Hydrodynamic and thermal characteristics of a Herschel-Bulkley fluid's flow in a circular duct

Comportements hydrodynamique et thermique lors de l'écoulement d'un fluide de Herschel-Bulkley dans une conduite cylindrique

N. LABSI[a], Y.K. BENKAHLA[a], E. BRUNIER[b]

a. Laboratoire des Phénomènes de Transfert,
Université des Sciences et de la Technologie Houari Boumediene
BP 32 El Alia, Bab Ezzouar 16111, Alger, Algérie
b. Département de Génie Chimique, Université de Technologie de Compiègne,
BP. 20529 – 60205 Compiègne, France.

Résumé :

La présente étude numérique concerne l'écoulement laminaire et stationnaire d'un fluide viscoplastique incompressible au sein d'une conduite cylindrique maintenue à une température pariétale uniforme. Le fluide obéit au modèle rhéologique de Herschel-Bulkley. En outre, en plus de la chaleur fournie au fluide par la paroi de la conduite, la dissipation visqueuse est prise en compte.
Les résultats montrent que l'augmentation du nombre de Herschel-Bulkley entraine la diminution de la vitesse centrale et par conséquent, l'augmentation du coefficient de frottement. Ce dernier croît également avec l'augmentation de l'indice d'écoulement du fluide. Il a été noté en outre que le transfert thermique est amélioré par l'accroissement du nombre d'Herschel-Bulkley. Cette amélioration est très visible dans la zone de l'établissement thermique et ce, loin de l'entrée. Quant à l'introduction de la fonction de dissipation, les résultats montrent une amélioration notable du transfert thermique en comparaison avec le cas où celle-ci est négligée.

Abstract:

The present study concerns the numerical analysis of hydrodynamic and thermal characteristics of the flow of an incompressible Herschel-Bulkley fluid of constant physical and rheological properties. The flow takes place within a pipe of circular cross section. The pipe is maintained at uniform parietal temperature. Because of the viscous character of this type of fluid, viscous dissipation is taken into account.
The results show that both the friction factor of Fanning and the Nusselt number increase with the increase of the Herschel-Bulkley number. However, the increase of the flow index leads to the increase of the friction factor on one hand and to the decrease of the Nusselt number on the other hand. In addition, the extent of the plug flow region increases when the Herschel-Bulkley number increases. Taking into account viscous dissipation improves significantly heat transfer comparing to the case where viscous dissipation is neglected.

Key words: Herschel-Bulkley fluid, yield stress, forced convection, constant wall temperature, viscous dissipation, finite volume method.

1 Introduction

The Herschel-Bulkley model is one of rheological models which describe the rheological behaviour of viscoplastic non Newtonian fluids. The fluids obeying the Herschel-Bulkley model are encountered in many industrial applications. They are characterized by a yield stress from which they start moving. Mitsoulis [1] studied, by means of finite element method, the friction factor evolution during the laminar and steady flow of a Bingham fluid past a circular cylinder kept between parallel plates. He found that the results are independent of the studied geometry. He proposed also a usual correlation, relating the friction factor to the

Bingham number. Nouar [2] studied free and forced convection heat transfer of a Herschel-Bulkley fluid flow in a horizontal circular pipe subjected to a constant parietal heat flux. He analyzed the effect of thermodependency of the fluid consistency and its density on the secondary flows and gave correlations for the Nusselt number and the wall shear stress.

The present work deals with a numerical study of laminar forced convection of a Herschel-Bulkley fluid, for which the power law index is taken equal to 0.5 and 1. The fluid is incompressible and of constant physical and rheological properties. The flow takes place within a circular pipe, subjected to a constant parietal temperature. The study focuses on the effect of the fluid's viscoplasticity on hydrodynamic and thermal characteristics of the flow as well as the effect of viscous dissipation, since it has not yet been deeply investigated for this category of fluids, especially for negative values of the Brinkman number.

2 Governing equation and numerical procedure

Let's consider the laminar steady flow of an incompressible Herschel-Bulkley fluid through a circular pipe of length L and diameter D maintained at constant wall temperature T_w. The physical and rheological properties of the fluid are constant and uniform. The study concerns the case of a shear thinning Herschel-Bulkley fluid (n = 0.5) and a Bingham fluid (n = 1).

