Prediction of suspended sediment concentrations in river flows

A. TERFOUS, M. SABAT, A. GHENAIM, J. B. POULET, S. CHIBAN

INSA Graduate School of Engineering, 24 Boulevard de la Victoire, Strasbourg France 67084
Icube UMR 7357 – Laboratoire des Sciences de l'Ingénieur, de l'Informatique et de l'Imagerie

Résumé :

Nous proposons un modèle pour calculer la concentration des sédiments transportés en suspension dans un écoulement à surface libre en régime uniforme quand la concentration moyenne est connue. Le coefficient de diffusion obtenu par l'application de la correction d'Itakura et Kishi à un profil parabolique constant est implémenté dans une forme simplifiée de l'équation de convection diffusion. Le modèle proposé est validé par des résultats de mesures expérimentales.

Abstract :

This paper suggests a method for profiling suspended sediments concentration (SSC) through the water column for uniform open channel flows when the mean value of concentration is known. A new sediment diffusivity coefficient is defined by applying the Itakura and Kishi correction method to the well known parabolic constant profile. This coefficient is implemented into a simplified version of the convection diffusion equation. The proposed profile is validated through comparison with experimental data and shows an enhanced prediction of SSC near the free surface.

Mots clefs: open channel flows, suspended sediment concentration, vertical distribution, sediment diffusivity.

1 Introduction

Various numerical and analytical formulas of different order of complexity are used to represent suspended sediments concentration (SSC), and knowing the complexity of transport in rivers, it is still impossible to have a universal profile. To state some examples, for steady problems, many analytical formulas are proposed [1, 2, 3]. For unsteady flows uniform in width and flow directions, modelers used one-dimensional vertical simulation with finite differences schemes [4]. For unsteady flows uniform in the width direction, two dimensional simulations are used along with a finite differences method [5], finite volumes method [6]. For non-uniform and non-steady flow conditions, full 3D models are developed using finite differences [7], using finite volumes CFD softwares (sediment simulation in intakes with multiblock option SSIM), finite elements (Fluent).

In this context, we are interested into establishing an innovative model to predict SSC in open channel flows. In a previous work [8] we studied the main parameters influencing the suspended sediment concentration distribution and we compared the simplest method in order to combine them later on into new hybrid ones that have some of the properties of the parent models and their own originality. The model suggested in the present study is obtained by combining the properties of the sediment diffusivity coefficients of the parabolic constant model [9] and of the model presented by Itakura and Kishi [2].

2 Transport equation

We consider the vertical distribution of suspended sediment concentration in a steady and uniform flow and under equilibrium conditions. For small concentrations, the convection diffusion equation giving the transport of suspended sediment concentration in the vertical direction can be obtained by equating the downward fluxes caused by gravity and the upward fluxes caused by turbulence :

$$\varepsilon_s \, {dc}/{dz} + w_s c = 0 \tag{1}$$

2.1 Sediment diffusivity coefficient

In order to keep the non zero property of the sediment diffusivity at the free surface of an open channel flow, we use the following sediment diffusivity model :

$$\varepsilon_s = \begin{cases} 0.25\,\kappa u_* h \left(1+\varphi_2\, z/h\right)^{-1} & for \;\; z/h > 0.5 \quad (2a) \\ \kappa u_* h\, z/h \left(1-z/h\right)\left(1+\varphi_2\, z/h\right)^{-1} & for \;\; z/h \le 0.5 \quad (2b) \end{cases}$$

Where $\varphi_2 = \alpha\, h/L$, L being the Monin Obukhov length scale given by:

$$\frac{1}{L} = \frac{\kappa\, g\,(s-1)\, w_s\, \bar{c}}{u_*^{\,3}} \tag{3}$$

$\alpha \cong 7$ is the Monin Obukhov coefficient [2], $\bar{c}$ is the given mean concentration.

The graph of this function is shown in figure 1 along with the profiles of Rouse [1], Kerssens *et al*. [9] and Itakura and Kishi [2].

