Probabilistic approach for the design of an Equal-Leaf Spring

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Résumé :

Les ingénieurs créatifs dans le domaine de construction des éléments de suspension, cherchent à optimiser le dimensionnement en service des ressorts à lames durant la conception. Dans ce papier, on utilise une approche probabiliste pour le dimensionnement d'un ressort à lames égales en vérifiant le critère de dimensionnement à une fiabilité donnée ce qui nous permet d'introduire des diagrammes de dimensionnement conservateurs. La fiabilité de dimensionnement est déterminée en utilisant la fonction de performance avec simulation numérique à travers la méthode de Monte Carlo. La méthode proposée permet d'obtenir un dimensionnement d'un ressort à lames égales plus fiable que le dimensionnement basé sur le critère déterministe.

Abstract :

The creative engineers in the suspension parts manufacturing field seek to optimize the design in a service of leaf springs they want during its conception. In this paper, we use a probabilistic approach to design an Equal-Leaf Spring (ELS) by verifying the design criterion for a given reliability that allows introducing design conservator diagrams for several ELS characteristics. The reliability of the design is determined using the performance function with numerical Monte Carlo Simulation (MCS). This proposed method leads to a more reliable design of an ELS compared to deterministic design criterion.

Mots clefs: **ressort à lames, dimensionnement, fiabilité, méthode de Monte Carlo.**

1 Introduction

A spring is a mechanical part that uses elastic properties of some materials to absorb mechanical energy. It produces a movement or exerts an effort or a couple over a system. They are variously employed in motor vehicle suspension systems, the actual choice made being determined largely by versatility in application and best economy of material in terms of energy storage per unit volume. The goal of the use of such a spring is to fulfill a best suspension: the stability of vehicle, the bearing of contact of the wheel on the ground, the safety of the automotive passenger and transported material against the impacts and vibrations that can be found during the path. We distinguish many types of springs: leaf spring, helical spring and torsion spring. Leaf springs are among springs that are used for the suspension in a wheeled vehicle because of the flexibility of one or many super-imposed metallic leaves. This flexibility orders the oscillation of spring due to the vehicle load. There are many types of leaf springs [1]: laminated leaf spring; parabolic leaf spring; single, equal or multi-leaf spring. The equal-leaf spring (ELS) is constituted of super-imposed leaves having the same dimension parameters.

The modelisation of this spring is basically carried out from the modelisation of a flexible beam with uniform rectangular section with a fixed-one-end point (a cantilevered beam) which serves to find the design in a service criterion for this spring [1]. The design in a service criterion is expressed by a mathematical inequality. When it is verified, the leaf spring keeps its form and performance. This criterion depends on many parameters defining the dimension and the metallurgy of the leaf spring. Varying the parameters values, the service behaviour of the leaf changes; these parameters influence the leaf flexibility (oscillation period). We consider the parameters defining the design in a service criterion of a leaf as random variables. They are expressed by a sampling drawn following a probabilistic distribution[3,4]. In this work, the service reliability is computed by using Monte Carlo method as the ratio of the secured events number and the total size. In order to stabilize the reliability response of the modelling, the size number should be optimized.

2 Mathematical modelling of an equal-leaf spring

In our work, we consider an ELS defined by the following dimensional data: length l, width B, leaf thickness e and number of equal leaves n .

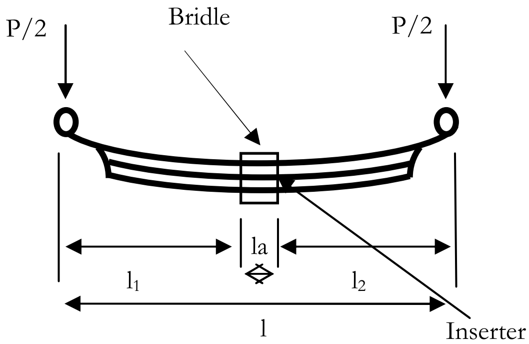

Figure 1. Equal-leaf spring

A section part of the ELS is fitted in the bridle stopping the flexure and is then eliminated from the length calculation as expressed in [1]:

$$L=l_1+l_2=l-l_a \tag{1}$$

The load P is uniformly distributed on the whole section of the leaves. However, the first leaf called master leaf, which is directly rested on the supports, has to rest on the longitudinal loading due to the engine acceleration and breaking, as well as on the lateral loading. The master leaf contains reinforced extremities called eyes which allow the spring handling.

