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Accepted Version

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To link to this article DOI: http://dx.doi.org/10.1016/j.econlet.2019.01.019

Publisher: Elsevier

www.reading.ac.uk/centaur

Portfolio Management with Cryptocurrencies: The Role of Estimation Risk

Abstract

This paper contributes to the literature on cryptocurrencies, portfolio management and estimation risk by comparing the performance of naïve diversification, Markowitz diversification and the advanced Black-Litterman model with VBCs that controls for estimation errors in a portfolio of cryptocurrencies. We show that the advanced Black-Litterman model with VBCs yields superior out-of-sample risk-adjusted returns as well as lower risks. Our results are robust to the inclusion of transaction costs and short-selling, indicating that sophisticated portfolio techniques that control for estimation errors are preferred when managing cryptocurrency portfolios.

1. Introduction

The mean-variance portfolio optimization framework of Markowitz (1952) is highly sensitive to estimation errors in the input parameters, and this has been extensively documented in the literature (e.g. Kan and Zhou, 2007; Levy and Levy, 2014). Hence, several studies have investigated whether other naïve strategies (such as the 1/N) can beat mean-variance optimal portfolio diversification and its extensions in the out-of-sample. For example, Board and Sutcliffe (1994) document that there is very little to select between 1/N and other more sophisticated estimation methods for portfolio selection, while DeMiguel et al (2009) show that the 1/N is superior to 14 different portfolio optimization models across a range of markets in the out-of-sample setting.

Interest in cryptocurrencies is growing, especially as an investment where Baur et al (2018) show that Bitcoin accounts are mainly used as a speculative investment and not as an alternative currency and medium of exchange.[1] The diversification benefits of Bitcoin to other financial assets has been reported by Bouri et al (2017) and Corbet et al (2018a), while recently Kajtazi and Moro (2018) and Platanakis and Urquhart (2018) both report substantial benefits from including Bitcoin in traditional portfolios. Platanakis et al (2018a) show that there is very little to select between optimal mean-variance diversification and 1/N for a portfolio of cryptocurrencies. However, cryptocurrencies have been found to be highly volatile (Chaim and Laurini 2018) and therefore have higher potential estimation errors in their parameters that may make portfolio theory particularly problematic when applied to a portfolio of cryptocurrencies. In this paper, we attempt to highlight this issue and add to the debate of optimal versus naïve diversification in cryptocurrencies by applying a more advanced and sophisticated portfolio optimization technique. This technique uses alternative estimates for the input parameters and imposes tighter constraints to the weights of assets with higher potential estimation errors and we find that this technique outperforms both 1/N and the Markowitz portfolio optimization framework when applied to a portfolio of cryptocurrencies. Therefore our paper furthers the findings by Platanakis et al (2018a).

The rest of this paper is organized as follows. Section 2 presents the data and methodology, Section 3 contains the empirical results. We conclude in Section 4.

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[1] For an up-to-date review of the literature of cryptocurrencies, see Corbet et al (2018b).

2. Data and Methodology

2.1. Data

We collect weekly data on Bitcoin, Litecoin, Ripple and Dash over the period 21st February 2014 to 4th May 2018 (220 weekly observations in total) from www.coinmarketcap.com, as well as for the risk-free rate from the Kenneth French web-site.[2] Correlations are reported in Table 1 where we find the highest correlation between Litecoin and Ripple to be 0.5588, while the lowest correlation is between Dash and Ripple at 0.0294.

2.2. Methodology
2.2.1. 1/N Model

Initially, we employ the 1/N rule which does not require any optimization and assigns a portfolio weight of 1/N to each asset. We use 1/N with re-balancing as in DeMiguel et al (2009).

