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(revised December 1974)

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In recent years interest in applying quantitative methodologies to policy-related problems has increased markedly throughout the social sciences. This paper outlines and contrasts two approaches to estimating the effects of reforms, policy innovations, and similar discontinuous "interventions" or "treatments" on phenomena that are observed through time. The sorts of substantive problems for which the intervention analysis techniques developed here are applicable include, for example, the impact of the introduction or repeal of capital punishment on murder rates; the effect of government incomes policies on wage and price inflation; and the contribution of women's suffrage, personal registration and residency laws, new ballot forms, and so on to the secular decline in American electoral turnout since the 1890's.

The first scheme for intervention analysis treated in this paper is based on the time-series models of Box, Jenkins, and Tiao (Box and Jenkins, 1970; Box and Tiao, 1965, 1973). This approach owes much to the conceptual work of D.T. Campbell (Campbell, 1963, 1969; Campbell and Stanley, 1966) which emphasizes that post-hoc time-series analysis can be viewed quasi-experimentally to evaluate the impact of interventions by government agencies

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* I am grateful to James Bennett, Arthur Goldberger, and Peter Lemieux for comments on an earlier draft of this paper.

and other institutional actors. Box-Tiao and Box-Jenkins models[1] represent time-series observations as the realization of a linear stochastic process of autoregressive, moving average, or mixed autoregressive - moving average form. Hence no attempt is made to model the causal structure generating the time-series data. Intervention occurrences are represented by binary variables (0,1) and intervention effects (changes in the slope and/or level of the time-series) are assessed by estimating intervention "transfer functions."

The second approach to intervention analysis considered here is the so-called structural equation method. Structural techniques were developed primarily in economics but have subsequently gained wide acceptance in all of the social sciences. (See, for example, Blalock, ed., 1971; Goldberger, 1972; and Goldberger and Duncan, eds., 1973.) The principal difference between structural equation and Box-Tiao modelers is that the former specify and estimate intervention effects in the context of a system of equations designed to represent the causal relationships underlying the realizations of endogenous time-series. The structural approach is therefore geared to determining how and to what extent reforms or policy innovations influence endogenous phenomena as they are transmitted through a dynamic causal structure.

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[1] These models have been explicitly linked to Campbell's methodological perspective by the educational methodologists Glass, Gottman, Maguire and Willson (Glass, 1968, 1972; Glass, Willson, and Gottman, 1972; Maguire and Glass, 1967). A number of studies by political scientists have also employed Campbell's perspective, but these analyses have relied on statistical procedures that have weak justification in the time-series context. See, for example, Caporaso and Pelowski, 1971; and Duvall and Welfling, 1973, which are conveniently collected along with related studies in Caporaso and Roos, eds., 1973.

We develop what appear to be lines of convergence between the two approaches in the final section.

I. The Box-Tiao (Box-Jenkins) Approach

Imagine that we are dealing with a time-series of equally spaced observations on some endogenous or dependent variable Y_t, $t = 1, 2, \ldots, T$, and that we want to determine the impact of some exogenous treatment or policy motivated intervention that occurs, say, at the nth period, $Y_1, \ldots, Y_n, \ldots, Y_t$. The Box-Tiao approach employs a model of the general form

$$(1) \quad Y_t = f(\underline{\kappa}, \underline{I}) + N_t \qquad t = 1, 2, \ldots, T$$

where $f(\underline{\kappa}, \underline{I})$ denotes an unspecified function of unknown parameters and associated binary intervention variables, respectively, and N_t denotes stochastic noise and deterministic time effects.[2]

Suppose that the intervention occurring at the nth period is sustained thereafter. Only the pre-intervention Y_t series is therefore driven entirely by stochastic noise and deterministic time trends. Hence:

$$(2) \quad N_t = Y_t - f(\underline{\kappa}, \underline{I}) = Y_t \qquad \text{for } t < n.$$

Since these Y_t observations are unperturbed by external intervention, they may be analyzed to determine a time-series model for the N_t component of

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[2] Actually Box and Tiao (1973) include deterministic time effects (functions of time) in the first term on the right-hand side of (1), rather than in the N_t component. The specification given here will make matters clearer later on.

(1).[3] This N_t model provides a stochastic benchmark against which the intervention effects function $f(\underline{\kappa}\ \underline{I})$ can be specified and estimated.
In the Box-Tiao framework the N_t process takes the form of an autoregressive, moving average, or mixed autoregressive - moving average model of order p, d, q:

(3) $\phi_p(B)(1 - B)^d Y_t = \theta_0 + \theta_q(B)a_t$

where: B is a backshift operator such that $BY_t = Y_{t-1}$, $B^i Y_t = Y_{t-i}$,

$(1 - B)^d = \nabla^d$ is a backward difference operator such that

$(1 - B)Y_t = Y_t - Y_{t-1} = \nabla Y_t$, $(1 - B)^2\ Y_t(1 - 2B + B^2)Y_t$

$= \nabla Y_t - \nabla Y_{t-1} = \nabla^2 Y_t$, etc.,

$\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$

$\theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$ are autoregressive and moving average polynomials in B of order p and q, respectively (with roots outside the unit circle),

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[3] In some situations the model for N_t can be developed by analyzing all Y_t observations that are not in the immediate vicinity of the initial intervention point, Y_n. This would apply, for example, in cases in which a sustained intervention is believed to produce an immediate change in the level of the series, or in cases in which a one-shot, nonsustained intervention produces an effect that dies out quickly. See the discussion below.

θ_0 is a constant which indexes a deterministic polynomial time trend of degree d in the Y_t. (Notice that this implies that $\nabla^d Y_t$ has a non-zero mean equal to $\theta_0/1 - \phi_1 - \ldots - \phi_p$), and

a_t is a sequence of independently distributed, random variables with mean zero and variance σ_a^2.

Defining Y_t^* as the d-th difference of Y_t, so that $Y_t^* = (1 - B)^d Y_t = \nabla^d Y_t$, the autoregressive - moving average model may be written in the familiar form of a multiple regression equation

$$(4) \quad \phi_p(B)Y_t^* = \theta_0 + \theta_q(B)a_t$$

$$(1 - \phi_1 B - \ldots - \phi_p B^p)Y_t^* = \theta_0 + (1 - \theta_1 B - \ldots - \theta_q B^q)a_t$$

$$Y_t^* - \phi_1 Y_{t-1}^* - \ldots - \phi_p Y_{t-p}^* = \theta_0 + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q}$$

$$Y_t^* = \phi_1 Y_{t-1}^* + \ldots + \phi_p Y_{t-p}^* + \theta_0 + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q}$$

$$Y_t^* = \theta_0 + \sum_p \phi_p Y_{t-p}^* - \sum_q \theta_q a_{t-q} + a_t .$$

Despite the somewhat formidable notation, equation (4) shows that the Box-Tiao scheme for the stochastic noise component in the general intervention model of (1) is actually quite simple. It asserts that the d-th difference (∇^d) of the unperturbed Y_t series, denoted Y_t^*, is generated by a linear combination of autoregressive terms and moving average shocks. Hence, Y_t^* depends on p lagged terms Y_{t-p}^* with coefficients $(\phi_1, \ldots, \phi_p)$ and on a moving linear sum of q random shocks a_t with coefficients

$(1, -\theta_1, \ldots, -\theta_q)$. A nonzero constant term θ_0 accomodates deterministic time trends of order d in the undifferenced Y_t, i.e., if $Y_t = \mu + \theta_0 t^d$ + stochastic terms, then $Y_t^* = \nabla^d Y_t = \theta_0$ + stochastic terms.

The first task in the Box-Tiao method is to derive a model for the N_t component in (1) by fitting an appropriately specified version of (3) or (4) to the Y_t observations that are not perturbed by external interventions. As developed by Box and Jenkins (1970), this involves an iterative process of tentative model specification, preliminary estimation, a series of diagnostic checks, possible model respecification, and so on.

Specification

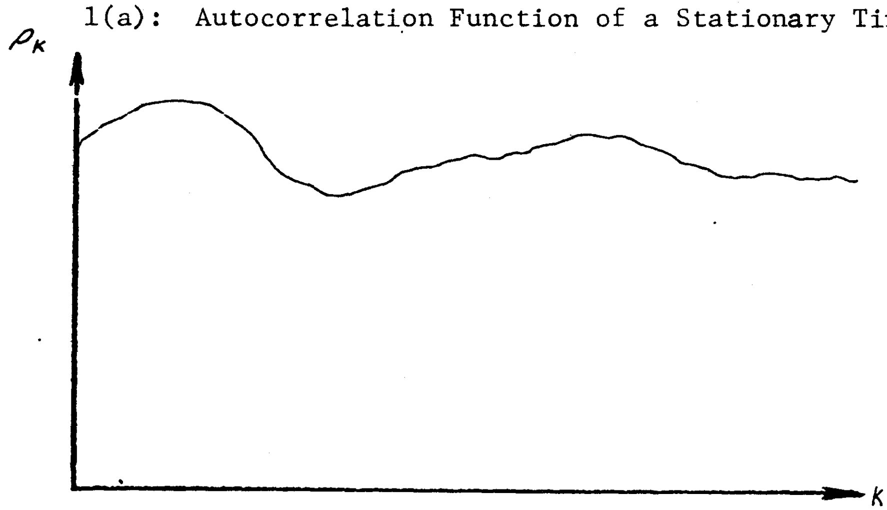
The basic noise model in (3) or (4) is fully specified by choosing the degree of differencing, d, the order of the autoregressive component, p, and the order of the moving average component, q.

