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CAPITAL FORMATION AND ECONOMIC GROWTH:
A THEORETICAL AND EMPIRICAL ANALYSIS

R. S. Eckaus and L. Lefeber

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I. Introduction

Many of the issues of economic growth and development must be analysed within the framework of capital theory. In this paper we develop a model for inter-temporal choice in production which, though relatively simple, nonetheless contains some of the most important elements commonly encountered in economic planning and in the analysis of capital accumulation. We then employ the results of our model for the empirical estimation of the marginal rate of return over cost and other quantitative features of the U.S. economy.

We work at a high level of aggregation, distinguishing only between consumption and capital goods on the output side and between labor and capital on the input side. However, both the theory and the estimation procedures, as discussed below, would allow further disaggregation in the analysis. Nevertheless, without trying to minimize the usefulness of more disaggregated models, we have purposely decided to present a two-sector approach. We have done so for several reasons. First, its concepts approximate the framework most frequently employed to analyse problems of economic growth. Secondly, at this stage the gaps in the data are such that multi-sector comparison may not be warranted. Finally, our approach facilitates the analytical exposition which otherwise might not lend itself to simplified presentation.

The theoretical framework and analysis are described in Part II. Part III discusses the relationship between our approach and other models in capital theory developed in the past. The empirical application of our theoretical framework is presented in Part IV with the details of the calculations summarized in the Appendix.

II. The Analytical Framework

The framework consists of a simple non-linear program based on the customary assumptions of convexity and continuity in the constraints. We consider the production of a consumption good $X^1(t)$ and of a homogeneous capital good $X^2(t)$ in each of two discrete time periods, given stocks of labor $\bar{L}(t)$ in each period and given an initial stock of capital $\overline{KS(0)}$. Both labor and capital are used in both lines of production. The capital available in period t being restricted to the capital stock existing at the end of the previous period KS (t-1). Capital is non-substitutable for the consumption good; i.e., $X^2(t)$ is a distinctly different product from $X^1(t)$ and is not consumable in the conventional sense of the term. The consumption good produced in any one period is fully consumed in the same period: inventories of $X^1(t)$ are assumed to be zero.[1]

Capital, however, depreciates in use. Depreciation in this context means that part of the capital employed in any one activity is consumed in some given proportion to the capital input itself. If $K^1(t)$ and $K^2(t)$ denote capital employed in the production of the two goods in any one period, $\beta^1(t)K^1(t)$ and $\beta^2(t)K^2(t)$ are the amounts depreciated in that period. We stipulate that $0 \leq \beta^i(t) \leq 1$. Depreciation in any one line of production cannot exceed the capital employed in that line of activity.[2]

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1. This is strictly an assumption to simplify the exposition. Inventories can be readily built into the framework. The motivation for carrying inventories is twofold: a) to overcome various frictions in the flow of goods, b) to satisfy future demand at higher prices than those of today. We abstract from both of these possibilities.

2. A fully general treatment of capital consumption should take into account time depreciation and the effect of variations in any one of the factors. To avoid cluttering up our equations we have abstracted from both of these considerations. However, for purposes of estimation in Part IV, time depreciation is implied by the retention of the time designation on the β's. As far as the effects of variations in either factor are concerned, the following relationships provide the general case. Denoting depreciation in the production of the i-th good in each period by $D^i(t)$ we have $D^i(t) = \beta^i(t)X^i(t)$. However, the rate of depreciation, $\beta^i(t)$, must

In order to derive a transformation surface corresponding to any stipulated level of terminal capital stock, $\overline{KS(2)}$, we maximize the objective function,[1]

$$W_1X^1(1) + W_2X^1(2) \qquad (1)$$

where the W's are arbitrarily selected, constant weights, subject to the following conditions:

$$X^i(t) = F^i[L^i(t), K^i(t)]; \; i = 1,2; \; t = 1,2; \qquad (2)$$

$$\sum^i L^i(t) \leq \overline{L(t)} \quad ; \; i = 1,2; \; t = 1,2; \qquad (3)$$

$$\sum^i K^i(t) \leq KS(t-1) \quad ; \; i = 1,2; \; t = 1,2; \qquad (4)$$

$$D(t) = \sum^i \beta^i(t)K^i(t) \quad ; \; i = 1,2; \; t = 1,2; \qquad (5)$$

$$KS(o) = \overline{KS(o)} \qquad (6)$$

$$KS(1) - KS(o) - X^2(1) + D(1) \leqq o; \qquad (7)$$

$$KS(2) = \overline{KS(2)} \quad ; \qquad (8)$$

$$KS(2) - KS(o) - X^2(1) - X^2(2) + D(1) + D(2) \leqq 0. \qquad (9)$$

All variables are stipulated to be greater than or equal to zero.[2]

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itself be a function of factor proportions; i.e. $\beta^i(t) = \beta^i(t)\left[\frac{K^i(t)}{L^i(t)}\right]$.

Hence $D^i(t) = \beta^i(t)\left[\frac{K^i(t)}{L^i(t)}\right] X^i(t)$.

In this case, however, if the overall, first order homogeneity requirement in the constraints is to be preserved, the function must be homogeneous of order zero. $\beta^i(t) = \beta^i(t) \left[\frac{K^i(t)}{L^i(t)}\right]$

1. Alternatively and equivalently we could stipulate the output of one of the variables, $X^1(1)$ or $X^1(2)$, and maximize the output of the other.

2. There is one added set of constraints which we do not introduce explicitly but mention now for future reference. This set represents the Hawkins-Simon conditions for our model which ensure the viability of the growth system. The conditions can be formalized by writing.

$$F_K{}^2(t) - \beta^2(t) > 0 \quad ; \quad t = 1,2;$$

i.e., the net marginal productivity of capital in producing capital must be greater than zero. We shall return to these conditions on page 10.

The constraints under (2) are the production functions of the two goods in the two periods. Constraints (3) show the distribution of labor in each period. The constraints under (4) show the distribution of capital in each period and that the total available is confined to the capital stock existing at the end of the previous period. The constraints under (5) describe the total depreciation of capital in each period. Constraint (6) indicates the initial stock of capital available. Constraint (7) shows the capital stock at the end of the first period after the initial capital stock is changed by production and depreciation of capital in that period. Constraint (8) indicates the stipulated terminal capital stock and constraint (9) shows the capital stock at the end of the second period after the initial endowment is changed by production and depreciation in the subsequent two periods.

For any given terminal capital stock KS(2) the variation of the arbitrarily chosen W's of the objective function will trace out a feasibility surface for consumption in the two periods. By assuming alternative values for KS(2), itself, we can trace out a feasibility surface for all choices of consumption and capital goods which are open to society in the two periods under consideration. [1]

This two period feasibility surface represented in Figure 1 is completely descriptive of <u>all</u> the alternative choices which exist in these two periods, <u>irrespective of whether the time horizon consists of two, or of any number of finite or infinite periods</u>. Since this surface is of crucial importance, we shall present a geometrical as well as an analytical interpretation.

Figure 1 shows the interrelationships between $X^1(1)$, $X^1(2)$ and the net addition to the total capital stock over both periods, $\overline{KS(2)} - \overline{KS(0)}$; where $\overline{KS(2)}$ and $\overline{KS(0)}$ denote the terminal and initial capital stocks, respectively. In the geometric derivation we shall follow the procedure already established: we stipulate alternative levels of terminal capital stocks and then ask what consumption

