A numerical simulation of compressible mixing layers

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Résumé :

Dans ce travail, le modèle au second ordre de Favre basé sur les équations des tensions de Reynolds est utilisé pour étudier une turbulence compressible évoluant en présence d'une couche de mélange dans différent cas de nombres de Mach convectif. Dans ce contexte, le modèle compressible proposé par Park et al. Pour la corrélation pression-déformation est examiné sous forme originale et sous une forme corrigée(la version corrigée correspond au modèle original auquel la constante C_1 de la partie de retour à l'isotropie de la corrélation pression-déformation est modifiée, elle est devenue fonction du nombre de Mach turbulent). L'évaluation de ce modèle est faite par référence aux résultats expérimentaux de Goebel et al. En général les résultats obtenus sont relativement encourageants.

Abstract :

In this work, the Favre Reynolds stress closure model is used to study spatially evolving compressible mixing layers at different convective Mach numbers. Regarding the compressibility correction model of Park and al. for the pressure strain correlation, the coefficient C_1 is taken as in the incompressible model of Launder Reece and Rodi (LRR). Correction of this coefficient using the turbulent Mach number is proposed. Application of the model with and without correction of C_1 is examined by comparison with the experience of Goebel and Dutton and with other DNS results. Simulations at the low convective Mach number show that the two model forms have nearly similar behaviors. At high convective Mach number, Simulations show that there are differences between the models in the predictions of the decrease of the growth rate and of the turbulence intensities with the increase of the convective Mach number. In general, the predictions are in acceptable accordance with the experimental and DNS results.

Mots clefs: turbulence compressible, mixing layers, models .

1 Introduction

Compressible turbulence mixing layers play an important role in many industrial applications such as aerospace, combustion and engineering problems related to the environnemental domains . The direct extension of incompressible models was used in simulating different compressible flows. That one was observed when the Reynolds stress closures were extended to compressible flows with an explicit account of compressibility effects, by considering dilatational terms models. It has been shown that this practice of modelling, called compressibility correction models, may be able to reproduce compressibility phenomenon at small values of Mach number. But, when the compressibility effects are more significant, the extended models do neither predict correctly the decrease in spreading rate of mixing layers, as it is observed in the experiments of Goebel and al.[3] and Samimy and al.[7], nor the reduction in the growth rate of turbulent kinetic energy Sarkar[6]. The deficiencies of such closure is due principally to the use of the incompressible models of the pressure strain correlation which controls the level of Reynolds stress anisotropy. However, new models taking into account structural compressibility effects are needed for the pressure strain correlation. Many DNS and experiment results have been carried out on compressible turbulent flows, most of which show the significant compressibility effects on the pressure-strain correlation via the pressure field. Consequently, the pressure-strain correlation requires a careful modeling in the Reynolds stress turbulence model. In this context, many compressible models have been developed for the pressure-strain correlation. Hereafter, most of all these models are generated from a simple extension of its incompressible counter-part. Adumitroiae and al.[1] assumed that incompressible modeling approach of the pressure-strain can be used to develop turbulent models taking into account compressibility effects. In their approach we have considered a non zero divergence for the velocity fluctuations called the compressibility continuity constraint and used different models for the pressure dilatation which is proportional to the trace of the pressure-strain. Park and al.[2] proposed that the compressibility effects on the pressure strain correlation affected only the rapid part of this term. They introduced an empirical function in term of the turbulent Mach number to modify the standard coefficients model[4], the C_1- coefficient which affect the slow part of the pressure – strain correlation is conserved as in the LLR model[4]. Khlifi and al.[10-11] have modified this coefficient which become function of the turbulent Mach number.

In the present work, we examine the ability of the two model forms of Park and al.[2](with and without correction of C_1) in predicting different characteristic parameters of the compressible mixing layers by considering the experimental results of Goebel and al.[3] for different convective Mach numbers.

