On Aggregation Process in Linguistic Decision Making Framework

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Abstract. When solving a problem, human beings must face situations in which they should choose among different alternatives by means of reasoning and mental processes. Many of these decision problems are under uncertain environments including vague, imprecise and subjective information that is usually modeled by fuzzy linguistic approach. This approach uses linguistic information or natural language words and its relation to mental reasoning processes of the experts when expressing their assessments. In a decision process multiple criteria can be evaluated which involving multiple experts with different degrees of knowledge. Such process can be modeled by using Multi-granular Linguistic Information (MGLI) and Computing with Words (CW) processes to solve the related decision problems. Once decision makers (experts) provided their opinions, it is necessary to combine all these opinions to obtain a single overall result that can be interpreted. An aggregation operator allows accomplishing this objective calculating a global value in different ways. In this paper we study the use of aggregation operators in multi-criteria decision-making processes comparing them and obtaining conclusions about their use in our framework. Furthermore, we propose a new aggregation operator taking into account the criteria importance to evaluate the alternatives, and then an illustrative example shows its outcomes.

Keywords: Multi-granular Linguistic Information, Computing with Words, Aggregation operator, Decision Making.

1 Introduction

The decision making is a day-to-day activity for human beings. The multiple facets of real world decision problems are well addressed by Multi-Criteria Decision Making (MCDM) [1]. The crucial point of interest within the MCDM is the analysis and the modeling of the multiple decision makers' preferences giving rise to Multi-Expert Decision Making (MEDM) [2].

In many situations, context involves vague and probably incomplete information. In these cases, information cannot be assessed precisely in a quantitative form; experts may feel more comfortable employing other approaches. To overcome this problem, information is normally modeled by using a fuzzy linguistic approach [3][4][5] allowing the experts to express their opinions with words rather than numbers (e.g. when evaluating the comfort or design of a car, terms like good, medium, bad can be used). Therefore, the fuzzy linguistic approach is a technique that represents qualitative information as linguistic values by means of linguistic variables [3], that is, variables whose values are no numbers but words or sentences in a natural language. Each linguistic value is characterized by its syntax (label) and semantic (meaning). The label is a word or a sentence belonging to a linguistic term set and the meaning is a fuzzy subset in a universe of discourse. The concept of linguistic variables provides an estimated measure since words are less precise than numbers. This is more effective because the experts may feel more comfortable using words they really know and understand in accordance with the context of use of these words. Also, when offering different expression domains or different linguistic term sets (multi-granular information) to the experts, this solution would be suitable to adjust the degree of experience of each one [6][7].

An important aspect of the MCDM is the aggregation process. In order to obtain a unique final result, the assessments of each expert involved must be taken in account. An aggregation operator allows accomplishing this objective calculating a global value. The aggregation is the operation that transforms a set of elements, such as individual opinions on a set of alternatives, into a single element that is representative of the whole. Different ways of carrying out the combination of preferences have led many authors to study and design different aggregation operators. Depending on the problem different types of aggregation operators can be used.

In this paper, we focus on the aggregation process when dealing with complex decisions under uncertainty using decision analysis process. We will study the results of applying different aggregation operators on the same decision problem in order to obtain relevant conclusions about their use in complex decision systems.

This paper is organized as follows. Section 2 reviews basic concepts about linguistic background that will be used to model uncertain information and multi-granular information in our framework. Section 3 presents the phases in order to analyze decisions, with special emphasis on aggregation process. Then, section 4 proposes an example of use on investment decisions in a company. Finally, section 5 shows some conclusions.

2 Preliminaries

Normally the decision analysis depends highly on subjective, vague and ill-structured information must have a model to manage this kind of information. Therefore, we consider the use of the fuzzy linguistic approach [3] to model and manage the inherent uncertainty in this kind of problems and the 2-tuple linguistic model to represent linguistic information [8]. Additionally, it is useful to manage multiple linguistic scales (multi-granular information) giving more flexibility to the different experts involved in the problem and, to manage this, we use Extended Linguistic Hierarchies (ELH) method. For this reason, in this section we review in short the concepts and methods such as the fuzzy 2-tuple linguistic model, ELH and its computational method (aggregation process).

