Robust Fault Tolerant Tracking Controller Design for a VTOL aircraft

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Abstract- **This paper deals with the fault tolerant control (FTC) design for a Vertical Takeoff and Landing (VTOL) aircraft subject to external disturbances and actuator faults. The aim is to synthesize a fault tolerant controller ensuring trajectory tracking for the nonlinear uncertain system represented by Takagi-Sugeno (T-S) model. In order to design the FTC law, a proportional integral observer (PIO) is adopted which estimate both of the faults and the faulty system states. Based on Lyapunov theory and $\mathcal{L}_2$ optimization, the trajectory tracking performance and the stability of the closed loop system are analyzed. Sufficient conditions are obtained in terms of linear matrix inequalities (LMI). Simulation results show that the proposed controller is robust with respect to uncertainties on the mechanical parameters that characterize the model and secures global convergence.**

Keywords: **TS fuzzy systems, fault and state estimation, PI, $\mathcal{L}_2$ norm, VTOL aircraft, LMI.**

1. Introduction

In the last three decades, the need for increased flight safety and aircraft reliability has been and will continue to be an important issue in commercial aviation industry. All pilots undergo widespread training to help them to be able to react to unexpected difficulties that may happen during a flight in uncertain conditions. Furthermore, advanced fault-tolerant control (FTC) systems are designed to help pilots overcome abnormal situations that previously might have resulted in catastrophic events.

Aircrafts today handle fault detection and isolation via redundant actuators and sensors. Voting schemes based on the health of independent channels are used to detect component failures. Command and control often have triplex or quad redundancy of critical flight control hardware including actuators, sensors and the flight control computer ensure a fault tolerant architecture. The importance of FTC has helped to simulate a growing body of research work in the area. A recent paper by Zhang and Jiang [1] provides a classification and bibliographical review of FTC in general, especially for the so-called active FTC [2]. In terms of flight control applications, the survey paper of [3] describes the latest development in this subarea. The paper [4], represents some of the most important recent research in the field of flight fault tolerant control using sliding-mode techniques. In [5], the problem of the FT control of aircraft in the presence of both unknown input disturbance and sensor failure is presented. The sensor faults are detected by a full-order unknown input observer (UIO), and then the faults are isolated by a bank of UIOs in the framework of the generalized observer scheme.

FTC is a control technique that allows the ability to conserve overall system stability and satisfactory performance in the occurrence of component failures [6-8]. FTC problem for linear systems have been widely studied [7], [9] and have been extended to the nonlinear and descriptor systems [10-12]. Regrettably, the design of FTC for nonlinear systems is far more complicated. Fortunately, as shown in [13], Takagi-Sugeno (T-S) fuzzy modeling concepts can be used to overcome this defect for the nonlinear plant systems [35-36]. Consequently fault tolerant controls, for several kind of T-S fuzzy model have been strongly investigated and a lot of works, involving various specifications, are now available. Among this literature we find FTC for uncertain and disturbed models [14-15], time delay models with and without uncertainties [16-17], uncertain descriptor delay models [18-19].

Despite numerous works presented, a few authors have dealt with the tracking problem for uncertain T-S and faulty models. For example, in [20] an adaptive fault tolerant tracking control scheme is developed based on the online estimation of actuator faults. In [21] a fault tolerant control law is designed for T-S models with unmeasurable premise variable using a proportional integral observer. The aims were to compensate the actuator faults and allowing the system states to track a reference corresponding to a fault free situation. The objective of this study is to exploit the effectiveness of the FTC law for the trajectory tracking problem of a VTOL aircraft system such that the closed loop fuzzy uncertain system can maintain stability and performances for the actuator fault case. The main contribution of the paper is the proposition of a LMI formulation to derive the proposed FTC law for an aircraft system with respect to uncertainties on the mechanical parameters that characterize the model.

The paper is organized as follows: in the next section, an uncertain T-S fuzzy representation for the VTOL aircraft system is obtained from nonlinear model. Hence, the entire VTOL system can be structured as several interconnected subsystems. The problem of fault tolerant controller design is formulated in section 3. The reference model, observer and T-S fuzzy uncertain faulty models are then presented. An active FTC approach is considered where the stability conditions for the whole closed-loop system derived in LMI formulation are developed. Finally, simulation results, showing the tracking performance of the VTOL aircraft model are given in the last section.

2. Plant Model and T-S Modeling

2.1. VTOL aircraft model

The purpose of this paper is the design of fault tolerant control law for a VTOL aircraft model. As in [22] a simplified model describing the motion of this aircraft in the vertical lateral plane is the following one. Let x, y and θ denote, respectively, the horizontal and vertical position of the center of mass C and the roll angle of the aircraft with respect to the horizon, as in fig.1. The control inputs are the thrust T directed out the bottom of the aircraft and the rolling moment produced by a couple equal forces F acting at the wingtips.

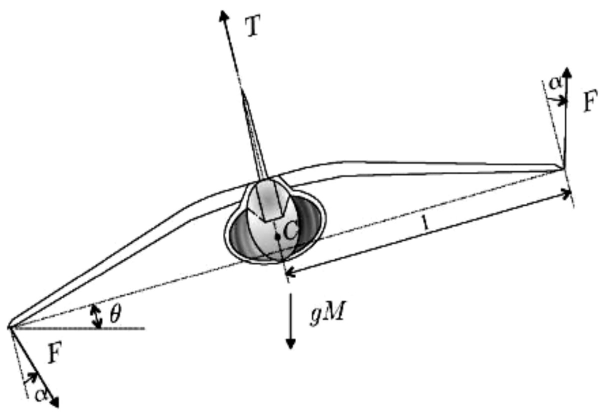

Fig.1. Forces acting on the aircraft.

Their direction is not perpendicular to the horizontal body axis, but titled by some fixed angle $\boldsymbol{\alpha}$. if $\boldsymbol{M}$ denotes the mass of the aircraft, $\boldsymbol{J}$ the moment of inertia about the center of mass, $\boldsymbol{l}$ the distance between the wingtips and $\boldsymbol{g}$ the gravitational acceleration, the motion of the aircraft on the lateral-vertical plane is modeled by the equations:

$$\begin{cases} M\ddot{x} = -sin(\theta)T + 2cos(\theta)sin(\alpha)F \\ M\ddot{y} = cos(\theta)T + 2sin(\theta)sin(\alpha)F - gM \\ J\ddot{\theta} = 2lcos(\alpha)F \end{cases} \tag{1}$$

Choosing $x_1 = x$, $x_2 = \dot{x}$, $y_1 = y$, $y_2 = \dot{y}$, $\theta_1 = \theta$, $\theta_2 = \dot{\theta}$ yields the sixth dimensional system

$$
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -sin(\theta_1)\dfrac{T}{M} + cos(\theta_1)\dfrac{2sin(\alpha)}{M}F \\
\dot{y}_1 = y_2 \\
\dot{y}_2 = cos(\theta_1)\dfrac{T}{M} + sin(\theta_1)\dfrac{2sin(\alpha)}{M}F - g \\
\dot{\theta}_1 = \theta_2 \\
\dot{\theta}_2 = \dfrac{2l}{J}cos(\alpha)F
\end{cases} \tag{2}
$$

where $\theta_1$is in the range $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Equations (2) can be then concisely written as

$$
\dot{x}(t) = Ax(t) + B\big(\theta_1(t)\big)u(t) + D \tag{3}
$$

where

$$
x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ y_1(t) \\ y_2(t) \\ \theta_1(t) \\ \theta_2(t) \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ B\big(\theta_1(t)\big) = \begin{bmatrix} 0 & 0 \\ -sin\big(\theta_1(t)\big) & cos\big(\theta_1(t)\big) \\ 0 & 0 \\ cos\big(\theta_1(t)\big) & sin\big(\theta_1(t)\big) \\ 0 & 0 \\ 0 & b \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -g \\ 0 \\ 0 \end{bmatrix} \tag{4}
$$

$$
u(t) = \begin{bmatrix} \dfrac{T(t)}{M} & \dfrac{2sin(\alpha)}{M}F(t) \end{bmatrix}^T \tag{5}
$$

and

$$
b = \frac{lMcos(\alpha)}{Jsin(\alpha)} \tag{6}
$$

In order to obtain the best possible performance from this highly nonlinear system, the following sub-section gives a T-S fuzzy representation of the system (3)