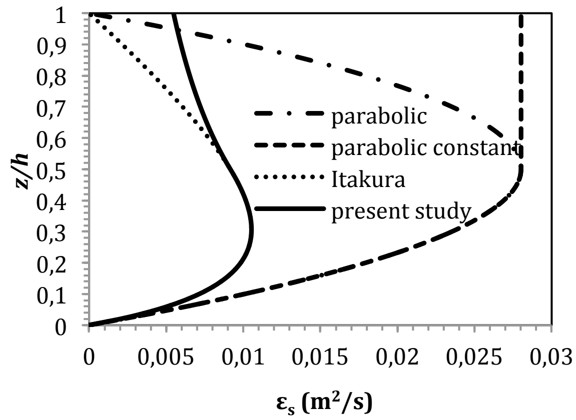

FIG. 1 – Sediment diffusivity coefficients: parabolic [1], parabolic constant [9], Itakura and Kishi [2] and present study.

2.2 Suspended sediment concentration profile

The vertical suspended sediment distribution is given by the Rouse profile (Eq. 4):

$$\frac{c}{c_a} = \left[\frac{a\left(h-z\right)}{z\left(h-a\right)}\right]^{Z_1} \tag{4}$$

With : $Z_1 = \dfrac{w_s}{\kappa u_*}$

Kerssens *et al*. [9] introduced the parabolic constant concentration profile with momentum diffusivity equal to the sediment diffusivity:

$$\frac{c}{c_a}=\begin{cases}\left[\dfrac{a(h-z)}{z(h-a)}\right]^{Z_1} & for\ \ z/h\le 0.5 \quad (5a)\\ \left[\dfrac{a}{h-a}\right]^{Z_1}\exp\left(-4Z_1\left(\dfrac{z}{h}-\dfrac{1}{2}\right)\right) & for\ \ z/h>0.5 \quad (5b)\end{cases}$$

Itakura and Kishi [2] introduced a correction factor on the parabolic diffusion coefficient based on the Monin Obukhov length scale to better account for open channel flows with suspended sediment. They considered the sediment diffusivity and the momentum diffusivity to be equal:

$$\frac{c}{c_a}=\left[\frac{a}{z}\left(\frac{h-z}{h-a}\right)^{(1+\varphi_2)}\right]^{Z_1} \tag{6}$$

In the present study we apply thee modifications of Itakura and Kishi [9] on the parabolic constant profile of Kerssens *et al*. [9].

We develop the vertical suspended sediment concentration profiles using Eqs. (2) and we validate it by comparison with some experimental data.

The reference concentration is considered as the concentration at the lowest measurement point a. The reference concentration and the reference elevation are considered for each experiment separately.

We consider two domains of study: $z/h\le 0.5$ and $z/h>0.5$

2.2.1 Sediment concentration profile in the region $z/h \le 0.5$

In this region the sediment diffusivity is the one of Itakura and Kishi

$$\varepsilon_s=\kappa u_* z(1-z/h)(1+\varphi_2\, z/h)^{-1} \tag{7}$$

and Eq. (2b).

After separating variables, the convection diffusion equation (Eq. (1)) reduces to:

$$\frac{dc}{c}=\frac{-w_s(1+\varphi_2\, z/h)}{\kappa u_* z(1-z/h)}dz=-Z_1\frac{(1+\varphi_2\, z/h)}{z(1-z/h)}dz \tag{7}$$

By using partial fractions:

$$\frac{(1+\varphi_2\, z/h)}{z(1-z/h)}=\frac{1}{z}+\frac{1}{h}\frac{1+\varphi_2}{(1-z/h)}$$

Then:

$$\frac{dc}{c}=-Z_1\left(\frac{1}{z}+\frac{1}{h}\frac{1+\varphi_2}{(1-z/h)}\right)dz$$

Integrating between the reference elevation "*a*" and "*z*" an arbitrarily chosen position in the lower half of the flow:

$$\ln\frac{c}{c_a}=-Z_1\ln\left[\frac{z}{a}\left(\frac{h-a}{h-z}\right)^{(1+\varphi_2)}\right] \tag{8}$$

Or also:

$$\frac{c}{c_a}=\left[\frac{a}{z}\left(\frac{h-z}{h-a}\right)^{(1+\varphi_2)}\right]^{Z_1} \tag{9}$$

2.2.2 Sediment concentration profile in the region *z/h* > 0.5

In this region the sediment diffusivity is given by Eq. (2a).