3 Design criterion of an equal-leaf spring

In order to ensure a good design of an ELS, the maximal stress must be always inferior to the admissible stress. This condition is given by the following inequality:

$$\sigma_{\max} = \frac{3.P.L}{2.B.n.e^2} < \sigma_{adm} \tag{2}$$

σ_{adm} is the admissible stress which defines the state just before failure and it is expressed as follow:

$$\sigma_{adm} = \frac{R_e}{s} \tag{3}$$

where s is the safety coefficient and R_e is the yield stress of the material.

In this paper, we consider the 55Cr3 steel that is the most useful in the leaf spring construction. This material has the yield stress of 1250N/mm^2 at 0.2%. We use a safety coefficient of 5 corresponding for this case of material and application [2]. The applied stress is expressed as follows:

$$\sigma_{app} = \frac{P}{B.L} \tag{4}$$

the criterion is equivalent to the strict inequality below:

$$\sigma_{\max} = \frac{3.\sigma_{app}.L^2}{2.n.e^2} < \sigma_{adm} \tag{5}$$

This design criterion in a service may be interpreted graphically on the figure 2:

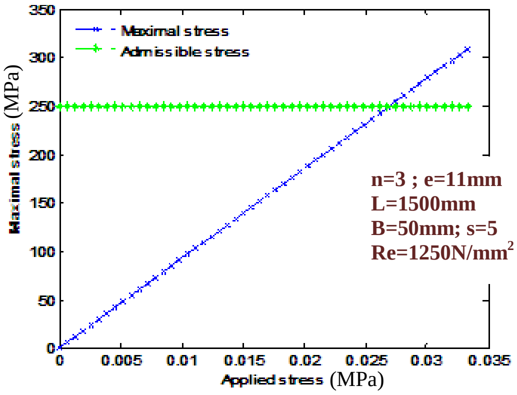

Figure 2. The curve of the maximal stress as a function of the applied stress.

The horizontal straight line represents the admissible stress and the inclined straight line represents the maximal stress as a function of the applied stress. The zone in which the maximal stress is strictly less than the admissible stress (the point is situated below the horizontal straight line) is called design in a service security zone. The ELS is then designed and it is performant. Whereas, the zone in which the maximal stress is strictly superior to the admissible stress is called the weakness zone and the spring is defective.
The equality condition is a condition of doubt: we cannot decide whether it is the state of security or weakness. The slope of the oblique straight line is $\frac{3.L^2}{2.n.e^2}$.
The variation of the slope is deduced from the design variation of the ELS. When varying the values of e and L parameters, we get a beam of straight lines with different slopes dispersed from the origin instead of one inclined straight line. This is geometrically interpreted by the figure 3:

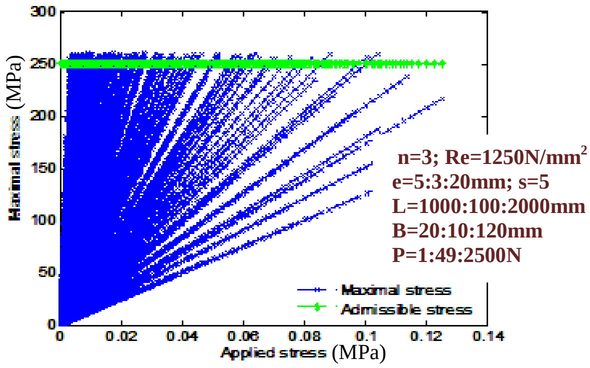

Figure 3. Curve of the maximal stress as a function of applied stress for different design types.

Higher values of a slope mean that the length value of the spring is big and the thickness value is small. A small load applied on the ELS produces a big deflection and strong oscillations. The behavior of the spring in a service rapidly decreases with increasing the slope. Otherwise, when the slope is small it means that the length value is small and the thickness one is big. A heavy load applied on the ELS lead to a small deflection and the spring will be stable. Then, the behavior in a service of the ELS improves.