2 Governing equations

The general equations governing the motion of a compressible fluid are the Navier Stokes equations. They can be written as follows for mass, momentum and energy conservation :

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x_i}\rho u_i = 0 \tag{1}$$

$$\frac{\partial}{\partial t}\rho u_i + \frac{\partial}{\partial x_j}\rho u_i u_j = \frac{\partial}{\partial x_j}\sigma_{ij} \tag{2}$$

$$\frac{\partial}{\partial t}\rho e + \frac{\partial}{\partial x_j}\rho e u_j = \frac{\partial}{\partial x_j}\sigma_{ij} u_i - \frac{\partial}{\partial x_j}(\kappa T_{,j}) \tag{3}$$

where, $e = c_v T$, $\sigma_{ij} = -p\delta_{ij} + \tau_{ij}$ and $\tau_{ij} = 2\mu(u_{i,j} + u_{j,i})$.

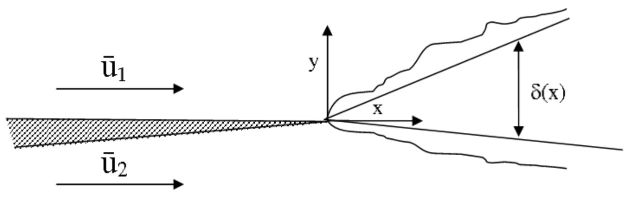

Fig. 1 : scheme of mixing layer

The fully developed stationary compressible mixing layers (Fig.1) is governed by the averaged Favre equations deduced from eqs.(1,2,3), such equations are associated to those described the continuity, momentum energy, Reynolds stress and turbulent dissipation with the forms as:

$$\frac{\partial}{\partial x_i}(\bar{\rho}\tilde{U}_i) = 0 \tag{4}$$

$$\frac{\partial}{\partial x_j}(\bar{\rho}C_v\tilde{T}\tilde{U}_j) = -\frac{\partial}{\partial x_j}(C_v\overline{\rho u_i'' T''}) + \varepsilon_s + \varepsilon_c - \overline{p' u'_{i,i}} \tag{5}$$

$$\frac{\partial}{\partial x_m}(\bar{\rho}\tilde{U}_m R_{ij}) = -(R_{im}\tilde{U}_{j,m} + R_{jm}\tilde{U}_{i,m}) + (\overline{\rho u_i'' u_j'' u_m''})_{,m} + \phi^*_{ij} + \frac{2}{3}\overline{p' u'_{i,i}}\delta_{ij} - \frac{2}{3}\varepsilon\delta_{ij} \tag{6}$$

$$\frac{\partial}{\partial x_k}(\bar{\rho}\varepsilon_s\tilde{U}_k) = \bar{\rho}\frac{\varepsilon_s}{K}\left(C_{\varepsilon 1}R_{km}\frac{\partial}{\partial x_m}\tilde{U}_k - C_{\varepsilon 2}\varepsilon_s\right) - \frac{\partial}{\partial x_k}\left(C_{\varepsilon 3}\bar{\rho}\frac{K}{\varepsilon_s}R_{km}\frac{\partial}{\partial x_m}\varepsilon_s\right) \tag{7}$$

In the above mentioned transport equations, different terms should be modeled, the gradient diffusion hypothesis is used to represent :

-The turbulent heat flux[8]:

$$\overline{\rho u_i'' T''} = -C_T \frac{K}{\varepsilon} \overline{\rho u_i'' u_m''} \frac{\partial}{\partial x_m} \tilde{T} \quad (8)$$

-The diffusion term[8]:

$$\overline{\rho u_i'' u_j'' u_m''} = -C_s \frac{K}{\bar{\rho}\varepsilon} \overline{\rho u_i'' u_m''} \frac{\partial}{\partial x_m} \overline{\rho u_j'' u_m''} \quad (9)$$

For the turbulent dilatational part of the dissipation and the correlation pressure-dilatation, we chose the models proposed by Sarkar[5], namely:

$$\varepsilon_d = 0.5\, M_t^2\ \varepsilon_s, \quad \overline{p' u'_{i,i}} = 0.15 M_t \bar{\rho}(R_{ij} - \frac{2}{3} K\delta_{ij}) + 0.2\bar{\rho} M_t^2 \varepsilon_s \quad (10)$$

