Non-intrusive generalized polynomial chaos takes uncertain parameters into account in the stability analysis of a clutch system

M.H. TRINH, S. BERGER, E. AUBRY

Laboratoire MIPS, Université de Haute-Alsace, 12 rue des Frères Lumière, 68093 Mulhouse, France
minh-hoang.trinh@uha.fr - sebastien.berger@uha.fr evelyne.aubry@uha.fr

Abstract:

In the transmission systems of vehicles, unforced vibrations can be observed during the sliding phase of clutch engagement. These vibrations are due to frictional forces and may generate noise. Several studies have shown that the stability of such friction systems is highly sensitive to the parameters (friction law, damping...) which admit significant dispersions. Therefore, the uncertain parameters must be considered in the stability analysis of a clutch system. This paper investigates the ability of generalized polynomial chaos to take an increasing number of uncertain parameters into account in the stability analysis of a clutch system ; it focuses on accuracy, on the criterion for the choice of the order of truncation and on the computation quantity. The objective is to propose a low cost, high accuracy model, compared to the Monte Carlo method.

Keywords: Clutch, friction, mode coupling, stability, generalized polynomial chaos

1 Introduction

In vehicles with manual transmission systems, unforced vibrations can be observed during the sliding phase of clutch engagement. These vibrations are caused by the frictional forces and may generate noise. Several studies have focused on the mechanisms responsible for these self-excited friction-induced vibrations [1]. Various mechanisms have been defined to explain the friction-induced vibration phenomenon. They are classified into two main families. The first one is related to the tribological aspects of friction systems and includes the stick-slip and speed dependent friction force mechanisms, while the second family is related to geometrical and structural properties and includes the so-called sprag-slip and mode coupling mechanisms. Low frequency phenomena such as judder (10–20 Hz) can often be attributed to tribological properties [2]. However, high frequency phenomena, such as squeal noise (up to several kHz), cannot be related to stick-slip behaviour because of the speed range of the vibrations measured. Consequently, mode coupling instabilities due to the intrinsic structure of the system are more likely to be responsible for this phenomenon [3]. Moreover, numerous studies have demonstrated that the dynamic behaviour of dry friction systems is highly sensitive to design parameters, in particular to the friction coefficient, which has been shown to admit dispersions which may be due to the manufacturing process. It is therefore necessary to take account of the dispersion of the uncertain parameters to ensure the robustness of the analysis of friction systems and thus the robustness of the design of this system class. The Monte-Carlo approach which is classically used to reach this aim requires prohibitive calculation time. The polynomial chaos formalism has been proposed as an alternative to take account of the uncertainties of the friction coefficient in the study of the dynamic behaviour of friction systems [4], [5]. However, these studies were carried out on the model of a braking system with two degrees of freedom (DOF) and there were only one or two uncertain parameters. Therefore, the main objective of this paper is to investigate the ability of generalized polynomial chaos (GPC) to take account of an increasing number of uncertain parameters (up to 8) in the stability analysis of a clutch system which has numerous DOF. This paper focuses on accuracy, the criterion for choosing the order of truncation, and the computation quantity, with the aim to propose a low cost, high accuracy model compared to the Monte Carlo approach. This paper is organized as follows: section 2 presents the methods for the analysis of the stability of uncertain systems and section 3 the GPC formalisms; the friction system is described in section 4 and the results of the stability analysis are presented in sections 5 and 6. A conclusion is given in section 7.

2 Stability analysis of dynamic systems using the indirect Lyapunov approach

The equilibriums are steady static states of a system. Their investigations aim at understanding the reasons why a dynamic state arises preferentially to equilibrium. The eigenvalue approach of the stability analysis is presented in order to highlight how equilibrium can become a repulsive state [4].