2.2. Takagi-Sugeno model representation

Note that a T-S model is not unique for a given nonlinear system. Using the well-known sector nonlinearity approach [23], a T-S model structure is obtained where the nonlinear entries of the input matrix are considered as "premise variables" and denoted$\xi_j(.)(j = 1,\cdots,q)$. For q premise variables, $r = 2^q$ submodels will be obtained. The above model is constituted by two nonlinearities:

$$
\begin{aligned}
\xi_1(t) &= sin\big(\theta_1(t)\big) \\
\xi_2(t) &= cos\big(\theta_1(t)\big)
\end{aligned} \tag{7}
$$

Under the assumptions:

$$
\begin{cases}
-1 \le \xi_1(t) \le 1 \\
-1 \le \xi_2(t) \le 1
\end{cases} \tag{8}
$$

For the premise variables choice (7), $\xi(t) = [\xi_1(t) \quad \xi_2(t)]^T$ is measurable. The local weighting functions are defined by

$$
W_1^1\big(\xi_1(t)\big) = \frac{\xi_1(t) + 1}{2}, \qquad W_1^2\big(\xi_1(t)\big) = \frac{1 - \xi_1(t)}{2}
$$

$$W_2^1(\xi_2(t)) = \frac{\xi_2(t)+1}{2}, \qquad W_2^2(\xi_2(t)) = \frac{1-\xi_2(t)}{2} \tag{9}$$

Finally, the weighting functions of the derived T-S model are given by:

$$\begin{aligned}
\mu_1(\xi(t)) &= W_1^1(\xi_1(t))W_2^1(\xi_2(t)) \\
\mu_2(\xi(t)) &= W_1^2(\xi_1(t))W_2^1(\xi_2(t)) \\
\mu_3(\xi(t)) &= W_1^1(\xi_1(t))W_2^2(\xi_2(t)) \\
\mu_4(\xi(t)) &= W_1^2(\xi_1(t))W_2^2(\xi_2(t))
\end{aligned} \tag{10}$$

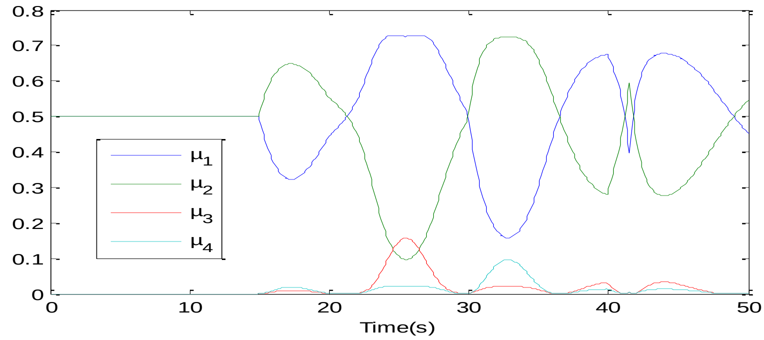

Fig.2. Membership functions evolution

The constant matrices B_i defining the 4 submodels, are determined by replaycing the premise variables ξ_j in the matrices $B(\xi(t))$ with the scalars $\xi_j^{\partial_i^j}$, $i = 1, \cdots, 2^q$ and $j = 1, \cdots, q$:

$$B_i = B\left(\xi_1^{\partial_i^1}\right) \qquad\qquad i = 1, \cdots, 4 \tag{11}$$

In definitions (11), the indexes ∂_i^j ($i = 1, \cdots, 4$ and $j = 1, \cdots, 2$) are equal to min or max and indicates which partition of the j^{th} premise variable $\left(W_j^0 \text{ or } W_j^1\right)$ is involved in the i^{th} submodel. Consequently the nonlinear model (3) can be proposed as:

$$\begin{cases} \dot{\boldsymbol{x}}(\boldsymbol{t}) = \sum_{i=1}^{4} \boldsymbol{\mu_i}(\boldsymbol{\xi}(\boldsymbol{t}))\,(\boldsymbol{Ax}(\boldsymbol{t}) + \boldsymbol{B_i u}(\boldsymbol{t}) + \boldsymbol{D}) \\ \boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{Cx}(\boldsymbol{t}) \end{cases} \tag{12}$$

where

$$B_1 = \begin{bmatrix} 0 & 0 \\ -1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & b \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & -1 \\ 0 & 0 \\ 0 & b \end{bmatrix}, B_3 = \begin{bmatrix} 0 & 0 \\ -1 & -1 \\ 0 & 0 \\ -1 & 1 \\ 0 & 0 \\ 0 & b \end{bmatrix}, B_4 = \begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 0 & 0 \\ -1 & -1 \\ 0 & 0 \\ 0 & b \end{bmatrix} \tag{13}$$

Consider the model (12), we assume that only $(x_1(t), y_1(t), \theta_1(t))$ are measurable. This gives the following matrix C:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{14}$$

2.3. Faulty uncertain T-S model

In order to point up the proposed approach additional actuator faults are used, and are injected to the T-S model (12) representing the VTOL aircraft. We assumed that at $t = 15(sec)$, due to the occurrence of fault, the actuator u_2 starts providing half of the thrust it is required to. Independently, at $t = 25(sec)$, the control action provided by the other actuator is reduced to 95% of its nominal value. We model these two faults with the signals [32]

$$f_1(t) = \begin{cases} 0 & for\ t \in [0,20] \\ 0.05T & elsewhere \end{cases}$$
$$f_2(t) = \begin{cases} 0 & for\ t \in [0,15] \\ 0.5F & elsewhere \end{cases} \quad (15)$$

The control strategy described in this paper is aimed-among other things-at offsetting the effect of major parameter uncertainties, such as those regarding the mass M of the aircraft (and hence its moment of inertia J about the center of mass). in view of this, we set in what follows:

$$M = M_0 + \Delta M, J = J_0 + \Delta J \quad (16)$$

where M_0, J_0 represent nominal values. Consequently, the structure of the T-S model (12) representing the VTOL aircraft model involved parameter uncertainties of M and J in the coefficient b of the matrix B. The variation of these parameters is 20% and 25% for the nominal values of M and J respectively. The uncertain part ΔB separated from the perfectly known part B is given by:

$$\Delta \boldsymbol{B} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.2\Delta M + 0.25\Delta J} \end{bmatrix}$$
(17)

Considering the uncertainties structure ΔB is written under the form $\Delta B = M^b F^b N^b$ with:

$$M^b = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}, F^b = \begin{bmatrix} 0.2\Delta M \\ 0.25\Delta J \end{bmatrix}, N^b = [0 \quad 1] \quad (18)$$

where $F^b(t)$ has the following property $F^b(t)F^{bT}(t) \leq I$. Thus, the equation (12) is modified as follows:

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{4} \mu_i(\xi(t)) \left(Ax_f(t) + (B_i + \Delta B)(u(t) + f(t)) + D\right) \\ y_f(t) = Cx_f(t) \end{cases} \quad (19)$$

where

$$\boldsymbol{f(t)} = [\boldsymbol{f_1(t)} \quad \boldsymbol{f_2(t)}]^{\boldsymbol{T}}$$
(20)

Let us see in the next section the proposed FTC approach.

