Curling dynamics of naturally curved ribbons: from high to low Reynolds numbers

Octavio E. Albarrán Arriagada[a], Gladys Massiera[a], Robin Clavé[a] and Manouk Abkarian[a]

a. *Laboratoire Charles Coulomb, UMR 5221 CNRS/Université Montpellier 2, Place Eugène Bataillon, Bât 11, cc26, 34095 cedex 5, Montpellier, France*

Abstract :

When straight rods or ribbons are bent and suddenly released, a burst of flexural waves propagates down the material. However, for naturally curved ribbons of radius a_0, geometrical constrains couple bending and stretching deformations leading to a strain localized region. Here, we study the curling dynamics of such ribbons with a width W and show that the buckled region dictates the velocity of curling propagation. At high Reynolds numbers Re, we show that curling proceeds in a cylindrical rolling up deformation which has a constant velocity predicted by a balance between drag dissipation and the variation of elastic, kinetic and gravitational energies. The normalized rolling curvature a_0/R depends both on the elasto-gravitational length and the Cauchy number C_Y. It reaches a limiting value of 0.48 when gravity is negligible and $C_Y \ll 1$ and is close to 1 when $C_Y \sim 1$ as observed in water. Finally, at low Re, we find that curling velocity decreases in time and it is controlled by the interlayers lubrication forces.

Mots clefs : naturally curved ribbon ; curling ; rolling

1 Introduction

Curling deformation of thin elastic sheets appears in numerous structures in nature, such as membranes of red blood cells [1] and artificial polymersomes [2], epithelial tissues [4] or green algae colonies [3] to cite just a few examples (Fig. 1A-D). However, despite its ubiquity, the dynamics of curling propagation in a naturally curved material remains still poorly investigated.

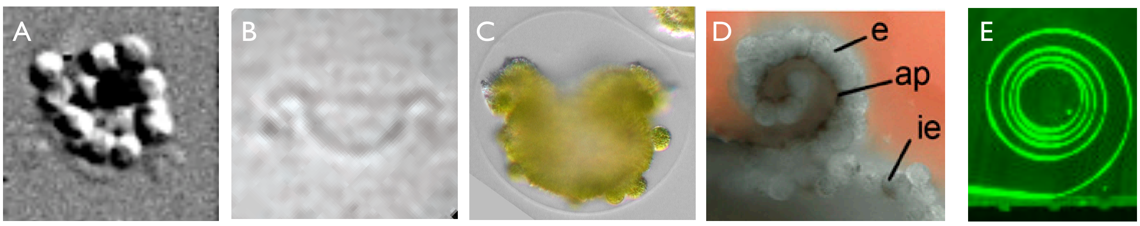

Figure 1 – Curling observed for A) Malaria infected red blood cells [1] B) Artificial polymersomes [2] C) Volvox [3] D) Epithelial tissue [4] E) a naturally curved PVC ribbon in Silicone oil [5].

Here, we present a coupled experimental and theoretical study of the dynamical curling deformation of naturally curved ribbons (see Fig. 1E). Using thermoplastic and metallic ribbons molded on cylinders of different radii, we tune separately the natural curvature and the geometry to study curling dynamics in air, water and in viscous oils, thus spanning a wide range of Reynolds numbers.

Our theoretical and experimental approaches separate the role of elasticity, gravity and hydrodynamic dissipation from inertia and emphasize the fundamental differences between the curling of a naturally curved ribbon and a rod described by the classical Kirchhoff equations [6].

Ribbons are indeed an intermediate class of objects between rods, which can be totally described by one-dimensional deformations, and sheets. Since Lord Rayleigh, it is known that a thin sheet can easily be bent but not stretched. As a result, large deformations in thin sheets usually lead to the localization of deformations into small peaks and ridges as observed by crumpling a simple piece of paper. These elastic defects induce critical buckling situations studied in detail statically in the literature, while experimental and theoretical studies on their dynamics are scarce. Our work shows evidence for the propagation of such a single instability front, selected by a local buckling mechanism. Finally, we show that depending on gravity, and both the Reynolds and the Cauchy numbers, the curling speed and shape are modified by the large scale drag and the local lubrication forces, shedding a new light on microscopic experiences where curling is observed.