Consider the equation of motion of a dynamic system:

$$\dot{X} = f(X, d) \tag{1}$$

where X represents the instantaneous state of the system (its coordinates in the phase space), the upper dot denotes time derivation and f is a function of X parameterized by the elements of d. According to the Hartman–Grobman theorem, the linearization of Eq. (1) in the vicinity of $X_e(d_0)$ preserves its nonmarginal stability nature. Therefore, the determination of the stability nature of equilibriums only requires the knowledge of the linearized equation of motion in their vicinity in most cases. Because of the form of the solutions, the stability nature of $X_e(d_0)$ is expressed by the eigenvalues $\lambda(d_0)$ of the Jacobian $Df(X_e(d_0),d_0)$. Following Lyapunov's indirect method, if all the eigenvalues show a strictly negative real part then the equilibrium is asymptotically stable; if at least one eigenvalue shows a strictly positive real part then the equilibrium is unstable. So, in the classic Monte Carlo procedure, to analyze the stability of a system which has uncertain parameters ξ, the samples are first generated following the probabilistic support of parameters, then the eigenvalues $\lambda(d)$ corresponding to each sample are calculated. This sampling based method is known to be time-consuming since it requires a high number of samples to ensure reasonable accuracy with high confidence. The resulting computing cost is exorbitant since the system's eigenvalues must be calculated for each sample, an operation which is difficult, especially for systems with numerous DOF. Therefore, the generalized polynomial chaos formalism can be used instead of the classic Monte Carlo procedure.

3 Generalized polynomial chaos theory

Generalized polynomial chaos establishes a separation between the stochastic components of a random function and its deterministic components [4]. In the dynamic system, if the uncertain parameters d are uniform, all the eigenvalues $\lambda_i(i=1,...,n)$ are also random functions. According to the Askey scheme [4], the Legendre polynomials L_j are best suited to deal with uniform uncertainties, so the random eigenvalues are given by:

$$\lambda_i(\xi) \approx \sum_{j=0}^{P} \bar{\lambda}_{i,j} L_j(\xi) \tag{2}$$

Here ξ is a vector of r independent random variables, distributed uniformly within the orthogonality interval [-1,1]. The truncation order P is shown to be dependent on the polynomial chaos order p and stochastic dimension r denoting the number of uncertain parameters:

$$P = \frac{(p+r)!}{p!r!} - 1 \tag{3}$$

Then, the computing λ_i is turned into the problem of finding the coefficients $\bar{\lambda}_{i,j}$ of its truncated expansion. These coefficients called stochastic modes can be computed by the non-intrusive spectral projection (NISP) or the regression technique. The principal advantage of these techniques is related to the fact that no modification is performed on the system, only the calculations of the eigenvalues of the completed clutch system for a limited number of samples are required. The NISP technique exploits the orthogonality property of the Legendre polynomials L_j to calculate $\bar{\lambda}_{i,j}$ of the equation (2) (Gauss collocation methods) as follows:

$$\bar{\lambda}_{i,j} \approx \left(1/\langle L_j, L_j\rangle\right) \sum_{k=1}^{Q} \lambda_i\left(\xi^{(k)}\right) L_j\left(\xi^{(k)}\right) W\left(\xi^{(k)}\right) \tag{4}$$

where $\xi^{(k)}$ and $W(\xi^{(k)})$ (k=1,...,Q) are given by the well known Gauss collocation points and their corresponding weights [2].

The accuracy of the proposed method depends on the accuracy of the polynomial chaos (2). Therefore, it is important to choose an appropriate order p which ensures an acceptable level of accuracy.