3. Fault Tolerant Controller Design

Let us consider the following T-S model corresponding to the reference model with measurable premise variable $\xi(t)$:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(Ax(t) + B_i u(t)\right) \\ y = Cx(t) \end{cases} \tag{21}$$

where: r is the number of fuzzy rules, $\mu_i(\xi(t))$ are the weighting functions verifying the convex sum property $0 \leq \mu_i(\xi(t)) \leq 1$ and $\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$.
$x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^p$, and $u(t) \in \mathbb{R}^m$ represent respectively the state, the measured output and the bounded input vectors, $\{A_i, B_i, C\}$ are the submodels matrices. Recall that actually different ways to perform the T-S model (21) from non linear models existed. An interesting approach is the well-known nonlinear sector transformation [23]. In fact this technique allows obtaining an exact T-S representation without information loss on a compact set of the state space.

In the sequel, $\mathcal{H}(T)$ denotes the Hermetian of the matrix T, i.e. $\mathcal{H}(T) = T + T^T$. The single or double sums can be rewritten as:

$$\mathfrak{F}_\mu = \sum_{i=1}^{r} \mu_i(\xi(t))\mathfrak{F}_i \tag{22}$$

and

$$\mathfrak{F}_{\mu\mu} = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi(t))\mu_j(\xi(t))\mathfrak{F}_{ij} \tag{23}$$

The symbol $*$ indicates the transposed element in the symmetric positions of a matrix and $diag(S_1, \cdots, S_r)$ is a block diagonal matrix which diagonal entries are defined by $S_1, \cdots, S_r$. To obtain our result we need the following lemmas.

Lemma 1 [26]: Consider two real matrices Π and Y with appropriate dimensions, for any positive scalar σ the following inequality is verified:

$$\Pi^T Y + Y^T \Pi \leq \Pi^T \sigma \Pi + Y^T \sigma^{-1} Y \qquad \sigma > 0 \tag{24}$$

Lemma 2 [27]: The inequality $\mathfrak{F}_{\mu\mu} < 0$ holds if:

$$\begin{cases} \mathfrak{F}_{ii} < 0, i = 1, \dots, r \\ \frac{2}{r-1}\mathfrak{F}_{ii} + \mathfrak{F}_{ij} + \mathfrak{F}_{ji} < 0, i,j = 1, \dots, r, i \neq j \end{cases} \tag{25}$$

The faulty uncertain disturbed system is inferred as follows:

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left((A_i + \Delta A_i)x_f(t) + (B_i + \Delta B_i)\left(u_f(t) + f(t)\right) + E_i w(t)\right) \\ y_f(t) = Cx_f(t) + G_i w(t) \end{cases} \tag{26}$$

In here, $x_f(t) \in \mathbb{R}^n$,$y_f(t) \in \mathbb{R}^p$, $u_f(t) \in \mathbb{R}^m$ and $w(t) \in \mathbb{R}^{d \leq n}$ represent respectively the faulty state, faulty measured output vectors, the fault tolerant control signal, and the bounded input disturbance vectors. f depict fault directly affecting the input. ΔA_i and ΔB_i are the uncertainty matrices (with appropriate dimensions) corresponding to the i^{th} subsystem.

Assumption: the parameter uncertainties considered here are norm-bounded, in the form: $\Delta Z_i = M_i^z F_i^z N_i^z$, where $Z \in \{A, B\}, M_i^z$ and N_i^Z are known real constant matrices of appropriate dimension. F_i^z is a known Lebesgue measurable matrix satisfy:

$$\forall\, t \geq 0: F_i^{zT}(t)F_i^z(t) \leq I \tag{27}$$

In which I is the identity matrix of appropriate dimension. The aim is to design a fault tolerant controller ensuring the tracking trajectory performance of the faulty uncertain system to the reference one. The FTC law is given by the following structure:

$$u_f(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) K_i \left(x(t) - \hat{x}_f(t) \right) + u(t) - \hat{f}(t) \tag{28}$$

where: $K_i \in \mathbb{R}^{m \times n}$ are the state feedback gain matrices to be determined. In order to derive the FTC law an additional PI observer is added and has the usual form:

$$\begin{cases} \dot{\hat{x}}_f(t) = \sum_{i=1}^{r} \mu_i\left(\xi(t)\right)\left(A_i \hat{x}_f(t) + B_i\left(u_f(t) + \hat{f}(t)\right) + H_i^1\left(y_f(t) - \hat{y}_f(t)\right)\right) \\ \dot{\hat{f}}(t) = \sum_{i=1}^{r} \mu_i\left(\xi(t)\right)\left(H_i^2\left(y_f(t) - \hat{y}_f(t)\right)\right) \\ \hat{y}_f(t) = C\hat{x}_f(t) \end{cases} \tag{29}$$

where $H_i^1 \in \mathbb{R}^{n \times p}$ and $H_i^2 \in \mathbb{R}^{m \times p}$ are the observer's gain matrices to be determined to estimate $f(t)$ and $x_f(t)$. A first solution to this problem without uncertainties was proposed in [24]. For simplification we assume that:

$$\bar{A}_i = (A_i + \Delta A_i); \bar{B}_i = (B_i + \Delta B_i) \tag{30}$$

The FT controller design methodology is illustrated by the following scheme.

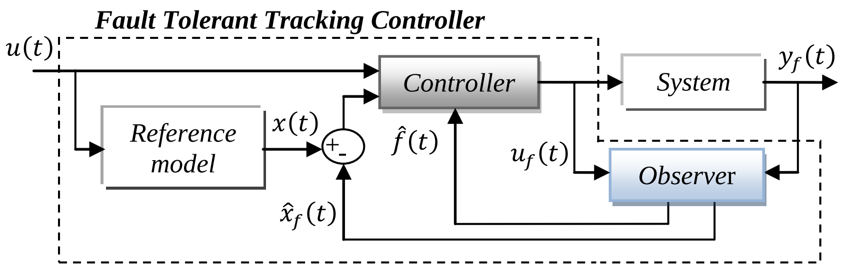

Fig.3. Tracking fault tolerant controller design methodology

With this controller structure, one can remark that fault detection and isolation are performed since an estimate of the fault affecting the system is available.

