

# Numerical study of conjugate natural convection in a square enclosure with top active vertical wall

A. BELAZIZIA<sup>a,b</sup>, S. ABBOUDI<sup>a</sup>, S. BENISSAAD<sup>b</sup>

*a. institut IRTES, Laboratoire M3M, site de Sévenans, UTBM, 90010 Belfort Cedex France  
Phone : 33(3)84583036, Email : nacer.belazizia@gmail.com*

*b. Laboratoire d'Energétique Appliquée et de la Pollution. Université Mentouri Constantine*

## Abstract

*In this paper, we consider a two-dimensional numerical study of laminar conjugate natural convection in a square enclosure. The right vertical wall has constant temperature  $\theta_c$ , while the outer surface of the left vertical wall is partially heated in the top  $\theta_h$ , with  $\theta_h > \theta_c$ . The remaining left portion in the vertical wall and the two horizontal walls are considered adiabatic. The obtained results show that for a given thickness,  $D$  either increasing the Rayleigh number  $Ra$  and the thermal conductivity ratio  $Kr$ , can increase the average Nusselt number  $\overline{Nu}$ , the interface temperature and the flow velocity. Generally it is found that the increase of the thickness of the bounded wall can decrease  $\overline{Nu}$ , especially for  $Kr > 1$  and  $Ra < 10^4$   $\overline{Nu}$  increases with the increase of  $D$ . The wall-fluid interface temperature is found to be quite non-uniform. This non uniformity tends to make the flow pattern in the enclosure asymmetric.*

**Keywords:** conjugate natural convection, square enclosure, top active vertical wall, finite volumes method, thermal conductivity ratio.

## Résumé

*Dans cet article, nous présentons une étude bidimensionnelle sur la convection naturelle laminaire conjuguée dans une enceinte carrée. La paroi droite est soumise à la température  $\theta_c$ , tandis que la surface extérieure de la paroi gauche est partiellement chauffée sur la moitié supérieure  $\theta_h$ , avec  $\theta_h > \theta_c$ . La moitié inférieure et les autres parois horizontales sont considérées adiabatiques. Les résultats obtenus montrent que pour une épaisseur donnée  $D$ , l'accroissement des paramètres nombre de Rayleigh  $Ra$  et le rapport des conductivités thermiques  $Kr$  entraînent un accroissement du nombre de Nusselt moyen  $\overline{Nu}$ , de la température de l'interface et de la vitesse de l'écoulement. Généralement il a été trouvé que l'augmentation de l'épaisseur de la paroi solide réduit  $\overline{Nu}$  sauf pour le cas  $Kr > 1$  et  $Ra < 10^4$   $\overline{Nu}$  croît avec l'accroissement de  $D$ . la température de l'interface solide-fluide est non uniforme. Cette non uniformité rend la structure de l'écoulement non symétrique.*

**Mots clés:** convection naturelle conjuguée, enceinte carrée, paroi chauffée en haut, volumes finis, rapport des conductivités thermique.

## 1. Introduction

Natural convection in enclosures is a topic of considerable engineering interest. Applications range from thermal design of buildings, to cryogenic storage, furnace design, nuclear reactor design, and others. In many studies, the walls of the enclosure are assumed to be of zero thickness and conduction in the walls is not accounted for. In addition convection heat transfer is due to the imposed temperature gradient between the opposing walls of the enclosure taking the entire vertical wall to be thermally active. However, in many practical situations, especially those concerned with the design of thermal insulation. It is only a part of the wall which is thermally active and conduction in the walls can have an important effect on the natural convection flow in the enclosure [1-12].

This article presents a numerical study of steady laminar conjugate natural convection in a square enclosure. The outer surface of the thick vertical wall is partially heated in the top. The main focus is on examining the effect of conduction in the wall, Rayleigh number, and wall thickness on heat transfer and fluid flow.