4 A squeal model of the clutch system

The squeal model of the clutch system used in this paper has been defined by Wickramarachi P. [6]. This squeal model which is a masses/springs model with 6 DOF has been chosen because it is sufficient and efficient to study mode coupling instability and has been validated with experiments. In the model, the contact between the friction disc (1) and the flywheel (2) is created at points A', B', C', D' by a progressive spring k_p that is split into four springs k_A, k_B, k_C and k_D. In order to consider the nonlinear characteristic of the progressive spring, the stiffness k_A and k_B are respectively divided and multiplied by a ratio γ_1 and the stiffness k_C and k_D are respectively divided and multiplied by a ratio γ_2 (as shown by equation 9). γ_1 and γ_2 can be considered the impact coefficients differences around the axis x and axis y in the process of manufacture and assembly between the flywheel and the friction disc. Points A, B, C, D are the projections of contacts on the average surface of the flywheel. The flywheel is deformable and it is modelled by the bending stiffness (k_f). Points E, F, G, H are fixed. The DOF of the flywheel are the rotations θ_x, θ_y around the fixed axes x, y and the movement translations Z_A, Z_B, Z_C, Z_D of points A, B, C, D along the fixed axis z.

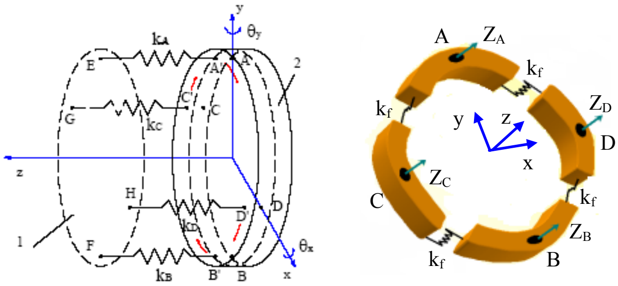


Figure 1. Clutch model ([6])

The linear model of an undamped clutch system can be expressed as follows:

$$M \cdot \ddot{U} + K \cdot U = 0 \tag{5}$$

$$\text{with } U = \begin{bmatrix} \theta_x & \theta_y & Z_A & Z_B & Z_C & Z_D \end{bmatrix}^T \tag{6}$$

$$M = diag\left(\begin{bmatrix} I_x & I_y & m/4 & m/4 & m/4 & m/4 \end{bmatrix}\right) \tag{7}$$

$$K = \begin{bmatrix} r^2(k_A + k_B) & \mu l r(k_C + k_D) & r(k_A + 2k_f) & -r(k_B + 2k_f) & \mu l k_C & -\mu l k_D \\ -\mu l r(k_A + k_B) & r^2(k_C + k_D) & -\mu l k_A & \mu l k_B & r(k_C + 2k_f) & -r(k_D + 2k_f) \\ r k_A & 0 & k_A + 2k_f & 0 & -k_f & -k_f \\ -r k_B & 0 & 0 & k_B + 2k_f & -k_f & -k_f \\ 0 & r k_C & -k_f & -k_f & k_C + 2k_f & 0 \\ 0 & -r k_D & -k_f & -k_f & 0 & k_D + 2k_f \end{bmatrix} \tag{8}$$

$$k_A = \gamma_1 k_p/4; \quad k_B = k_p/(4\gamma_1); \quad k_C = \gamma_2 k_p/4; \quad k_D = k_p/(4\gamma_2); \tag{9}$$

where $r = (r_1+r_2)/2$ with r_1, r_2 are the minimal and maximal radii of sliding; μ is the coefficient of friction and l is the thickness of the flywheel. In the case of a damped clutch system, in the context of this study, the dampings c_i are applied in the same locations as the springs k_p and k_f. Base nominal values set are $\mu \in [0,0.5]$; k_p=16.8MN/m; k_f= 7.35MN/m; $\gamma_1 = 0.9$; $\gamma_2 = 0.9$; r_1 = 75mm; r_2 = 120mm; l = 12.5mm; c_i = 1 N/m.

5 Study of the influence of parameters on the mode coupling of a clutch system in a deterministic approach

The deterministic calculations of the eigenvalues help to identify the mode coupling phenomena in the system. If there is a coalescence of two modes (their imaginary parts are equal) and the real part of a mode becomes positive, the system becomes unstable. Therefore, the deterministic approach helps to identify the pairs of modes which generate the mode coupling phenomena and to study the influence of parameters on the stability of the clutch system (Figures 2 and 3).