Let us respectively define the state and fault estimation errors defined by: $\begin{pmatrix} e_s(t) \\ e_f(t) \end{pmatrix} = \begin{pmatrix} x_f(t) - \hat{x}_f(t) \\ f(t) - \hat{f}(t) \end{pmatrix}$. Let us also define the state tracking error, and the output estimation error given by: $\begin{pmatrix} e_t(t) \\ e_y(t) \end{pmatrix} = \begin{pmatrix} x(t) - x_f(t) \\ y_f(t) - \hat{y}_f(t) \end{pmatrix}$. As a result , by adding and subtracting $K_\mu x_f(t)$ (28) can be rewritten as:

$$u_f(t) = K_\mu e_t(t) + K_\mu e_s(t) + u(t) - \hat{f}(t) \tag{31}$$

The dynamics of $e_t(t)$ and $e_s(t)$ are given by

$$\dot{e}_t(t) = \left(A_\mu - \bar{B}_\mu K_\mu\right) e_t(t) - \bar{B}_\mu K_\mu e_s(t) - \bar{B}_\mu e_f(t) - \Delta A_\mu x_f(t) - \Delta B_\mu u(t) - E_\mu w(t) \tag{32}$$

$$\dot{e}_s(t) = \left(A_\mu + \Delta B_\mu K_\mu\right) e_s(t) + \Delta B_\mu K_\mu e_t(t) + \bar{B}_\mu e_f(t) - H_\mu^1 e_y(t) + \Delta A_\mu x_f(t) + \Delta B_\mu u(t) + E_\mu w(t) \tag{33}$$

A "virtual dynamics" is introduced in the output error $e_y(t)$ to avoid the crossing terms resulting from the observer's gains H_i^1 and system matrices C multiplication [25]. This latter can be expressed as given by (34), where $0 \in R^{p \times p}$ is a zero matrix. Since the faults affecting the system in this approach are supposed to be constant $\left(i.e. \dot{f}(t) = 0\right)$, the dynamics of the fault estimation error is given by (35).

$$0\dot{e}_y(t) = Ce_s(t) - e_y(t) + G_\mu w(t) \tag{34}$$

$$\dot{e}_f(t) = -H_\mu^2 Ce_s(t) \tag{35}$$

The combination of (32), (33), (34), and (35) allows the formulation of the dynamics $e_y(t), e_t(t), e_s(t)$, and $e_f(t)$ with $x_f(t)$ in a descriptor form [28],[29]:

$$E\dot{\tilde{x}}(t) = \tilde{A}_{\mu\mu}\tilde{x}(t) + G_{\mu\mu}\tilde{\omega}(t) \tag{36}$$

with: $E = diag\ (I\ I\ I\ 0_m I), \tilde{\omega}^T(t) = \big(\ u(t)\ w(t)\big),\ \ \tilde{x}^T(t) = \Big(e_t(t)\ e_s(t)\ e_f\ (t)\ e_y(t)\ x_f(t)\Big)$,
and

$$\tilde{A}_{\mu\mu} = \begin{bmatrix} (A_\mu - \bar{B}_\mu K_\mu) & -\bar{B}_\mu K_\mu & -\bar{B}_\mu & 0 & -\Delta A_\mu \\ \Delta B_\mu K_\mu & (A_\mu + \Delta B_\mu K_\mu) & \bar{B}_\mu & -H_\mu^1 & \Delta A_\mu \\ 0 & -H_\mu^2\ C & 0 & 0 & 0 \\ 0 & C & 0 & -I & 0 \\ \bar{B}_\mu K_\mu & \bar{B}_\mu K_\mu & \bar{B}_\mu & 0 & \bar{A}_\mu \end{bmatrix} G_{\mu\mu} = \begin{bmatrix} -\Delta B_\mu & -E_\mu \\ \Delta B_\mu & E_\mu \\ 0 & 0 \\ 0 & G_\mu \\ \bar{B}_\mu & E_\mu \end{bmatrix} \tag{37}$$

The objective is now to compute the gains K_j and H_j from $\tilde{A}_{\mu\mu}$ described in (37) to ensure the stability of the closed loop model (36) guaranteeing the tracking performance for all $\tilde{\omega}(t)$. A basic result is summarized in the following theorem.

Theorem 1: The system (36) that generates tracking error $e_t(t)$, fault $e_f(t)$ and the state $e_s(t)$ estimation errors is stable and the $\mathcal{L}_2$-gain of transfer from $\tilde{\omega}(t)$ to $\tilde{x}(t)$ is bounded if there exists some matrices $X = X^T \geq 0$, $P_7 = P_7^T \geq 0,\ P_{19},\ P_{25} = P_{25}^T \geq 0, \bar{H}_i^1,\ \ \bar{H}_j^2,\ \ K_j$, jointly with positive scalars $\delta_{ij}^{1b}, \delta_{ij}^{2b}, \delta_{ij}^{3b}, \delta_{ij}^{6b}$, $\delta_{ij}^{7b}, \delta_{ij}^{10b}, \delta_{ij}^{11b}, \delta_{ij}^{13b}, \delta_{ij}^{2a}, \delta_{ij}^{3a}$ and $\bar{\gamma}$ solution to the following optimization problem:

$$\min_{X,P_7,P_{19},P_{25},,\bar{H}_i^1,\bar{H}_j^2,K_j} \bar{\gamma}\ \ s.t\ (25)$$

$$\mathfrak{F}_{ij} = \begin{bmatrix} \Sigma_{ij}^{(1,1)} & (*) \\ \Sigma_{ij}^{(2,1)} & \Sigma^{(2,2)} \end{bmatrix} < 0 \tag{38a}$$

with

$$\Sigma_{ij}^{(1,1)} = \begin{bmatrix} \Theta_{ij}^{(1,1)} & * & * & * & * & * & * \\ K_i^T B_j^T & \Theta_{ij}^{(2,2)} & * & * & * & * & * \\ -B_j^T & \Theta_{ij}^{(3,2)} & \Theta_{ij}^{(3,3)} & * & * & * & * \\ 0 & \Theta_j^{(4,2)} & 0 & \Theta_{ij}^{(4,4)} & * & * & * \\ 0 & 0 & P_{25}^T B_i & 0 & \Theta_{ij}^{(5,5)} & * & * \\ 0 & 0 & 0 & 0 & B_i^T P_{25} & \Theta_{ij}^{(6,6)} & * \\ -E_i^T & E_i^T P_7 & 0 & G_i^T P_{19} & E_i^T P_{25} & 0 & -\bar{\gamma} I \end{bmatrix}$$

$\Theta_{ij}^{(1,1)} = \mathcal{H}(A_i X) - \mathcal{H}(B_i K_j X) + M_i^a M_j^{aT} + \left(\delta_{ij}^{1b} + \left(\delta_{ij}^{4b}\right)^{-1} + I\right) M_i^b M_j^{bT}$
$\Theta_{ij}^{(1,1)} = \mathcal{H}(A_i X) - \mathcal{H}(B_i K_j X) + M_i^a M_j^{aT} + \left(\delta_{ij}^{1b} + \left(\delta_{ij}^{4b}\right)^{-1} + I\right) M_i^b M_j^{bT}$
$\Theta_{ij}^{(2,2)} = \mathcal{H}(P_7 A_i) + \delta_{ij}^{3b} M_i^b M_j^{bT} + I$
$\Theta_{ij}^{(3,2)} = -\bar{H}_j^2 C + B_i^T P_7$
$\Theta_j^{(4,2)} = P_{19}^T C - \bar{H}_j^1$
$\Theta_{ij}^{(3,3)} = \left(\delta_{ij}^{4b} + \delta_{ij}^{7b}\right) N_i^{bT} N_j^b + I$
$\Theta_{ij}^{(4,4)} = -\mathcal{H}(P_{19}) + I$

$$\Theta_{ij}^{(5,5)} = \mathcal{H}(P_{25}A_i) + \left(I+\delta_{ij}^{2a} + \delta_{ij}^{3a}\right)N_i^{aT}N_j^a + \delta_{ij}^{10b}N_i^{bT}N_j^b$$
$$\Theta_{ij}^{(6,6)} = -\bar{\gamma}I + \left(\delta_{ij}^{11b} + \delta\varepsilon_{ij}^{12b}+\varepsilon_{ij}^{13b}\right)N_i^{bT}N_j^b \tag{38b}$$