2 Curling at high Reynolds number ($10^3 - 10^4$)

When a Ribbon with natural curvature $c_0 = 1/a_0$ is uncoiled on a solid substrate a pronounced deformation in the cross-section appears as a consequence of the poisson ratio ν of the material (Fig.2). This deformation induces localizations of the longitudinal curvature which can not be explained using the standard mechanics of beams. A good geometrical parameter to characterize the localization phenomena that we name the Tapespring Number (TSN), is the ratio between the typical displacement in the cross-section ($\sim \frac{\nu W^2}{a_0}$) and its tickness h. When $\frac{\nu W^2}{h a_0}$ is much smaller than one, the system will behave like a perfect rod, otherwise, the bending mode selected by curling will depend on the presence of a buckled region and its propagation will follow the criteria of propagating instabilities [7, 8].

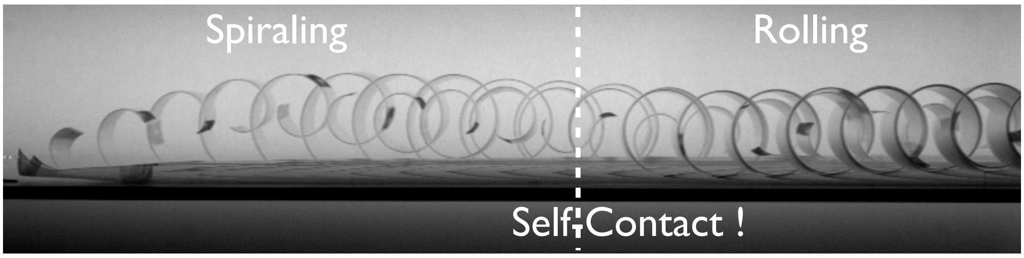

FIGURE 2 – Superposition of pictures of a typical experiment of a PVC ribbon ($W = 50$ mm, $h = 200$ μm and $a_0 = 11$ mm) obtained at 1000 fps

In a typical curling experiment, the ribbon rapidly curls on itself until self-contact is produced, forming a compact cylinder of radius R which moves with a constant velocity V_r (see Fig.2). At short time scale, the dynamics depends strongly on the TSN (see in Fig.3). Ribbons with low TSNs, disclose a decelerating curling front, while ribbons with TSN values present both a typical buckling time followed by an accelerating regime. This occurs basically because the localization of the planar deformations generates an extra inertia compared to the low TSN case. During curling, not only the curled part grows while moving, but the inertia associated to lateral unfolding of the cross-section modifies the dynamics.

At long time scale however, the rolling speed V_r becomes independent of the TSN. The normalized curvature of rolling $e = a_0/R$ remains constant and depends only on $1/a_0$. Considering a balance between the different energies involved in the problem (Kinetic, Elastic, Gravitational and inner and outer Dissipated energies), we can find the following relation between V_r , e, and the mechanical properties of the material [5] :

$$\left(\frac{V_r}{V_{flex}}\right)^2 = \frac{\lambda^2}{2(1+D)} - \frac{12(1-\nu^2)}{e(1+D)}\left(\frac{a_0}{L_g}\right)^3 \quad (1)$$

, where $V_{flex} = \frac{1}{a_0}\sqrt{\frac{B}{\sigma}}$ is the characteristic speed of flexion (B is the bending stiffness, σ is the surface density of the material) $\lambda^2 = 2e - e^2 - \nu^2$ is a geometrical prefactor in the variation of elastic energy,

$D = \frac{C_D \rho_f a_0}{e\sigma}$ is a number proportional to the drag forces (C_D is the drag coeficient of the cylinder, ρ_f is the density of the outer fluid) and L_g is the elastogravitational length of the ribbon.