The degree of differencing, d, is chosen such that the differenced series is stationary and hence varies about a fixed mean or equilibrium level with variance independent of displacements in time and autocovariance dependent only on the magnitude of lags in time. Stationarity therefore means that $E(Y_t) = E(Y_{t-m})$ and $\mathrm{Cov}(Y_t, Y_{t-k}) = \mathrm{Cov}(Y_{t-m}, Y_{t-m-k})$ for all t, k, and m. A stationary series will exhibit an autocorrelation function (ρ_k) that dies out after moderate-to-large lag. (A "large" lag is on the order of k = T/5.) Figure 1 shows hypothetical examples of the autocorrelation functions of a nonstationary and a stationary time-series. Sample estimates of the lag k autocorrelations $(\hat{\rho}_k)$ are given by

$$\hat{\rho}_k = \frac{\sum_{t=k}^{T} (Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{T} (Y_t - \bar{Y})^2} \qquad k = 1, 2, \ldots, T/5$$

Figure 1: HYPOTHETICAL AUTOCORRELATION FUNCTIONS FOR NON-STATIONARY AND STATIONARY TIME-SERIES

1(a): Autocorrelation Function of a Stationary Time-Series



1(b): Autocorrelation Function of a Non-Stationary Time-Series and those of successive differences, $\nabla^d Y_t$, $d = 1, 2, \ldots,$ are calculated analogously. In practice it is rarely necessary that d exceed 2 and, typically, $d = 1$ is sufficient to induce stationary behavior.[4]

Having settled upon a degree of differencing sufficient to ensure stationarity, the orders of the moving average and autoregressive components of (3) are tentatively specified by comparing the sample autocorrelation and partial autocorrelation functions of $\nabla^d Y_t$ to the theoretical functions of various autoregressive-moving average models. The theoretical behavior of autocorrelation and partial autocorrelation functions, denoted as ρ_k and ϕ_{kk}, respectively, are readily derived through algebraic manipulation of (4) for varying values of p and q. Such manipulations show that:

(1) Purely autoregressive processes of order p [AR(p)] have autocorrelation functions that tail off and partial autocorrelation functions that cut off after lag p. Hence, ρ_k tails off and $\phi_{kk} = 0$ for $k > p$ in autoregressive models.

(2) Purely moving average processes of order q [MA(q)] have autocorrelation functions that cut off after lag q and partial autocorrelation functions that tail off. Hence, $\rho_k = 0$ for $k > q$ and ϕ_{kk} tails off in moving average models.

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[4] All homogeneous nonstationary series will exhibit stationary behavior after suitable differencing, that is, the autocorrelations of the differences $\nabla^d Y_t$ will "damp off", "tail off" or "cut off" as the lag k becomes large. Occasionally it may be necessary to apply a transformation to the Y_t in order to obtain a well-behaved series. For example, a series driven by an exponential function of time is nonhomogeneous nonstationary and therefore should be logarithmically transformed prior to specification and estimation.

(3) Mixed autoregressive-moving average processes of order p,q [ARMA(p,q)] have autocorrelation functions that are a mixture of exponential and damped sine waves after the first q-p lags and partial autocorrelation functions that are dominated by a mixture of exponentials and damped sine waves after the first p-q lags. Hence, neither ρ_k nor ϕ_{kk} cut off in mixed models.

Since AR, MA, and ARMA time-series models are distinguishable by their autocorrelation and partial autocorrelation functions, sample estimates of these functions facilitate preliminary identification of p and q and permit calculation of initial values of the parameters ϕ_p and θ_q. Figure 2 and Table 1 (derived from Box and Jenkins, 1970) put this into somewhat sharper focus by displaying the autocorrelation functions, partial autocorrelation functions, and related theoretical properties of some simple autoregressive, moving average, and mixed autoregressive-moving average models.

Estimation

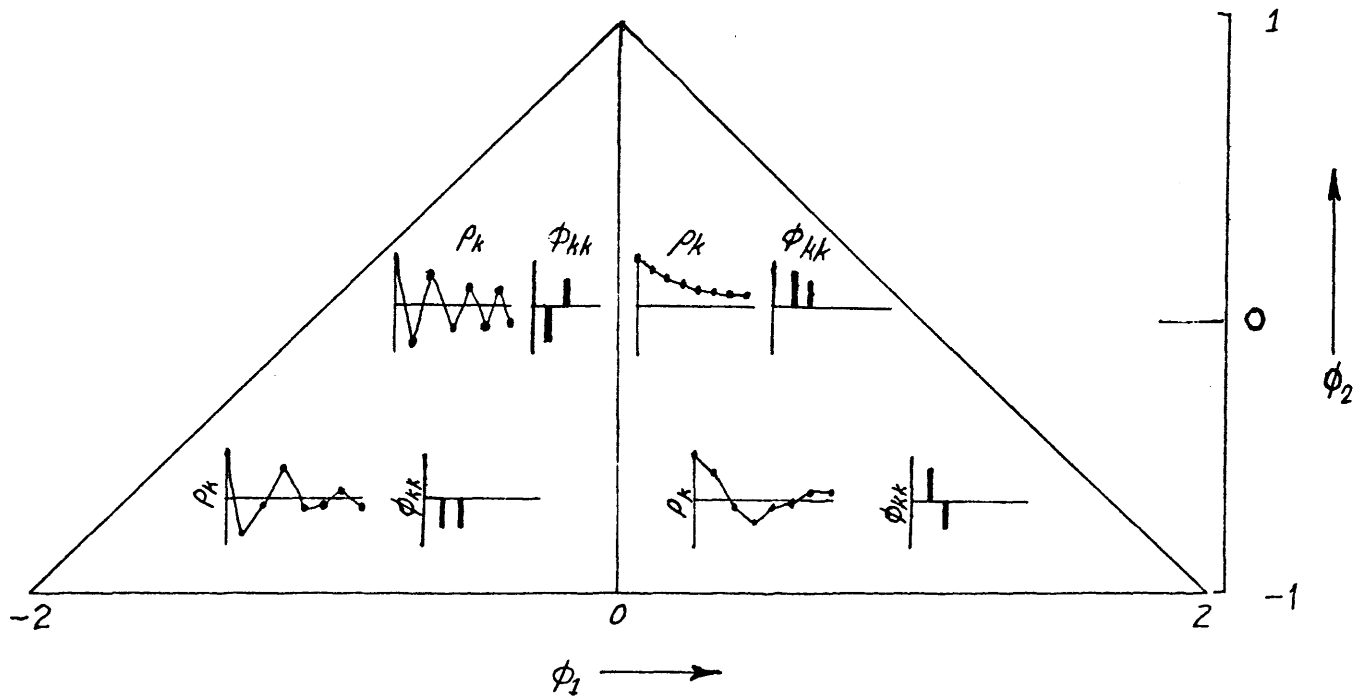
The specification process outlined above leads to a tentative choice of p, d, and q, and yields preliminary guesses of the parameters ϕ_p and θ_q. Recall that the autoregressive-moving average model is written most generally as

(5) $\phi_p(B)\ \nabla^d Y_t = \theta_0 + \theta_q(B) a_t.$

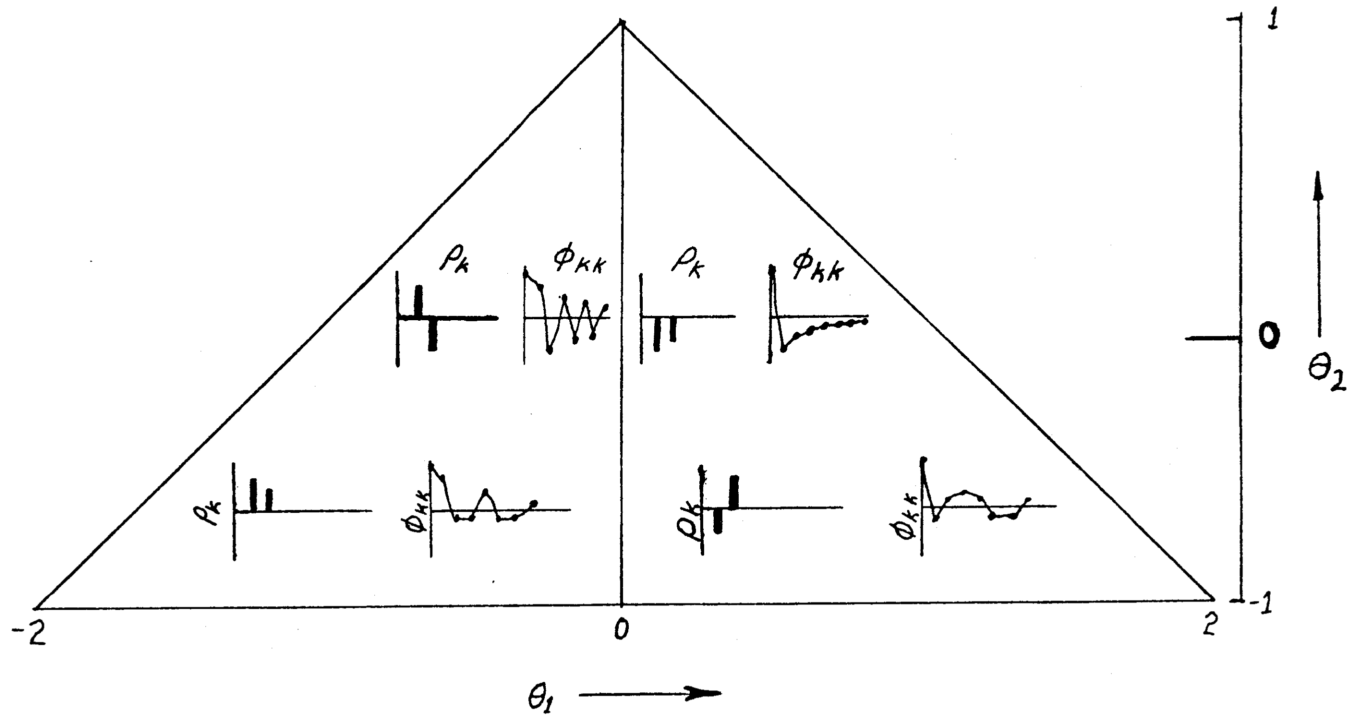
Again, let $Y_t^* = \nabla^d Y_t = (1 - B)^d Y_t$, and rewrite (5) as

