On the Expected Utility Objection to the Dutch Book Argument for Probabilism*

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Abstract

The Dutch Book Argument for Probabilism assumes Ramsey's Thesis (RT), which purports to determine the prices an agent is rationally required to pay for a bet. Recently, a new objection to Ramsey's Thesis has emerged (Hedden, 2013; Wroński & Godziszewski, 2017; Wroński, 2018)—I call this the Expected Utility Objection. According to this objection, it is Maximise Subjective Expected Utility (MSEU) that determines the prices an agent is required to pay for a bet, and this often disagrees with Ramsey's Thesis. I suggest two responses to Hedden's objection. First, we might be permissive: agents are permitted to pay any price that is required or permitted by RT, and they are permitted to pay any price that is required or permitted by MSEU. This allows us to give a revised version of the Dutch Book Argument for Probabilism, which I call the Permissive Dutch Book Argument. Second, I suggest that even the proponent of the Expected Utility Objection should admit that RT gives the correct answer in certain very limited cases, and I show that, together with MSEU, this very restricted version of RT gives a new pragmatic argument for Probabilism, which I call the Bookless Pragmatic Argument.

Aarav believes the Eiffel Tower is over 200m tall more strongly than he believes that it is over 100m tall. Cináed is 60% confident that it will rain tomorrow and 20% confident it will not. Intuitively, Aarav and Cináed are irrational. You are irrational if you believe one proposition more strongly than another it entails; and you are irrational if you believe a proposition and its negation to degrees that sum to less than 100%. Both of these credal principles follow from the more general principle of Probabilism, which says that your credences should obey the axioms of the probability calculus. We specify Probabilism more precisely in Section 1 below.

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The two most popular arguments for Probabilism are the Dutch Book Argument—originally due to Ramsey (1931) and de Finetti (1937) and bolstered by Kemeny (1955) and Shimony (1955)—and the Accuracy Dominance Argument—originally due to Rosenkrantz (1981) and Joyce (1998) and bolstered by Predd et al. (2009) and Pettigrew (2016). The Accuracy Dominance Argument begins from the observation that one of the roles our credences should play is to represent the world—this is their epistemic role. They are thus better or worse depending on how well they represent the world. The Accuracy Dominance Argument then purports to show that credences that do not satisfy Probabilism play this role suboptimally—there will be alternative credences that are guaranteed to represent the world better. The Dutch Book Argument, on the other hand, begins from the observation that another of the roles our credences should play is to guide our actions and help us make good decisions—this is their pragmatic role. They are thus better or worse depending on the value of the outcomes of the decisions they lead us to make. The Dutch Book Argument then purports to show that credences that do not satisfy Probabilism play this role suboptimally—there will be actions you might face where these credences will lead you to choose in a way that gets you bad outcomes for sure.

In this paper, we focus on the Dutch Book Argument and a recent sort of objection to it raised by Hedden (2013) and again by Wroński & Godziszewski (2017) and Wroński (2018, Chapter 2)—I call this *the expected utility objection*. We specify Probabilism precisely in Section 1. We sketch the Dutch Book Argument for that norm in Section 2, focussing particularly on the first premise. Then, in Section 3, we lay out the objection, which targets that first premise. We respond to the objection in Sections 4 and 5. In Section 4, we show how to tweak the existing Dutch Book Argument to avoid the expected utility objection; in Section 5, we offer a new style of pragmatic argument for Probabilism. In Section 6, we conclude. We state a couple of theorems along the way, and we prove these in the Appendix (Section 7).

1 What is Probabilism?

As I said above, Probabilism says that your credences should obey the axioms of the probability calculus. In this section, we spell this out more precisely. Suppose $\mathcal{F}$ is the algebra of propositions to which you assign a credence. Then we let 0 represent the lowest possible credence you can assign, and we let 1 represent the highest possible credence you can assign. We then represent your credences by your credence function $c : \mathcal{F} \to [0, 1]$, where, for each A in $\mathcal{F}$, $c(A)$ is your credence in A.

> **Probabilism** If $c : \mathcal{F} \to [0, 1]$ is your credence function, then rationality requires that:

(P1a) $c(\bot) = 0$, where $\bot$ is a necessarily false proposition;

(P1b) $c(\top) = 1$, where $\top$ is a necessarily true proposition;

(P2) $c(A \vee B) = c(A) + c(B)$, for any mutually exclusive A and B in $\mathcal{F}$.

Now, you might think that we have missed out a condition that is often stated amongst the axioms of probability, namely, Non-Negativity:

(P0) $c(A) \geq 0$, for any A in $\mathcal{F}$.