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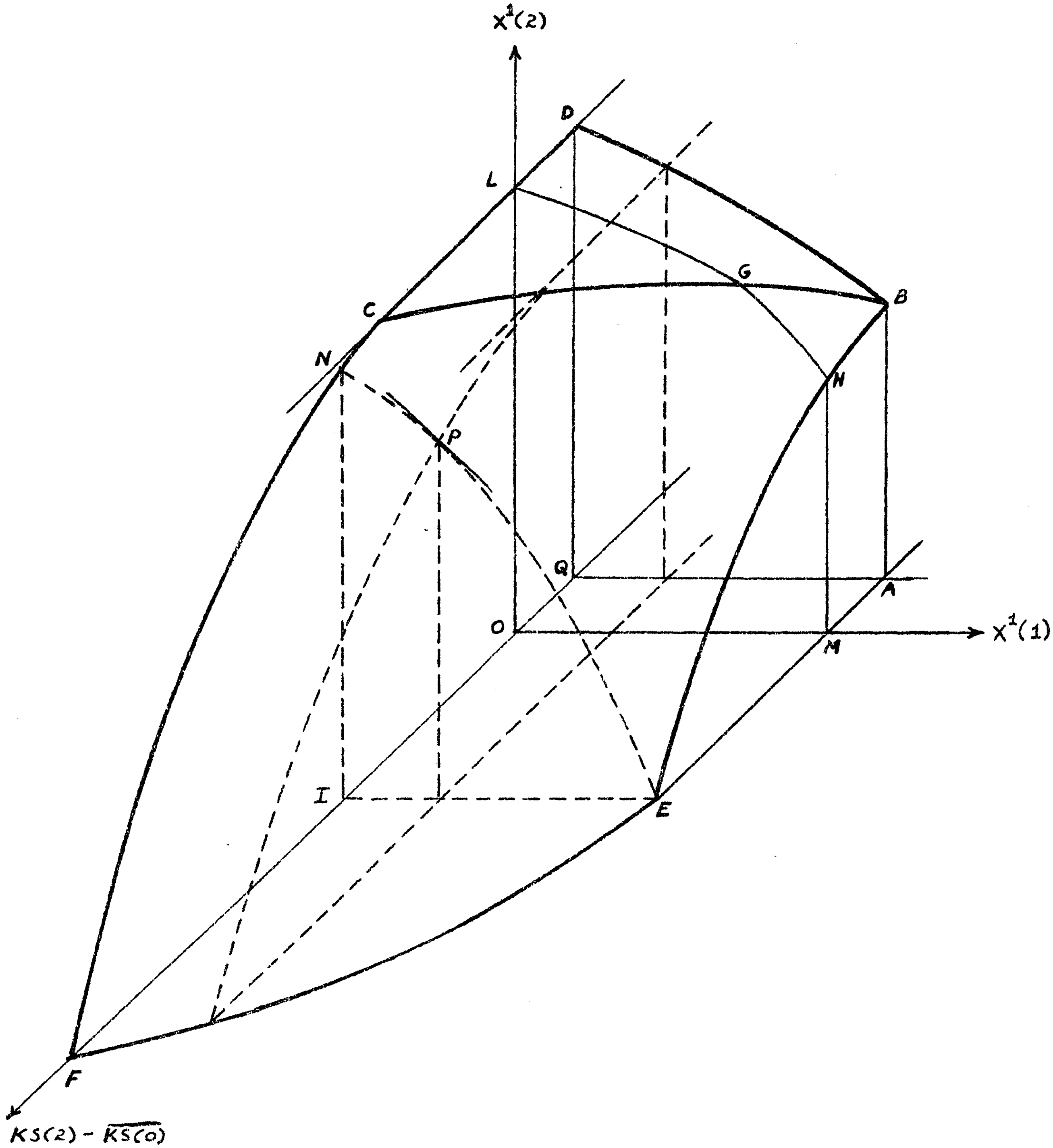
1. Alternatively we could have included in the objective function the terminal capital stock as a variable. Had we done that, the shadow price, corresponding to the fixed terminal capital stock in the above system, would have been replaced by an arbitrary weight in the objective function. Otherwise everything would remain the same.

Figure 1



alternatives exist in the two periods consistent with these terminal goals and full and efficient use of resources. For each stipulated terminal stock these consumption alternatives will be an arc formed by the intersection of the three-dimensional feasibility surface with a plane parallel to the $X^1(1)$ and $X^1(2)$ axes cutting through the $[\overline{KS(2)} - \overline{KS(0)}]$ axis at the specified level. These arcs are the transformation curves between today's and tomorrow's consumption for given amounts of terminal stocks. They must be convex because of our original assumptions of convexity in the constraints. The slopes along the arcs represent the rates at which today's consumption can be transformed into tomorrow's, with the appropriate shifting of factors between consumption and capital goods production to insure also the achievement of the specified terminal capital conditions.

To trace out the feasibility surface we first note that there must be an upper limit to the amount by which the capital stock can be increased over the original endowment. The limit is reached when all resources in both periods are devoted to the production of capital goods, and, hence, the production of consumer goods must, throughout, be zero. This is the case at the vertex of the feasibility surface on the $[\overline{KS(2)} - \overline{KS(0)}]$ axis, at point F in Figure 1.

Stipulation of a somewhat smaller terminal capital stock than that represented by point F would make some resources available for the production of consumer goods in either the first or second periods. Such is the case, for example, at $\overline{KS(2)} - \overline{KS(0)} = I$. There might be IE of $X^1(1)$ produced and nothing of $X^1(2)$ or IN of $X^1(2)$ and nothing of $X^1(1)$, or a range of combinations between these extremes represented by the arc NE which is appropriately convex. Notice that IE equals OM, the maximum mmount of consumer goods which can be produced in the first period when no resources are allocated to producing capital. Figure 1 shows that to reach the desired terminal stock indicated by I and have $X^1(1) = IE$, all resources must be allocated to produce capital goods in the second period and the production of $X^1(2)$ must be zero. If we chose to produce $X^1(1) < IE$ in the first period, some resources could produce capital in that period and, correspondingly, some resources would then be available to produce $X^1(2)$ in addition to the resources needed to complete the stipulated terminal stock in the second period.

To reach terminal capital stocks corresponding to the range between points I and F $(I < \overline{KS(2)} - \overline{KS(0)} < F)$ we must avoid concentration of all our resources on the production of consumption goods in the first period. In this range the required total addition to capital stock exceeds the maximum amount of capital which can be produced in the second period alone. Hence, some capital must be produced in the first period. This accounts for the curvature of the feasibility surface from point E toward point F in the $X^1(1)$ and $\overline{KS(2)} - \overline{KS(0)}$ plane.

In the range of terminal stocks between point Q and point I $(Q \leq \overline{KS(2)} - \overline{KS(0)} < I)$ the required total addition to capital stock is sufficiently small that there is more leeway in the use of resources. Even if all factors were concentrated in the production of consumer goods in the first period, the stipulated additions to capital can be obtained in the second period and some consumers good production as well. This explains the curvature of the arc EH.

Suppose the required total net addition to capital stock, $\overline{KS(2)} - \overline{KS(0)}$, is set at zero and the maximum amount of $X^1(1)$, OM, is produced. Then in the second period we can produce both $X^1(2) = HM$ <u>and</u> enough capital to offset the depreciation incurred by the production of consumer goods in the first period and of consumer and capital goods in the second periods.

If we continue to reduce the production of $X^1(1)$ and move from M toward 0, we can, correspondingly, increase the output of $X^1(2)$ along the arc GH, using the factors released in the first period for capital formation. However, by further reducing $X^1(1)$ in order to increase $X^1(2)$, we build up so much capital in the first period that even after depreciation in both periods the terminal capital stock will exceed the amount stipulated by the condition $\overline{KS(2)} - \overline{KS(0)} = 0$. This point is reached at G; beyond this on the arc GH the terminal capital stock becomes a free good.

It is possible that the required terminal stock may be smaller than the initial one. This is the case in the range $Q < \overline{KS(2)} - \overline{KS(0)} < 0$. If only consumer goods are produced in both periods, the decrement in capital stock, $\overline{KS(2)} - \overline{KS(0)}$, exactly equals the total depreciation which takes place. This position is located at point B on Figure 1. If 0 represents the required terminal stock but some amount of capital is produced in either period, the terminal stock becomes redundant. The area in which this occurs is shown on the surface by the triangular section BCD. In this region the intersection of the surface with planes parallel to the $X^1(2)$ and $[\overline{KS(2)} - \overline{KS(0)}]$ axes will define straight lines of zero slope in such planes; i.e. the price of the terminal stock relative to $X^1(2)$ is zero.

If positive weights are attached to $X^1(1)$ and $X^1(2)$ and if the terminal capital stock is not considered a free good the relevant surface is described by the convex surface of continuous slope in all directions marked BCFE. The two regions ABE and CDB are reflections of the fixed nature of capital: they can be deduced as the vertical edge AB was obtained in Figure 1. In the plane ABE both $X^1(2)$ and the terminal stock are free goods; in CDB only the terminal stock is valueless.[1]

1. We abstract from disposal problems.

The surface in Figure 1 can now be related to our programming framework. Assume that we fix $\overline{KS(2)} - \overline{KS(0)}$ in constraint (9), at I. Then by maximizing the objective function for alternative values for the W's we obtain the arc NE. Or, if we fix $\overline{KS(2)} - \overline{KS(0)}$ at zero $[\overline{KS(2)} = \overline{KS(0)}]$, we can derive HGL. However, the terminal condition will be binding only for the arc GH and not binding for the section GL. This is so because increasing production of $X^1(2)$ at the expense of $X^1(1)$ beyond G requires so much capital formation in the first period that the left over capital at the end of the second period is in excess of $\overline{KS(2)} - \overline{KS(0)}$, depreciation notwithstanding.

The method of fixing the amount $[\overline{KS(2)} - \overline{KS(0)}]$ and maximizing the objective function as described above, makes it possible to develop the entire surface shown in Figure 1.[1] By stipulating increasing values for $\overline{KS(2)}$ we, of course, necessarily have to narrow the consumption choices open to society in the two periods. If $\overline{KS(2)}$ is chosen sufficiently large, no consumption is possible: all effort must go into capital creation in both periods (see F in Figure 1).

The selection of $\overline{KS(2)}$ is nothing but the expression of a long-term saving goal for society. But any stipulated terminal capital stock, barring the extreme one corresponding to F, leaves society with a second choice to make: how to allocate effort in the short run between capital creation and consumption in a manner consistent with the long run goal of a desired capital stock. If the stipulated terminal stock corresponds to $\overline{KS(2)} - \overline{KS(0)} =$ I in Figure 1, that goal can be reached by a two-dimensional infinity of choices lying along the arc NJ. The slope of that arc at any given point indicates the rate whereby tomorrow's consumption can be increased by

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1. See footnote 1 on page 3.

diminishing today's consumption and by utilizing the factors thus released in the production of capital goods in the first period. By denoting this marginal rate of transformation of today's into tomorrow's consumption by $(1 + r)$, we can identify r as Irving Fisher's famous concept of "marginal rate of return over cost" which in equilibrium must be equal to the market rate of interest.[1] In our programming framework equilibrium is established if $(1 + r)$ is brought into equality with the ratio of the arbitrary weights W_1/W_2, which in turn is equal to $(1 + i)$ where i is the market rate of interest.