(6) $a_t = \theta_q^{-1}(B)[\theta_p(B)\ Y_t^* - \theta_0]$

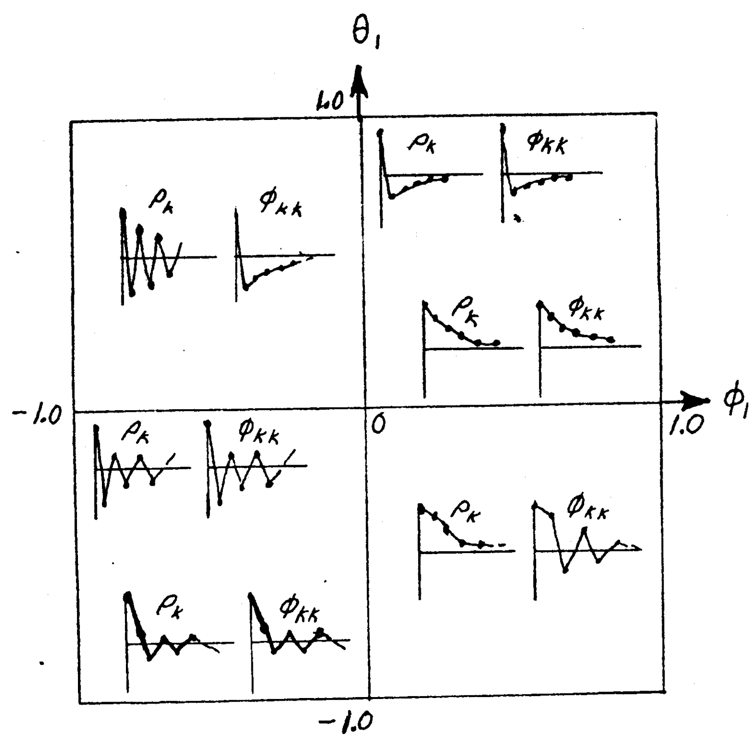
Figure 2: TYPICAL AUTOCORRELATION (ρ_k) AND PARTIAL AUTOCORRELATION (ϕ_{kk}) FUNCTIONS FOR VARIOUS STATIONARY AR, MA, AND ARMA MODELS



2(a): Autocorrelation and Partial Autocorrelation Functions for Various AR(2) Models



2(b): Autocorrelation and Partial Autocorrelation Functions for Various MA(2) Models



2(c): Autocorrelation and Partial Autocorrelation Functions for Various ARMA(1,1) Models

TABLE 1: SOME PROPERTIES OF SIMPLE ARMA MODELS OF ORDER (p, q)

Order	AR (1)	MA (1)
Behavior of ρ_k	$\rho_k = \phi_1^k$	$\rho_k = 0$, for $k > 1$
Behavior of ϕ_{kk}	$\phi_{kk} = 0$, for $k > 1$	tails off, dominated by damped exponential
Preliminary estimates from	$\phi_1 = \rho_1$	$\rho_1 = -\theta_1/(1 + \theta_1^2)$
Admissible region	$-1 < \phi_1 < 1$	$-1 < \theta_1 < 1$

Order	AR (2)	MA (2)
Behavior of ρ_k	$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2}$ (mixture of exponentials or damped sine wave)	$\rho_k = 0$, for $k > 2$
Behavior of ϕ_{kk}	$\phi_{kk} = 0$, for $k > 2$	tails off, mixture of exponentials or damped sine wave
Preliminary estimates from	$\phi_1 = \dfrac{\rho_1(1 - \rho_2)}{1 - \rho_1^2}$	$\rho_1 = \dfrac{-\theta_1(1 - \theta_2)}{1 + \theta_1^2 + \theta_2^2}$
	$\phi_2 = \dfrac{\rho_2 - \rho_1^2}{1 - \rho_1^2}$	$\rho_2 = \dfrac{-\theta_2}{1 + \theta_1^2 + \theta_2^2}$
Admissible region	$-1 < \phi_2 < 1;\ \phi_2 + \phi_1 < 1;$	$-1 < \theta_2 < 1;\ \theta_2 + \theta_1 < 1;$
	$\phi_2 - \phi_1 < 1$	$\theta_2 - \theta_1 < 1$

Order	ARMA (1, 1)	
Behavior of ρ_k	$\rho_k = \phi_1^{k-1}\ \rho_1$, for $k > 1$ (decays exponentially after first lag)	
Behavior of ϕ_{kk}	$\phi_{11} = \rho_1$, thereafter tails off dominated by damped exponential	
Preliminary estimates from	$\rho_1 = \dfrac{(1 - \phi_1\theta_1)\ (\phi_1 - \theta_1)}{1 + \theta_1^2 - 2\phi_1\theta_1}$	$\rho_2 = \rho_1\phi_1$
Admissible region	$-1 < \phi_1 < 1;\ -1 < \theta_1 < 1$	

thereby obtaining an equation for the (independently distributed) "error" term a_t. The model is estimated by choosing $(\phi_p, \theta_q, \theta_0)$ in the admissible parameter space [5] such that the sum of the squared errors

(7) $S(\phi_p, \theta_q, \theta_0) = \Sigma a_t^2$ is minimized.

Estimates $(\hat{\phi}_p, \hat{\theta}_q, \hat{\theta}_0)$ corresponding to a minimum of (7) are least-squares estimates, and evaluating (6) at $(\hat{\phi}_p, \hat{\theta}_q, \hat{\theta}_0)$ generates the residuals $\hat{a}_t$. (Notice that we ignore the problem of initializing the series.) In practice, the minimization of (7) may be undertaken by a number of acceptable nonlinear least-squares procedures, such as grid search, steepest descent, successive

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[5] Admissibility requires that the roots of the characteristic equations

$\phi_p(B) = 1 - \phi_1 B - \ldots - \phi_p B^p = 0,$

$\theta_q(B) = 1 - \theta_1 B - \ldots - \theta_q B^q = 0$

(with B treated as an algebraic quantity) have roots outside the unit circle, i.e., the solutions

$B_1, B_2, \ldots, B_p$

$B_1, B_2, \ldots, B_q$

must all be greater than one in absolute value. This means that the process is stationary (if autoregressive) and invertible (if moving average) and therefore converges to an equilibrium level. Notice that Table 1 gives the admissible coefficient values for some simple ARMA models. For further discussion see Box and Jenkins, 1970 or Nelson, 1973a.

linearizations or some combination thereof.[6] (Marquardt's (1963) compromise between the latter two methods has been popular.)

Diagnostic Checks

If the fitted model is adequate, then the calculated residuals $\hat{a}_t$ should behave as independently distributed random variates. This may be formally tested by computing the residual autocorrelations

$$r_k(\hat{a}) = \frac{\Sigma\, \hat{a}_t\, \hat{a}_{t-k}}{\Sigma\, \hat{a}_t^2}$$

and evaluating the test statistic (developed by Box and Pierce, 1970)

$$Q = (T-d) \sum_{k=1}^{K} r_k^2\, (\hat{a}) \qquad\qquad K \geq 20$$

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[6] Computer programs for Box-Jenkins ARMA model specification, estimation, and forecasting are described in Box and Jenkins, 1973, appendix (batch process programs are distributed by the Data and Program Library Service, Social Systems Research Institute, University of Wisconsin, Madison); Nelson, 1973a, appendix (batch process programs available by writing to the author, Professor C.R. Nelson, Graduate School of Business, University of Chicago); TSP/DATATRAN manual (Cambridge Project, M.I.T., interactive computer system accessible via the ARPA national network); and the TROLL Reference Manual (available from Support Staff Coordinator, NBER Computer Research Center, 575 Technology Square, Cambridge, Mass., interactive computer system accessible via the NBER's national network).

which for large K is distributed as χ^2 with (K-p-q) degrees of freedom. Q serves as a general or "portmanteau" criterion of model adequacy. A large value is evidence of significant lack of fit and indicates that model respecification is necessary. Patterns in the residual autocorrelations are usually informative about the nature of the misspecification and should be analyzed along the lines proposed earlier for specification of p and q.

A well-specified ARMA model should of course also satisfy more conventional statistical criteria of adequacy. Thus the coefficient estimates $\hat{\phi}_p$, $\hat{\theta}_q$, $\hat{\theta}_0$ should be significantly different from zero, and the estimated error variance, $\hat{\sigma}_a^2$, should be less than that of alternative ARMA specifications.

Intervention Transfer Functions

The techniques outlined so far pertain to the specification, initial estimation, and diagnostic checking of the N_t component of the general Box-Tiao model in (1). As I noted earlier, the N_t process provides a stochastic benchmark against which intervention-induced changes in the slope and/or level of the endogenous Y_t series can be determined. Recall that the intervention effects part of the general model was expressed as $f(\underline{\kappa}, \underline{I})$, and that for the moment attention is confined to the case of a single intervention occurring at the n-th period, which is sustained thereafter. Such an intervention would be represented by the binary variable,

$$I_t = 0 \qquad \text{for } t < n$$

$$= 1 \qquad \text{for } t \geq n.$$

Let $\underline{y}_t$ denote the "dynamic transfer" from I_t to Y_t, that is the influence of the intervention on the endogenous variable, and replace $f(\underline{\kappa}, \underline{I})$ in (1) with

(8) $\underline{y}_t = \frac{\omega(B)}{\delta(B)} I_{t-b}$

where: $\omega(B) = \omega_0 - \omega_1 B - \dots - \omega_s B^s$

$\delta(B) = 1 - \delta_1 B - \dots - \delta_r B^r$

are polynomials in B of degree s and r,

b is a delay (lag) parameter, and the system is stable.[7]

Hence, the impact of the intervention or the "transfer" from I_t is represented by the linear difference equation

(9) $\delta(B)\, \underline{y}_t = \omega(B)\, I_{t-b}$

$$\underline{y}_t - \delta_1 \underline{y}_{t-1} - \dots - \delta_r \underline{y}_{t-r} = \omega_0 I_{t-b} - \omega_1 I_{t-b-1} - \dots - \omega_s I_{t-b-s}$$

$$\underline{y}_t = \delta_1 \underline{y}_{t-1} + \dots + \delta_r \underline{y}_{t-r} + \omega_0 I_{t-b} - \omega_1 I_{t-b-1} - \dots - \omega_s I_{t-b-s}$$

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[7] Stability requires that the roots of the characteristic equation