Nomenclature	
$D$	dimensionless wall thickness, $d/H$
$Kr$	thermal conductivity ratio, $Kr = k_w/k_f$
$L$	cavity length, $m$
$H$	wall height, $m$
$h$	height of the active part of the wall $m$
$Nu$	local Nusselt number, Eq(7)
$\overline{Nu}$	average Nusselt number, Eq(7)
$P_r$	Prandtl number of the fluid, $(\nu/\alpha)$
$Ra$	Rayleigh number, $g\beta H^3(T_h - T_c)/\nu\alpha$
$t$	dimensionless time, $t^*/(H^2/\alpha)$
$U, V$	dimensionless velocity components, $u/(\alpha/H), v/(\alpha/H)$
$P$	dimensionless pressure, $p/(\alpha/H)^2$
$X, Y$	non-dimensional Cartesian coordinates $x/H, y/H$
<b>Greek symbols</b>	
$\alpha$	thermal diffusivity, $m^2s^{-1}$
$\alpha^*$	thermal diffusivity ratio, $\alpha_w/\alpha_f$
$\theta$	non-dimensional temperature $(T - T_c)/(T_h - T_c)$
$\psi$	non-dimensional stream function, $U = \partial\psi/\partial Y$
<b>Subscripts</b>	
$f$	fluid
$w$	wall
$wf$	solid-fluid interface

## 2. Problem geometry

The geometry of the problem is shown in fig.1. The left vertical wall has a thickness  $d$ , the other walls are assumed to be of zero thickness. The outer vertical surface of the left wall is partially heated ( $h=H/2$ ) in the top  $T_h$ . The right wall is maintained at cooled temperature  $T_c$  such that  $T_h > T_c$ , while the horizontal walls are insulated. The problem is solved for air with  $P_r = 0.70$ .

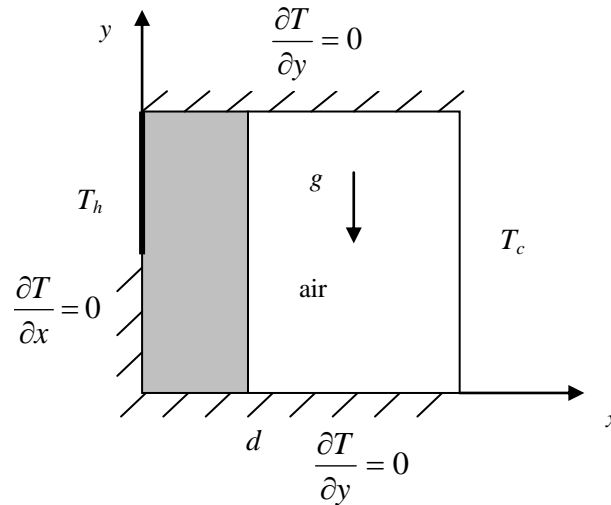


Fig 1: Physical configuration.

## 3. Governing equations

The flow in the enclosure is assumed to be two-dimensional. All fluid properties are constant. The fluid is considered to be incompressible and Newtonian. The Boussinesq approximation is applied. Viscous dissipation, heat generation and radiation effects are neglected. The dimensionless form of the governing equations can be written as:

- at  $t = 0$ ;  $U = V = \theta_w = \theta_f = 0$

- For  $t > 0$

• **fluid part**

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + P_r \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (2)$$

$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + P_r \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra \cdot P_r \cdot \theta \quad (3)$$

$$\frac{\partial \theta_f}{\partial t} + U \frac{\partial \theta_f}{\partial X} + V \frac{\partial \theta_f}{\partial Y} = \left( \frac{\partial^2 \theta_f}{\partial X^2} + \frac{\partial^2 \theta_f}{\partial Y^2} \right) \quad (4)$$

• **solid part**

$$\frac{\partial \theta_w}{\partial t} = \alpha^* \left( \frac{\partial^2 \theta_w}{\partial X^2} + \frac{\partial^2 \theta_w}{\partial Y^2} \right) \quad (5)$$

Where the subscript  $f$  for fluid layer and  $w$  for the wall,  $Ra$  is the Rayleigh number,  $\alpha^*$  is the diffusivity ratio and  $P_r$  is the Prandtl number.