In fact, we haven't missed it out, because we have specified that 0 is the lowest possible credence you can assign, 1 is the highest, and your credence function therefore takes values at least 0 and at most 1. So (P0) is automatically satisfied by any credence function; it is not a feature that some credence functions have and others lack and that we wish to say all rational credence functions have.

However, this might seem like a cheat. Instead of establishing Non-Negativity, as some take the Dutch Book Argument to do, we have simply assumed it in our representation of credences. But I think it must be this way. Probabilism combines a metaphysical claim with some normative claims. The metaphysical claim is that there is a lowest possible credence and a highest possible credence. Having made that metaphysical claim, Probabilism then makes some normative claims: it says that you should have that lowest possible credence in necessary falsehoods, the highest possible credence in necessary truths, and your credence in a disjunction of two incompatible propositions should be the sum of your credences in the disjuncts. The Dutch Book Argument for Probabilism purports to establish the normative claims, not the metaphysical claims.

Consider the alternative, where it is possible for you to have negative credences—that is, credences lower than 0. If that's the case, those who endorse Probabilism would no longer want to say that rationality requires you to have credence 0 in necessary falsehoods—that is, they would abandon (P1a). They want to say that rationality requires you to have as low a credence as you possibly can in a necessary falsehood—there is nothing special about 0 other than that it is how we chosen to represent lowest possible credence. So either we say that 0 represents the lowest possible credence you can have, in which case Non-Negativity (i.e. (P0)) holds of all credence functions and is not a normative claim to be established; or we say that it is possible for you to have a negative credence, in which case the proponent of Probabilism will not want to demand that $c(\bot) = 0$, and we abandon (P1a). I opt for the first. It is an interesting question what the rational requirements for credences would be if there were no lowest possible credence or no highest possible credence. Whatever is the answer, it won't be Probabilism.

Finally, before we leave this section, let me state an equivalent formulation of Probabilism, which it will be useful to have in mind in Section 5:[1]

> **Partition Probabilism** If $c : \mathcal{F} \to [0, 1]$ is your credence function, then rationality requires that, for any two partitions $\mathcal{X} = \{X_1, \ldots, X_m\}$ and $\mathcal{Y} = \{Y_1, \ldots, Y_n\}$,
>
> $$\sum_{i=1}^{m} c(X_i) = 1 = \sum_{j=1}^{n} c(Y_j)$$

2 The Dutch Book Argument for Probabilism

The Dutch Book Argument for Probabilism has three premises. The first, which I will call *Ramsey's Thesis* and abbreviate *RT*, posits a connection between your credence in a proposition and the prices you are rationally permitted or rationally required to pay for a bet on that proposition. The second, known as the *Dutch Book Theorem*, establishes that, if you violate Probabilism, there is a set of bets you might face, each with a price attached, such that (i) by Ramsey's Thesis, for each bet, you are rationally required to pay the attached price for it, but (ii) the sum of the prices of the bets exceeds the highest possible payout of the bets, so that, having paid each of those prices, you are guaranteed to lose money. The third premise, which we might call the *Domination Thesis*, says that credences are irrational if they mandate you to make a series of decisions (i.e, paying certain prices for the bets) that is guaranteed to leave you worse off than another series of decisions (i.e., refusing to pay those prices for the bets)—in the language of decision theory, paying the attached price for each of the bets is *dominated* by refusing each of the bets, and credences that mandate you to choose dominated options are irrational. The conclusion of the Dutch Book Argument is then Probabilism. Thus, the argument runs:

The Dutch Book Argument for Probabilism

(DBA1) Ramsey's Thesis

(DBA2) Dutch Book Theorem

(DBA3) Domination Thesis

Therefore,

(DBAC) Probabilism

[1] A subset $\mathcal{X} \subseteq \mathcal{F}$ is a partition iff

(i) $\bigvee_{X \in \mathcal{X}} X \equiv \top$ and

(ii) for any $X \neq X'$ in $\mathcal{X}$, $X \,\&\, X' \equiv \bot$.

The argument is valid. The second premise is a mathematical theorem. Thus, if the argument fails, it must be because the first or third premise is false, or both. In this paper, we focus on the first premise, and the expected utility objection to it. So, let's set out that premise in a little more detail.

In what follows, we assume that (i) you are risk-neutral, and (ii) that there is some quantity such that your utility is linear in that quantity—indeed, we will speak as if your utility is linear in money, but that is just for ease of notation and familiarity; any quantity would do. Neither (i) nor (ii) is realistic, and indeed these idealisations are the source of other objections to Ramsey's Thesis. But they are not our concern here, so we will grant them.