The following dimensionless variables are considered for respectively, the radial and axial coordinates, the axial and radial velocities, pressure and temperature:

$$R=\frac{r}{D} \qquad X=\frac{x}{D} \qquad U=\frac{V_x}{V_0} \qquad V=\frac{V_r}{V_0} \qquad P^*=\frac{p^*}{\rho V_0^2} \qquad \theta=\frac{T-T_w}{T_0-T_w} \tag{1}$$

Where x and r are respectively the axial and the radial coordinates, V_x and V_r represent respectively the axial and the radial velocity components, V_0 is the inlet velocity, p^* is the pressure, T_0 and T_w are respectively the inlet and the wall temperatures.

Thus, the dimensionless governing equations, i.e. continuity, momentum and energy equations are respectively given by:

$$\frac{1}{R}\frac{\partial(RV)}{\partial R}+\frac{\partial U}{\partial X}=0 \tag{2}$$

$$\begin{aligned}\frac{1}{R}\frac{\partial(RVV)}{\partial R}+\frac{\partial(UV)}{\partial X}=&-\frac{\partial P^*}{\partial R}+\frac{1}{Re}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(\eta_{app}R\frac{\partial V}{\partial R}\right)+\frac{\partial}{\partial X}\left(\eta_{app}\frac{\partial V}{\partial X}\right)\right]\\&+\frac{1}{Re}\left[\frac{V}{R}\frac{\partial\eta_{app}}{\partial R}-\eta_{app}\frac{V}{R^2}+\frac{\partial\eta_{app}}{\partial X}\frac{\partial U}{\partial R}+R\frac{\partial\eta_{app}}{\partial R}\frac{\partial}{\partial R}\left(\frac{V}{R}\right)\right]\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{1}{R}\frac{\partial(RVU)}{\partial R}+\frac{\partial(UU)}{\partial X}=&-\frac{\partial P^*}{\partial X}+\frac{1}{Re}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(\eta_{app}R\frac{\partial U}{\partial R}\right)+\frac{\partial}{\partial X}\left(\eta_{app}\frac{\partial U}{\partial X}\right)\right]\\&+\frac{1}{Re}\left[\frac{\partial\eta_{app}}{\partial R}\frac{\partial V}{\partial X}+\frac{\partial\eta_{app}}{\partial X}\frac{\partial U}{\partial X}\right]\end{aligned} \tag{4}$$

Taking into account viscous dissipation and assuming that the physical properties of the fluid (ρ, C_p and k) are constant, result in the following dimensionless energy equation:

$$\begin{aligned}\frac{1}{R}\frac{\partial(RV\theta)}{\partial R}+\frac{\partial(U\theta)}{\partial X}=&\frac{1}{Pe}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)+\frac{\partial^2\theta}{\partial X^2}\right]\\&+\frac{Br}{Pe}\eta_{app}\left[2\left\{\left(\frac{\partial V}{\partial R}\right)^2+\left(\frac{\partial U}{\partial X}\right)^2+\left(\frac{V}{R}\right)^2\right\}+\left(\frac{\partial U}{\partial R}+\frac{\partial V}{\partial X}\right)^2\right]\end{aligned} \tag{5}$$

This dimensional analysis generates the following dimensionless numbers: $Re=\rho V_0^{2-n}D^n/K$ (Reynolds number), $\mathrm{Pe}=\rho C_p V_0 D/k=\mathrm{Pr}\,\mathrm{Re}$ (Peclet number) and $Br=KV_0^2/k(T_0-T_w)$ (Brinkman number). The latter compares the dissipation term with the conduction term in the energy equation. It represents the

viscous dissipation function. A negative value of the Brinkman number means that the fluid is heated (heating case) whereas a positive value indicates that the fluid is cooled down (cooling case).