The boundary conditions are:

$$U(D, Y) = U(1, Y) = U(X, 0) = U(X, 1); V(D, Y) = V(1, Y) = V(X, 0) = V(X, 1) \quad (6a)$$

$$\theta_w(0, Y) = 1; \theta_f(1, Y) = 0; \frac{\partial \theta_w(X, 0)}{\partial Y} = 0, \frac{\partial \theta_f(X, 0)}{\partial Y} = 0; \frac{\partial \theta_w(X, 1)}{\partial Y} = 0, \frac{\partial \theta_f(X, 1)}{\partial Y} = 0 \quad (6b)$$

$$\theta_f(D, Y) = \theta_w(D, Y); \left. \frac{\partial \theta_f(D, Y)}{\partial X} \right|_{\text{fluide}} = Kr \left. \frac{\partial \theta_w(D, Y)}{\partial X} \right|_{\text{solide}} \quad (6c)$$

Where  $Kr = k_w/k_f$  the thermal conductivity ratio, and  $D$  is the dimensionless wall thickness of the solid part.

The local and average Nusselt numbers are defined by:

$$Nu = \left( -\frac{\partial \theta}{\partial X} \right)_{X=D,1}; \overline{Nu} = \int_0^1 Nu dY \quad (7)$$

## 4. Numerical method

Equations (1) to (5) subjected to the boundary conditions (6) are integrated numerically using the finite volume method described by Patankar [13]. A uniform mesh of (90x90) is used in  $X$  and  $Y$  directions. A hybrid scheme and first order implicit temporally discretisation are used. The iteration process is terminated under the following conditions:

$$\sum_{i,j} |\phi_{i,j}^n - \phi_{i,j}^{n-1}| / \sum_{i,j} |\phi_{i,j}^n| \leq 10^{-5}$$

For wall side  $\overline{Nu}|_{X=0} = \overline{Nu}|_{X=D}$ ; For fluid side  $\overline{Nu}|_{X=D} = \overline{Nu}|_{X=1}$

Where  $\phi$  represents:  $U$ ,  $V$  and  $\theta$ ;  $n$  denotes the iteration step.

## 6. Numerical validation

In order to validate our results a comparison was made with the results obtained by Kaminski and Prakash [1] (see table 1). A good agreement between the obtained and reported results can be observed.

**Table 1:** Comparison of  $\overline{Nu}$  with Kaminski solution [1].

$Ra$	$Kr$	Kaminski	Present Study
$7.10^2$	1	0.87	0.867
	5	1.02	1.017
	10	1.04	1.040
	$\infty$	1.06	1.063
$7.10^4$	1	2.08	2.084
	5	3.42	3.417
	10	3.72	3.719
	$\infty$	4.08	4.076