> **Ramsey's Thesis (RT)** Suppose your credence in X in $c(X)$. Consider a bet that pays you £S if X is true and £0 if X is false, where S is a real number, either positive, negative, or zero—S is called the *stake* of the bet. You are offered this bet for the price £x, where again x is a real number, either positive, negative, or zero. Then:
>
> (i) If $x < c(X) \times S$, you are rationally required to pay £x to enter into this bet;
>
> (ii) If $x = c(X) \times S$, you are rationally permitted to pay £x and rationally permitted to refuse;
>
> (iii) If $x > c(X) \times S$, you are rationally required to refuse.

Roughly speaking, Ramsey's Thesis says that, the more confident you are in a proposition, the more you should be prepared to pay for a bet on it. More precisely, it says: (a) if you have minimal confidence in that proposition (i.e. 0), then you should be prepared to pay nothing for it; (b) if you have maximal confidence in it (i.e. 1), then you should be prepared to pay the full stake for it; (c) for levels of confidence in between, the amount you should be prepared to pay increases linearly with your credence.

3 The Expected Utility Objection

We turn now to the objection to Ramsey's Thesis (RT) we wish to treat here. Hedden begins by pointing out that we have a general theory of how credences and utilities should guide action:

> Given a set of options available to you, expected utility theory says that your credences license you to choose the option with the highest expected utility, defined as:
>
> $$\mathrm{EU}(A) = \sum_i P(O_i|A) \times U(O_i)$$

> On this view, we should evaluate which bets your credences license you to accept by looking at the expected utilities of those bets. (Hedden, 2013, 485)

He considers the objection that this only applies when credences satisfy Probabilism, but rejects it:

> In general, we should judge actions by taking the sum of the values of each possible outcome of that action, weighted by one's credence that the action will result in that outcome. This is a very intuitive proposal for how to evaluate actions that applies even in the context of incoherent credences. (Hedden, 2013, 486)

Thus, Hedden contends that we should always choose by maximising expected utility relative to our credences, whether or not those credences are coherent. Let's call this principle *Maximise Subjective Expected Utility* and abbreviate it *MSEU*. He then observes that MSEU conflicts with RT. Consider, for instance, Cináed from the beginning of the paper. He is 60% confident it will rain and 20% confident it won't. According to RT, he is rationally required to sell for £65 a bet in which he pays out £100 if it rains and £0 if is doesn't. But the expected utility of this bet for him is

$$0.6 \times (-100 + 65) + 0.2 \times (-0 + 65) = -8$$

That is, it has lower expected utility than refusing to sell the bet, since his expected utility for doing that is

$$0.6 \times 0 + 0.2 \times 0 = 0$$

So, while RT says you must sell that bet for that price, MSEU says you must not. So RT and MSEU are incompatible, and Hedden claims that we should favour MSEU. There are two ways to respond to this. On the first, we try to retain RT in some form in spite of Hedden's objection—I call this the *permissive response* below. On the second, we try to give a pragmatic argument for Probabilism using MSEU instead of RT—I call this the *bookless response* below. In the following sections, I will consider these in turn.

4 The Permissive Response

While Hedden is right to say that maximising expected utility in line with Maximise Subjective Expected Utility (MSEU) is intuitively rational even when your credences are incoherent, so is Ramsey's Thesis (RT). It is certainly intuitively correct that, to quote Hedden, "we should judge actions by taking the sum of the values of each possible outcome of that action, weighted by one's credence that the action will result in that outcome." But it is also intuitively correct that, to quote from our gloss of Ramsey's Thesis above, "(a) if you have minimal confidence in that proposition (i.e. 0), then you should be prepared to pay nothing for it; (b) if you have maximal confidence in it (i.e. 1), then you should be prepared to pay the full stake for it; (c) for levels of confidence in between, the amount you should be prepared to pay increases linearly with your credence." What are we to do in the face of this conflict between our intuitions?

One natural response is to say that choosing in line with RT is rationally permissible and choosing in line with MSEU is also rationally permissible. When your credences are coherent, the dictates of MSEU and RT are the same. But when you are incoherent, they are sometimes different, and in that situation you are allowed to follow either. In particular, faced with a bet and proposed price, you are permitted to pay that price if it is permitted by RT *and* you are permitted to pay it if it is permitted by MSEU.