2.2.2. Markowitz Model

The mean-variance portfolio optimization framework of Markowitz can be viewed as the choice of portfolio weights $(\mathbf{x})$ that maximize the Sharpe ratio. We also impose additional constraints to prohibit short selling $(\mathrm{x}_i \geq 0)$, and for the normalization of portfolio weights $\left(\sum_{i=1}^{N} \mathrm{x}_i = 1 \right)$. The optimization problem is written as follows:-

$$
\begin{aligned}
\max_{\mathbf{x}} \quad & \left\{ \frac{\mathbf{x}^{\mathrm{T}} \boldsymbol{\mu} - \bar{r}_f}{\sqrt{\mathbf{x}^{\mathrm{T}} \boldsymbol{\Sigma} \mathbf{x}}} \right\} \\
s.t. \quad & \mathrm{x}_i \geq 0, \qquad \forall i \\
& \sum_{i=1}^{N} \mathrm{x}_i = 1
\end{aligned}
\tag{1}
$$

---

[2] Bitcoin, Litecoin, Ripple and Dash stand as the most liquid cryptocurrencies. The starting date has been determined by the availability of all the cryptocurrencies used in this study. We choose weekly data since monthly data would not provide enough observations for a robust analysis while daily prices would result in a large turnover and thus high transaction costs. The Kenneth French database can be found at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

where the parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ denote the sample estimates for the means and the covariance matrix respectively, while $\bar{r}_f$ represents the average risk-free rate over the estimation period.

2.2.3. Black-Litterman with VBCs

The Black-Litterman (BL) portfolio optimization approach is an alternative portfolio optimization framework for dealing with estimation risk which has recently gained great attention, see for instance Bessler et al (2017), Platanakis and Sutcliffe (2017) and Oikonomou et al (2018), amongst others. The BL technique combines the subjective estimates (views) on returns and the benchmark portfolio to compute the implied returns.

Black and Litterman (1992) compute the vector of implied excess-returns $\left(\mathbf{H}\right)$ as follows:-

$$\mathbf{H} = \lambda \boldsymbol{\Sigma} \mathbf{x}^{Benchmark}. \tag{2}$$

The vector $\mathbf{x}^{Benchmark}$ represents the benchmark portfolio and λ denotes the relative risk aversion[3].

The column vector of mean returns $\left(\boldsymbol{\mu}_{\mathrm{BL}}\right)$ is computed as follows:-

$$\boldsymbol{\mu}_{\mathrm{BL}} = \left[\left(c\boldsymbol{\Sigma}\right)^{-1} + \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\boldsymbol{P}\right]^{-1}\left[\left(c\boldsymbol{\Sigma}\right)^{-1}\mathbf{H} + \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\mathbf{Q}\right], \tag{3}$$

where the column vector $\mathbf{Q}$ contains the subjective returns and $\boldsymbol{P}$ represents a binary matrix with only non-zero elements in its diagonal. We also follow Platanakis and Sutcliffe (2017) and set the parameter c, which represents the overall level of confidence in the implied asset returns, to 0.1625. Meucci (2010) defines the diagonal matrix $\boldsymbol{\Omega}$ as follows:-

$$\boldsymbol{\Omega} = \frac{1}{\delta}\boldsymbol{P}\boldsymbol{\Sigma}\boldsymbol{P}^{\mathrm{T}}, \tag{4}$$

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3 $\mathbf{x}^{Benchmark}$ is set to the global minimum-variance portfolio as in Bessler et al (2017), and many others. Since we maximize the Sharpe ratio, the parameter λ is set to unity.

where $1/\delta$ is set to unity as in Meucci (2010). We also follow Platanakis et al (2018B) and Bessler et al (2017) and use the mean returns in each estimation period for setting the subjective returns in $\mathbf{Q}$. The covariance matrix $\left(\mathbf{\Sigma}_{\text{BL}}\right)$ is computed as follows (Satchell and Scowcroft, 2000):-

$$\mathbf{\Sigma}_{\text{BL}} = \mathbf{\Sigma} + \left[\left(c\mathbf{\Sigma}\right)^{-1} + \mathbf{P}^{\text{T}}\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1} \tag{5}$$

We additionally impose variance-based constraints (VBCs) of Levy and Levy (2014) to control further the negative impact of estimation errors in the input parameters. VBCs impose tighter constraints on the weights of the assets with the higher potential estimation risk, and are described as follows:-

$$\left|\mathbf{x}_i - \frac{1}{N}\right|\frac{\sigma_i}{\bar{\sigma}} \leq a, \quad \forall i \tag{6}$$

where $\bar{\sigma} = \dfrac{1}{N}\displaystyle\sum_{i=1}^{N}\sigma_i$, and the threshold a is set to 20%.