$\delta(B) = 1 - \delta_1 B - \dots - \delta_r B^r = 0$

(with B treated as an algebraic quantity) lie outside the unit circle, i.e., the solutions $B_1, B_2, \dots, B_r$ must all be greater than unity in absolute value, which implies that the system eventually converges to an equilibrium level. This exactly parallels the stationarity and invertibility conditions of the ARMA model in (3) and means that the admissible regions for the parameters are the same as those given in Table 1.

which admits a wide variety of possibilities. Notice that when the intervention is sustained indefinitely ($I_t = 1$ for all $t \geq n$), the response or effect will eventually reach the equilibrium or steady state value[8]

$$\frac{\omega_0 - \omega_1 - \dots - \omega_s}{1 - \delta_1 - \dots - \delta_r} .$$

Figure 3 shows a few simple examples of transfer function responses (taken from Box and Tiao, 1973) for s = 0; r = 0, 1; and b = 1. Suppose that the (sustained) intervention is anticipated to produce a change in the level of the endogenous series immediately following a one period delay. The appropriate transfer function would be

(10) $\underline{y}_t = \omega_0 I_{t-1}$. (Figure 3a)

An intervention that generated a gradual change in the level of a series could be represented by the model

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[8] Since $\underline{y}_t$ is a stable or stationary process $E(\underline{y}_t) = E(\underline{y}_{t-1}) = \dots = E(\underline{y}_{t-r})$ equals a constant, say $\underline{y}^*$. Taking $\underline{y}^*$ as the initial conditions of (9) gives

$$\underline{y}^* - \delta_1 \underline{y}^* - \dots - \delta_r \underline{y}^* = \omega_0 I_{t-b} - \dots - \omega_s I_{t-b-s}.$$

Hence, if I_t is held indefinitely at the value +1, then

$$\underline{y}^* = \frac{\omega_0 - \omega_1 - \dots - \omega_s}{1 - \delta_1 - \dots - \delta_r}$$

is the equilibrium value of $\underline{y}_t$. Structural equation modelers will recognize this as the equilibrium multiplier, which is discussed in the next section.

(11) $$\underline{y}_t = \delta_1 \underline{y}_{t-1} + \omega_0 I_{t-1}$$

$$\underline{y}_t(1 - \delta_1 B) = \omega_0 I_{t-1} \qquad \text{(Figure 3b)}$$

$$\underline{y}_t = \frac{\omega_0}{1 - \delta_1 B} I_{t-1}$$

in which the rate of adjustment to a new equilibrium depends on δ_1. A slope change intervention effect can be represented by taking δ_1 to unity, which gives

(12) $$\underline{y}_t = \underline{y}_{t-1} + \omega_0 I_{t-1}$$

$$\underline{y}_t(1 - B) = \omega_0 I_{t-1} \qquad \text{(Figure 3c)}$$

$$\underline{y}_t = \frac{\omega_0}{1 - B} I_{t-1}.$$

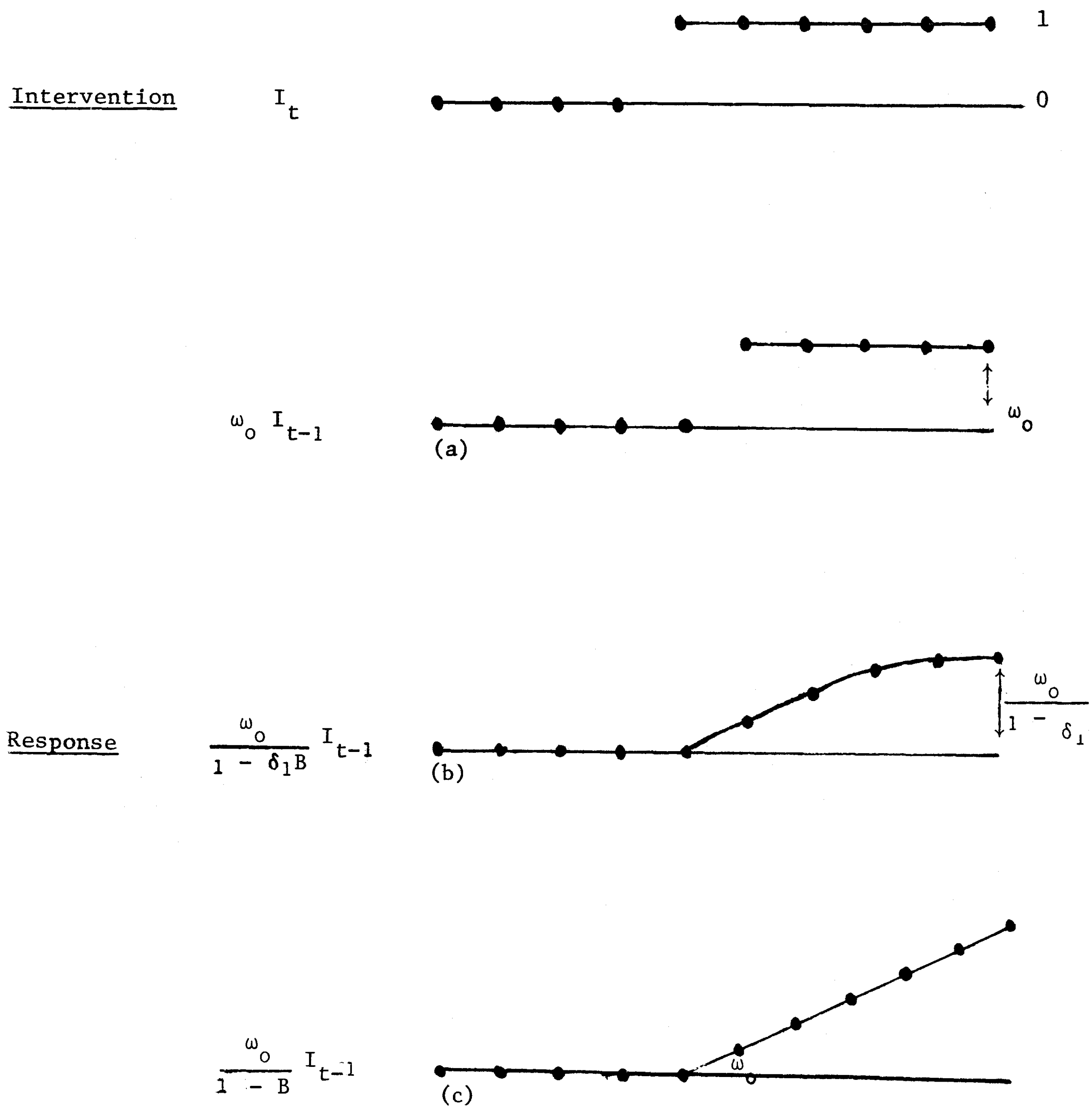
This model never adjusts to a new equilibrium level and might be used to characterize empirical situations in which convergence is very slow and occurs far beyond the period of observation.

After a theoretically plausible intervention transfer function has been specified (in practice, several alternatives might be entertained), it should be adjoined to the noise model whose functional form has been established by the procedures outlined previously. The parameters of the complete model can then be estimated simultaneously in order to make inferences about the impact of interventions. For example, given the general model

(13) $$Y_t = \underline{y}_t + N_t$$

$$= \frac{\omega(B)}{\delta(B)} I_{t-b} + \frac{\theta_0 + \theta_q(B)a_t}{\phi_p(B)(1 - B)^d} \qquad \text{(by eqs. 3 and 8)}$$

FIGURE 3: RESPONSES TO A SUSTAINED INTERVENTION FOR SOME SIMPLE TRANSFER FUNCTIONS



suppose that analysis of the unperturbed Y_t series establishes that the noise process satisfies a first-order moving average in the first difference

$$(14) \quad \nabla N_t = a_t - \theta_1 a_{t-1}$$

$$(1 - B)N_t = (1 - \theta_1 B)a_t$$

$$N_t = \frac{1 - \theta_1 B}{1 - B} a_t \ .$$

Furthermore, suppose that the intervention event is believed to have produced a gradual change in the level of the series which commences after a lag of 1 period and is parsimoniously expressed as a first-order dynamic response:

$$(15) \quad \underset{\sim}{y}_t = \delta_1 \underset{\sim}{y}_{t-1} + \omega_0 I_{t-1}$$

$$= \frac{\omega_0}{1 - \delta_1 B} I_{t-1} \ .$$

This leads to the full model

$$(16) \quad Y_t = \frac{\omega_0}{1 - \delta_1 B} I_{t-1} + \frac{1 - \theta_1 B}{1 - B} a_t \qquad t = 1, 2, \ldots T \ .$$

Clearing (16) of denominators (by treating the shift operator B as an algebraic quantity) yields

$$(17) \quad (1 - B)(1 - \delta_1 B)Y_t = \omega_0(1 - B)I_{t-1} + (1 - \delta_1 B)(1 - \theta_1 B)a_t$$

$$(1 - B - \delta_1 B + \delta_1 B^2)Y_t = \omega_0(1 - B)I_{t-1} + (1 - \theta_1 B - \delta_1 B + \delta_1 \theta_1 B^2)a_t$$

$$Y_t - Y_{t-1} - \delta_1 Y_{t-1} + \delta_1 Y_{t-2} = \omega_0(1 - B)I_{t-1} + a_t$$
$$- \theta_1 a_{t-1} - \delta_1 a_{t-1} + \delta_1 \theta_1 a_{t-2}$$

$$\nabla Y_t = \delta_1 \nabla Y_{t-1} + \omega_0 \nabla I_{t-1} + a_t - (\theta_1 + \delta_1)a_{t-1}$$
$$+ \delta_1 \theta_1 a_{t-2},$$

which can be estimated from the entire time-series by using the nonlinear least-squares methods noted earlier. In particular, (17) implies regressing ∇Y_t on ∇Y_{t-1} and ∇I_{t-1} in the presence of a second-order moving average error [MA(2)] and, therefore, may be approached as a generalized least squares estimation problem (cf. Hibbs, 1974). Since the errors $\hat{a}_t$ from the generalized estimation equation are assumed to be white noise,[9] hypothesis tests may be undertaken in the usual way.