If this is right, then we can resurrect the Dutch Book Argument with a permissive version of RT as the first premise:

> **Permissive Ramsey's Thesis** Suppose your credence in X in $c(X)$. Consider a bet that pays you £S if X is true and £0 if X is false. You are offered this bet for the price £x. Then:
>
> (i) If $x \leq c(X) \times S$, you are rationally permitted to pay £x to enter into this bet.

And we could then amend the third premise—the Domination Thesis (DBA3)—to ensure we could still derive our conclusion. Instead of saying that credences are irrational if they *mandate* you to make a series of decisions that is guaranteed to leave you worse off than another series of decisions, we might say that credences are irrational if they *permit* you to make a series of decisions that is guaranteed to leave you worse off than another series of decisions. In the language of decision theory, instead of saying only that credences that *mandate* you to choose dominated options are irrational, we say also that credences that *permit* you to choose dominated options are irrational. We might call this the *Permissive Domination Thesis*.

Now, by weakening the first premise in this way, we respond to Hedden's objection and make the premise more plausible. But we strengthen the third premise to compensate and perhaps thereby make it less plausible. However, I imagine that anyone who accepts one of the versions of the third premise—either the Domination Thesis or the Permissive Domination Thesis—will also accept the other. Having credences that *mandate* dominated choices may be worse than having credences that *permit* such choices, but both seem sufficient for irrationality. Perhaps the former makes you *more* irrational than the latter, but it seems clear that the ideally rational agent will have credences that do neither. And if that's the case, then we can replace the standard Dutch Book Argument with a slight modification:

The Permissive Dutch Book Argument for Probabilism

(PDBA1) Permissive Ramsey's Thesis

(PDBA2) Dutch Book Theorem

(PDBA3) Permissive Domination Thesis

Therefore,

(PDBAC) Probabilism

In the following three subsections, I wish to offer two considerations in favour of the permissive version of Ramsey's Thesis (Sections 4.1 and 4.3) and one against it (Section 4.2).

4.1 Competing partitions in MSEU

Quite independently of Ramsey's Thesis, it seems that we might want a permissive version of MSEU. Consider the following example:

Cináed is asked to consider the following two bets:

- **Bet 1** £10 if it rains, £0 if it doesn't.
- **Bet 2** £10 if it doesn't rain, £0 if it does.

He must either pay £4 for Bet 1 or £4 for Bet 2—this is a forced choice situation. Let R be the proposition that it will rain. The payoffs are as follows:

	R	$\overline{R}$
Bet 1	6	-4
Bet 2	-4	6

Let W be the proposition that it will be windy. Then suppose Cináed has credences in the following four states of the world:

RW	$R\overline{W}$	$\overline{R}W$	$\overline{R}\overline{W}$
0.2	0.2	0.3	0.3

And he has credences in the following two more coarse-grained propositions:

R	$\overline{R}$
0.6	0.4

> So his credences are incoherent: his credences in RW and $R\overline{W}$ don't sum to his credence in R; and his credences in $\overline{R}W$ and $\overline{RW}$ don't sum to his credence in $\overline{R}$.
>
> Now, MSEU says that Cináed should choose whether to bet on rain or no rain by calculating which will maximise his expected utility. But this raises the question: calculated relative to which partition? The more fine-grained one, namely, RW, $R\overline{W}$, $\overline{R}W$, and $\overline{RW}$? Or the more coarse-grained one, namely, R, $\overline{R}$? It's easy to check that, if he calculates relative to the first, he should choose to pay for Bet 1, since its expected utility (i.e., 2) is higher than the expected utility of Bet 2 (i.e. 0); but if he calculates relative to the second, he should choose to pay for Bet 2, since its expected utility (i.e. 2) is higher than the expected utility of Bet 1 (i.e. 0).

A natural conclusion to draw from this is that there can't be any general norm that says you should maximise expected utility. As we have just seen, in certain situations, such a norm leads to incompatible pairs of demands, and rationality surely satisfies an logical ought-can principle—that is, principles of rationality must never make two demands that, as a matter of logic, cannot both be fulfilled.

The upshot is that, quite independently of any considerations of Ramsey's Thesis, we have reason to give a permissive reading of MSEU. We must not take MSEU to say that you are *required to* choose an option if it maximises subjective expected utility, for that will give rise to incompatible demands; rather, we must take it to say that you are *permitted to* choose such an option. And this makes it more plausible that there might be other permitted actions besides those that maximise expected utility relative to some partition—for instance, actions permitted by the permissive version of Ramsey's Thesis.