Underlying the maximization is a set of eight differential inequalities, one corresponding to each of the input variables. Denoting the partial change in $X^1(t)$ and $X^2(t)$ with respect to the two factors of production as $F^1_L(t)$, $F^1_K(t)$ and $F^2_L(t)$ and $F^2_K(t)$ identifying the λ's as Lagrangean multipliers, we have:

$$W_1 F^1_L(1) \leqq \lambda_L(1) \quad ; \quad (10)$$

$$\left[\lambda_K(2) + \lambda^k\right] F^2_L(1) \leqq \lambda_L(1) \quad ; \quad (11)$$

$$W_2 F^1_L(2) \leqq \lambda_L(2) \quad ; \quad (12)$$

$$\lambda^k F^2_L(2) \leqq \lambda_L(2) \quad ; \quad (13)$$

$$W_1 F^1_K(1) - \left[\lambda_K(2) + \lambda^k\right] \beta^1(1) \leqq \lambda_K(1) \quad ; \quad (14)$$

$$\left[\lambda_K(2) + \lambda^k\right]\left[F^2_K(1) - \beta^2(1)\right] \leqq \lambda_K(1) \quad ; \quad (15)$$

$$W_2 F^1_K(2) - \lambda^k \beta^1(2) \leqq \lambda_K(2) \quad ; \quad (16)$$

$$\lambda^k \left[F^2_K(2) - \beta^2(2)\right] \leqq \lambda_K(2) \quad ; \quad (17)$$

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1. We shall refer to this same concept loosely as the internal transformation rate or the internal production substitution rate.

The set of the first four relationships (10 to 13) refers to the allocation of labor; the remaining ones (14 to 17) refer to the allocation of capital in the two periods. λ^k is the shadow price of the terminal stock $\lambda_L(i)$ and $\lambda_K(i)$ are the net shadow prices of the two factors in the i^{th} period.

It is interesting to observe the rent relationships contained by the differential inequations. The value of the marginal product of any factor is, of course, the product of the marginal physical product of that factor and of a price term. In this sense relationships (10), (12) and (13) are straightforward. However, notice, for example, in (11) that the price term itself is a sum of two prices, i.e., that of the rent of capital in the second period and of the shadow price of the terminal stock. This is so because the capital produced in the first period will be available for use in the second period and will also contribute to the desired terminal capital stock. In addition, in the constraints governing the allocation of capital, it should be noted that the Lagrangean multipliers on the right hand sides of the inequalities represent net rents of capital. As capital depreciates with use the value of the depreciation in response to a marginal increase in the capital input must be subtracted from the market value of the marginal product of capital in order to obtain the net value of its marginal productivity. Depreciation resulting in the first period is valued by a composite price term, since the "lost" capital will not be available in the second period for producing either good, nor will it become a component part of the required terminal stock.

In the relationships (15) and (17), that is the ones which refer to the marginal productivity of capital in producing capital, the depreciation factor is subtracted directly from the marginal product in physical terms. This can be done because of our assumption that capital produced and depreciated is homogeneous. The difference of these two magnitudes, i.e., of $F_K^2(t)-\beta^2(t)$ is the net marginal physical product of capital in producing capital. We have referred to this concept in footnote 2 on page 2 above when we stipulated that $F_K^2(t)-\beta^2(t)>0$ as an absolute requirement for the viability of the growth system. This requirement is the Hawkins-Simons condition of the model.

The condition ensures that the marginal rate of transformation between different combinations of consumptions, corresponding to the same terminal capital stock, exceed one for at least some portions of the feasibility surface. It also ensures that the rents of capital are non-negative. This, the rationale of the condition can be readily understood by a glance at the structure of the surface in Figure 1. Take for instance the transformation that corresponds to $\overline{KS(2)} - \overline{KS(0)} = I$. Here, if the net marginal productivity of the factors producing capital is positive, then IN > IE; hence the slope of a straight line between N and E must exceed one (in absolute value); hence, the arc NE must at least in the portion adjacent to E have a slope greater than one.[1]

Assuming that we are somewhere on the surface bounded by CFED, that portion where both consumption and the terminal stock has a positive finite value, all differential relationships must exactly satisfy the equality. Making use of all the relationships, we eliminate the multipliers and obtain the following equality as a condition of a maximum

$$\frac{W_1}{W_2} = \frac{F_L^1(2)}{F_L^1(1)} \frac{F_L^2(1)}{F_L^2(2)} \left[1 + F_K^2(2) - \beta^2(2)\right] \quad ; \qquad (18L)$$

$$\frac{W_1}{W_2} = \frac{F_K^1(2)}{F_K^1(1)} \frac{F_K^2(1) - \beta^2(1) + \beta^1(1)}{F_K^2(2) - \beta^2(2) + \beta^1(2)} \left[1 + F_K^2(2) - \beta^2(2)\right] \quad . \qquad (18K)$$

1. On a transformation curve cutting between E and B such as the one corresponding to $\overline{KS(2)}-\overline{KS(0)}=0$ described by LGHM, the section GH does not have necessarily an internal rate greater than one. While OL>OM, the arc itself is "incomplete" in the sense that it is cut short by the vertical constraint HM.

Both equations have a common, multiplicative term, i.e., one plus the net marginal product of capital in producing capital. Otherwise equation (18L) is composed of marginal productivities of labor and equation (18K) of marginal productivities of capital. W_1/W_2 is, of course, identically equal to one plus the market rate of interest; hence, both ratios are expressions of the transformation rate of the curve between consumptions corresponding to any given terminal capital stock.

Notice that if fixed proportions prevail and if the β's do not change in either period, after cancellation of terms in both numerator and denominator of (18L) and (18K) we have $1 + F_K^2(2) - \beta^2(2)$ as the rate of transformation; in this case the marginal rate of return over cost is equal to one plus the net marginal product of capital in producing capital.[1]

Equations (18L) and (18K) are significant as they show the nature of the dependence of the internal transformation rate on the different marginal productivities and depreciation. By combining them we obtain one of the familiar overall efficiency conditions on the marginal productivities of capital and labor:[2]

$$\frac{F_L^1(2)}{F_L^1(1)} \; \frac{F_L^2(1)}{F_L^2(2)} = \frac{F_K^1(2)}{F_K^1(1)} \; \frac{F_K^2(1) - \beta^2(1) + \beta^1(1)}{F_K^2(2) - \beta^2(2) + \beta^1(2)} \quad . \qquad (19)$$

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1. This statement is strictly true only if capital is not redundant in the linearized (fixed proportion) model.

2. It should be noted that (19) is different from the general intertemporal conditions yielded by the Dorfman, Samuelson, Solow model (Chapter 12, op. cit.) with many produced goods and factors. In the model above, with the initial and terminal stocks of capital given, the total net output of capital is also determined. Then, if either consumption or capital goods is specified in any one of the periods, everything else is also determined in both periods. Hence, all that is left is the maintenance of intratemporal efficiency. The condition obtained from the combination of (18L) and (18K) is essentially a result of the combination of two separately identifiable intratemporal conditions

[cont'd]

It may improve the understanding of (18K) and (18L) if we come at them by way of common-sense reasoning about the marginal adjustments which are needed to maintain an equilibrium position on the transformation surface. Holding the output of $X^1(1)$, the capital stock at the end of the first period and the variables of the second period, constant, one unit of capital, K, is shifted in the first period from producing capital to producing the consumption good. Thus

$$\Delta X^1(1) = F_L^1(1) \cdot \Delta L^1(1) + F_K^1(1) \cdot \Delta K^1(1) = 0 \quad .$$

Setting $\Delta K^1(1) = 1$, the offsetting change in the labor input becomes

$$\Delta L^1(1) = -\left(F_K^1(1) / F_L^1(1) \right) \quad .$$

Since

$$\Delta L^2(1) = -\Delta L^1(1) \quad \text{and} \quad \Delta K^1(1) = -\Delta K^2(1) = 1 \quad ,$$

the change in the production of capital can be written as

$$\Delta X^2(1) = F_L^2(1) \cdot \Delta L^2(1) + F_K^2(1) \cdot \Delta K^2(1) = F_L^2(1) \left(F_K^1(1) / F_L^1(1) \right) - F_K^2(1) \, .$$

The capital stock at the end of the first period must remain unaffected by marginal changes in the production and use of capital in the various lines;

[cont'd from previous page] obtainable from elimination of the λ's in equations (10)-(17) such as:

$$\frac{F_L^1(2)}{F_K^1(2)} = \frac{F_L^2(2)}{F_K^2(2) - \beta^2(2) + \beta^1(2)}$$

Introduction of another capital factor would create the need for satisfaction of essential intertemporal efficiency conditions. We would obtain an additional condition like (18K) with the new capital factor appearing in the final term which then would not cancel out.

hence these changes must all balance out. Thus

$$\Delta KS(1) = \Delta X^2(1) + \beta^2(1)\,\Delta K^2(1) - \beta^1(1)\,\Delta K^1(1) = 0\,.$$

Substituting for $\Delta X^2(1)$, $\Delta K^2(1)$ and $\Delta K^1(1)$, we have

$$F_L^2(1)\left(F_K^1(1)/F_L^1(1)\right) - F_K^2(1) + \beta^2(1) - \beta^1(1) = 0\,.$$

This can be rearranged in the form:

$$\left(F_K^1(1)/F_L^1(1)\right)\cdot\left(F_L^2(1)/F_K^2(1) - \beta^2(1) + \beta^1(1)\right) = 1\,.$$

By similar reasoning for the second period we can obtain the analogous condition

$$\left(F_K^1(2)/F_L^1(2)\right)\cdot\left(F_L^2(2)/F_K^2(2) - \beta^2(2) + \beta^1(2)\right) = 1\,.$$

By equating the two left-hand sides of the conditions for each period and rearranging terms we obtain the conditions expressed in (19).