and

$$\Sigma^{(2,2)} = -diag\Big[I \quad I \quad \left(\delta_{ij}^{1b}+\delta_{ij}^{2b} + \delta_{ij}^{5b}\right) \quad \delta_{ij}^{3b} \quad \left(\delta_{ij}^{6b}+\delta_{ij}^{9b}\right) \quad \left(\delta_{ij}^{2b}\right)^{-1} \left(\delta_{ij}^{13b}+\delta_{ij}^{7b}+\left(\delta_{ij}^{6b}\right)^{-1}\right) \quad \delta_{ij}^{2a} \quad \delta_{ij}^{10b} \quad 2I \quad \delta_{ij}^{3a} \quad \left(\delta_{ij}^{11b}+\left(\delta_{ij}^{9b}\right)^{-1}+\left(\delta_{ij}^{5b}\right)^{-1}\right)\Big] \tag{38c}$$

$$\Sigma_{ij}^{(2,1)} = \begin{bmatrix} X & 0 & 0 & 0 & 0 \\ B_iK_jX & 0 & 0 & 0 & 0 \\ N_i^bK_jX & 0 & 0 & 0 & 0 \\ N_i^bK_j & 0 & 0 & 0 & 0 \\ 0 & N_i^bK_j & 0 & 0 & 0 \\ 0 & M_i^{bT}P_7 & 0 & 0 & 0 \\ 0 & M_i^{aT}P_7 & 0 & 0 & 0 \\ 0 & 0 & M_i^{bT}P_{25} & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{25} \\ 0 & 0 & 0 & 0 & M_i^{aT}P_{25} \\ 0 & 0 & 0 & 0 & M_i^{bT}P_{25} \end{bmatrix} \tag{38d}$$

The observer gains are computed from:

$$H_{i1} = P_7^{-1}\bar{H}_{i1}$$
$$H_{i2} = \bar{H}_{i2} \tag{39}$$

The attenuation level of transfer from $\tilde{\omega}(t)$ to $\tilde{x}(t)$ is obtained by $\gamma = \sqrt{\bar{\gamma}}$.

Proof*:* Let us choose the following candidate quadratic Lyapunov function as:

$$V\big(\tilde{x}(t)\big) = \tilde{x}^T(t)EP\tilde{x}(t) \tag{40}$$

with $$EP = P^TE \geq 0 \tag{41}$$

and $P = diag[P_1 \quad P_7 \quad P_{13} \quad P_{19} \quad P_{25}]$. According to (40), it follows that $P_1 = P_1^T \geq 0, P_7 = P_7^T \geq 0, P_{13} = P_{13}^T \geq 0,\ P_{25} = P_{25}^T \geq 0$ and P_{19} is a free slack matrix. The time derivative of the function $V\big(\tilde{x}(t)\big)$ is then:

$$\dot{V}\big(\tilde{x}(t)\big) = \dot{\tilde{x}}^T(t)EP\tilde{x}(t) + \tilde{x}^T(t)EP\dot{\tilde{x}}(t) < 0 \tag{42}$$

With (36) and (40), the inequality (42) becomes:

$$\tilde{x}^T(t)\big(\tilde{A}_{\mu\mu}^TP + P^T\tilde{A}_{\mu\mu}\big)\tilde{x}(t) + \tilde{x}^T(t)PG_{\mu\mu}\tilde{\omega}(t) + \tilde{\omega}^T(t){G_{\mu\mu}}^TP\tilde{x}(t) < 0 \tag{43}$$

We note here that the term $\tilde{\omega}(t)$ depends on $u(t), w(t)$ which are bounded, then it is also bounded. So, the objective is to minimize the $\mathcal{L}_2$-gain of the transfer from $\tilde{\omega}(t)$ to the state vector $\tilde{x}(t)$, this is formulated by:

$$\|\tilde{\omega}\|_2 \neq 0, \frac{\|\tilde{x}\|_2}{\|\tilde{\omega}\|_2} < \gamma \tag{45}$$

Then we are seeking to ensure asymptotic convergence toward zeros if $\tilde{\omega}(t) = 0$ and to guarantee a bounded $\mathcal{L}_2$-gain if $\tilde{\omega}(t) \neq 0$. This problem can be formulated as follows:

$$\dot{V}\big(\tilde{x}(t)\big) + \tilde{x}^T(t)Q\tilde{x}(t) - \gamma^2\tilde{\omega}^T(t)\tilde{\omega}(t) < 0 \tag{46}$$

where $Q = diag[I \ \ I \ \ I \ \ I \ 0\,]$. By substituting $\dot{V}\big(\tilde{x}(t)\big)$ we obtain:

$$\tilde{x}^T(t)\left(\tilde{A}_{\mu\mu}^T P + P^T \tilde{A}_{\mu\mu}\right)\tilde{x}(t) + \tilde{x}^T(t)P^T G_{\mu\mu}\tilde{\omega}(t) + \tilde{\omega}^T(t){G_{\mu\mu}}^T P\tilde{x}(t) + \tilde{x}^T(t)Q\tilde{x}(t) - \gamma^2\tilde{\omega}^T(t)\tilde{\omega}(t) < 0 \qquad (47)$$

Then the inequality (47) is negative if the following condition is fulfilled:

$$\begin{pmatrix} \mathcal{H}\left(P^T\tilde{A}_{\mu\mu}\right) + \mathrm{Q} & * \\ {G_{\mu\mu}}^T P & -\gamma^2 I \end{pmatrix} < 0 \qquad (47)$$

After developing inequality (47) using (37) some easy manipulations can be performed. Hence Multiplying inequality (47) left and right by $diag(X \quad I \quad I \quad I \quad I \quad I \quad I)$, with $X = P_1^{-1}$, and considering $P_{13} = P_{13}^T = I > 0$, with the bijective variable changes $\left(H_\mu^1\right)^T P_7 = \bar{H}_\mu^1$, $P_{13}H_\mu^2 = \bar{H}_\mu^2$. One obtains:

$$\psi_{\mu\mu} + \Delta\psi_{\mu\mu} < 0 \qquad (48)$$

with

$$\psi_{\mu\mu} = \left(\begin{array}{ccccc|c} \varphi_{\mu\mu}^{(1,1)} & * & * & * & * & \\ -K_\mu^T B_\mu^T & \varphi_\mu^{(2,2)} & * & * & * & \\ -B_\mu^T & \varphi_{\mu\mu}^{(3,2)} & I & * & * & \\ 0 & \varphi_\mu^{(4,2)} & P_{19}^T D_\mu & \varphi^{(4,4)} & * & (*) \\ \varphi_{\mu\mu}^{(5,1)} & \varphi_{\mu\mu}^{(5,2)} & P_{25}^T B_\mu & 0 & \varphi_\mu^{(5,5)} & \\ \hline 0 & 0 & 0 & 0 & B_\mu^T P_{25} & -\gamma^2 I \\ -E_\mu^T & E_\mu^T P_7 & 0 & G_\mu^T P_{19} & E_\mu^T P_{25} & \end{array}\right) \qquad (49)$$