Implementing this equation in Eq. (1), the following differential equation is obtained:

$$\frac{dc}{c}=\frac{-4w_s\left(1+\varphi_2\, z/h\right)}{\kappa u_* h}dz=-4\frac{Z_1}{h}\left(1+\varphi_2\, z/h\right)dz \tag{10}$$

For all height $h/2 < z < h$, we get the concentration by integrating:

$$\ln\frac{c}{c_{h/2}}=-4\frac{Z_1}{h}\left(z-h/2\right)\left[1+\frac{\varphi_2}{2h}\left(z+h/2\right)\right] \tag{11}$$

Or also:

$$\frac{c}{c_{h/2}}=\exp\left\{-4Z_1\left(z/h-1/2\right)\left[1+\frac{\varphi_2}{2}\left(z/h+1/2\right)\right]\right\} \tag{12}$$

$c_{h/2}$ is the sediment concentration at height $h/2$. It can be calculated from Eq. (9):

$$c_{h/2}=c_a\left[\frac{2a}{h}\left(\frac{h/2}{h-a}\right)^{(1+\varphi_2)}\right]^{Z_1}=c_a\left[\frac{a\left(h/2\right)^{\varphi_2}}{\left(h-a\right)^{(1+\varphi_2)}}\right]^{Z_1} \tag{13}$$

Then the sediment concentration for $z/h > 0.5$ is given by:

$$\frac{c}{c_a}=\left[\frac{a\left(h/2\right)^{\varphi_2}}{\left(h-a\right)^{(1+\varphi_2)}}\right]^{Z_1}\exp\left\{-4Z_1\left(z/h-1/2\right)\left[1+\frac{\varphi_2}{2}\left(z/h+1/2\right)\right]\right\} \tag{14}$$

3 Validation and discussion

The obtained concentration model was compared to the data of Itakura and Kishi [2] and Vanoni and Brooks [10]. It shows good agreement with these experimental data.

Fig. 2 and Fig. 3 show the parabolic profile [1], the parabolic constant profile [9], Itakura and Kishi [2] profile and the profile of the present study given by Eq. (9) and Eq. (14) along with the above mentioned experimental data. We can remark in all these figures that the Itakura and Kishi and present study models are in better agreement with data then the parabolic and parabolic constant profiles.

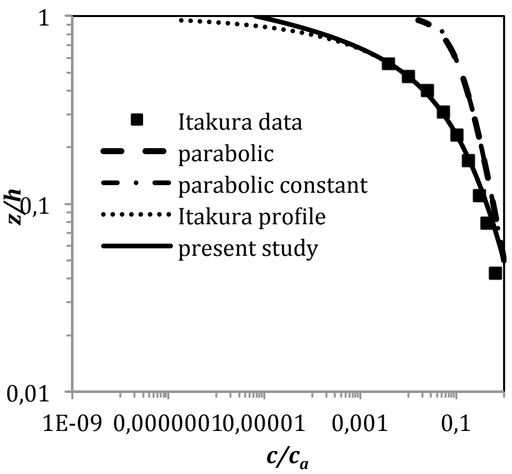

FIG. 2. The concentration profiles compared to the data of Itakura and Kishi (1980)

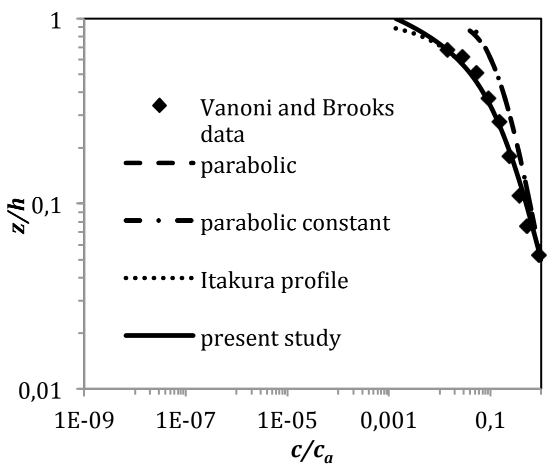

FIG. 3. The concentration profiles compared to the data of Vanoni and Brooks (1957).