In system (5), there are 12 non-zero eigenvalues including 4 single frequencies which depend on the values μ, k_p, k_f, (Modes 1, 2, 3 and 4) and one double frequency (Modes 5, 6) which depends on the values μ, k_p. Modes 4, 5 and 6 are always decoupled; the coalescence phenomena occur between 3 modes (1, 2 and 3).

The results in Figures 1, 2, 3 show that, if k_p is small (13 MN/m), 2 modes (1 and 2) are decoupled, 2 modes (2 and 3) become coupled from a value of μ. When k_p increases (14 MN/m), all 3 modes are decoupled. If k_p

increases (16 MN/m), 2 modes (1 and 2) are coupled from a value of μ, 2 modes (2 and 3) are decoupled. So, the system becomes more unstable if μ increases. If damping increases, the real parts become increasingly negative (Figure 3).

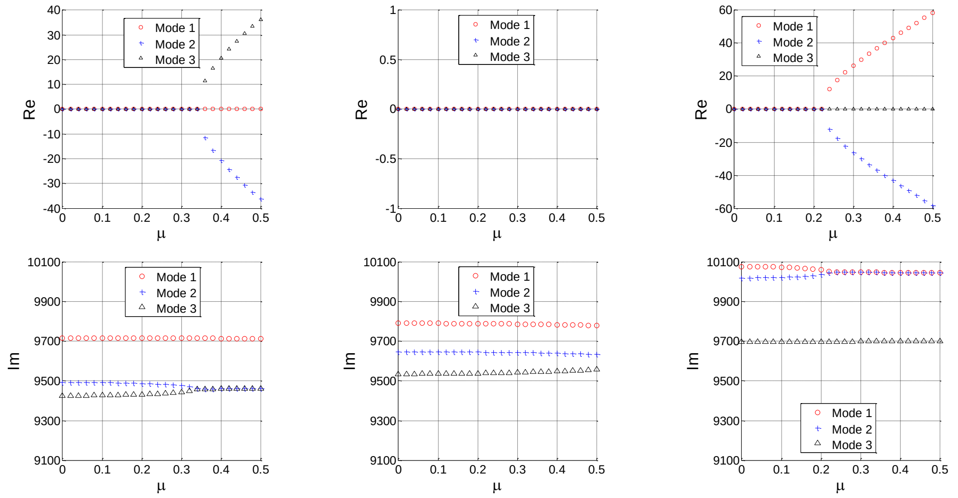


Figure 2. The real and imaginary parts of the eigenvalues of the 3 modes 1, 2, 3 (with k_p = 13, 14 and 16 MN/m, undamped)

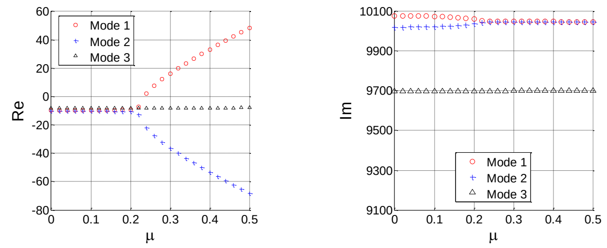


Figure 3. Real and imaginary parts of the eigenvalues of the 3 modes 1, 2, 3 (with k_p = 16 MN/m, the damping c_i = 1 N/m)

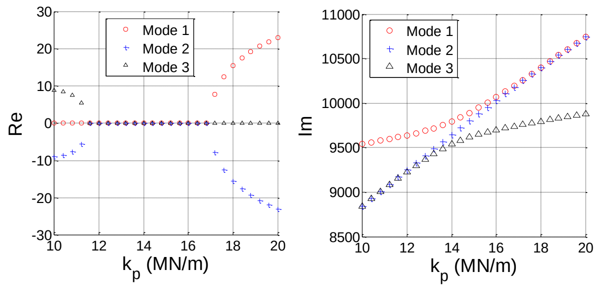


Figure 4. Real and imaginary parts of the eigenvalues of the 3 modes 1, 2, 3 with μ = 0.15