2.1 The 2-tuples linguistic model

When using linguistic information to solve a problem it is necessary the use of computing with words CW. The main limitation with this approach is the "loss of information" suffered in the most used computational techniques that implies the lack of precision in the final results. These computational models are: The semantic model [9] and the symbolic model [10]. In these two models an approximation process must be developed to express the result in the initial expression domain, here is when the information gets lost.

The 2-tuples linguistic model [11] is a representation model that overcomes the loss of information. It represents the linguistic information with a pair of values, that we call 2-tuple, composed by a linguistic term and a number.

Definition 1. The Symbolic Translation of a linguistic term $s_i \in S = \{s_0,...,s_g\}$ is a numerical value assessed in $[-0.5, 0.5)$ that supports the "difference of information" between an amount of information $\beta \in [0, g]$ and the closest value in $\{0,...,g\}$ that indicates the index of the closest linguistic term in $S(s_i)$, being $[0, g]$ the interval of granularity of S.

From this concept a new linguistic representation model was developed, which represents the linguistic information by means of a linguistic 2-tuple. It consists of a pair of values namely, $(s_i, \alpha) \in \overline{S} \equiv S \times [-0.5, 0.5)$, being $s_i \in S$ a linguistic term and $\alpha \in [-0.5, 0.5)$ a numerical value representing the symbolic translation. This representation model defined a set of transformation functions between numeric values and linguistic 2-tuples to facilitate linguistic computational processes.

Definition 2. Let $S = \{s_0,...,s_g\}$ be a linguistic terms set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation. The 2-tuple set associated with S is defined as $\overline{S} = S \times [-0.5, 0.5)$. A 2-tuple that expresses the equivalent information to β is then obtained as follow:

$$\Delta : [0, g] \to \overline{S}$$

$$\Delta(\beta) = (s_i, \alpha), with \begin{cases} s_i, & i = round(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0,5) \end{cases} \tag{1}$$

being round $(\cdot)$ the usual round operation, i the index of the closest label, s_i, to "β", and "α" the value of the symbolic translation.

It is noteworthy to point out that Δ is a one to one mapping and $\Delta^{-1} : \overline{S} \to [0, g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$. In this way the 2-tuple of $\overline{S}$ is identified by a numerical value in the interval $[0, g]$.

Remark 1. The transformation of a linguistic term into a linguistic 2-tuples consists of adding value 0 as symbolic translation: $s_i \in S \Rightarrow (s_i, 0) \in \bar{S}$. On other hand, $\Delta(i) = (s_i, 0)$ and $\Delta^{-1}(s_i, 0) = i, \forall i \in \{0,1,..,g\}$.

If $\beta = 3.25$ is the value representing the result of a symbolic aggregation operation on the set of labels,
$S = \{s_0 = Nothing, s_1 = VeryLow, s_2 = Low, s_3 = Mediums, s_4 = High, s_5 = VeryHigh, s_6 = Perfect\}$
, then the 2-tuple that expresses the equivalent information to β is $(medium, .25)$.

This model has a linguistic computational technique based on the functions Δ and Δ^{-1}, for a further detailed see Ref. [12].

2.2 Extended Linguistic Hierarchies:

A flexible expression domain with several linguistic scales is necessary to express the assessments for experts according to their degree of knowledge about the problem. Different approaches dealing with multi-granular linguistic information have been proposed. In this paper shall use the ELH [13] approach to model and manage multi-granular linguistic information because of its features of flexibility and accuracy in the processes of computing with words (CW) in multi-granular linguistic contexts. An ELH is a set of levels, where each level represents a linguistic term set with different granularity from the remaining levels of the ELH. Each level belongs to an ELH is denoted as $l(t, n(t))$ being t a number that indicates the level of the ELH and $n(t)$ the granularity of the terms set of the level t . To build an ELH have been proposed a set of extended hierarchical rules:

Rule 1: A finite set of levels, $l(t, n(t))$ with $t = 1,..,m$, that defines the multi-granular linguistic context required by experts to express their assessments are included.

Rule 2: to obtain an ELH a new level, $l(t^*, n(t^*))$ with $t^* = m+1$, should be added. This new level must have the following granularity:

$$n(t^*) = (L.C.M.(n(1)-1,...,n(m)-1))+1 \qquad (2)$$

being L.C.M. the Least Common Multiple.