4 Probabilistic approach for design an equal-leaf spring

The aim of this probabilistic approach is to eliminate the use of safety coefficient (s) and to replace it by defining different reliability values with corresponding confidence levels.

4.1 Reliability computation

Many experiences have been carried out on the ELS. We vary the dimensions of the ELS small variations; we verify therefore the design criterion and we test the behaviour of the leaves spring. In this paper, we verify the design criterion and then we compute the reliability using the Monte Carlo method [4]. This probabilistic approach is used to find the dimensions that carry a way to the best reliability: we speak about design criterion with a given reliability.

Then, we define the following random variables: $\underline{R_e}$, $\underline{L}$, $\underline{e}$ with a coefficient of variation (cov) for each one.

These variables can be represented by a vector. For each random variable, we carry out an N random sampling following a Gaussian normal distribution defined by a mean value and a coefficient of variation. These vectors allow the computation of the reliability in a service.

The proposed probabilistic approach for this problem of design is based on the couple « maximal stress-yield stress » that consist in determining the probability that the yield stress of ELS will be superior to the maximal stress. For a function of yield stress we have

$$\text{YieldS}=R_e \tag{6}$$

and for a function of maximal stress we have $\text{Sigmax}=\sigma_{\max}=\dfrac{3.\sigma_{app}.L^2}{2.n.e^2}$ (7)

We define a performance function G that separates the design in a service field: a zone where the spring fulfills the best suspension and the weakness area by $\text{G}(x_i)=\text{YieldS}(x_i)-\text{Sigmax}(x_i)$, where xi is an element of random vector {X}, having [3,4] a joint probability distribution function $f\{x_i\}(\{X\})$. $\text{G}(x_i)>0$ presents a best suspension of the spring field (spring designed in a service) F_s for the ELS; $\text{G}(x_i)<0$ presents a weakness field F_d for the same spring.

The security design probability P_S is determined by:

$$P_S=\int_{\mathbb{R}^n} 1_{\{G(x_i)>0\}}\, f_{\{x_i\}}(\{X\})dx_1...dx_n=\Pr ob(YieldS(x_i)>Sig\max(x_i))=\Pr ob(G(x_i)>0) \tag{8}$$

When n is the number of parameters defining the function G.

This probability can be computed by numerical integration which remains most of time complicated due to the existence of many integration variables. Consequently, the Monte Carlo method is very practical and easy for use. In this paper, the design probability P_S is approximated by numerical resolution using the Monte Carlo method.

For a total number of simulations N, we admit that the frequency of performance events of the spring where $G(x_i)>0$ stretch to design probability P_S when $N\to+\infty$:

$$P_S=\lim_{N\to+\infty}\frac{Number\ of\ performant\ events\ (G(x_i)>0)}{Total\ number\ of\ events} \tag{9}$$

This procedure presents the simplicity of the reliability computing. The speed of the method convergence is very slow as it is proportional [5] at $\sqrt{N}$. A number of simulations equal to 10^4 is required in order to ensure the estimation [6] of the reliability which represents the security design probability P_S with a variation coefficient of 10%. The reliability response begins to stabilize from the number $N=10^5$ which will be chosen as a standard size applied in our work [4]. The used random variables for the following application are defined by a cov of 2%. In order to explore the proposed design service criterion, we use a graphic representation (figure 4) showing the yield stress as a function of the applied stress which both are considered as random variables.

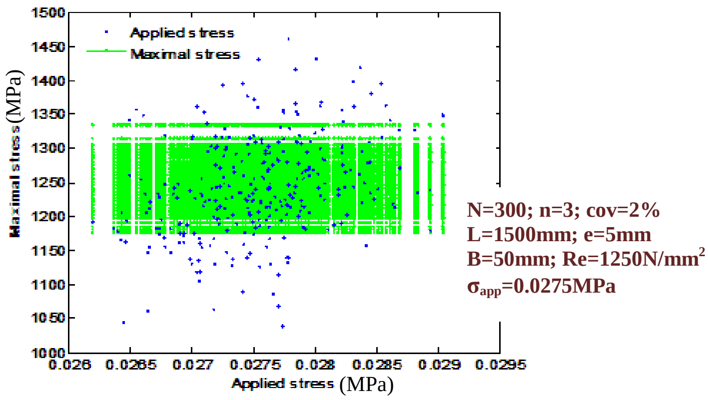

Figure 4. Surface of dispersion and cloud of points of an ELS.