In addition to conditions of the form in equations (18) we can find other interesting relationships among the different rents and prices. The rate at which current consumption is given up for terminal capital stock is important as an indication of societies' "long-term saving" goals.[1] This rate is given by

$$\frac{\lambda^k}{W_2} = \frac{F_L^1(2)}{F_L^2(2)}\,; \qquad (20L)$$

$$\frac{\lambda^k}{W_2} = \frac{F_K^1(2)}{F_K^2(2) - \beta^2(2) + \beta^1(2)}\,. \qquad (20K)$$

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1. It should be noted that as a statistical concept the problem of dimensions arises; i.e., the ratio will not be a pure number. However, it may be possible to give it a meaningful interpretation if its value is compared intertemporally. As such it may indicate whether society is climbing up or down along the slope of a slice in the $[\overline{KS(2)} - \overline{KS(0)}] - X^1(2)$ plain; that is, if the changes in technology or in capital formation are marginal from period to period.

These are ratios between the marginal products of the factors in producing consumer goods on the one hand and their marginal effect on the size of the capital stock on the other hand. The latter, in the case of the application of labor is equal to the marginal product of labor in producing capital; in the case of the application of capital, however, its depreciation in producing consumer goods must be also added into account for the entire marginal change in the capital stock. No corresponding rate needs to be written out for consumption in period 1 (λ^k/W^{11}) as (18L) or (18K) and (20L) and (20K) jointly determine it. Apart from indicating the relative weight put on having a given terminal stock vis-a-vis consumption, equations (20) are important if the terminal stock is considered a variable rather than a parameter. In that case, instead of λ^k we would have an arbitrary weight but the expression itself would remain the same, and it would serve to fix our exact position jointly with either (18L) or (18K) on the feasibility surface of Figure 1.

Another relationship, the internal rate of capital, is obtained by dividing the net rent of capital by its price in any one period. By doing so we find (from either (16) or (17) that the internal rate of capital is $F_K^2(t)-\beta^2(t)$, i.e., the net marginal product of capital in producing capital, as we would expect.

The analysis makes explicit that if capital as a factor is fixed rather than circulating and if the "terminal" stock is itself a variable, the marginal rate of return over cost, or the rate of interest, between consumption goods in different periods, is not sufficient to yield a unique, optimum solution. To specify a point on the feasibility surface we also need the rate at which society is willing to exchange current consumption in any one of the two periods for terminal capital stock. Once the terminal stock is fixed, however, the market rate of interest will yield the consumptions in each period consistent with the accumulation of the terminal stock. Considered in this light the role of the rate of interest is to guide allocation of effort between producing consumption goods and capital in each time period toward the fulfillment of an ultimate goal determined separately though not independently from the interest rate itself.

To illustrate the latter point, assume the existence of a social welfare function or community indifference curve, which includes the terminal capital stock in addition to consumptions, as follows:

$$W = W\left[X^1(1),\ X^1(2),\ KS(2)\right].$$

Then the ratio of the partials

$$\frac{\dfrac{\partial W}{\partial X^1(2)}}{\dfrac{\partial W}{\partial X^1(1)}} \quad \text{and} \quad \frac{\dfrac{\partial W}{\partial KS(2)}}{\dfrac{\partial W}{\partial X^1(2)}}$$

brought into equality with the corresponding ratios on the supply side, (i.e., along the transformation surface) yields the maximum solution together with a corresponding rate of interest and a price ratio between consumption in any one period and the final capital stock.

III. Analytical Background

The objective of our analysis was to determine the available choices open to society in the intertemporal allocation of resources to production of consumption and capital goods. In deriving the intertemporal feasibility surface we have obtained all-important conditions for the optimal distribution of resources given final prices in each time period or a social welfare function.

In order to analyze the choices between consumption and the formation of fixed capital, we have drawn an absolute distinction between the production of capital and consumption goods. We did not permit any stocks to be carried over in the hands of producers from one to the next period though we could have done so. We ruled them out because in our case they do not enter into the production of fixed capital as productive inputs. We have also assumed away the use of such stock as intratemporal inputs.

Our approach, of course, was determined by the objective of our investigations. In terms of analytical purpose our effort is most closely related to the work of Irving Fisher.[1] However, we do not deal with the demand side where Fisher's most important contribution lies. We confine ourselves to the derivation of the alternatives open to society and the optimality conditions on the supply side and we represent demand only by relative weights. It is probably fair to say that had Fisher concerned himself with fixed rather than circulating capital he would have constructed for the analysis of production a framework not dissimilar from ours.

While our purpose most closely agrees with that of Fisher, our analytical framework is rooted in the approach that was recently given its general

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1. Irving Fisher, The Theory of Interest.

development particularly by the contributions of Dorfman, Samuelson, and Solow.[1] In fact, the model presented here can be interpreted as a special case of the Dorfman-Samuelson-Solow analysis of capital theory.

In addition it is interesting to note the similarities and differences between the structural details of our analysis and other capital models. As our analysis focuses on the alternatives open on the production side, we do not "close" our model by determining consumption as some given fixed proportion of output or by providing some alternative arbitrary decision rule. By comparison the analytically important Ramsey model, in which there is only one good used either for consumption and investment, is closed by a condition on consumption.[2] The Ramsey objective, however, was the determination of an optimum savings program and not the derivation of the range of alternative production programmes made possible by investment in durable capital. Similarly the advanced, many-sector theory presented by Samuelson and Solow in their paper, "A Complete Capital Model Involving Heterogeneous Capital Goods," is "closed" on the consumption side.[3]

The many-sector model of von Neumann is also closed; hence, it does not make possible the exploration of the full range of feasible patterns of output over time.[4] Von Neumann investigated the characteristics of a special type of balanced growth equilibrium given the condition that all goods must have the dual character of being both produced inputs and outputs.

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1. R. Dorfman, P. A. Samuelson, and R. M. Solow, Linear Programming and Economic Analysis, 1958, especially Chapters 11 and 12. Other references are given below.

2. Frank Ramsey, "A Mathematical Theory of Saving," Economic Journal, Vol. XXXVIII, No. 152, December 1928, pp. 543-559.

3. P. A. Samuelson and R. M. Solow, "A Complete Capital Model Involving Heterogeneous Capital Goods," Quarterly Journal of Economics, Vol. LXX, November, 1956, pp. 537-562.

4. John von Neumann, "A Model of General Equilibrium," Review of Economic Studies, Vol. XIII, No. 33, pp. 1-9; this is a translation of a German article which appeared in 1938 and was first presented in 1932.

Consumption goods in his model are, in fact, no more than inputs necessary to produce the labor factor. If this assumption is dropped we move toward the framework of the model presented in Part II above; but then some of the striking results of the von Neumann analysis disappear, specifically the uniqueness of the "balanced" and "optimal" growth rate.[1]

Our treatment of capital can certainly not claim to cover all of its aspects. We operate with a simplified concept of depreciation (see footnote 2, p. II/1) in which, unlike in Wicksell or in Solow's capital model, durability is not explicitly a choice variable.[2] Nevertheless it is the system which determines the optimal amount of depreciation for each time period as we assume different depreciation coefficients for each industry in each period. Depreciation occurs as an element in the von Neumann and some other capital models in a form which cannot easily be related to reality: depreciated capital is one of the joint products of production processes into which the undepreciated capital is an input.[3] Of course, in some capital models depreciation is avoided completely by operating only with net productivities as the result of an implicit depreciation deduction from gross productivities or by dealing only with "working" or "circulating" capital concepts.

In addition to allowing no variation in durability with respect to

1. See J. G. Kemeny, O. Morganstern, and G. L. Thompson, "A Generalization of the von Neumann Model of an Expanding Economy," Econometrica, Vol. 24, No. 2, April 1956.

2. Knut Wicksell, Lectures on Political Economy, Vol. I. The, as yet unpublished, paper by Prof. Solow of M.I.T., "Notes Toward a Wicksellian Model of Distributive Shares," has been particularly informative both about Wicksell's capital theory and interesting extensions of it.

3. Actually in the relationships (9) to (16) we could have given a "joint product" interpretation to capital rents. However, this would add nothing to clarity or empirical applicability.

inputs given a certain output, the model contains only a simple lag structure. Inputs in one period result in the production and use of consumption goods in the very same period but capital goods produced in one period become available for use only in the succeeding period. Variations in the lag structure by type of capital could be, of course, introduced into the model if we would work with several capital goods.

By confining the model to a single capital good we have, of course, exposed ourselves to recent criticism of the type advanced by Mrs. Robinson of the "capital" concept.[1] We could take refuge in such devices as Trevor Swan's meccano sets[2] but we prefer not to do so and to defend ourselves on other grounds.[3] It is only a matter of analytical and empirical convenience for us to have one capital and one consumption good; otherwise we could not have drawn the diagram in Figure 2 or presented the empirical results in the form as they appear in section IV. Many capital and consumption goods would reveal more about internal structure, a la Leontief, but we forego that type of analysis.[4] The Dorfman, Samuelson, Solow model of Chapter 12 in their Linear Programming and Economic Analysis is admirably suited to investigation of such matters.