In fact, Hedden considers cases of partition-sensitive decisions—such as Cináed's forced choose between the bets on rain above—and comes to a different conclusion (Hedden, 2013, Footnote 17). He concludes that we should take MSEU to say that you are *required to* choose an option if it maximises expected utility, and he responds to cases in which our credences lead, via this version of MSEU, to deliver two incompatible verdicts by saying that, in such cases, we should declare the credences irrational, rather than declaring the norm faulty.

So, while these cases of partition-sensitivity might give us some reason to be permissive about the actions that are rational for someone with incoherent credences, other responses to these cases are available.

4.2 An error theory for our RT intuitions

Above, I claimed that, while Hedden is right to say that MSEU is intuitively correct, RT also has intuitive support—and I asked what to do in the face of this. One way to argue that we should favour MSEU over RT would then be to undermine the intuitions in favour of RT.

We might begin by noting cases in which the verdict of MSEU is more intuitive than the verdict of RT. Thus, for instance, suppose I have credence 0.99 in a proposition and also 0.99 in its negation. Then RT says that I must pay £98 for a bet that pays me £100 if the proposition is true and £0 if it is false, while MSEU says that I must pay at most £50 for that bet. Now, intuition is on the side of MSEU here, I think. It seems irrational to choose an action that will result in such a large loss (£98) in one situation and such a small gain (£2) in the other, when you are equally confident in both situations.

What's more, it seems that we can give an error theory for our intuitions in favour of RT. Our intuitions, we might say, are acquired in an environment in which, while many individuals we encounter are not probabilistically coherent, they at least tend to become less confident in a proposition as they become more confident in its negation. Thus, when we find RT intuitively correct, we are giving voice to intuitions that only really apply in these sorts of situations. As soon as we consider other sorts of situation, such as the case in which I have credence 0.99 in a proposition and 0.99 in its negation, the intuitive support for RT evaporates.

I have some sympathy with this line of argument. However, as we will see in the next subsection, there are also situations in which RT gives what is clearly the more intuitive recommendation.

4.3 More intuitions in favour of RT

Consider Dima and Esther. They both have minimal confidence—i.e. 0—that it won't rain tomorrow. But Dima has credence 0.01 that it will rain, while Esther has credence 0.99 that it will. If we permit only actions that maximise expected utility, then Dima and Esther are required to pay exactly the same prices for bets on rain—that is, Dima will be required to pay a price exactly when Esther is. After all, if £S is the payoff when it rains, £0 is the payoff when it doesn't, and x is a proposed price, then $0.01 \times (S - x) + 0 \times (0 - x) \geq 0$ iff $0.99 \times (S - x) + 0 \times (0 - x) \geq 0$ iff $S \geq x$. So, according to MSEU, Dima and Esther are rationally required to pay anything up to the stake of the bet for such a bet. But this is surely wrong. It is surely at least permissible for Dima to refuse to pay a price that Esther accepts. It is surely permissible for Esther to pay £99 for a bet on rain that pays £100 if it rains and £0 if it doesn't, while Dima refuses to pay anything more than £1 for such a bet, in line with Ramsey's Thesis. Suppose Dima were offered such a bet for the price of £99, and suppose she then defended her refusal to pay that price saying, 'Well, I only think it's 1% likely to rain, so I don't want to risk such a great loss with so little possible gain when I think the gain is so unlikely'. Then surely we would accept that as a rational defence.

In response to this, defenders of MSEU might concede that RT is sometimes the correct norm of action when you are incoherent, but only in very specific cases, namely, those in which you have a positive credence in a proposition, minimal credence (i.e. 0) in its negation, and you are considering the price you might pay for a bet on that proposition. In all other cases—that is, in any case in which your credences in the proposition and its negation are both positive, or in which you are considering an action other than a bet on a proposition—you should use MSEU. Again, I have some sympathy with this, and I will appeal to it when I offer my new pragmatic argument for Probabilism in Section 5.

Thus, at the end of this section, we have an amended version of the Dutch Book Argument that is based on the Permissive Ramsey's Thesis and the Permissive Domination Thesis—I called this the Permissive Dutch Book Argument above. In the last three subsections, we have considered arguments for and against Permissive Ramsey's Thesis. None of the objections are decisive, but they are sufficiently worrying that we would surely prefer to have a more secure foundation for the Dutch Book Argument.