where

$\varphi_{\mu\mu}^{(1,1)} = \mathcal{H}\left(A_\mu X\right) - \mathcal{H}\left(B_\mu K_\mu X\right) + XX$; $\varphi_\mu^{(2,2)} = \mathcal{H}\left(P_7 A_\mu\right) + I$; $\varphi_{\mu\mu}^{(3,2)} = -\bar{H}_\mu^2 C + B_\mu^T P_7$; $\varphi_{\mu\mu}^{(4,2)} = P_{19}^T C - \bar{H}_\mu^1$
$\varphi_{\mu\mu}^{(5,1)} = P_{25}^T B_\mu K_\mu X$; $\varphi_{\mu\mu}^{(5,2)} = P_{25}^T B_\mu K_\mu$; $\varphi^{(4,4)} = -\mathcal{H}(P_{19}) + I$; $\varphi_\mu^{(5,5)} = \mathcal{H}\left(P_{25}A_\mu\right)$

and

$$\Delta\psi_{\mu\mu} = \left(\begin{array}{ccccc|c} \bar{\varphi}_{\mu\mu}^{(1,1)} & * & * & * & * & \\ \bar{\varphi}_{\mu\mu}^{(2,1)} & \bar{\varphi}_{\mu\mu}^{(2,2)} & * & * & * & \\ \bar{\varphi}_{\mu\mu}^{(3,1)} & \bar{\varphi}_{\mu\mu}^{(3,2)} & 0 & * & * & (*) \\ 0 & 0 & 0 & 0 & * & \\ \bar{\varphi}_{\mu\mu}^{(5,1)} & \bar{\varphi}_{\mu\mu}^{(5,2)} & \bar{\varphi}_{\mu\mu}^{(5,3)} & 0 & \bar{\varphi}_\mu^{(5,5)} & \\ \hline -\Delta B_\mu^T P_1 & \Delta B_\mu^T P_7 & 0 & 0 & \Delta B_\mu^T P_{25} & (0) \\ 0 & 0 & 0 & 0 & 0 & \end{array}\right) \qquad (50)$$

with

$\bar{\varphi}_{\mu\mu}^{(1,1)} = -\mathcal{H}\left(P_1\Delta B_\mu K_\mu X\right)$; $\bar{\varphi}_{\mu\mu}^{(2,1)} = P_7\Delta B_\mu K_\mu X - K_\mu^T\Delta B_\mu^T$; $\bar{\varphi}_{\mu\mu}^{(3,1)} = -\Delta B_\mu^T$; $\bar{\varphi}_{\mu\mu}^{(5,1)} = -\Delta A_\mu^T + P_{25}^T\Delta B_\mu K_\mu X$
$\bar{\varphi}_{\mu\mu}^{(2,2)} = \mathcal{H}\left(P_7\Delta B_\mu K_\mu\right)$; $\bar{\varphi}_{\mu\mu}^{(3,2)} = \Delta B_\mu^T P_7$; $\bar{\varphi}_{\mu\mu}^{(5,2)} = \Delta A_\mu^T P_7 + P_{25}^T\Delta B_\mu K_\mu$; $\bar{\varphi}_{\mu\mu}^{(5,3)} = P_{25}^T\Delta B_\mu$; $\bar{\varphi}_\mu^{(5,5)} = \mathcal{H}\left(P_{25}\Delta A_\mu\right)$

Using the uncertainties structure defined in (27) and the well-known lemma 1, $\Delta\psi_{\mu\mu}$ can be bounded as follows:

$$\Delta\psi_{\mu\mu} \leq diag\left[\Pi 1_{\mu\mu} \quad \Pi 2_{\mu\mu} \quad \Pi 3_{\mu\mu} \quad 0 \quad \Pi 5_{\mu\mu} \quad \Pi 6_{\mu\mu}\right] \qquad (51)$$

where

$$\Pi 1_{\mu\mu} = \left(\left(\delta_{\mu\mu}^{1b}\right)^{-1} + \left(\delta_{\mu\mu}^{2b}\right)^{-1} + \left(\delta_{\mu\mu}^{5b}\right)^{-1}\right) X K_\mu^T N_\mu^{bT} N_\mu^b K_\mu X + \left(\delta_{\mu\mu}^{3b}\right)^{-1} K_\mu^T N_\mu^{bT} N_\mu^b K_\mu + \left(\delta_{\mu\mu}^{1a}\right)^{-1} M_\mu^a M_\mu^{aT} + \left(\delta_{\mu\mu}^{1b} + \left(\delta_{\mu\mu}^{4b}\right)^{-1} + \left(\delta_{\mu\mu}^{12b}\right)^{-1}\right) M_\mu^b M_\mu^{bT}$$

$$\Pi 2_{\mu\mu} = \left(\left(\delta_{\mu\mu}^{6b}\right)^{-1} + \left(\delta_{\mu\mu}^{9b}\right)^{-1}\right) K_\mu^T N_\mu^{bT} N_\mu^b K_\mu + \left(\delta_{\mu\mu}^{2a}\right)^{-1} P_7 M_\mu^a M_\mu^{aT} P_7 + \delta_{\mu\mu}^{3b} M_\mu^b M_\mu^{bT} + \left(\delta_{\mu\mu}^{2b} + \delta_{\mu\mu}^{6b} + \left(\delta_{\mu\mu}^{7b}\right)^{-1} + \left(\delta_{\mu\mu}^{13b}\right)^{-1}\right) P_7 M_\mu^b M_\mu^{bT} P_7$$

$$\Pi 3_{\mu\mu} = \left(\delta_{\mu\mu}^{4b} + \delta_{\mu\mu}^{7b}\right) N_\mu^{bT} N_\mu^b + \delta_{\mu\mu}^{4d} N_\mu^{dT} N_\mu^d + \left(\delta_{\mu\mu}^{10b}\right)^{-1} P_{25} M_\mu^b M_\mu^{bT} P_{25}$$

$$\Pi 5_{\mu\mu} = \left(\delta_{\mu\mu}^{3a}\right)^{-1} P_{25} M_\mu^a M_\mu^{aT} P_{25} + \left(\delta_{\mu\mu}^{5b} + \delta_{\mu\mu}^{9b} + \left(\delta_{\mu\mu}^{11b}\right)^{-1}\right) P_{25} M_\mu^b M_\mu^{bT} P_{25} + \left(\delta_{\mu\mu}^{1a} + \delta_{\mu\mu}^{2a} + \delta_{\mu\mu}^{3a}\right) N_\mu^{aT} N_\mu^a + \delta_{\mu\mu}^{10b} N_\mu^{bT} N_\mu^b$$

$$\Pi 6_{\mu\mu} = \left(\delta_{\mu\mu}^{11b} + \delta_{\mu\mu}^{12b} + \delta_{\mu\mu}^{13b}\right) N_\mu^{bT} N_\mu^b$$

Applying lemma 1 on the terms, $\varphi_{\mu\mu}^{(5,1)}, \varphi_{\mu\mu}^{(5,2)}$ by considering $\Omega_1 = \Omega_2 = I$, we obtain the inequality (52) with:

$$\Lambda_{\mu\mu}^{(1,1)} = \mathcal{H}\left(A_\mu X\right) - \mathcal{H}\left(B_\mu K_\mu X\right) + XX + \Omega_1^{-1} X K_\mu^T B_\mu^T B_\mu K_\mu X + \Pi 1_{\mu\mu}$$
$$\Lambda_{\mu\mu}^{(2,2)} = \varphi_{\mu\mu}^{(2,2)} + \Omega_2^{-1} K_\mu^T B_\mu^T B_\mu K_\mu + \Pi 2_{\mu\mu}$$
$$\Lambda_{\mu\mu}^{(3,3)} = \varphi_{\mu\mu}^{(3,3)} + \Pi 3_{\mu\mu} \; ; \; \Lambda_{\mu\mu}^{(4,4)} = \varphi_{\mu\mu}^{(4,4)}$$
$$\Lambda_{\mu\mu}^{(5,5)} = \varphi_{\mu\mu}^{(5,5)} + (\Omega_2 + \Omega_1) P_{25}^T P_{25} + \Pi 5_{\mu\mu}$$
$$\Lambda_{\mu\mu}^{(6,6)} = -\gamma^2 I + \Pi 6_{\mu\mu}$$

