]

(i) *Success*: $f_c(\varphi, w) \in \varphi$.

(ii) *CSO*: If $f_c(\varphi, w) \in \psi$ and $f_c(\psi, w) \in \varphi$, then $f_c(\varphi, w) = f_c(\psi, w)$.

(iii) *Strong Centering*: If $w \in \varphi$, then $f_c(\varphi, w) = w$.

(iv) *Indicative Constraint*: If $\text{CG}_c \cap \varphi \neq \varnothing$ and $w \in \text{CG}_c$, then $f_c(\varphi, w) \in \text{CG}_c$.

Then we say that an indicative conditional $\ulcorner A \to C \urcorner$, asserted at c, expresses the proposition $\{w : f_c([\![A]\!]^c, w) \in [\![C]\!]^c\}$: in other words, $\ulcorner A \to C \urcorner$ is true at a world just in case the closest $[\![A]\!]^c$-world is a $[\![C]\!]^c$-world. (Note that *Indicative Constraint* only applies to indicative conditionals. This is crucial, since this constraint is essential to ensuring that *Local Preservation* is valid for indicative conditionals; whereas *Local Preservation* is not valid for counterfactuals, for the reasons noted in Footnote 3.)

These conditions suffice to validate *Local Preservation*. Suppose $P_c([\![A]\!]^c) > 0$ and $P_c([\![C]\!]^c) = 0$. Then it follows (from the assumption that P_c assigns non-zero measure to all and only the elements of CG_c) that $[\![A]\!]^c \cap \text{CG}_c \neq \varnothing$, and thus that the closest $[\![A]\!]^c$-world to any world in CG_c will also be in CG_c (by *Indicative Constraint*); since $P_c([\![C]\!]^c) = 0$, there are no $[\![C]\!]^c$-worlds in CG_c (again, by our assumption about the relation between P_c and CG_c), and thus for any world

[9] The 'if. . . then. . . ' in these conditions is intended to be interpreted as the material conditional. We include an absurd world λ which makes every sentence true so that *Success* can be satisfied even when φ is inconsistent, and stipulate that $f_c(\varphi, w) = \lambda$ only if φ is inconsistent.

in the common ground, the closest $[\![A]\!]^c$-world to that world cannot be a $[\![C]\!]^c$-world; thus by our truth-conditions, $\ulcorner A \to C \urcorner$ will be false at every world in CG_c, and thus $P_c([\![\ulcorner A \to C \urcorner]\!]^c) = 0$.

Moreover, this theory of the conditional is non-trivial, in Bradley's sense. To show this it suffices to construct a partial model of Stalnaker's semantics which is non-trivial. Let $\text{CG}_c = \{w, w', w''\}$ with $w \in [\![A \wedge \neg C]\!]^c, w' \in [\![A \wedge C]\!]^c$, and $w'' \in [\![\neg A \wedge \neg C]\!]^c$. Let $f_c([\![A]\!]^c, w'') = w'$. Let entailment be the subset relation. Then $[\![A]\!]^c$ fails to entail $[\![C]\!]^c$ (since the former is true at w and the latter is not); and $[\![\ulcorner A \to C \urcorner]\!]^c$ fails to entail $[\![C]\!]^c$ (since the former is true at w'' and the latter is not). Thus there are non-trivial truth-conditional theories of the conditional which validate *Local Preservation*; and Stalnaker's is one of them.

Furthermore, it is clear that *Local Preservation* does not lead to the absurd result discussed in §4: to obtain that result, we had to assume that *Preservation* held not only for a starting probability measure, but also for the probability measure which resulted from conditionalizing on a certain content; but this does not follow from *Local Preservation*.

We conclude this discussion by working through the **Fundraiser** case to show in more detail how this semantics predicts exceptions to *Preservation*, even while validating *Local Preservation*. We'll make one key assumption about the selection function made salient by the initial context in **Fundraiser**, namely that, for any A and w, the closest $[\![A]\!]^c$-world from w will agree with w on whether the last ticket in w is a winner (whenever this is possible given the constraints above). Now consider again (1), which we'll abbreviate $\ulcorner B \to W \urcorner$:

(1) If Ginger buys the last ticket, she will win.

