

Energy release rate evaluation from marks tracking method and J-integral

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Abstract:

In this paper, an experimental analysis of energy release rate via the J-integral and the optical marks tracking methods is presented. In order to evaluate the invariant J, the experimental displacement fields were measured by the marks tracking method, while the stress and strain fields were calculated using the finite element method. The finite element model is generated from the experimental measurements in terms of sample geometry, loading and boundaries conditions. Moreover the experimental displacement fields were implemented in finite element model. Afterwards, the stress and strain fields will be calculated via a constitutive law. The experimental test was carried out using a sample made in Douglas fir loaded in opening mode.

Key words: Wood, Fracture Mechanics, Marks tracking, Finite Elements, Energy release rate, J-Integral

1 Introduction

To keep the structure timbers integrity, it's important to develop the appropriate tools and models in order to evaluate the failure risks and the damage level [1], [2], [3], [4]. It's known that the crack occurrence, in timber elements, can modify the structure behavior and lead to structure failure. Among the fracture parameters allow evaluating the fracture process the energy release rate, is one of them. The advantage of this method is the possibility to use the mechanical fields distant from the crack tip. The energetic approaches allowing characterizing the global behavior of media [5], [6], [7], [8]. Moreover the mechanical fields are not affected by the crack tip singularity.

In this context, we propose to evaluate the energy release rate from J-integral approach by coupling the experimental measurements by marks tracking method with a numerical analysis by finite elements method. While, the marks tracking method is used to measure the displacement field evolution in the experimental test. In parallel, the stress and strain fields are calculated using a finite element model generated from the experimental measurements by mark tracking method. The experimental sample is made of Douglas fir and its geometry is a mix between the single edge notch (SEN) and wedge splitting (WS) specimens.

2 J-integral

Our approach is based on J-integral. It is associated to energy release rate, or work per unit of crack area development. The theoretical concept of J-integral was developed by Cherepanov [9], [10] and Rice [11], [12]. Taking into account an arbitrary counter-clockwise path around the crack tip, as illustrated in figure 1, the J integral can be expressed in the following form:

$$J = \int_r \left(W \cdot n_l - T_i \cdot \frac{\partial u_i}{\partial x_l} \right) \cdot ds \quad (1)$$

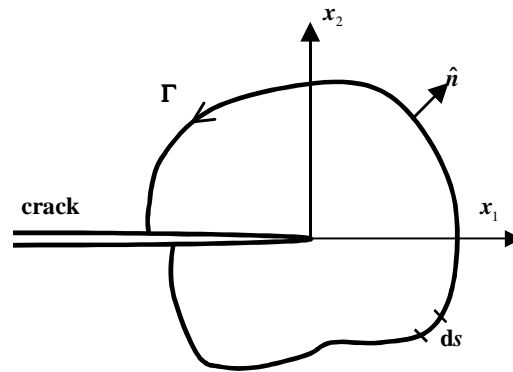


FIG. 1 – Domain of integration

W is the elastic strain energy density, T_i is the traction along the contour Γ with being \vec{n} the unit normal of the contour path.

$$W = \frac{1}{2} \cdot \sigma_{ij} \cdot \varepsilon_{ij} \quad (2)$$

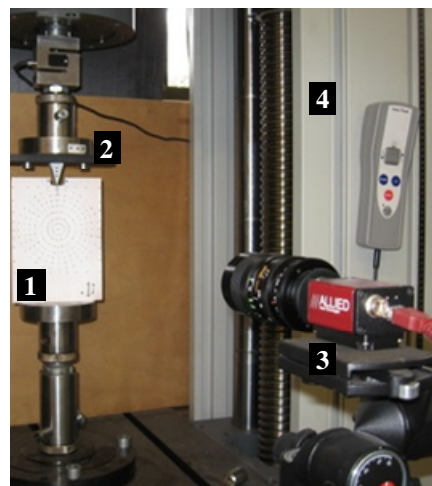
σ_{ij} and ε_{ij} are the stress and strain tensors, respectively. The traction vector is given by:

$$T_i = \sigma_{ij} \cdot n_j \quad (3)$$

By analyzing the analytical forms (1), (2) and (3), we observe that the J-integral is expressed in terms of displacement, strain and stress mechanical fields. In this context our propose is to evaluate the displacement fields from the experimental measurements by marks tracking method and afterwards to evaluate the strain and stress fields from a finite element analysis based on the experimental data.

3 Experimental test

As shown in figure 2, the experimental setup is composed by a Zwick testing machine and a loading system allowing obtaining a symmetric opening mode. The loading system comprises a metallic pyramid, interdependent with the crossbar of the testing machine, and two metallic bearings. The test is realized under displacement control (0.01 mm/s).



- 1 – Cracked sample with black marks
- 2 – Loading system
- 3 – CCD camera
- 4 – Electromechanical press

FIG. 2 – Experimental setup

The experimental test was performed using a sample made in Douglas fir. The geometry and sample

dimensions are illustrated in figure 4. The crack length is 80mm and this one is parallel with the grains orientation (L – direction). The sample moisture content is fixed at 11% into climatic room.

