

THE MATHEMATICAL FRAMEWORK OF THE THIRD
FIVE YEAR PLAN

S. Chakravarty

I. Introduction

The purpose of this note is threefold: (a) it seeks to formalize the various hypotheses of a systematic nature in models such as have been previously used in Indian development planning. Similar planning models have been used in certain other countries, e.g., Italian ten-year development plan and are used at present by Messrs. Pant and Little in their respective memoranda concerning the forthcoming Indian plan. In thus formalizing, it reveals the essential interconnections implicit in the planning calculations, e.g., the way in which the various exogenous and endogenous variables hang together, once certain basic parameter values have been assumed to be known.

(b) Having drawn up the formal scheme or the 'model' if we prefer to use this expression, several numerical illustrations are presented in the following order:

- (i) The Pant-Little case using their model structure and parameter values;
- (ii) The modified Pant-Little case with a capital coefficient equal to 3;
- (iii) The same case with an increased amount of foreign aid, all the assumed parameters values being the same as in (ii).

These comparisons will serve the important purpose of highlighting the degree of economic realism implicit in these numerical exercises.

(c) Once the implications of the 'model' as it stands have been sufficiently worked out, the natural thing to do would be to suggest some simple extensions,

¹ Numerical models as well as the question of 'resource gap' relating to the Third Five Year Plan are discussed in the Paper C/59-20, Alternative Numerical Models of the Third Five Year Plan of India, by Professor P.N. Rosenstein-Rodan. I am indebted to Professors P.N. Rosenstein-Rodan and R.S. Eckaus for valuable suggestions.

particularly when the original version is an overly simple one. We try to indicate a few of the extensions that would make the 'model' a more realistic one from the point of view of decision-making in the present Indian context.

II. Before taking up the various structural equations constituting the type of 'model' generally involved in such plans, it may be useful to add some methodological observations concerning its general nature.

To start with, it should be borne in mind that the 'model' we are after is a type of 'decision model.' This implies that it must necessarily have more unknowns than equations. In other words, the system must possess certain 'degrees of freedom.' The reason for this is that a model which is completely 'locked' (e.g. one for which the number of equations is the same as that of unknowns) cannot serve as a decision model, since it cannot discriminate between the results of alternative policy constellations.^{1,2} The models implicit in the previous Indian plans as well as the Pant-Little papers satisfies this criterion of having some open ends and hence must be regarded as 'decision models.'

In the usual literature on 'decision models,' a distinction is normally drawn between models having fixed targets and those having 'flexible targets.' In the fixed targets case, certain values are ascribed by the planners to the target variables they have in mind. The optimization procedure is generally hidden behind these specified target values. In the flexible targets case, however, the optimization procedure is an integral part of the planning problem. The problem is determinate only after the optimization procedure has been worked out. In general the 'flexible targets' problem is more difficult to work out than the 'fixed targets' problem and thus, in many planning situations, 'fixed targets' are chosen as best indicative of the planners' preferences. The type of model we shall be discussing here is one having "fixed targets."

¹Each policy constellation is defined by the way the degrees of freedom are filled up.

²Any 'locked' model may be easily 'unlocked' by just dropping one equation. Whether it is meaningful to do so depends on the economic nature of the model.

Further, such 'models' are normally 'linear.' This implies that the relationships indicate the constancy of marginal quantities like marginal savings rate, etc., while they may or may not imply constancy of the average quantities. This is not a very restrictive assumption if one confines oneself to a limited period of time, as is usually done in many of these plans.

Finally, although the model is concerned with a period of five years, the present orientation is towards results achieved over the whole period, rather than any year-to-year variation. Thus, even if it is possible, in principle, to cast the model in terms of difference equations, it is more useful to present it in the form of a set of algebraic equations in which only the initial and final values are related. No essential feature of the model would be lost in this form of presentation.