The general model of Herschel-Bulkley is given by the following rheological law, which relates the shear stress τ to the shear rate $\dot{\gamma}$:

$$\begin{cases} \tau = K\dot{\gamma}^n + \tau_0 & \tau \geq \tau_0 \\ \dot{\gamma} = 0 & \tau < \tau_0 \end{cases} \tag{6}$$

K is the fluid consistency, n is the flow index and τ_0 is the yield stress. In order to avoid numerical instabilities in the low shear rate region, many authors [1,3] recommend to use the following constitutive law proposed by Papanastasiou, for which they advised to take m = 1000 s:

$$\eta_{app} = \frac{\eta}{K} = \dot{\gamma}^{*n-1} + \frac{HB}{\dot{\gamma}^*}\left[1 - exp\left(-M\,\dot{\gamma}^*\right)\right] \tag{7}$$

Where $\dot{\gamma}^*$ is the dimensionless shear rate, M = mV_0/D represents the dimensionless exponential growth parameter and $HB = \tau_0\, D^n / K V_0^n$ is the Herschel-Bulkley number which represents the ratio of the yield stress to the nominal shear stress.

The boundary conditions consist of uniform axial velocity and temperature at the inlet (U = θ = 1, V = 0), no-slip condition and a uniform wall temperature along the wall (U = V = θ = 0) as well as fully developed velocity and temperature at the outlet $\left(\frac{\partial U}{\partial X} = V = \frac{\partial \theta}{\partial X} = 0\right)$.

The governing equations quoted previously, are solved numerically using the finite volume method proposed by Patankar [4]. They are discretized and put in the form of an algebraic equation which is solved using a computing code based on the SIMPLER algorithm, by considering a 250x50 non uniform mesh.

3 Validation of the computing code

To validate our computing code, we consider the limit case of a Bingham fluid ($\tau_0 \neq 0$ and n = 1) and we compare the friction factor's variation according to the Reynolds number values obtained from this code with the ones obtained by Malin [5], by taking a Hedstrom number $\left(He = Re\; HB^{\left(\frac{2}{n}-1\right)}\right)$ equal to 100. The comparison in figure 1, shows that the results seem to be in good agreement.

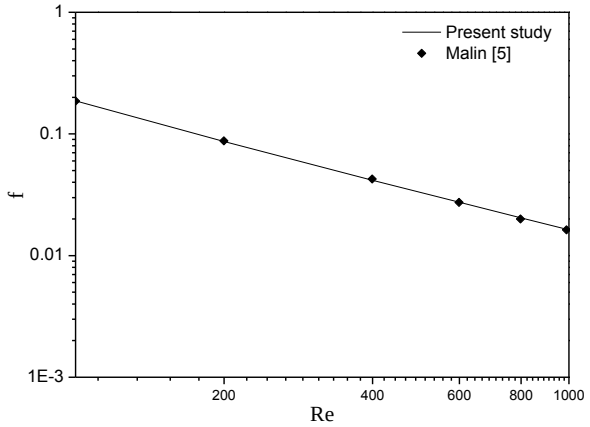

FIG. 1 – Variation of the friction factor according to the Reynolds number. n = 1, He = 100.

4 Results and discussion

The results concern the effect of the fluid's viscoplasticity, represented by the Herschel-Bulkley number, on hydrodynamic and thermal properties of the flow.

Since the momentum and energy equations are decoupled in relation to the velocity field, viscous dissipation affects only thermal properties of the flow. This result has been confirmed by [3]. For that purpose, the effect of the Brinkman number will be studied only for the evolution of the Nusselt number.

Furthermore, we consider for all the study, a great value of the Prandtl number (Pr = 50) in order to get close to industrial applications concerning this category of viscous fluids.

4.1 Effect of the Herschel-Bulkley number

Figures 2-a and 2-b show the fully developed velocity profiles (at X = 1000) for various values of the Herschel-Bulkley by considering n = 0.5 and n = 1, respectively. We note that all the curves, except the one corresponding to HB = 0 (shear thinning power law fluid when n = 0.5 and Newtonian fluid when n = 1), present two regions: a region close to the wall characterized by a parabolic velocity profile and a zone around the centreline which represents a uniform velocity distribution, called "plug flow region" or "unyielded" region. In this region, the shear stress is less than the yield stress, the fluid resists consequently to deformations and moves like a rigid solid. It is to be noted also that the centreline velocity decreases when the Herschel-Bulkley number increases and the flow index decreases whereas the extent of the unyielded region increases with the increase of both the Herschel-Bulkley number and the flow index.