5 The Bookless Response

Suppose you refuse even the permissive version of RT, and insist that coherent and incoherent agents alike should choose in line with MSEU. Then what becomes of the Dutch Book Argument? As we noted above, Hedden shows that it fails—MSEU is not sufficient to establish the conclusion. In particular, Hedden gives an example of an incoherent credence function that is not Dutch Bookable via MSEU. That is, there are no sets of bets with accompanying prices such that (a) MSEU will demand that you pay each of those prices, and (b) the sum of those prices is guaranteed to exceed the sum of the payouts of that set of bets. However, as we will see, accepting individual members of such a set of bets is just one way to make bad decisions based on your credences.

Consider Hedden's example. In it, you assign credences to propositions in the algebra built up from three possible worlds, w_1, w_2, and w_3. Here are some of your credences:

$$c(w_1 \vee w_2) = 0.8 \qquad c(w_3) = 0$$
$$c(w_1) = 0.7 \qquad c(w_2 \vee w_3) = 0$$

Now, consider the following two options, A and B, whose utilities in each state of the world are set out in the following table:

	w_1	w_2	w_3
A	78	77	77
B	74	74	75

Then notice first that A dominates B—that is, the utility of A is higher than B in every possible state of the world. But, using your incoherent credences, you assign a higher expected utility to B than to A. Your expected utility for A—which must be calculated relative to your credences in w_1 and $w_2 \vee w_3$, since the utility of A given $w_1 \vee w_2$ is undefined—is $0.7 \times 78 + 0 \times 77 = 54.6$. And your expected utility for B—which must be calculated relative to your credences in $w_1 \vee w_2$ and w_3, since the utility of B given $w_2 \vee w_3$ is undefined—is $0.8 \times 74 + 0 \times 75 = 59.2$. So, while Hedden might be right that MSEU won't leave you vulnerable to a Dutch Book, it will leave you vulnerable to choosing a dominated option. And since what is bad about entering a Dutch Book is that it is a dominated option—it is dominated by the option of refusing the bets—the invulnerability to Dutch Books should be no comfort to you.

Now, this raises the question: For which incoherence credences is it guaranteed that MSEU won't lead you to choose a dominated option? Is it *all* incoherent credences, in which case we would have a new Dutch Book Argument for Probabilism from MSEU rather than RT? Or is it some subset? Below, we prove a theorem that answers that. First, a weakened version of Probabilism:

> **Bounded Probabilism** If $c : \mathcal{F} \to [0, 1]$ is your credence function, then rationality requires that:

(BP1a) $c(\bot) = 0$, where $\bot$ is a necessarily false proposition;

(BP1b) There is $0 < M \leq 1$ such that $c(\top) = M$, where $\top$ is a necessarily true proposition;

(BP2) $c(A \vee B) = c(A) + c(B)$, if A and B are mutually exclusive.

Bounded Probabilism says that you should have lowest possible credence in necessary falsehoods, some positive credence—not necessarily 1—in necessary truths, and your credence in a disjunction of two incompatible propositions should be the sum of your credences in the disjuncts.

Theorem 1

$$c \text{ satisfies Bounded Probabilism}$$
$$\Leftrightarrow$$
$$\textit{For all options } A, B, \textit{ if } A \textit{ dominates } B, \textit{ then } \mathrm{EU}_c(A) > \mathrm{EU}_c(B).$$

The proof is in the Appendix. Thus, even without Ramsey's Thesis or the permissive version described above, you can still give a pragmatic argument for a norm that lies very close to Probabilism, namely, Bounded Probabilism. On its own, this argument cannot say what is wrong with someone who gives less than the highest possible credence to necessary truths, but it does establish the other requirements that Probabilism imposes. To see just how close to Probabilism lies Bounded Probabilism, consider the following two norms, which are equivalent to it:

> **Scaled Probabilism** If $c : \mathcal{F} \to [0,1]$ is your credence function, then rationality requires that there is $0 < M \leq 1$ and a probability function $p : \mathcal{F} \to [0,1]$ such that $c(-) = M \times p(-)$.
>
> **Bounded Partition Probabilism** If $c : \mathcal{F} \to [0,1]$ is your credence function, then rationality requires that, for any two partitions $\mathcal{X} = \{X_1, \ldots, X_m\}$ and $\mathcal{Y} = \{Y_1, \ldots, Y_n\}$,
>
> $$\sum_{i=1}^{m} c(X_i) = \sum_{j=1}^{n} c(Y_j)$$

Then

Lemma 2 *The following are equivalent:*

(i) *Bounded Probabilism*

(ii) *Scaled Probabilism*

(iii) *Bounded Partition Probabilism*

As before, the proof is in the Appendix.