We have the following variables:

- (1) I : total investment
- (2) S_t : Savings in period 't'
- (3) F : the amount of 'net' foreign aid
- (4) ΔY : the increase of income over the five-year period
- (5) D_A : demand for agricultural output
- (6) ΔY_A : increase in agricultural production
- (7) T_t : total amount of tax revenue in year 't'
- (8) T_t : part of tax revenue dependent on income or some component of income
- (9) ΔY_{NA} : non-agricultural production
- (10) C_t : consumption in period 't'
- (11) E_t : total government expenditure over the period
- (12) D : increase in government debt, tax rates remaining unchanged
- (13) I_A : investment in agriculture
- (14) I_{NA} : investment in non-agriculture
- (15) α : the annual increment in savings

The following are the data of the system:

- (a) P_t : population in year 't'
- (b) $E_{c,t}$: government current expenditure in period 't'
- (c) R_t : operating surplus from public enterprises in period 't'
- (d) T''_t : the part of tax revenue, which is roughly autonomous with respect to income

The data of the system are those variables which are always determined from outside the model.

We have the following set of parameters:

- (a) β : the global output-capital ratio
- (b) β_a : the output-capital ratio in agriculture
- (c) ρ_i : the proportion of investment expenditure undertaken by the government
- (d) $\gamma_1, \gamma_2, \gamma_3$: the proportions in which existing tax revenue is earned from current consumption, current agricultural and non-agricultural incomes
- (e) η : the income elasticity of demand for agricultural production.

We have the following set of equations:

- (1) $I = \Sigma I_t = \Sigma S_t + F$
- (2) $\Delta Y = \beta I$
- (3) $S_t = S_0 + t\alpha$
- (4) $D_A(t) = P_t \left(\frac{Y}{P}\right)_t^\eta$
- (5) $\Delta Y_A = \beta_a I_A$
- (6) $T_t = T'_t + T''_t$
- (7) $T_t^d = C_t^{\gamma_1} \cdot Y(NA)_t^{\gamma_2} \cdot Y(A)_t^{\gamma_3}$
- (8) $\Sigma E_t = \Sigma E_{c,t} + \rho_i I$
- (9) $\Delta D = \Sigma E_t - (\Sigma T'_t + \Sigma R_t)$
- (10) $\Delta Y_A + \Delta Y_{NA} = \Delta Y$
- (11) $I = I_A + I_{NA}$
- (12) $\Delta C_t + \Delta S_t = \Delta Y_t$
- (13) $\Delta D_A = \Delta Y_a$

Thus, we have 15 unknowns and 13 equations. Of these 13 equations, equations (1), (6), (9), (10), (11) are either definitional equations or balance equations, while the remaining equations are composed of behaviour equations, technological equations, or institutional equations (e.g. tax equations). The above counting shows that the system has two degrees of freedom. The values of any two variables may, therefore, be set arbitrarily from outside, and the remaining variables will be determined from within the system. This gives us a number of alternative policy constellations from which a choice may be made. In the Pant-Little case, these two open ends were filled up by assuming I and F to be given from outside. Certain other possibilities of filling up the open ends are discussed later towards the end of this section.

The following explanations may be offered regarding the structural equations, although a large number of them are self-explanatory. Equation (1) states that total investment over the whole period is equal to total domestic savings plus net foreign aid.

Equation (2) indicates the increase in national income over the whole five-year period as obtained from investment over the period multiplied by the incremental output-capital ratio. Equation (3) is a more interesting one. It gives savings at time 't' as a linearly increasing function of time 't', starting from a certain base period.

This is just the mathematical equivalent of the numerical projection made in the Pant paper. Although it looks quite arbitrary in itself, taken together with other equations, it defines although implicitly, how savings change with respect to change in income. Thus, the marginal savings propensity is a derived figure. Equation (4) indicates that demand for agricultural production is a function of the level of population as well as the per capita income. Thus, on the logarithmic axis, this will read as follows:

$$\frac{\Delta D_a}{D_a} = \frac{\Delta P}{P} + \eta \frac{\Delta(Y/P)}{Y/P}$$

In other words, relative increase in demand for agriculture is the sum of the relative rate of increase in population plus the relative rate of increase in per capita income multiplied by the income-elasticity of demand.