ELH building process then consists of two processes: i) It adds m linguistic scales used by the experts to express their information. And ii) then it adds the term set $l(t^*, n(t^*))$, with $t = m+1$, according to Eq. (2). Therefore, the ELH is the union of all levels required by the experts plus the new level $l(t^*, n(t^*))$.

$$ELH = \bigcup_{t=1}^{t=m+1} (l(t, n(t)))$$

The use of multi-granular linguistic information makes the processes of CW more complex. ELH computational model needs to make a three-step process.

1. Unification phase. The multi-granular linguistic information is conducted into only one linguistic term set, that in ELH is always $S^{n(t^*)}$, by means of a transformation function $TF_b^a(\cdot)$:

Definition 3. Let $S^{n(a)} = \{s_0^{n(a)},...,s_{n(a)-1}^{n(a)}\}$ and $S^{n(b)} = \{s_0^{n(b)},...,s_{n(b)-1}^{n(b)}\}$ be two linguistic term sets, with $a \neq b$. The linguistic transformation function is defined as:

$$TF_b^a : \bar{S}^{n(a)} \rightarrow \bar{S}^{n(b)}$$

$$TF_b^a(s_j^{n(a)},\alpha_j^{n(a)}) = \Delta_S\left(\frac{\Delta^{-1}(s_j^{n(a)},\alpha^{n(a)})\cdot(n(b)-1)}{n(a)-1}\right) = (s_k^{n(b)},\alpha_k^{n(b)}) \tag{2}$$

2. Computational process. Once the information is expressed in only one expression domain $S^{n(t^*)}$, the computations are carried out by using the linguistic 2-tuple model.

3. Expressing results. In this step the results might be transformed into any level, t , of ELH in a precise way by using Eq. (3) to improve the understanding of the results if necessary.

Remark 2. In the processes of CW with information assessed in an ELH, the linguistic transformation function, TF_b^a , performed in the unification phase, a , might be any level in the set $\{t = 1,..,m\}$ and the computational processes are carried out in the level b that it is always the level t^* (See Eq. (3)).

It was proved in [13] that the transformation functions between linguistic terms in different levels of the Extended Linguistic Hierarchy are carried out without loss information.

2.3 Aggregation process:

Aggregation operators allow accomplishing a global value from a set of values in order to obtain a unique final value. Here we have analyzed four kinds of aggregation operators, Geometric Mean Aggregation Operator (GMAO), Arithmetic Mean Aggregation Operator (AMAO) Weighted Aggregation Operator (WAO). WAO is based on the weight of the experts (WAO) or criteria (WAOC).

Definition 6. GMAO. Let $((l_1,\alpha_1),..,(l_m,\alpha_m)) \in \bar{S}^m$ be a 2-tuples linguistic vector, geometric mean operator is defined as follows: $G : \bar{S}^m \rightarrow \bar{S}$

$$G:\left[((l_1,\alpha_1),..,(l_m,\alpha_m))\right] = \left[\prod_{i=1}^{m}\Delta^{-1}\left[(l_i,\alpha_i)\right]\right]^{\frac{1}{m}} = \left[\prod_{i=1}^{m}\beta_i\right]^{\frac{1}{m}} \tag{3}$$

Definition 5. AMAO: Let $((l_1,\alpha_1),..,(l_n,\alpha_n)) \in \bar{S}^n$ be a 2-tuples linguistic vector, arithmetic mean operator is defined as follows: $\bar{G} : \bar{S}^n \rightarrow \bar{S}$

$$\bar{G}[(l_1,\alpha_1),..,(l_n,\alpha_n)] = \Delta(\sum_{j=1}^{n}\frac{1}{n}\Delta^{-1}(r_j,\alpha_j)) = \Delta(\frac{1}{n}\sum_{j=1}^{n}\beta_j) \tag{4}$$

A rational assumption about the resolution process could be associating more importance to the experts who have more "knowledge" or "experience". These values can be interpreted as *importance degree*, *competence*, *knowledge* or *ability* of the experts. In addition some experts could have some difficulties in giving all their assessments due to lack of knowledge about part of the problem. Besides the use of different scales, the expert should be carried out in different way with weighted aggregation operator.