Finally, the Box-Tiao scheme readily accommodates multiple interventions, a wide variety of effect patterns (transfer function responses), and seasonal or cyclical movements in a time-series. As an illustration of a more complex problem, consider a situation in which two sustained interventions were initiated at different periods and are hypothesized to have produced a level change and a slope change, respectively, in the endogenous variable. Let these interventions be represented by the binary variables

$$I_{1t} = 0 \qquad \text{for } t < n$$
$$= 1 \qquad \text{for } t \geq n$$
$$I_{2t} = 0 \qquad \text{for } t < n + k$$
$$= 1 \qquad \text{for } t \geq n + k$$

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[9] Diagnostic checks applied to the residuals test this assumption and serve also to evaluate the overall adequacy of the model. Cf. the previous discussion.

and specify the hypothetical transfer function

$$(18)\quad \underline{y}_t = \omega_{01} I_{1t} + \frac{\omega_{02}}{1 - B} I_{2t} .$$

Equation (18) allows an immediate change in the level of the endogenous series (of magnitude ω_{01}), which commences at time n, and a slope change (defined by the parameter ω_{02}), which commences at time n + k. If we again let the noise process governing the stochastic behavior of the Y_t series to be given by (14), we arrive at the complete model

$$(19)\quad Y_t = \underline{y}_t + N_t$$

$$Y_t = \omega_{01} I_{1t} + \frac{\omega_{02}}{1 - B} I_{2t} + \frac{1 - \theta_1 B}{1 - B} a_t .$$

Rewriting (19) yields an estimating equation with ∇Y_t as a function of ∇I_{1t}, I_{2t} and a first-order moving average disturbance

$$(20)\quad (1 - B) Y_t = \omega_{01}(1 - B) I_{1t} + \omega_{02} I_{2t} + (1 - \theta_1 B) a_t$$

$$\nabla Y_t = \omega_{01} \nabla I_{1t} + \omega_{02} I_{2t} + a_t - \theta_1 a_{t-1} .$$

Equation (20) is estimable by standard nonlinear, generalized least-squares techniques and thus the level change and slope coefficients, ω_{01} and ω_{02}, are readily determined.

If the pre-intervention series or noise component N_t exhibited a linear time trend, the post-intervention slope coefficient ω_{02} would index the time trend <u>change</u> induced by the intervention event I_{2t}. For example, suppose that

$$(21) \quad N_t = \mu + \theta_0 t + \frac{1 - \theta_1 B}{1 - B} a_t \ ,$$

where $\theta_0 t$ denotes a deterministic linear trend. Notice that (21) implies the model

$$(22) \quad (1 - B)N_t = \theta_0 + (1 - \theta_1 B)a_t$$

$$N_t = \frac{\theta_0}{1 - B} + \frac{1 - \theta_1 B}{1 - B} a_t \ ,$$

which is compatible with the ARMA notation introduced previously. Adjoining (22) to the hypothetical intervention transfer function of (18) gives the complete model

$$(23) \quad Y_t = \omega_{01} I_{1t} + \frac{\omega_{02}}{1 - B} I_{2t} + \frac{\theta_0}{1 - B} + \frac{1 - \theta_1 B}{1 - B} a_t$$

$$\nabla Y_t = \omega_{01} \nabla I_{1t} + \omega_{02} I_{2t} + \theta_0$$

$$+ a_t - \theta_1 a_{t-1} \ .$$

The Y_t series therefore trends linearly at a rate of θ_0 units per period prior to the intervention event I_{2t}, whereas, after I_{2t} the series trends at the average rate of $(\theta_0 + \omega_{02})$ units per period.

Extensions of the basic underlying model to even more complex situations involving multiple interventions sustained over varying periods (including 1 period, "pulse" interventions) are handled straightforwardly by appropriate application of previous results.

II. The Structural Equation Approach

In contrast to the Box-Tiao scheme, which employs a sophisticated noise model as the point of reference for assessing intervention effects, the structural equation approach attempts to represent explicitly the behavioral processes generating movements in endogenous variables. Stochastic noise in structural models is usually given little attention, and is typically specified as a sequence of additive, independently distributed random variates perturbing each equation in the model.

Consider a model of M simultaneous (jointly dependent) equations taking the structural form

$$(24)\quad p_{11}y_{1(t)} + \cdots + p_{1m}y_{m(t)} + \sum_i a_{11(i)}y_{1(t-i)} + \cdots + \sum_i a_{1m(i)}y_{m(t-i)} +$$
$$b_{11}x_{1(t)} + \cdots + b_{1k}x_{k(t)} + \sum_j c_{11(j)}x_{1(t-j)} + \cdots + \sum_j c_{1k(j)}x_{k(t-j)} = u_{1(t)}$$

$$\vdots$$

$$p_{m1}y_{1(t)} + \cdots + p_{mm}y_{m(t)} + \sum_i a_{m1(i)}y_{1(t-i)} + \cdots + \sum_i a_{mm(i)}y_{m(t-i)} +$$
$$b_{m1}x_{1(t)} + \cdots + b_{mk}x_{k(t)} + \sum_j c_{m1(j)}x_{1(t-j)} + \cdots + \sum_j c_{mk(j)}x_{k(t-j)} = u_{m(t)}$$

where: $y_{m(t)}$, $y_{m(t-i)}$ denote current and lagged endogenous variables, respectively, $x_{k(t)}$, $x_{k(t-j)}$ denote current and lagged exogenous variables, respectively, and $u_{m(t)}$ denotes stochastic disturbances.

Without sacrificing the generality of subsequent analyses, it is convenient to confine lags in endogenous and exogenous variables to one period (any higherorder system of difference equations can be translated to a first-order system) and to write the model more compactly in matrix notation as

$$PY_t + AY_{t-1} + BX_t + CX_{t-1} = U_t \qquad (t = 1, 2, \ldots T) \tag{25}$$

where: P and A are M x M coefficient matrices; B and C are M x K coefficient matrices; Y_t and Y_{t-1} are M-component column vectors of current and lagged endogenous variables, respectively; X_t and X_{t-1} are K-component column vectors of current and lagged exogenous variables, respectively; and U_t is an M-component column vector of current disturbances.

If identification conditions are satisfied (see Fisher, 1966 for an exhaustive analysis), simultaneous equation models can be estimated by a variety of consistent methods; the most common being two-stage least-squares.[10] In the special case of recursive models in which P is triangular (there are no simultaneous relationships) and the cross-equation disturbance covariance

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[10] Note, however, that the appearance of lagged endogenous variables in (24) and (25) introduces additional complications. Briefly, consistency is not ensured unless the U_t are serially uncorrelated. If this condition fails, there are essentially two options: (1) treat the Y_{t-1} as endogenous for estimation purposes (which has obvious implications for identification); or (2) combine two-stage least-squares with generalized least-squares so that the transformed disturbances are properly behaved. On the latter procedure see Fair, 1970. A general review of this and related problems is provided by Fisher, 1970a and Hibbs, 1973, Appendix 3.

matrix is diagonal (the disturbances are uncorrelated across equations), ordinary least-squares regression yields consistent parameter estimates. Throughout the discussion in this section it is assumed that the functional form of the hypothetical structural model is well established and hence that model validation is not an issue.[11] Attention will be confined therefore to techniques for intervention effects analysis in the context of a well-defined model.

Interventions and Direct Manipulation of Exogenous Variables

Intervention analysis is least problematic when the intervention or policy change is known to have been implemented by direct manipulation of exogenous variables or policy parameters. Notable examples are policy motivated, exogenously induced changes in government spending, tax rates, and the like, which figure prominently in econometric analyses of macroeconomic policy experiments.

If the structural model consists of a relatively small number of *linear*, simultaneous difference equations, the response of endogenous ("target") variables to exogenous interventions can be assessed analytically by the method of multiplier analysis. (See Goldberger, 1959; and Thiel and Boot, 1962.) The first step in multiplier analysis is to derive the "reduced form"

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[11] It is hardly necessary to mention that establishing the functional form of a structural model is a substantial task, particularly in areas in which theory is not well developed and the processes under investigation are behaviorally complex. A very useful review of model evaluation procedures (which is geared to econometric systems) is given by Dhrymes *et al.*, 1972.

of the system by solving all right-hand side current endogenous variables as functions of the predetermined lagged endogenous and exogenous variables. Thus, given the estimated structural form

$$(26) \quad \hat{P}Y_t = -\hat{A}Y_{t-1} - \hat{B}X_t - \hat{C}X_{t-1} + \hat{U}_t$$

the reduced form can be secured by premultiplying by $\hat{P}^{-1}$

$$(27) \quad Y_t = -(\hat{P}^{-1}\hat{A})Y_{t-1} - (\hat{P}^{-1}\hat{B})X_t - (\hat{P}^{-1}\hat{C})X_{t-1} + \hat{P}^{-1}\hat{U}_t$$

which for convenience may be rewritten as

$$(28) \quad Y_t = A^*Y_{t-1} + B^*X_t + C^*X_{t-1} + V_t$$

$$\text{where: } A^* = -\hat{P}^{-1}\hat{A};\ B^* = -\hat{P}^{-1}\hat{B};\ C^* = -\hat{P}^{-1}\hat{C};\ \text{and } V_t = \hat{P}^{-1}\hat{U}_t\ .$$

Notice that every predetermined variable appears in each reduced form equation. Hence, derivation of the reduced form of the model makes explicit what is implied by the structural form; namely, that all predetermined variables directly and/or indirectly influence all endogenous variables.[12] The effects of policy motivated interventions now can be readily assessed by analyzing the reduced form in (28). Assuming that the expectation of $U_t = V_t = 0$, the immediate effects of induced changes in exogenous variables (x_{kt}) on the expected values of endogenous variables (y_{mt}) taking account of all contem-

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[12] This of course also means that the reduced form parameters can be (consistently) estimated by regressing each endogenous variable on all predetermined variables. The trade-offs between the derived reduced form estimation procedure shown in (24) and the unrestricted least-squares method mentioned here are developed in Fisher, 1965; and Goldberger, 1964, chapter 7.9.

poraneous feedbacks in the system, are given by elements of B^*--the so-called <u>impact multiplier</u> matrix. The elements of B^* therefore estimate the instantaneous impact of a unit change in x_{kt} on the (conditional) expectation of Y_{mt} with the remaining exogenous variables held constant. Thus the impact multipliers correspond to the reduced form derivatives $\partial y_{m(t)}/\partial x_{k(t)} = b^*_{mk}$. Since the model at hand is linear, endogenous responses to multiple interventions (packages of policy changes) are determined by summing over the appropriate elements of B^*, that is by calculating

$$\sum_k \partial y_{m(t)}/\partial x_{k(t)} = \sum_k b^*_{mk}.$$ [13]

Typically, interest will not be confined to the immediate consequences of policy treatments or interventions but will center instead on the dynamic, long-run implications of exogenously induced change. This amounts to investigating how the time-paths of endogenous variables are affected by external manipulation of exogenous policy instruments. Lagged, cross-temporal feedbacks in the system are now of central importance.