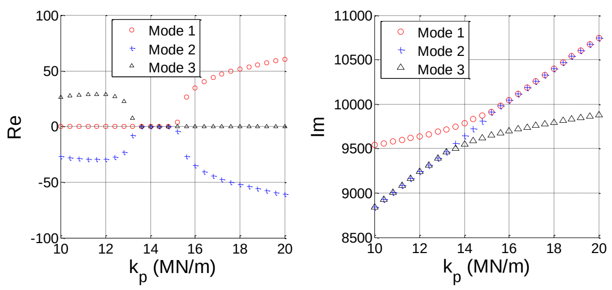


Figure 5. Real and imaginary parts of the eigenvalues of the 3 modes 1, 2, 3 with μ = 0.35

The results in figure 4 show that, if μ = 0.15 (low value), there are three stability areas for the system: unstable, stable and unstable. The results in figure 5 show that if μ = 0.35 (higher value), there are three stability zones : unstable, stable and unstable. Other studies show that if there is a little damping (c_i = 2.8 Ns/m), there are four stability zones : stable, unstable, stable and unstable. If damping increases, there are two zones of stability - stable and unstable. If damping has a high value, all real parts are negative, and the system is stable. So, these different studies show that the dynamic behaviour of the clutch system is highly nonlinear and highly sensitive to design parameters (friction, stiffness).

6 Generalized polynomial chaos for the stability analysis of a clutch system

As mentioned in section 3, to obtain acceptable accuracy, the choice of the truncation order P of generalized polynomial chaos is of great importance. Moreover, the criterion for the choice of the optimal order P should only depend on the results obtained with GPC. The eigenvalues are complex, so the assessment error between the modules obtained with two developments in successive order P_l is used to select the optimal order P. In a complex system, the interaction between modes is very important, so that the optimal P must be chosen to satisfy the requirements that all the average errors of the eigenvalue modules do not pass a threshold ε_{mod} and the relative error of the stability proportion does not pass a threshold of stability ε_{stable}. The stability of the equilibrium point is analyzed for each of the N samples generated according to the probabilistic law considered for uncertain parameters. N was set to ensure a confidence level of 99% with an accuracy margin of 1% for the stability proportion [7].

The average relative error of the module of the eigenvalue λ_i (in the least square sense), between two developments in successive order p_l, p_{l+1} ($l = 1, 2,...$) of the polynomial chaos, is defined by:

$$e_{i,rel} = \frac{1}{N}\sqrt{\sum_{k=1}^{N}\left(\frac{\mathrm{mod}\left(\lambda_{i,p_l}\left(\xi^{(k)}\right)\right)-\mathrm{mod}\left(\lambda_{i,p_{l+1}}\left(\xi^{(k)}\right)\right)}{\mathrm{mod}\left(\lambda_{i,p_l}\left(\xi^{(k)}\right)\right)}\right)^2}\text{, (N number of samples)} \quad (9)$$

The relative error of the stability proportion of the system between two successive developments in order P_l, P_{l+1} of polynomial chaos is defined by:

$$e_{stab,rel} = \frac{n_{stable,p_l} - n_{stable,p_{l+1}}}{n_{stable,p_l}}\text{, } (n_{stable} \text{ is the number of stable points}) \quad (10)$$

To choose P optimal, the truncation order has been increased until all average relative errors $e_{i,rel}$ of module of each eigenvalues and the relative error of the stability proportion of the system between two successive developments are successively less than the threshold of module (ε_{mod}) and the threshold of the stability proportion (ε_{stable}). In addition, the number n_c of eigenvalues of the complete system - which must be calculated to evaluate the stochastic modes of polynomial chaos - must be less than the number of calculations of the complete system using the Monte Carlo method (N).

$$\max\{e_{i,rel}, (i = 1,...,n)\} \leq \varepsilon_{\mathrm{mod}}; \quad e_{stable,rel} \leq \varepsilon_{stable}; \quad n_c \leq N \quad (11)$$

Figures 6 show the percentages of the relative error of the eigenvalues modules and the percentages of the relative error of the stability proportion of the system between two successive orders, using polynomial chaos (NISP) (blue curves) and between the calculated results, using polynomial chaos (with the order p_l) and the calculated results directly with the complete system (DC) (red curves). The results show that if the value of p is either too low or too high, the errors increase. The figures show the real and imaginary parts of the eigenvalues, which have 1 uncertain parameter (7a) and 3 uniform uncertain parameters (7b).