Interpretation of equation (5) is similar to equation (2). It may be regarded as a supply equation for agriculture. Equation (6) states that total tax revenue is composed of those taxes which show the same rate of increase as income or some components of income and those which remain roughly invariant with respect to income. Note, however, that this equation presupposes the existing taxes as well as the tax rates to remain unchanged. Equation (7) may be more meaningfully stated in incremental form:

$$\frac{\Delta T^d}{T^d} = \delta_1 \frac{\Delta C}{C} + \delta_2 \frac{\Delta Y_{NA}}{Y_{NA}} + \delta_3 \frac{\Delta Y_A}{Y_A}$$

In other words, the relative rate of increase in derived tax revenue is equal to the weighted average of the rates at which consumption, agricultural income, and non-agricultural income are increasing, the weights being the proportions in which the existing derived tax revenue is earned from current consumption, current agricultural income, and current non-agricultural income. This approximates very closely the arithmetical procedure used by Little in computing the amount of derived tax revenue, although there is a slight discrepancy in as much as a few items of excise revenue are projected on a somewhat different assumption. Equation (8) describes total government expenditure over the period as equal to the sum of current expenditure plus the proportion of total investment expenditure that is to be undertaken by the government. ρ_i is naturally a policy parameter. The total increase in government debt is by definition equal to the total expenditure minus total tax revenue and total amount of operating surpluses of public enterprise. This expression has to be modified if we assume that part of the deficits will be covered by introducing new taxes and/or increasing the rate of existing taxes. This implies adding a separate equation to indicate the tax revenues from new sources.

The next three equations are clearly definitional equations in incremental form. The final equation is an equilibrium relation that enables one to decide how total investment will have to be distributed between agriculture and non-agriculture, by equating incremental demand to incremental supply. Since no systematic hypothesis is made concerning the distribution of non-agricultural investment within the different sub-sectors constituting the non-agricultural sector, no additional equations are given.

We shall take up the investment alleviation problem once again, while discussing the possibilities of extending the above decision model.

It has been remarked previously that we can arbitrarily choose any two of the variables as exogenous and accordingly work out the values of the remaining variables. We may choose among the following possibilities: These are naturally a sample of all the possibilities that exist:

(a) ΔY given and R prescribed from outside. In this case, I will be determined as a consequence and so also will be the marginal savings rate.

(b) ΔY given, and the savings programme is prescribed, namely, α , the required amount of foreign aid and the total amount of investment appears as derived magnitudes.

(c) I and F are "given" from outside. In this case, the rate of growth of income, etc. and the marginal savings ratio will be determined in consequence. A few other examples could be given. But the above is sufficiently indicative of what we may do by varying the model. In addition, parameter values may also be assumed to be different from case to case.

III. Let us take up the numerical illustrations of the above algebraic setup.

The following situations will be considered:

(a) the Pant-Little case

- (i) Exogenous Variables:
 - $I = 10,000$ crores
 - $F = 1,000$ "
- (ii) initial values:
 - $Y_0 = 12,500$ "
 - $S_0 = 1,050$ "
- (iii) parameters

- $h = .75$
- $R = .45$
- $P_1 = .67$
- $\delta_1 = .567$
- $\delta_2 = .323$
- $\delta_3 = .007$

Derivation:

$$I = \sum S_t + F$$

$$\therefore \sum S_t = 9000$$

$$\therefore = 5S_0 + 15\alpha = 9000$$

$$\therefore \alpha = 250$$

$$\therefore \Delta S = 1250$$

$$\Delta Y = .45 \times 10,000$$

= 4500. This implies that the relative rate of growth of income

per annum is 6% and

$$\frac{\Delta S}{\Delta Y} = \frac{1250}{4500} = 28\%$$

It should be obvious from the calculations that $\frac{\Delta S}{\Delta Y} = \frac{5\alpha}{\beta I}$. Thus, it is determined once α , β , and I are determined.