3 Turbulence models

Park and al.[2] used the concept of moving equilibrium in homogeneous shear flow to modify the linear pressure strain term part model of LRR[4]:

$$\phi_{ij}^* = -C_1 \bar{\rho}\varepsilon_s b_{ij} + C_2 \bar{\rho} K (\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{ll}\delta_{ij}) + C_3 \bar{\rho} K [b_{ik}\tilde{S}_{jk} + b_{jk}\tilde{S}_{ik} - \frac{2}{3} b_{ml}\tilde{S}_{ml}\delta_{ij}] + C_4 \bar{\rho} K [b_{ik}\tilde{\Omega}_{jk} + b_{jk}\tilde{\Omega}_{ik}] \quad (11)$$

Table 1 Model constants

	LRR[4]	Park and al.[2] : model1	Park and al.[2] : model2
C_1	3	3	$3.(1-2.5M_t^2)$
C_2	0.8	$C_2 = 0.8 + \lambda_1 F$	$C_2 = 0.8 + \lambda_1 F$
C_3	1.75	$C_3 = 1.25 + \lambda_2 F$	$C_3 = 1.25 + \lambda_2 F$
C_4	1.31	$C_4 = 1.25 - \lambda_3 F$	$C_4 = 1.25 - \lambda_3 F$

Table 2 Initial conditions [3]

M_C	0.2	0.46	0.69	0.86	1
$r = \frac{U_2}{U_1}$	0.78	0.57	0.18	0.16	0.16
$s = \frac{\rho_2}{\rho_1}$	0.76	1.55	0.57	0.6	1.14

$$\lambda_1 = \frac{4}{3}(1-\alpha) - \frac{6}{5},\ \lambda_2 = 2(1-\alpha),\ \lambda_3 = 2(1+\alpha) \text{ and } F = \frac{0.54}{2+\alpha}(1-\exp(-(4M_t)^2)) .$$

4 Results and discussions

The two free streams of the fully developed compressible mixing layer are characterized typically by the convective Mach number M_C, the parameters $s = \frac{\rho_2}{\rho_1}$ and $r = \frac{U_2}{U_1}$ are respectively the density and velocity ratios, the experiments conditions of Goebel and al.[3] are listed in Table 2. The values of the constants models used in the present simulation are: $C_{\varepsilon 1} = 1,4$, $C_{\varepsilon 2} = 1,8$, $C_\mu = 0,09$, $C_\varepsilon = 0,25$, $C_T = 0,26$. The averaged equations are solved using a finite difference scheme. In this study, two versions of the Park and al. model will be examined: the original version(without correction of the C_1-coefficient) noted model1 and the version where the proposed M_t –correction of C_1 is included, this version is referred to model 2 as it is indicated in Table 1. The fundamental parameter characterizing the effects of compressibility on the mixing layer is the growth rate $\frac{d\delta}{dx}$, δ denotes the momentum thickness of the mixing layer.

Figure 2 shows the comparison between the computed normalized growth rate by its incompressible counterpart $G = \frac{d\delta}{dx} / (\frac{d\delta}{dx})_{M_C=0}$ with different experiment results available in the literature and with those obtained by empirical formula of Dimotakis [9]: $G = 0.8.\exp(-M_C^2) + 0.2$.

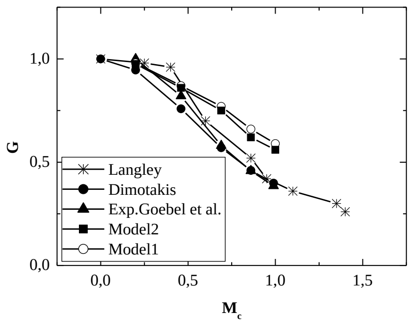

Fig.2: Normalized growth rate G versus M_c

The calculated growth rate G decreases with increasing convective Mach number: a phenomenon which has often been observed in experimental studies of compressible mixing layers. The park and al. model[2] overpredicts the growth rate G. With the proposed correction of the slow part of the pressure-strain correlation, the model of Park and al. the results are relatively acceptable for the growth rate G.