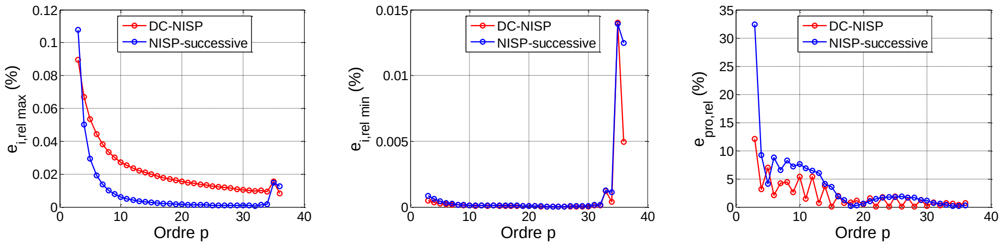
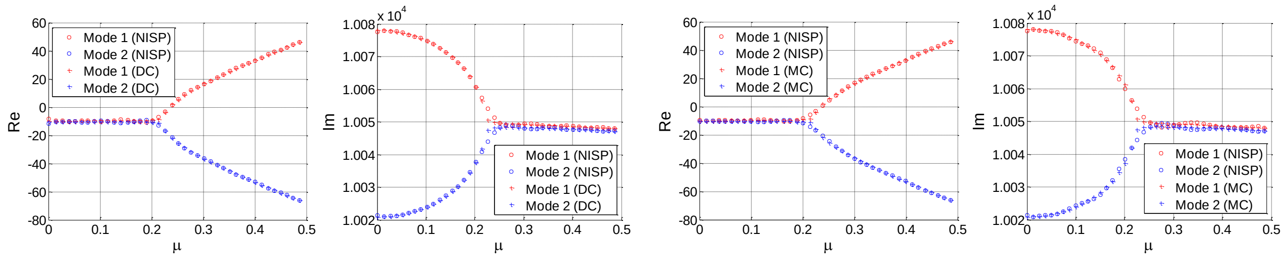


Figure 6. Evolution of the percentage of the maximum and minimum average errors of the eigenvalue modules and the percentage of the relative error of the stability proportion



(a) with 1 uniform uncertain parameter: μ (b) with 3 uniform uncertain parameters: μ, k_p, k_f

Figure 7. Real and imaginary parts of the eigenvalues of the two modes 1, 2 using the direct calculation of the complete system (DC) and with polynomial chaos (NISP)

Table 1 shows the comparison of the results of the stability analysis of the clutch system based on the analysis of the real and imaginary parts of eigenvalues of the two modes (1, 2) obtained successively through the direct calculation of the complete system and with generalized polynomial chaos, the optimal order p being calculated with ε_{mod} = 0.01%, ε_{stable} = 1% and N = 10,000). The results concern the Hopf bifurcation point (with 1 uncertain parameter), the stability proportion, and the number of calculations of the eigenvalues. The uncertain parameters are k_p, k_f, γ_1, γ_2, r_1, r_2, l in uniform ranges [0.95*nominal value, 1.05*nominal value] and $\mu \in [0, 0.5]$. The percentage of error for the stability proportion between direct calculation and GPC is less than 6% up to r = 7, and a little more than 10% up to r = 8. So, polynomial chaos will be efficient for up to 7 uncertain parameters. Moreover, with r = 3, $p_{optimal} = 10$, the number of direct calculations of the eigenvalues of the complete system - which is a complicated and expensive task - must reach 10,000. However, with polynomial chaos, 1000 calculations of the eigenvalues of the complete system are sufficient. Therefore, the computational cost is considerably reduced. These results show that the application of polynomial chaos in the stability analysis of dynamic systems with numerous DOF and several uncertain parameters can be effective.