From the above it follows that $\frac{\Delta C}{C} = \frac{Y - S}{Y_0 - S_0} = \frac{3250}{11500} = 28\%$

This implies that the relative rate of growth of consumption per annum is 5%.

Since population is assumed to increase at a rate of 2% per annum, per capita national income increases at 4% per annum. Increase in the demand for agricultural production per annum is equal to $.75 \times .04 + .02 = .05$ or 5 per cent per annum. Agricultural production thus must increase by 5%. This means an increase of non-agricultural production by roughly 7%.

Relative increase in derived tax revenue is a weighted average of the rates of growth of consumption, agricultural output and non-agricultural output. With the above values of the rates of growth and the given values of $\delta_1, \delta_2, \delta_3$, it works out at roughly 6 per cent per annum.

This together with the estimates of autonomous tax revenue gives nearly the total of 6568 crores over the five-year period. There is, however, a slight discrepancy in as much as Little projects a few of the indirect taxes on bases somewhat different from the above. The difference however is negligible in relation to the totals involved.

Putting $\sum E_t = 15,700$ crores and $\sum T_t + \sum R_t$ equal to nearly 9000 crores, it follows that the balance of Rs 6700 crores has to come from borrowing if no new taxes are introduced, or the existing tax rates are not increased. Assuming further that external borrowing amounts to 1000 crores, we are left with the remainder of Rs 5700 crores. This total will presumably be distributed into three parts: additional taxes, borrowing, and the 'deficit financing' so called. Interesting questions arise in this connection as to how best to fill this gap in the required amount of resources.

Since this problem is already considered in the paper on "Alternative Numerical Models of the Third Five Year Plan of India", we do not discuss it here.

(b) For the second case, exogenous variables: the same as before

initial values: the same

parameter values: $\beta = .33$,

all other parameters remain unchanged.

ΔY in this case is equal to Rs 3,300 crores. This implies that national income grows at the rate of 4.8% per annum. Per capita national income goes up at the rate of 2.8% per annum.

$\frac{\Delta C}{\Delta Y} = \frac{S_t}{\beta I}$ = Since $\sum S_t = 9000$ crores as before, it follows that α is equal to Rs 250 crores. Thus, $\frac{\Delta S}{\Delta Y} = \frac{1250}{3300} = .38$ or 38% per annum. Hence, $\frac{\Delta C}{C} = \frac{\Delta Y - \Delta S}{C} = \frac{3300 - 1250}{11450} = \frac{2050}{11450} = 18\%$.

This implies that consumption increases roughly at the rate of 3.5 per cent per annum. Consumption per capita increases at the rate of 1.5% per annum.

Relative increase in agricultural demand = $.028 \times .75 + .02 = .041$ or 4.1 per cent per annum. Non-agricultural output increases roughly at 5.5% per annum.

Relative increase in tax revenue in this case is equal to nearly 4.1% per annum. Total revenue over the whole period in this case amounts to roughly 6270 crores.

Thus, there is a decline in tax revenue by 300 crores. This is a small amount compared with the total, but is significant as a proportion of the incremental tax revenue, which is only 700 crores on the present assumption as against the Pant-Little assumption of 1000 crores.

The marginal savings ratio implied in this programme is very high indeed and raises questions of feasibility in view of the existing sectoral propensities and the marginal income shares of the different sectors in the proposed investment allocation. This point as well as the related question of 'resources gap' is further elaborated in the paper of Prof. Rosenstein-Rodan quoted earlier.

Case 'c'

Exogenous Variables: I = 10,000

F = 2,500

Initial values: unchanged

Parameter values: $\beta = .33$. The rest as in (a).

Derivations:

$$\sum s_t = 7500$$

$$\text{Hence, } \alpha = 150$$

$$\text{Hence } \frac{\Delta S}{\Delta Y} = \frac{5\alpha}{\beta I} = \frac{750}{3300} = .23\%$$

National income in this case increases at the rate of 4.8 per cent per annum as in case (b). Per capita income accordingly increases at the rate of 2.8 per cent.