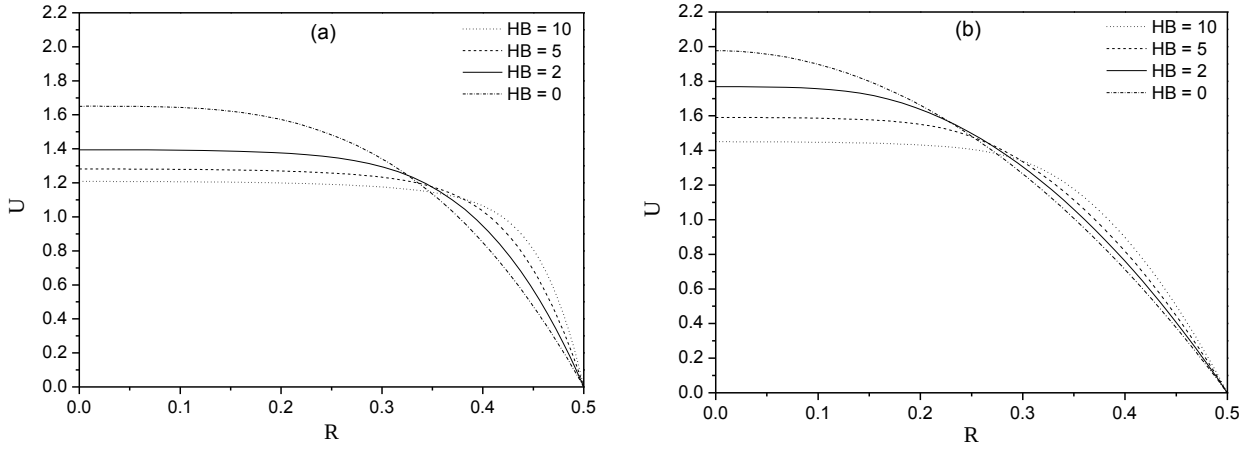

FIG. 2 – Fully developed velocity profiles according to the Herschel-Bulkley number.
Pe = 1000. (a) n = 0.5 (b) n = 1.

The effect of the Herschel-Bulkley number on the velocity profile has a significant consequence on the friction factor of Fanning (fRe), which is directly related to the pressure drop. Indeed, figure 3 illustrate this effect, by considering the axial evolution of fRe for both n = 0.5 and n = 1. We can see that the increase of the Herschel-Bulkley number leads to the increase of fRe, since the wall velocity gradient increases too (figure 2). It is interesting to note that the values of fRe are more important for n = 1 (figure 3-b) than for n = 0.5 (figure 3-a). This can be explained by the fact that the shear thinning Herschel-Bulkley (n < 1) are less viscous than Bingham fluids (n = 1).

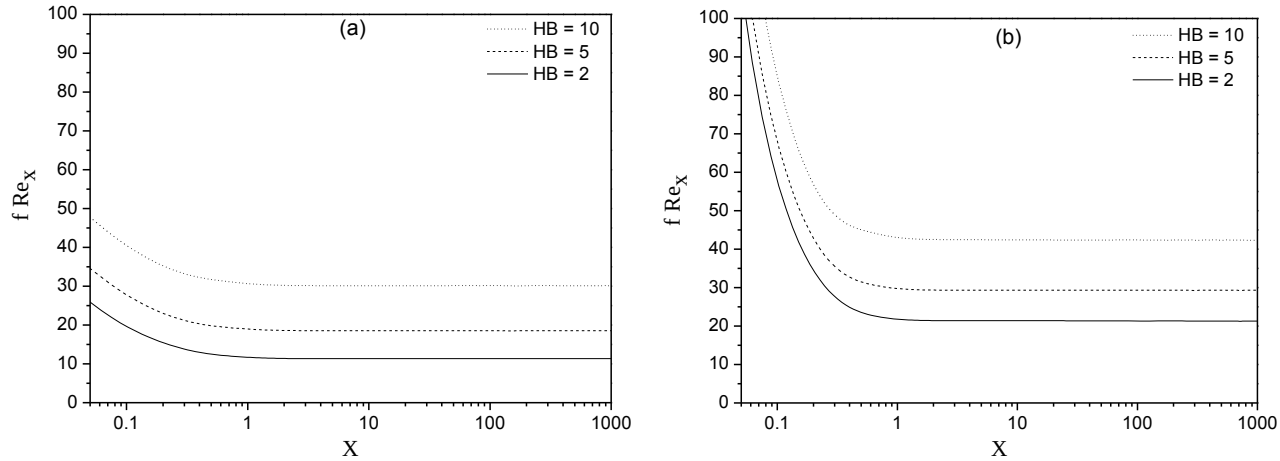

FIG. 3 – Axial evolution of the friction factor of Fanning according to the Herschel-Bulkley number.
Pe = 1000. (a) n = 0.5 (b) n = 1.