Both Itakura and Kishi profile and ours show the same level of agreement with Itakura and Kishi's measurements.

These measurements are concentrated in the lower half of the water height were the profile of the present study coincides with the one of Itakura and Kishi.

4 Conclusions

Knowing the difficulty of measuring the distribution of suspended sediment concentration over the water depth for uniform open channel flows and the non existence of a universal numerical profile, we suggest a method for predicting concentrations when the mean concentration is known.

The model proposed is obtained by applying the Itakura and Kishi correction method to the parabolic constant profile. The profile is based on a sediment diffusivity coefficient that increases in the lower half of the height and decreases in the upper half without reaching zero at the free surface which is physically more realistic. It represents smaller values than the parabolic constant one at the free surface. This profile is validated with a range of experimental data.

References

[1] Rouse H., Modern conception of the mechanics of fluid turbulence. Trans. ASCE, 102, 463-543, 1937.

[2] Itakura T., Kishi T., Open channel flow with suspended sediments. J. Hydr. Div. ASCE, 106 (HY8), 1325 – 1343, 1980.

[3] Umeyama M., Vertical distribution of suspended sediment in uniform open – channel flow. J. Hydr. Eng. ASCE, 118(6), 936 – 941, 1992.

[4] Yu D., TIAN C., Vertical distribution of suspended sediment at the Yangtze river estuary. In: Proceedings of the International conference on estuaries and coasts, Hangzhou, China, 214-220, 2003.

[5] Zhang J., Liu, H., A vertical 2-D numerical simulation of suspended sediment transport. J. of hydrodynamics, 19(2), 217-224, 2007.

[6] Peña E., Fe J., Sánchez-Tembleque F., Puertas J., A 2D numerical model using finite volume method for sediment transport in rivers. Proceedings of International Conference on Fluvial Hydraulics, Louvainla-Neuve, Belgium, 693–698, 2002.

[7] Lin B., Namin M. M., Modelling suspended sediment transport using an integrated numerical and ANNs model. Journal of Hydr. Research, 43(3), 302-310, 2005.

[8] Sabat M., Terfous A., Ghenaim A., Poulet J. B., The wicked problem of suspended sediment profile: a choice criterion. Hydrocomplexity: New Tools for Solving Wicked Water Problem IAHS, 338, 251- 252, 2010.
[9] Kerssens P., Prins A., Van Rijn L. C., Model for suspended sediment transport. J. Hydr., ASCE, 105(HY5), 461 – 476, 1979.
[10] Vanoni V. A., Brooks N. H., Laboratory studies of the roughness and suspended load of alluvial streams. Ed. U.S. Army Engineer Division, Missouri River Division Collection, 1957.

Notations

a = water height corresponding to the reference concentration (m)
c = volumetric concentration ($0 \leq c \leq 1$)
$\overline{c}$ = mean concentration (-)
c_a = reference concentration (-)
g = gravity acceleration (m^2/s)
h = water height(m)
L = Monin Obukhov length scale (m)
s = sediments relative density (-)
u_* = bed shear velocity (m/s)
w_s = particle fall velocity (m/s)
z = vertical coordinate (m)
Z_1 = Rouse suspension number (-)
α =Monin Obukhov coefficient (-)
ε_m = momentum diffusivity coefficient (m^2/s)
ε_s = sediment diffusivity coefficient (m^2/s)
φ_2 = coefficient (-)
κ =Von Karman constant = 0.4 (-)