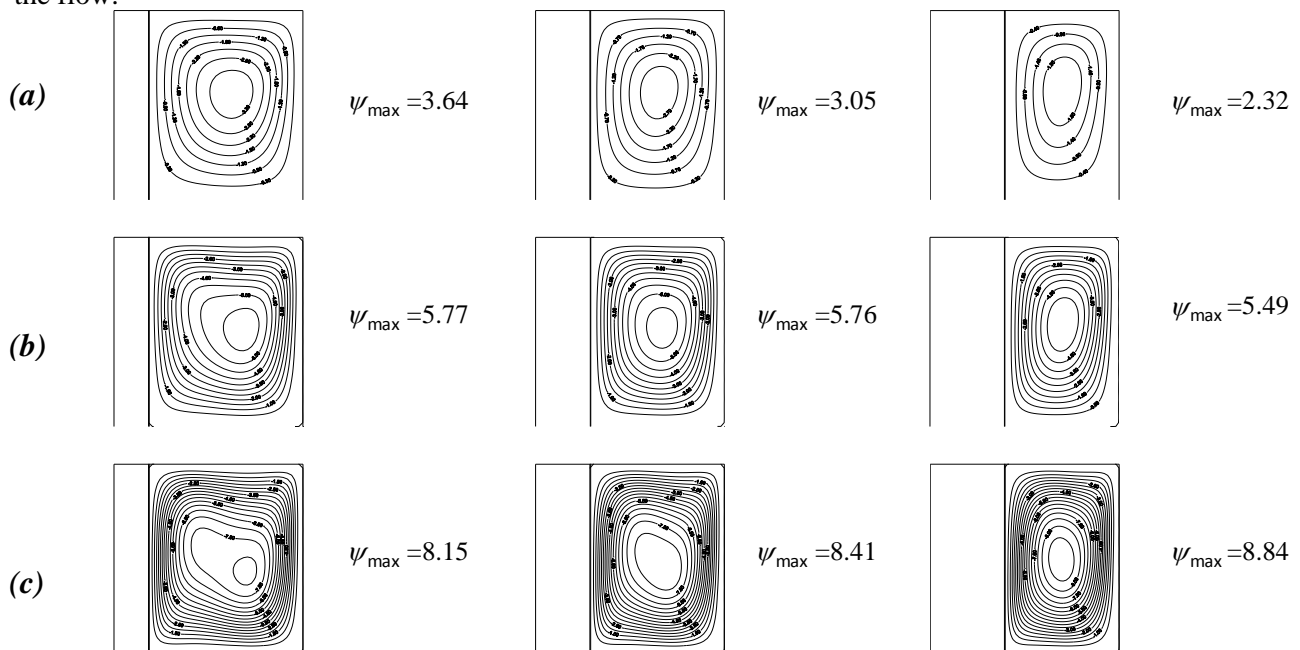
## 7. Results and discussion

The tests The results are generated for different values of governing parameters:  $\alpha^* = 1$ ;  $0.2 \leq D \leq 0.5$ ,  $500 \leq Ra \leq 10^6$  and  $0.1 \leq Kr \leq 100$ .

### 7.1. Streamlines plots

For  $Ra=10^5$ , Fig. 2 show the effect of both wall thickness  $D$  and thermal conductivity ratio  $Kr$ , on fluid motion in the enclosure. The circulation pattern is in clockwise direction, with flow upward at the hot left solid-fluid interface and downward at the cold right wall.

Panel (a) represents a poorly conducting wall ( $Kr = 0.1$ ), Panel (b) equal fluid/wall conductivity ( $Kr = 1$ ) and panel (c) a high wall conductivity ( $Kr = 10$ ). We can see for  $Kr=0.1$  and  $Kr=10$  that the increase of wall thickness  $D$  leads to reduce the maximum values of the dimensionless stream function  $\psi_{\max}$ . The convection in this case becomes weaker because of the temperature drop in the wall. In contrast to that, for  $Kr=10$   $\psi_{\max}$  increase with the increase of wall thickness. It can be seen also from Fig. 2 that the strength of the fluid circulation is increasing with the increase of the thermal conductivity ratio. As a result the convection becomes more important because of the increase of the effective temperature difference driving the flow.



**Fig 2:** Streamlines: from left to right  $D = 0.2, 0.3, 0.4, 0.5$ . (a)  $Kr = 0.1$ ; (b)  $Kr = 1$ ; (c)  $Kr = 10$ .

### 7.2. Isotherms plots

The isotherms are shown in fig.3, for different values of wall thickness  $D$  and constant Rayleigh number ( $Ra=10^5$ ). A comparison is made for three cases of thermal conductivity ratio:  $Kr = 0.1, 1$  and  $10$ .

For poor conductive wall (Panel a  $Kr = 0.1$ ), the average Nusselt number have low values comparing with those in panel (b) and (c). This is a logical result since reducing the thermal conductivity of the wall leads to the increase in thermal resistance of the overall system and therefore reducing the Nusselt number. Heat transfer is mainly by heat conduction for all the values of  $D$ .

As  $Kr$  increases ( $Kr=1$  Panel b), conduction in the solid wall is more important and so that for convection heat transfer in fluid part. The average Nusselt  $\overline{Nu}$  becomes higher than that in case (a).

The third case (Panel c) represents a high and good conductive wall ( $Kr = 10$ ). The convection heat transfer is very important comparing with cases (a) and (b). The values of  $\overline{Nu}$  are decreasing with the increase of wall thickness, and increasing with the increase of  $Kr$ .