Regarding the effect of the Herschel-Bulkley number on heat transfer (the Nusselt number, Nu), figure 4 shows the case when viscous dissipation is neglected. We can see, for n = 0.5 (figure 4-a) and n = 1 (figure 4-b), that the increase of the Herschel-Bulkley number improves heat transfer especially in the fully developed region. However, this effect is not very significant at the inlet and is less noticeable for n = 1.

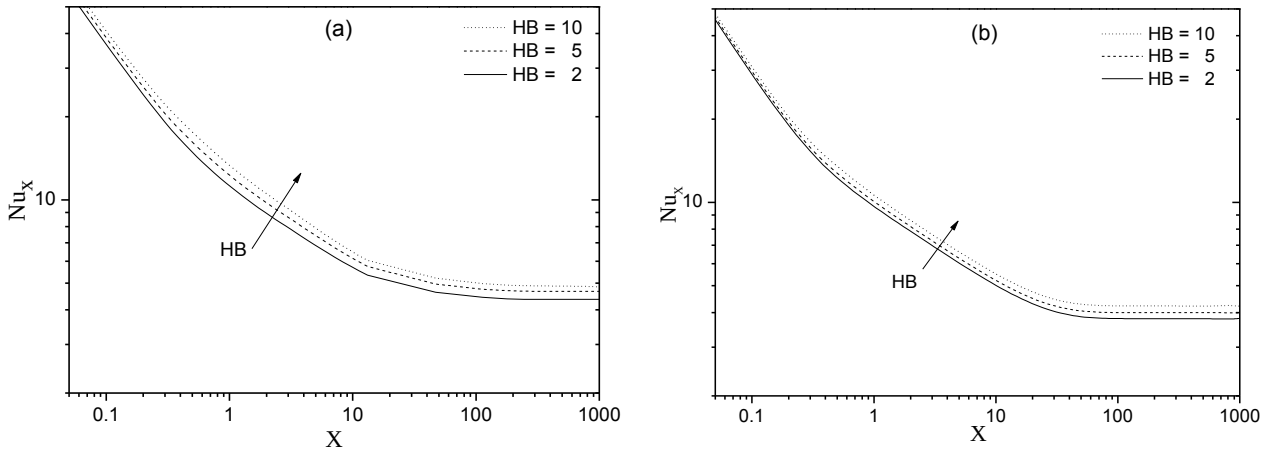

FIG. 4 – Axial evolution of the Nusselt number according to the Herschel-Bulkley number.
Pe = 1000, Br = 0. (a) n = 0.5 (b) n = 1.

4.2 Effect of viscous dissipation

Viscous dissipation is an energy source, represented by the Brinkman number. Taking this function into account in the energy equation, leads to modifications on heat transfer behaviour.

Figures 5 and 6 show the effect of the Brinkman number on the axial evolution of the Nusselt number for n = 0.5 and n = 1, respectively, by considering both heating (Br < 0) and cooling (Br > 0) cases.