Definition 6. WAO: Let $((l_1,\alpha_1),..,(l_m,\alpha_m)\in\bar{S}^m$ be a vector of linguistic 2-tuples, and $w=(w_1,...,w_m)\in[0,1]^m$ be a weighting vector such that $\sum_{i=1}^{m} w_i=1$. The 2-tuple WAO associated with w is the function $G^w:\langle\bar{S}\rangle^m\rightarrow\langle\bar{S}\rangle$ defined by

$$G^w[(l_1,\alpha_1),..,(l_m,\alpha_m)]=\Delta_{\bar{s}}\left(\sum_{i=1}^{m} w_i\beta_i\right) \tag{5}$$

In the same way that experts have importance, criteria also may have it. In this sense we use the process of obtaining the importance of criteria based on the potencies method. This method takes in account the importance for each criterion in the problem solution using a vector of importance with defined values for every criterion involved. When working with linguistic information we just don't have a method for comparing criteria in order to obtain this vector of importance. According to this, it is necessary to obtain the comparison matrix between criteria and then calculate the weighted vector based on criteria importance. The matrix $[A]_{nxn}$ that represents the matrix comparison between criteria is obtained from the experts judgments about criteria. Then the weighted vector ω that represents the weight held by each criterion in the decision process and is obtained using $[A]_{nxn}$ as explained in [14].

Definition 7. WAOC: Let $((l_1,\alpha_1),..,(l_n,\alpha_n)\in\bar{S}^n$ be a vector of linguistic 2-tuples, and $\omega=(\omega_1,...,\omega_n)\in[0,1]^n$ be a weighting vector based on the criteria importance such that $\sum_{j=1}^{n}\omega_j=1$. The 2-tuple aggregation operator associated with ω is the function $G^\omega:\langle\bar{S}\rangle^n\rightarrow\langle\bar{S}\rangle$ defined by:

$$G^\omega[(l_1,\alpha_1),..,(l_n,\alpha_n)]=\Delta_{\bar{s}}\left(\sum_{j=1}^{n} w_j\beta_j\right) \tag{6}$$

3 Decision analysis process

Linguistic decision analysis process consists of several phases described below:

Phase 1. Data definition: It defines the evaluation context in which experts will express their preferences. Linguistic descriptors and their semantics are chosen as well as each problem potential solution (alternative) is identified. It also determines the criteria to evaluate every alternative and the experts who are involved in decision process. In order to allow different expression domain for multiple experts, linguistic terms sets used are organized into an ELH. Therefore, let consider:

A finite set of alternatives $X = \{x_k, k = 1,...,q\}$.

A finite set of criteria $C = \{c_j, j = 1,...,n\}$.

A finite set of experts $E = \{e_i, i = 1,...,m\}$ that express their assessments by using different linguistic scales of information in ELH.

Phase 2. Information gathering: Experts provide their linguistic assessments in utility vectors for each criterion of the evaluated alternatives. The experts express their assessments on every criterion considering every alternative using their linguistic term set in ELH. Due to the fact that our Framework will use linguistic 2-tuple computing model the linguistic preferences provided by the experts will be transformed into linguistic 2-tuples according to the Remark 1.

Phase 3. Computational process: This phase consists of three steps to obtain a global value for each alternative:

-Unification of MGLI. Due to experts provide their assessments in different linguistic scales; it is necessary to transform each assessment in a unique expression domain so called t^* whose granularity is given by Eq. (2). Thus, transformation must be the last level of the ELH according to Eq. (3). Once the information has been unified, it will be expressed by means of linguistic 2-tuples in $S^{n(t^*)}$.

In order to obtain the global value for each alternative the information must be aggregated. In our framework we use four different aggregation operators and the process is performed in two levels:

- Expert Aggregation Level: The first aggregation step it obtains a collective value for all experts' assessments. Here is possible to choose between GMAO and WAO.

- Criteria Aggregation Level: The second one computes a global value for each alternative from results obtained in previous step. Here is possible to choose between AMAO and WAOC. Figure 1 shows the possible combinations of operators.