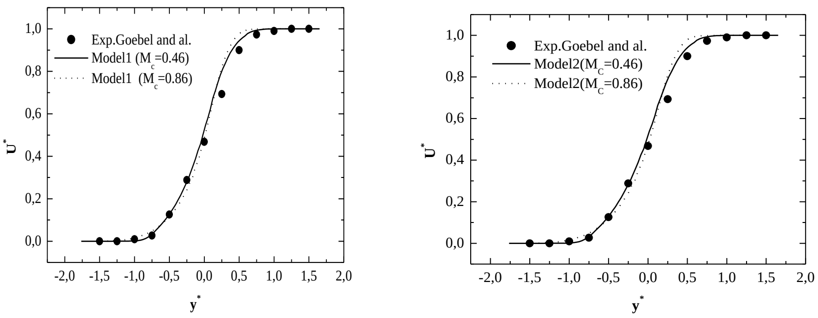

Fig.3: Similarity profiles of the mean velocity.

The normalized stream mean velocity $U^* = \dfrac{\tilde{U} - U_2}{U_1 - U_2}$ is represented in relation to the similarity variable $y^* = (y - y_c)/\delta$ in Fig. 3, where y is the local cross stream coordinate and y_c is the cross-stream coordinates corresponding to $U^* = 0.5$. The calculated velocity profiles with the model 1 and model 2 are in reasonable agreement with experimental data[3]. Fig.4,5 and 6 show the computed results of the Reynolds similarity intensities: the streamwise intensity $r_{11} = \sqrt{R_{11}/(U_1 - U_2)^2}$, the transverse intensity $r_{22} = \sqrt{R_{22}/(U_1 - U_2)^2}$,and the shear stress $r_{12} = R_{12}/(U_1 - U_2)^2$ respectively, obtained from the models 1 and 2 are compared with experiments results of Goebel and Dutton[3]. It is clear that the two models lead to similar results for small value of convective Mach number ($M_C = 0.46$). But, when the compressibility effects are more significant $M_C = 0.86$, the effects of the proposed correction model are clearly manifested on the normal turbulent Reynolds stress , particularly on the transverse similarity intensity. Figure 6 show the behavior of the normalized pressure-strain correlation $phij = \phi_{ij}^* / \bar{\rho}(U_1 - U_2)^2 d\tilde{U}/dy$ for $M_C = 0.46$ and $M_C = 0.86$. It is clear that there is a systematic decrease with increasing the convective Mach number for the all components of the pressure-strain correlation as it is believe observed in several experiment and DNS results. The two models are similar for $M_C = 0.46$. But when the compressibility increases($M_C = 0.86$),the model 2 affect significantly the shear component of the pressure-strain correlation, apparently the other components do not sensitive to this model.

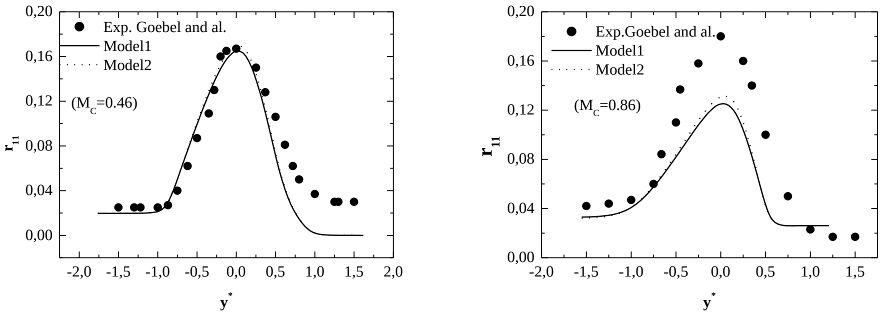

Fig.4: Similarity profiles of longitudinal turbulence intensity.