During the test, the mechanical behavior of the sample is recorded using two measurement devices. The global behavior is recorded using a LVDT transducer and a load cell. In parallel, an optical system, which comprises a CCD camera, is added to measure the sample deformation by marks tracking method. This method allows measuring the displacement and the strain fields around the crack tip. And then, use these experimental data to evaluate the fracture parameters through the energetically approaches by the integral invariants.

As shown in figure 3, the principle of the marks tracking method is based on the comparison between two images acquired before and after sample deformation [13], [14], [15]. The algorithm of method detects the local displacement of the marks barycenter in x_1 and x_2 directions.

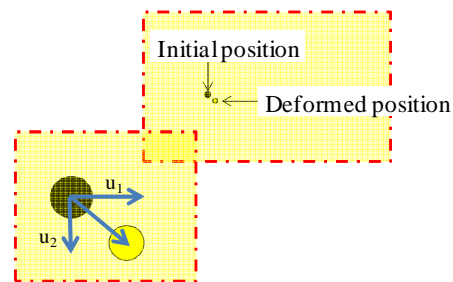


FIG. 3 – Marks tracking method principle

For the experimental measurements, several black marks are positioned manually on the sample surface and loading setup. As can be seen in figure 4, the marks are positioned concentrically around the crack tip, in order to define some integration domains Γ defined in figure 1.

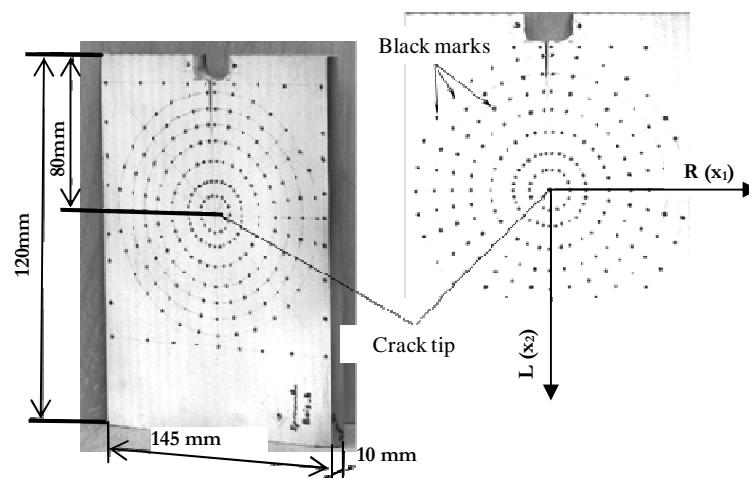


FIG. 4 – Sample made in Douglas fir with marks tracking method

4 Results and discussions

The load-displacement evolution recorded during the test is plotted in figure 5. The evolution puts in evidence the linear behavior of sample and the crack length invariance during the test.

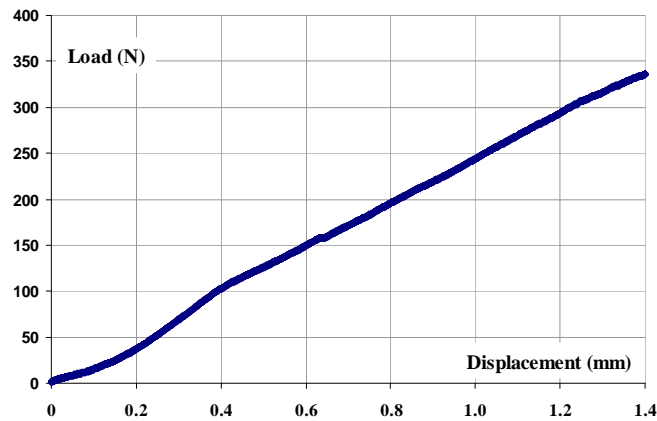
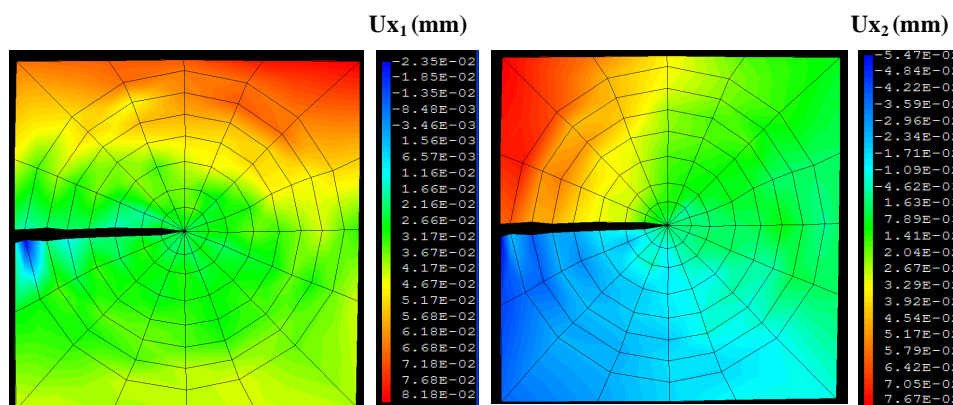


FIG. 5 – Load-displacement curve

After fracture opening test, the displacement field, in the crack tip vicinity, was calculated by marks tracking method [15]. One example of displacement field obtained by analyzing the evolution of the black marks position is illustrated in figure 6. These cartographies correspond to a loading level of $F = 163.4\text{N}$.