The effects of exogenous interventions, as they are transmitted dynamically through the model, are evaluated by lagging (28) repeatedly and substituting in for lagged endogenous variables. For example, lagging (28) one period gives

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[13] The response to a change of any order is simply $\Delta y_{m(t)} = \sum_k b^*_{mk} \cdot \Delta x_{k(t)}$. Sociologists and political scientists will recognize that algebraic computation of the reduced form coefficients is the simultaneous equation analog of compound path analysis, which is commonly applied to static, recursive models. See, for example, Stokes, 1971.

(29) $$Y_{t-1} = A^{*}Y_{t-2} + B^{*}X_{t-1} + C^{*}X_{t-2} + V_{t-1}$$

which upon substitution yields

(30) $$Y_{t} = A^{*2}Y_{t-2} + B^{*}X_{t} + (C^{*} + A^{*}B^{*})X_{t-1} + A^{*}C^{*}X_{t-2} + V_{t} + A^{*}V_{t-1} .$$

Applying this procedure s times, we obtain:

(31) $$Y_{t} = A^{*s+1}Y_{t-s-1} + B^{*}X_{t} + \sum_{\tau=1}^{s} A^{*\tau-1} (C^{*} + A^{*}B^{*})X_{t-\tau} + A^{*s}C^{*}X_{t-s-1} + \sum_{\tau=0}^{s} A^{*\tau}V_{t-\tau} .$$

If the system is stable [14] (in which case $\lim_{s\to\infty} A^{s} = 0$), letting s go to infinity yields

(32) $$Y_{t} = B^{*}X_{t} + \sum_{\tau=1}^{\infty} A^{*\tau-1}(C^{*} + A^{*}B^{*})X_{t-\tau} + \sum_{\tau=0}^{\infty} A^{*\tau}V_{t-\tau}$$

---

[14] The stability assumption is identical to that of the previous section and essentially means that the system cannot grow or oscillate explosively without growth in exogenous variables and/or without impulses from the disturbances. Introductory accounts of the formal conditions for stability of simultaneous difference equations are given by Baumol, 1970; and Goldberg, 1958. Samuelson, 1947, provides an advanced treatment.

which is known as the final form of the model.[15]

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[15] The final form of the model can also be derived by applying the algebra of lag operators. Previously we introduced the backshift operator B, however, to avoid confusion here with the coefficient matrices B and B^* we define a new lag operator L, such that $L^i Y_t = Y_{t-i}$. The reduced form of the system given in (28) can therefore be expressed

$$(I - A^* L) Y_t = B^* X_t + C^* X_{t-1} + V_t$$

$$Y_t = (I - A^* L)^{-1} B^* X_t + (I - A^* L)^{-1} C^* X_{t-1} + (I - A^* L)^{-1} V_t \ .$$

Since $(I - A^* L)^{-1}$ is the limit of the convergent geometric series $(I + A^* L + A^{*2} L^2 + \ldots)$ we have

$$Y_t = (I + A^* L + A^{*2} L^2 + \ldots) B^* X_t + (I + A^* L + A^{*2} L^2 + \ldots) C^* X_{t-1} + (I + A^* L + A^{*2} L^2 + \ldots) V_t$$

$$Y_t = B^* \sum_{\tau=0}^{\infty} A^{*\tau} X_{t-\tau} + C^* \sum_{\tau=0}^{\infty} A^{*\tau} X_{t-\tau-1} + \sum_{\tau=0}^{\infty} A^{*\tau} V_{t-\tau}$$

$$Y_t = B^* X_t + \sum_{\tau=1}^{\infty} A^{*\tau-1} (C^* + A^* B^*) X_{t-\tau} + \sum_{\tau=0}^{\infty} A^{*\tau} V_{t-\tau} \ .$$

The period-by-period responses of endogenous variables to induced shifts in exogenous variables, which are known as dynamic multipliers, now can be obtained from (31) and (32). If the exogenous change is sustained for only one period, the effects on subsequent (expected) values of endogenous variables are given by the delay multiplier matrices. Hence the estimated influence of a one-shot exogenous intervention s periods later is

$$(33) \quad A^{*s-1}(C^* + A^*B^*) \qquad s \geq 1 .$$

The responses of endogenous variables to one-shot exogenous impulses can therefore be tracked through time by evaluating the impact and successive delay multiplier matrices B^*, $(C^* + A^*B^*)$, $A^*(C^* + A^*B^*)$, $A^{*2}(C^* + A^*B^*)$, ..., the elements of which correspond to the reduced form derivatives $\partial y_{m(t+s)}/\partial x_{k(t)}$. Since the assumption of system stability implies that $\lim_{s\to\infty} A^s = 0$, it is clear that unsustained exogenous interventions will produce responses (displacements from equilibrium) in the Y_t that die out after sufficiently long lag.

Frequently, however, policy motivated interventions will be sustained through time. The dynamic implications of (unit) changes in exogenous variables that are continued, say, over s periods are given by the cumulated multiplier matrices

$$(34) \quad B^* + \sum_{\tau=1}^{s} A^{*\tau-1}(C^* + A^*B^*) = B^* + (I + A^* + A^{*2} + \ldots + A^{*\tau-1})(C^* + A^*B^*) ,$$

which have elements corresponding to the summed reduced form derivatives

$$\sum_{\tau=0}^{s} \partial y_{m(t+\tau)}/\partial x_{k(t)} .$$

The time paths of endogenous responses to sustained exogenous interventions can therefore be determined by evaluating (34) over the index τ.

Finally, by taking $\tau \to \infty$ we obtain the equilibrium multiplier matrices

$$(35)\quad B^* + \sum_{\tau=1}^{\infty} A^{*\tau-1}(C^* + A^*B^*)$$

$$= B^* + (I + A^* + A^{*2} + \ldots)(C^* + A^*B^*)$$

$$= B^* + (I - A^*)^{-1}\,(C^* + A^*B^*) \qquad \text{(by the convergence rule for a geometric series)}$$

$$= (I - A^*)^{-1}\,[(I - A^*)\,B^* + C^* + A^*B^*]$$

$$= (I - A^*)^{-1}\,[B^* - A^*B^* + C^* + A^*B^*]$$

$$= (I - A^*)^{-1}\,(B^* + C^*)\ .$$

The elements of (35) give the equilibrium or steady-state responses of endogenous variables to unit changes in exogenous variables that are sustained indefinitely.

Perhaps a simple analytic example will help clarify the results of this section. Suppose that the system under investigation is adequately represented as a pair of simultaneous equations in the structural form

$$(36a)\quad y_{1(t)} + p_{12}\,y_{2(t)} + a_{11}\,y_{1(t-1)} + b_{11}\,x_{1(t)} = u_{1(t)}$$

$$(36b)\quad p_{21}\,y_{1(t)} + y_{2(t)} + a_{22}\,y_{2(t-1)} + b_{22}\,x_{2(t)} = u_{2(t)}\ .$$

In order to determine the consequences of hypothetical policy manipulations of the exogenous variables x_1 and x_2 we need to derive the reduced form of the system, which is obtained by applying simple matrix operations in the manner of (27) or, alternatively, by solving algebraically the current endogenous variables as functions of the lagged endogenous and exogenous variables. Either approach yields the reduced form equations [16]

$$\text{(37a)} \quad y_{1(t)} = \frac{-a_{11}}{1 - p_{12}\, p_{21}} y_{1(t-1)} + \frac{p_{12}\, a_{22}}{1 - p_{12}\, p_{21}} y_{2(t-1)} - \frac{b_{11}}{1 - p_{12}\, p_{21}} x_{1(t)} + \frac{p_{12}\, b_{22}}{1 - p_{12}\, p_{21}} x_{2(t)} + \frac{1}{1 - p_{12}\, p_{21}} (u_{1(t)} - p_{12}\, u_{2(t)})$$

$$\text{(37b)} \quad y_{2(t)} = \frac{p_{21}\, a_{11}}{1 - p_{12}\, p_{21}} y_{1(t-1)} - \frac{a_{22}}{1 - p_{12}\, p_{21}} y_{2(t-1)} + \frac{p_{21}\, b_{11}}{1 - p_{12}\, p_{21}} x_{1(t)} - \frac{b_{22}}{1 - p_{12}\, p_{21}} x_{2(t)} + \frac{1}{1 - p_{12}\, p_{21}} (u_{2(t)} - p_{21}\, u_{1(t)}) \,.$$

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[16] The results in (37a) and (37b) are obtained as follows. Rearrange terms in (36a) and (36b) so that the structural form is written

$$\text{(i)} \quad y_{1(t)} = -p_{12}\, y_{2(t)} - a_{11}\, y_{1(t-1)} - b_{11}\, x_{1(t)} + u_{1(t)}$$

$$\text{(ii)} \quad y_{2(t)} = -p_{21}\, y_{1(t)} - a_{22}\, y_{2(t-1)} - b_{22}\, x_{2(t)} + u_{2(t)} \,.$$

Now simply solve $y_{1(t)}$ and $y_{2(t)}$ as functions of the lagged endogenous and exogenous variables. Solving (i) gives

$$\text{(iii)} \quad y_{1(t)} = -p_{12}[-p_{21}\, y_{1(t)} - a_{22}\, y_{2(t-1)} - b_{22}\, x_{2(t)} + u_{2(t)}] - a_{11}\, y_{1(t-1)} - b_{11}\, x_{1(t)} + u_{1(t)}$$