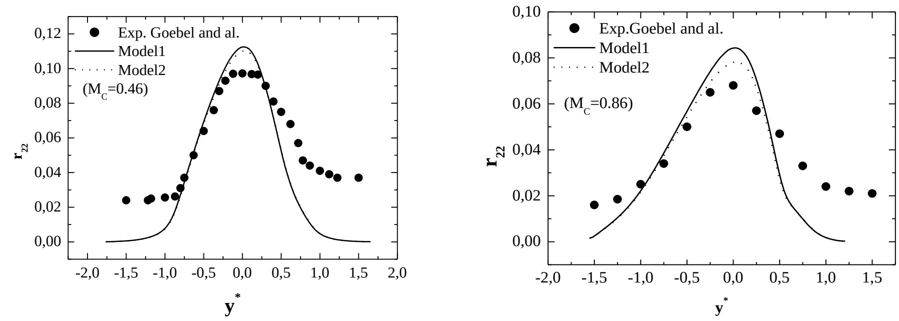

Fig.5: Similarity profiles of transverse turbulence intensity.

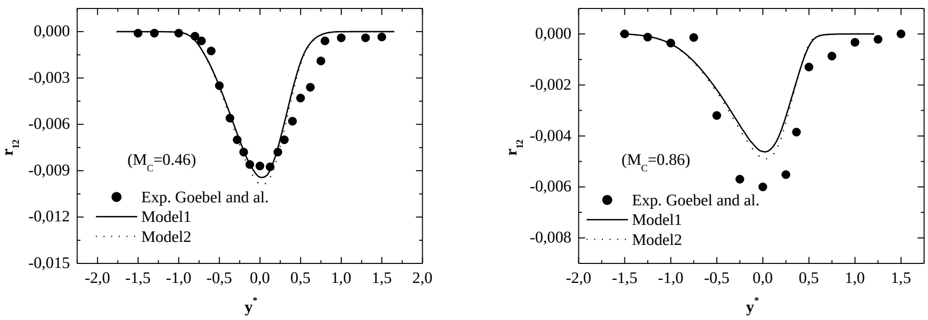

Fig.6: Similarity profiles of Reynolds stress.

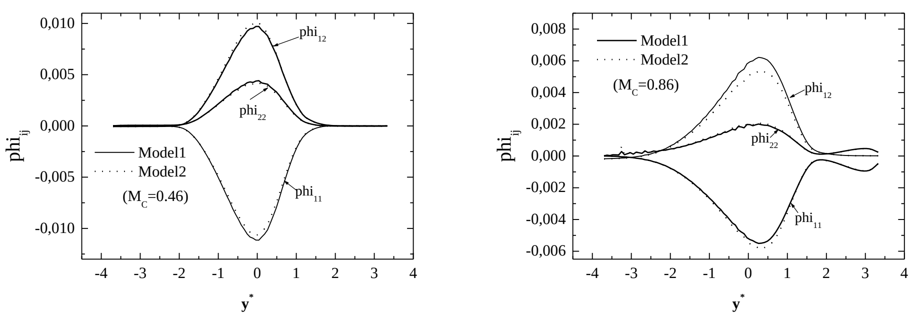

Fig.7: Similarity profiles of pressure-strain correlation.

5 Conclusion

In this study, the widely used second order closures has been used for the prediction of compressible mixing layers. The standard –stress closure with the addition of the dilatational terms: the pressure dilatation correlation and the turbulent dissipation of the dilatation yields very poor predictions of the changes in the Reynolds stress anisotropy magnitude. The deficiencies of this closure is due to the use of the incompressible models of the pressure-strain correlation. This term controls the structural compressibility effects on the turbulence. A modification of the standard[4] model of the slow part of the pressure strain correlation has been made by making the usual coefficient C_1 depend on the turbulent Mach number M_t. In general, the proposed model with the Park and al. model.[2] of the mean part of the pressure strain successfully predict important parameters which in general characterize the compressible mixing layers.

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