FIG. 6 – Displacement field cartography corresponding at $F=163.4\text{N}$

As mentioned above, the energy release rate evaluation, via the J-integral, is expressed in terms of displacement, strain and stress mechanical fields. While once the displacement field is evaluated, we propose to evaluate the strain and stress fields from a finite element analysis [16], [17], [18], [19].

In the first time, the finite element mesh is generated from the experimental data. In this case, the mesh nodes are created using the barycenters of black marks applied on the sample surface (see figure 7).

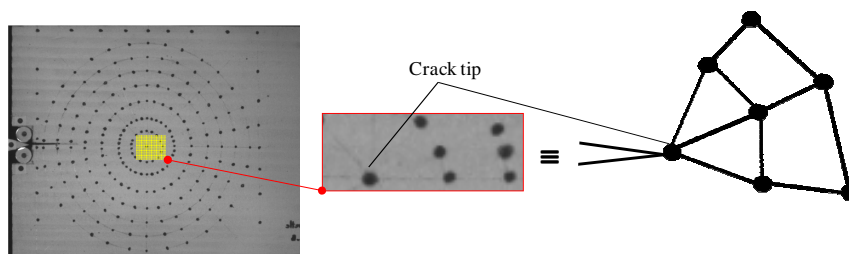


FIG. 7 – Principle of finite element mesh construction from the experimental data

In the second time, in order to define the loading and the boundaries conditions, the experimental displacements measured by mark tracking method are implemented in the finite element analysis. In this case, the displacement of each mark is assigned to the corresponding node of finite element mesh.

The elastic properties of constitutive law implemented in finite element mesh are those given by Guitard [20] ($E_R = 1370\text{ MPa}$, $E_L = 15600\text{ MPa}$, $G_{LR} = 1020\text{ MPa}$, $\nu_{LR} = 0.39$). Finally, the strain and stress fields were calculated from the numerical model. By replacing the mechanical fields identified by experimental and numerical approaches into equation (1) the energy release rate can be estimated on arbitrary counter-

clockwise path defined around the crack tip. In order to verify the path independence of J-integral, several integration domains are chosen, Fig. 8.

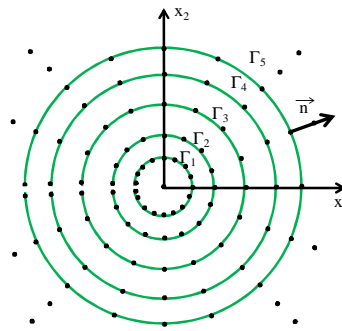


FIG. 8 – Integration domain of J

The J-integral values obtained from equation (1) and the mechanical fields identified by experimental and numerical approaches are resumed in table 1. These values correspond to different loading level and to different integration domain sizes.

Applied Force (N)	J (J/m ²)						
	Γ ₁	Γ ₂	Γ ₃	Γ ₄	Γ ₅	Average	CV
126	4.69	4.76	4.64	4.68	4.67	4.68	0.009
163	5.41	5.72	5.70	5.63	5.71	5.63	0.023
204	11.40	11.39	11.55	11.48	11.43	11.45	0.006
285	17.77	16.62	17.03	17.11	17.42	17.19	0.025
324	22.14	23.21	23.23	23.08	23.20	22.97	0.018
338	25.21	26.24	25.43	25.63	25.34	25.57	0.016

TABLE 1 – J-integral values

The results of table 1 show a low variation of the energy release in terms of integration domain size. This variation can be associated to experimental noise observed on the experimental data. In this case, a correction of the displacement fields could be considered in order to increase the accuracy of the experimental fields.

5 Conclusion

In this study, an experimental approach is proposed in order to characterize the fracture parameters in wood specimens via the J-integral. The marks tracking method was employed in order to measure the displacements fields on the specimen surface. Using the experimental displacements, the strain and stress fields were calculated via a constitutive law. For this, the experimental data were implemented in a finite element code in order to generate a finite element mesh. In this case, the strain and stress fields are calculated by using the experimental data and by imposing real boundary conditions. The implementation of experimental data in a finite elements code allows obtaining a realistic discretization of sample and the loading and experimental boundaries conditions. This possibility can be explored to generate the finite element mesh corresponding to the specimens with complex geometry.

Finally, using the mechanical fields, the J-integral was calculated for several loading steps and for different integration domains defined around the crack tip. The J-integral evolution versus the crown size shows the energy release rate invariance. This approach could be easily generalized for mixed mode configurations by using the formulation of the M-integral.

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