(footnote continued on p. 33a)

(footnote 16 continued)

$$y_{1(t)} - p_{12}\, p_{21}\, y_{1(t)} = p_{12}\, a_{22}\, y_{2(t-1)} + p_{12}\, b_{22}\, x_{2(t)} - p_{12}\, u_{2(t)}$$

$$- a_{11}\, y_{1(t-1)} - b_{11}\, x_{1(t)} + u_{1(t)}$$

$$y_{1(t)} = \frac{-a_{11}}{1 - p_{12}\, p_{21}}\, y_{1(t-1)} + \frac{p_{12}\, a_{22}}{1 - p_{12}\, p_{21}}\, y_{2(t-2)}$$

$$- \frac{b_{11}}{1 - p_{12}\, p_{21}}\, x_{1(t)} + \frac{p_{12}\, b_{22}}{1 - p_{12}\, p_{21}}\, x_{2(t)}$$

$$+ \frac{1}{1 - p_{12}\, p_{21}}\, (u_{1(t)} - p_{12}\, u_{2(t)}\ .$$

Similarly, solving (ii) gives

(iv) $$y_{2(t)} = -p_{21}[-p_{12}\, y_{2(t)} - a_{11}\, y_{1(t-1)} - b_{11}\, x_{1(t)}$$

$$+ u_{1(t)}] - a_{22}\, y_{2(t-1)} - b_{22}\, x_{2(t)} + u_{2(t)}$$

$$y_{2(t)} - p_{21}\, p_{12}\, y_{2(t)} = p_{21}\, a_{11}\, y_{1(t-1)} + p_{21}\, b_{11}\, x_{1(t)}$$

$$- p_{21}\, u_{1(t)} - a_{22}\, y_{2(t-1)} - b_{22}\, x_{2(t)} + u_{2(t)}$$

$$y_{2(t)} = \frac{p_{21}\, a_{11}}{1 - p_{12}\, p_{21}}\, y_{1(t-1)} - \frac{a_{22}}{1 - p_{12}\, p_{21}}\, y_{2(t-1)}$$

$$+ \frac{p_{21}\, b_{11}}{1 - p_{12}\, p_{21}}\, x_{1(t)} - \frac{b_{22}}{1 - p_{12}\, p_{21}}\, x_{2(t)}$$

$$+ \frac{1}{1 - p_{12}\, p_{21}}\, (u_{2(t)} - p_{21}\, u_{1(t)})\ .$$

In accordance with the earlier convention (cf. eq. 28) these equations are conveniently rewritten as

(38a) $y_{1(t)} = a^*_{11}\, y_{1(t-1)} + a^*_{12}\, y_{2(t-1)} + b^*_{11}\, x_{1(t)} + b^*_{12}\, x_{2(t)} + v_{1(t)}$

(38b) $y_{2(t)} = a^*_{21}\, y_{1(t-1)} + a^*_{22}\, y_{2(t-1)} + b^*_{21}\, x_{1(t)} + b^*_{22}\, x_{2(t)} + v_{2(t)}$.

The instantaneous effects of unit changes in the exogenous x_{kt} on the conditional expectations of the endogenous y_{mt}, the impact multipliers, are given directly by the b^*_{mk}, which as (37) makes apparent are nonlinear functions of the underlying structural coefficients in (36). The responses of the y_{mt} s periods later to interventions (which, for simplicity, are again taken to be unit changes in the x_{kt}) that are initiated at time t and sustained only 1 period are given by the delay multipliers. For example, taking s = 2 and applying (33) to the model at hand yields

$$(39)\quad \frac{\partial y_{m(t+2)}}{\partial x_{k(t)}} = A^{*2}\, B^*$$

$$= \begin{bmatrix} a^*_{11} & a^*_{12} \\ a^*_{21} & a^*_{22} \end{bmatrix}^2 \begin{bmatrix} b^*_{11} & b^*_{12} \\ b^*_{21} & b^*_{22} \end{bmatrix}$$

$$= \begin{bmatrix} (a^{*2}_{11} + a^*_{12}\, a^*_{21})\, b^*_{11} + (a^*_{11}\, a^*_{12} + a^*_{12}\, a^*_{22})\, b^*_{21} & (a^{*2}_{11} + a^*_{12}\, a^*_{21})\, b^*_{12} + (a^*_{11}\, a^*_{12} + a^*_{12}\, a^*_{22})\, b^*_{22} \\ (a^*_{21}\, a^*_{11} + a^*_{22}\, a^*_{21})\, b^*_{11} + (a^*_{21}\, a^*_{12} + a^{*2}_{22})\, b^*_{21} & (a^*_{21}\, a^*_{11} + a^*_{22}\, a^*_{21})\, b^*_{12} \quad (a^*_{21}\, a^*_{12} + a^{*2}_{22})\, b^*_{22} \end{bmatrix}$$

Hence, the impact of a one-shot (or pulse) unit increment in x_{1t} on $y_{1(t+2)}$ is given by the northwest entry of the matrix, that of x_{1t} on $y_{2(t+2)}$ by the southwest entry, that of x_{2t} on $y_{1(t+2)}$ by the northeast entry, and that of x_{2t} on $y_{2(t+2)}$ by the southeast entry. Delay multipliers for longer lags (leads) and/or for more complex models will obviously require even more tedious calculations if undertaken analytically.

The cumulative effects of interventions that are sustained over some finite period are determined by application of (34), that is by simply summing the impact and delay multiplier matrices over the appropriate time index. The ultimate impact of induced changes in exogenous variables that are maintained indefinitely are calculated by employing the equilibrium multiplier expression in (35). To illustrate for the system of (36) - (38) we derive

$$(40)\quad \sum_{\tau=0}^{\infty} \frac{\partial y_{m(t+\tau)}}{\partial x_{k(t)}} = (I - A^{*})^{-1} B^{*}$$

$$= \begin{bmatrix} 1 - a^{*}_{11} & -a^{*}_{12} \\ -a^{*}_{21} & 1 - a^{*}_{22} \end{bmatrix}^{-1} \begin{bmatrix} b^{*}_{11} & b^{*}_{12} \\ b^{*}_{21} & b^{*}_{22} \end{bmatrix}$$

$$= \frac{1}{D} \begin{bmatrix} (1 - a^{*}_{22})\, b^{*}_{11} + a^{*}_{12}\, b^{*}_{21} & (1 - a^{*}_{22})\, b^{*}_{12} + a^{*}_{12}\, b^{*}_{22} \\ a^{*}_{21}\, b^{*}_{11} + (1 - a^{*}_{11})\, b^{*}_{21} & a^{*}_{21}\, b^{*}_{12} + (1 - a^{*}_{11})\, b^{*}_{22} \end{bmatrix}$$

where: $D = (1 - a^{*}_{11})(1 - a^{*}_{22}) - a^{*}_{21}\, a^{*}_{12}$.

The elements of (37) give the equilibrium responses of the y_m to unit changes in the x_k that are sustained forever.

Although the analytic approach taken thus far has considerable heuristic value, the dynamic multipliers associated with changes in exogenous variables are in practice usually derived numerically by computer simulation. The reason, of course, is that simulated solutions are vastly more convenient computationally, even for relatively small models.[17] Simulation-based estimates of the intervention multipliers are secured by simulating the model dynamically in order to obtain an "intervention" endogenous series and a "nonintervention" endogenous series.[18] The intervention or "policy-on" solution is designed to depict actual endogenous outcomes during the post-intervention periods. Accordingly, these outcomes, which we denote as $\hat{y}^i_{mt}$, are generated by supplying initial conditions (values) for the y_{mt} and allowing exogenous variables and parameters to take on their historical, post-intervention values. On the other hand, the nonintervention or "policy-off" solution attempts to replicate endogenous outcomes that would have occurred in the absence of exogenously induced change. These outcomes, which we denote as $\hat{y}^n_{mt}$, are obtained by imposing values on exogenous variables and/or

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[17] Multiplier analysis in nonlinear models (i.e., models in which one or more endogenous variables appear in two or more linearly independent functional forms) requires a simulation approach, since explicit analytical solutions for the reduced form equations are difficult, if not impossible, to obtain. For further discussion see Howrey and Kelejian, 1969.

[18] Dynamic simulation simply means that actual historical values of lagged endogenous variables are used only for initial conditions; all subsequent endogenous values are generated sequentially by the model. Thus, the current period's endogenous calculations form the lagged inputs to the next period, and so on.

parameters that would have prevailed without the external manipulation (i.e., by subtracting the known magnitude of induced changes from the historical values).

Comparison of the differences $(\hat{y}^i_{mt} - \hat{y}^n_{mt})$, $(\hat{y}^i_{mt+1} - \hat{y}^n_{mt+1})$, ..., $(\hat{y}^i_{mt+\tau} - \hat{y}^n_{mt+\tau})$ yields estimates of the endogenous responses to exogenous interventions. Simulation estimates of the intervention multipliers corresponding to a sustained exogenous variable change of, say, δ--$(x_{kt} + \delta)$, ..., $(x_{kt+\tau} + \delta)$--would be given the ratios $(\hat{y}^i_{mt} - \hat{y}^n_{mt})/\delta$, ..., $(\hat{y}^i_{mt+\tau} - \hat{y}^n_{mt+\tau})/\delta$. Studies of the consequences of specific policy changes (hypothetical and actual) that have been undertaken in this way include Fromm and Taubman's (1967) analysis of the effects of the U.S. excise tax cut of 1965, Klein's (1969) similar investigation of the U.S. income tax cut of 1964, and Klein's (1968) study of the economic consequences of Vietnam peace.

It is clear that static and dynamic multipliers are the structural equation equivalents of the Box-Tiao transfer function response schemes. However, an important advantage of the structural method is that endogenous responses to exogenous interventions can be interpreted causally in the light of structural information. That is, the behavioral mechanisms underlying intervention multipliers are made apparent by inspection of the interdependent structure of the system. Naturally, such multipliers have meaning only within the framework of the model from which they are derived. If the model does not square well with reality, then the estimated multipliers cannot be informative about real-world intervention effects.

