Optimization of geometrical parameters of covers and hoops of metal packaging boxes

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Abstract :

Shape optimization has became one of the most important tasks in the process of developing a new product. This work presents an approach for optimization of geometrical parameters of packaging metal boxes. The objective of this study is to find the optimal combination of geometric parameters of contact between cover and hoop to prevent the opening of the cover subjected to internal pressure in the box.

Mots clefs : Metal packaging ; Sequential Quadratic Programming (SQP) ; Response Surface Methodology (RSM)

1 Introduction

Numerical optimization methods have been developed mainly after the Second World War, in parallel with the improvement of computers, and have never ceased to grow. In nonlinear optimization, we can distinguish several waves of penalization methods, augmented Lagrangian method (1958), quasi-Newton methods (1959), Newtonian methods or Sequential quadratic programming (SQP) (1976), interior-point algorithms (1984),...etc . Each method does not erase the previous one but can provide better answers. [1].
Bahloul *et al.* [2], have presented a study to optimize the die radius and the gap between the punch and die in the bending process of thin sheets. 3D numerical modeling with ABAQUS code, assuming an elasto-plastic behavior of the material is chosen to predict the stress and the bending force. Finally, a Response Surface Methodology (RSM) based on design of experiments is used to optimize the forming process with regard to stresses and the maximum punch load during the bending operation.
In recent study developed by Chihara and Yamazaki [3], to evaluate the usability of consumers when they drink a beverage directly from the opening of aluminum bottles, which have recently been launched on the Japanese market. A RSM based on radial basis function networks was used to minimize the objective function, which is consider as a difference between the actual volume of the liquid in the mouth and the volume expected ideal, to make drinking easier. The variables are : the opening diameter, the volume remaining in the bottle and the height of consumers.

2 Experimental device and numerical simulations

2.1 Experimentations

These experiments are done by a test machine (Fig. 1). This device allows leak detection, usually due to defects in welding or crimping. The use of this later is limited to the observation of the mechanical behavior of metal box (hoop, body & bottom) subjected to internal pressure. The principle of operation of this device is to maintain an open box between a support and an actuator, inject inside a compressed air at a pressure between 0.1 and 0.6MPa, and then immerse in a basin containing water to observe the emergence of air bubbles indicating the existence of leaks.[4]. A swelling of the bottom is generally noted. (Fig. 2). We notice also, folds on the bottom of box which generate a thickening or a thinning of sheet.

Figure 1 – leakage Test device

Figure 2 – Inside & outside of one box tested

2.2 Simulations

The simulations are performed on a box of $83mm$ of diameter composed of three parts : Hoop, body and bottom. The material is a thin plate shell (TH415) of $0.2mm$ thickness. (Table. 1) display the mechanical properties of the studied material according to Pepelnjak and Barisic [5] & Struyven [6].

Table 1 – Tin plate (TH415) properties

Designation	Symbol	Value	Unit
Density	ρ	7800	Kg/m^3
Young's Modulus	E	210000	MPa
Poisson's ratio	ν	$0,3$	–
Friction coefficient	μ	$0,0426$	–
Constant of Hollomon	K	$667,90$	MPa
Constant of Hollomon	n	$0,1626$	–
yield strength	R_e	415	MPa

For symmetry reasons, only 1/4 of the box is modeled by S3R shell elements in ABAQUS code. The box is supposed to flush on the bottom and both ends of the hoop. The Crimping is supposed to be a contact with friction coefficient of 0.8. A maximal of internal pressure of $0.6MPa$ is applied. The (Fig. 3) show the distribution of the resultant displacement on the bottom of the deformed box.

The following graph (Fig. 4) present the variation of the shell thickness in the radial direction from the center.

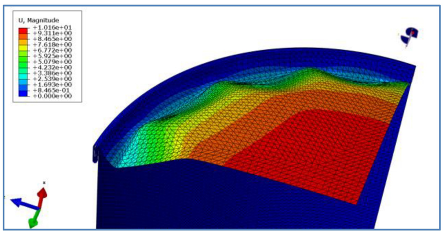

Figure 3 – Resulting displacements of the bottom

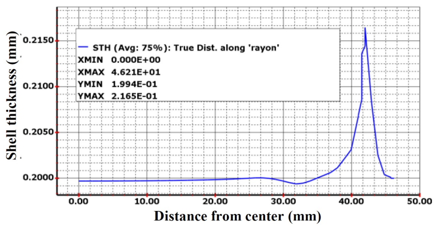

Figure 4 – Shell thickness variation of the bottom of the box

At the end of the experimental tests, we have shown that the deformation of the body of the boxes is almost negligible but the deformation of the bottom is significant without compromising the seal of the crimp. we can approximate the general phenomenon to a couple made only by Hoop & Cover subjected to internal pressure.

3 Optimization strategy adopted

We opted for an approximation method based on design of experiments coupled with the finite elements code ABAQUS. It is divided into two steps :

A first step will allow us to express the objective function which is the contact pressure between the lid and hoop by a quadratic polynomial formula with the Box-Behnken method.