The straight line representing the limit state of the design criterion ($G(x_i)=0$) is transformed into a beam of straight lines. Instead of the loading representative point defined by the applied stress, we get a cloud of points. The surface dispersion corresponding to the criterion and to the loading depends on the fixed size of the random sampling. The obtained beam of lines defines a new zone called the uncertainty zone which separates both security and weakness zones.

4.2 Iso probabilistic design in a service criterion

In order to obtain an iso probabilistic design in a service criterion for a given reliability, we increment the stress amplitude loading and we compute the reliability in each incrementation step until obtaining the target one [4]. The whole random variables are used with a cov of 2%. Several reliability values (99,90,75,50,25,10,1%) for the iso probabilistic design in a service criterion are determined in this application. These results are illustrated in figure 5:

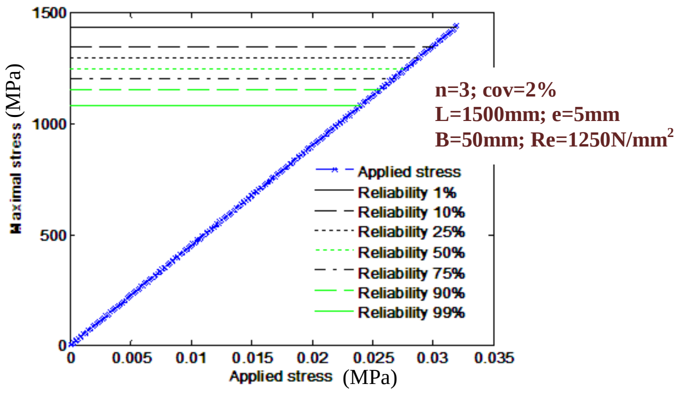

Figure 5. Design diagrams of a given reliability.

By applying the iso probabilistic design in a service approach for different reliability values, we obtain several confidence level zones instead of both unique security and weakness zones, characterizing the classic deterministic approach. Particularly, the result of this last application is fundamentally based on the hypothesis that iso reliability criteria, in-sighted by horizontal lines (figure 5), are obtained for a specific ELS leading to increment the loading amplitude according to a unique direction. In order to obtain an iso probabilistic design criterion valid to many different types of ELS, many increment directions have been used corresponding each one to specific design characteristics. In each direction, we increment the load until obtaining the desirable reliability value. We record the points corresponding each one a particular direction. Fitting these recorded points, we obtain the iso probabilistic design in a service criterion curve.
An application for 99% reliability is shown on figure 6:

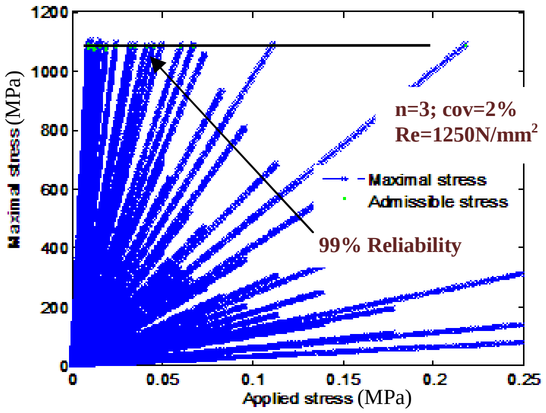

Figure 6. 99% iso reliability diagram for many ELS characteristics.

5 Conclusions

The proposed probabilistic approach permits improving the design in a service criterion of an ELS, in the sense that we increase the security confidence level. Geometrically, this can be interpreted as: the probabilistic approach allows obtaining a beam of straight lines and a cloud of points leading to get an uncertainty zone. Furthermore, the probabilistic design in a service criterion for a given reliability allows plotting iso probabilistic diagrams (conservative diagrams). Two applications of the iso probabilistic design criterion are studied: the first one is for a specific dimension of ELS following then a unique increment direction, and the second one is for several ELS characteristics following so many increment directions.

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