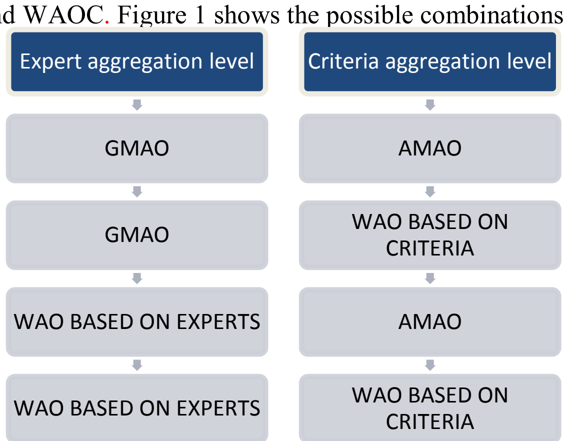

Fig. 1. Framework aggregation operators

Phase 4. Results presentation: Final values are presented in an ordered scale as a ranking of preferences from the most suitable to the less convenient alternative.

4 Illustrative example

We consider the decision process to acquire software in one organization. The decision from where get software implies decide the supply channel option. There are advantages and drawbacks for particular acquire channels and experts many times do not reach to an agreement. In order to satisfy this need, the CEO has arranged a meeting with the three main experts in software solution in the organization: CIO, Head of development department and Head of data management department. The objective of this meeting is to determine which one of the three channels available for software supplying is the most suitable for the company. There are three main channels to obtain software: internal development, external development and buy a standard packet.

In the Internal development, the organization IT department builds the needed software solution.

The External development means acquire by external software development consulting.

Buy a standard package. One of the fastest way for satisfying software needs is by acquiring a standard software package of general purpose. To obtain software 4 criteria should be evaluated, how well it meets the necessary requirements, ease of changes and growths and development time.

Therefore, in phase 1 we have the following:

$$\text{A set of experts } = \left\{ \begin{array}{l} E_1 = \text{CIO}, E_2 = \text{Head of development department}, \\ E_3 = \text{Head of data management department} \end{array} \right\}$$

$$\text{A set of alternatives} = \left\{ \begin{array}{l} A_1 = \text{Internal development}, A_2 = \text{External development}, \\ A_3 = \text{Buy a standard package} \end{array} \right\}$$

$$\text{A set of criteria} = \left\{ \begin{array}{l} C_1 = \text{Satisfied requirements}, C_2 = \text{Facility implementing changes}, \\ C_3 = \text{Development time} \end{array} \right\}$$

An ELH with two linguistic term sets:

$$S_1 = \{VB = \text{Very Bad}, B = \text{Bad}, M = \text{Medium}, G = \text{Good}, VG = \text{Very Good}\}$$

$$S_2 = \left\{ \begin{array}{l} W = \text{Worst}, VB = \text{Very Bad}, B = \text{Bad}, M = \text{Medium}, G = \text{Good}, \\ VG = \text{Very Good}, E = \text{Excellent} \end{array} \right\}$$

Besides a new level, t^*, in accordance with Eq. (2).

Table 1. Phase 2. Information gathering

Experts	Assessments								
	A_1			A_2			A_3		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
E_1	(E,0)	(VG,0)	(B,0)	(VG,0)	(M,0)	(VG,0)	(M,0)	(W,0)	(E,0)
E_2	(VG,0)	(VG,0)	(VB,0)	(E,0)	(G,0)	(M,0)	(M,0)	(B,0)	(VG,0)
E_3	(VG,0)	(G,0)	(VB,0)	(G,0)	(M,0)	(M,0)	(M,0)	(VB,0)	(VG,0)

Table 2. Criteria comparison

	E_1			E_2			E_3		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
C_1	1	5	1 / 3	1	7	5	1	7	4
C_2	1 / 5	1	1 / 9	1 / 7	1	1 / 2	1 / 7	1	1 / 2
C_3	3	9	1	1 / 5	2	1	1 / 4	2	1

Table 3. Criteria weight vector

ω vector	
Criterion	Weight
C_1	0.2654
C_2	0.0629
C_3	0.6716

Table 4. Experts weight vector

w vector	
Expert	Weight
E_1	0.5
E_2	0.3
E_3	0.2

Table 5. GMAO and AMAO results

Alternative	Percentage
A_2	38,54%
A_1	33,67%
A_3	27,79%

Table 6. WAO with experts weighting and AMAO results

Alternative	Percentage
A_2	37,12%
A_1	33,73%
A_3	29,15%

Table 7. GMAO and WAO based on criteria importance

Alternative	Percentage
A_3	44,34%
A_2	38,25%
A_1	17,42%

Table 8. WAO with experts weighting and WAO based on criteria importance

Alternative	Percentage
A_3	42,62%
A_2	35,98%
A_1	21,41%

Bearing in mind the first step of aggregation, our framework allows use GMAO and WAO in accordance with the weighting vector showed in Table 4. Then, the second aggregation steps we use AMAO and WAO based on criteria importance (see Table 3).