According to our semantics, in its initial context of assertion c_1, (1) expresses the proposition $[\![\ulcorner B \to W \urcorner]\!]^{c_1} = \{w : f_{c_1}([\![B]\!]^{c_1}, w) \in [\![W]\!]^{c_1}\}$. This proposition is true at any world where the last ticket is a winner and Ginger buys the ticket (by *Strong Centering*). At any world where the last ticket is a winner and Ginger *doesn't* buy the ticket, given our assumption that the selected world matches the ticket status whenever possible, the closest world where Ginger does buy the ticket is one where she wins (note that the common ground of c_1 still contains worlds in which Ginger wins, so this doesn't violate *Indicative Constraint*), and so the conditional is true. By *Strong Centering*, the conditional is false at any world where the last ticket is a loser and Ginger buys it; and, by our assumption about the selection function, the conditional is likewise false at any world where the last ticket is a loser and Ginger doesn't buy it. So—in accord with the intuitions we elicited at the outset—(1), *as asserted in its original context*, expresses the same proposition as 'The last ticket is a winner' expresses.

This is helpful to see for two reasons. First, this shows that our theory matches the intuitions elicited in our discussion of (1) at the outset, where we noted that these are intuitively equivalent in the context in question. Second, it helps explain what happens when, from the posterior context c_2 (in which we have learned that Ginger did not win), we evaluate what (1) expressed *in context* c_1. Intuitively, we should still assign non-zero credence to this content—in violation of *Preservation*—because, in the posterior context, we still assign non-zero credence to the proposition that the last ticket was a winner, which we have just shown is the same proposition as $[\![\ulcorner B \rightarrow W \urcorner]\!]^{c_1}$.

By contrast, things are different if we consider, from c_2, not the content which (1) expressed as asserted in c_1, but rather the content which the corresponding past conditional (2) expresses as asserted in c_2:

(2) If Ginger bought the last ticket, she won.

Since we validate *Local Preservation*, (2), as asserted in this posterior context, has zero probability. We have already shown that this is true in general. In this case, this follows because there are no worlds in the common ground of c_2 where the last ticket is a winner and Ginger bought it; there are only worlds where (a) the ticket was a loser or (b) the ticket was a winner and Ginger didn't buy it. In (a)-worlds, (2) will be false for the same reasons as in the initial context. But what about in (b)-worlds? In those worlds, in the initial context, we said that the closest world where Ginger did buy the ticket was one where the status of the ticket remained the same, and thus where she won; and so that the conditional was true at those worlds. But, thanks to *Indicative Constraint*, this does not hold for (2) (as uttered in c_2) *since there are no worlds in the common ground of* c_2 *where*

Ginger bought the ticket and won. Instead, the selection function must, by *Indicative Constraint* plus *Success*, take any (b)-world to an (a)-world where Ginger bought the ticket—and thus lost. So, as uttered in c_2, (2) expresses a proposition which is false throughout the common ground of c_2.

This discussion brings out the crucial role of *Indicative Constraint* in validating *Local Preservation*. *Indicative Constraint* ensures that, when we get new information, what proposition a given conditional expresses can change. This ensures that if we have zero credence in the consequent and non-zero credence in the antecedent in that context, we will have zero credence in what the conditional expresses *in that context*; even though we may still have non-zero credence in what the conditional expressed *in a different context*.

7 Conclusion

Bradley's *Preservation* condition looks universally valid at first sight. But the example we have given, and the absurd corollary we have pointed to, show that it is only valid in a limited case: when we are considering a conditional *in the context in which it is asserted*. And, while no non-trivial Boolean theory of the conditional validates *Preservation*, there *are* non-trivial Boolean theories, like Stalnaker's, which validate *Local Preservation*. *Pace* Bradley and some of his followers, considerations about *Preservation* thus do not tell against Boolean approaches to the conditional. On the contrary, they tell *against* any theory which validates *Preservation* full stop, and in favor of any theory (including Boolean approaches like Stalnaker's) which invalidates *Preservation* in general but still validates *Local Preservation*.[10]

Appendix

Here we add numerical probabilities to **Fundraiser**. Let:

- $T =$ 'The last ticket is a winning ticket'

[10]Many thanks to an anonymous editor and referee for this journal, and to Richard Bradley, Ian Rumfitt, and Paolo Santorio for very helpful comments.