Finally applying Schur complement [30] on the BMI terms of (52) the sufficient LMI conditions proposed in the theorem 1 are obtained.

$$\begin{bmatrix} \Lambda_{\mu\mu}^{(1,1)} & * & * & * & * & * & * \\ -K_\mu^T B_\mu^T & \Lambda_{\mu\mu}^{(2,2)} & * & * & * & * & * \\ -B_\mu^T & \varphi_{\mu\mu}^{(3,2)} & \Lambda_{\mu\mu}^{(3,3)} & * & * & * & * \\ 0 & \varphi_\mu^{(4,2)} & P_{19}^T D_\mu & \Lambda_{\mu\mu}^{(4,4)} & * & * & * \\ 0 & 0 & P_{25}^T B_\mu & 0 & \Lambda_{\mu\mu}^{(5,5)} & * & * \\ 0 & 0 & 0 & 0 & B_\mu^T P_{25} & \Lambda_{\mu\mu}^{(6,6)} & * \\ -E_\mu^T & E_\mu^T P_7 & 0 & G_\mu^T P_{19} & E_\mu^T P_{25} & 0 & -\gamma^2 I \end{bmatrix} \quad (52)$$

Remark: At last recall that using weighting functions depending on the nominal control input $u(t)$ as a measurable premise variable for the faulty uncertain system seems to be critical especially in the case of actuator faults affecting the system. A further solution is to consider faulty premise variables as illustrated by [31], [34].

4. Simulation results

In this section, numerical simulations have been performed on the VTOL aircraft model (2) with numerical values given in Table 1. The T-S model constructed in Section 2 representing the aircraft model with premise variables depend on measurable input variable is used to build the observer. An unknown disturbance $w(t)$ with band-limited white noise as given by fig.4 is considered where E_i and G corresponding matrices are given by:

$$E_i = \begin{bmatrix} 0.1 \\ 0 \\ -0.1 \\ 0.2 \\ 0 \\ 0.2 \end{bmatrix}, i = 1, \cdots, 4 \text{ and } G = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \\ 0 \\ 0 \\ 0.1 \end{bmatrix} \quad (54)$$

We considered the case in which the aircraft is controlled so as to move from initial steady state hover $(x_1, y_1, \theta_1) = (0,0,0)$, to another steady state hover $(x_1, y_1, \theta_1) = (35,0,0)$ assuring a predefined trajectory tracking. For that reason, we used a normalized control law [32]:

$$u_1(t) = \frac{k_3(y_1(t) - y_{ref}) + k_4 y_2 + 1}{cos(\theta_1(t))} \tag{55}$$

$$u_2(t) = k_5\left(\theta_1(t) + tan^{-1}\left(\frac{k_1(x_1(t) - x_{ref}) + k_2 x_2(t)}{k_3(y_1(t) - y_{ref}) + k_4 y_2(t) + 1}\right)\right) \tag{56}$$

System failure is modeled by an actuator piecewise constant fault $f(t)$ normalized as given by fig.5. Notice that even if the assumption $\dot{f}(t) = 0$ is not satisfied, the PIO is able to reconstruct time varying signals with slow variation [33].

Table1
Numerical values of a VTQL aircraft

Parameters	Description	Numerical value
M	Mass of the aircraft	$5 \times 10^4 kg$
J	Moment of the aircraft inertia	$1.25 \times 10^4 kg$
l	The distance between the wingtips and the center of mass	$5m$
g	The gravitational acceleration	9.81m/s^2
α	The angle between the direction of application of the forces F and the vertical body axis	4°

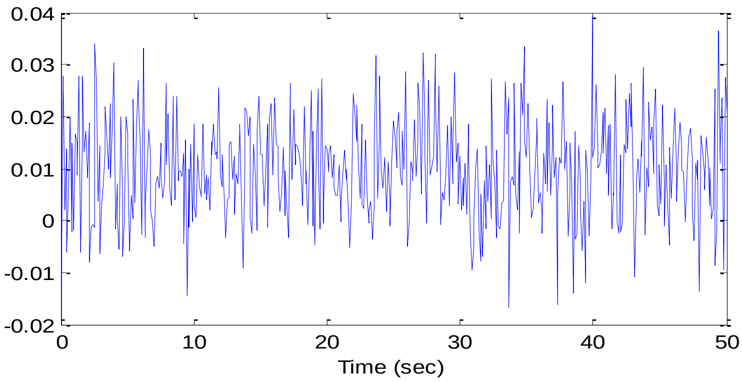

Fig.4. disturbance $w(t)$

Applying Theorem 1, the observer (29) and the fault tolerant controller (28) are designed by finding symmetric and positive definite matrices X, P_7, P_{25}, matrices $P_{19}, \overline{H}_i^1, \overline{H}_j^2, K_j$, jointly with positive scalars $\varepsilon_{ij}^{1b}, \varepsilon_{ij}^{2b}, \varepsilon_{ij}^{3b}, \varepsilon_{ij}^{6b}, \varepsilon_{ij}^{7b}, \varepsilon_{ij}^{10b}, \varepsilon_{ij}^{11b}, \varepsilon_{ij}^{13b}, \varepsilon_{ij}^{2a}, \varepsilon_{ij}^{3a}$, that are not given here-such that the convergence conditions given in Theorem 1 hold. The value of the attenuation rate from the input vector $\widetilde{\omega}(t)$ to the state vector $\tilde{x}(t)$ is $\bar{\gamma} = 1.74$.

The top of the fig.5 shows the time evolution of the faults with their estimate values, whereas the bottom part illustrates the nominal control inputs together with the FTC algorithm. Both the state estimation errors and the state tracking errors are given by Fig.6. Due to the convergence property of the observer, the reconstructed states fully represent the state of the process. Simulations of Fig.7 allow the comparison of the reference model states, to the faulty uncertain and estimated model states. These simulation results show the effectiveness of the synthesized observer and FTC controller, since the fault and the system states are estimated and the tracking between the faulty system states and the reference model ones is ensured.