However, there is no point in claiming more or less generality for the

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1. J. Robinson, The Accumulation of Capital, 1956.

2. T. Swan, "Economic Growth and Capital Accumulation," Economic Record, 1957.

3. See P. A. Samuelson and R. Solow, "A Complete Capital Model Involving Heterogeneous Capital Goods," Quarterly Journal of Economics, Vol. LXX, No. 4, November 1956, pp. 537-562.

4. The model of Part II is also related analytically to the Marx and Ricardo-like systems developed by Prof. Samuelson in "Wages and Interest: Marxian Economic Models," American Economic Review, Vol. XLVII, December 1957, pp. 884-912, and "A Modern Treatment of the Ricardian Economy," Quarterly Journal of Economics, Vol. LXXIII, Nos. 1 and 2, February and May, 1959, pp. 1-35 and 217-231, respectively.

approach of Part II. It is intended to be a simple approximation to choice problems as they are often seen in projections of economic development. That it has empirical relevance must be demonstrated; this we shall attempt in the next section.

IV. The Empirical Application (Preliminary Draft)

One of the end results of the analysis of Part II was the derivation of several transformation rates such as those of equations (18L) and (18K). These rates, which are slopes on the feasibility surface of Figure 1 along arcs such as NE, indicate the amount of one output to be gained by the sacrifice of a unit of some other output. These rates reflect, of course, the allocation of resources between production of consumption goods and capital goods. In the analysis of economic growth requirements this allocation is quite naturally seen as one of the key decisions.[1] The intrinsic interest of these transformation ratios has motivated the theoretical analysis and instigated an attempt at their measurement for the U.S. economy.

The issues of resource allocation have been investigated empirically before this at the highest levels of aggregation in terms of the proportions of total output which take the form of consumption or investment goods. Measurement over time of the amount of resources employed in the production of the one or the other has, to our knowledge, not previously been attempted in a dynamic framework and, therefore, all of the practical implications of such allocation could not be explored.[2]

The degree of approximation which has been required in the estimation process which has had to be followed may help explain the lack of previous

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1. This point hardly needs citation, but we may refer to the Indian Five Year Plans and the Italian Plan of Employment and Income in the Decade, 1955-64 as practical examples. In the theoretical literature on growth and development this is one of the most frequently recurring themes.

2. The input-output matrices which have been constructed for a number of countries may be considered an exception. These do provide a means of determining, for the year which a particular matrix represents, the proportions of various types of resources used for capital creation or the provision of consumption goods. Since, however, input-output matrices are seldom available on a year-to-year basis, that device cannot be used for successive, annual estimates of resource allocation, except in rare cases as, for example, The Netherlands.

effort along these lines. However, the significance of the transformation rates warrants, we believe, the attempt at measurement which we have made.

In order to estimate, year by year, the transformation ratios for the U.S. economy, we must ascribe to it the characteristics of our mode. The ratios are then measured as "observed" slopes on successive inter-period feasibility surfaces which mount up like the layers of a piece of an onion. This involves not only the aggregation of all inputs and outputs to two each, but also the ascription to the economy of the assumptions of continuity and convexity necessary to perform the mathematical analysis. Unfortunately, we would have to know much more than we do in order to form a judgment as to the extent to which these assumptions are justified and the effect of any actual deviations on the estimated quantities. But, by making the assumptions, we justify our use of U.S. data, if, in turn, empirical quantities can be found which correspond to the marginal productivities and depreciation factors necessary to estimate the transformation ratios.

In this section we shall outline our estimation procedure briefly, describe our results, and speculate on their meaning. Details of the calculations and some of the adjusted series are left to an appendix.

Strictly speaking, of course, one cannot find from existing information estimates of marginal productivities or depreciation factors. Since we have assumed first order homogeneity we could estimate marginal productivities by the average rate of return to the factors.[1]

1. This is the procedure used by Prof. Robert Solow in his pathbreaking article, "Technical Change and the Aggregate Production Function," Review of Economics and Statistics, XXXIX, August, 1957.

Use of this approach in turn requires estimates of the total amounts of factors allocated to production of capital and consumption goods and of the total returns to the factors in their various uses. Construction of such estimates has been the major part of the empirical effort.

The first step was to settle on the appropriate aggregate concept to be used and to divide it between consumption and capital goods production. Since we must work with factor shares to estimate marginal productivities we use the "national income" concept in constant 1954 dollars as estimated by the Department of Commerce. Because of problems of valuation of output and measurement of inputs we omit from the total national income that produced by "government" (except government enterprises) and the "rest-of-the-world." Thus, our income concepts are "private" and "domestic." This income had next to be divided between income generated in consumption and investment goods production. The following procedure was used. The conventional gross national product was converted into a "private, domestic" concept and the proportions going into consumption and investment goods recorded.[1] These proportions were, in turn, applied year by year to the derived private, domestic national income totals to estimate the proportions of national income created in producing the two types of output. The income generated in agriculture, wholesale and retail trade, finance, insurance and real estate, and part of construction, was credited

1. Government purchases from the private sector are included as privately produced outputs.

entirely to consumption production. The remainder of the national income earned in producing consumption was then allocated to the mining, manufacturing, transportation, construction, communication and public utility sectors. The rest of the national income created in these sectors was credited to capital production.

Total labor force figures were readily taken from Department of Commerce sources but capital stock estimates had to be developed sector by sector, relying mainly on the Department of Commerce estimates, the National Bureau of Economic Research Studies in Capital Formation and Financing, and other sources. The allocation of the labor force and capital stock between consumption and capital production was done sector by sector according to the proportions corresponding to the division of incomes in each sector.

Wage bill information, sector by sector, was available from Income and Output, with some necessary imputations of wages in the non-corporate sector. The total returns allocable to capital were estimated sector by sector as the residual after subtracting wages from total national income plus the recorded depreciation. Wages and the gross rents were then distributed between earnings in capital and consumption goods production by the same procedures with which labor force and capital stocks were distributed.

These assumptions, allocations and adjustments finally permitted calculation of the marginal productivities of labor and capital in consumption and capital goods production. The depreciation factors were calculated as the ratio of capital consumption allowances to the sum of income generated and capital consumption.

To avoid unwarranted interpretations and excessive reliance on the absolute magnitudes the model assumptions and the calculation methods must be kept in the forefront when examining the implications of the results which have been obtained. Inevitably in such calculations a number of assumptions must be made which are open to question and data must be used which are not entirely suitable. The assumptions and data sources are specified fully in the Appendix.

In turning now to discussion of the results actually obtained, we shall frequently mention their sensitivity to computational procedures and model assumptions.

For estimation of the rates of return over cost, two alternative procedures were possible, as shown by the equations; each was used to provide a check. The results are shown in the accompanying tables and charts.

The marginal rate of return over cost for the years covered averages .1887 when computed with reliance on marginal capital productivities, i.e., by substitution in equation (18K), and .2074 when computed "on the labor side," using equation (18L). The year-by-year results are shown in columns 1 and 2 of Table 1 and in Chart 1. The difference in the averages of the ratios calculated by the two approaches is about ten per cent over the entire period covered. Most of that is due to the disparities of the first three pairs of years; in the last seven years the difference is only about four per cent.

It can be seen from equations (18) that all the marginal productivities enter the equations in year-to-year ratios. The net marginal productivity of capital in producing capital also enters in its absolute value but in both equations. Hence, annual changes in the factor marginal productivities