Relative rate of growth of consumption in this case is, however, higher than in case (b) because $\frac{\Delta C}{C} = \frac{3300 - 750}{11450}$ is roughly 22%. This means that consumption will increase at the rate of 4.1% per annum.

Relative increase in demand for agricultural products is the same as in case (b) e.g., 4.1%. Tax revenue now increases at a relatively higher rate in as much as the rate of growth of consumption is now higher. Tax revenue increases at the rate of 4.5% per annum. This compares with the 4.1% in the previous case.

Total tax revenue now aggregates, over the whole period to roughly 6350 crores. This means a slight improvement over the preceding situation. This present calculation necessarily implies that no new taxes are introduced or tax rates remain unchanged.

Case (c) appears to be somewhat more realistic in as much as the marginal savings ratio is considerably lower and the burden of new taxes needed to balance the government budget lower.

IV. It appears that in the interest of more satisfactory decision-making the model as discussed above should be in a somewhat more disaggregated form. Disaggregation should proceed further both on the 'horizontal' as well as the 'vertical' levels. The 'horizontal' level here refers to the distribution of expenditure between final consumer goods while the 'vertical' relations are those involving input-output considerations.

It may be said that the model so far described deals entirely with relationships on the horizontal level. But even there it is very far from being complete. While the model indicates how an increment of per capita national income will be distributed between food and non-food items, it does not say anything either about the distribution within the different items belonging to this food category, or of those falling within the non-food category. Demand for consumer goods, particularly textiles and housing services in the urban area, are likely to be the two most important categories on the level of final consumer demand. Insertion of corresponding equations would thus enable us to determine the ratios in which investment should be allocated to these sectors.

Allocating investment resources to sectors whose products are mostly used for interindustrial purposes, namely, those sectors where circularity is apt to be very important, requires information concerning the input-output relations. Even for an underdeveloped country like India, a partial input-output analysis may be extremely important, when investment is being channeled into the steel-fuel-metal complex. Having estimated the sum total of incremental demand for each of these sectors, the total being composed both of direct and derived demands, we may be in a position to suggest how much should be allocated to these sectors. This is all the more possible because the knowledge of capital-output ratios obtained from the data regarding more advanced countries may be applicable here, since the technological process would roughly be the same.

It may, however, be true that in the detailed preparation of the programs for industrial development, these cross-effects have been taken into account, in which input-output considerations are implicitly considered. In the present model, this will be reflected in the value of the global capital-coefficient. Even so, it may be better to put them more explicitly, so as to make for more consistent allocation of investment between the various industrial sectors.

No reference has been made in the above to the employment aspects of the problem. Pant gives an estimate of the incremental employment effects of the investment allocation scheme he suggests. These are essentially derived magnitudes. What is important is that a few of the allocation ratios may be so chosen as to attain certain desired increases in employment. This is possible because (a) the sectors differ with regard to their labor intensity; (b) and secondly, within some sectors, labor intensities of different processes may be different. The employment aspect of the question was one of the main issues of the second plan. It is still important and should not be treated only as a derived phenomenon.

It should also be apparent from the model that the foreign trade aspects of the problem are nowhere explicitly integrated with the other aspects of the economy. As a matter of fact, the model so formulated shows clearly only the inflow of capital that is needed in order to finance the import surplus that is expected to arise over the plan. But how the import surplus is expected to arise is mentioned only very briefly by the plan and this will undoubtedly be the subject of an additional study. Especially here the choice of investment ratios is of importance in as much as the export-expanding or import saving industries appear to be particularly important in the present Indian context.

To sum up, if there are 'n' sectors, then there are (n-1) allocation ratios of investment which may be used as (n-1) instruments. It follows from an elementary principle of economic policy that we can attain (n-1) targets with the help of (n-1) instruments. The suggestions above are mainly intended to show how these (n-1) targets may be chosen by means of extending the scope of the present model through incorporating a few new relationships.