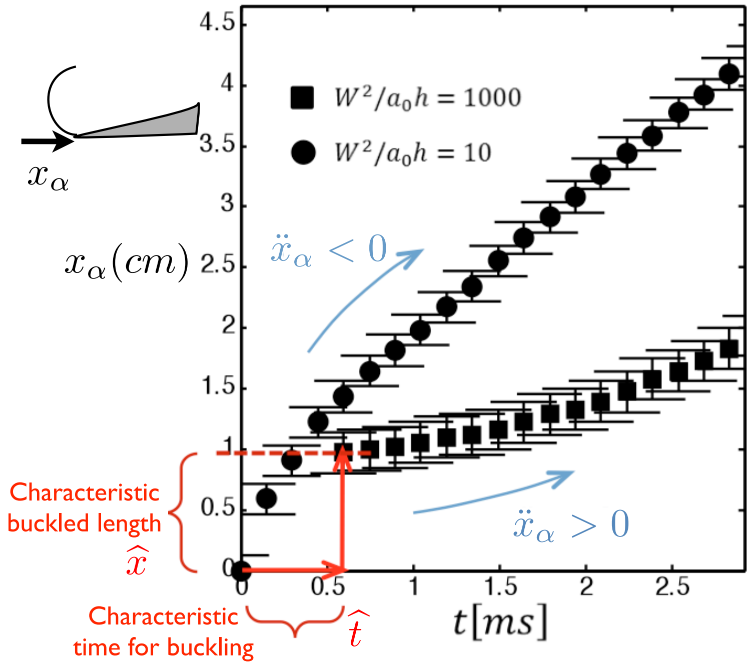

FIGURE 3 – kinematics at short time scale : Curled length as a fuction of time for two different ribbon with different width (PVC 200 μm thick and $a_0 = 11$ mm).

The parameter e reflects an equilibrium between three principal interactions : elastic, drag, gravitational and centrifugal forces. When drag and gravitation are neglected, e reaches its minimum value ≈ 0.50 which is compatible with a rolling solution of the elastica problem associated [5]. However, we characterized the gravitational effects in terms on the critical natural radius $a_0^* \approx 0.28L_g$ for static equilibrium with gravity (when $a_0 > 0.28L_g$ curling cannot propagate along the material). Experimentally, when a_0 is slightly less than a_0^*, e gets its maximum experimental value ≈ 1.1.

With large scale drag, the balance of energy is modified and the Cauchy number $C_Y = \frac{D}{1+D}$ (it is the ratio between drag and elastic forces, noteworthy it cannot be larger than 1) becomes important in the problem. This is illustrated with experiments we realized in water, where the rolling solution is still observed. However, at $C_Y \approx 1$ and since $a_0 \sim 2 - 3 \times a_0^*$, we find $e \approx 1$ and the velocity is close but lower than the one we predict with equation 1. We believe this discrepancy arises from the corner flow between the cylindrical body to the rolling ribbon and its uncoiled region which is not taken into account in our analysis. Moreover, we find that at larger speed when $a_0 \ll a_0^*$, the theoretical estimates becomes worst and worst with the observed one. We observe indeed that the ribbon enters a new dynamical state and vortices are shed, inducing an extra mode of deformation in the system (see Fig. 4), affecting the way to consider the drag dissipation [5].

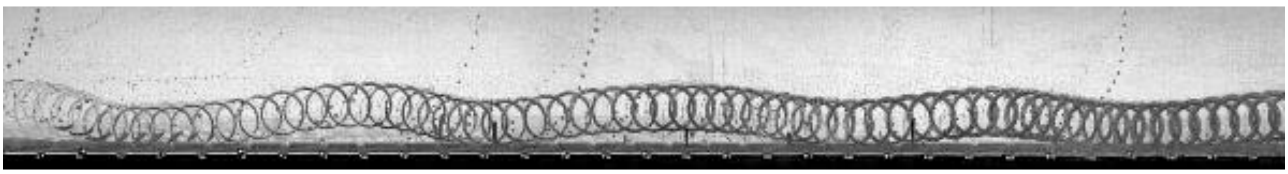

FIGURE 4 – Superposition of images associated with the curling process in water, the images were taken at 100 fps (PVC 100 μm thick and $a_0 = 5$ mm). The figure shows oscillations produced by the elastic response to the shedding vortices generated by the curling itself.