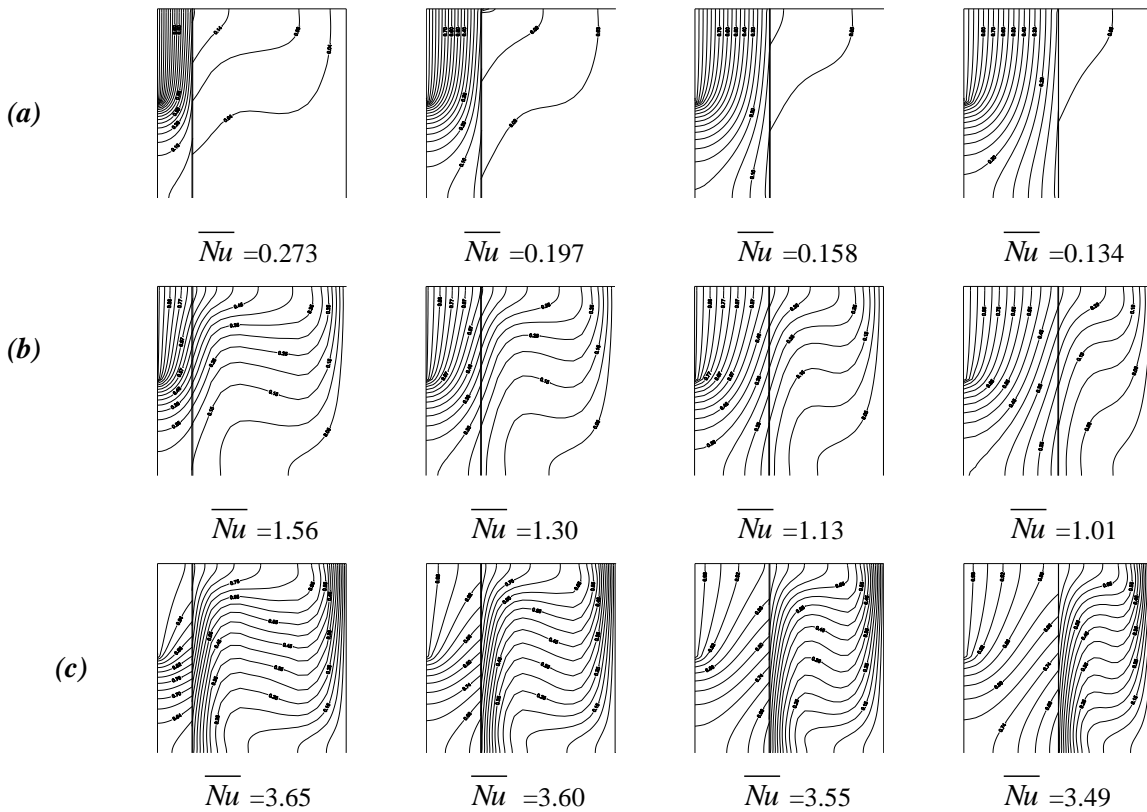


Fig 3: Isothermes: from left to right  $D = 0.2, 0.3, 0.4, 0.5$ . (a)  $Kr = 0.1$ . (b)  $Kr = 1$ . (c)  $Kr = 10$ .  $Ra = 10^5$

### 7.3. Interface temperature

The variation of wall/fluid interface temperature for different values of the thermal conductivity ratio and at constant  $D=0.2$  and  $Ra = 10^7$  is shown in Fig.4. A comparison is made with the standard enclosure ( $D = 0, Kr = \infty$ ). Natural convection inside the fluid part is driven by the temperature difference between the interface and the cold wall. This difference is lower for walls with poor thermal conductivity. It becomes more important with the increase of  $Kr$ , and lead to increase the average Nusselt number. For high values of  $Kr$  ( $Kr \rightarrow \infty$ ) the interface temperature distribution tend to become uniform which represents the standard enclosure with isothermal vertical walls ( $\theta = 1$ ) and zero wall thickness ( $D = 0$ ).