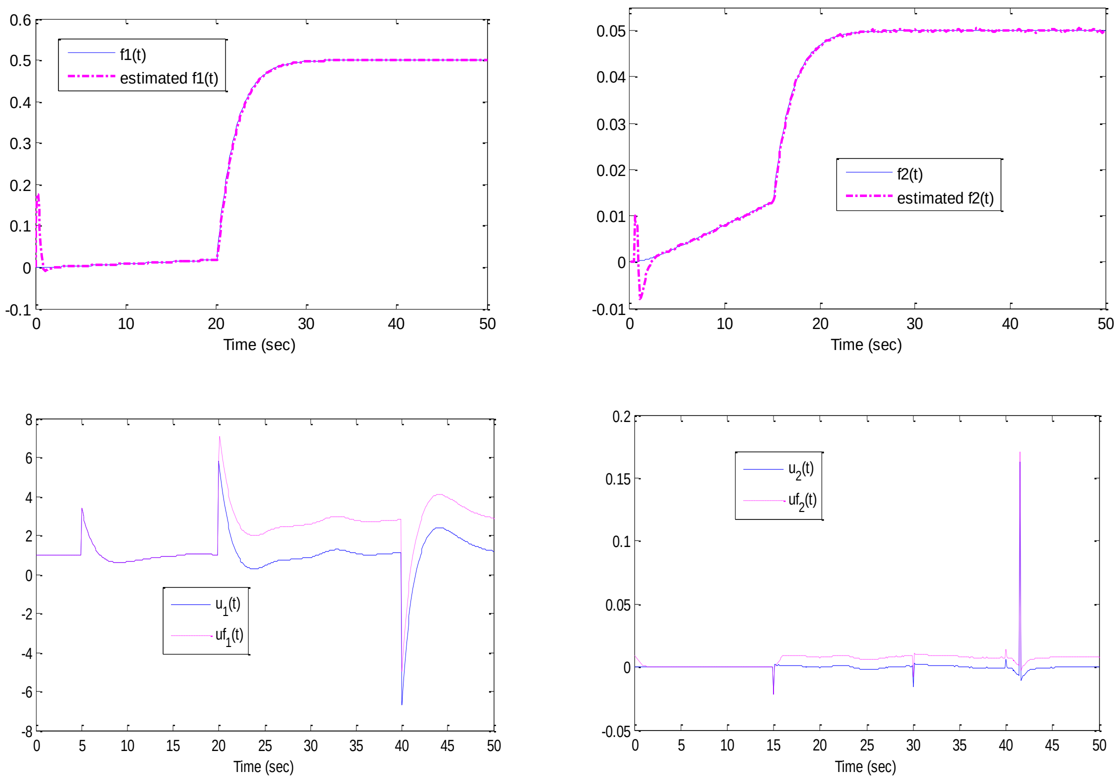

Fig. 5. Faults and their estimates (Top), Nominal control and FTC (bottom)

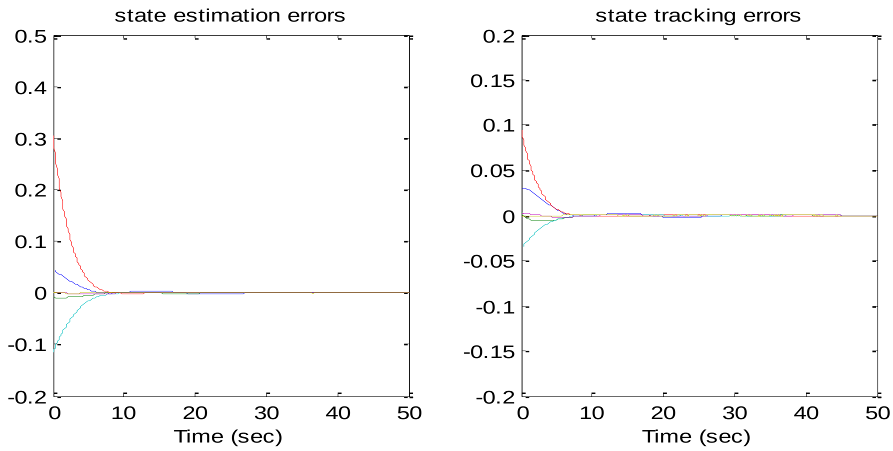

Fig.6. State estimation errors (left), State tracking errors (right)

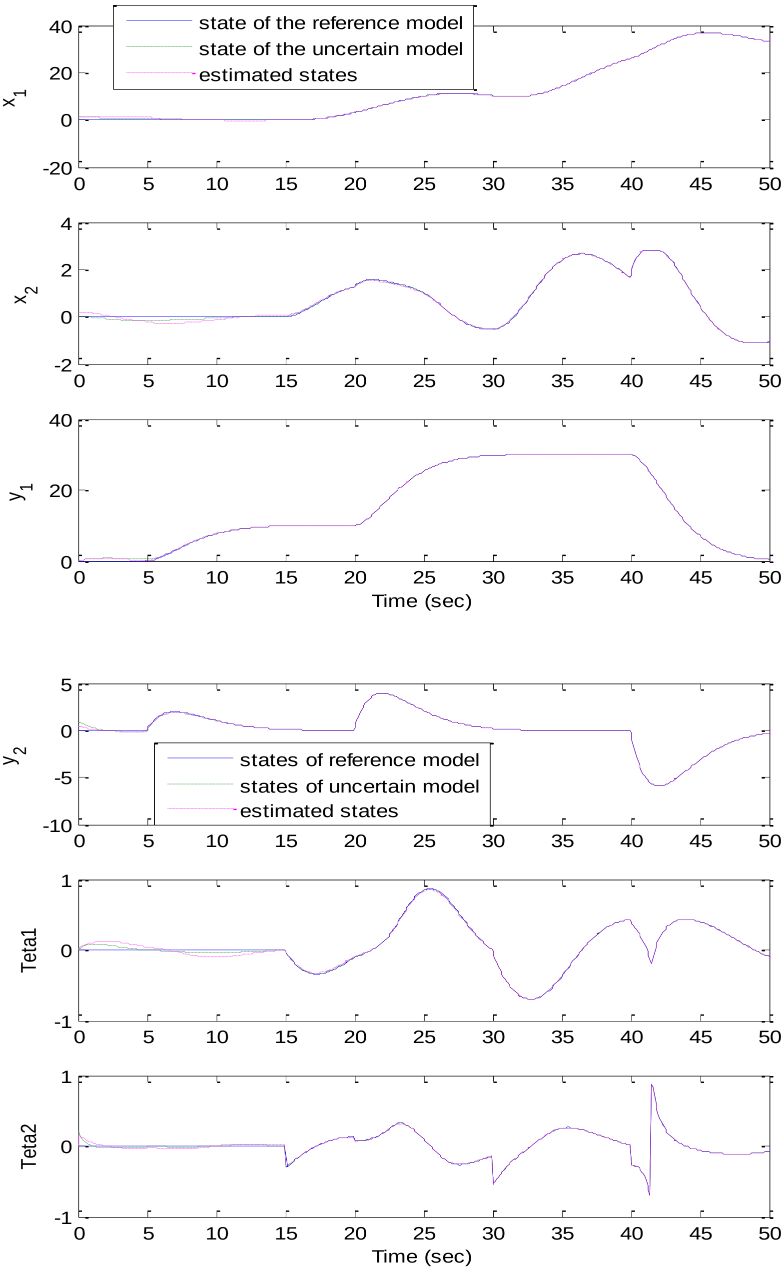

Fig.7. Comparison between reference model states, states of the uncertain faulty system with FTC and states of the estimated system

5. Conclusion

This paper has presented a fault tolerant tracking controller for a VTOL aircraft flight in uncertain conditions. The considered system contains structured uncertainties which affect the mechanical parameters of the air vehicle. The VTOL aircraft system is then presented as a faulty T-S uncertain disturbed model. An efficient control law is designed in order to ensure from one side the tracking between the faulty uncertain system and one healthy reference model, and the stability convergence of the closed loop system from the other side. Using Lyapunov theory and $\mathcal{L}_2$ optimization, the LMI formalism used virtual dynamics on the output error which allows decoupling the observer gains and system matrices. Results obtained under simulation show that the proposed approach is able to cope with the actuators faults occurrence during the motion control of the aircraft on the lateral vertical.

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