Hence, the optimization model is expressed as follows:-

$$\begin{aligned}
\max_{\mathbf{x}} \quad & \left\{\frac{\mathbf{x}^{\text{T}}\mathbf{\mu}_{\text{BL}} - \bar{r}_f}{\sqrt{\mathbf{x}^{\text{T}}\mathbf{\Sigma}_{\text{BL}}\mathbf{x}}}\right\} \\
s.t. \quad & \mathbf{x}_i \geq 0, \qquad \forall i \\
& \sum_{i=1}^{N}\mathbf{x}_i = 1, \\
& \left|\mathbf{x}_i - \frac{1}{N}\right|\frac{N\sigma_i}{\displaystyle\sum_{i=1}^{N}\sigma_i} \leq a, \quad \forall i
\end{aligned} \tag{7}$$

where $\mathbf{\mu}_{\text{BL}} = \left[\left(c\mathbf{\Sigma}\right)^{-1} + \mathbf{P}^{\text{T}}\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1}\left[\left(c\mathbf{\Sigma}\right)^{-1}\mathbf{H} + \mathbf{P}^{\text{T}}\mathbf{\Omega}^{-1}\mathbf{Q}\right]$ and $\mathbf{\Sigma}_{\text{BL}} = \mathbf{\Sigma} + \left[\left(c\mathbf{\Sigma}\right)^{-1} + \mathbf{P}^{\text{T}}\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1}$.

2.3. Transaction Costs

The total transaction costs $\left(\mathrm{TC}_t\right)$ are estimated as follows:-

$$\mathrm{TC}_t = \sum_{i=1}^{N} \mathrm{T}_i \left(\left| \mathrm{x}_{i,t} - \mathrm{x}_{i,t-1}^{+} \right| \right), \tag{8}$$

where $\mathrm{x}_{i,t-1}^{+}$ represents the proportion of the asset i at the end of the period t-1. We set the proportionate transaction cost (T_i) per transaction to 50 bps for all cryptocurrencies as in Lintilhac and Tourin (2017). The total transaction costs are subtracted from portfolio returns when measuring performance.

3. Empirical Results

Figure 1 reports the out-of-sample and net of transaction costs annualized Sharpe ratio for the 1/N rule, the Markowitz mean-variance portfolio model and the Black-Litterman technique with VBCs by using a 110-week rolling estimation window (half of the entire sample period) and re-balanced every week.[45] We observe that the Black-Litterman approach with VBCs outperforms the both the 1/N and Markowitz benchmarks indicating that the advanced portfolio optimization model offers higher risk-adjusted returns for a cryptocurrency portfolio, inclusive of transaction costs. In Figure 2 we report the standard deviation of the of each portfolio and apart from the first 3 months of the reported out-of-sample period, the Black-Litterman model has a lower portfolio risk than the 1/N and Markowitz models highlighting the risk reduction from the more advanced optimization technique. For robustness, in Figure 3 we present the re-estimation of the 3 portfolio models with a 100% increase in the transaction cost estimates (e.g. 100 bps rather than 50 bps) and again show that the Black-Litterman model offers higher Sharpe ratios throughout the out-of-sample period. Finally in Figure 4 we allow for short-selling and consistent with the previous findings, the more advanced portfolio optimization model offers higher risk-adjusted returns throughout the reported out-of-sample period.

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[4] All analysis is conducted in Matlab.

[5] We allow for some weeks (out-of-sample observations) until we start reporting the out-of-sample Sharpe ratio in each figure, since its estimation with just a few observations may cause instability problems.

4. Conclusions

We contribute to the literature on cryptocurrencies and estimation risk management by highlighting the fact that portfolio theory may face significant difficulties when applied to a portfolio of cryptocurrencies given the higher potential estimation errors in their parameters. To this end, we use an advanced portfolio optimization methodology and show that the Black-Litterman model with VBCs yields superior out-of-sample performance than other traditional benchmarks when applied to a portfolio of cryptocurrencies. This indicates that investors should use more sophisticated portfolio techniques that control for estimation errors in the input parameters when managing cryptocurrency portfolios.

Tables

Table 1: Correlation matrix of the returns of the cryptocurrencies employed in this study.