A second step is to maximize under constraints, the contact pressure at the last increment before final opening of the cover, this maximization is solved by the SQP algorithm.

3.1 SQP Optimization Algorithm

The basic principle of this method is to transform the original optimization problem into a sequence of equivalent optimization problems [7].

3.2 Box-Behnken design

There are many experimental designs to suit all cases encountered by an experimenters. [8]. In our case, we will implement Box-Behnken design for response surfaces, Box and Wilson (1951) have introduced the method of Response Surface Methodology (RSM)) based on the Design Of Experiment (DOE) of the second degree. They are used to continuous variables, we have (eqn. 1) for two factors :

$$y = a_0 + a_1x_1 + a_2x_2 + a_{12}x_1x_2 + a_{11}(x_1)^2 + a_{22}(x_2)^2 + e. \qquad (1)$$

4 Implementation of the optimization strategy

Three forms of hoops and covers are used, the diameter 56, 83 and 108.5mm. The modeling of system are shown in (Fig. 5).

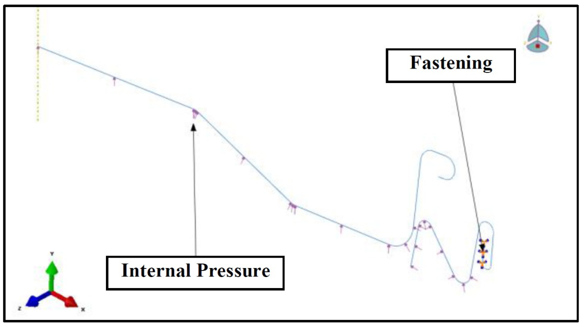

FIGURE 5 – Cover and hoop of box diameter 83mm

The resolution is performed in two steps : Cover closing with general static mode. Application of internal pressure with dynamic implicit mode for the opening phase. The mesh is made with axi-symmetric SAX1 ABAQUS type elements. A pressure of $0.1MPa$ was applied in the second step on all internal walls.

4.1 Definition of the objective function

The objective function is the contact pressure P_c between cover and hoop in last increment before final opening of the cover. The objective is to maximize this pressure to prevent the opening of cover.

The mathematical expression of the optimization problem can be written as follows :

$$Max\ P_c(x_i) \quad = \quad Min\ (-P_c(r_1, r_2, h)). \qquad (2)$$

Under constraints :

$$\begin{cases} g_i(r_1, r_2, h) \leq 0. \\ hj(r_1, r_2, h) = 0. \\ r_{1min} \leq r_1 \leq r_{1max}. \\ r_{2min} \leq r_2 \leq r_{2max}. \\ h_{min} \leq h \leq h_{max}. \end{cases} \tag{3}$$

Where (r_1, r_2, h) are the design variables retained (Fig. 6).

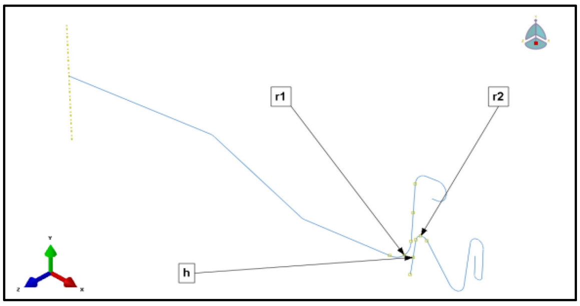

FIGURE 6 – Design variables for box diameter 83mm

4.2 Optimization constraints

Geometric data (Fig. 7) for each pair Cover/Hoop are given below (Table. 2).

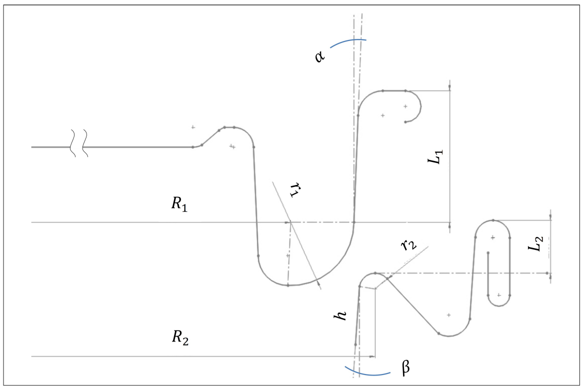

FIGURE 7 – Geometrical parameters Cover/Hoop for box diameter 108.5mm

TABLE 2 – Geometrical parameters of Cover/Hoop

$\O(mm)$	$R_1(mm)$	$R_2(mm)$	$L_1(mm)$	$L_2(mm)$	$\alpha(^o)$	$\beta(^o)$
56	21.94	24.17	6.20	1.55	2.59	3.92
83	33.89	36.92	5.70	1.60	2.49	7.94
108, 5	45.05	49.19	6.61	1.67	2.12	4.01

For industrial considerations, functional and even ergonomic designs ; the variables (r_1, r_2, h) are bounded, we give the lower and upper limits in (Table. 3).