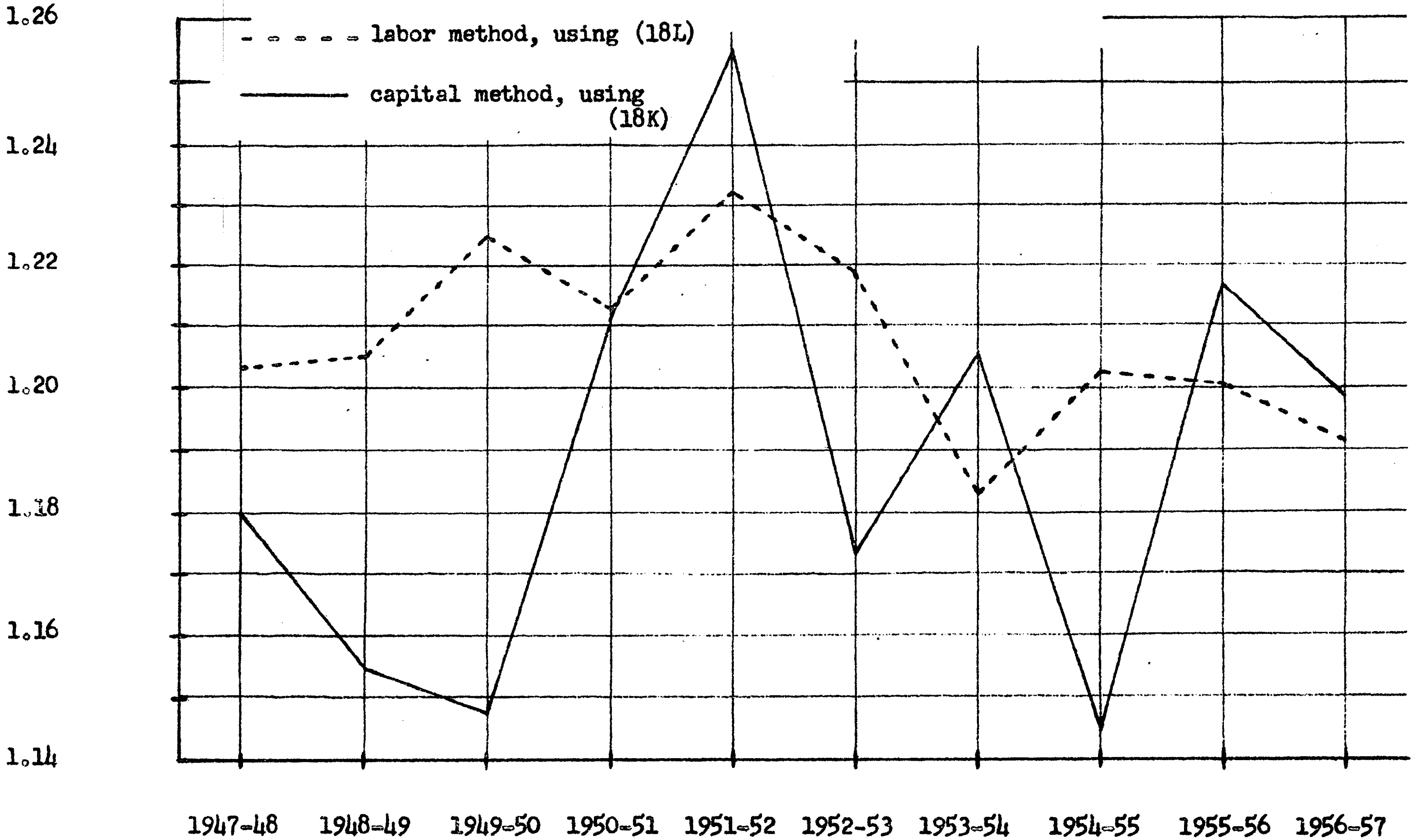
TABLE 1

Transformation Rates

	Marginal Rate of Return Over Cost		Between Terminal Capital Stock and Final Consumption	
	Computed from equation		Computed from equation	
	(18L) (1)	(18K) (2)	(20L) (3)	(20K) (4)
1947			.8122	.5627
	.2036	.1800		
1948			.8137	.5506
	.2052	.1545		
1949			.8287	.5576
	.2251	.1478		
1950			.8341	.5158
	.2125	.2107		
1951			.8230	.5061
	.2320	.2551		
1952			.8424	.5401
	.2186	.1731		
1953			.8552	.5367
	.1827	.2056		
1954			.8585	.5656
	.2024	.1448		
1955			.8560	.5306
	.2004	.2165		
1956			.8590	.5432
	.1910	.1986		
1957			8626	.5543

Chart 1

$\frac{W_1}{W_2}$, 1 + the marginal rate of return over cost for the U.S.



are more important than absolute values in determining the differences in the levels of the two alternative estimates of the marginal rate of return over cost.

The factor marginal productivities are listed in Table 2 and plotted in Charts 2 and 3. One of the explanations for the different trend patterns in the marginal productivities can be found in the fact that the capital-labor ratio rose more rapidly in the production of consumption goods than in capital goods in the period covered.

To demonstrate the sensitivity of all of the results to absolute value of the net marginal productivity of capital it may be noted, for example, that a ten per cent change in the average estimate of capital used in producing capital would lead to an approximately ten per cent change in the opposite direction in the gross marginal productivity of capital. In turn, the factor $\left[1 + F_K^2(2) - \beta^2(2)\right]$ would change by only about two per cent. This in turn would be the approximate percentage change in the quantity $(1 + r)$, however, the estimate of r, the marginal rat rate of return over cost, would itself change by approximately ten cent again.

The marginal rate of return over cost as calculated here is a "before tax and after depreciation" concept. Thus, to some extent, it is not surprising that the average level is frequently above the rates of return

TABLE 2

Marginal Productivities

	In Capital Goods		In Consumption Goods	
	Of Capital (1)	Of Labor (2)	Of Capital (3)	Of Labor (4)
1947	0.2275	3.509	0.1180	2.850
1948	0.2414	3.473	0.1229	2.826
1949	0.2288	3.462	0.1157	2.869
1950	0.2616	3.641	0.1244	3.037
1951	0.2732	3.723	0.1281	3.064
1952	0.2524	3.806	0.1239	3.206
1953	0.2530	3.979	0.1220	3.403
1954	0.2365	3.964	0.1169	3.403
1955	0.2679	4.104	0.1248	3.513
1956	0.2571	4.169	0.1226	3.581
1957	0.2447	4.170	0.1194	3.597

Chart 2

Marginal Physical Productivities of Capital

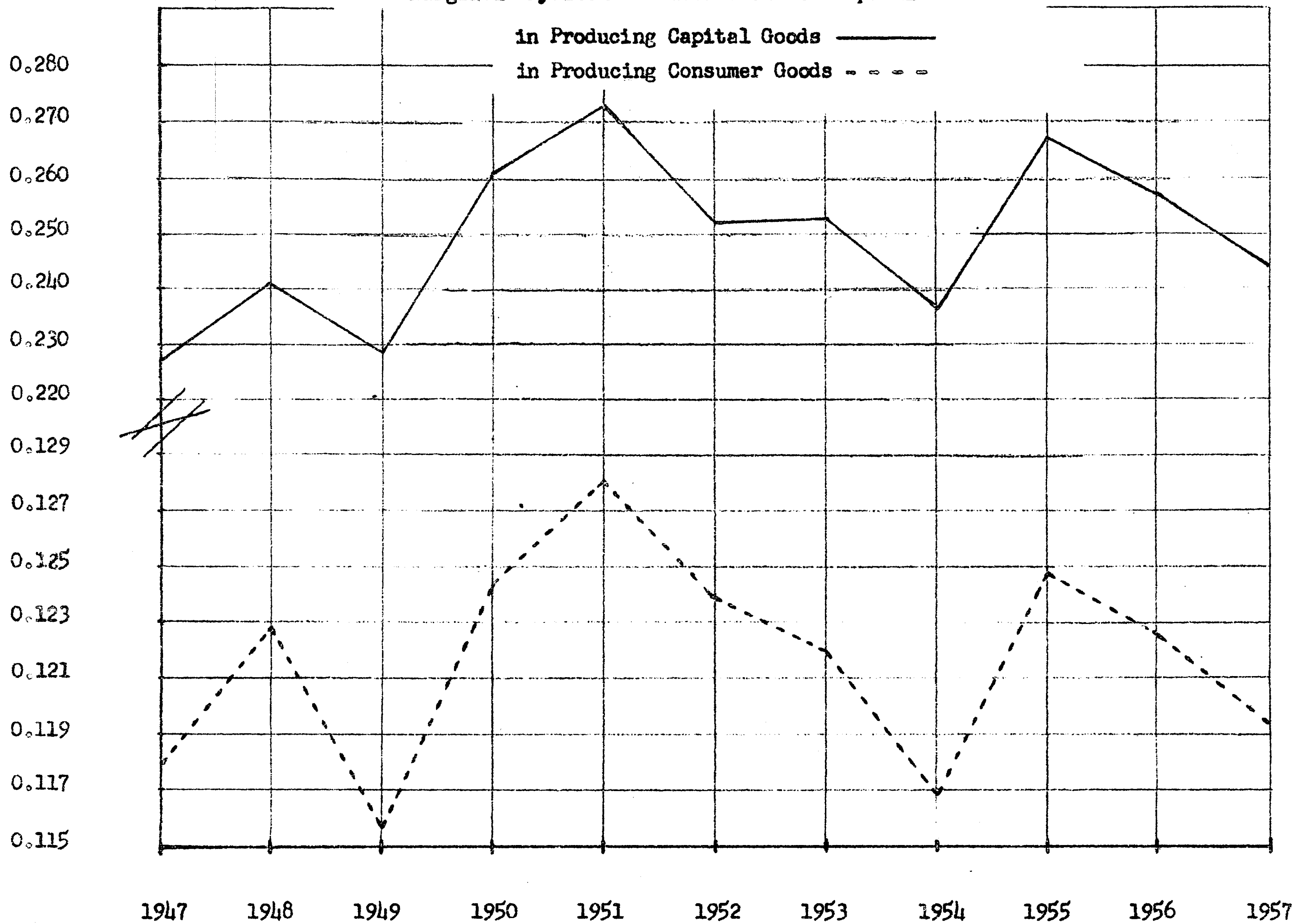
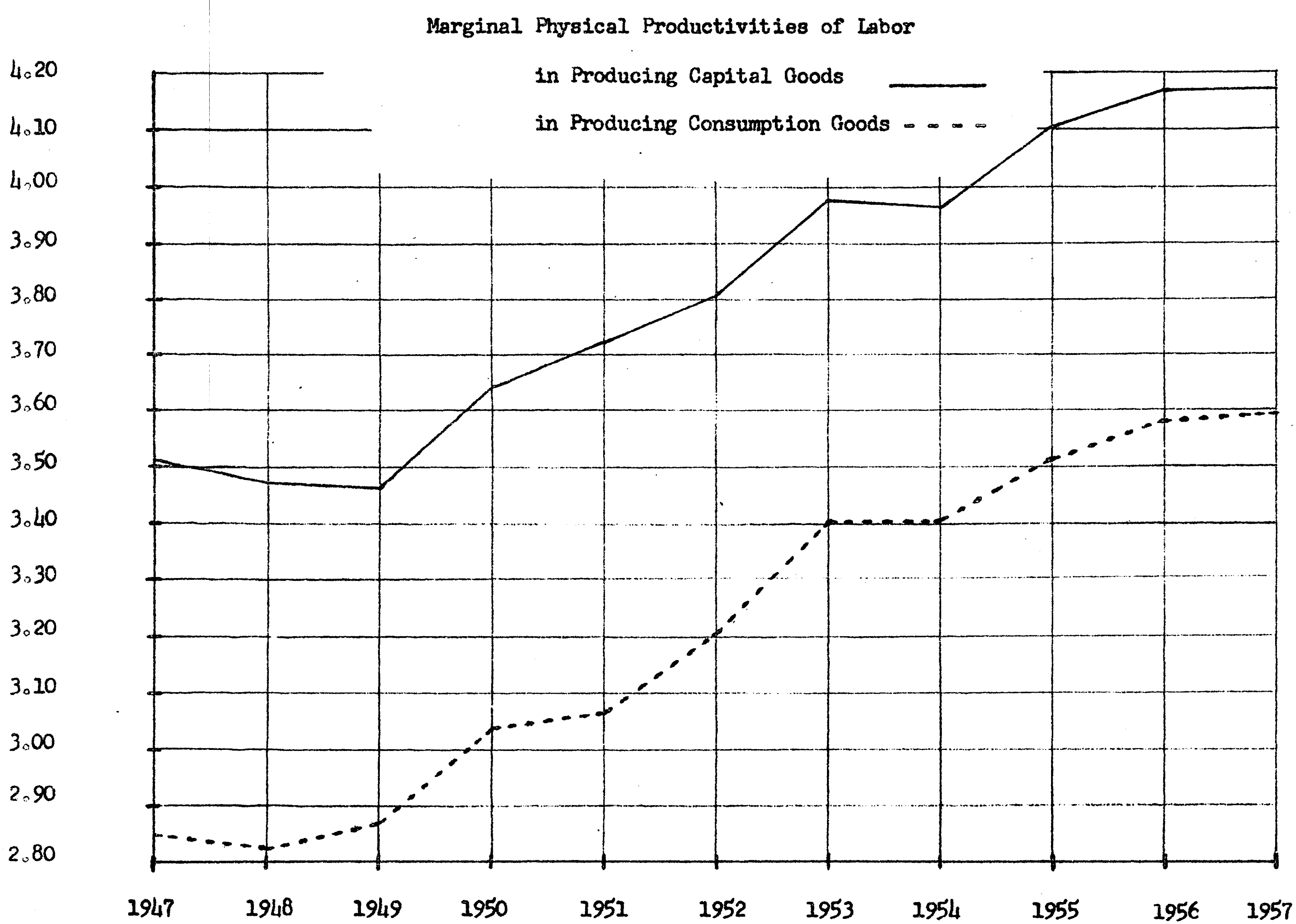