Correlation Matrix	Bitcoin	Litecoin	Ripple	Dash
Bitcoin	1.0000			
Litecoin	0.5202	1.0000		
Ripple	0.2465	0.5588	1.0000	
Dash	0.1658	0.1415	0.0294	1.0000

Figures

Figure 1: The out-of-sample Sharpe ratio over time of the advanced portfolio optimization model and the 2 benchmarks (1/N and Markowitz), inclusive of transaction costs, for a 110-week rolling window. Short selling is prohibited.
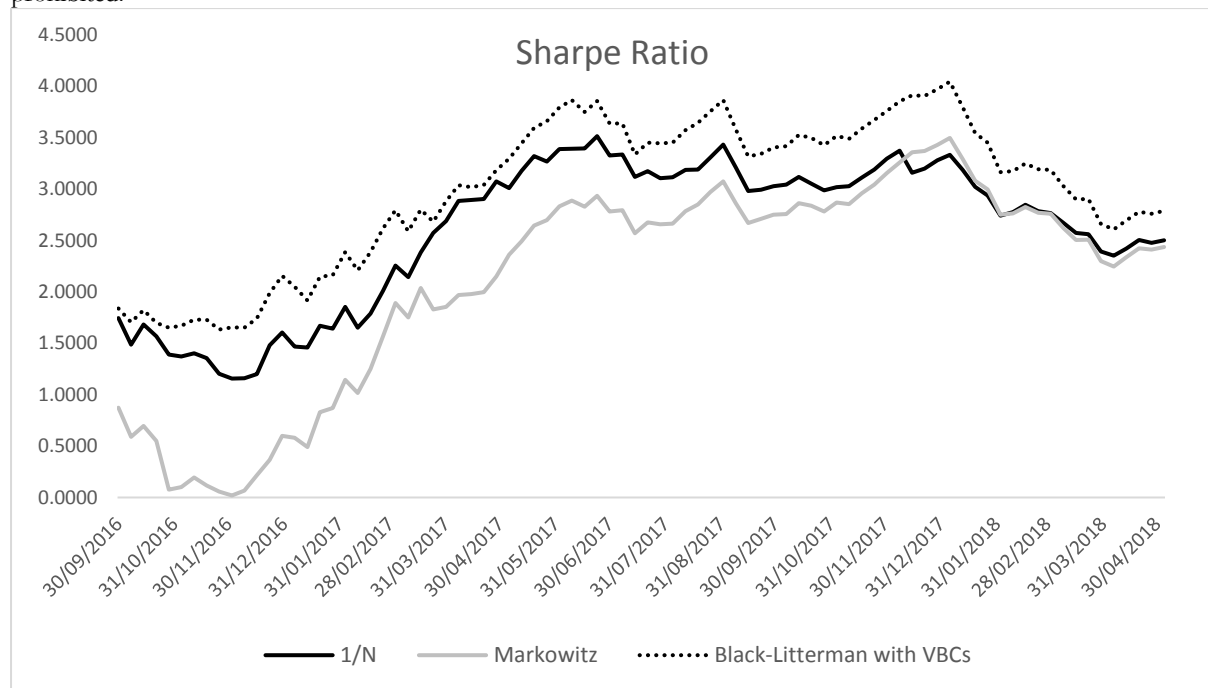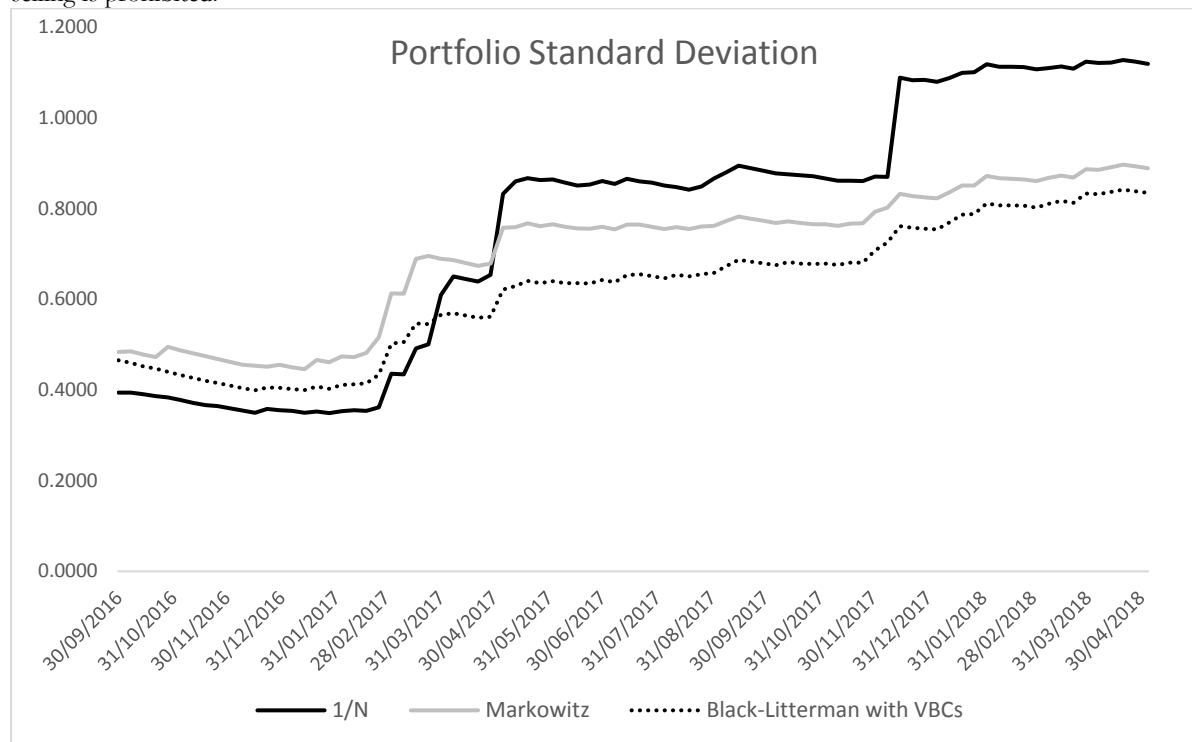