Intervention and Structural Shifts

In most empirical situations, at least outside of macroeconomics, policy interventions are not likely to consist of direct manipulation of exogenous variables or policy parameters. On the contrary, the typical intervention will involve a change in law, government regulation or administrative procedure, or perhaps, a dramatic event such as a war, strike, critical electoral outcome, important international agreement, and so on. In such situations the manner in which an exogenous intervention or event potentially affects a particular endogenous variable or an entire system of variables is not known a priori.

If it can be assumed that the intervention does not perturb the values of exogenous variables, but affects only the parameters of the model, the problem is readily approached by structural shift estimation. Recall that the general (M equation) linear dynamic structural model was expressed previously as

$$(41)\quad PY_t + AY_{t-1} + BX_t + CX_{t-1} = U_t \ .$$

Rearranging terms and normalizing for left-hand side endogenous variables allows the equations of the model to be written in the scalar form

$$(42)\quad y_{m(t)} = -\sum_{1}^{M\neq m} p_{mm} y_{m(t)} - \sum_{1}^{M} a_{mm} y_{m(t-1)} - \sum_{1}^{K} b_{mk} x_{k(t)} - \sum_{1}^{K} c_{mk} x_{k(t-1)} + u_{m(t)} \ .$$

Suppose that the intervention event under investigation occurs at the n-th period and continues thereafter. Shifts in the structural parameters associated with the intervention can be determined by defining a binary variable, say, D

$$D = 0 \quad \text{for } t < n$$

$$= 1 \quad \text{for } t \geq n$$

and estimating the revised, unrestricted equation(s)

$$(43) \quad Y_{m(t)} = - \sum_{1}^{M \neq m} p_{mm} y_{m(t)} - \sum_{1}^{M \neq m} p'_{mm} [y_{m(t)} \cdot D] - \sum_{1}^{M} a_{mm} y_{m(t-1)} - \sum_{1}^{M} a'_{mm} [y_{m(t-1)} \cdot D]$$

$$- \sum_{1}^{K} b_{mk} x_{k(t)} - \sum_{1}^{K} b'_{mk} [x_{k(t)} \cdot D] - \sum_{1}^{K} c_{mk} x_{k(t-1)} - \sum_{1}^{K} c'_{mk} [x_{k(t-1)} \cdot D]$$

$$+ u^{*}_{m(t)} \ .$$

Equations in the form of (43) allow detection of structural shifts or breaks induced by the exogenous intervention by permitting all parameters to have different values in the pre- and post-intervention periods.[19] Of course any prior (theoretical) information about the location of intervention shift effects should be exploited by setting the relevant cross-product terms equal to zero. The t ratios of p'_{mm}, a'_{mm}, b'_{mk} and c'_{mk} provide direct tests of the null hypotheses that the post-intervention parameters are not significantly different from the corresponding pre-intervention parameters. The joint hypothesis that all coefficients (or some subset thereof) are common across

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[19] Intercept-constants are not shown explicitly in (42) or (43) but may be considered to be among the b_{mk}. Also, there are alternative ways to set up the problem, for example, one might estimate equations in the model separately for the pre- and post-intervention periods in the spirit of analysis of covariance.

the pre- and post-intervention observations may be evaluated by computing the F ratio(s) [20]

$$F_{(m)} = \frac{[\sum_t \hat{u}^2_{m(t)} - \sum_t \hat{u}^{*2}_{m(t)}]/r}{[\sum_t \hat{u}^{*2}_{m(t)}]/T-J}$$

which is (are) distributed with r, and T-J degrees of freedom, where:

> $\sum_t \hat{u}^2_{m(t)}$ and $\sum_t \hat{u}^{*2}_{m(t)}$ are estimates of the restricted and unrestricted residual sums of squares, respectively, and are derived by applying the structural coefficient estimates to the original data,[21] r denotes the number of restrictions or constraints in (42), and T-J denotes the degrees of freedom of the residual sum of squares in (43)--J being the number of parameters in that equation.

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[20] If the number of parameters to be estimated exceed the available post-intervention observations, the F test of Chow, 1960, should be used in place of that given above. A unifying exposition of these and related tests is given by Fisher, 1970b.

[21] Because the residual sums of squares are necessarily calculated in this way in simultaneous equation models, t and F statistics do not have full classical justification in the sample. They might be viewed as tests of "quasi-significance." If the model under investigation consists of a single equation that can be estimated consistently by ordinary least-squares (rather than by a simultaneous equations estimator such as two-stage least-squares), $\sum_t \hat{u}^2_{m(t)}$ and $\sum_t \hat{u}^{*2}_{m(t)}$ are of course computed directly from the residuals of the estimating equation.

Once the magnitude of parameter shifts attributable to the exogenous intervention(s) has been determined for each equation in the model, the methods outlined previously for calculating intervention effects can be employed. However, the structural equation approach would appear to be of little value in situations in which the external intervention not only perturbs the model's parameters but also affects the values of exogenous variables. Unless the investigator knows which exogenous variables are affected, and how much (a case which was treated earlier), there is simply no way to determine the consequences of an intervention within the structural framework. This is an important limitation which we will return to in the next section.

III. Limitations and Lines of Convergence

Limitations

Box-Tiao or Box-Jenkins methods are essentially models for "ignorance" which are not based on theory and are therefore void of explanatory power. Although these models are in many situations likely to yield good estimates of endogenous responses to external interventions, they provide no insight into the causal structure underlying the transmission of exogenous impulses through a dynamic system of interdependent social, economic, or political relationships. Moreover, the Box-Tiao approach appears to be very susceptible to errors of inference due to "omitted variables". Discontinuous movements in endogenous variables that are actually responses to discontinuous changes in omitted exogenous variables are easily attributed by Box-Tiao methods to external interventions that happen to covary with the omitted variables. However, the multiple contrast design ("multiple-group time-series design") proposed by Campbell (1963; 1966) to deal with this problem and implemented in the Box-Tiao framework by Glass, Willson, and Gottman (1972) is likely to be at least somewhat effective in coping with this potential source of spurious inference.[22]

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[22] A good example of this design is provided by Glass' (1968) study of the effectiveness of Governor Abraham Ribicoff's 1955 "crackdown" on speeding in reducing traffic fatalities in Connecticut. In order to ensure that the effects attributed to this intervention were genuine, Glass analyzed the fatality rates of four "control" states that did not experience a comparable alteration of law enforcement practice.

Perhaps the most obvious constraint on the use of the structural approach to intervention analysis is that many areas of inquiry are simply not sufficiently rich in theory and/or data to permit specification and estimation of adequate structural models. In such situations, the causally naive Box-Tiao scheme--which merely requires time-series observations on endogenous variables, knowledge of the time-spans of external interventions, and some hunches about the form of endogenous responses--would appear to have no serious rival.

However, even in areas in which acceptable structural models have been developed, the empirical data cannot be informative about intervention effects unless it can be assumed that exogenous variables do not respond to external treatments (or at least do not respond in ways that are not fully known a priori). An illustration of this problem in macroeconomics, a field in which theory is comparatively well developed and structural models have enjoyed great success, is provided by Feige and Pearce's (1943) recent study of the impact of wage and price controls on the rate of inflation in the United States. Feige and Pearce reject standard econometric procedures in favor of Box-Jenkins and Box-Tiao methods because of the general "unreliability" of the former in generating accurate forecasts. They go on to argue that variables such as the unemployment rate, which are normally taken to be exogenous in the estimation of structural models of inflation, are themselves potentially affected by the policy intervention and therefore are likely to contaminate "counterfactual" forecasts (policy-off simulations) of the endogenous series.[23] The situation encountered by Feige and Pearce is likely to be common in applied work and serves to underscore the limitations of the structural approach to intervention analysis.

Lines of Convergence

It has been noted several times that Box-Jenkins and Box-Tiao methods are essentially sophisticated noise models that make no attempt to represent the behavioral structure generating endogenous time-series. However, recent Box-Tiao papers hint at the need to elaborate the basic "noise plus intervention transfer function" model to incorporate additional exogenous variables and interdependent relationships among "output" or endogenous variables. Work on this is apparently underway and clearly points in the structural equation direction.

Conversely, the structural equation tradition has placed great emphasis on behavioral sophistication but has given much less attention to noise or disturbance processes. Error models other than first-order autoregressive schemes are rarely entertained in empirical studies; indeed, in simultaneous equation models the disturbances are nearly always assumed at the outset to be white noise. This may in part underlie the rather poor short-term forecasting performance of econometric models in relation to that of naive,

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[23] "Exogenous" variables that respond in this way to external interventions are, in a sense, really endogenous. Hence, a more committed structural modeler might argue that such a study should have been framed in the context of a "large" macroeconomic model that explicitly treats unemployment and related variables as endogenous. However, this argument can only be pushed so far, and we are still left with the general problem.

especially Box-Jenkins, alternatives. (Cf. Cooper, 1972; Naylor *et al*., 1972; Nelson, 1973; Stekler, 1968.)

However, structural modelers are becoming more sensitive to the need for stochastic sophistication. A number of recent state of the art papers have urged that greater attention be given to error processes (Dhrymes, *et al*., 1972; Klein, 1971) and work on the specification and estimation of more complex disturbance models in the structural context is beginning to appear with regularity in the technical literature. (See, for example, Chow and Fair, 1973; Fair, 1970, 1973; Hannan and Nichols, 1972; Hibbs, 1974; Sarris and Eisner, 1974; and Schmidt, 1971.) As I have tried to show in an earlier paper (Hibbs, 1974), Box-Jenkins techniques are ideally suited to the characterization of structural disturbance processes, which, after all, represent our ignorance. Finally, the traditional econometric commitment to the maintained hypothesis and strong axiomatization of models appears to be giving way to a renewed emphasis on experimentation with functional forms. These developments in the structural equation camp have much in common with the explicit empiricism of the ARMA approach and clearly point in the Box-Jenkins direction. Indeed, Box-Jenkins techniques applied to the fundamental dynamic equations of econometric models have been shown to be useful in validating the adequacy of the presumed causal structure. (See Pierce and Mason, n.d.; and Zellner and Palm, 1973.)

Convergence will be fully realized when structural models corrupted by ARMA noise are used routinely in empirical work.

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