Chart 3


Marginal Physical Productivities of Labor
in Producing Capital Goods
in Producing Consumption Goods
4.20
4.10
4.00
3.90
3.80
3.70
3.60
3.50
3.40
3.30
3.20
3.10
3.00
2.90
2.80
1947
1948
1949
1950
1951
1952
1953
1954
1955
1956
1957

commonly cited as effective in the U.S. economy.[1] As an observed return it also includes all the effects of technological progress in improving factor productivities. These cannot ever be fully foreseen in undertaking investment so that a difference between the "normally expected" rates of return and the observed values would, in turn, be expected.

The year-to-year variability is another striking feature of the marginal rate of return over cost estimates. As can be seen from Chart 1 the estimate relying exclusively on capital marginal productivities possesses this feature to a greater degree than the estimate using ratios of labor marginal productivities. This, in turn, reflects the greater year-to-year changes of the marginal productivities of capital as compared to labor productivities as shown in Chart 2. We operate on the assumption that, in the years chosen for investigation, we are observing a point on the intertemporal transformation surface and that there is full employment of the factors measured. Although there was no "great depression" during the period covered, there were several minor recessions, and we would expect that cyclical variability would show up more in the variability of the marginal productivities of capital than of labor. This is true partly because capital is a fixed factor whose inputs cannot be so easily adjusted to cyclical changes in demand as can labor use. Secondly, profits, a major part of the returns to capital, are a residual. This, taken together with the postwar downward inflexibility of wage rates, means that the average returns to capital have been more variable than to labor.

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1. The rate calculated here approximates the profit rates on stockholders' equity reported in Quarterly Financial Report for Manufacturing Corporations published by the Federal Trade Commission and the Securities and Exchange Commission. However, a rate which would be more comparable would be the return to all assets and, thus, include the bond return. Moreover, the rate calculated here covers all sectors. If, as seems generally agreed, the rate of return in manufacturing runs higher than in other sectors, the conclusion above is warranted.

The cyclical changes in the marginal rates of return over cost estimates are undoubtedly related to cycles in the U.S. economy but not in any simple way. For example, it is reasonable to suppose that, for the reasons cited just above, the computed marginal productivities of capital would vary directly with the business cycle. There are, however, conflicting cyclical influences in the marginal productivities of labor and in addition strong trend effects in the period covered. Reference to equations (18L) and (18K) again will indicate that the marginal rate of return over cost is largely the result of comparison of present and future productivities. Improvement of capital and labor marginal productivities will have opposite effects depending on whether they occur in capital or consumer goods production.

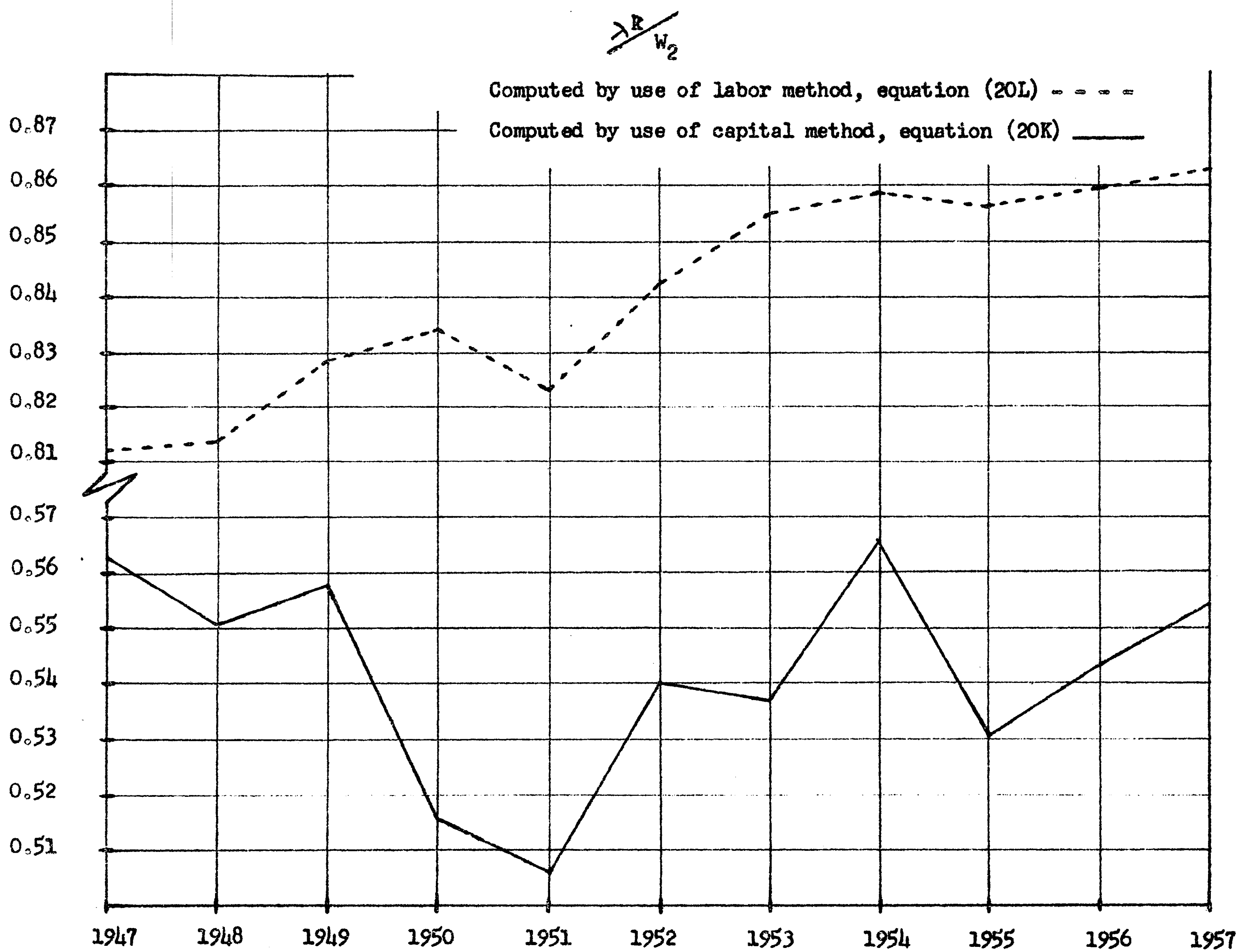
A cyclical improvement in the marginal productivity of capital due to greater utilization of capacity will have somewhat offsetting effects in the equations on the capital side but straight-forward positive effects in equation (17L) on the labor side. Cyclical improvement in the marginal productivities of labor will have opposite effects depending on whether they are greater in the production of consumer goods or capital goods.