- $B =$ 'Ginger buys a ticket'
- $W =$ 'Ginger wins'

Mark's credences, prior to learning that Ginger did not win, are given by the function P:

- $P(B) = 0.1$, since Mark thinks it unlikely that Ginger will buy a ticket; and since the likelihood of winning is independent of buying, we have: $P(B) = P(B|T) = P(B|\overline{T}) = 0.1$
- $P(T) = 0.99$
- $P(\overline{T}) = 1 - P(T) = 0.01$
- $P(\ulcorner B \rightarrow W \urcorner) = P(T) = 0.99$, since these are intuitively equivalent.

We can now calculate Mark's conditional probability that the last ticket was a winning ticket, given that Ginger didn't win (calculation below, rounding to three decimal places):

- $P(T|\overline{W}) = 0.989$

Assuming that Mark's credence that Ginger wins if she buys the ticket remains equivalent to his credence that the final ticket is a winning ticket, Mark's posterior credence that Ginger wins if she buys a ticket should thus be 0.989.

Calculation: $\boxed{P(T|\overline{W}) = \underbrace{P(T|\overline{W}B)}_{0} \cdot P(B|\overline{W}) + \underbrace{P(T|\overline{W}\overline{B})}_{0.99} \cdot \underbrace{P(\overline{B}|\overline{W})}_{0.999} = 0.989}$

- $P(T|\overline{W}\overline{B}) = 0.99$ because learning that Ginger did not buy a ticket (and hence did not win) should not impact how likely you think that the final ticket was a winning ticket.

- $P(\overline{B}|\overline{W}) = \dfrac{P(\overline{W}|\overline{B}) \cdot P(\overline{B})}{P(\overline{W})} = \dfrac{1 \cdot P(\overline{B})}{P(\overline{W})} = \dfrac{1 \cdot 0.9}{0.901} = 0.999$

- $P(\overline{W}) = P(\overline{W}|B) \cdot P(B) + P(\overline{W}|\overline{B}) \cdot P(\overline{B})$

$$= P(\overline{T}) \cdot P(B) + 1 \cdot P(\overline{B})$$

$$= 0.01 \cdot 0.1 + 1 \cdot 0.9 = 0.901$$

References

Adams, E. (1975). *The Logic of Conditionals*, volume 86 of *Synthese Library*. D. Reidel, Boston.

Bennett, J. (2003). *A Philosophical Guide to Conditionals*. Oxford University Press, Oxford.

Bradley, R. (2000). A preservation condition for conditionals. *Analysis*, 60(267):219–222.

Bradley, R. (2002). Indicative conditionals. *Erkenntnis*, 56(3):345–378.

Bradley, R. (2012). Multidemonsional possible-world semantics for conditionals. *Philosophical Review*, 121(4):539–571.

Douven, I. (2007). On Bradley's preservation condition for conditionals. *Erkenntnis*, 67:111–118.

Douven, I. (2013). The probabilities of conditionals revisited. *Cognitive Science*, pages 1–20.

Edgington, D. (1995). On conditionals. *Mind*, 104(414):235–329.

Kaufmann, S. (2004). Conditioning against the grain: Abduction and indicative conditionals. *Journal of Philosophical Logic*, 33(6):583–606.

Lewis, D. (1976). Probabilities of conditionals and conditional probabilities. *The Philosophical Review*, 85(3):297–315.

McGee, V. (2000). To tell the truth about conditionals. *Analysis*, 60(1):107–111.

Ramsey, F. P. (1931). *The Foundations of Mathematics and other Logical Essays*. Kegan Paul, Trench, Trubner & Co., London.

Rothschild, D. (2013). Do indicative conditionals express propositions? *Nous*, 47(1):49–68.

Rumfitt, I. (2013). Old Adams buried. *Analytic Philosophy*, 54(2):157–188.

Stalnaker, R. (1968). A theory of conditionals. In Rescher, N., editor, *Studies in Logical Theory*, pages 98–112. Oxford: Blackwell.

Stalnaker, R. (1970). Probability and conditionals. *Philosophy of Science*, 37(1):64–80.

Stalnaker, R. (1975). Indicative conditionals. *Philosophia*, 5(3):269–86.