	DC	Generalized polynomial chaos with r uncertain parameters							
		r=1	r=2	r=3	r=4	r=5	r=6	r=7	r=8
Order optimal p		17	23	10	8	5	4	3	3
Hopf bifurcation point μ_0	0.234	0.235	-	-	-	-	-	-	-
Percentage of error of μ_0 (%)	-	0.427	-	-	-	-	-	-	-
Percentage of error of the stability proportion between direct calculation and polynomial chaos (%)	-	0.841	1.11	0.753	0.635	4.755	5.736	5.966	11.72
Percentage of error of the stability proportion between 2 successive orders (%)	-	0.129	0.386	0.353	0.696	0.540	5.036	5.635	18.63
Number of calculations of the complete system	10,000	17	529	1000	4096	3125	4096	2187	6561

Table 1. Comparison between the direct calculation (DC) of the complete system and polynomial chaos

7 Conclusion

This paper has shown that mode coupling phenomena may occur in clutch systems and generate instabilities. Deterministic studies show that parameters such as the friction coefficient, pressure (represented by progressive stiffness) and damping are factors which significantly affect the stability of the system. Therefore, the dynamic behaviour of a clutch system is highly nonlinear and highly sensitive to design parameters. In order to circumvent the drawback of the classic Monte-Carlo method which is prohibitive for industrial systems, generalized polynomial chaos has been proposed. The ability of generalized polynomial chaos to take an increasing number of uncertain parameters into account in the stability analysis of a clutch system has been investigated. The eigenvalue has been built using a non-intrusive spectral approach. Two criteria for the choice of the truncation order have been defined: the average relative error of the module of the eigenvalue λ_i (in the least square sense) and the relative error of the stability proportion between two developments in successive order. Polynomial chaos must be efficient for up to 7 uncertain parameters and computation time will then be reduced considerably. For a greater number of uncertain parameters, the truncation order required to obtain good accuracy with generalized polynomial chaos leads to a higher number of direct calculations of the eigenvalues of the complete system than the number required in the classic Monte-Carlo approach. The results show that the application of polynomial chaos in the stability analysis of dynamic systems with numerous degrees of freedom and 7 uncertain parameters can be effective.

References

[1] B. Hervé, J.-J. Sinou, H. Mahé and L. Jézéquel, Analysis of Squeal Noise in Clutches and Mode Coupling Instabilities including Damping and Gyroscopic Effects, European Journal of Mechanics A-Solids, 27(2), 141–160, 2008.
[2] G. Chevallier, D.L. Nizerhy, F.R. Valloire, F. Macewko, Chattering instabilities: study of a clutch system. Structural dynamics (EURODYN), Millpress, Paris (France) 2005.
[3] B. Hervé, J.-J. Sinou, H. Mahé, L. Jézéquel, Extension of the destabilization paradox to limit cycle amplitudes for a nonlinear self-excited system subject to gyroscopic and circulatory actions, Journal of Sound and Vibration 323 (2009) 944–973.
[4] Lyes Nechak, Sébastien Berger, Evelyne Aubry, Non-intrusive generalized polynomial chaos for the robust stability analysis of uncertain nonlinear dynamic friction systems, Journal of Sound and Vibration 332 (2013) 1204–1215.
[5] E. Sarrouy, O. Dessombz and J.-J. Sinou, Stochastic study of a non-linear self-excited system with friction, European Journal of Mechanics A/Solids, 40, 1-10, 2013.
[6] Wickramarachi P., Singh R. Analysis of friction induced vibration leading to Eek Noise in a dry friction clutch. Proc. Internoise 2002.
[7] G. Baillargeon, Méthodes statistiques de l'ingénieur, Editions SMG