As shown in fig.4 the temperature profile across the solid/fluid interface is quite non uniform. This non uniformity has a noticeable effect on the flow field. The flow structure is asymmetric as shown in fig.2.

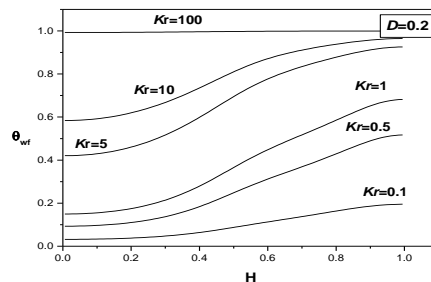


Fig 4: Variation of wall-fluid interface temperature.  $Ra = 10^5$ .

### 7.4. Average Nusselt Number

Fig.5 shows the effect of Rayleigh number, thermal conductivity ratio and wall thickness on the rate of heat transfer across the enclosure. For the three cases of  $Kr$ , we note that  $\overline{Nu}$  increases with the increase of  $Ra$ . For low conductive wall ( $Kr=0.5$ ), where the solid part is an insulate material,  $\overline{Nu}$  has small values comparing with those with  $Kr=1$  and  $Kr=5$ . In this case temperature difference driving the flow between the

interface and the cold boundary is very small, so most of heat transfer is by heat conduction. As the conductivity ratio increases  $Kr = 1$ , reducing the wall thickness leads to enhance heat transfer by natural convection and so that for  $\overline{Nu}$ . In addition for  $Ra < 10^3$ ,  $\overline{Nu}$  is almost constant so no effect of wall thickness in this case. For high conductive walls ( $Kr = 10$ ) and low values of  $Ra$  ( $Ra < 10^4$ ) we found that  $\overline{Nu}$  is reducing with reducing the wall thickness. This means that the thermal resistance of the wall is less than that of fluid medium for highly conductive walls and  $Ra < 10^4$ .

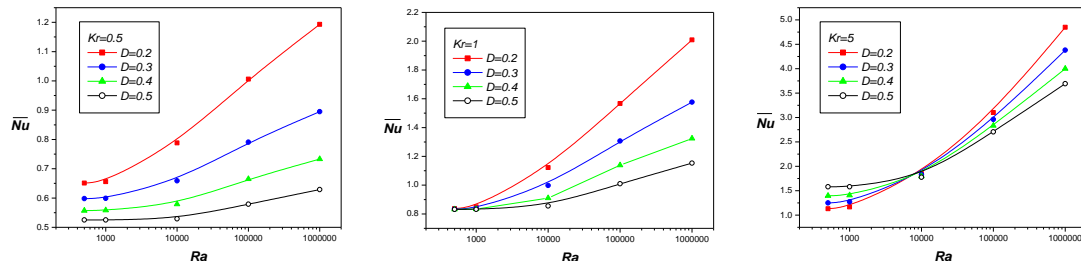


Fig 5: Variation of  $\overline{Nu}$  with Rayleigh number  $Ra$ .

## 8. Conclusion

A numerical model was employed to analyze the flow and heat transfer of air filled in a square enclosure with partially heated and conductive vertical wall. The following conclusions are drawn: conduction and wall thickness make a strong effect on natural convection in the fluid part. It is found that the rate of heat transfer increases and the fluid moves with greater velocity when the values of Rayleigh number and conductivity ratio increase. It is observed that the temperature difference between the interface and the cold boundary is reducing with increasing the wall thickness and therefore reducing the average Nusselt number. It is found that as the wall thickness increases the average Nusselt number decreases and the maximum value of the dimensionless stream function in the fluid part are higher with thin walls. It is found for special cases at low  $Ra$  ( $Ra < 10^4$ ) and high conductive walls ( $Kr = 10$ ), the values of  $\overline{Nu}$  are increasing with the increase of the wall thickness. For low values of  $Kr$  (poor conductive wall), where the solid wall is insulation material,  $\overline{Nu}$  has low values with those at high values of  $Kr$  because of the increase in the thermal resistance of the overall system and vas-versa. The heat is transferred mainly by conduction in both wall and fluid layer for small values of  $Ra$  and the average Nusselt is approximately constant.

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