Other optimization constraints are needed, as functions of inequalities ; The first class constraints (eqn. 4) & (eqn. 5) is to ensure contact between the Cover and Hoop.

$$R_1 + r_1.cos(\alpha) \quad < \quad R_2 - r_2.cos(\beta). \tag{4}$$

Table 3 – Limitation of design variables

Ø(mm)	Variables	Lower limit	average	Upper limit
	r_1	1,00	1,50	2,00
56	r_2	0,32	0,52	0,72
	h	1,92	2,92	3,92
	r_1	1,00	2,00	3,00
83	r_2	0,40	0,60	0,80
	h	2,11	3,11	4,11
	r_1	2,20	3,20	4,20
108,5	r_2	0,50	0,80	0,90
	h	1,86	2,86	3,86

$$R_1 + r_1.cos(\alpha) \quad \geq \quad R_2 - r_2.cos(\beta) - h.tg(\beta). \tag{5}$$

The second category of constraints is to ensure sufficient penetration of the cover when closing box (eqn. 6).

$$r_1 + L_1 - L_2 - h.cos(\beta) - r_2.(sin(\beta) - 1) \geq r_1.(1 - sin(\alpha)). \tag{6}$$

4.3 Application of Box-Behnken design

In this step, we apply Box-Behnken method to approximate the function P_c. Fifteen different configurations for each type of box are introduced into ABAQUS to calculate for each configuration, using the software MINITAB, an interpolation following a quadratic nonlinear model (eqn. 7) is applied for each type of box.

$$P_c = \beta_0 + \beta_1 r_1 + \beta_2 r_2 + \beta_3 h + \beta_{11} r_1^2 + \beta_{22} r_2^2 + \beta_{33} h^2 + \beta_{12} r_1 r_2 + +\beta_{13} r_1 h + \beta_{23} r_2 h. \tag{7}$$

For example for the box diameter 56mm, the function $P_c = f(r_1, r_2, h)$ represented by (eqn. 8) :

$$\begin{aligned} P_{c1} = -11.9633 - 1.8621 r_1 + 12.3580 r_2 + 6.8758 h + 5.0555 r_1^2 + 12.1766 r_2^2 + \\ 0.1150 h^2 - 5.3888 r_1 r_2 - 3.0658 r_1 h - 5.8260 r_2 h. \end{aligned} \tag{8}$$

To graphically visualize in three dimensions, these results, we fix the value of h for example and we plot graphs $P_c = f(r_1, r_2)$ for box 56mm for example on the figure (Fig. 8).

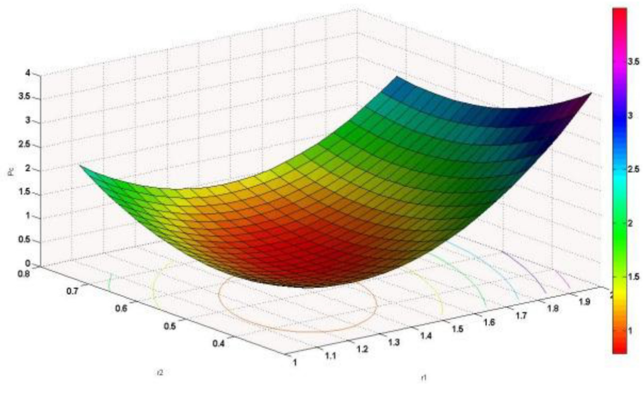

Figure 8 – $P_c = f(r_1, r_2)$ for box diameter 56mm

Graphically, we note that for the three types of boxes, the optimum is sought when the radius of the hoop, remains close to its minimum value.

4.4 Application of SQP optimization algorithm

By applying the SQP optimization algorithm, we obtain the following results respectively for the box diameter 56, 83 and 108.5mm :

$$\text{Ø56mm}\left\{\begin{array}{lcl} (r_{1.opt}, r_{2.opt}, h_{opt}) & = & (1.913, 0.320, 1.920) \\ P_{c.opt} & = & 3.66224 Mpa \\ lightweighting & = & 3.33\% \end{array}\right.$$

$$\text{Ø83mm}\left\{\begin{array}{lcl} (r_{1.opt}, r_{2.opt}, h_{opt}) & = & (2.360, 0.400, 2.110) \\ P_{c.opt} & = & 6.81348 Mpa \\ lightweighting & = & 3.02\% \end{array}\right.$$

$$\text{Ø108.5mm}\left\{\begin{array}{lcl} (r_{1.opt}, r_{2.opt}, h_{opt}) & = & (3.240, 0.729, 1.860) \\ P_{c.opt} & = & 3.03042 Mpa \\ lightweighting & = & 1.62\% \end{array}\right.$$

For the three types of boxes, the maximum contact pressure was always found when the length of the lip of the hoop is at the minimum. This can be explained by the high stiffness of the shorter hoop.

5 Conclusion

The contribution of this paper is to find the optimal values of geometric variables of metal packaging boxes. The developed approach consists in the modeling of the system (Cover/Hoop) by the finite elements code ABAQUS and the Box-Behnken design. The objective function P_c relying different parameters such as (r_1, r_2, h) is maximized using SQP algorithm to find optimal values of the geometric dimensions (r_1, r_2, h). This ensure a better quality of service of the product and optimum closing.

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