So, on its own, MSEU can deliver us very close to Probabilism. But it cannot establish (P1b), namely, $c(\top) = 1$. However, I think we can appeal to an insight from above to secure (P1b) and push us all the way to Probabilism. Recall our example of Dima and Esther from Section 4.3. At the end, we concluded that, even if the proponent of MSEU is not moved to permit also the actions recommended by RT in general, they should concede that RT gives the correct answers in a very specific sort of case, namely, one in which you have a positive credence in a proposition, the lowest possible credence (i.e. 0) in its negation, and you are considering a bet on that proposition. But of course it is precisely by applying Ramsey's Thesis to such a case that we can produce a Dutch Book against someone with $c(\bot) = 0$ and $c(\top) < 1$—we simply offer to pay them £$c(\top) \times 100$ for a bet in which they will pay out £100 if $\top$ is true and £0 if it is false; this is then guaranteed to lose them £$100 \times (1 - c(X))$, which is positive. Thus, we end up with a disjunctive pragmatic argument for Probabilism: if $c(\bot) = 0$ and $c(\top) < 1$, then RT applies and we can produce a Dutch Book against you; if you violate Probabilism in any other way, then you violate Bounded Probabilism and we can then produce two options A and B such that A dominates B, but your credences, via MSEU, dictate that you should choose B over A. This, then, is our bookless pragmatic argument for Probabilism:

Bookless Pragmatic Argument for Probabilism

(BPA1) If c violates Probabilism, then either (i) $c(\bot) = 0$ and $c(\top) < 1$, or (ii) c violates Bounded Probabilism.

(BPA2) If $c(\bot) = 0$ and $c(\top) < 1$, then RT applies, and there is a bet on $\top$ such that you are required by RT to pay a higher price for that bet than its guaranteed payoff. Thus, there are options A and B (namely, *refuse the bets* and *pay the price*), such that A dominates B, but RT demands that you choose B over A.

(BPA3) If c violates Bounded Probabilism, then by Theorem 1, there are options A and B such that A dominates B, but RT demands that you choose B over A.

Therefore, by (BPA1), (BPA2), and (BPA3),

(BPA4) If c violates Probabilism, then there are options A and B such that A dominates B, but rationality requires you to choose B over A.

(BPA5) Dominance Thesis

Therefore,

(BPAC) Probabilism

6 Conclusion

The Dutch Book Argument for Probabilism assumes Ramsey's Thesis, which determines the prices an agent is rationally required to pay for a bet. Hedden argues that Ramsey's Thesis is wrong. He claims that Maximise Subjective Expected Utility determines those prices, and it often disagrees with RT. In Section 4, I suggested that, in the face of that disagreement, we might be permissive: agents are permitted to pay any price that is required or permitted by RT and they are permitted to pay any price that is required or permitted by MSEU. This allows us to give a revised version of the Dutch Book Argument for Probabilism, namely, the Permissive Dutch Book Argument. In Section 5, I then explored what we might do if we reject this permissive response and insist that only prices permitted or required by MSEU are permissible. I showed that, in that case, we can give a pragmatic argument for Bounded Probabilism, which comes close to Probabilism, but doesn't quite reach; and I showed that, if we allow RT in the very particular cases in which it agrees better with intuition than MSEU does, we can give a pragmatic argument for Probabilism, namely, the Bookless Pragmatic Argument.

7 Appendix: Proof of Lemma 2 and Theorem 1

Lemma 2 The following are equivalent:

(i) Bounded Probabilism

(ii) Scaled Probabilism

(iii) Bounded Partition Probabilism

Proof of Lemma 2.

(i) $\Rightarrow$ (ii). Suppose c satisfies Bounded Probabilism. Then consider the credence function $\frac{1}{M} \times c(-)$. We show that it is a probability function:

(P1a) $\frac{1}{M} \times c(\bot) = \frac{1}{M} \times 0 = 0$.

(P1b) $\frac{1}{M} \times c(\top) = \frac{1}{M} \times m = 1$.

(P2) $\frac{1}{M} \times c(A \vee B) = \frac{1}{M} \times (c(A) + c(B)) = \frac{1}{M}c(A) + \frac{1}{M}c(B)$.

So $c(-) = M \times (\frac{1}{M} \times c(-))$, and $\frac{1}{M} \times c(-)$ is a probability function, so c satisfies Scaled Probabilism.