From Table 5 to Table 8 results are expressed in percentage way to better understanding. When it compute GMAO and AMAO (see Table 5) the results are similar to the Table 6 that uses WAO with experts weighting and AMAO (see Table 6). However, a slight difference it can be seen between both but the order of importance is the same. Weighted operator (WAO) introduces a new parameter, the weight of importance of experts, allowing greater differentiation between the final results to eliminate equal importance between opinions. Thus, decision makers have more accurate values with better differentiation between them.

On the contrary, in Tables 7 and 8, the operator for the second level of aggregation used was the weighted vector based on criteria importance. Here, the priority ranking changes significantly. It is because importance vector modifies last criteria aggregation step, allocating highest values for the most important criterion and reducing values for the others. Furthermore, obtained values in Table 8 take into account the weight of the experts.

5 Conclusion

Aggregation refers to the process of combining several values into a single one, so that the final result of aggregation takes into account in a given manner all the individual values. Such an operation is used in many well-known disciplines such as Multi-Criteria Multi-Expert Decision Making. In order to reach good results for decision process, classical synthesizing functions have been proposed: arithmetic mean, geometric mean, median and many others. In this papers we present a linguistic framework developed that allows analyze different decision results by using several aggregation operators. In this regard we also propose compute criteria importance based on the potencies method with Saaty scale.

Currently, the framework computation capability is expanded by using different aggregation operators such as Ordered Weighted Averaging (OWA) aggregation operators' family. In addition, we are comparing different methodologies and decision making approaches such as Analytic Hierarchy Process (AHP).

References

1. Figueira, J., Greco, S., Ehrgott, M.: Multiple Criteria Decision Analysis: State of the Art Surveys. Kluwer Academic Publishers, Boston/Dordrecht/London (2005)
2. Clemen, R.: Making Hard Decisions. An Introduction to Decision Analisys. Duxbury Press (1995)
3. Zadeh, L.: The concept of a linguistic variable and its applications to approximate reasoning. Information Sciences, Part I, II, III (8,9) (1975) 199–249,301–357,43–80
4. Dong, Y., Xu, Y., Yu, S.: Linguistic multiperson decision making based on the use of multiple preference relations. Fuzzy Sets and Systems 160(5) (2009) 603–623
5. Delgado, M., Verdegay, J., Vila, M.: Linguistic decision-making models. International Journal of Intelligent Systems 7(5) (1992) 479–492
6. Herrera, F., Herrera-Viedma, E., Martínez, L.: A fusion approach for managing multigranularity linguistic term sets in decision making. Fuzzy Sets and Systems 114(1) (2000) 43–58
7. Herrera, F., Herrera-Viedma, E., Verdegay, J.: A linguistic decision process in group decision making. Group Decision and Negotiation 5 (1996) 165–176
8. Martínez, L., Herrera, F.: An overview on the 2-tuple linguistic model for computing with words in decision making: Extensions, applications and challenges. Information Sciences 207 (2012) 1–18

9. Degani, R., Bortolan, G.: The problem of linguistic approximation in clinical decision making. International Journal of Approximate Reasoning 2 (1988) 143–162
10. Delgado, M., Verdegay, J., Vila, M.: On aggregation operations of linguistic labels. International Journal of Intelligent Systems 8(3) (1993) 351–370
11. Herrera, F., Martínez, L.: A 2-tuple fuzzy linguistic representation model for computing with words. IEEE Transactions on Fuzzy Systems 8(6) (2000) 746–752
12. Herrera, F., Martínez, L.: The 2-tuple linguistic computational model. Advantages of its linguistic description, accuracy and consistency. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 9 (2001) 33–48
13. Espinilla, M., Liu, J., Martínez, L.: An extended hierarchical linguistic model for decision-making problems. Computational Intelligence 27(3) (2011) 489–512
14. Saaty, R. W.: The analytic hierarchy process: What it is and how it is used. Math Modelling, Vol. 9, No. 3-5, pp. 161-176 (1987).