3 Curling at low Reynolds number $(10^{-4} - 10^{-3})$

When the Reynolds number is low, the scenario is very different. Lubrication prevents self contact and the rolling regime does not exist. The constant speed of propagation is never reached. In Fig.5A we show the experimental measurement of the outer diameter d of the coiled spiral as a function of time. We observe that the increase of d is faster for larger values of the width W of the ribbon. The curled length (position) as a function of time appears in Fig.5B ; at short time scale, the tendency of the curve is compatible with a power law $\sim t^{1.7}$, while for long time scale, the suitable relationship is $\sim t^{0.7}$. This result is intriguing and shows the effect of such non compact curling on the kinematics of curling of ribbons. Indeed, a simple argument as used in the earlier paragraph predicts a constant velocity for a ribbon of constant width. This preliminary result suggests that not only the drag at large scale should be considered but also the important role lubrication forces play in curling dynamics.

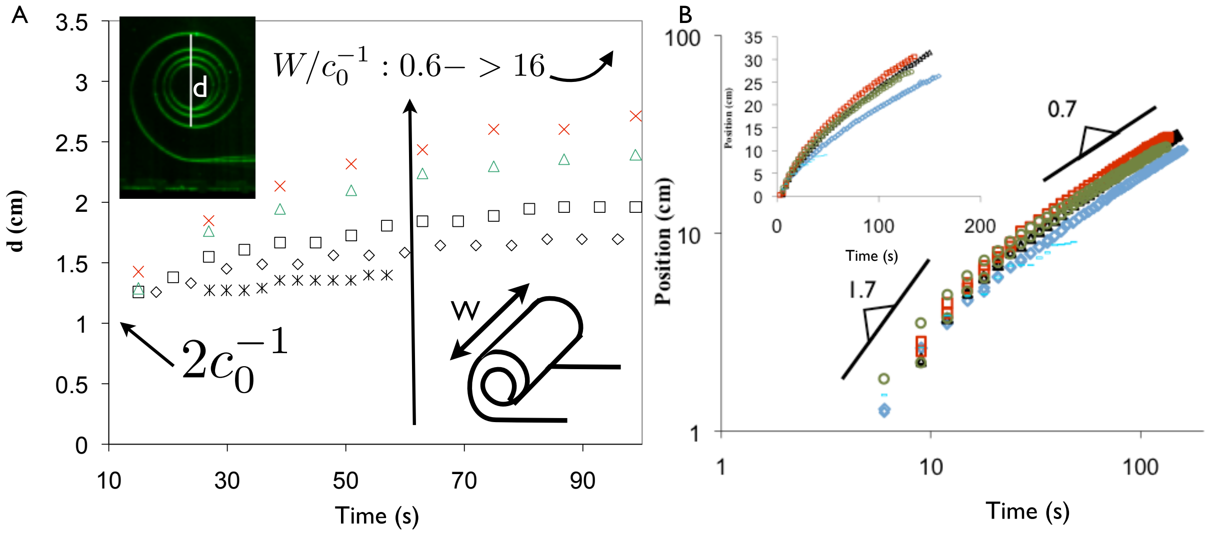

FIGURE 5 – Polypropylene curling ribbon : (A) size d and (B) position as a function of time. The ribbon is 90 μm in thickness moving in a Silicon oil of 12 500 cSt. Re estimated at 8×10^{-4}. $c_0^{-1} = 0.5$ cm : ($\diamond$) $W = 1.0$ cm, ($\square$) $W = 2.0$ cm, ($\triangle$) $W = 4.0$ cm, ($\times$) $W = 8.0$ cm, ($*$) $W = 3$ mm (from thesis of Octavio Albarran Arriagada).

4 Conclusions

In conclusion, curling belongs to the class of propagating instabilities in mechanics, where a localized deformation travels along the material. This property originates from the geometric extension of ribbons, which selects a single front of propagation by a local buckling mechanism and lead to a rolling regime when only inertia and elasticity are important. Moreover, we show that depending on gravity, and both the Reynolds and the Cauchy numbers, the curling speed and shape are modified by the large scale drag and local lubrication forces.

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