(ii) $\Rightarrow$ (iii). Suppose c satisfies Scaled Probabilism. Then $c(-) = M \times p(-)$, for some $0 < M \leq 1$ and some probability function p. Now, we know from the equivalence of Probabilism and Partition Probabilism that, for any two partitions, $\mathcal{X}$ and $\mathcal{Y}$,

$$\sum_{i=1}^{m} p(X_i) = 1 = \sum_{j=1}^{n} p(Y_j)$$

Thus,

$$\sum_{i=1}^{m} c(X_i) = \sum_{i=1}^{m} Mp(X_i) = M\sum_{i=1}^{m} p(X_i) = M = M\sum_{j=1}^{n} p(Y_j) = \sum_{j=1}^{n} Mp(Y_j) = \sum_{j=1}^{n} c(Y_j)$$

So c satisfies Bounded Partition Probabilism.

(iii) $\Rightarrow$ (i). Suppose c satisfies Bounded Partition Probabilism. Then we show that c is a bounded probability function:

(BP1a) Consider the two partitions $\{\top\}$ and $\{\bot, \top\}$. By Bounded Partition Probabilism, $c(\top) = c(\bot) + c(\top)$. So $c(\bot) = 0$.

(BP1b) Consider the partition $\{\top\}$. By Bounded Partition Probabilism, $c(\top) = M$.

(BP2) Consider the two partitions $\{A, B, \overline{A \vee B}\}$ and $\{A \vee B, \overline{A \vee B}\}$. Then, by Bounded Partition Probabilism,

$$c(A) + c(B) + c(\overline{A \vee B}) = M = c(A \vee B) + c(\overline{A \vee B})$$

So $c(A) + c(B) = c(A \vee B)$.

Thus, c satisfies Bounded Probabilism.

This completes the proof. □

Theorem 1

$$\begin{gathered} c \text{ satisfies Bounded Probabilism} \\ \Leftrightarrow \\ \text{For all options } A, B, \text{ if } A \text{ dominates } B, \text{ then } \mathrm{EU}_c(A) > \mathrm{EU}_c(B). \end{gathered}$$

Proof of Theorem 1.

($\Rightarrow$) Suppose c satisfies Bounded Probabilism. Then, by Lemma 2, there is $0 < M \leq 1$ and a probability function p such that $c(-) = M \times p(-)$. Now suppose A and B are actions. Then

- $\mathrm{EU}_c(A) = \mathrm{EU}_{M \times p}(A) = M \times \mathrm{EU}_p(A)$
- $\mathrm{EU}_c(B) = \mathrm{EU}_{M \times p}(B) = M \times \mathrm{EU}_p(B)$

Thus, $\mathrm{EU}_c(A) > \mathrm{EU}_c(B)$ iff $\mathrm{EU}_p(A) > \mathrm{EU}_p(B)$. And we know that, if A dominates B and p is a probability function, then $\mathrm{EU}_p(A) > \mathrm{EU}_p(B)$.

($\Leftarrow$) Suppose c violates Bounded Probabilism. Then there are partitions $\mathcal{X} = \{X_1, \ldots, X_m\}$ and $\mathcal{Y} = \{Y_1, \ldots, Y_n\}$ such that

$$\sum_{i=1}^{m} c(X_i) = x < y = \sum_{j=1}^{n} c(Y_j)$$

We will now define two acts A and B such that A dominates B, but $\mathrm{EU}_c(A) < \mathrm{EU}_c(B)$.

- For any X_i in $\mathcal{X}$,
$$U(A, X_i) = y - i\frac{y - x}{2(m+1)}$$
- For any Y_j in $\mathcal{Y}$,
$$U(B, Y_j) = x + j\frac{y - x}{2(n+1)}$$

Then the crucial facts are:

- For any two $X_i \neq X_j$ in $\mathcal{X}$,
$$U(A, X_i) \neq U(A, X_j)$$
- For any two $Y_i \neq Y_j$ in $\mathcal{Y}$,
$$U(B, Y_i) \neq U(B, Y_j)$$

- For any X_i in $\mathcal{X}$ and Y_j in $\mathcal{Y}$,

$$x < U(B, Y_j) < \frac{x+y}{2} < U(A, X_i) < y$$

So A dominates B, but

$$\text{EU}_c(A) = \sum_{i=1}^{m} c(X_i)U(A, X_i) < \sum_{i=1}^{m} c(X_i) \times y = xy$$

while

$$\text{EU}_c(B) = \sum_{j=1}^{n} c(Y_i)U(B, Y_j) > \sum_{j=1}^{n} c(Y_j) \times x = yx$$

So $\text{EU}_c(B) > \text{EU}_c(A)$, as required. □

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