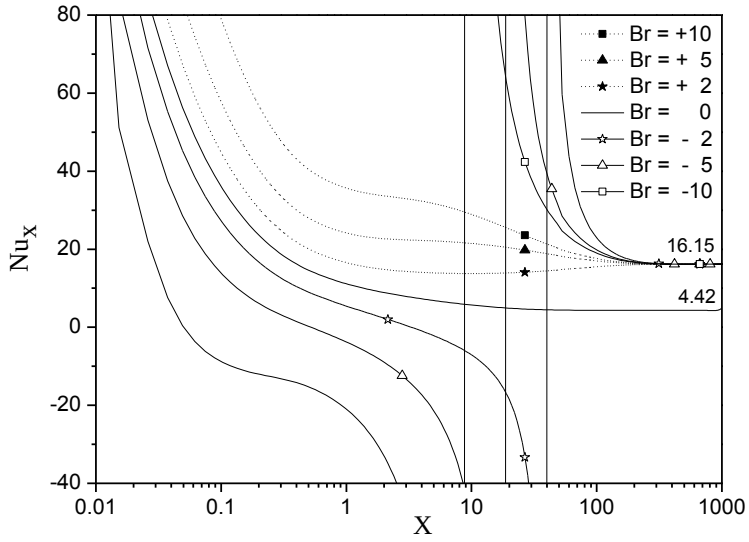

FIG. 5 – Axial evolution of the Nusselt number according to the Brinkman number.
Pe = 1000, HB = 2, n = 0.5.

The curves display a sharp decrease of the Nusselt number near the inlet, before reaching an asymptotic value downstream, which corresponds to the fully developed flow. It is interesting to note that for both heating and cooling, when viscous dissipation is taken into account, this asymptotic value is independent of the Brinkman number. It is equal to 16.44 and 10.66 for n = 0.5 and n = 1, respectively. Thus, the Nusselt number increases when the Brinkman number increases. Therefore, neglecting viscous dissipation leads to underestimate heat transfer by about 256% and 180% for n = 0.5 and n = 1, respectively.

It is interesting to note also, that in the case of heating (Br ≺ 0), the curves present a discontinuity. We notice moreover, the existence of negative values of the Nusselt number due to the change in heat direction.

For the limit case of a Bingham fluid (figure 6), the results are compared to those obtained from the numerical study of Min et *al.* [3]. The agreement is good whether or not viscous dissipation is neglected or taken into account, since the deviation between the two studies does not exceed 3%.

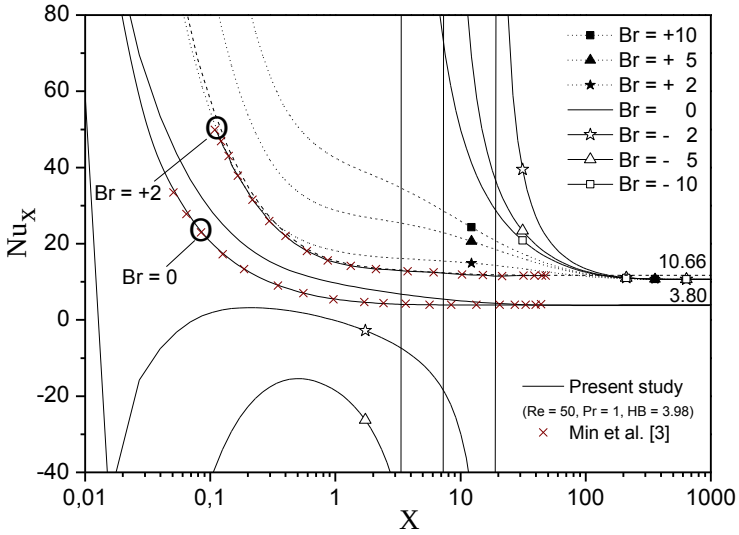

FIG. 6 – Axial evolution of the Nusselt number according to the Brinkman number.
Pe = 1000, HB = 2, n = 1.

5 Conclusion

A numerical study based on finite volume method was carried out. It consisted on the laminar forced convection flow of an incompressible Herschel-Bulkley fluid in a circular pipe maintained at uniform wall temperature, by taking viscous dissipation into account.

The results show that the increase of the Herschel-Bulkley number leads to the decrease of the centerline velocity and the increase of the extent of the plug flow region. Heat transfer is consequently enhanced but the friction factor of Fanning increases too and, so does the pressure drop. However, the increase of the flow index leads to the increase of the friction factor on one hand and to the decrease of the Nusselt number on the other hand.

Taking into account viscous dissipation improves significantly heat transfer since the asymptotic value of the Nusselt number is notably greater than the one corresponding to the case where viscous dissipation is neglected. Thus, in order to have a good design of industrial equipments dealing with the flow of viscoplastic fluids, it is necessary to take into account viscous dissipation in computation.

References

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