Figure 2: The out-of-sample portfolio standard deviation over time of the advanced portfolio optimization model and the 2 benchmarks (1/N and Markowitz), inclusive of transaction costs, for a 110-week rolling window. Short selling is prohibited.

Figure 3: The out-of-sample Sharpe ratio over time of the advanced portfolio optimization model and the 2 benchmarks (1/N and Markowitz), for a 110-week rolling window. The proportional transaction cost estimates are set to 100 bps (100% increase).
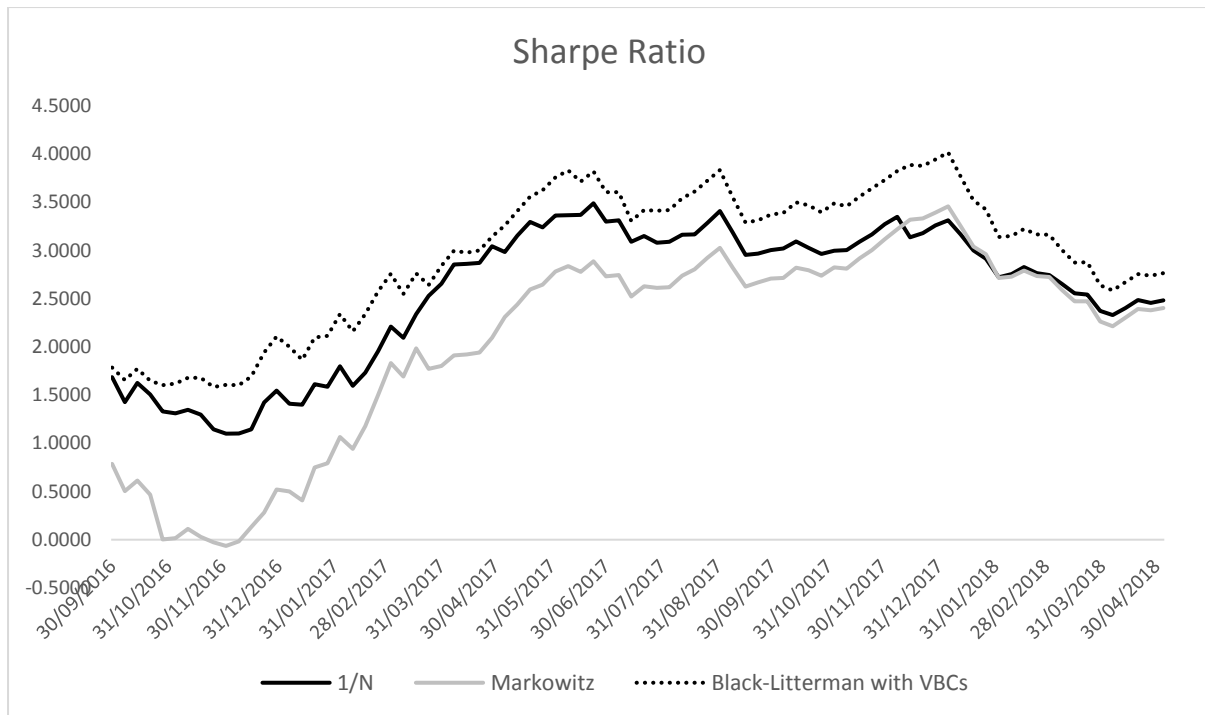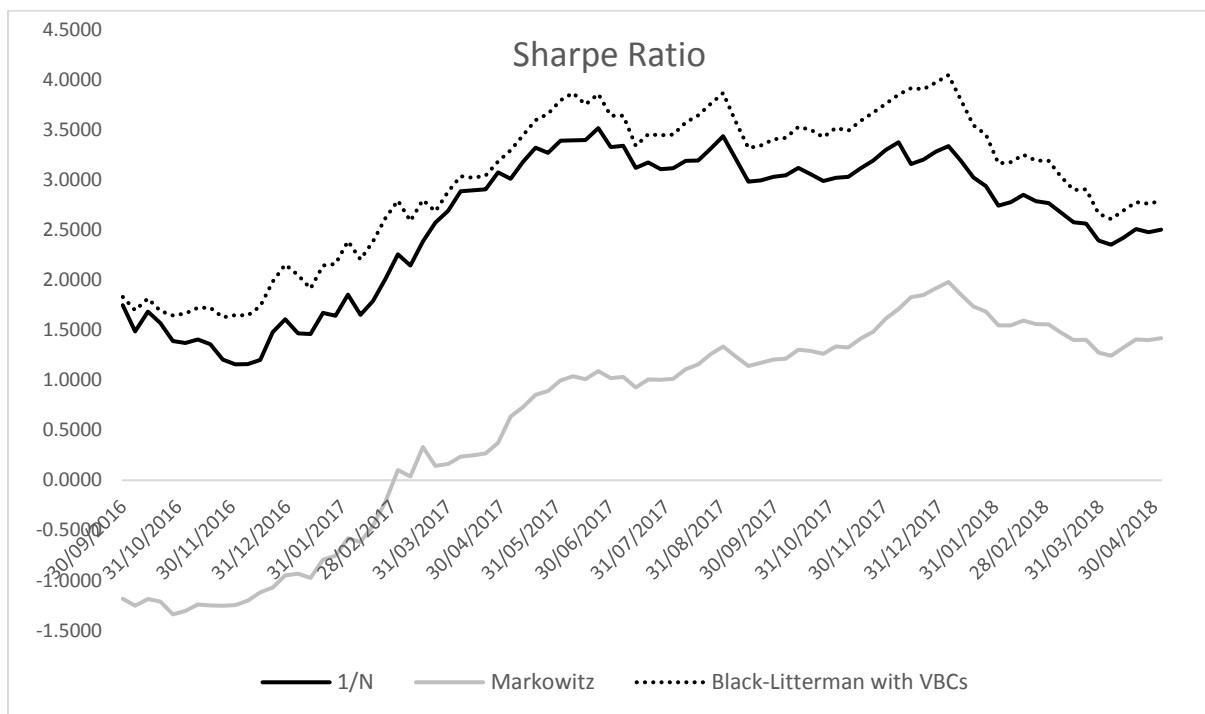

Figure 4: The out-of-sample Sharpe ratio over time of the advanced portfolio optimization model and the 2 benchmarks (1/N and Markowitz) by allowing for short-selling, inclusive of transaction costs, and for a 110-week rolling window.

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