If the U.S. economy corresponded to the model of Part II and complete data were available, the computation of the marginal rate of return over cost by (18L) or (18K) would give the same results. The observed discrepancies reflect both real departures from the conditions of the theoretical model and inadequacies of data. Some of the data problems arise from cyclical variability, e.g., the difficulty of adjusting capital stock inputs when there is less than full utilization of capital. This type of discrepancy can be reduced by smoothing the series or some other type of adjustment. However, the extent to which the remaining differences in the two estimates are due to data problems or "imperfections" cannot be fully known. If there are no systematic changes in data availabilities, changes in the discrepancies between the two types of estimates would reflect changes in the importance of imperfections in the labor and capital markets in the U.S. economy.

As pointed out in Part II two slopes are required to determine a point on the intertemporal transformation surface of Figure 2. In estimating W_1/W_2 we have found one of the slopes. Columns 3 and 4 of Table 1 and Chart 4 present estimates for λ^k/W_2, the slope of successive transformation surfaces in an X_1 plane, i.e., the rate of substitution between terminal capital stock and consumer goods production in the final period. The average rate computed by use of equation (20L) is .8405; when computed by use of equation (20K) it is .5421.

Reference to Charts 2 and 3 again reveals the sources of the disparities in the estimated ratios to be in the relative behavior of the marginal productivities of labor and capital in producing consumption and capital. In particular the relatively rapid rise of the marginal productivity of labor in the consumption goods sector and the fall of the marginal productivity of capital in this sector accounts for the behavior of the ratios in Chart 4 over most of the period. The discrepancies in the results computed by the two methods again reflect both data errors and differences in the degree of "perfection" in the markets for the various factors. The latter is even more clearly indicated in these ratios than in the estimates of the marginal rate of return over cost as only capital or labor marginal productivities occur in each estimating equation.

Chart 4



λ^R / W_2
Computed by use of labor method, equation (20L) - - - -
Computed by use of capital method, equation (20K) ____
0.87
0.86
0.85
0.84
0.83
0.82
0.81
0.57
0.56
0.55
0.54
0.53
0.52
0.51
1947
1948
1949
1950
1951
1952
1953
1954
1955
1956
1957

V. Conclusion

The analysis underlying the theoretical and empirical findings of this paper is a continuation of existing models in capital theory. In particular, it provides a link between the analyses of the type advanced by Irving Fisher and the Dorfman-Samuelson-Solow type.

The transformation surface derived in Part II is fully descriptive of the choices available to an economy and the underlying relationships yield the conditions for efficient resource utilization. The dual interpretation of the latter gives intricate price and rent relationships connecting factor returns, depreciation and marginal productivities. The surface itself shows the discontinuities of slope reflecting the constraints imposed by fixed capital and depreciation. The limit on depreciation incurred in producing new capital is set by the Hawkins-Simon conditions.

One important conclusion of the theoretical analysis can be summed up as follows. Given fixed capital not substitutable for consumption goods, for any fixed interest rate there are an infinite number of combinations in which consumption and capital goods can be produced. The explanation for this conclusion is that each particular output combination corresponding to the same rate of interest will define different price ratios between capital and consumer goods such that the present discounted values of the alternative expansion programs are equal to each other. The particular program chosen is determined both by the rate of interest <u>and</u> the price ratio between consumption and capital goods.

The empirical results of Part IV are interesting in their own right. In spite of the inherent weaknesses of the data underlying the computation and the shortcomings of our statistical procedures, the outcome summarizes some significant characteristics of the economy. Moreover the empirical exercise demonstrates the need for more accurate information on the participation of different industries in capital formation as a basic datum in the empirical approach to capital theory. Furthermore, the method presented here opens up alternative approaches to the investigation of factor markets, international comparisons of the structure of production and capital movements. One virtue of the analysis is that it points ahead to a variety of further theoretical and empirical extensions.

Appendix

It will be noted that while ratios of marginal productivities are important in the equations for the transformation rates the absolute values of the marginal products of capital employed in producing capital play an important role. These marginal productivities in turn rely on an estimate of capital stock. Since only the manufacturing, mining, transportation and communication, public utility and contract construction sectors were assumed to produce capital, the capital stock estimates in these sectors are of especial importance. The fixed capital components for all except the last sector can be said to be the result of judicious procedures as the citations will show. Since no similar set of fixed capital estimates could be found for the construction sector, a fixed capital-output ratio was applied to the national income generated in this sector. But all the capital estimates must still be subject to qualification due to the nature of the concept. For example, however careful and precise are the annual estimates of investment, the cumulation of these to obtain capital stock figures involves difficult problems of deflation in order to value all stock components consistently. Then, too, there are the many persistent conceptual and empirical difficulties in the treatment of depreciation.

The appropriate inventory stock figures were not available for any of the sectors as the existing inventory data necessarily reflect cyclical influences of all types rather than the "technical requirements" estimate desired. Our way out was to find the average inventory-output ratio in the years covered and apply this annually to determine the inventory component of capital used in production. All capital stock estimates were converted to 1954 prices.

The distribution of outputs, factor inputs and factor returns between capital and consumer goods production was another crucial stage in the estimation procedure. The method followed, as described above, certainly leaves out the possible contributions of agriculture, wholesale and retail trade, and finance, insurance and real estate to the production of capital and thus possibly overstates the contribution of the remaining sectors in this respect and understates their output of consumption goods. However, given the different characteristics of capital and consumer goods production it is both theoretically and empirically important to distinguish the contribution of each sector to capital or consumer goods production. Our judgment was that the error involved in the above division of sectorial output was less important than that involved in lumping all outputs together as a single, homogeneous capital-consumption good. For example, the real estate sector is a particularly capital-intensive sector and this, undoubtedly, is very largely due to the predominance in the capital totals of that real estate, especially residential, whose output is a consumption service. Not being confident of our ability to extract the appropriate capital producing component from all real estate and believing anyway it was relatively small we put the output of the whole sector into consumption goods.[1]

The accompanying Appendix Table presents the final numerical results and calculations for the various transformation rates.

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1. Inspection of an input-output table such as TABLE 1 prepared as part of the interindustry research program sponsored by the Department of the Air Force bears out these approximate allocations.

Appendix Table

		1947	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957
(1)	$F_L^1(t)$	2.850	2.826	2.869	3.037	3.064	3.206	3.403	3.403	3.513	3.581	3.597
(2)	$F_L^2(t)$	3.509	3.473	3.462	3.641	3.723	3.806	3.979	3.964	4.104	4.169	4.170
(3)	$F_K^2(t)$	0.2275	0.2414	0.2288	0.2616	0.2732	0.2524	0.2530	0.2365	0.2679	0.2571	0.2447
(4)	$F_K^1(t)$	0.1180	0.1229	0.1157	0.1244	0.1281	0.1239	0.1220	0.1169	0.1248	0.1226	0.1194
(5)	$\beta^2(t)$	0.0392	0.0401	0.0454	0.0444	0.0444	0.0487	0.0527	0.0583	0.0620	0.0609	0.0588
(6)	$\beta^1(t)$	0.0214	0.0219	0.0241	0.0240	0.0243	0.0257	0.0270	0.0285	0.0293	0.0295	0.0295
(7)	$F_K^2(t)-\beta^2(t)+\beta^1(t)$	0.2097	0.2232	0.2075	0.2412	0.2531	0.2294	0.2273	0.2067	0.2352	0.2257	0.2154
(8)	$F_K^2(t+1)+1-\beta^2(t+1)$	1.1883	1.2013	1.1834	1.2172	1.2288	1.2037	1.2003	1.1782	1.2059	1.1962	1.1859
(9)	$F_L^1(t+1)/F_L^1(t)$	0.9916	1.0152	1.0586	1.0089	1.0463	1.0614	1.0000	1.0323	1.0194	1.0045	
(10)	$F_L^2(t)/F_L^2(t+1)$	1.0104	1.0032	0.9508	0.9780	0.9782	0.9565	1.0038	0.9659	0.9844	0.9998	
(11)	W_1/W_2 = (8)·(9)·(10), from (18L)	1.2036	1.2052	1.2251	1.2125	1.2320	1.2186	1.1827	1.2024	1.2004	1.1910	
(12)	$\dfrac{F_K^2(t)-\beta^2(t)+\beta^1(t)}{F_K^2(t+1)-\beta^2(t+1)+\beta^1(t+1)}$	0.9395	1.0757	0.8603	0.9530	1.1033	1.0092	1.0997	0.8788	1.0421	1.0478	
(13)	$F_K^2(t+1)/F_K^2(t)$	1.0455	0.9069	1.0961	1.0339	0.9451	0.9684	0.9305	1.0803	0.9759	0.9646	
(14)	W_1/W_2 = (8)·(13)·(14), from (18K)	1.1800	1.1545	1.1478	1.2107	1.2551	1.1731	1.2056	1.1448	1.2165	1.1986	
(15)	λ^k/W_2 = (1)/(2), from (20L)	0.8122	0.8137	0.8287	0.8341	0.8230	0.8424	0.8552	0.8585	0.8560	0.8590	0.8626
(16)	λ^k/W_2 = (4)/(7), from (20K)	0.5627	0.5506	0.5576	0.5158	0.5061	0.5401	0.5367	0.5656	0.5306	0.5432	0.5543